HANDBOOK
OF
BUILDING CONSTRUCTION

VOLUME I

Courtesy of Holabird & Roche, Architects *Frontispiece*

UNIVERSITY CLUB OF CHICAGO

HANDBOOK

OF

BUILDING CONSTRUCTION

DATA FOR
ARCHITECTS, DESIGNING AND CONSTRUCTING
ENGINEERS, AND CONTRACTORS

VOLUME I

COMPILED BY A STAFF
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PREFACE

These volumes have been prepared to provide the architect, engineer, and builder with a reference work covering thoroughly the design and construction of the principal kinds and types of modern buildings with their mechanical and electrical equipment. Since the art of building is now highly specialized, an unusually large number of associate editors were engaged in order to cover the field in a reliable and comprehensive manner.

The Editors-in-Chief desire here to express their appreciation of the spirit of coöperation shown by the Associate Editors and the Publishers. They desire also to express their indebtedness to Mr. Clifford E. Ives for his excellent work in preparing the drawings from which all the zinc etchings were made.

G. A. H.
N. C. J.

September, 1920.

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USED THROUGHOUT THIS VOLUME
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HANDBOOK
OF
BUILDING CONSTRUCTION

PART I—DESIGN AND CONSTRUCTION

SECTION 1

ELEMENTS OF STRUCTURAL THEORY

DEFINITIONS

1. Structure.—A *structure* is a part, or an assemblage of parts, constructed to support certain definite loads. Structures are acted upon by external forces and these external forces are held in equilibrium by internal forces, called *stresses.*

2. Member.—A *member* or piece of a structure is a single unit of the structure, as a beam, a column, or a web member of a truss.

3. Beam.—A *beam* is a structural member which is ordinarily subject to bending and is usually a horizontal member carrying vertical loads. In a framed floor, beams are members upon which rest directly the floor plank, slab, or arch.

A *simple beam* is one which rests on supports at the ends. A *cantilever beam* is a beam having one end rigidly fixed and the other end free. Extending a simple beam beyond either support gives a combination of a simple beam and a cantilever beam. A beam with both ends free and balanced over a support is also called a cantilever beam. A *restrained beam* is one which is more or less fixed at one or both points of support. A *built-in* or *fixed beam* is a beam rigidly fixed at both ends. A *continuous beam* is one having more than two points of support.

4. Girder.—A *girder* is a beam which receives its load in concentrations. In a framed floor it supports one or more cross beams which in turn carry the flooring. The term "girder" is also applied to any large heavy beam, especially a built-up steel beam or plate girder. In Bethlehem steel sections the terms "beam" and "girder" are used to denote rolled sections of different proportions (see Sect. 2, Art. 2*b*).

5. Column.—A *column, strut* or *post* is a structural member which is compressed endwise. A strut is usually considered of smaller dimensions than either a column or post.

6. Tie.—A *tie* is a structural member which tends to lengthen under stress.

7. Truss.—A *truss* is a framed or jointed structure. It is composed of straight members which are connected only at their intersections, so that if the loads are applied at these intersections the stress in each member is in the direction of its length. Each member of a truss is either a *tie* or a *strut.*

The *span* of a roof truss is the horizontal distance in feet between the centers of supports. The *rise* is the distance from the highest point of the truss to the line joining the points of support. The *pitch* is the ratio of the rise of the truss to its span. The *upper* or *top chord* consists of the upper line of members. The *lower chord* consists of the lower line of members. The *web members* connect the joints of the upper chord with those of the lower chord.

8. Force.—*Force* is that which tends to change the state of motion of a body, or it is that which causes a body to change its shape if it is held in place by other forces.

9. Outer Forces.—The *external* or *outer forces* acting upon a structure consist of the applied loads and the supporting forces, called *reactions.*

10. Inner Forces.—The *internal* or *inner forces* in a structure are the stresses in the different members which are brought into action by the outer forces and hold the outer forces in equilibrium.

11. Dead Load.—*Dead load* is the weight of a structure itself plus any permanent loads. In design, the weight of the structure must be assumed; and the design corrected later if the assumed weight is very much in error. Brick and concrete construction have the largest dead load relative to the total load.

12. Live Load.—*Live load* is any moving or variable load which may come upon the structure—as, for example, the weight of people or merchandise on a floor, or the weight of snow and the pressure of wind on a roof. The *total load* or dead load plus live load must be used in design. In addition the dynamic effect or impact of the live load must often be considered.

13. Statically Determinate Structures.—A structure is *statically determinate* when both outer and inner forces may be determined by the aid of statics. If all the outer forces may be found by statics, the structure is said to be *statically determinate with respect to the outer forces* whether or not it is possible to determine the inner forces by the same means (see definition of "Statics," Art. 30).

Wooden beams, pin-connected trusses, and steel beams resting on horizontal supports are ordinarily statically determinate. Small riveted trusses and steel beams in a framed floor are commonly assumed in design as statically determinate.

14. Statically Indeterminate Structures.—Structures which cannot be statically determined are those which the equations of statics will not suffice to design. All rigidly connected building frames are statically indeterminate.

STRESS AND DEFORMATION

By Walter W. Clifford

15. Stress.—*Stress* is the cohesive force in a body which resists the tendency of an external force to change the shape of the body. For example, if a steel rod supports a load or force of 30,000 lb., it has in it a stress of 30,000 lb. This is called the *total stress.*

If a force tends to stretch a member, the resulting stress is called *tension* or *tensile stress.* If a force tends to shorten a member, the resulting stress is called *compression* or *compressive stress.*

If the above-mentioned rod has a cross-sectional area perpendicular to its axis of 2 sq. in., and the load is uniformly distributed, it has a *unit stress* or *intensity of stress* of 15,000 lb. per sq. in.—that is, the unit stress is the total uniformly distributed stress divided by the cross-sectional area, or $f = \frac{P}{A}$.

If the load on a member is increased until the member fails, the highest unit stress sustained is called the *ultimate stress.* Some materials, notably steel, after being stressed to the ultimate, sustain a gradually lessening load until failure. The unit load at failure is called the *rupture stress* (see Fig. 1).

16. Deformation.—Whenever any material is subjected to the action of a force, it changes shape. This change in shape is called *deformation* or *strain.* The former term will be used in this book. The deformation per unit of length is called the *unit deformation.*

All structural materials, within the limits of working stresses, follow very closely *Hooke's Law* which is that deformation is proportional to stress. Thus, if a force of 1000 lb. stretches a rod 1 in., a force of 2000 lb. will stretch the same rod 2 in.

17. Modulus of Elasticity.—The ratio between stress and deformation is commonly called the *modulus of elasticity,* which term will be used in this book. *Coefficient of elasticity* and *Young's modulus* are synonymous with modulus of elasticity. The value of the modulus of elasticity varies with different materials, but in any case $E = \frac{f}{\delta}$, where f is the unit stress and δ is the deformation per unit of length. The same linear unit must be used in computing the unit stress as for measuring the deformation. This unit is commonly the inch, except where the metric system is used. It may be noted from the curves (Figs. 1–4) that the modulus of elasticity is the tangent of the angle which the stress-deformation curve makes with the horizontal axis.

18. Elastic Limit and Yield Point.—The *elastic limit* is the stress at which the ratio of stress to deformation ceases to be constant. *Yield point* is the stress at which deformation increases without additional load. These terms are best illustrated in the curve for steel (Fig. 1). They are not clearly defined in the curves of other materials.

19. Stress and Deformation Curves.—The typical curves shown (Figs. 1–4) indicate graphically the relation between stress and deformation for four common building materials. The portions of the curves above the horizontal axis are for tension; the portions below are for compression. It will be noted that the concrete curve (Fig. 4) is curved throughout. Within working stresses, however, the curve varies so little from a straight line that the modulus of elasticity is assumed constant.

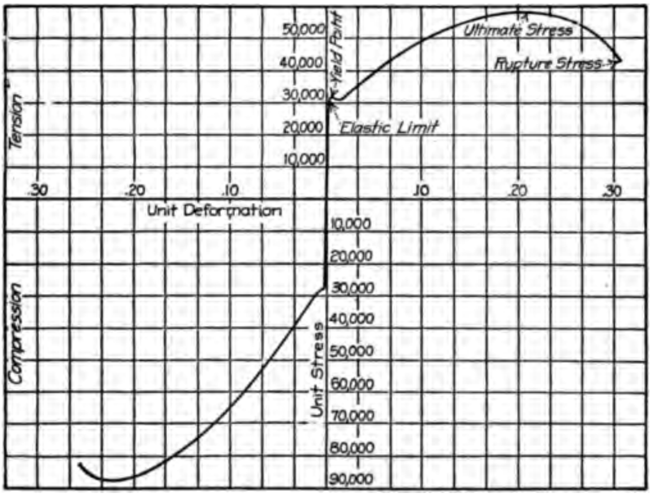

FIG. 1.—Stress-deformation diagram for steel.

20. Shear and Torsion. In addition to direct stresses, namely tension and compression, bodies may be subjected to shear and torsion. *Shear* is caused by a force tending to make the part of a body on one side of a plane slide by the other part. This is an important stress to consider in beam design and occurs in other members. *Torsion* is twisting stress. It is seldom of importance in structural design although it may occur in such members as spandrel beams with rigidly connected slabs.

21. Axial and Combined Stresses.—When a force acts parallel to the axis of a member and at the center of gravity of its cross-section, it produces what is called *axial stress.* Such stress is uniformly distributed over the cross-section. A force parallel to the axis of a member but not acting along this axis is called an *eccentric force.* It is equivalent to an axial force of like amount and a couple whose moment is equal to the product of the force by the normal distance from the force to the axis of the member. Thus an eccentric force as described above produces ***combined***

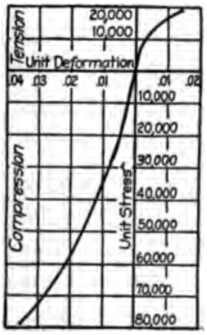

FIG. 2.—Stress-deformation diagram for cast iron.

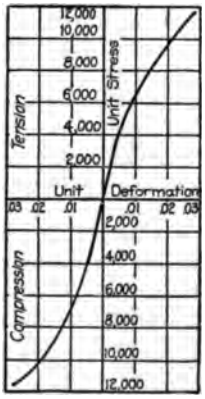

FIG. 3.—Stress-deformation diagram for timber.

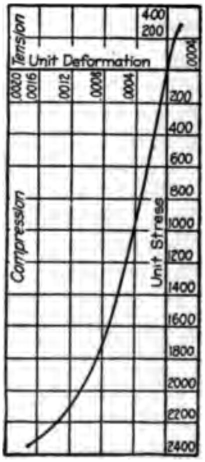

FIG. 4.—Stress-deformation diagram for concrete.

stresses. The axial stresses may be considered separately from those due to moment, and the resulting stresses added to obtain the total stress at any point. For cases of combined stresses which are not parallel, as horizontal and vertical shear, or shear and direct stress, the combined stress must be figured by methods given in the chapter on "Simple and Cantilever Beams."

22. Bending Stress and Modulus of Rupture.—*Bending stresses* are stresses induced by loads perpendicular to the member. *Modulus of rupture* is the maximum bending stress computed on the assumption that elastic conditions exist until failure. Bending stress is discussed in the chapter on "Simple and Cantilever Beams."

23. Stiffness.—*Stiffness* is a term used with reference to the rigidity of structural members. In columns or struts it refers to their lateral stability; *i.e.*, by a stiff column is meant one with a small ratio of length to least radius of gyration, as compared to a slender column. In the case of beams, stiffness refers to lack of deflection rather than to strength.

24. Factor of Safety and Working Stress.—The stress used in design is called the *working* or *allowable stress.* It is obtained by dividing the ultimate stress by the *factor of safety.*

The working stresses usually employed apply to static loads only. Proper allowance for the dynamic effect of the live load should be taken into account by adding the desired amount to the live load to produce an equivalent static load before applying the unit stresses in proportioning parts. An allowance for impact will be necessary only in special cases, as in the case of floors supporting heavy machinery. The amount to add to the live load because of impact will vary from 25 to 100% depending upon the proportion of the specified live load which may be subject to motion.

The factor of safety is dependent upon many things. Among the most important are: the reliability of the material, type of failure, kind of loading, and consequences of failure.

24*a*. Reliability of the Material.—There is always the possibility of the individual piece of the material falling below the average strength of test pieces. Steel, manufactured under almost laboratory conditions, is the most reliable of materials. In common practice it is used with a factor of safety of about 4. Timber, on the other hand, varies greatly in strength and there is difficulty in inspecting and testing it thoroughly. It has therefore been considered as somewhat unreliable and, for this and other reasons, safety factors as high as 10 have commonly been used. At the present time, recent tests of the U. S. Forest Service and other laboratories, together with the branding of timbers by some lumber associations to insure its quality, have greatly reduced the need of a high factor of safety on timber. Cast iron is commonly used with a factor of safety as high as 10, partly on account of uncertainties in its manufacture and partly on account of its method of failure. Concrete is used in the best practice with safety factors varying from about 3 for bending to about 5 for diagonal tension. The factor of safety of concrete, however, is complicated by another factor; namely, the increase in the strength of the material with age. Working stresses are based upon ultimate strengths of 30-day old concrete. At the end of a year the strength of concrete is about 50% more than that at 30 days.

Possible deterioration of materials, such as reduction of section in exposed steel work, due to rust, must be considered in connection with reliability.

24*b*. Type of Failure.—Materials which fail gradually and with plenty of warning like steel are obviously entitled to a lower factor of safety than brittle materials like cast iron. Lumber is about midway in this range. Concrete, well reinforced, can be classed with steel in method of failure, while plain concrete is distinctly in a class with cast iron.

24*c*. Kind of Loading.—A large proportion of dead load, or of live load fixed in amount and point of application, will require a smaller safety factor than loads largely live and uncertain. Also the possibility of the maximum combination of loads occurring, and the probable duration and frequency of this combination must be considered. A common illustration of this point is the allowance of a higher fiber stress (thus lower factor of safety) in buildings, for stresses due to a·combination of maximum live and wind loads.

24*d*. Consequences of Failure.—Where loss of life would be the result of failure, the factor of safety must be such as to make work safe beyond reasonable doubt, but where the loss due to failure would be material only, it is a question of balancing amount of loss in case of failure and probability of failure, against the saving by using a higher fiber stress. Thus temporary construction will have a smaller factor of safety than permanent construction, and concrete forms a lower factor than floor beams.

25. Working Load or Safe Load.—The product obtained by multiplying the cross-sectional area of a column or tie by the working or allowable unit stress is called the *working load* or *safe load* of a member. For a beam, the safe load is that load which will stress the most-stressed fibers to the allowable unit stress.

26. Ratio of Moduli of Elasticity in Combination Members.—When two materials, rigidly joined, are used in a structural member, it is obvious that their deformations must be equal. By definition, $E = \frac{f}{\delta}$ or $f = E\delta$. Therefore, the deformations being equal, the stresses must be proportional to the relative moduli of elasticity. The once-common Flitch girder, composed of wood and steel, is an illustration of the use of two materials in the same member. A concrete member reinforced with steel is a more common illustration. It is plain that in a reinforced-concrete column the vertical steel rods and the concrete shaft are compressed an equal amount. Let this unit deformation be denoted by δ. The concrete stress then is $f_c = \delta E_c$, and the steel stress $f_s = \delta E_s$. Thus $\frac{f_s}{f_c} = \frac{\delta E_s}{\delta E_c} = \frac{E_s}{E_c}$, and $f_s = f_c\frac{E_s}{E_c}$. The ratio $\frac{E_s}{E_c}$ is called n. The modulus of elasticity of steel is fairly constant at 30,000,000 lb. per sq. in. while E for concrete varies from 750,000 to 3,000,000 lb. per sq. in., giving values of n from 40 to 10. The most used values are $n = 15$ for 1:2:4 concrete, and $n = 12$ for 1:1½:3 concrete.

27. Bond Stress.—The combined action of steel and concrete is dependent upon the grip of concrete upon steel, called *bond*. Denoting the allowable bond stress per square inch by u, the load which a rod can take from the concrete per lineal inch is $u\pi d$ for a round rod, and $4ud$ for a square rod. The allowable stress in the rod is $f_s\frac{\pi d^2}{4}$ for round rods and $f_s d^2$ for square rods. The length of embedment of a straight rod necessary to develop its allowable strength is therefore $\frac{f_s d}{4u}$ (in inches) for both round and square rods. For given stresses the necessary length of embedment is easily computed. For example, let $f_s = 10{,}000$ lb. per sq. in. and $u = 80$, then $l = \frac{10{,}000d}{4 \times 80} = 31+$ diameters. Bond stress in reinforced concrete beams is considered in the chapter on "Simple and Cantilever Beams."

28. Shrinkage and Temperature Stresses.—Shrinkage is a function of materials which are poured in a semi-liquid state and then harden by cooling or by chemical action. Such materials are cast iron and concrete. A cast-iron member should be designed so that in cooling it will not shrink unequally and cause stresses which may crack it. For this reason adjacent parts should be made of nearly equal thickness, and filets should be used at all angles and corners.

Concrete shrinks when setting in air and expands when setting under water If the ends of a concrete structure be rigidly fixed, stress will be developed equal to that required to change the length by the amount of the deformation which would occur if the ends were free, or $f = \delta E$.

All bodies change in length with changes in temperature, expanding with heat and contracting with cold. The *coefficient of expansion* is the change in length, per unit of length, per degree change in temperature. The total change in length of a body for a given change of temperature may be found by multiplying this coefficient by the length and the change of temperature in degrees. The fact that the coefficient of expansion is practically alike for both steel and concrete is an important factor in their combined use. As in the case of shrinkage stresses, a tendency to change of length in a member fixed at the ends induces stress equal to that which would cause the computed change in length; that is $f = \delta E$. This may be an important factor to consider in almost any form of steel or concrete construction. In wood construction there is usually sufficient play at columns to take up any expansion.

29. Poisson's Ratio.—Whenever bodies elongate under stress, they shrink laterally; and conversely when they are compressed, under a load, they expand at right angles to the direction of the load. The ratio of deformation normal to stress, to deformation parallel to stress is called *Poisson's ratio*. This is commonly taken as about ¼ for metals and ⅛ for concrete.

PRINCIPLES OF STATICS

By George A. Hool

30. Statics.—*Definition.*—*Statics* is the science which treats of forces in equilibrium.

31. Elements of a Force.—A force acting upon a body is completely known when its *general direction*, *point of application* and *magnitude* are given.

A straight line with arrowhead may be used in representing these elements, as shown in Fig. 5. The angle that the line makes with the vertical and the arrowhead determine the general direction of the force exerted upon the body B The general direction and the point of application completely determine the *line of action*.

The external effect of a force upon a rigid body is the same, no matter at what point of the body along the line of action the force is applied.

Forces are given in pounds and the length of lines are measured in inches. If the scale of force be 5000 lb. to the inch, a line 0.20 in. long would represent a force of 1000 lb.; that is, $5000 \times 0.20 = 1000$. A line 1.55 in. long would represent a force of 7750 lb.; or, vice versa, 7750 lb. would be represented by a line $\frac{7750}{5000} = 1.55$ in. long.

An engineer's scale should be used in laying off the lengths of lines to represent the magnitude of forces, or in scaling such lines. For example, assuming the scale of force to be 4000 lb. to the inch and using the scale divided into 40ths, a force of 1750 lb. would be represented by a line 17½ divisions in length. If the scale of force is assumed to be 400 lb. to the inch, the same force would be represented by 175 divisions.

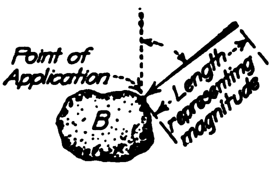

Fig. 5.

32. Concentrated Force.—A *concentrated* force is one whose place of application is so small that it may be considered to be a point.

33. Distributed Force.—A *distributed* force is one whose place of application is an area. A distributed force may often be considered as a concentrated force acting at the center of the contact area.

34. Concurrent and Non-concurrent Forces.—Forces are said to be *concurrent* when their lines of action meet in a point; *non-concurrent* when their lines of action do not meet in this manner.

35. Coplanar and Non-coplanar Forces.—Forces may lie in the same plane or in different planes; that is, they may be either *coplanar* or *non-coplanar* forces.

36. Equilibrium of Forces.—When a number of forces act upon a body and the body does not move, or if moving does not change its state of motion, then the forces considered are said to be in *equilibrium*. Any one of the forces balances all the other forces and it is called the *equilibrant* of those other forces.

37. Resultant of Forces.—A single force which would produce the same effect as a number of forces is called the *resultant* of those forces. The process of finding the single force is called *composition*.

It is evident from the above that the equilibrant and resultant of a number of forces are equal in magnitude, act along the same line, but are opposite in direction.

38. Components of a Force.—Any number of forces whose combined effect is the same as that of a single force are called *components* of that force. The process of finding the components is called *resolution*.

39. Moment of a Force.—The moment of a force with respect to a point is the measure of the tendency of the force to produce rotation about that point. It is equal to the magnitude of the force multiplied by the perpendicular distance of its line of action from the given point. The point about which the moment is taken is called the *origin* (or *center*) *of moments*, and the perpendicular distance from the origin to the line of action is called the *lever arm* (or *arm*) of the force. When a force tends to cause rotation in the direction of the hands of a clock, the moment is usually considered *positive*, and in the opposite direction, *negative*.

40. Couple.—A couple consists of two equal and parallel forces, opposite in direction, and having different lines of action. The perpendicular distance between the lines of action of the two forces is called the *arm* of the couple. The *moment of a couple* about any point in the plane of the couple is equal to the algebraic sum of the moments of the two forces, composing the couple, about that point. (Algebraic sum of the moments means the sum of the moments of the forces, considering positive moments *plus* and negative moments *minus*.)

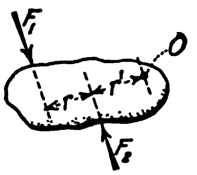

Fig. 6.

In Fig. 6 assume F_1 equal and parallel to F_2, and consider these forces to act upon the body shown. F_1 and F_2 will cause rotation of the body and this rotation will occur about any point in the same plane as the couple, provided the body is pivoted at that point. Consider the body to be pivoted at O in the same plane with the forces. The moment of F_1 about the point O is $F_1(r + r')$, and the moment of F_2 about the same point is F_2r'. The moment of F_2 is positive and the moment of F_1 is negative. Then the moment of the couple is equal to $F_2r' - F_1(r + r') = -F_1r$. The *moment of a couple* is thus equal to one of the forces multiplied by the perpendicular distance between the lines of action of the forces. Since O is any point in the plane of the couple, it is evident that the moment of the couple is independent of the origin of moments: that is, a couple may be transferred to any place in its plane or rotated through any angle and its effect will remain the same. It follows also that any couple may be replaced by another of the same moment in the same plane.

41. Space and Force Diagrams.—In solving problems in statics graphically, it is convenient, in all except the most simple problems, to draw two separate figures, one showing the lines of action and the other the magnitudes and directions of the forces. The former is called the *space diagram*, and the latter the *force diagram*.

Notation used in the graphical solution of all problems in this chapter is explained in Art. 42*d*, p. 9.

42. Composition, Resolution and Equilibrium of Concurrent Forces.

42*a*. Composition of Two Concurrent Forces.—In Fig. 7 let forces F_1 and F_2 which are concurrent forces acting at the point O, be represented in magnitude and direction by OA and OB respectively. From B draw BC parallel to OA, and from A draw AC parallel to OB. Join the point of intersection C with O. The line OC represents the magnitude of a single force R which would produce the same effect as the forces F_1 and F_2. Thus R is the resultant of F_1 and F_2. A force equal and opposite in direction to R and with the same line of action would be the equilibrant of F_1 and F_2, since it would hold them in equilibrium. F_1 and F_2 are components of R.

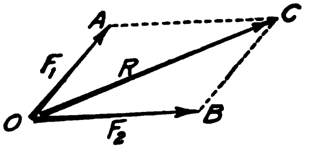

Fig. 7.

It is not necessary to construct the entire parallelogram since either triangle OAC or OBC will suffice. Either of these triangles is called a force triangle and either one, if constructed, is sufficient to give the value of the resultant and the equilibrant of forces F_1 and F_2.

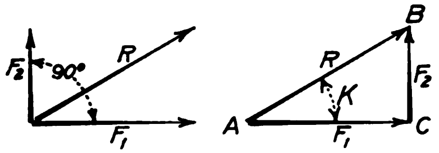

Fig. 8.

It is convenient to solve the force triangle algebraically where the angle between the lines of action of two forces is 90 deg. In Fig. 8 the angle between the lines of action of F_1 and F_2 is 90 deg. It is required to find the value of the resultant R. Since ABC is a right triangle

$$\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$$

or

$$R = \sqrt{F_1^2 + F_2^2}$$

The direction of the resultant R is decided by the angle K. K may be determined as follows:

$$\tan K = \frac{BC}{AC} = \frac{F_2}{F_1}$$

42*b*. Resolution of a Force into Components.—If the resultant R is given at the point O, Fig. 9, and it is desired to obtain two components of R parallel to the lines $o'a'$ and

$o'b'$, then OC is first drawn equal in magnitude and parallel to R, OB is drawn from O parallel to $o'b'$, and CB is drawn from C parallel to $o'a'$ and the lengths of the lines OB and BC, when scaled from the drawing, give the magnitudes of the two components desired.

When components are required making 90 deg. with each other, the magnitude of these forces may easily be determined algebraically. Thus, if R in Fig. 8 is known and the components F_1 and F_2 are required,

$$F_1 = R \cos K$$
$$F_2 = R \sin K$$

42c. Equilibrium of Three Concurrent Forces.—If R in Fig. 8 or Fig. 9 had the opposite direction to that shown, the direction of the forces would follow in order around the sides of the triangle. A force opposite in direction to R and with the same line of action would be the equilibrant of the forces F_1 and F_2 and the three forces would be in equilibrium. Thus, if three forces be represented, in magnitude and direction, by the three sides of a triangle taken in order, then, if these forces be simultaneously applied at one point, they will balance each other. Conversely, three forces which, when simultaneously applied at one point, balance each other, can be correctly represented in magnitude and direction by the three sides of a triangle taken in order.

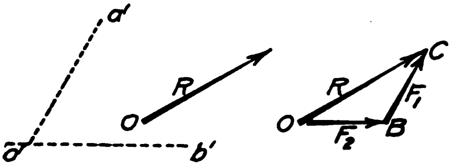

FIG. 9.

42d. Composition of Any Number of Concurrent Forces.—In Fig. 10 assume that the resultant of the four concurrent forces F_1, F_2, F_3, and F_4 is to be found. This may be done by finding the resultant of two forces, then by combining this resultant with a third force to find a second resultant, and so on until all the forces are combined and the resultant of all the forces determined.

The resultant of the force F_1 and F_2 is R_1, determined by the force triangle $R_1F_1F_{2a}$, F_{2a} being drawn parallel to F_2. In the same manner R_2 is the resultant of R_1 and F_3, also R_3 is the resultant of R_2 and F_4. R_3 or R is then the resultant of the four forces, F_1, F_2, F_3, and F_4. F_1, F_{2a}, F_{3a}, F_{4a}, and R_3 form a closed polygon. F_{2a}, F_{3a}, and F_{4a} are parallel and equal in magnitude to forces F_2, F_3, and F_4 respectively, being drawn so. A closed polygon called the *force polygon* can, therefore, be drawn by drawing in succession, lines parallel and equal to the given forces, each line beginning where the preceding one ends and extending in the same direction as the force it represents. The line joining the initial to the final point represents the resultant in magnitude and direction. The diagram $ABCDE$ shows the polygon as it is generally drawn with the diagonals omitted. It makes no difference in what order forces are arranged in the force polygon since the magnitude and direction of the resultant obtained will be the same.

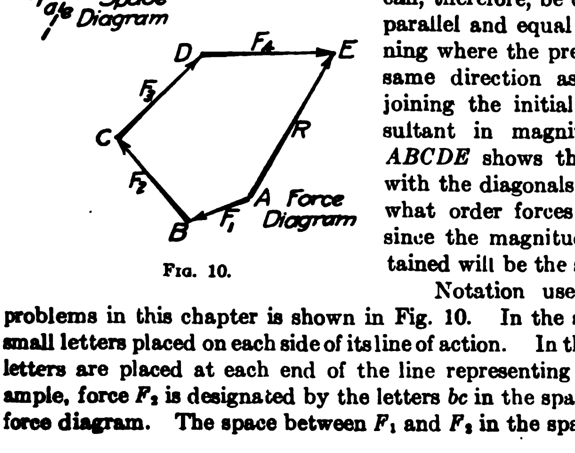

FIG. 10.

Notation used in the graphical solution of all problems in this chapter is shown in Fig. 10. In the space diagram a force is designated by small letters placed on each side of its line of action. In the force diagram corresponding capital letters are placed at each end of the line representing the magnitude of the force. For example, force F_2 is designated by the letters bc in the space diagram and by the line BC in the force diagram. The space between F_1 and F_2 in the space diagram is known as the space b.

The resultant of any number of concurrent forces may be found algebraically in the following manner: Resolve each force algebraically into components F_x and F_y parallel to lines X and Y respectively, lines X and Y being any lines at right angles to each other and called rectangular axes. Let R represent the resultant of all forces acting at the given point; ΣF_x the algebraic sum of the components along the line X; and ΣF_y the algebraic sum of all the forces along the line Y. ΣF_x will then be the component of R along the line X and ΣF_y will be the component along the line Y. The magnitude of R is then given by the formula

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and its direction by

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

θ being the angle between the resultant R and the line X. Particular attention should be paid to the signs of ΣF_x and ΣF_y in order to properly determine the direction of the resultant.

42e. Equilibrium of any Number of Concurrent Forces.—The arrow of the resultant R in Fig. 10 opposes the arrows of the other forces in following around the force polygon. A force equal and opposite to R would be the equilibrant of the forces or, in other words, the forces would be in equilibrium. Thus if a closed force polygon can be drawn for a system of concurrent forces, the forces considered are in equilibrium; and conversely, that for a system of concurrent forces in equilibrium the force polygon must close.

Suppose a number of forces in equilibrium and acting at a single point on a given body be resolved into components in two directions at right angles to each other; horizontal and vertical, for example. The body will evidently be in equilibrium under the action of these component forces since they produce the same effect as their resultants. Moreover, the component forces along each line must balance or the body would move along that line. The condition of equilibrium may now be stated in a different way than above, by saying that the algebraic sums of the components of the forces along each of two lines at right angles to each other must equal zero. (By *algebraic sum* is meant the sum of the forces considering one direction plus and the opposite direction minus.)

Let ΣH represent the algebraic sum of the components along a horizontal line and let ΣV represent the algebraic sum of the components along a vertical line. Then a special case of the above condition of equilibrium would be $\Sigma H = 0$ and $\Sigma V = 0$.

Problems in the equilibrium of concurrent forces may be solved either graphically or algebraically if the number of unknowns is not greater than two. In the graphical method the two unknowns may be determined by the closure of the force polygon, while in the algebraic method the two unknowns may be found by means of two independent equations made possible by the conditions above stated. The two unknowns which may be determined in any given case are the magnitude and direction of one force, the magnitudes or directions of two forces, or the magnitude of one and the direction of the other.

Illustrative Problem.—A boom AB, Fig. 11, is supported in a horizontal position by a cable AC which makes an angle of 30 deg. with the boom. A load of 3000 lb. is carried at point A. Determine the compression in the boom AB and the tension in the cable AC.

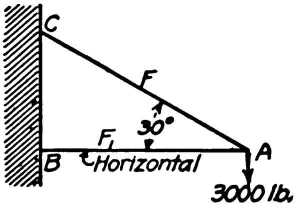

Fig. 11.

The concurrent forces at A are in equilibrium and these forces are all known in direction. Two are unknown in magnitude.

Since F_1 is horizontal, the vertical component of F must equal 3000 lb. in order that ΣV may equal zero at the point A.

$$F \sin 30^\circ = 3000$$
$$F = 6000 \text{ lb.}$$

In order that $\Sigma H = 0$

$$F_1 = F \cos 30^\circ$$
$$F_1 = 5200 \text{ lb.}$$

Illustrative Problem.—The crane truss shown in Fig. 12 is loaded with 3000 lb. at L. Determine the stresses in the boom ac; the tie ab; the mast ad; and the stay bd.

$\overline{LM}^2 = \overline{8}^2 + \overline{15}^2$ $\quad$ $\overline{LN}^2 = \overline{20}^2 + \overline{15}^2$ $\quad$ $\overline{MP}^2 = \overline{12}^2 + \overline{9}^2$

$LM = 17$ $\quad$ $LN = 25$ $\quad$ $MP = 15$

At the point L three forces are acting; namely, the 3000-lb. load, the stress F in the tie ab, and the stress F_1 in the boom ac. Draw the force polygon ABC by laying off the vertical line BC equal to 3000 lb. (since weight always acts vertically) and drawing BA and CA parallel to F and F_1 respectively.

Since there is equilibrium in the crane truss, the forces acting at the point L are in equilibrium. Hence, the force polygon should close and the forces should act in order around the polygon. If the drawing is made to scale, the lines BA and CA represent directly the magnitude and direction of F and F_1. It should be noticed that triangle ABC is similar to triangle LMN and it is not necessary to construct a separate force polygon if the crane truss is drawn to some scale in the first place. For example, if the scale used for drawing the truss is 1 in. = 2 ft. then MN = 6 in. But MN represents a force of 3000 lb., hence, the scale used for determining the forces should be 1 in. = 500 lb.

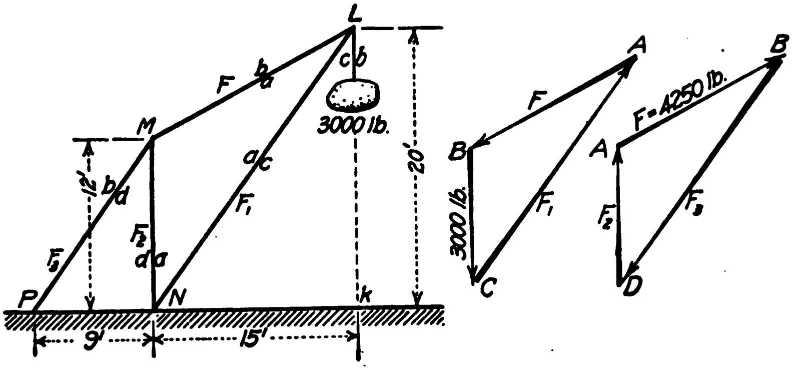

Fig. 12.

F and F_1 may also be solved algebraically as follows:

$$\frac{LM}{MN} = \frac{17}{12} = \frac{F}{3000}$$

$$F = 4250 \text{ lb.}$$

$$\frac{LN}{MN} = \frac{25}{12} = \frac{F_1}{3000}$$

$$F_1 = 6250 \text{ lb.}$$

It will be noticed that the stress F_1 acts toward the point L or, in other words, it is the stress acting against the shortening of the member LN, thus denoting compression. The force F is the stress acting against the lengthening of the member LM, thus denoting tension. We know this to be true, and we have then a general rule, that, when a force is shown by the force polygon to act toward the point of application of the forces, the stress caused is compression, and, when a force is shown to act away from the point of application of the forces, the stress caused is tension.

A force polygon ABD should next be drawn for the forces at the point M. The force F is now known and the two unknown forces F_2 and F_3 may be found in the same manner as the forces F and F_1 were obtained from the force 3000. In fact it should be remembered that when the forces of a concurrent system in equilibrium are all known except two, the magnitudes and directions of these two forces may be determined if only their lines of action are known.

Since the tangents of the two angles MPN and LNK are each equal to $\frac{4}{3}$, the angles themselves are equal and MP is parallel to LN. Thus, the force polygon drawn for the three forces F, F_2, and F_3, is similar to triangle LMN. If the crane truss is drawn to scale, no separate force polygon is needed. MN and LN, if properly scaled, will give the magnitude and direction of F_2 and F_3. However, it is not even necessary to scale the forces in this case since it is evident that F_1 and F_3 are equal in magnitude and that F_2 is equal to the weight; that is, 3000 lb.

We know F to be tension, hence, we should represent it as acting away from the point M. The arrows must follow in order around the force triangle ABD, consequently, F_2 is compression and F_3 is tension.

F_2 and F_3 may also be solved independently as follows:

$$\frac{LM}{MN} = \frac{17}{12} = \frac{4250}{F_2}$$

$$F_2 = 3000 \text{ lb. (same as the weight).}$$

$$\frac{LM}{LN} = \frac{17}{25} = \frac{4250}{F_3}$$

$$F_3 = 6250 \text{ lb. (same as } F_1\text{).}$$

Answers
F = 4250 lb. (tension)
F_1 = 6250 lb. (compression)
F_2 = 3000 lb. (compression)
F_3 = 6250 lb. (tension)

43. Composition and Equilibrium of Non-concurrent Forces.

43a. Graphical Method.—When several forces lying in the same plane and acting on a given body have different points of application, so that their lines of action do not intersect in the same point, the magnitude of the resultant may be found graphically by compounding the forces in the same manner as in concurrent systems. Two of the forces may be produced until they intersect and their resultant found, then the resultant of these two forces compounded with a third, then the resultant of the first three compounded with the fourth, and so on until the resultant of all has been found.

For example, it is required to determine the resultant of the four forces shown in Fig. 13 (*a*) which act on a given body. Produce forces F_1 and F_2 until they meet at the point *o*. The resultant of these forces is R_1, the magnitude and direction of which is determined by the force triangle *ABC* in Fig. 13 (*b*). Produce R_1 until it intersects the third force F_3 at *m*. R_2 is the resultant of F_3 and R_1, determined by the force triangle *ACD*. Produce R_2 until it intersects the force F_4 at *n*. *R* is the resultant of F_4 and R_2, determined by the force triangle *ADE*, and, consequently, *R* is the resultant of the four given forces.

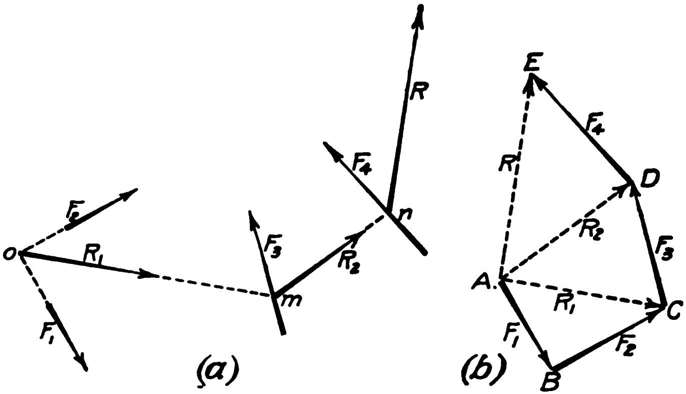

Fig. 13.

It will be noticed that Fig. 13 (*b*) is a force polygon for the given forces, and the resultant of all the forces is represented by the closing line *AE*. There is, then, the same general rule for non-concurrent forces as for concurrent forces; namely, that the magnitude of the resultant of any number of forces acting in the same plane may be found by constructing the force polygon and scaling the closing side. The line *AE* also shows the direction of the resultant *R*, but note that it does not give a point on its line of action. A point in the line of action of the resultant cannot be determined unless the construction of Fig. 13 (*a*) (or its equivalent) is made. A force equal and opposite to *R* and having the same line of action would balance the forces acting and the system would be in equilibrium.

Forces Nearly Parallel.—The graphical method already explained for finding a point such as *n*, Fig. 13 (*a*), on the line of action of the resultant, cannot always by conveniently used. If the forces are parallel, or nearly so, it is not easy to obtain the intersection of the forces and, consequently, a different construction is necessary. The diagram that is used for such cases is called the *equilibrium polygon*. The force polygon, however, is needed to find the magnitude and direction of the resultant, the same as before.

Consider the four forces shown in Fig. 14 (*a*). The force polygon *ABCDE* for these forces is reproduced in Fig. 14 (*b*). The line *AE* gives the magnitude and direction of the resultant R. Select any point *O* and draw the lines *OA*, *OB*, *OC*, *OD*, and *OE* to the vertices of the force polygon.

In the force triangle *ABO*, *BO* and *OA* represent the magnitudes and directions of two forces *bo* and *oa* which balance F_1. (The notation used is explained in Art. 42*d*.) Select some point 1 on the line of action of F_1 and draw the lines *bo* and *oa* parallel to *BO* and *OA* respectively. The force *bo* intersects the force F_2 at the point 2. In the triangle *BCO*, forces *CO* and *OB* hold F_2 in equilibrium. At the point 2 draw *co* parallel to *CO* until it meets the force F_3 at 3. In the triangle *CDO*, forces *DO* and *OC* balance the force F_3. At the point 3 draw *do* parallel to *DO* until it meets the force F_4 at the point 4. At the point 4, draw *eo* parallel to *EO* until it meets the line of action of *oa* at point 5. It should be noted that forces *eo* and *oa* are the only forces in the equilibrium polygon which so far have not been balanced by equal and opposite forces. As shown by the force polygon *OABCDE*, these two forces hold in equilibrium the four forces F_1, F_2, F_3, and F_4. The force triangle *AEO* shows these forces to hold also the resultant *R* in equilibrium. Therefore a line drawn through the point 5 in the equilibrium polygon parallel to *AE* of the force polygon gives the line of action of *R*.

The point *O* in Fig. 14 (*b*) is called the *pole; OA, OB, OC*, etc., are called *rays;* and the lines 1–2, 2–3, etc., in Fig. 14 (*a*) are called *strings.*

Since *O* is any point that may be selected, it should be taken so that it will be most convenient for the solution of the given problem and never on the closing line *AE* since then the strings *oa* and *oe* become parallel to *AE* and hence parallel to each other. It should be remembered that the magnitude and direction of the resultant of any number of non-concurrent forces is given by the force polygon and a point on its line of action by the equilibrium polygon. The force polygon must first be drawn and the resultant determined in both magnitude and direction by the closing side. The pole *O* should next be selected and the rays drawn, to which the strings of the equilibrium polygon should be made respectively parallel. The line through the intersection of the first and last strings parallel to the direction of the resultant in the force polygon is the line of action of the resultant.

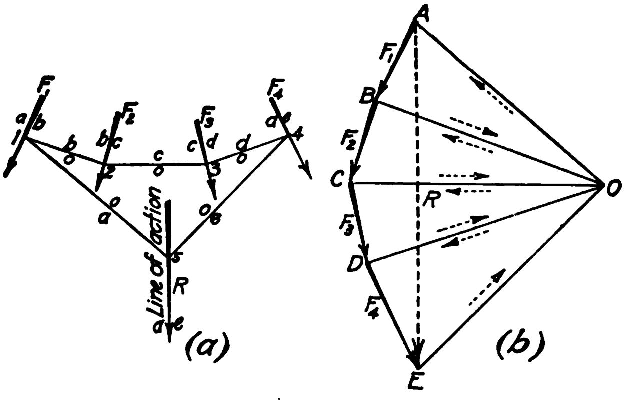

Fig. 14.

If the force *R* acted in the opposite direction, the system would be in equilibrium and the forces would follow in order around the force polygon. The system in equilibrium would then be forces F_1, F_2, F_3, and F_4 and a force equal and opposite to *R* acting through the point 5. If the force equal and opposite to *R* should be placed to one side or the other of the point 5, but still parallel to its direction as shown by the force polygon, the intersection of *oe* and *oa* would not fall on its line of action. We would then say that the equilibrium polygon did not close. Thus, it is easily seen for a given system of forces that, even if the force polygon closes, the equilibrium polygon may not close.

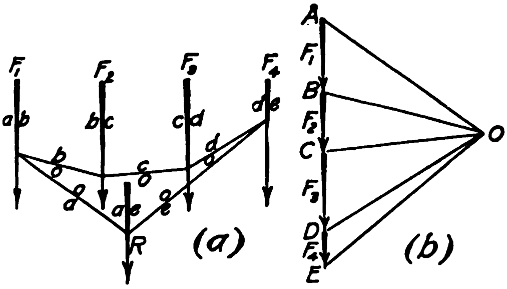

Fig. 15.

When the force polygon closes and the equilibrium polygon does not, the result is that of couple. For such a case the resultant of the forces F_1, F_2, F_3, and F_4 would not be in the same line of action as the remaining force and equilibrium could not result. Equilibrium exists when the moment of the couple is zero.

Parallel Forces.—The method is the same as shown for forces nearly parallel (Fig. 14). Fig. 15 shows the construction necessary to find the resultant of the four parallel forces F_1, F_2, F_3, and F_4.

43b. Algebraic Method.—The resultant of any number of non-concurrent forces may be found algebraically in the following manner: Resolve each force algebraically into components F_x and F_y, parallel respectively to X and Y axes. Then according to Art. 42d, the magnitude of R is given by the equation

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and the angle it makes with the X axis is given by

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

Its line of action is found by placing its moment about any point equal to the algebraic sum of the moments of the forces with respect to the same point. If the moment arm of the resultant is denoted by a, and the moment arms of the several forces by a_1, a_2, etc., then

$$Ra = F_1a_1 + F_2a_2 + \text{etc.}$$

If a force is applied equal and opposite to R and in the same line of action, the system of forces will be in equilibrium. Let ΣM represent the algebraic sum of the moments about any point. For equilibrium, then,

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M = 0$$

In practice it is common to use horizontal and vertical axes, for which case the above equations may be written:

$$\Sigma H = 0 \qquad \Sigma V = 0 \qquad \Sigma M = 0$$

Problems in the equilibrium of non-concurrent forces may be solved if the number of unknowns is not greater than three. Three independent equations may be written; employing the three algebraic conditions above stated, and solving these equations simultaneously in any given case gives the three unknowns. It is often convenient to use two moment equations and either $\Sigma H = 0$ or $\Sigma V = 0$. A new moment center must be taken each time $\Sigma M = 0$ is used.

The three unknowns usually desired may be classed under three general cases; namely, where the following unknowns are required: (1) point of application, direction and magnitude of one force (that is, the force is wholly unknown); (2) magnitudes of two forces and the direction of one of these forces; and (3) magnitude of the three forces. The first case is nothing more than the finding of the resultant of a system of non-concurrent forces.

A special case in the solution of non-concurrent forces occurs when all the forces considered are parallel. Then the number of independent equations reduces to two and it is possible, therefore, to determine but two unknowns, namely: (a) point of application and magnitude of one force; and (b) magnitude of two forces.

Illustrative Problem.—Find the resultant of the three vertical forces shown in Fig. 16.

Since the forces are all vertical, $\Sigma H = 0$, and the resultant must also act in a vertical direction. Consider downward forces positive and upward forces negative. The magnitude of the resultant may be found as follows:

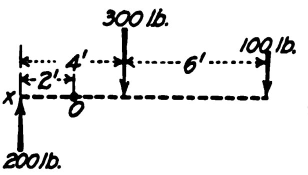

Fig. 16.

$$R = 300 + 100 - 200$$
$$= 200 \text{ lb., acting down (since the result is positive).}$$

It will be noticed that a force equal and opposite to R would make the forces in equilibrium.

It is now necessary to find the point of application of the resultant R. By the point of application in this case is meant a point on the line of action of the resultant.

The algebraic sum of the moments about the point o is equal to $(300)(2) + (100)(8) + (200)(2) = 1800$ ft.-lb. The resulting force is 200 lb. and the problem resolves itself into finding how far from the point o the 200 lb. should be placed to have the same effect as the three loads shown, or, in other words, how far away from o a load equal and opposite to the 200-lb. resultant should be placed in order to cause equilibrium. Thus, $\Sigma M = 0$ may be used to find this distance

$$\frac{1800 \text{ ft.-lb.}}{200 \text{ lb.}} = 9 \text{ ft. to the right of } o.$$

It should be noted that the computations would have been more simple if the point x had been selected instead of the point o—that is, the work would have been simplified by taking the origin on the line of action of one of the forces. The computations for that case would be arranged as follows:

$$\frac{(300)(4) + (100)(10)}{200} = 11 \text{ ft. to the right of } x.$$

Illustrative Problem.—The beam AB (Fig. 17) is 14 ft. long and loaded as shown. It is simply supported at A and C. (a) Determine the supporting forces due to the three given loads. (b) Determine the supporting forces, including the weight of the beam which is 50 lb. per lin. ft.

(a) $R = 200 + 300 + 400 = 900$ lb., acting down.
$F + F_1 = R = 900$ lb.

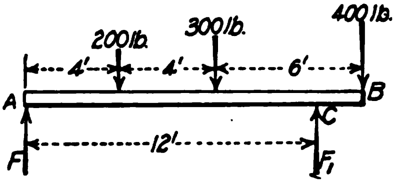

FIG. 17.

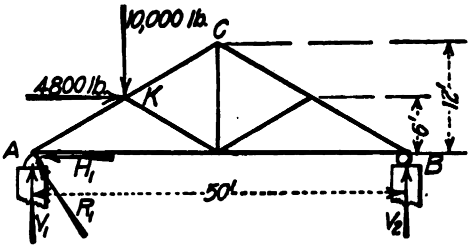

FIG. 18.

Origin at A:

$$(200)(4) + (300)(8) + (400)(14) = 12F_1$$
$$F_1 = 733 \text{ lb.}$$
$$F = 900 - 733 = 167 \text{ lb.}$$

Answers $\begin{cases} F = 167 \text{ lb.} \\ F_1 = 733 \text{ lb.} \end{cases}$

(b) Wt. of beam $= (50)(14) = 700$ lb.
$R = 900 + 700 = 1600$ lb.

$$(200)(4) + (300)(8) + (400)(14) + (700)(7) = 12F_1$$
$$F_1 = 1142 \text{ lb.}$$
$$F = 1600 - 1142 = 458 \text{ lb.}$$

Answers $\begin{cases} F = 458 \text{ lb.} \\ F_1 = 1142 \text{ lb.} \end{cases}$

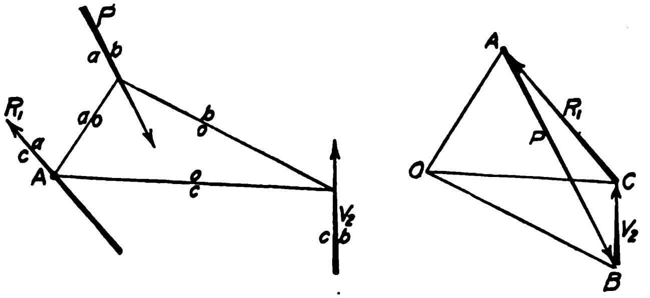

FIG. 19.

Illustrative Problem.—Find the reactions of the roof truss shown in Fig. 18 for the loads assumed. Solve by both the algebraic and graphical methods. The truss is fixed at A. Rollers are placed at B so that the reaction at the right end acts at right angles to the supporting surface—that is, vertically.

$\Sigma M = 0$. Origin at A.

$$10{,}000\frac{(25)}{2} + 4800\ (6) - 50V_2 = 0.$$

$V_2 = 3080$ lb., acting up.

$\Sigma V = 0$.

$$3080 + V_1 - 10{,}000 = 0$$

$V_1 = 6920$ lb., acting up.

$\Sigma H = 0$.

$$4800 - H_1 = 0.$$

$H_1 = 4800$ lb., acting toward the left.

$R_1 = \sqrt{6920^2 + 4800^2} = 8420$ lb., acting as shown.

Fig. 19 shows how the reactions are obtained by means of the force and equilibrium polygons. The construction is as follows: Draw P, the resultant of the 10,000 and 4800 lb. loads, in the force polygon. Choose pole

O. Draw rays OA and OB. Draw strings oa and ob so that oa passes through the point of support A, A being a known point in the line of action of R_1. Draw the closing line oc of the equilibrium polygon. Draw ray OC in force polygon corresponding to the closing line oc. Knowing V_2 to be vertical, its magnitude is easily determined. R_1 is the closing side of the force polygon in magnitude and direction. Draw a line through A parallel to R_1 of the force polygon, thus giving the line of action of the left reaction.

Fig. 20 shows how the reactions are obtained by producing the forces until they intersect. In many cases the intersection method cannot be used because the point of intersection lies outside the limits of the drawing.

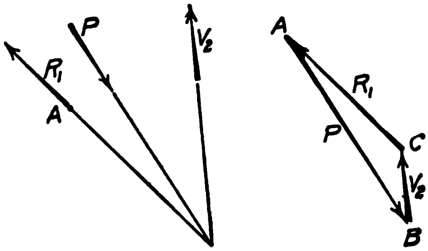

FIG. 20.

44. Center of Gravity.—The *center of gravity* of a body is the point through which the resultant of all the parallel forces of gravity, acting upon the body, passes for every position of the body. The resultant of any set of these parallel forces of gravity is the *weight* of the body. If a force equal and opposite in direction to this resultant is applied in a line passing through the center of gravity of the body, the body will be in equilibrium. A force of gravity exists for each particle composing the body.

In designing structures it is frequently necessary to deal with the center of gravity, or centroid, of areas. The center of gravity may usually be found by some simple geometrical construction but for irregular figures it is convenient to divide the area into sections whose gravity centers may be easily obtained, such as rectangles and triangles. By treating these sectional areas as a system of parallel coplanar forces, the center of gravity may be found since it is the point through which the line of action of the resultant passes in whatever direction the parallel forces are assumed to act. It is only necessary to find the line of action of the resultant with respect to two axes at right angles to each other since the intersection of the two resultants so found will give the center of gravity of the area for all axes.

The center of gravity of a rectangle is evidently at the intersection of the diagonals. The center of gravity of a circle or regular polygon is at the geometrical center of the figure. To find the center of gravity of a triangle draw a line from each of two vertices to the middle of the opposite side. The point of intersection of the two bisectors is the center of gravity of the triangle and lies at a distance from any vertex equal to two-thirds of the length of the corresponding bisector.

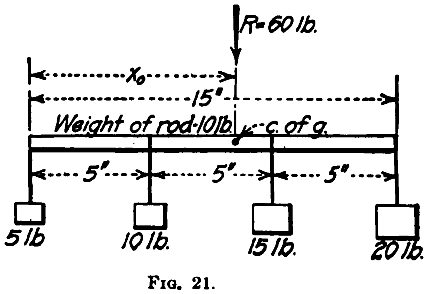

FIG. 21.

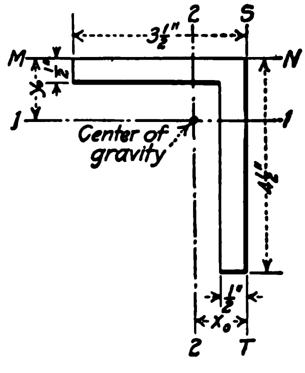

FIG. 22.

Illustrative Problem.—A rod of uniform section, 15 in. long and weighing 10 lb., supports weights of 5 lb., 10 lb., 15 lb., and 20 lb. The 5-lb. and 20-lb. weights are supported at the ends and the other two weights are equally spaced along the rod in the order shown (Fig. 21). Find the point at which the rod will balance.

The weight of the rod may be assumed to be concentrated at its center. Taking moments about the end at which the 5-lb. weight is hung, we have

$$Rx_0 = 5(0) + 10(5) + 10(7.5) + 15(10) + 20(15) = 575 \text{ in.-lb.}$$

$$R = 5 + 10 + 10 + 15 + 20 = 60 \text{ lb.}$$

$$x_0 = \frac{575}{60} = 9.58 \text{ in.}$$

Illustrative Problem.—Locate the center of gravity, or centroid, of section shown in Fig. 22.

Divide the figure into two rectangles and denote total area by A. The center of gravity of each rectangle is at its center. The gravity axis 1-1 may be located by taking moments about MN, or

$$Ay_o = (4\tfrac{1}{2} \times \tfrac{1}{2})(2\tfrac{1}{4}) + (3 \times \tfrac{1}{2})(\tfrac{1}{4}) = 5.44 \text{ in.}^3$$
$$A = (4\tfrac{1}{2} \times \tfrac{1}{2}) + (3 \times \tfrac{1}{2}) = 3.75 \text{ in.}^2$$
$$y_o = \frac{5.44}{3.75} = 1.45 \text{ in.}$$

The gravity axis 2-2 may be located in a similar manner by taking moments about ST, or

$$x_o = \frac{(3\tfrac{1}{2} \times \tfrac{1}{2})(1\tfrac{3}{4}) + (4 \times \tfrac{1}{2})(\tfrac{1}{4})}{3.75} = 0.95 \text{ in.}$$

The intersection of axes 1-1 and 2-2 determines the centroid of section.

45. Moments of Forces.—The moment of a system of forces about a given point is equal to the algebraic sum of the moments of the forces composing the system about the same point.

The moment of a system of forces about a given point may be found graphically in the following manner:

Let F_1, F_2, F_3, and F_4, Fig. 23, be the given system of forces and let k be the point about which the moment is required. Draw the force and equilibrium polygons as described in Art. 43*a* and determine the resultant R in both magnitude, direction, and line of action. The distance H in the force polygon is called the *pole distance* of the resultant R. Draw through k a line parallel to R and intersecting the strings oa and oc at A' and E' respectively. The triangles AOE and $A'O'E'$ are similar (sides respectively parallel) and

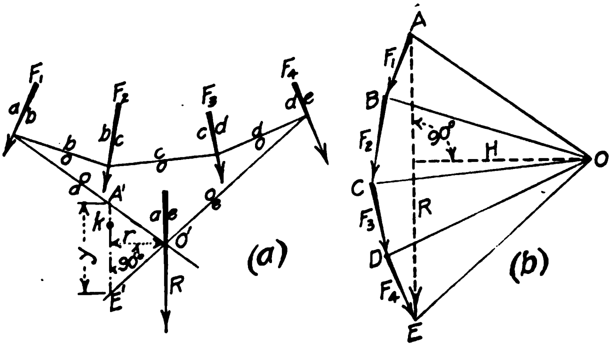

Fig. 23.

$$\frac{r}{y} = \frac{H}{R} \text{ or } Rr = Hy$$

Therefore

$$M = Rr = Hy.$$

H is measured in pounds to the scale of the force diagram and y is measured in units of length to the scale of the space diagram.

For parallel forces the method is the same as given above.

REACTIONS

By George A. Hool

46. General Considerations.—The finding of the reactions of a structure having two points of support—such as the simple beam, girder or truss—is a problem in the equilibrium of non-concurrent forces. As shown in Art. 43*b*, the problem may be solved if the number of unknowns is not greater than three. Three independent equations may be written employing the following three equations of statics:

$$\Sigma H = 0 \qquad \Sigma V = 0 \qquad \Sigma M = 0$$

Solving these equations simultaneously in any given case gives the three unknowns. The three unknowns may also be found graphically as explained in Art. 43*a*.

Instead of the three equations of statics as given above, it is often convenient to use two moment equations and either $\Sigma H = 0$ or $\Sigma V = 0$. A new moment center must be taken each time $\Sigma M = 0$ is used.

Referring to Fig. 24, it will be seen that six conditions are needed in order to completely determine the two reactions R_1 and R_2; namely, their points of application, their directions (direction determined for each reaction by the angle made with the vertical), and their magnitudes. Three of these conditions may be determined by statics if the other three conditions are determined by the manner in which the structure is supported. The three conditions generally known are the points of support and the direction of one of the reactions.

If there are less than three unknown conditions in regard to the manner in which a structure is supported, then the structure is in general unstable and will tend to move bodily under the applied loads. For example, suppose the supporting forces to have only their magnitudes unknown. Then unless the resultant of these reactions is in the same line of action as the resultant of the applied loads, equilibrium cannot exist. The structure, therefore, will move and is termed unstable.

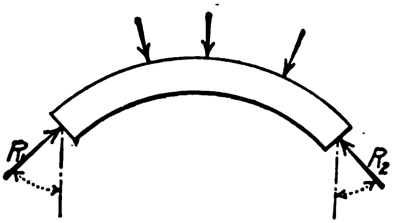

FIG. 24.

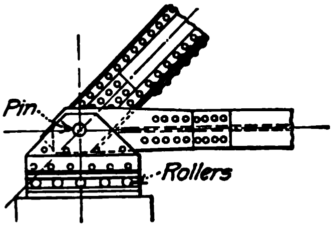

FIG. 25.

When one end of a structure is placed on rollers, the reaction at that end is made to act at right angles to the supporting surface since the rollers, if in good condition, cannot offer resistance to motion along this surface. If a structure is hinged at a support, the line of action of the reaction at that support passes through the hinge. (A hinge generally is a steel cylindrical shape of short length and but a few inches in diameter, and called a *pin*. When used at a support it rests upon a shoe which in turn rests upon the support.) When a hinge is placed at the same support where rollers are used (Fig. 25), the reaction is at once determined in both direction and point of application.

Rollers not only cause a reaction to act at right angles to the supporting surface but also serve the purpose of allowing structures to expand and contract with changes in temperature and thus prevent additional stresses in different members.

Structures supported at one end by a tie-rod should be considered as having the reaction at that point fixed in direction. A tie-rod is incapable of carrying compression or bending, and thus the reaction which it carries must act along its axis and produce tension in the rod.

It is seldom found in practice that the point of application of a reaction is definitely fixed. For short beams which deflect but little and which rest at the ends upon steel bearing plates (inserted in order to distribute the load over the masonry supports), it is usually sufficient to consider the reaction as applied at the center of bearing, but this assumption is by no means an exact one. For long girders, especially, the deflection would be so great that the center of bearing would be brought near the edge of support and the assumption would not hold. However, if a pin bearing is used with rollers, a uniform bearing on the support is ensured. The reaction is then considered to pass through the pin center, but this will not be quite true if the pin is badly turned or the bearing surface of the shoe upon which it rests is imperfect.

The method of finding the reactions of restrained and continuous beams is explained in Art. 71.

47. Determination of Reactions.

47a. Forces Parallel.—As explained in Art. 43*b*, a special case in the solution of non-concurrent forces occur when all the forces are parallel. For forces all vertical $\Sigma H = 0$ is not needed, and the number of independent equations reduces to two. It is possible, therefore, to determine but two unknowns: namely, (*a*) point of application and magnitude of one force; and (*b*) magnitude of two forces.

47b. Forces Not Parallel.—Reaction problems when solved algebraically will generally be simplified by finding the horizontal and vertical components of the reactions and then obtaining the magnitude of either reaction by computing the square root of the sum of the squares of its two components. With one end on rollers and resting upon a horizontal surface, the vertical component at that support is the reaction required, and the horizontal component is zero. With a roller end resting upon an inclined surface, the reaction at that support will have both a vertical and a horizontal component, but there is at once a relation between them due to the fact that the reaction must act at right angles to the supporting surface.

Reaction problems may also be simplified when solving algebraically by resolving inclined loads into horizontal and vertical components.

If a load is distributed over a considerable area, as wind pressure for example, instead of being applied at a point, the resultant of this load may be used in the reaction computations as a concentrated load. For example, in Fig. 26, only the resultant wind pressure P needs to be considered and it will act at the center of AC. The horizontal and vertical components of P may be found in the following convenient manner:

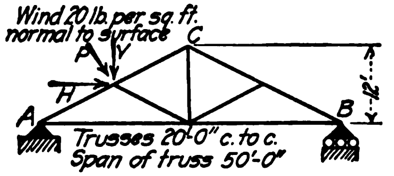

Fig. 26.

Consider first the wind pressure acting on a strip of roof surface having a length AC and a width of one foot. Normal pressure on this strip $= 20 \times AC = P_n$. Denote horizontal and vertical components of P_n by H_s and V_s respectively. Then

$$\frac{H_s}{P_n} = \frac{12}{AC}$$

or

$$H_s = \frac{12(P_n)}{AC} = 12 \times 20$$

Similarly,

$$V_s = 25 \times 20$$

Thus, from the above it follows that these H_s and V_s components can be determined by multiplying the normal pressure in pounds per square foot by the projection of the upper chord (AC in this case) on a plane at right angles to the direction of the desired component. Since the trusses are 20 ft. center to center, the H and V components of the total normal pressure P acting on the truss are as follows:

$$H = H_s(20) = 12(20)(20) = 4{,}800 \text{ lb.}$$
$$V = V_s(20) = 25(20)(20) = 10{,}000 \text{ lb.}$$

Roof trusses of short span are generally fixed at both ends to the walls of the building, thus becoming statically indeterminate with respect to the outer forces. In this case the reactions for the wind load are determined separately from those caused by the dead and snow loads. Dead and snow loads cause only vertical reactions (Fig. 27). The wind load causes the reactions to be inclined and the horizontal components tend to overturn the walls of the building. One of two assumptions is usually made, either (a) that the horizontal components of the two wind reactions are equal, or (b) that the direction of the wind reactions are parallel to the resultant wind load (Fig. 28).

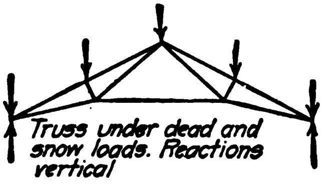

Fig. 27.

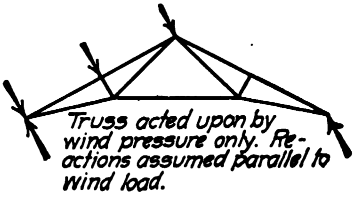
Truss acted upon by wind pressure only. Reactions assumed parallel to wind load.

Fig. 28.

In the following illustrative problems, the reactions at points shown thus are considered to have both a horizontal and vertical component. This symbol for a fixed end is not intended to represent a knife bearing but simply means that the point of application is determined and that the reaction may act in any direction. With rollers added to this symbol as here shown the reaction is considered as determined in both direction and point of application. When solving algebraically, the horizontal and vertical components of the reactions are represented thus: Where the value of H_1 comes out negative, the horizontal component of the reaction acts in the opposite direction to that assumed.

$H_1 \rightarrow$
V_1
V_2

For finding the reactions of simple beams and trusses, see also illustrative problems on pp. 15 and 16.

Illustrative Problem.—A beam is loaded as shown in Fig. 29. Find the reactions at A and B by both algebraic and graphical methods. Neglect weight of beam.

$\Sigma H = 0 \qquad \therefore H_1 = 0$

$\Sigma M = 0 \qquad$ Origin at A.

$$(6)(6) + (20)(22.5) - 15V_2 + (10)(5) = 0$$

$V_2 = 29.1$ tons, acting up, since result is positive.

$\Sigma V = 0$

$$10 + 6 + 20 - 29.1 = V_1$$

$V_1 = 6.9$ tons, acting up.

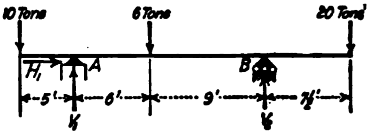

Fig. 29.

(If a check on V_1 is desired, it may be obtained by applying $\Sigma M = 0$ about B as an origin.)

In Fig. 30, the force polygon is drawn for the given forces. The forces are designated by letters instead of by weight. It can easily be seen that $H_1 = 0$ or the forces would not be in equilibrium. The force polygon, consequently, becomes a straight line since the forces are all vertical. $AB = F_1$, $BC = F_2$, $CD = F_3$, $DE = V_2$, $EA = V_1$. It is not possible to determine the point E until after the equilibrium polygon is drawn. The string od intersects V_2 at t. The string oa intersects V_1 at k. The line OE in the force polygon drawn parallel to kt in the equilibrium polygon divides the line AD into two parts, DE and EA, which represent V_2 and V_1 respectively. kt is drawn in the equilibrium polygon because the forces are in equilibrium and the equilibrium polygon should close.

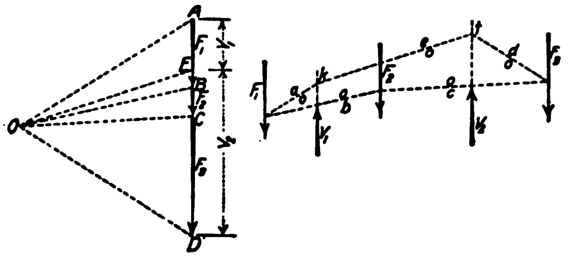

Fig. 30.

Illustrative Problem.—Find the horizontal and vertical components of the reactions at A and B, Fig. 31, by the algebraic method. Neglect weight of beam.

Considerable labor will be saved by resolving the inclined forces into horizontal and vertical components and using these components only in the computations. The lever arms of the horizontal components about either point of support is zero, leaving only the vertical components to be considered when applying $\Sigma M = 0$. Components are shown dotted in Fig. 31.

$\Sigma M = 0$ Origin at A.

$$(17.32)(7) - (5)(2) + (7.07)(8) + (10)(20) - 15V_1 = 0$$

$V_1 = 8.36$ tons.

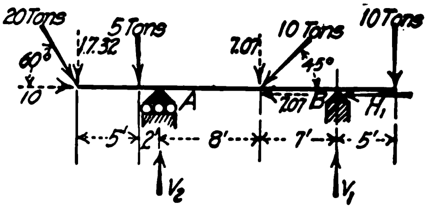

Fig. 31.

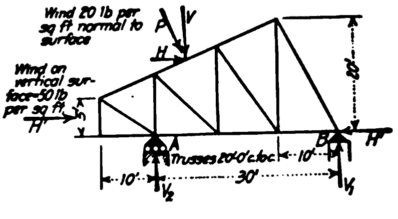
Wind 20 lb per sq ft normal to surface
P
V
H
Wind on vertical surface=50 lb per sq ft.
H'
20'
A
B
H'
Trusses 20'-0" c. to c.
10'
10'
30'
V_2
V_1

Fig. 32.

$\Sigma V = 0$

$$-17.32 - 5 + V_2 - 7.07 + 8.36 - 10 = 0$$

$V_2 = 31.03$ tons.

$\Sigma H = 0$

$$7.07 + H_1 - 10 = 0$$

$H_1 = 2.93$ tons.

Illustrative Problem.—Compute horizontal and vertical components of the reactions for the truss shown in Fig. 32 for the wind pressure shown.

As explained in Art. 47b, the components of the total wind pressure may be readily found as follows:

$$V = (20)(30)(20) = 12{,}000 \text{ lb.}$$
$$H = (20)(15)(20) = 6{,}000 \text{ lb.}$$
$$H' = (5)(20)(50) = 5{,}000 \text{ lb.}$$

$\Sigma M = 0$. Origin at A

$$(5000)\frac{5}{2} + (6000)\frac{25}{2} + 12{,}000(5) - 30V_1 = 0$$
$$V_1 = 4920 \text{ lb.}$$

$\Sigma V = 0$

$$4920 + V_2 - 12{,}000 = 0$$
$$V_2 = 7080 \text{ lb.}$$

$\Sigma H = 0$

$$6000 + 5000 - H_1 = 0$$
$$H_1 = 11{,}000 \text{ lb.}$$

Fig. 33 shows how the reactions are obtained by means of the force and equilibrium polygons. Since point B is a known point in the line of action of R_1, the string oa is drawn starting from this point.

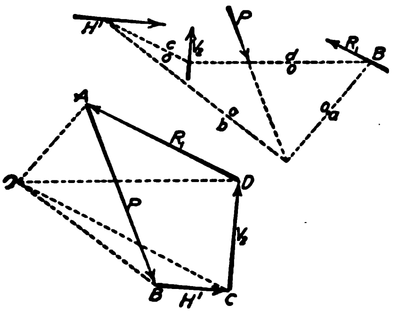

FIG. 33.

Illustrative Problem.—Fig. 34 represents a Howe bridge truss of 120-ft. span, with 12 equal panels. Neglecting the dead load on the end panel points, determine the reactions algebraically for a dead load of 9000 lb. on each intermediate panel point and a live load of 20,000 lb. on panel points marked a, b, and c.

Reactions A and B are both vertical since the loads are vertical, which is generally the case on bridge trusses. Then again, since the panels are all equal the algebraic method is by far the more convenient one to use. The stringers at each end either rest directly upon the abutments or upon end floor beams. In either case the load on an end panel point is fully carried by the support beneath, thus causing no reaction at the other support and hence no stresses in the truss. This is the reason for the omission of the dead load on the end panel points in this problem. In designing the details at A and B, however, the loads at these points must be considered.

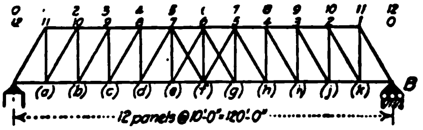

FIG 34.

Reactions A and B each receive one-half the dead load, or $9000 \times 5\frac{1}{2} = 49{,}500$ lb.

Reaction A for the live load is

$$\frac{(90)(20{,}000) + (100)(20{,}000) + (110)(20{,}000)}{120} \text{ (origin at } B)$$
$$= \frac{(20{,}000)(90 + 100 + 110)}{120} = \frac{(20{,}000)(9 + 10 + 11)}{12} = 50{,}000 \text{ lb.}$$

This may be more conveniently calculated by obtaining the last equation directly, which means that we take the panel as a unit of length. Thus, the B reaction for the live load is

$$(20{,}000)\frac{(1 + 2 + 3)}{12} = 10{,}000 \text{ lb.} \quad \text{(Origin at } A)$$

$$\text{Total reaction } A = 49{,}500 + 50{,}000 = 99{,}500 \text{ lb.}$$
$$\text{Total reaction } B = 49{,}500 + 10{,}000 = 59{,}500 \text{ lb.}$$

Illustrative Problem.—Find the horizontal and vertical components of the reactions of the three-hinged arch, Fig. 35, for loads F_1 and F_2 placed as shown; hinges at points a, b, and c.

From $\Sigma M = 0$ about the point a

$$F_1(20) + F_2(90) - V_2(120) = 0$$
$$V_2 = \frac{2F_1 + 9F_2}{12}$$

From $\Sigma V = 0$

$$F_1 + F_2 = V_1 + V_2$$

or

$$V_1 = \frac{10F_1 + 3F_2}{12}$$

From $\Sigma H = 0$

$$H_1 = H_2.$$

In order to obtain the value of H_1 and H_2, it is necessary to equate the sum of the moments about the center hinge b of all forces on *either* side of the hinge to zero. Considering the part of the arch to the left of the center hinge

$$V_1(60) - H_1(100) - F_1(40) = 0$$

or

$$H_1 = H_2 = \frac{3V_1 - 2F_1}{5} = \frac{2F_1 + 3F_2}{20}$$

It should be noted that four independent equations have been used to give four unknowns.

If tie rods should be placed as shown, the tension in these rods would be equal to $H_1 = H_2$, and only vertical pressure would be brought upon the supports.

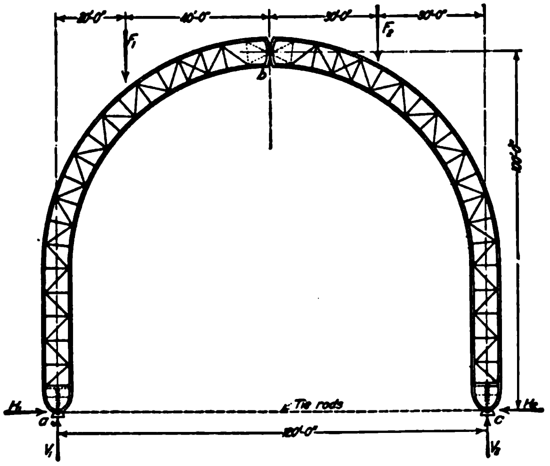

Fig. 35.

SHEARS AND MOMENTS

By George A. Hool

48. Shear.—Consider the forces acting on a beam to be resolved into horizontal and vertical components. Then the shear at any section is the algebraic sum of the vertical forces acting on either side of the section, and is the force which tends to cause the part of the beam on one side of the section to slide by the part on the other side. This tendency is opposed by the resistance of the material to transverse shearing.

When the resultant force acts upward on the left of the section, the shear is called *positive*, and when it acts downward on the same side of the section, it is called *negative*. Since $\Sigma V = 0$ when we consider the forces on both sides of the section, then the resultant of the forces on the right of the section must be equal and opposite in direction to the resultant of the forces on the left of the section. Thus, it makes no difference which side of the section we consider, the shear is *positive* when the resultant on the left is upward and when the resultant on the right is downward. Also the shear is *negative* when the resultant on the left is downward and when the resultant on the right is upward.

At the section *ab*, Fig. 36, the shear, since there are no loads between the section and the left support, equals the left reaction and is positive. This is true of any section between the left support and the section *cd*. The shear to the right of *cd* is negative and is equal to the right hand reaction.

49. Bending Moment.—The bending moment (or moment) at any section of a beam is the algebraic sum of the moments of the forces acting on either side of the section about an axis through the center of gravity of the section, and is the moment which measures the tendency of the outer forces to cause the portion of the beam lying on one side of the section to rotate about the section. This tendency to bend the beam is opposed by internal fiber stresses of tension and compression.

When the resultant moment on the left of the section is clockwise, the moment is called *positive*, and when it is counter-clockwise on the same side of the section, it is called *negative*. Since $\Sigma M = 0$ when we consider the forces on both sides of the section, then the resultant moment of the forces on the left of the section is equal and opposite to the resultant moment of the forces on the right of the section. Thus, it makes no difference which side of the section we consider, the moment is *positive* when the resultant moment of the forces on the left is clockwise and when the resultant moment of the forces on the right is counterclockwise. Also, the moment is *negative* when the resultant moment of the forces on the left is counterclockwise and when the resultant moment of the forces on the right is clockwise.

At the section *ab*, Fig. 36, the moment is $\frac{P}{2}(x)$. It increases uniformly from the left support where it is zero to the section *cd* where it is $\left(\frac{P}{2}\right)\left(\frac{L}{2}\right) = \frac{PL}{4}$.

Positive bending moment causes compression in the upper fibers of a beam, and tension in the lower fibers. The reverse is true for negative bending moment.

50. Shear and Moment Diagrams.—The variation in the shear or bending moment from section to section for fixed loads may be well represented by means of diagrams, called shear and moment diagrams. The diagrams are constructed by laying off a *base-line* equal to the length of the beam and marking off on this line the positions of the loads and the reactions. Positive shear and moment at given points should be represented above the base-line and negative shear or moment below this line. Points are plotted vertically above or below given points on the base-line, and the distance these plotted points are from the base-line should represent to some scale the magnitude of the shear or moment at these given points on the beam. The line joining the points plotted in this way is called the shear or moment line, depending upon whether a shear or moment diagram is being drawn.

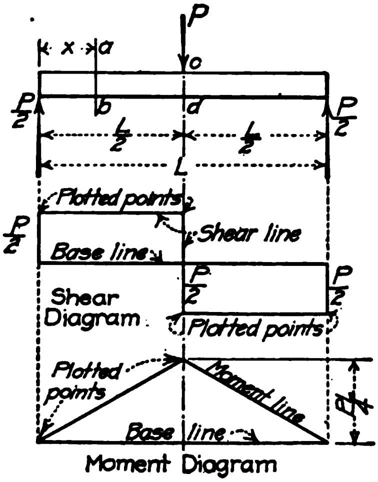

FIG. 36.

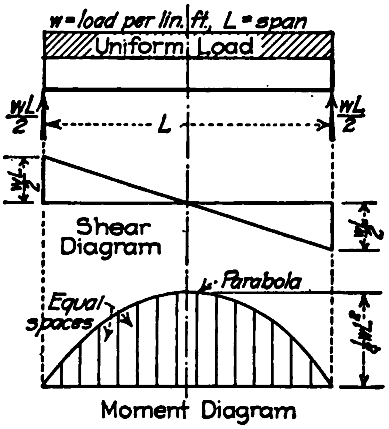

FIG. 37.

To illustrate, in Fig. 40, the ordinate *ab* represents the value of the shear at the point *b* of the beam and the ordinate *cd* represents the value of the moment at the point *d*.

In shear diagrams for uniform loading, ordinates need only be erected at the ends of the beam and at the points of support. If concentrated loads are also applied to the beam, ordinates must also be plotted at their points of application.

In moment diagrams for uniform loading, ordinates should be erected and points plotted at the reactions and every foot or two along the beam. If concentrated loads are also applied to the beam, ordinates must also be plotted at their points of application.

If the shear or moment lines are not completely determined by the above rules, additional points should be taken.

A *cantilever* beam is a beam having one end fixed and the other end free (see Art. 3, p. 2). The reaction at the fixed end is indeterminate, but the shear or bending moment at a given section may be easily found by considering the loads between the section and the free end.

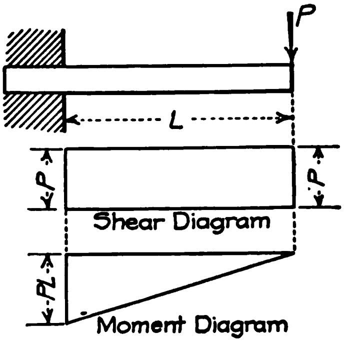

Fig. 38.

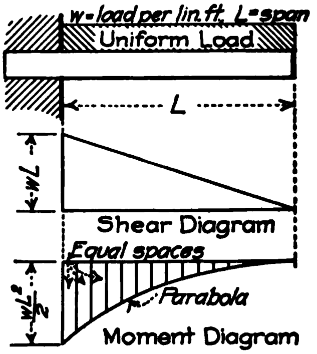

Fig. 39.

Shear and moment diagrams for both simple and cantilever beams with various loadings are shown in Figs. 36 to 41 inclusive. In all cases the weight of the beam is neglected.

51. Maximum Shear.—It is always desirable in proportioning beams to know the greatest or maximum value of the shear in a given case. The following rules apply:

1. In cantilevers fixed in a wall, the maximum shear occurs at the wall.
2. In simple beams, the maximum shear occurs at the section next to one of the supports.

These rules can be verified by examining the shear diagrams in Figs. 36 to 41 inclusive.

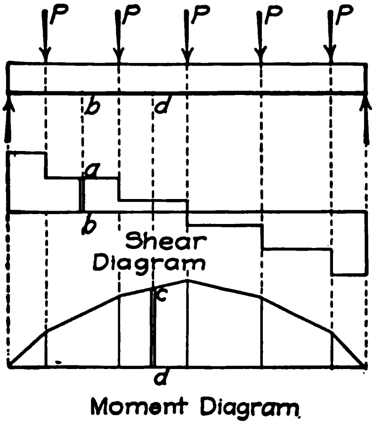

Fig. 40.

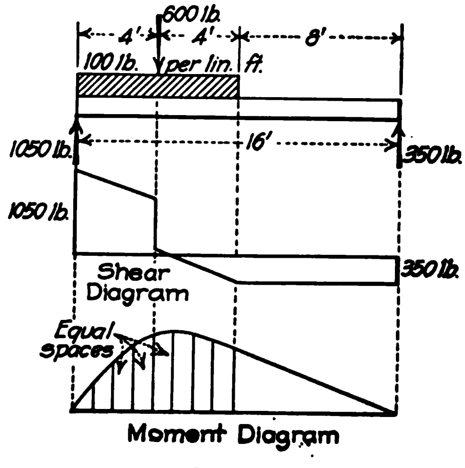

Fig. 41.

52. Maximum Moment.—By comparing the corresponding shear and moment diagrams in Figs. 36 to 41 inclusive, it will be found that the maximum moment occurs where the shear changes sign; that is, where the shear line crosses the base-line. This could also be shown algebraically.

By the help of this principle it is necessary to construct only the shear line and observe from it where the shear changes sign; then compute the bending moment for that section.

Illustrative Problem.—Construct shear and moment diagrams for a 20-ft. beam supported at the ends and loaded as shown in Fig. 42. Also, find the maximum shear and maximum moment, and the sections where they occur.

$$\text{Reaction } A = \frac{(5000)(5) + (4000)(10 + 15)}{20} + 8000$$
$$= 14{,}250 \text{ lb.}$$
$$\text{Reaction } B = 13{,}000 + 16{,}000 - 14{,}250$$
$$= 14{,}750 \text{ lb.}$$
$$\text{Shear at } A = 0$$
$$\text{Shear at section just to right of } A = 14{,}250$$
$$\text{Shear at } a \begin{cases} \text{to left} = 14{,}250 - (800)(5) = 10{,}250 \\ \text{to right} = 10{,}250 - 4000 = 6250 \end{cases}$$
$$\text{Shear at } b \begin{cases} \text{to left} = 6250 - (800)(5) = 2250 \\ \text{to right} = 2250 - 4000 = -1750 \end{cases}$$
$$\text{Shear at } c \begin{cases} \text{to left} = -1750 - (800)(5) = -5750 \\ \text{to right} = -5750 - 5000 = -10{,}750 \end{cases}$$
$$\text{Shear at section just to left of } B = -14{,}750$$
$$-10{,}750 - (800)(5) = -14{,}750 \text{ (check)}$$
$$\text{Shear at } B = 0.$$

We shall determine the moment at points A, a, b, c and B. Moments should also be found at sections 2 ft. apart on this beam to completely determine the moment curve.

$$\text{Moment at } A = 0.$$
$$\text{Moment at } a = (14{,}250)(5) - (800)(5)\left(\frac{5}{2}\right) = 61{,}250.$$
$$\text{Moment at } b = (14{,}250)(10) - (8000 + 4000)(5) = 82{,}500.$$
$$\text{Moment at } c = (14{,}750)(5) - (800)(5)\left(\frac{5}{2}\right) = 63{,}750.$$
$$\text{Moment at } B = 0.$$

The maximum shear = − 14,750 lb. at a section just to the left of the right support.

The shear changes sign at section b, consequently the moment is a maximum at that point = 82,500 ft.-lb.

In some cases the shear does not change sign at the point of application of a concentrated load and in such a case the position of the section, where the bending moment is a maximum, must be scaled or computed from the shear diagram to the nearest one-tenth of a foot.

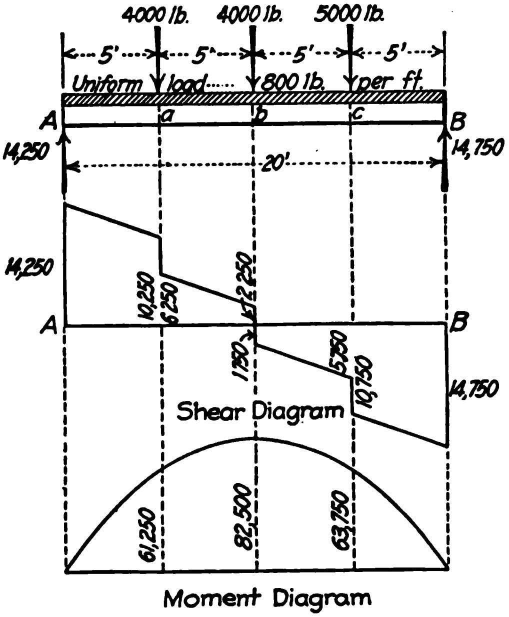

FIG. 42.

53. Moment Determined Graphically. The bending moment at any section of a beam due to concentrated loads may readily be determined by means of the force and equilibrium polygons. The method used is the same as that for finding the moment of a system of forces about a given point, described in Art. 45.

Let the bending moment M be required at any section of the beam shown in Fig. 43, such as the point k. Draw a vertical line through the section, cutting two sides of the equilibrium polygon, and let the ordinate intercepted between these sides be called r. The intersection of these sides produced gives the point of application of the resultant of the forces P_1 and R_1, the magnitude of which is represented by EB in the force polygon; that is, $R_1 - P_1 = AE - AB = EB$. It should be noticed that R_1 and P_1 act in opposite directions, and consequently the resultant of these two forces is their difference. Let this resultant be called R and its horizontal distance from k be called x. Then, $M = Rx$.

The triangle *OBE* is similar to the triangle which has a base r and an altitude x (sides respectively parallel) and, since *EB* is equal to *R*, we have $\frac{x}{H} = \frac{r}{R}$ or $Rx = Hr$.

Therefore the bending moment of the forces on the left of the section is

$$M = Hr$$

Since *H* is constant, the bending moment at any point in the span is proportional to the vertical ordinate of the equilibrium polygon at that point.

Suppose in the equilibrium polygon ¼ in. = 1 ft., and H = 2000 lb., then ¼ in. in the equilibrium polygon represents 2000 ft.-lb. That is, each inch on the vertical ordinate of the equilibrium polygon represents 2000 × 4 = 8000 ft.-lb. of bending moment. For instance, if a vertical ordinate at a given section scales 2.45 in., the bending moment of that section under the above conditions is 8000 × 2.45 = 19,600 ft.-lb.

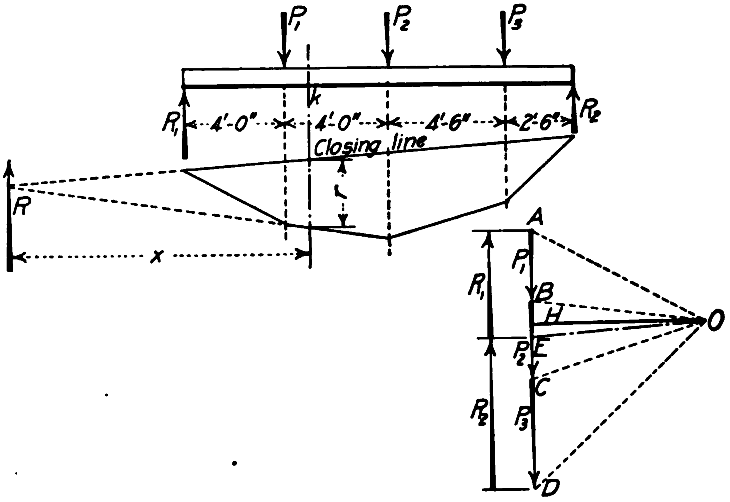

FIG. 43.

Inclined forces acting on beams should be resolved into horizontal and vertical components. The horizontal components cause no moment so that only the vertical components need be considered.

The graphical representation of bending moment at every point in the span can be applied to cases of uniform loading, but the construction is difficult and the algebraic method is much more simple. When a beam is subjected to both uniform and concentrated loads, it is sometimes convenient to find the bending moment for the concentrated loads by the graphical method, and the bending moment for the uniform load by the algebraic method. The algebraic sum of the two moments at any given section will give the correct moment at that section.

54. Effect of Floor Beams in Bridge Construction.—Since bridges are frequently used to connect factories and other buildings, the effect of using floor beams in bridge construction on the shears and moments in the supporting girders or trusses, will be considered in this book. The principles involved apply to a number of other special cases in building construction.

Floor beams are ordinarily riveted to the sides of girders. For clearness in presentation, however, the floor beams will be shown as resting upon the girders and the stringers upon the floor beams (Fig. 45). The shears and moments are identical for the two cases. Girders are usually placed parallel to each other and any load coming upon the planking or rails (or whatever the flooring may be) is transmitted by means of the stringers to the floor beams and thence to the girders, each girder receiving a proportional part. The loads given in each case will be the proportional part of the total load considered which is actually transmitted to the given girder.

Let F be the proportional part of an applied load which is transmitted to a given girder. As shown in Figs. 44 and 45 it will be transmitted at panel points 2 and 3. Panel point 3 will receive $F\frac{a}{p}$ and panel point 2 will receive $F\frac{(p-a)}{p}$ or, in other words, these panel points receive the reactions of a simple beam one panel in length, the stringers not being continuous over the floor beams.

In Fig. 45 considering only the applied load shown, the left hand reaction R_1 equals $F\frac{a+b}{L}$ and the right hand reaction R_2 equals $F\frac{L-(a+b)}{L}$, the same as if there were no floor beams. To prove this, it is only necessary to distribute a proportional part of the load F to the panel point 3 and also the proper amount to the panel point 2, and determine the reactions.

Load at 3 $= F\frac{a}{p}$

Load at 2 $= F\frac{(p-a)}{p}$

Left hand reaction $= \dfrac{F\frac{a}{p}(b+p) + F\frac{(p-a)}{p}b}{L}$

$= F\dfrac{(a+b)}{L}$ (same as without floor beams)

Right hand reaction $= F - \dfrac{F(a+b)}{L}$

$= F\dfrac{L-(a+b)}{L}$ (same as without floor beams)

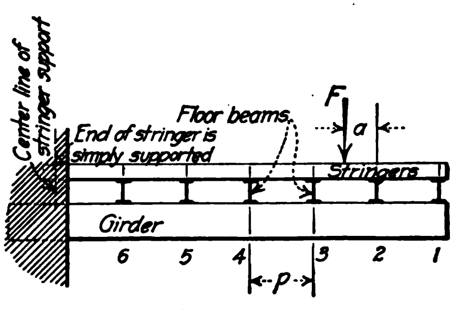
Center line of stringer support
Floor beams
End of stringer is simply supported
F
a
Stringers
Girder
6
5
4
3
2
1
p

FIG. 44.

In bridges carrying tracks, the stringers and rails are generally equally spaced about the center line between girders or trusses. If the bridge is single-track, a girder (or truss) thus receives one-half the total live load; that is, the weight coming upon one rail. The above discussion applies directly to such a case, the load F being any wheel load which may come upon one rail.

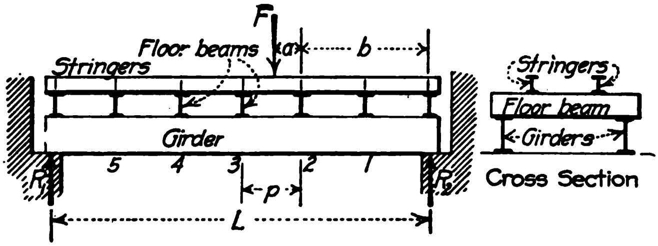

FIG. 45.

The following statements may be made pertaining to the effect of using floor beams. The first four statements refer to a girder supported at one or both of its ends. Statements 5 and 6 explain themselves. The load considered is the proportional part of the floor load (live and dead) which is transmitted to the girder in question. Statements 1 and 3 are of use in designing trusses.

(The only load applied to a girder between floor beams is its own weight. This is a uniform load and can be considered by itself, according to method previously stated. The following statements do not include this.)

1. Shear is constant between any two adjacent floor beams.
2. Moment varies uniformly between any two adjacent floor beams.
3. Moment at any floor beam is the same as it would be if there were no floor beams.
4. If no load is applied in a given panel, the moment at any point in that panel is the same as it would be if there were no floor beams.
5. If a load is applied in a given panel of a cantilever girder, the moment at any point in that panel is *greater* than it would be if the girder had no floor beams.
6. If a load is applied in a given panel of a girder supported at its two ends, the moment at any point in that panel is *less* than it would be if the girder had no floor beams.

55. A Single Concentrated Moving Load.—For a single concentrated moving load the maximum positive live shear on a simple beam at any section as A, Fig. 46, occurs when the load is just to the *right* of the section. This statement is readily verified by considering how the shear varies at the section as a load passes across the beam from the right to the left support. The left reaction, and consequently the positive shear, is increased as the load P is moved from the right support up to the section, being greatest when the load is just to the right of the section. Now move the load to the left of A. The shear is equal to the difference btween the left reaction and the load P and, since a load is always greater than either reaction (the load being equal to the sum of the reactions), the shear with the load to the left of A is negative, proving that the positive shear is a maximum with the load just to the right of the section. In practice the load is always placed *at* the section. This same line of reasoning might be followed through for negative shear, moving a load from the left abutment to the section and considering how the shear varies to the right of the section. The maximum negative shear is found to occur when the load is just to the *left* of the section. The value of the maximum positive shear for the load P is $P\frac{x}{L}\cdot$ and the maximum negative shear is $P\frac{L-x}{L}\cdot$

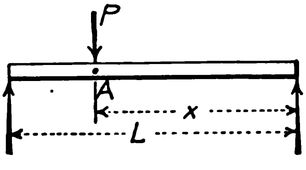

FIG. 46.

The maximum live moment at A occurs with the load at A, for a movement to either side reduces the opposite abutment reaction and consequently the moment. The maximum moment is $P\frac{x}{L}(L-x)$.

At any point on a cantilever beam, such as at A, Fig. 47, the shear is a maximum when the load is anywhere to the right of the point. When the load is on the left, the shear is zero. The moment is a maximum at the section when the load is at B and equals $P \times x$. When the load is to the left of A, the moment is zero.

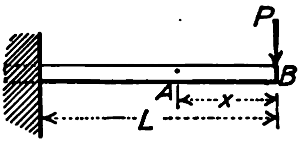

FIG. 47.

Now consider a bridge girder supported at both ends and carrying floor beams. Required the maximum live shear in any panel as EF, Fig. 48. As previously mentioned, the load shown is the proportional part of the total load in the panel which is transmitted to the girder in question. The shear is constant in EF for any loading. Let V denote this shear. Then, when the load P is in the panel EF, the shear

$$V = (\text{left reaction}) - (\text{load at } E) = P\left(\frac{a+b}{L} - \frac{a}{p}\right)$$

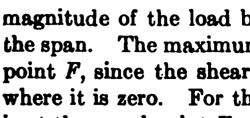

FIG. 48.

If the load is so placed that $\frac{a+b}{L} = \frac{a}{p}$ then the shear in $EF = 0$. This point is called the *neutral* point in the panel. A load to the right of this neutral point causes positive shear and to the left causes negative shear. Every panel has a neutral point which can be found by using the equation

$$\frac{a+b}{L} = \frac{a}{p} \text{ which gives } a = \frac{pb}{L-p}$$

It can be seen from the equation that the position of the neutral point does not depend upon the magnitude of the load but simply upon the length of panel and the position of the panel in the span. The maximum positive shear in panel EF will occur when the load P is at the panel point F, since the shear decreases as the load is moved from that point to the neutral point where it is zero. For the same reason the maximum negative shear will occur when the load is at the panel point E.

As stated in Art. 54 the moment at any point in a panel, as EF, for a load P in that panel is less than it would be if there were no floor beams, while with the load P outside of EF, the moment is the same as for a simple beam. At the floor beams the moment is the same as if there were no floor beams. In designing structures maximum moment only is usually desired, consequently it is sufficient to compute the moments only at the floor beams and to do it just as if there were no floor beams. Fig. 49 represents a cantilever girder supporting floor beams. Maximum shear in *EF* occurs when the load is anywhere to the right of *F* and equals *P*. Maximum moment at any panel point, as *E*, occurs with *P* at *B* and equals $P \times x$.

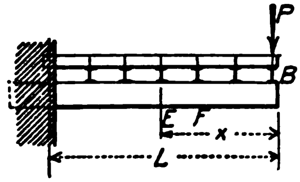

FIG. 49.

56. Moving Uniform Load.—For a moving uniform load the maximum positive live shear on a simple beam at any section as *A*, Fig. 50, occurs when the right hand section of the beam is loaded up to the point considered. This is seen to be true when we consider that adding a load to the right of *A* increases the left reaction and therefore the positive shear, while adding a load to the left of *A* increases the left reaction by an amount less than the load which is added, and hence decreases the positive shear. The maximum positive shear at *A* in Fig. 51 for a uniform load of *w* lb. per ft. $= \frac{1}{2} w \frac{x^2}{L}$.

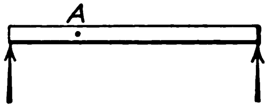

FIG. 50.

From similar reasoning to the above, the maximum negative shear at any section as *A*, Fig. 50, is found by loading to the left of the point. Maximum negative shear at *A*, Fig. 52, for a uniform load of *w* lb. per ft. $= \frac{1}{2} w \frac{(L - x)^2}{L}$ (considering the right hand reaction).

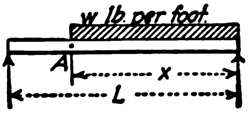

FIG. 51.

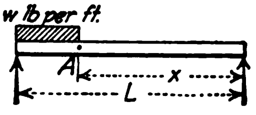

FIG. 52.

The maximum moment at any section as *A* occurs when the beam is *fully loaded*, for the addition of a load anywhere on the beam will add a positive moment at the section. For a load of *w* lb. per ft., the

$$\text{maximum } M = \frac{wL}{2}(L - x) - \frac{w(L - x)^2}{2} = \frac{w}{2}(L - x)(L - L + x) = \frac{w}{2}(x)(L - x)$$

If the section is at the center of the beam, the

$$\text{maximum } M = \frac{1}{8} wL^2$$

The above formulas for maximum moment give results in foot pounds, since *w* represents the load in pounds per foot and *L* the span of the beam in feet. To get inch pounds, multiply by 12 or insert for *w* in the formulas the load in pounds per inch and for *L* the span of the beam in inches.

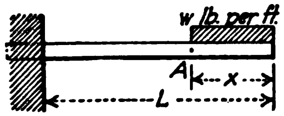

FIG. 53.

At any point on a cantilever beam, such as at *A*, Fig. 53, the maximum shear occurs for either a full load over the entire length, or for full load on the portion of the beam between the section and the free end, and equals wx. The moment is always negative and the maximum moment occurs for the same loading giving maximum shear; *i.e.*,

$$\text{maximum } M = \frac{wx^2}{2}.$$

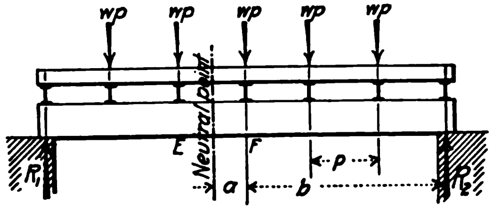

FIG. 54.

Now consider the case of a uniform load of *w* lb. per ft. on a bridge girder supported at its two ends and carrying floor beams. If the girder is fully loaded, the load on each floor beam is *wp*, except on the end floor beams which carry ½ *wp*. These end floor beam loads are usually supported directly on walls or abutments, and may be neglected in determining shear and moment. R_1, Fig. 54, then equals 2½ *wp* and R_2 equals 2½ *wp*. The maximum positive shear in any panel, such as *EF*, occurs when the load extends from the right to the neutral point in the pane (Fig. 55). Thus

$$\text{maximum } V = \frac{w(a+b)^2}{2L} - \frac{wa^2}{2p}.$$

In practice, the assumption is generally made that for maximum positive shear in a panel, all panel points up to and including the one at the right of the panel are fully loaded, and the ones to the left without any load. It is not possible to get this loading, but the assumption is convenient and a little on the safe side. It is obvious that in order for panel point *F*, Fig. 55, to have a full load, the load must extend to the panel point *E* and then *E* would have half a panel load. A load at *E* would reduce the positive shear in *EF*, so by omitting this we are on the safe side; that is, we are providing for a little greater positive shear than actually exists. For this loading the shear in *EF* is

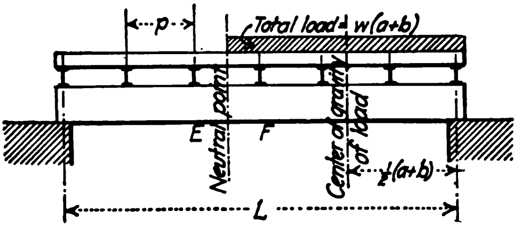

FIG. 55.

$$\frac{(1+2+3)}{6}(pw)$$

The maximum negative shear is likewise

$$\frac{(1+2)}{6}(pw).$$

The moments at the floor beams are the same as they would be if there were no floor beams. Maximum moment occurs as before for full loading and is positive at every point. The maximum moment at a floor beam distant x from the right abutment is (as in a simple beam)

$$\frac{wL}{2}(L-x) - \frac{w(L-x)^2}{2} = \frac{w}{2}(x)(L-x)$$

Fig. 56 represents a cantilever girder supporting floor beams. Maximum shear in *EF* occurs when *BE* is loaded and equals $w(b + \frac{1}{2}p)$. Maximum moment at *E* occurs for either full loading or for full load on *BE*, and equals (in this particular figure),

$$p(1+2+3)wp + 4p\left(\frac{1}{2}wp\right) = 8p^2w$$

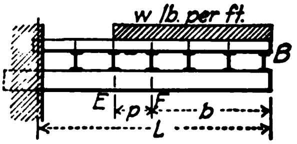

FIG. 56.

57. Influence Lines.—As a load moves over a beam, the shear and moment at a given section will vary. If the value of moment at any point A is plotted as an ordinate at the point where the load is applied, and this process repeated for each position of the load, the result is called an *influence diagram* for the moment at point A; and the curve generated by the extremities of all ordinates is called an *influence line* for the moment at point A. Similar lines may be drawn for shear and for deflections. In structures, influence lines may also be drawn for stress intensities at a given point. The curve gets its name because of the fact that for any chosen point, it gives the influence on a certain function at that point, for varied positions of the load.

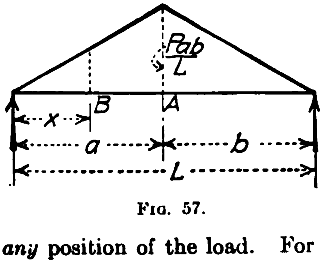

FIG. 57.

It should be noted that the influence line for moment—for a simple beam, for instance—differs from the moment diagram for that beam. The moment diagram gives the *moment at any point* for *one* position of the load; while the influence line for moment gives the moment at *one* point for *any* position of the load. For each point in the beam there may be drawn an influence line, but each influence line is descriptive of but one point. In Fig. 57 there is drawn an influence line for moment at A. The moment at A is $\frac{Pab}{L}$, and that is the value of the ordinate at A The ordinate at B is $\frac{Pxb}{L}$ and is the moment at A when the load P is at B.

Suppose the beam to have a load of 1 lb. moving across it. The ordinate at A is then $\frac{ab}{L}$. Usually influence lines are drawn for unit loads. The ordinate at B is then the moment at A when a unit load is placed at B. If the load at B is not unity, then the moment at A will be equal to the load times the ordinate at B for the 1-lb. load.

If the beam is loaded with a uniform load, the moment at A is equal to the load per foot times the area of the influence diagram for the moment at A. In Fig. 57 this is $\left(w \cdot \frac{ab}{L} \cdot L \cdot \frac{1}{2}\right)$ or $\frac{w}{2} \cdot ab$, which is readily recognized as the moment at A for a uniform load. For a partial uniform loading, the load per foot multiplied by the area of the influence diagram for the loaded portion will give the moment at A.

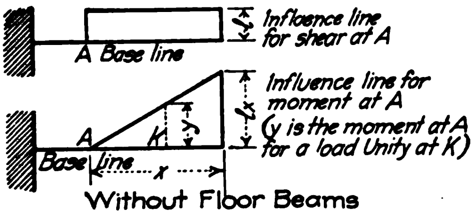
Influence line for shear at A
A Base line
Influence line for moment at A
(y is the moment at A for a load Unity at K)
A
K
y
Base line
x
Without Floor Beams

FIG. 58.

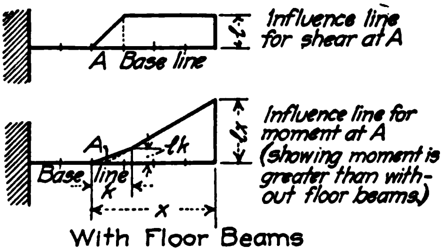
Influence line for shear at A
A Base line
Influence line for moment at A (showing moment is greater than without floor beams)
A
k
Base line
K
x
With Floor Beams

FIG. 59.

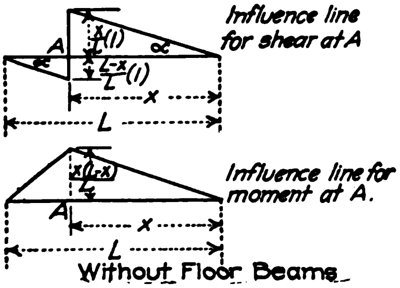

FIG. 60.

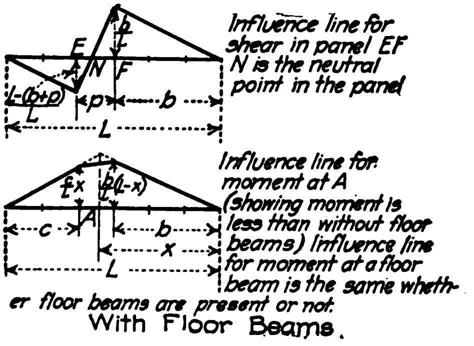
Influence line for shear in panel EF N is the neutral point in the panel
E
F
N
$\frac{b}{L}$
$\frac{L-(b+p)}{L}$
p
b
L
Influence line for moment at A (showing moment is less than without floor beams) Influence line for moment at a floor beam is the same whether floor beams are present or not.
$\frac{c}{L}x$
$\frac{b}{L}(L-x)$
A
c
b
x
L
With Floor Beams.

FIG. 61.

Influence lines for shear and moment on cantilever and simple beams and girders are shown in Figs. 58, 59, 60, and 61.

The influence line shows three things:

1. The effect on the function under consideration for a single load at any point on the structure.
2. Where a single load must be placed in order to produce the maximum or minimum effect.
3. With a uniform live load, the part (or parts) of the structure which must be loaded in order to produce the maximum positive or the maximum negative effect.

Influence lines are not generally used for determining values of functions for simple beams, girders, or trusses, because the algebraic methods are more simple, but the use of influence lines leads to a better understanding of the effect of moving loads and in many complicated structures the influence line affords the simplest and best solution of a problem. It is freely used in methods of analysis; that is, finding the position of loads to give maximum shear or moment or whatever the function may be which is under consideration.

58. Concentrated Load Systems.

58a. Maximum Shear Without Floor Beams.—In order to determine the value of the maximum shear at a given section due to a series of concentrated loads in a load system, it is first necessary to find just how the loads must be placed in order to give this maximum shear.

Suppose the maximum shear is required at any section on a structure without floor beams, such as Section *A*, Fig. 60. Place some load just to the right of *A*, which for convenience we shall call P_1. Let G_1 then represent the sum of the loads to the left of, and including P_1, and G_2 the sum of the loads to the right of P_1. Also, let *G* equal the total load on the structure when P_1 is at *A*, and *b* the distance between P_1 and the next load to the right which we shall call P_2.

Now suppose the system of loads be moved a distance *b* to the left thus bringing P_2 to *A*. The effect upon the positive shear is first to decrease it suddenly by an amount P_1, after which it is gradually increased. The increase due to G_2 may be expressed by

$$G_2\, b \tan \alpha \text{ (see Fig. 60)}$$

and the increase due to G_1 (decrease in negative shear) may likewise be expressed by

$$G_1 b \tan \alpha$$

The net change in shear due to the entire movement is

$$G_1 b \tan \alpha + G_2 b \tan \alpha - P_1$$

or

$$G\frac{b}{L} - P_1$$

If this expression is positive, then the second position gives the greater shear and, if negative, the first position. For equal shears we have, therefore

$$\frac{G}{L} = \frac{P_1}{b}$$

The slight increase in shear due to additional loads that may come upon the structure from the right has been neglected. The above expression means that to increase the shear we move to the left provided the average load per foot on the whole span is greater than the load at the section divided by the distance between this load and the next load to the right.

Since the slight increase in shear due to additional loads that may come upon the structure from the right has been neglected in deriving the above criterion for maximum shear, the effect of such loads must be investigated. If G' be the total load on the structure when P_2 is at *A*, then the increase in shear when moving up P_2 will be somewhere between $G\frac{b}{L} - P_1$ and $G'\frac{b}{L} - P_1$. It may be possible for the first expression to be negative and the latter positive. Such a circumstance would result in causing $\frac{G}{L}$ to be less than $\frac{P_1}{b}$ for two succeeding loads and both positions would have to be tried.

58b. Maximum Moment Without Floor Beams.—In order to determine maximum live moment at any section of a structure for a system of concentrated loads, it is first necessary to find the position of the loads to give this moment.

Consider the determination of maximum moment at a section of a simple beam, such as *A*, Fig. 62.

Let P_L = resultant of all loads to the left of *A*.
x_L = its distance from the section.
P = total load on span.
x_R = its distance from right support.
x = distance of section from right support.

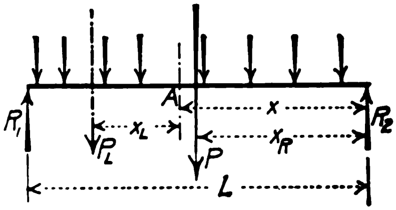

FIG. 62.

Then the moment at *A* is

$$M = P\frac{x_R}{L}(L - x) - P_L x_L$$

Let the system of loads be moved a small distance Δ to the left, the distance being so small that the distribution of the loads will not be changed. Then the new moment is

$$M = P\frac{x_R + \Delta}{L}(L - x) - P_L(x_L + \Delta)$$
$$= \left[P\frac{x_R}{L}(L - x) - P_L x_L\right] + P\frac{\Delta}{L}(L - x) - P_L\Delta$$

The moment has increased by so doing provided

$$P\frac{\Delta}{L}(L - x) > P_L\Delta$$

or

$$\frac{P}{L} > \frac{P_L}{L - x}$$

In other words, the moment at a given section will keep increasing by moving the loads to the left until the sign of inequality is changed. That is, the maximum moment is obtained when with a load to the right of the section

$$\frac{P}{L} > \frac{P_L}{L - x}$$

and with the same load moved to the left of the section

$$\frac{P}{L} < \frac{P_L}{L - x}$$

During this slight movement $\frac{P}{L}$ passes the value $\frac{P_L}{L - x}$.

Thus, for maximum moment

$$\frac{P}{L} = \frac{P_L}{L - x}$$

It follows from this that the moment will be increased by moving the loads to the left provided the average load per foot on the whole span is greater than the average load on the left of the section. Thus, the maximum moment at any section, as A, will occur when some load lies at that point, and that load must be such that when it lies just to the right of the section, the average load on the whole span will be greater than the average on the left, while if it lies to the left of the section, the average load on the left will be the greater.

It sometimes happens that with a load just to the left of the section, the average load on the whole span is just equal to the average load on the left of the section. This means that the moment which has been increasing by moving the loads to the left, will now remain the same until some load either comes on the span, passes the section, or goes off the span. If a load comes on the span, the moment is increased and the loads should be kept moving to the left. If a load should go off the span before a load reaches the section, then the average load on the whole span is still greater than the average load on the left, and the moment will keep increasing until some load reaches the section. Thus it follows from the above, that when the average load on the whole span is equal to the average load on the left of the section, the resulting moment is not necessarily a maximum. It is a maximum only when no load comes on or goes off the span in the process of moving up the next load to the section. In such a case the same maximum moment is obtained for the two loads in succession.

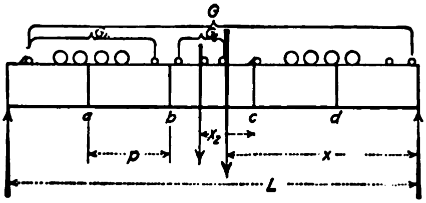

Fig. 63.

58c. Maximum Shear With Floor Beams.—The position of loads to give maximum shear in any given panel of a girder or truss must first be determined before the value of this maximum shear can be found. Let Fig. 63 represent a system of concentrated loads on a bridge having floor beams. Suppose the maximum shear from the live load is required in panel bc. Let G_1 be the total load on the bridge to the left of the panel in question, G_2 the sum of the loads in the panel bc, and G the total load on the span. Also let x equal the distance from G to the right abutment, and x_2 the distance from G_2 to the point c.

Then the shear

$$V = \frac{Gx}{L} - \frac{G_2x_2}{p} - G_1$$

Let the system of loads be moved a distance Δ to the left; then the new shear is

$$V' = \frac{G(x + \Delta)}{L} - \frac{G_2(x_2 + \Delta)}{p} - G_1$$

The shear has been increased by the operation provided

$$\frac{G(x + \Delta)}{L} - \frac{G_2(x_2 + \Delta)}{p} - G_1 > \frac{Gx}{L} - \frac{G_2x_2}{p} - G_1$$

or

$$\frac{G}{L} > \frac{G_2}{p}$$

The above expression means that to increase the shear we move to the left if the average load per foot on the whole span is greater than the average load in the panel in question, and vice versa. Hence, we find that the maximum shear in the panel will occur when some load is at the panel point at the right of the panel, and that load must be such that when it lies just to the right of the panel point, the average load on the whole span will be greater than the average in the panel, while if it lies to the left of the panel point, the average load in the panel will be the greater. More than one maximum may be found under each set of heavy loads.

58*d*. Maximum Moment With Floor Beams.—As shown in Fig. 61, the moment between floor beams is always less than if there were no floor beams. Hence, it is only necessary to compute the maximum moments at the floor beams and to do it as if there were no floor beams.

58*e*. Absolute Maximum Moment.—When a series of concentrated loads passes over a structure without floor beams, the bending moment under a given wheel load will vary and will be a maximum when the wheel is near the center of the beam. There will, consequently, be a maximum moment considering each wheel load and the greatest of these moments is called the *absolute maximum moment.*

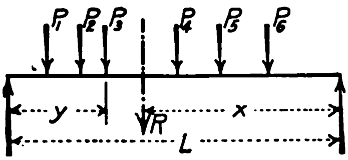

FIG. 64.

Suppose the maximum moment is required at the load P_3, Fig. 64, as the load system passes over the span. Let R equal the resultant of all the loads on the span when P_3 is somewhere near the center of the beam. The moment at P_3 is

$$M_3 = R\frac{xy}{L} - \text{(moments of loads } P_1 \text{ and } P_2\text{)}$$

In order for M_3 to be a maximum, xy must be a maximum; that is, x must equal y. In other words, the center of the beam must be half way between P_3 and R. Thus, the method of determining the maximum moment under any one of the concentrated loads is to place the loads so that the load in question is near the center of the beam and then find the line of action of the resultant of the loads which are on the span. (It is more convenient to move a line representing the length of the beam than it is to move the loads.) The beam should then be placed so that its center will come midway between R and the load in question, and the maximum moment at the load computed. The maximum moment should next be found at each of the heavy loads in the same manner as above. The greatest moment will be the absolute maximum.

SIMPLE AND CANTILEVER BEAMS

By Walter W. Clifford

59. General Method of Design.—The maximum bending moment and maximum shear in a beam should first be computed as explained in the preceding chapter. Then the problem in the design of beams is to select one of such section that the maximum unit stresses induced in the beam will be satisfactory and will not exceed the allowable working stresses. Formulas for unit stresses are used, one in terms of maximum moment and the other in terms of maximum shear.

60. Bending.—When a beam supported at each end deflects under a load, the upper fibers shorten and the lower fibers elongate. In a simple beam, therefore, the upper fibers are in compression and the lower fibers in tension. With a cantilever beam the reverse is true.

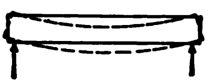

Fig. 65.

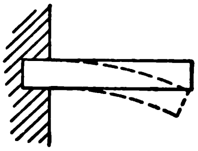

Fig. 66.

Figs. 65 and 66 show, much exaggerated, the effect of bending on a simple beam and cantilever beam respectively. The full lines represent the position of the beam before bending and the dash lines after bending. In each beam there is a horizontal plane or section, perpendicular to the elevations shown, in which the fibers neither elongate nor shorten. This is called the *neutral plane*. The line of intersection of the neutral plane with a vertical cross section is called the *neutral axis* of the section.

61. Fundamental Bending Formula.

61*a*. Assumptions.—In order to get an expression for fiber stress in terms of bending moment, certain assumptions must be made.

1. It is assumed that a plane cross section before bending remains a plane after bending—that is, the two planes shown in Fig. 67 by the full heavy lines remain planes when they assume their dotted positions after bending. Above the neutral axis the planes move toward each other an amount varying uniformly from the neutral axis to a maximum at the top of the sections. Below the neutral axis they move away from each other in a similar manner. This assumption is shown by tests to be true within the precision of ordinary structural work.

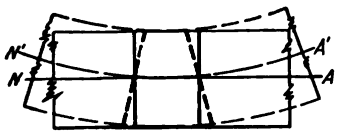

Fig. 67.

2. It is assumed that stress varies as deformation. This is also borne out by experiments within working limits (see Art. 19).

From the first assumption it follows that deformation varies from the neutral axis to a maximum at the outside fiber, and from the second assumption it follows that the stress varies in the same way. There is, therefore, uniformly varying compression on one side of the neutral axis and uniformly varying tension on the other. The moment of this compression and tension constitutes the resisting moment.

In standard treatises on mechanics it is demonstrated from the above assumptions that the neutral axis in homogeneous beams passes through the center of gravity of the section.

61*b*. Derivation of Formula.—The "unit" stress diagram for any section of a beam is given in Fig. 68, and shows the unit stress to vary uniformly from the neutral axis. If the fiber stress at the outside fiber, distant c from the neutral axis, be denoted by f, then the fiber stress at any point distant x from the neutral axis is $\frac{x}{c}f$; and the moment about the neutral axis of the stress on an infinitely small area, distant x from the neutral axis, is $a \cdot \frac{x}{c} \cdot f \cdot x$, or $M_x = \frac{afx^2}{c}$; and the moment for the whole section is $M = \frac{f}{c}\Sigma ax^2$.

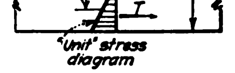

Fig. 68.

The term Σ represents summation and the quantity Σax^2 means the sum of the products obtained by multiplying each infinitesimal area by the square of its distance from the neutral axis. In rectangular sections, $c = \frac{d}{2}$.

61*c*. Moment of Inertia.—The quantity Σax^2 is called the moment of inertia of the section about the neutral axis, and is denoted by I. The general term *moment of inertia*, however, refers to *any* axis so the moment of inertia of a section with respect to an axis may be defined as the sum of the products obtained by multiplying each infinitesimal area of the section by the square of its distance from the given axis. Values of I for various sections are given in "Carnegie" and other handbooks. Substituting I in the formula of the preceding article we have

$$M = \frac{fI}{c}$$

which is the general formula for resisting moment in beams. $\frac{I}{c}$ is called the *section modulus.*

61*d*. Design of Wooden Beams for Moment.—From the standpoint of moment computation the wooden beam is simplest. It is homogeneous and of rectangular section. The "total" stress diagram is therefore similar in shape to the "unit" stress diagram (compare Figs. 68 and 69). I for a rectangle is $\frac{bd^3}{12}$. Substituting this in the general formula,

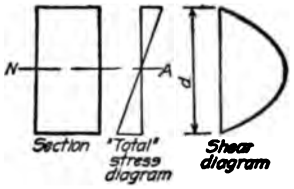

FIG. 69.—Wood beam.

$$M = \frac{f\frac{bd^3}{12}}{d/2} = \frac{fbd^2}{6}, \text{ or } bd^2 = \frac{6M}{f}$$

The above formula may also be derived as follows: The total compression equals the total tension (Fig. 68) or $C = T = b \cdot \frac{d}{2} \cdot \frac{f}{2}$, knowing $\frac{f}{2}$ to be the average stress. The moment arm is the distance between the centers of gravity of the two triangles, or $\frac{2d}{3}$. Then $M = \frac{bdf}{4} \cdot \frac{2d}{3} = \frac{fbd^2}{6}$.

To design a wooden beam for moment the only procedure necessary is to substitute, in the formula $bd^2 = \frac{6M}{f}$, the allowable fiber stress and the maximum bending moment (since the resisting moment must equal the external bending moment) and choose values of b and d which will make bd^2 equal to or greater than $\frac{6M}{f}$. Some handbooks give the allowable bending moments and section moduli for dressed timber (see Sect. 2, Art. 2*a*).

From the foregoing, it is evident that the strength of homogeneous rectangular beams in moment varies as the square of the depth and as the first power of the breadth.

61*e*. Design of Steel Beams for Moment.—Steel beams are most commonly I or channel shape. The bulk of the metal is, for economy, at the top and bottom where it will have higher fiber stresses. The "total" stress diagram for these sections, instead of being the same shape as the "unit" stress diagram, is as shown in Fig. 70. Handbooks giving the properties of standard steel sections are published by steel companies and are universally used (see chapter on "Steel Shapes and Properties of Sections" in Sect. 2).

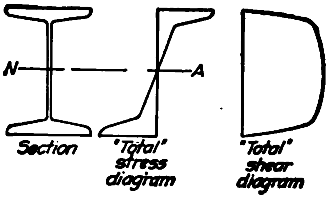

FIG. 70.—Steel beam.

61*f*. Design of Cast-iron Beams for Moment. Cast-iron beams, as such, are almost never seen. In the common uses of cast iron, such as bases, covers, etc., various parts, and often the whole must be designed as a beam. This is done by the general formula $f = \frac{Mc}{I}$. Such sections are usually irregular in shape and the center of gravity and the moment of inertia must be computed.

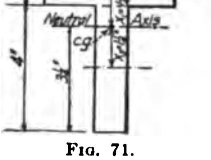

FIG. 71.

Computations for locating the center of gravity are explained in Art. 44.

61*g*. Moment of Inertia of Compound Sections.—The following rule, developed in treatises on mechanics, applies to any area: The moment of inertia of an area with respect to any axis equals the moment of inertia with respect to a parallel axis through the center of gravity, plus the product obtained by multiplying the given area by the square of the distance between the two parallel axes. Expressed by formula $I_1 = I + Ax^2$. Finding I for a built-up section is, therefore, a question of dividing the section into simple geometrical areas, or areas for which properties can be obtained from a handbook, and then finding the moment of inertia of each of these areas about the neutral axis of the entire section by applying the above rule. A summation of the moment of inertias so found gives the moment of inertia of the entire section. For example, to find the moment of inertia of the cast-iron section shown in Fig. 71, divide the section into two rectangles as shown.

$$
\begin{aligned}
&I \text{ for the upper rectangle is } \frac{bd^3}{12} = \frac{(4)(1)(1)(1)}{12} &&= 0.33\\
&I \text{ for the lower rectangle is } \frac{(1)(4)(4)(4)}{12} &&= 5.33\\
&Ax_1^2 \text{ for the upper rectangle is } (4)(1.25)^2 &&= 6.25\\
&Ax_2^2 \text{ for the lower rectangle is } (4)(1.25)^2 &&= 6.25\\
&\qquad\qquad I \text{ of entire section} &&= 18.16 \text{ in.}^4
\end{aligned}
$$

62. Bending Formulas for Concrete.—In concrete beams the general principles are the same as for wooden beams but, on account of the combination of materials, the neutral axis is not at the center of gravity of the concrete section. The assumption will be made in deriving formulas for concrete beams that the concrete takes no tension. This assumption is not strictly true, but the error is slight and on the safe side. In the early stages of loading all the concrete on the tension side takes tension but as the loading increases, the concrete cracks. The cracks start at the bottom of beam and extend toward the neutral axis.

Referring to Fig. 72, let Δs and Δc represent the deformations of the steel and concrete respectively, as shown.

Then $\dfrac{\Delta s}{\Delta c} = \dfrac{d - kd}{kd}$. But $\Delta c = \dfrac{f_c}{E_c}$ and $\Delta s = \dfrac{f_s}{E_s}$, or $\dfrac{\Delta s}{\Delta c} = \dfrac{f_s}{nf_c}$.

Therefore $\dfrac{\Delta s}{\Delta c} = \dfrac{d - kd}{kd} = \dfrac{f_s}{nf_c} = \dfrac{1-k}{k}$ or

$$k = \frac{nf_c}{f_s + nf_c}$$

If we let $\dfrac{f_s}{f_c} = m$, then $\dfrac{1-k}{k} = \dfrac{m}{n}$ and

$$k = \frac{n}{m+n}.$$

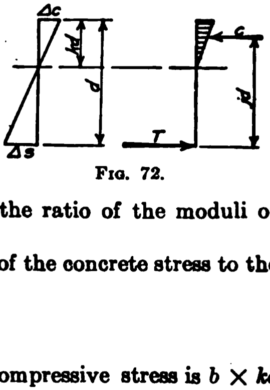

FIG. 72.

The depth of the neutral axis is therefore dependent only upon the ratio of the moduli of elasticity and the fiber stresses of the steel and concrete.

The arm of the resisting moment is from the center of gravity of the concrete stress to the center of the steel, or

$$jd = d - \frac{kd}{3}$$

The ratio of steel area to total area is called p. The total compressive stress is $b \times kd \times \frac{f_c}{2}$ and the total tension is $pbdf_s$. The allowable resisting moment is therefore $bkd \cdot \frac{f_c}{2} \cdot jd$ or $pbdf_s jd$—that is,

$$M = \tfrac{1}{2} f_c kjbd^2, \text{ or } f_s pjbd^2$$

according as to whether the steel or concrete is the weaker. It is obvious that good design will make the two moments as nearly equal as possible, or $\frac{1}{2} f_c kjbd^2 = f_s pjbd^2$, whence

$$p = \frac{kf_c}{2f_s}$$

Values of f_c, f_s and n are assumed for concrete design and from these k, j, and p can be computed by using the above formulas. Then by placing the term for internal moment equal to the actual external bending moment, values for b and d can be selected to satisfy the equation. The area of steel is equal to pbd and suitable rods can be selected to give the required area. The coefficient of bd^2 in the term for the resisting moment (*i.e.*, $pf_s j$ and $\frac{1}{2} f_c kj$) is a constant for any selected values of f_s, f_c and n, and is usually denoted by K. Table giving the value of K as well as values of k for various stresses is shown on p. 150.

For investigating concrete beams already designed, the formulas may be put in the following form:

$$p = \frac{A_s}{bd}$$

$$k = \sqrt{2pn + (pn)^2} - pn$$

$$j = 1 - \frac{k}{3}$$

$$f_c = \frac{M}{\frac{1}{2}kjbd^2}$$

$$f_s = \frac{M}{pjbd^2} \text{ or } f_s = \frac{f_c k}{2f_s}$$

It is interesting to note that for $f_s = 16{,}000$, $f_c = 650$ and $n = 15$, and for other values giving the same k, the formula $f = \frac{6M}{bd^2}$ as used for wooden beams is true within lessthan 1%, and gives an easily remembered method for the design of simple concrete beams knowing $p = 0.0077$. But is must be remembered that it is merely a mathematical coincidence that the simple beam formula applies since the error increases greatly with other unit stresses.

63. Shear.

63*a*. Vertical Shear.—Consider a beam with a single concentrated load at the center and cut away the left-hand third of the beam, as shown in Fig. 73. By the principles of statics the internal forces acting on the section cut must balance the external forces acting on the left-hand portion of the beam. It will be seen that C and T, the resultants of the compressive and tensile stresses respectively acting on the section, do not satisfy the conditions of equilibrium and there is required in addition the vertical shear V. In other words, each vertical section must resist the external vertical shear at that section.

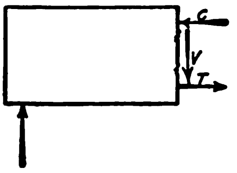

Fig. 73.

63*b*. Horizontal Shear.—It is quite evident, and easily demonstrated by experiment, that if a beam be made of boards laid flat one on another, and then loaded, it will assume the condition shown in Fig. 74. This demonstrates that a horizontal shear or force acts along the fibers of a solid beam at different depths tending to cause movement on horizontal planes. This longitudinal shearing stress is due to the change of horizontal fiber stresses along a beam. For example, if AC and BD in Fig. 75 are the "unit" stress diagrams at two sections, a unit distance apart, the cross-hatched area evidently represents a difference in stress to be resisted by the beam in horizontal shear. It is evident that a force is induced at every longitudinal layer tending to slide it past the next section above it; and this sliding or shearing force, which increases at every layer, attains its maximum intensity at the neutral axis.

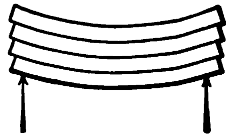

Fig. 74.

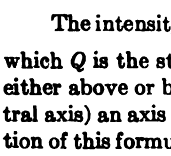

Fig. 75.

63*c*. Shear Variation in Wooden Beams.—The intensity of shear along a vertical cross-section for a rectangular beam varies as the ordinates to a parabola, as shown graphically in Fig. 69. The maximum intensity is $\frac{3}{2}$ times the average.

The intensity of shear at any point in a beam is given by the general formula $v = \frac{VQ}{bI}$, in which Q is the statical moment about the neutral axis of that portion of the cross-section lying either above or below (depending upon whether the point in question is above or below the neutral axis) an axis drawn through the point in question parallel to the neutral axis. The derivation of this formula is given in standard text books on mechanics. It can be easily demonstrated that the values for v so computed will fall on a parabola for a rectangular section.

63*d*. Shear Variation in Steel Beams.—In a steel I-beam most of the tensile and compressive stresses are taken by the flanges. From consideration of the "total" stress distribution (Fig. 70) and from use of the formula $v = \frac{VQ}{bI}$, it will be seen that there is very little difference between the intensity of shear at the inner edge of flange and at the neutral axis. The "total" shear diagram is shown in Fig. 70. In steel beams the shear is assumed as uniformly distributed over the web. This assumption greatly simplifies computations, and is not seriously in error.

63e. Shear Variation in Concrete Beams.—The variation of shear in a concrete beam is shown in Fig. 76, assuming the concrete to take no tension. The upper half of the diagram is a parabola as for the homogeneous rectangular beam. The shear from the neutral axis to the steel is constant since no tension exists between these points. The maximum intensity of shear is $v = \frac{V}{bjd}$. The shear diagram, assuming the concrete to take tension for a short distance below the neutral axis, is shown in Fig. 77. The break in the curve is at the top of the tension cracks in the concrete.

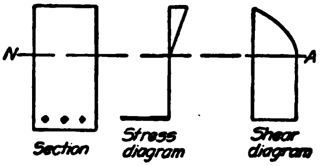

Fig. 76.—Concrete beam.

63f. Relation Between Vertical and Horizontal Shear.—At any point in a beam the intensity of the horizontal shear is equal to the intensity of the vertical shear. This may be seen by considering an infinitesimal cube from any part of a beam. The moment of the vertical shears must equal the moment of the horizontal shears for equilibrium. Therefore the intensity of the shears must be equal and the general formula and diagrams previously given are true for vertical as well as horizontal shear.

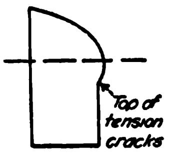

Fig. 77.

63g. Bond in Concrete Beams.—Bond in beam rods is a special case of horizontal shear, being the horizontal shear on the surface of the rods. As noted in a previous paragraph the maximum intensity of shear in a concrete beam is $v = \frac{V}{bjd}$. This is the value from the neutral axis to the steel, and the total bond for a unit of length must evidently be equal to this value multiplied by b. The unit bond is therefore $\frac{V}{jd}$ divided by the entire surface of all the rods per unit of length, or

$$u = \frac{V}{\Sigma ojd}$$

(See Notation in Appendix *A*.)

63h. Minimum Bar Spacing in Concrete Beams.—Spacing of reinforcing bars must evidently be such that the concrete on a horizontal section through the center of the rods can take, in shear, the amount of the bond on the lower half of the bars. Practical considerations as noted under "Reinforced Concrete Beams and Slabs," and "Concrete Detailing" in Sect. 2 call for a wider spacing than determined by theory.

64. Diagonal Compression and Tension.—It is proved in treatises on mechanics that if f represents the intensity of horizontal fiber stress and v the intensity of vertical or horizontal shearing stress at any point in a beam, the intensity of the inclined stress will be given by the formula

$$t = \frac{f}{2} \pm \sqrt{\tfrac{1}{4} f^2 + v^2}$$

and the direction of this stress by the formula

$$\tan 2K = \frac{2v}{f}$$

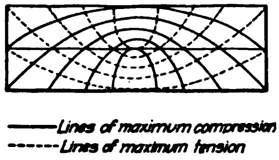
——Lines of maximum compression
------Lines of maximum tension

Fig. 78.

where K is the angle of the stress with the horizontal. These two formulas are general and apply when f is either tension or compression. The formula for K shows that two values of K, differing by 90 deg., will satisfy the equation; that is, at any point maximum compressive stress and maximum tensile stress make an angle of 90 deg. with each other. Fig. 78 shows approximately the directions of the maximum stresses for a uniformly loaded beam.

The following statements may be verified by using the above formulas:

(a) At the end of a simply supported beam where the shear is a maximum and the bending moment a minimum, the stresses lie practically at 45 deg. to the horizontal throughout the entire depth of beam.

(b) At the section of maximum moment, the shear is zero and the stresses are horizontal.

The fundamental bending formula—in other words, the common theory of flexure—is seen to give the unit fiber stress correctly at the important section of maximum moment and also for the extreme fibers in other sections, since at these points the shear is zero. Where the shear is not zero, an inclined stress is the result and the flexure formula gives only the horizontal component of this stress—namely, the *fiber stress.*

In homogeneous beams of rectangular section, the diagonal stresses are not of importance, but in steel beams, especially in the case of built-up plate girders, the web is thin, and although of sufficient strength to resist the diagonal tension near the end of beam (acting at approximately 45 deg. with the neutral axis) is often not stiff enough to take the diagonal compression without buckling. For this reason stiffener angles are used in plate girders (see Sect. 2, Art. 52).

In concrete beams, on the other hand, the material is amply strong in compression but weak in tension. Stirrups are therefore added to assist in taking this tension, and main steel is bent up near the supports. From Fig. 78 it is evident that shear reinforcement in concrete beams would be at various inclinations, from purely theoretical considerations, but this is not practical. The design of web reinforcement is discussed in Sect. 2, Art. 34. It should be noted in this connection that part of the horizontal reinforcement should always continue through to the end of the beam in order to avoid the occurrence of high tensile stresses near the end of beam where shear is a maximum. The steel stress must be kept low enough so that large cracks will not develop in the concrete.

65. Flange Buckling.—The top flange of a steel beam is in effect a column although it is stronger than a column standing alone because of its connection with the web. It is therefore necessary that its ratio of length to breadth be limited in a similar way to that of a column, if full working stress is to be used in design. It is usually specified that a beam must be supported laterally at distances not exceeding 20 times the flange width or the allowable fiber stress must be reduced. The reduction is usually specified to be in accordance with a modification of the formula for columns. Light ties or trussing may be used to hold the top flange, or the flange may be stiffened with a plate or a channel.

66. Deflection.—The general formula for deflection is derived in treatises on mechanics. From the general formula are developed the following formulas for homogeneous beams:

Simple beam uniformly loaded—Max. deflection $\frac{5}{384}\frac{Wl^3}{EI}$ at the center.

Simple beam with concentrated load in the center—$\frac{1}{48}\frac{Wl^3}{EI}$ at the center.

Cantilever with uniform load—$\frac{1}{8}\frac{Wl^3}{EI}$ at the end.

Cantilever with load at the end—$\frac{1}{3}\frac{Wl^3}{EI}$ at the end.

All terms must be in inches to give deflection in inches.

Formulas for other cases may be found in the steel manufacturers' handbooks. J. B. Kommers, in the *Engineering News-Record* for Jan 2, 1919, gives a very interesting method for computing "Beam Deflections under Distributed or Concentrated Loadings."

Deflection of supports for plastered ceilings is commonly limited to $\frac{1}{360}$th of the span. Deflection, or stiffness required, often limits plank floors. Steel beams supporting machines frequently have to be designed for deflection.

Deflection seldom needs to be computed for reinforced concrete beams on account of their great stiffness. G. A. Maney in a paper before the seventeenth annual meeting of the American Society for Testing Materials presented the following formula for the deflection of a reinforced concrete beam of whatever shape:

$$D = c\frac{l^2}{d}(e_c + e_s)$$

Where D = maximum deflection (inches).

l = span (inches).

d = depth of beam to the center of the steel (inches).

e_c = unit deformation in extreme fiber for the concrete $= \frac{f_c}{E_c}$.

e_s = unit deformation in extreme fiber for the steel $= \frac{f_s}{E_s}$.

$c = \frac{c_1}{c_2}$ in which

c_1 = the numerical coefficient in the formula for deflection of homogeneous beams, $D = c_1 \frac{wl^2}{EI}$, depending on the loading and method of support.

c_2 = the numerical coefficient in the formula for bending moment, $M = c_2wl^2$.

For a simple beam uniformly loaded, $c = 5/48$.
For a simple beam loaded at center, $c = 1/12$.
For a cantilever uniformly loaded, $c = 1/4$.
For a cantilever loaded at the end, $c = 1/3$.

67. Unsymmetrical Bending.—The most common case of oblique loading or unsymmetrical bending is that of I-beam and channel purlins on pitched roofs (see chapter on "Design of Purlins for Sloping Roofs" in Sect. 2, also the last chapter in this section).

68. Summary of Formulas for Internal Stresses.

Moment:

General (use for steel)

$$f = \frac{Mc}{I} = \frac{M}{S} \qquad M = \frac{fI}{c} = fS$$

Wood (use for homogeneous rectangular sections)

$$f = \frac{6M}{bd^2} \qquad M = \frac{fbd^2}{6} \qquad bd^2 = \frac{6M}{f}$$

Concrete

For design

$$n = \frac{E_s}{E_c} \qquad m = \frac{f_s}{f_c}$$

$$k = \frac{nf_c}{f_s + nf_c} = \frac{n}{n + m}$$

$$j = 1 - \frac{k}{3} \qquad p = \frac{kf_c}{2f_s}$$

$$M = \frac{bd^2kjf_c}{2} = f_spjbd^2 = Kbd^2$$

$$bd^2 = \frac{2M}{kjf_c} = \frac{M}{f_spj} = \frac{M}{K}$$

$$A_s = pbd$$

For investigation

$$p = \frac{A_s}{bd}$$

$$k = \sqrt{2pn + (pn)^2} - pn$$

$$j = 1 - \frac{k}{3}$$

$$f_s = \frac{M}{pjbd^2} = \frac{M}{A_sjd}$$

$$f_c = \frac{2M}{bjkd^2} = f_s\frac{k}{n(1 - k)}$$

$$k = \frac{3}{8} \text{ and } j = \frac{7}{8} \text{ (approx.)}$$

Shear:

General	Maximum for wood	Steel I	Concrete
$v = \frac{VQ}{bI}$	$v = \frac{3V}{2bd}$	$v = \frac{V}{th}$	$v = \frac{V}{bjd}$ (approx.) $v = \frac{8V}{7bd}$

Bond:

$$u = \frac{V}{\Sigma ojd}$$

RESTRAINED AND CONTINUOUS BEAMS

By Walter W. Clifford

69. General Information.—A *restrained beam* is one which is more or less fixed at one or both points of support. A *cantilever beam* is the most common example of a restrained beam. A *continuous beam* is one which extends over three or more supports. At the interior supports of a continuous beam, and also at the end supports if restrained, the curvature of the beam is concave downward—that is, like a cantilever, but just the opposite of a simple beam. In a continuous beam of approximately equal spans with uniform load, the curvature near the middle of a span is like that of a simple beam. The elastic curve (curve of the neutral plane) of a simple beam, a cantilever beam, a beam fixed at both ends, and a beam continuous over four spans, are shown in Fig. 79 in the order mentioned. It is assumed that the beams are uniformly loaded.

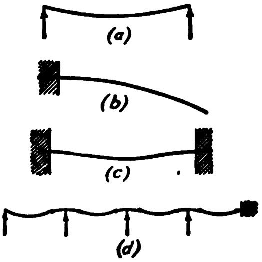

Fig. 79.

Where the curvature of the beam axis is concave downward, it is evident that the material in the lower part of the beam is compressed and that in the upper part is stretched, or in tension. This is opposite to the condition in a simple beam, but like that of the cantilever. The bending moment in a simple beam is commonly called *positive moment*. The bending moment in a cantilever is of the opposite sign and is called *negative moment*. The continuous beam has negative moment at the interior supports and usually positive moment at the center of span.

Fig. 80 shows graphically the moment variation and the deflection curve for a beam continuous over two spans and uniformly loaded. There are two points in the beam where the moment is zero for this loading. These points are called *inflection points* and are indicated by small circles. Inflection points are also indicated by small circles in Fig. 79(*d*).

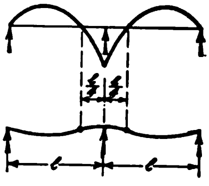

Fig. 80.

Since there is no moment at an inflection point, it is evident that a hinge might be placed at this point without changing the stresses anywhere. This is equivalent to saying that the part of a continuous beam from an interior support to an inflection point is in effect a cantilever; and the part of a span between inflection points acts as a simple beam. Practically a hinge at each inflection point would throw excessive bending into the supporting piers or columns, in the case of unsymmetrical loading. But if we put hinges at the inflection points of alternate bays, we have the variation of the continuous beam principle used for cantilever bridges (see Fig. 81). This form of construction is also used for girders, both concrete and steel.[1]

Fig. 81.

Considering the two-span beam in Fig. 80 as a cantilever at the center support with suspended spans on each side, it is evident that the reactions and shears are not the same as for simple beams. One-half the load on each suspended span goes to the end support adjoining and is equal in amount to the reaction at that support. The other half is the shear at the inflection point. The shear at the center support is the shear at the inflection point plus the loads between this point and the support. The shear at the center support is evidently greater than at the end supports. In the particular case shown in Fig. 80, the inflection point is $\frac{1}{4}\,l$ from the center. The shears are therefore $\frac{3}{8}\,wl$ and $\frac{5}{8}\,wl$ at the end and center supports respectively, instead of both being $\frac{1}{2}\,w$ as in the case of simple beams. Methods for computing shear in continuous beams are given in Art. 71.

70. Assumption Made in Design of Continuous Beams.—The moment of inertia, I, is usually assumed to be constant in value for the full length of the beam and the supports are assumed to be on the same level. Although the assumption with regard to I is not in error for

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[1] See article on Portland bridge, *Eng. Rec.*, Mar. 4, 1916, p. 319.

a wooden or steel beam, considerable variation in the value of I may occur in a concrete beam. For example, the moment of inertia is usually larger at the center of span for reinforced concrete T-beams, the ratio of I at center to I at support varying from 1 to 1.50 in typical cases of design, which causes about 10% variation in moment. This variation in the value of I increases the positive moment and decreases the negative moment from the values as computed, assuming I constant throughout.

With a rigid beam, as one of metal or wood, and with rigid supports, very precise work is required for each support to bear evenly on the undeflected beam. In a beam continuous over two equal spans, with uniform load, the center support carries $\frac{5}{8}$ of the load and the negative moment is $\frac{wl^2}{8}$. If the center support should be lowered by an amount equal to the deflection of a beam with a span of $2l$, the center support would take none of the load. The positive moment at that point would then be four times as great as the negative moment of $\frac{wl^2}{8}$. The end reactions would be increased 167%. For a steel beam with two 10-ft. spans, this lowering of the center support would need to be only $\frac{1}{2}$ in. in order to produce the above change in moments and reactions. From this illustration it should be clear that a slight change in elevation of a support of a continuous steel beam may cause a great change in the moments and shears as ordinarily computed.

With a concrete beam, the supports are automatically leveled when the concrete is poured—that is, so far as the beam itself is concerned. The only possible difference in elevation must come from unequal settlement of supports or deflection of members in the finished structure. In the case of well-designed columns and footings unequal settlement will be negligible. On the other hand, in the case of girders supporting continuous cross beams, the girders will deflect. When this occurs, the negative moments in the cross beams will be reduced, but the positive moment will be greater than the moment determined for supports on a level. Allowance is made for this in all concrete design specifications.

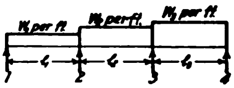

Fig. 82.

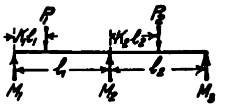

Fig. 83.

71. The Three-moment Equation.—The usual basis of continuous-beam design is the three-moment equation derived from the equation of the elastic curve. The mathematical derivation of this formula is found in standard text books on mechanics. The result is an equation for the moments at three adjacent supports in terms of the spans and loads. If the ends are free, the equations of the supports taken successively in groups of three are sufficient to solve for all the moments at the supports. If the ends are fixed, an extra span with a length of zero is assumed at each end of the beam to give the two needed extra equations. The common forms of the equations are as follows:

For uniform loads (see Fig. 82)

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = -\tfrac{1}{4}(w_1l_1^3 + w_2l_2^3) \qquad (a)$$

For concentrated loads (see Fig. 83)

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = -\Sigma P_1l_1^2(k_1 - k_1^3)$$
$$- \Sigma P_2l_2^2(2k_2 - 3k_2^2 + k_2^3) \qquad (b)$$

Both of these equations assume level supports and constant I.

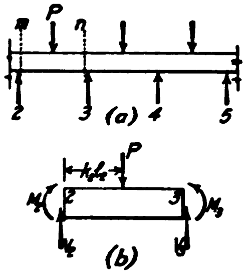

Fig. 84.

Having found the moments at the supports, the shears are found by considering each span of the beam (such as 2–3, Fig. 84a) after cutting it out close to the supports (as shown by the planes m and n), assuming the same shear and moment to act at each end of the cut portion as in its original position (Fig. 84b). By taking moments first about one end and then about the other, the values of the shears may be determined. The moments acting at the ends must be included in the moment equations.

The reaction at a support is the sum of the shears on each side of the support. Inflection points are at points of zero moment. Maximum positive moments are at points of zero shear.

The following typical example indicates the method of applying the three-moment equation to an actual problem.

Illustrative Problem.—Determine the shears, reactions, and moments at the supports for the beam of Fig. 85, loaded as shown.

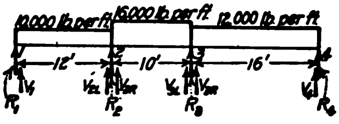

FIG. 85.

Using general Formula (a) and noting that $M_1 = 0$, we have

$$44M_2 + 10M_3 = -4{,}320{,}000 - 4{,}000{,}000 = -8{,}320{,}000 \text{ ft.-lb.} \quad (1)$$

For the next two spans

$$10M_2 + 52M_3 = -4{,}000{,}000 - 12{,}288{,}000 = -16{,}288{,}000 \text{ ft.-lb.} \quad (2)$$

Solving (1) and (2) for M_2 and M_3

$$M_2 = -123{,}000 \text{ ft.-lb.}$$
$$M_3 = -290{,}000 \text{ ft.-lb.}$$

For shear in span 1–2, consider this span cut out of the beam and take moments about 2. Consider clockwise moments plus.

$$+12V_1 - (10{,}000)(12)(6) - M_2 = 0$$

$$V_1 = \frac{720{,}000 - 123{,}000}{12} = 50{,}000 \text{ lb.}$$

Taking moments about 1,

$$V_{2L} = \frac{720{,}000 + 123{,}000}{12} = 70{,}000 \text{ lb.}$$

$$V_1 + V_{2L} = 120{,}000 = (12)(10{,}000) \text{ check.}$$

Shear in span 2–3. Taking moments about 2

$$-M_2 + (16{,}000)(10)(5) - 10V_{3L} + M_3 = 0$$
$$V_{3L} = 96{,}600 \text{ lb.}$$

Taking moments about 3

$$-M_2 - (16{,}000)(50) + 10V_{2R} + M_3 = 0$$
$$V_{2R} = 63{,}400 \text{ lb.}$$
$$V_{3L} + V_{2R} = 160{,}000 = (10)(16{,}000) \text{ check.}$$

Shear in span 3–4. Similarly

$$V_{3R} = 114{,}000 \text{ lb.}$$
$$V_4 = 77{,}500 \text{ lb.}$$

The reactions will be as follows:

$$R_1 = V_1 = 50{,}000 \text{ lb.}$$
$$R_2 = V_{2L} + V_{2R} = 133{,}000 \text{ lb.}$$
$$R_3 = V_{3L} + V_{3R} = 211{,}000 \text{ lb.}$$
$$R_4 = V_4 = \underline{78{,}000} \text{ lb.}$$
$$472{,}000 \text{ lb.} = \text{sum of loads (check).}$$

For span 1–2, zero shear and maximum moment is $\frac{50{,}000}{10{,}000} = 5.0$ from left support, and M at this point is

$$(50{,}000)(5) - (10{,}000)\left(\frac{5}{2}\right)^2 = -125{,}000 \text{ ft.-lb.}$$

For span 2–3, zero shear is $\frac{63{,}400}{16{,}000} = 3.96$ ft. from 2, and M at this point is

$$-123{,}000 + (3.96)(63{,}400) - \frac{(3.96)^2}{2}(16{,}000) = 2{,}600 \text{ ft.-lb.}$$

For span 3–4, the maximum positive moment is 253,000 ft.-lb. and occurs at a point 6.5 ft. from the right support.

Inflection points occur as follows:

Span 1–2. $$M_x = 0 = V_1x - \frac{10{,}000x^2}{2}$$

$$x = \frac{50{,}000}{5{,}000} = 10 \text{ ft. from left end.}$$

Span 2–3. $$M_x = 0 = -123{,}000 - \frac{x^2}{2}(16{,}000) + 63{,}600x$$

$$x^2 - 7.92x = -15.38, \text{ or } x = 3.96 \pm 0.55$$

Inflection points occur at 3.41 ft. and 4.51 ft. from 2.

Span 3–4. Inflection point is 13.0 ft. from 4.

The portions of the beams having positive moment may be considered simple beams as a check on the moment.

Span 1–2. $$M = \frac{(10{,}000)(10)(10)}{8} = 125{,}000 \text{ ft.-lb.}$$

Span 3–4. $$M = \frac{(12{,}000)(13)(13)}{8} = 253{,}000 \text{ ft.-lb.}$$

Span 2–3. $$M = \frac{(16{,}000)(1.10)(1.10)}{8} = 2{,}400 \text{ ft.-lb.}$$

In the span 2–3, the inexact check is due to lack of precision of the slide rule in the previous computations.

The checks given in the example are checks on certain portions of the mathematics only and a problem may be carried through incorrectly and all these checks used.

The shears and moments as computed above are shown in Fig. 86.

The foregoing example is typical, but computations are often long and laborious. Consequently, the opportunity for mathematical error is great and an error once made follows through succeeding calculations. Signs are the most common source of error. To avoid this as far as possible, the sum of the moments should be equated to zero instead of placing positive moments on one side of the equality sign and negative moments on the other side. Great care must be used in determining the sign of the various functions. It is well to call clockwise moments plus and counterclockwise moments minus.

Data on a great variety of continuous beams are given in Hool's "Reinforced Concrete Construction," Vol. II, and in "Concrete Engineers' Handbook" by Hool and Johnson.

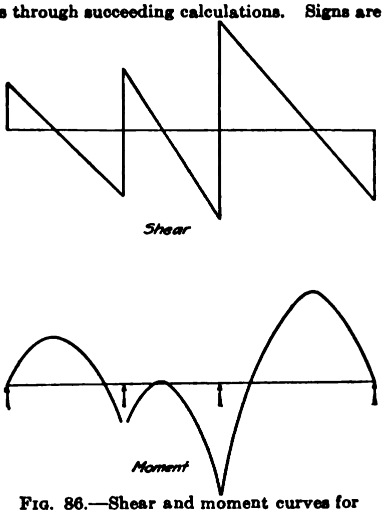

Fig. 86.—Shear and moment curves for beam shown in Fig. 85.

72. Continuous Beam Practice.

72a. Steel, Wood, and Cast Iron.—Steel beams are practically never designed as continuous in building construction on account of variation in the height of supports. They are ordinarily fixed to columns by riveted connections, but the columns are, however, often of little greater moment of inertia than the beams. The actual fixity of the beams, therefore, depends upon the stiffness of the column and adjacent beams. Except where wind loads are to be considered (see Chapter on "Wind Bracing of Buildings," Sect. 3), steel beams are usually assumed to have free ends, which is on the safe side as far as the beams are concerned.

Wooden beams are seldom continuous and in building construction usually have free ends. Cast-iron members or parts are often continuous and are sometimes fixed at the ends. Suitable reductions in moment factors should therefore be made.

It should be noted that beams of two spans have the same maximum moment, whether continuous or simple. If beams are of constant section, there is, therefore, no difference in section required. If shear or center reaction is the criterion, however, the excess of 25% in shear at the center support in the case of the continuous beam should be considered.

72b. Concrete.—The principal use of continuous-beam design in buildings is in concrete construction. Where spans are equal or very nearly so, the moments recommended by the Joint Committee[1] are commonly used. These specify double the strength theoretically required for positive moment in order to allow for deflection of supports.

Simply-supported ends are not common in concrete construction. They may occur when a concrete member is supported on steel or brick. Where concrete supports are used, there is always some degree of fixity, but seldom are the ends entirely fixed. Beams framing into heavy lower-story columns may to all practical purposes be considered as fixed. In other cases there is partial restraint at end supports, and part of the moment of eccentric loadings is taken by the columns at intermediate supports. This matter is well discussed by Edward Smulski in an article on "Design of Wall Columns and End Beams" in Journal American Concrete Institute for July, 1915.

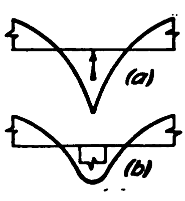

Fig. 87.

In practical construction, supports have considerable width. Thus moment curves over supports will actually be somewhat as shown in Fig. 87(*b*). This will tend to reduce the maximum negative moment. In the theoretical case, the maximum occurs at one point only (Fig. 87*a*). The Joint Committee allows higher unit stress in the concrete at a support because the actual negative moment is lower than that figured and occurs only for a short length of beam, and also because the section is enlarged due to the column.[2]

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[1] See Sect. 2, Art. 38.

[2] See Sect. 2, Art. 40*f*, and *Appendix J*.

72c. Concentrated Loads.—Uniform load is the common assumption in building design. For ordinary concentrated loads, it is common practice, and sufficiently accurate, to compute the maximum moment by considering the beam or girder simply supported, and then reducing this maximum moment by the same ratio used in the uniform loading. For example, suppose the maximum moment due to given concentrated loads is M, considering the beam simply supported, then if $\frac{1}{12}wl^2$ would be used in uniform loading instead of $\frac{1}{8}wl^2$ required for the simply supported beams, $\frac{8}{12}$ of M, or $\frac{2}{3}M$, may be used for the concentrated loads.

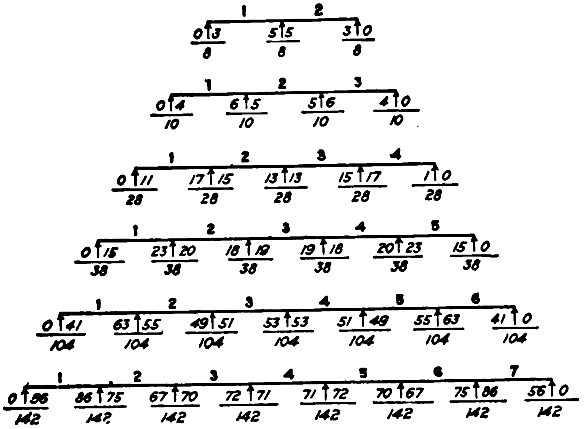

Fig. 88A.—Shears in continuous beams; supported ends; uniform loads on all spans; spans all equal. Coefficients of (wl).

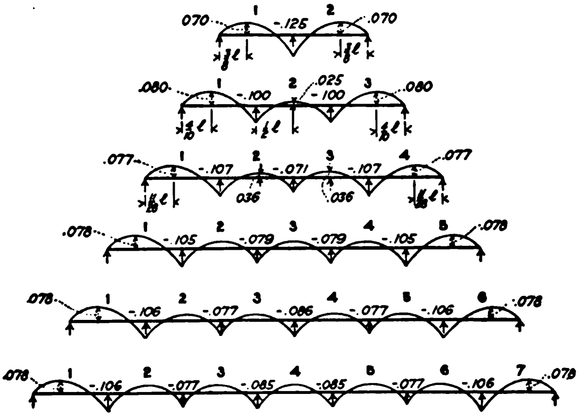

Fig. 88B.—Moments in continuous beams; supported ends; uniform load on all spans; spans all equal. Coefficients of (wl^2).

72d. Shear and Moment Considerations.—In the case of unimportant members or those which occur only once, it is often cheaper to design even for $\frac{wl^2}{8}$ at both center and support than to go to elaborate computations. Moment and shear factors for odd spans or unusual loads should not be assumed by any but experienced engineers.

Shears and moments in continuous beams with supported ends, uniform load on all spans, and with spans all equal, are shown in Figs. 88A and 88B respectively. The beam continuous over two spans is like two beams, each with one end fixed and one end supported. The beam fixed at both ends is like the center-span portion of a continuous beam of a large number of spans.

The moment curves of a fixed beam and a simple beam for uniform loading are the same but with the axis of zero moments shifted (see Fig. 89)—that is, the arithmetical sum of the center moment and the end moment equals $\frac{wl^2}{8}$.

Fig. 90 shows moments for center concentrated loads on two equal spans. Fig. 91[1] gives shears and moments for a uniform load on two continuous spans, one twice the other.

For important members, especially those which are typical and repeat many times, computations should be made, similar to the example given in Art. 71.

In concrete construction the dead load is usually a larger proportion of the total load than is true in other types of construction. This dead load is fixed and generally uniform. In computations, therefore, it is necessary to compute moments for the entire uniform dead load and then compute moments for live load with such spans loaded as will give maximum moments at

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[1] From paper by Frank S. Bailey on "Continuous Beams of Unequal Spans" in *Jour.* Boston Soc. C. E., Oct., 1917.

various points. The live and dead moments must then be so combined as to give maximum values.

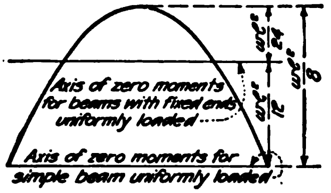

Axis of zero moments for beams with fixed ends uniformly loaded
Axis of zero moments for simple beam uniformly loaded
wl²/24
wl²/12
wl²/8

FIG. 89.

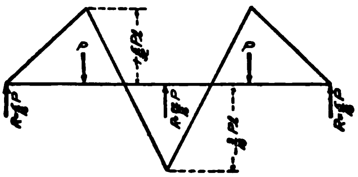


FIG. 90.—Moments for concentrated loads on two equal spans.

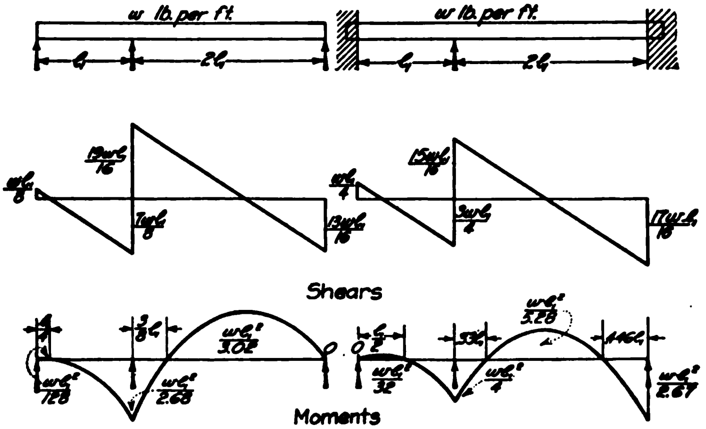


FIG. 91.—Shears and moments for a uniform load on two continuous spans, one twice the other.

The following functions were computed for a three-span beam, the center span being twice the side spans and a live load of $\frac{w}{2}$ lb. per ft. (Fig. 92):

Loading of $\frac{w}{2}$	R_1	R_2	V_{2L}	V_{2R}	M_2	Positive M_{1-2}		M_{2-3} at center
						Location of x	Value	
Full.....	0.11 wl	0.89 wl	0.39 wl	0.50 wl	$-0.14\ wl^2$	0.22	$0.012wl^2$	$0.11\ wl^2$
Center span..	$-0.125wl$	$0.625wl$	$0.125wl$	0.50 wl	$-0.125wl^2$	....	....	$0.125wl^2$
Both ends..	$0.235wl$	$0.265wl$	$0.265wl$	0.00 wl	$-0.016wl^2$	$0.469l$	$0.055wl^2$	$-0.016wl^2$
One end.	$0.227wl$	$0.289wl$	$-0.273wl$	$0.016wl$	$-0.023wl^2$	$0.456l$	$0.052wl^2$	$-0.0142l^2$
Maximum..	$\begin{pmatrix} +0.345wl \\ -0.015wl \end{pmatrix}$	1.78 wl	0.78 wl	$1.000wl$	$-0.28\ wl^2$	....	$0.067wl^2$	$0.235wl^2$

One-half of the beam only is shown. It will be noted that R_1 and M_{1-2} are maximum with live load on two end spans. R_2, V_{2L}, V_{2R} and M_2 are maximum with full load and M_{2-3} with live load on the center span only. Some parts of the beam may have either positive or negative moment.

Computations may be made directly for various combinations of dead and live loads as was done for a large school building. Loadings as indicated in Fig. 93 only were considered. The resulting maximum moments were:

Dead load ½ total
$$\begin{cases} M_{1-2} = 0.0894wl^2 & \text{Live load on two end spans} \\ M_2 = -0.0822wl^2 & \text{Live load on one end span} \\ M_{2-3} = -0.0602wl^2 & \text{Live load on two end spans} \end{cases}$$
(No positive moment in center span)

Dead load ⅔ total
$$\begin{cases} M_{1-2} = 0.0894wl^2 & \\ M_2 = -0.0822wl^2 & \text{Loadings as above} \\ M_{2-3} = -0.057wl^2 & \end{cases}$$

Max. $R_1 = R_4 = 0.42wl$....................Live load on end spans

Max. $R_2 = R_3 = 0.83wl$....................Full load

Max. $V_{2L} = 0.58wl$......................Live load on end spans

Max. $V_{2R} = 0.25wl$......................Live load on center span

The case of live load on center and one end span is not considered in these examples.

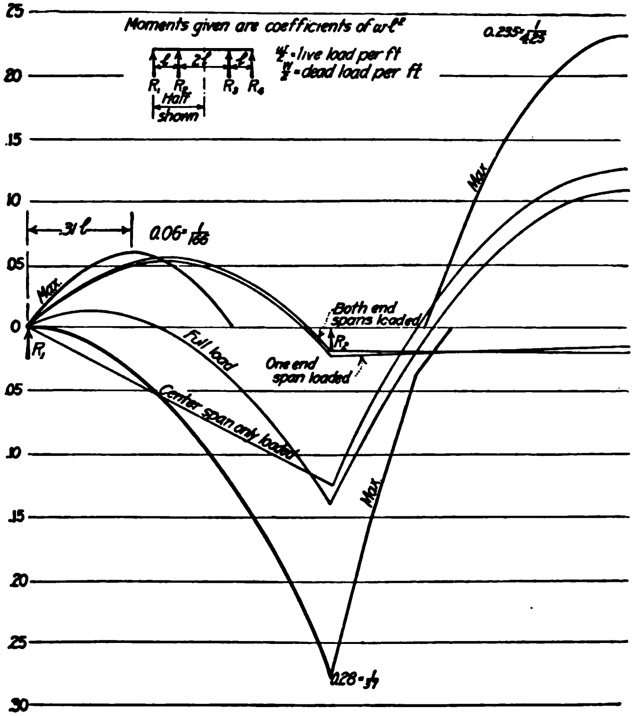


Fig. 92.

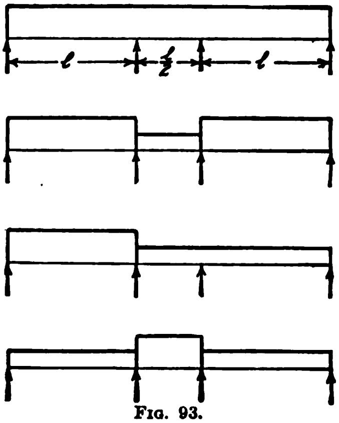


Fig. 93.

From the foregoing it is evident that a relatively short span between long spans may have negative moment throughout. In the case of a very short intermediate span, a practical method of design is to neglect it as a beam and treat it as a broad support for the adjacent beams.

72e. Shoring.—From a consideration of the moment curve for two spans (Fig. 80) it is evident that indiscriminate shoring of beams in the center may do more harm than good. Consider a span having a uniform load and introduce a support in the center at the same elevation as the original supports. The moment over this support is one quarter of what it was before, but of opposite sign. In the case of a concrete beam or of a truss the result will often be failure. The shear which was zero at the center becomes $\frac{5}{16}$th of the whole load, which may also be dangerous.

73. Deflection.—Continuous and fixed beams have less moment under similar conditions than simple beams and the deflection is therefore less. Some moments and shears as well as deflections are here repeated for comparison:

	Maximum positive moment	Maximum negative moment	Distance from support to inflection point	Maximum deflection
Simple beam; uniform load	$\frac{wl^2}{8}$	....	....	$\frac{5wl^4}{384EI}$
Simple beam; concentrated load	$\frac{Wl}{4}$	....	....	$\frac{Wl^3}{48EI}$
Cantilever; uniform load	....	$\frac{wl^2}{2}$	....	$\frac{wl^4}{8EI}$
Cantilever; load at end	....	Wl	....	$\frac{Wl^3}{3EI}$
Beam fixed one end, supported at other; uniform load	$\frac{9}{128}wl^2$	$\frac{1}{8}wl^2$	$\frac{1}{4}l$	$0.098\frac{wl^4}{EI}$
Beam fixed one end, supported at other; concentrated load at center	$\frac{5}{32}Wl$	$\frac{3}{16}Wl$	$\frac{3}{11}l$	$0.0054\frac{Wl^3}{EI}$
Beam fixed at both ends; uniform load	$\frac{wl^2}{24}$	$\frac{wl^2}{12}$	$0.211l$	$\frac{wl^4}{384EI}$
Beam fixed at both ends; concentrated load at center	$\frac{Wl}{8}$	$\frac{Wl}{8}$	$\frac{1}{4}$	$\frac{Wl^3}{192EI}$

74. Internal Stresses.—The formulas for internal moment and shear developed in the chapter on "Simple and Cantilever Beams" apply to continuous and restrained beams. In parts subjected to negative moment, compression will be at the bottom and tension at the top as in a cantilever. In the rest of the beam, stresses will be as in simple beams. The magnitude and direction of shear and diagonal tension is the same in relation to the external moment and shear in continuous and restrained beams as it is in simple beams.

GENERAL METHODS OF COMPUTING STRESSES IN TRUSSES

By George A. Hool

75. Two Methods Used.—The stresses in the members of a truss may be computed either by a "method of sections" or by a "method of joints." It is often convenient to compute the stresses in some of the members of a truss by one method and the stresses in the remaining members by the other method.

In either method the necessary procedure, in order to determine stresses for a given loading, is to separate the given truss into two parts by an imaginary section, either plane or curved; the part of the truss to one side of the section is removed (that is, considered so) together with all external forces, and the members that are cut by the section are replaced by the stresses acting in those members. By so doing, the part of the truss considered will be in equilibrium due to the outer forces acting on that portion of the truss and the stresses in the members cut. If the section is taken completely across the truss, as XX' or YY', Fig. 94(a), so that the members cut *do not* all intersect in one point, then the method used is the *method of*

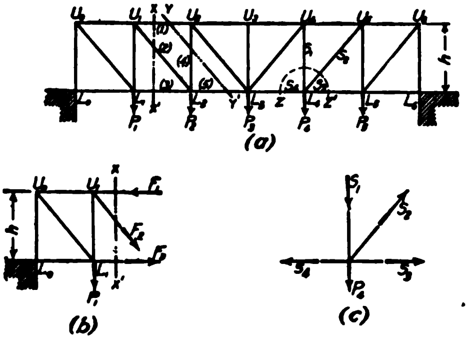


Fig. 94.

sections. If the section is so taken that the members *do* all intersect in one point, as ZZ', Fig. 94(a), then the method used is the *method of joints.*

76. Algebraic Treatment.—The algebraic treatment of the *method of sections* will be explained with reference to the truss shown in Fig. 94(a) which is subjected to moving loads transmitted to the lower panel points. Assume that the maximum stresses in members (1), (2) and (3) of the truss are required, these members being cut by the section XX'. Consider the portion of the truss shown in Fig. 94(b). For a definite loading the forces are all in equilibrium as explained above and, since only three members are cut, any or all of the three equations of equilibrium can be used; namely, $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$ (see Art. 43b). First use the equation $\Sigma M = 0$. This equation is true about any point in the plane of the truss but, in order to get the stress in a given member directly, it is necessary to take the center of moments at the intersection of the other two members. For example, the stress in F_3 for a given loading can be found by taking moments about the point U_1. It should be noticed that U_1 is vertically above L_1 and, since the loads are all vertical, the moments at U_1 and L_1 are equal. The *maximum* stress in F_3, then, occurs with the loading which gives maximum moment at the first panel point from the left support (see chapter on "Shears and Moments"). Call this maximum moment M_1. The moment of F_3 (when F_3 is a maximum) about the point U_1 must be equal and opposite to M_1 in order that ΣM may equal zero. Thus

$$(\text{max. } F_3)(h) = M_1$$

or

$$\text{max. } F_3 = \frac{M_1}{h}$$

In the same manner, calling M_2 the maximum moment at the second panel point,

$$\text{max. } F_1 = \frac{M_2}{h}$$

It should be observed (using $\Sigma M = 0$) that the stress in the upper chord acts toward the section, thus denoting compression, while the stress in the lower chord acts away from the section, thus denoting tension; that is, F_1 = compression and F_3 = tension. This is true of all the upper and lower chords throughout the truss.

The maximum stress F_2 remains to be found. This may be accomplished by using the equation $\Sigma V = 0$. The vertical component of the maximum stress in F_2 is equal to the maximum positive shear in the second panel from the left support Call this component V_2. Then

$$\text{max. } F_2 = V_2 \frac{U_1L_2}{h}$$

In using the equation $\Sigma V = 0$, observe that the stress acts away from the section, thus denoting tension.

Let the maximum stress be required in members (1), (4), and (5), Fig. 94(a). Take the section YY'. Using $\Sigma H = 0$, and knowing that the loads are all vertical, the stress in member (1) is seen to be equal and opposite to the stress in member (5). This applies for any loading, hence the loading giving maximum stress in member (1) will also give a maximum stress in member (5) of the same amount; that is, the loading giving the maximum moment at the second panel point from the left support will cause maximum stress in both members (1) and (5). The maximum stress (compression) in member (1) is, as before, $\frac{M_2}{h}$, using $\Sigma M = 0$. This same amount of tension, then, occurs in member (5). The maximum stress in member (4) is directly the maximum positive shear in the third panel from the left support, using the equation $\Sigma V = 0$. Stress in member (4) is compression.

In the method of sections, the section should always be taken so as to cut only three members whose stresses are unknown. If more than three members are cut, there are more unknown quantities than can be found by the principles of statics.

The method of joints is only a name given to the manner of determining stresses from the conditions of equilibrium of concurrent forces. The manner of using the algebraic conditions, namely, $\Sigma H = 0$ and $\Sigma V = 0$, is explained in an illustrative problem on p. 10, the stresses being determined in the members of a crane truss. It should be clear that this method can be applied to a joint only when there are two unknown stresses. In solving a truss by this method, it is evident that a joint must be selected where but two members meet and then proceed from this to other joints.

In the algebraic method of joints, if a maximum stress is desired in a certain member of a truss, all the joints from one end of the truss up to the member considered must be computed for the loading giving maximum stress in that member only. For this reason the algebraic method, although perfectly general, is too laborious to be employed in practice in determining the maximum stresses in all the members of an ordinary truss. It may be used with great advantage, however, for certain specific members, and should be understood. A graphical method based upon the same principles is well adapted for many types of trusses, particularly roof trusses with non-parallel chords. In roof trusses, the conditions for probable maximum stress in the given members are few, and usually all the stresses may be computed graphically for each loading in much shorter time than it would take to compute the stresses throughout the truss algebraically for any *one* condition of loading.

Illustrative Problem.—Roof truss of Fig. 95(*a*); loads as shown. (*a*) Required the stresses in all members algebraically by the method of sections. (*b*) By the method of joints.

(*a*) Method of Sections

To find the stresses in members L_0U_1 and L_0L_1, pass a section *a-b* cutting these members. Consider the truss to the left of the section. Fig. 95(*b*) shows the joint at L_0 removed and the known loads applied, together with the unknowns S_1 and S_2, assumed to act as shown. Consider upward forces and forces to the right as positive; downward forces and forces to the left as negative. The two equations, $\Sigma V = 0$ and $\Sigma H = 0$, may be employed to find the two stresses S_1 and S_2.

$$\Sigma V = 0. \quad 4000 - 1000 - S_1 \sin \theta = 0$$

$$S_1 = (3000)\left(\frac{22.36}{10}\right) = 6710 \text{ lb. (compression, as assumed, since result is positive).}$$

$$\Sigma H = 0. \quad S_2 - S_1 \cos \theta = 0$$

$$S_2 = (6710)\left(\frac{20}{22.36}\right) = 6000 \text{ lb. (tension, as assumed, since result is positive).}$$

To find the stresses in members U_1U_2, U_1L_2, and L_1L_2, pass a section *c-d* cutting these members and consider the portion of the structure to the left (Fig. 95*c*). The three equations of equilibrium may be used to determine the three unknown stresses, but the solution may be simplified by employing only $\Sigma M = 0$ three times. This equation should be applied at the intersection of two members to find the stress in the third. Thus, to determine the stress in U_1U_2, take moments about L_2, the intersection of U_1L_2 and L_1L_2. Then, considering clockwise moments as positive,

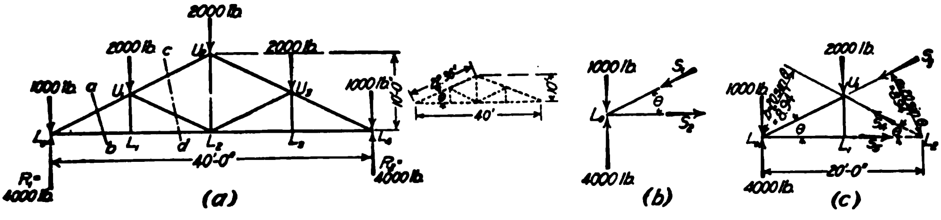


FIG. 95.

$$4000(20) - 1000(20) - 2000(10) - S_3(a) = 0$$

$$S_3 = 4470 \text{ lb. (compression)}$$

The stress in S_4 may be obtained by taking moments about L_0, the intersection of U_1U_2 and L_1L_2.

$$2000(10) - S_4(b) = 0$$

$$S_4 = 2240 \text{ lb. (compression)}$$

The stress in S_5 may be found by taking moments about U_1, the intersection of U_1L_2 and U_1U_2.

$$(4000 - 1000)(10) - S_5(5) = 0$$

$$S_5 = 6000 \text{ lb. (tension)}$$

Other sections should now be taken cutting only three members whose stresses are unknown and the moment equation again applied. Proceeding in this manner the stresses in all the members may be determined.

(*b*) Method of Joints

The stresses in members L_0U_1 and L_0L_1 are determined as for the method of sections and the solution will not be repeated here.

Passing now to the next joint at which only two unknowns exist, joint L_1 will be selected, shown in Fig. 96(*a*).

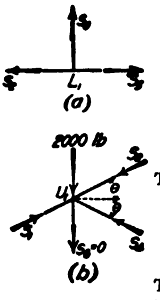


Fig. 96.

$$\Sigma V = 0. \quad S_5 = 0$$

$$\Sigma H = 0. \quad S_6 - S_2 = 0$$

$$\text{or } S_6 = S_2 = 6000 \text{ lb. (tension)}$$

Next pass to joint U_1, which is shown in Fig. 96(*b*). The two unknown forces are S_3 and S_4.

$$\Sigma V = 0. \quad S_1 \sin\theta + S_4 \sin\theta - S_3 \sin\theta - 2000 = 0$$

$$S_4 \sin\theta - S_3 \sin\theta = -1000$$

$$\Sigma H = 0. \quad S_1 \cos\theta - S_4 \cos\theta - S_3 \cos\theta = 0$$

$$S_4 \cos\theta + S_3 \cos\theta = 6000$$

These independent equations involve only the unknowns S_3 and S_4. Solving simultaneously

$$S_4 - S_3 = -2236$$

$$S_4 + S_3 = +6708$$

$$S_3 = 4470 \text{ lb. (compression)}$$

$$S_4 = 2240 \text{ lb. (compression)}$$

The stresses at joint U_1 are now completely determined. In the same way pass to the other joints until all the stresses in the members of the truss are determined.

77. Graphical Treatment.—In the graphical method of sections it is necessary to commence at one end of the structure and pass a section cutting but *two* members. The stresses in these members can be determined by the single condition that the force polygon, drawn from the forces on one portion of the structure, must close. Next a section is taken cutting *three* members, one of which has already been determined, and the two unknowns can be found by the force polygon method as before. By successive sections taken in this manner, all the stresses can be determined by simple force polygons.

The graphical construction resulting from the method of joints is identical with that resulting from the method of sections. The only difference is the sections taken and, consequently, the order in which the lines are drawn. The method of joints is generally preferred in practice on account of its simplicity and this method only will be illustrated here.

Illustrative Problem.—Required the stresses in all members of the roof truss shown in Fig. 97(*a*) by the graphical method of joints; loads as shown.

It will simplify matters to draw a sketch of the truss to some suitable scale and show on it all the outer forces including reactions. Also, to designate all the forces and members on this sketch by letters so located that each force and each member will lie between two letters and only two, as illustrated in 97(*a*).

Now any force, as *AB*, for example, in this figure may be designated in the graphical solution by a line having a length corresponding to the magnitude of the force and with the letter *a* at one end and the letter *b* at the other. By going through the graphical construction in this manner one letter only need be placed at each apex of a force polygon and the work is greatly simplified.

The next step is to draw a force polygon for the outer forces to a scale of sufficient size to give the desired accuracy. The force polygon is *abcdefga* in Fig. 97(*b*) and is a straight line, since all the forces are vertical. The external forces should be plotted in the order obtained by going around the figure in a clockwise direction. *ab* = 1000. *bc* = *cd* = *de* = 2000. *ef* = 1000. *fg* = R_2 = 4000. *ga* = R_1 = 4000. The right and left reactions must previously be computed either algebraically or graphically (see chapter on "Reactions").

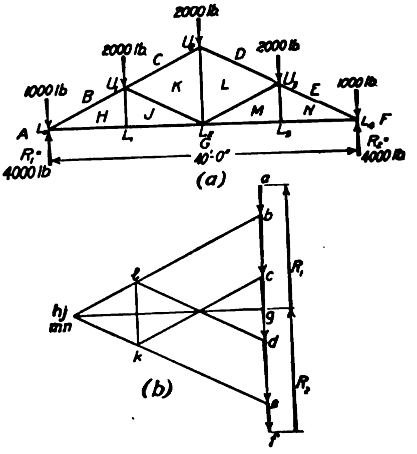


Fig. 97.

The force polygon should now be drawn for joint L_0. The unknown forces which act at this joint are the stress in *BH* and the stress in *HG*. *bh* and *hg* are known in direction but not in magnitude, hence, there are but two unknowns and these can be found by the polygon of forces. The figure *abhga*, Fig. 97(*b*), is this polygon obtained by drawing from *b* a line parallel to *BH*, and from *g* a line parallel to *HG*. The lines *bh* and *hg* may now be scaled from the force polygon to obtain the magnitude of the stresses in the two members intersecting at L_0. The character of these stresses must also be found. The forces at joint L_0, being in equilibrium, must follow in order around the corresponding force polygon. Reading around joint L_0 in a clockwise direction gives *bh* acting downward to the left, or toward the joint L_0, thus showing compression, and *hg* acting toward the right, or away from the joint L_0, showing tension.

The joint L_1 is the next one at which only two unknowns exist. The stress in *GH* is known from joint L_0, and the stresses in *HJ* and *JG* are unknown. The corresponding force polygon *hjg* for this joint must close. Since *gh* and *jg* have the same line of action, the line in the force polygon representing the magnitude of *hj* will be a point, thus having no length. The stress in HJ is, therefore, zero. This might have been seen by inspection, as there is no load at L_1 to cause stress in this member. In reading around joint L_1 in a clockwise direction, the line JG is from left to right, and the stress acts away from joint L_1, denoting tension.

Now pass to joint U_1. The stresses in CK and KJ are the unknowns. To obtain them draw ck and jk in the force polygon parallel respectively to the corresponding members in the truss. (The stress being zero in JH, the whole space occupied by J and H may conveniently be called J.) Reading around joint U_1 in a clockwise direction gives both ck and kj acting toward the joint U_1, hence, denoting compression in both these members. The polygon considered is *bckjb*. In a similar manner the stresses in the other bars may be determined.

STRESSES IN ROOF TRUSSES

By H. S. Rogers

78. Kinds of Stresses.—Stresses in roof trusses may be either *direct* or *combined*. The stress in a member is usually assumed to be direct unless the member is loaded at one or more points along its length or unless it is subjected to a distributed loading other than its own dead weight. For method of computing combined stresses see chapter on "Bending and Direct Stress—Wood and Steel." Direct stresses only are considered in this chapter.

79. Loads.—The loads upon a truss may be classified as (1) dead load, (2) wind load, (3) snow load, and (4) miscellaneous load. The dead load is vertical and includes the weight of the truss and all fixed loads of the completed structure bearing upon or suspended from the truss. For calculating direct stresses, the dead load is considered as concentrated at panel points of the truss. The wind load is concentrated at panel points and is usually taken normal to the plane of the roof. The snow load is vertical and treated in a manner similar to the dead load. The miscellaneous load may be due to mechanical equipment of a fixed or moving character suspended from or supported by the roof truss. If such loads exist, their effect should be carefully studied and provided for.

80. Reactions.—The reactions upon a truss together with the external loads form a complete system of forces in equilibrium. The reactions are vertical for dead and snow loads. Because the one-half dead panel load concentrated at the end of a truss has the same line of action and is opposite in direction to the total reaction, it may be subtracted from the total and the difference, called the "effective reaction," may be used in the solution of problems.

The direction and relative magnitude of wind load reactions depend upon the type of end supports. Three conditions for truss bearings are commonly used: (1) both ends fixed, (2) one end fixed and the other movable in a horizontal direction, (3) both ends equally free to move by elastic deflection in the columns supporting the truss. Condition (1) exists when both ends of the truss are rigidly anchored to solid masonry walls. For this condition the wind-load reactions are usually considered parallel to the wind load. Condition (2) exists when one end of the truss is placed upon a rocker, sliding plate, or rollers, and the reaction then at the free end may be considered vertical. Condition (3) exists in framed bents—that is, when roof trusses are attached to columns instead of being placed on masonry walls; for which condition the two horizontal components of the reactions at the points of inflection in the columns are considered equal. For stresses in framed bents, see Sect. 3, Art. 164. For methods of computing reactions, see chapter on "Reactions."

81. Methods of Computing Stresses.—The two general methods of computing stresses in trusses are the "method of sections" and the "method of joints," as explained in the preceding chapter.

82. Algebraic Method of Sections.—To determine the direct stress in the member of a truss, the following procedure should be used:

1. Pass a section through the unknown member and remove part of the truss to one side of the section.

2. Replace cut members by forces, assuming the directions of the forces.

3. Take moments about a point which is common to the lines of action of all unknowns but the one desired.

4. Determine the magnitude and direction of the unknown force by equating the algebraic sum of the moments to zero.

If the force which is to be determined acts toward the section, the member will be in compression; if it acts away from the section, the member will be in tension.

Illustrative Problem.—The stresses in the Pratt truss shown in Fig. 98 will be determined by the algebraic method, for the loads shown. Before beginning the determination of moments acting on sections of the truss, it will be convenient to determine the right-angle distances of upper chord members from lower panel points and the right-angle distances of web members from the heel joint, L_0.

The first section is taken through L_0U_1 and L_0L_1 and the part of the truss to the right of the section is removed, as shown in Fig. 98(*b*). The members are replaced by forces, as indicated by the arrows. In order to determine the stress in L_0U_1, the moments are taken about L_1, so as to eliminate the stress in L_0L_1, from the computations. In order to determine the stress in L_0L_1, the moments are taken about U_1 for a similar reason. The solutions of the equations give

$$L_0U_1 = (3000)\left(\frac{10}{4.47}\right) = 6710 \text{ lb.}$$

$$L_0L_1 = (3000)\left(\frac{10}{5}\right) = 6000 \text{ lb}$$

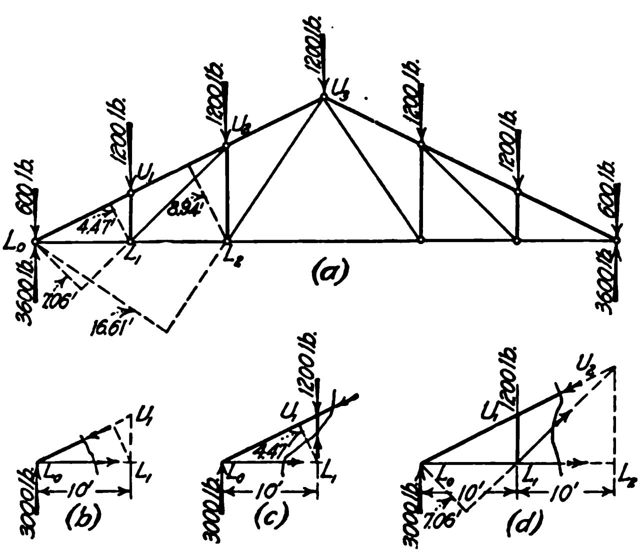


FIG. 98.

Because the sum of the moments about L_1 must equal zero, the force L_0U_1 must be directed toward the section; therefore the member L_0U_1 will be in compression. Because the sum of the moments about U_1 must equal zero, the force L_0L_1 must be directed away from the section; therefore, the member L_0L_1 will be in tension.

The second section is taken as shown in Fig. 98 (*c*), the cut members being replaced by forces. In order to determine the stress in U_1L_1 the moments are taken about L_0; and in order to determine the stress in U_1U_2 the moments are taken about L_1. The directions of the forces are determined as before.

The third section is taken as shown in Fig. 98(*d*) and the cut members are again replaced by forces. The stresses and their directions are determined as in the previous cases.

It should be observed that, if a section is passed through three unknowns, any one of them can be determined by taking moments of all the forces acting about the intersection of the other two unknowns.

The stresses in a symmetrical truss loaded symmetrically need be determined only for one-half the truss.

83. Methods of Equations and Coefficients.—The method of determining the stresses in symmetrical trusses, symmetrically loaded, by means of equations or coefficients involves the least amount of labor.

Equations for stresses in members can be determined in terms of the panel load and the ratio of span to height of truss, by the algebraic method of sections, the loads being expressed in panel loads and the moment arms in terms of span divided by height. These equations give constant values, or coefficients, for each member of a truss for each particular ratio of span divided by height. The value for any member, when multiplied by the panel load will give a product, which will be the stress in the member.

The equations for stresses and the coefficients of stresses for the standard simple types of symmetrical trusses are given in the Chapter on "Roof Trusses—Stress Data" in Sect. 3.

84. Graphical Method of Joints.—In the graphical method of computing stresses, joints are considered to be cut from the truss in consecutive order and a force polygon is drawn for the forces at each joint. The stresses should be determined by use of the following procedure:

(1) Draw a scaled diagram of the truss showing all the external forces, and letter each space between forces or members with a capital letter.

(2) Consider each joint separately as a "free body" acted upon by concurrent forces in equilibrium.

(3) Draw a force polygon for each joint showing the external and internal forces and letter each intersection of forces with a small letter corresponding to the space between the forces in the space diagram.

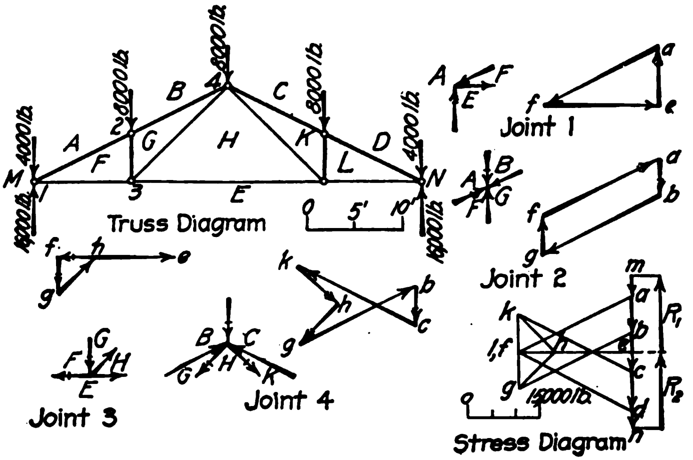
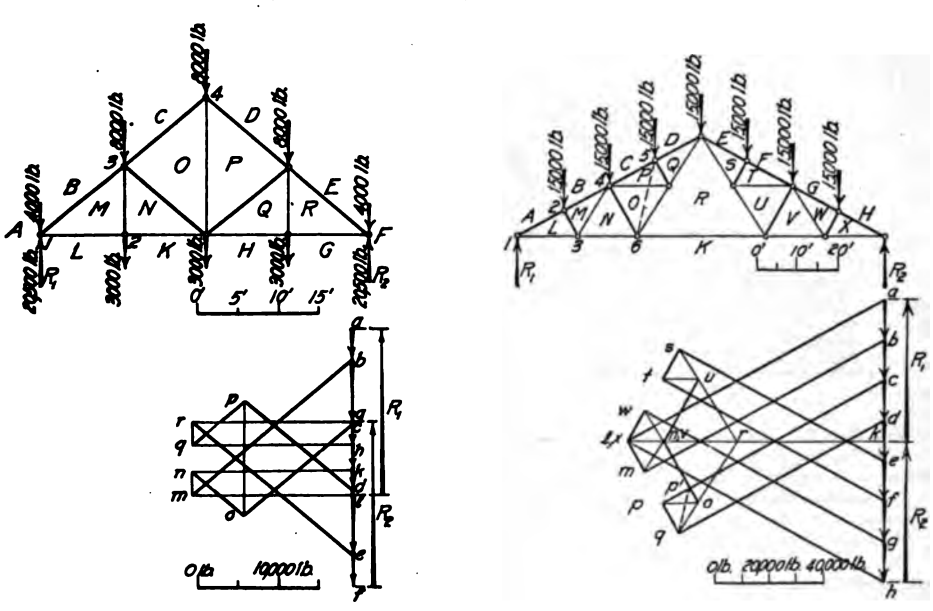


FIG. 99.

Illustrative Problem.—The stresses in the truss of Fig. 99 will be determined by the graphical method for the loads shown.

The heel joint, joint 1, is the first to be solved. The one-half panel load at the joint and the reaction are combined to give the effective reaction. The force polygon for the joint is drawn with the forces parallel to the lines of action shown in the space diagram. Since the sum of the horizontal components and the sum of the vertical components must equal zero for equilibrium, the polygon must close. The order of letters as read around the force polygon indicates the direction of the forces acting at the joint and thereby indicates whether a member is in compression or tension. If the force acts toward the joint, the member which transmits it must be in compression; if it acts away from a joint the member must be in tension.



(a) (b)

FIG. 100.

Joint 2 is the next joint to be solved. The procedure used in the solution of joint 1 is followed. The known forces are marked with a line across the arrow in the space and force diagrams. It should be noted that no more than two unknowns can be determined in the solution of any one joint.

The solutions of joints 3 and 4 follow in order and complete the solutions for the truss.

It is not necessary to draw separate space and force diagrams for each joint, as the *truss diagram* gives the space diagrams for all joints and the force diagrams may be combined into one *stress diagram* as shown in the figure.

Illustrative Problem.—The stresses in the King-rod truss of Fig. 100(*a*) for the roof and suspended-ceiling loads shown will be determined by the graphical method.

The truss diagram is first drawn to scale and all the external forces (loads and reactions) are indicated on the diagram. To construct the stress diagram, first plot to scale all the loads on the truss rafters, *i.e.*, *ab*, *bc*, *cd*, *de*, and *ef*. R_1 is then laid off from *a* and in opposite direction to *ab*, *bc*, etc., and R_2 is laid off from *f*. The two reactions are found to overlap because the suspended loads on the lower chord have the same line of action as the loads on the rafters at the panel points above. The left-hand heel joint is first considered by plotting the stresses in a clock-wise direction around the joint. The stress polygon is obtained by drawing *bm* and *ml*, from *b* and *l*, parallel to *BM* and *ML* respectively. Tracing this joint through by a continuous clockwise reading of the forces, *bm* is found to act toward the joint and *ml* to act away from the joint, which means that these stresses are compression and tension respectively.

The first lower-chord joint from the left reaction is next determined. The forces are again traced in a clock-wise direction beginning with the known force *kl*. In this force diagram it is found that *mn* and *nk* both act away from the joint and members *MN* and *NK* are, therefore, in tension.

Joints 3 and 4 are solved in the same manner, which completes the determination of stresses, as the stresses on the right-hand side of the truss are equal to those on the left. The stress diagram may be completed as a check on the work.

Illustrative Problem.—The dead-load stresses in the Fink truss shown in Fig. 100(*b*) will be determined by the graphical method. A special feature of this solution is the condition encountered at joint 4 which may at first appear to be an indeterminate condition.

The truss diagram is drawn to scale and the loads and effective reactions are plotted.

The joints are solved in the usual manner in the order indicated on the truss diagram. Bringing the solution from left to right, a condition which cannot at once be solved is met at joint 4. There are three unknowns *cp*, *pe*, and *on*. It is seen on inspection that the stress in the members *DQ*, *QR*, and *RK* will remain the same regardless of the web members toward the left. *OP* and *PQ* are, therefore, cut out and replaced by the dotted member *P′Q*. Joints 4, 5, and 6 are determined with this assumed member in place, and joint 6 is then corrected by throwing out the dotted member and replacing the members *OP* and *PQ*. The stresses in the members *OP* and *PQ* are then determined by the solution of joint at their intersection.

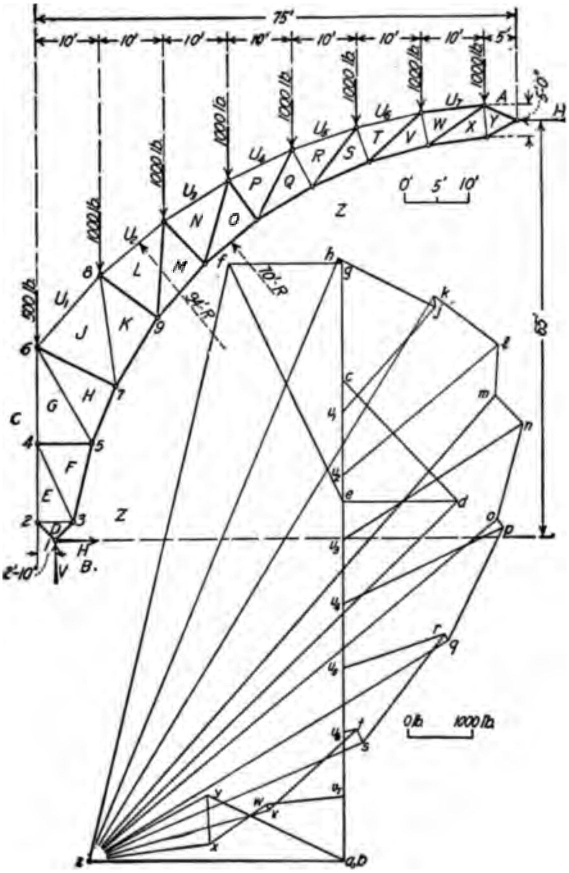


FIG. 101.

The solution may be obtained in another manner, by solving algebraically for the stress in *RK* and laying it off to scale on the stress diagram, so that joint 6 can be determined before joint 4.

Illustrative Problem.—The stresses are required in the three-hinged arch truss of Fig. 101.

The reactions may be found graphically but the algebraic solution is more simple (see Illustrative Problem, p. 21). After the components of the reactions are determined the stresses may be found by the usual stress diagram beginning at either reaction and determining stresses at consecutive joints, as shown in Fig. 101. The solution could, of course, be accomplished by beginning at the crown hinge.

Illustrative Problem.—The stresses are required in a cantilever truss loaded as shown in Fig. 102(*a*).

The reactions of the truss are determined graphically in Fig. 102(*a*), as explained in the chapter on "Reactions." The method of determining the stresses is the same as in the preceding illustrative problems.

85. Wind Load Stresses by the Graphical Method.—In the illustrative problems which follow, stresses will be found in trusses due to wind load under the following conditions: (1)

rollers on the leeward side of truss, (2) both ends fixed, and (3) rollers on the windward side of truss. The wind load is considered as that component of a horizontal wind force, normal to to the plane of the roof.

Illustrative Problem.—In Fig. 102(*b*), the external force polygon is first drawn with the loads parallel to the wind loads on the truss. The reaction, R_2, can be drawn vertically because it is transmitted through rollers, but the direction of R_1 is not known so the polygon cannot be completed. The reactions will, therefore, be determined by means of the force and equilibrium polygons. R_1 will be assumed as parallel to the wind load and the closing string will give the direction of the ray Oe'. Now because R_1 must take the entire horizontal component of the wind load and because R_2 acts vertically, a horizontal line drawn from e' to e will give the point of intersection of the two reactions. These reactions may be checked by considering the total wind load and the two reactions as three forces acting on the truss. Since the directions and points of application of the resultant of the wind load and the reaction R_2 are known, the two forces may be extended to their point of intersection, d; and, since the point of application of R_1 is known, the direction of the force will be from d to the point of left reaction. The determination of this direction makes it possible to complete the external force polygon and obtain a check on the first solution for reactions.

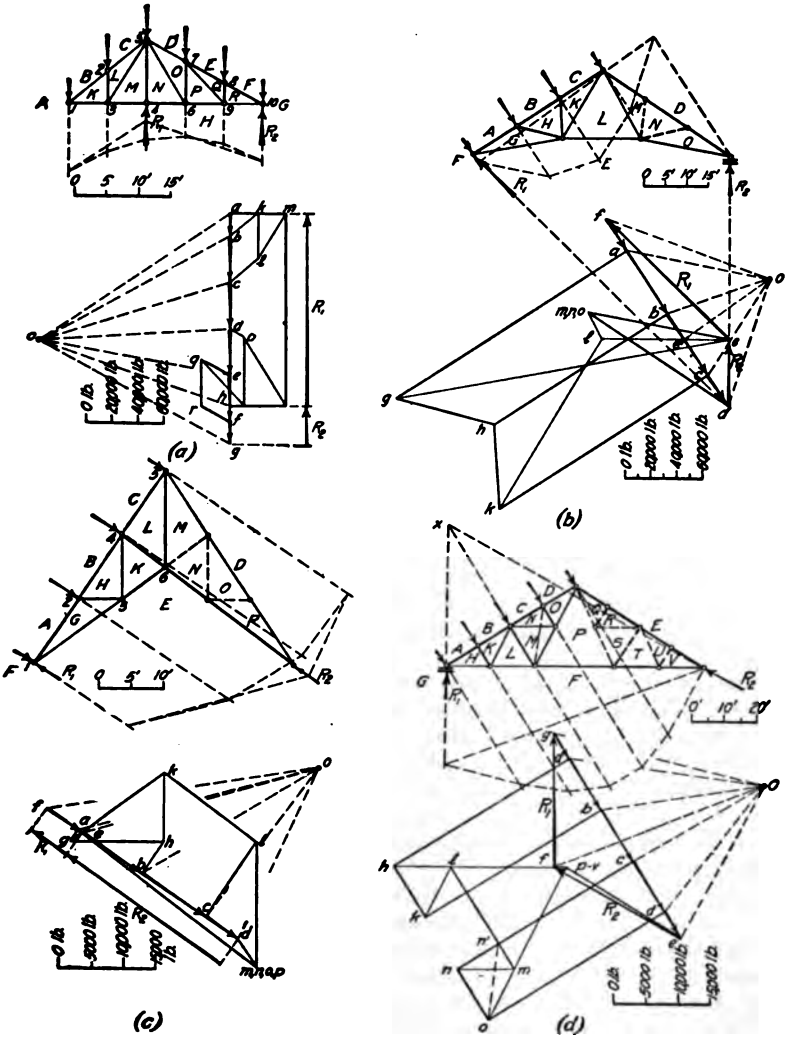


Fig. 102.

The stresses are now determined by drawing a force polygon for each joint. It should be noted that the web members in the leeward side received no stress.

Illustrative Problem.—The wind stresses in the Scissors truss of Fig. 102(*c*) will be determined by the graphical method under the assumption that the reactions are parallel when both ends of the truss are fixed by an anchorage to solid masonry walls.

The space diagram is drawn with the lines of action of the loads extended so that the equilibrium polygon can be drawn. The reactions are determined by the ray, *Oe*, which is parallel to the closing string of the equilibrium polygon.

The stresses are determined by beginning at the left-hand heel joint and following through in the order indicated. As in the previous problem no stress is found in the web members on the leeward side of the truss. Some stresses are produced in this truss due to wind load which are opposite in direction to those produced by dead loads. Stresses should be carefully determined in roofs of such extreme pitch.

Illustrative Problem.—The wind load stresses are required in the Fink truss of Fig. 102(*d*).

The wind-load reactions upon the Fink truss of Fig. 102(*d*) will be determined in a different manner than that used for the determination of the reactions in Fig. 102(*b*). The load line is plotted as usual and a pole from which the rays are drawn is selected. The line of action at the left support is known, but the point of application is the only element of the right reaction which is known. The equilibrium polygon, is, therefore, begun at the right-hand heel joint so that the intersection of the strings can be made on the line of action of the force. The string prallel to the ray *Oe* is first drawn. The others are drawn in consecutive order from that one parallel to *Od* to the one parallel to *Og*. Since the line of action at the left support is vertical, the point of intersection with the string can be obtained. The closing string between the forces which form the two reactions is then drawn and the ray, *Of*, is drawn parallel to it. The intersection at *f* with the vertical line through *g* gives the left reaction, *fg*. The force *ef*, which is the right reaction, is drawn to the point of intersection of the vertical force through *g* and the ray *Of*.

These reactions may be checked by extending the line of the left reaction and the line of the resultant of the wind loads to a point of intersection shown at *x*, and drawing the right reaction through the right-hand heel joint and point, *x*. Since the russ is in equilibrium the two reactions and the resultant of the wind loads must form a system of three concurrent forces. The extended forces drawn to point *x* give a space diagram from which the force diagram, *gef*, may be drawn.

The stress diagram is begun at the left-hand heel joint and the joints are taken in consecutive order until the joint at the middle point of the rafter is reached, at which the condition encountered in the Fink truss in Fig. 99(*b*) is again met. The difficulty is removed by replacing the members *NO* and *MN* by the dotted member shown and carrying the solution through until *fp* is determined, after which the corrections are made as before. It should be again noted that the web members on the leeward side of the truss take no stress.

COLUMNS

By H. S. Rogers

86. Column Loads.—The loads to be calculated in the design of columns may be divided into six classes: (1) dead load, including snow load, (2) live load, (3) true live load, (4) impact load, (5) wind load, and (6) earthquake load.

The dead load is produced by the weight of that portion of the completed structure which a column supports, and includes floors, curtain walls, roof, superimposed columns, and permanent fixtures. It can be accurately determined and should be computed with a good degree of precision. The snow load in effect is a dead load and may be considered as such. It may, however, be unsymmetrical and may be combined under certain conditions with wind load.

The live load on columns depends upon the use to which the building is put and includes such loads as the weight of people, furniture, goods, and equipment. Quite accurate data for determining the weights of furniture and mechanical equipment can be obtained, but in determining the loads due to occupancy of stores and office buildings, considerable judgment must be exercised. Since it is very improbable that the full live load on all floors will be imposed simultaneously, the uniform or concentrated loads used in calculating the strength of floor beams and girders may be reduced for the calculation of column stresses. The extent of the reduction of live loads in office buildings is usually specified in building codes, most of which permit a gradual reduction to some minimum for the assumed live load acting upon columns in consecutive lower stories.

Schneider's "Reduction of Live Load on Columns" is as follows:

For columns carrying more than five floors, these (Schneider's) live loads may be reduced as follows:

For columns supporting the roof and top floors, no reduction.

For columns supporting each succeeding floor, a reduction of 5% of the total live load may be made until 50% is reached, which shall be used for the columns supporting all remaining floors.

This reduction is not to apply to live load on the columns of warehouses, and similar buildings which are likely to be fully loaded on all floors at the same time.

The reduction of live load specified in the Seattle Building Code is as follows:

Reduction of live load shall not be permitted in determining the strength of any part of a building except in accordance with the following provisions:

Walls, piers, and columns, in buildings more than three stories high, used for stores, offices, places of habitation, refuge and detention shall be designed to carry besides the dead load not less than the following percentage of the required live load: Roof and top floor 100%, next lower floor 95%, and for each succeeding lower floor 5% less, until a minimum of 50% is reached and maintained for the remaining floors, if any. In all other buildings the full live load shall be taken.

The true live load is the dynamic load produced by machinery, cranes, elevators, telpherage systems, industrial railways or similar mechanical equipment. Detailed information concerning such loads should be obtained and provision should be made for the stresses which they produce in columns.

Impact load is produced by the shocks and vibrations caused by true live load. It should be thoroughly studied and should be provided for with judgment.

Wind load is produced by the horizontal pressure of the wind on exposed surfaces. The unit pressure is specified for various conditions in all building codes and is usually given as 30 lb. per sq. ft. The wind load produces an overturning moment which increases the compression in the columns on the leeward side of a building, decreases the compression in those on the windward side, and produces a moment in the columns by means of the truss and girder connections and wind bracing. Its effect is of great importance in high buildings and thorough study of the stresses produced by it should be made.

Earthquake load will produce stresses in columns which should be investigated in those localities where earthquakes are liable to occur.

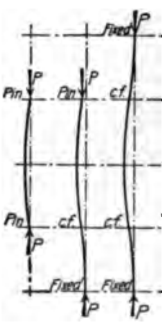
87. Columns and Struts.—A structural member which is acted upon by forces causing direct compression is called a *column*, a *pillar*, a *post*, or a *strut*. *Short columns* are those in which the ratio of length to least width is small. They fail by direct crushing of the material without appreciable bending or buckling.

An *ideal column* is one in which the axis is perfectly straight and the material absolutely uniform and in the same condition throughout, and to which the load is applied exactly on the axis. Such columns are not found in practice.

Practical columns fail by a combination of direct compression and bending. The bending in centrally loaded columns is caused by accidental eccentricities of the application of the load, by unavoidable imperfections in manufacture and nonuniformity of material, and by initial bends and stresses in the column shaft. Due to these imperfections, any column will immediately begin to deflect under load. This deflection increases the lever arm of the forces causing the bending, and the bending will continue to increase until a state of equilibrium is reached or until the column fails.

88. End Conditions.—One of the important factors governing the strength of columns is the degree of fixity of the ends. When the end of a column is perfectly free to turn, its end condition has no influence on its bending and it is said to be *pivoted*. A fixed end is one at which the axis of the column is held rigidly so that its direction cannot change.

Fig. 103 shows the flexure lines of three columns with different sets of end conditions and lengths such that their theoretical strengths are equal if their cross sections are the same. Fig. 103(*c*), with both ends fixed, has points of contraflexure (or zero moment) at the quarter points, so that the column between these points is essentially the same as the pivoted-end column in Fig. 103(*a*).



(*a*) (*b*) (*c*)

FIG. 103.

Conditions in practice are seldom such that a column may be considered as having fixed ends.[1] The usual end conditions are *pin* ends, *flat* ends, and *riveted* ends.[2] A *riveted* end fre

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[1] See article "Fixed End Columns in Practice," *Eng. News*, Nov. 2, 1911, vol. 66, p. 530.

Pin and riveted ends do not occur in concrete columns, see chapter on "Concrete Columns" in Sect. 2.

quently approaches the pivoted-end condition due to the influence of the flexure of other members connected to it, causing the point of contraflexure in the column to lie at or near the end.

The formulas in general use are applied to columns with any of the end conditions above mentioned.

89. Application of Column Loads.—The loads upon floors and roof are transmitted to columns by the girder and truss connections. They may be either *concentric* or *eccentric* according to the details of the connection. A concentric load is one which is applied axially along the column. The loads transmitted to columns by the usual girder connections should be considered as concentric. If, however, a girder is supported by a bracket on a column, the eccentricity of the load applied should be investigated and the column should be designed to withstand the bending stresses in addition to the direct stresses (see chapters on "Bending and Direct Stress"). In addition to concentric and eccentric loads, direct transverse loads may be applied to columns by cantilevers supporting platforms, roofs, and cranes, or by wind bracing. When such loads occur, the stresses produced by them should be considered in the design of the column.

90. Stresses Due to Concentric Loading.—There is no direct method which can be used to obtain the dimensions of a long-column section, but very short columns should be computed by using the safe compressive strength per square inch of the metal in short blocks. In the design of an ordinary column, which has no eccentric loading, the procedure which should be followed is: (1) select a column which will give the desired features in the detailing of connections, (2) determine the stresses which are produced by concentric loads acting upon the column, and then (3) correct the design of the section to bring the stresses within the allowed working intensities. There are two kinds of stresses produced by concentric loads to which a column may be subjected: (1) direct compressive stress distributed uniformly over the section; (2) transverse stress produced by the flexural action of the column and distributed with varying intensity from the neutral axis to extreme fibers so as to form a stress couple.

91. Column Formulas.[1]—There is no simple rigorous analytical method for determining the resultant stresses in a column. There are, however, two more or less rational and two empirical types of formulas for determining such stresses. These types are the Euler, the Gordon or Rankine, the Straight Line, and the Parabolic.

92. Euler's Formula.—Euler's formula is derived upon the assumptions, that the column is concentrically loaded, that it is subjected to direct compression, that it has fixed or square ends, and that it is free to bend laterally. It assumes that the material of the column is perfectly elastic and that the ultimate strength of the column is developed at a stress equal to the elastic limit of the material. The expression for the ultimate strength of columns with fixed ends is

$$p = \frac{4\pi^2 E}{\frac{L^2}{r^2}}$$

in which

p = intensity of stress within the limits of perfect elasticity.
E = modulus of elasticity.
L = length.
r = least radius of gyration.
$\frac{L}{r}$ is called the *slenderness ratio.*

Through the center of gravity of a cross-section there is always a pair of axes about one of which the moment of inertia is a maximum and about the other a minimum. These moments of inertia are called principal moments of inertia and the axes about which they are taken are called principal axes. An axis of symmetry which divides a cross-section symmetrically is always a principal axis. The least radius of gyration $\left(r = \sqrt{\frac{I}{A}}\right)$ and, consequently, the minimum moment of inertia is used in designing columns. A column bends in a direction at right angles to the axis about which the radius of gyration is a minimum, provided the column is not laterally supported in that direction.

Long columns with pivoted ends will act essentially as that part of the fixed column between

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[1] For "Concrete Columns" see chapter in Sect. 2.

the two points of contraflexure, which is equal to one-half the length of the column. The expression for the ultimate strength of columns with pivoted ends is therefore

$$p = \frac{\pi^2 E}{\frac{L^2}{r^2}}$$

Euler's formula is not used in specifications, as are formulas of the other types, because the ideal conditions upon which it is based are not met in practice. It is applicable to long columns with fixed ends which have a very large ratio of L/r and to columns with hinged ends which have an average ratio of L/r, but gives values up to infinity for short lengths, which is incompatible with actual conditions.

93. Gordon's Formula.—The Gordon formula is based upon the assumptions that the column is concentrically loaded, that it is subject to direct compression and flexural stresses, and that it is free to bend laterally. It assumes further that the column deflects laterally and that the bending stress is produced by the moment of the axial load about the point of maximum deflection.

Let p = allowable intensity of stress over the column section.
f_1 = the uniformly distributed stress due to the total load.
f_2 = the flexural stress due to the bending of column under the load.
f = the maximum allowable intensity of stress in short blocks.
P = the total load.
A = area of column section.
Δ = maximum deflection of column.
c = distance from neutral axis to the extreme fiber.
I = moment of inertia.
b = a constant depending upon the condition of column ends.

The direct stress $f_1 = \frac{P}{A}$; and the bending stress $f_2 = \frac{bP\Delta c}{I}$ from the common flexure formula (see Fig. 104).

Since $f = f_1 + f_2$

$$f = \frac{P}{A} + \frac{bP\Delta c}{I} \qquad (1)$$

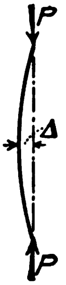


FIG. 104.

Now it can be shown by the theory of flexure that

$$\Delta = \frac{a_1 L^2}{c}$$

in which L = length of the column and a_1 = a constant depending upon f_2 and E.

Substituting in (1),

$$f = \frac{P}{A} + \frac{ba_1 PL^2}{I}$$

But $I = Ar^2$ (r = least radius of gyration).

$$\therefore f = \frac{P}{A}\left(1 + \frac{ba_1 L^2}{r^2}\right)$$

$$= \frac{P}{A}\left(1 + a\frac{L^2}{r^2}\right) \qquad (2)$$

in which a is a constant contingent upon the factors which influence b and a_1.

The allowable intensity of stress, p, over the column section will be

$$p = \frac{P}{A} = \frac{f}{1 + a\frac{L^2}{r^2}} \qquad (3)$$

Formulas of the Gordon type are used quite extensively in building specifications and codes. Those in use, however, do not all have the same values for f and a. A change of condition of the column ends produces a change in the constant, "a," as is evident from the derivation of the formula. Care should be exercised in selecting a formula which shall be applicable to the column under investigation.

94. Straight-line Formula.—The straight-line formula has been used because of the simplicity of its application and because it can be made to coincide very closely with the results o tests of columns having usual values of L/r. The equation is empirical and has the genera form

$$p = f - m\frac{L}{r}$$

in which f = maximum allowable compressive strength of the material, and m = a constant.

If the equation is made to coincide very closely with the values of safe stresses found b experiment in columns within the usual range of L/r, it will give large stresses for low values o L/r unless some limitation be placed upon L/r, and consequently upon the allowable uni stresses. A number of the column formulas in general use fix this maximum allowable stres for low ratios of L/r and also fix a maximum ratio of L/r.

95. Parabolic Formula.—The parabolic type of formula has been introduced to correc the large values of unit stresses allowed by the straight-line formula for very low or high ratio of L/r, and at the same time give a continuous equation. The equation is also empirical an has the general form

$$p = f - n\frac{(L)^2}{(r)^2}$$

in which n is an empirical constant. The curve given by the formula is a parabola with th origin on the stress axis at f. Some of the recently adopted specifications, notably that of th Engineering Institute of Canada, have embodied this type of column formula.

96. Formulas in General Use.—Formulas of either the straight-line or Gordon type ar usually embodied in specifications and building codes. Both are found in specifications fo stresses in structural steel and cast iron but the straight-line formula alone seems to be universally used in specifications for stresses in timber columns.

97. Steel Column Formulas.—A diagram of the allowed unit stresses for structural-stee columns as given by the principal column formulas which have received general sanction amon engineers is shown in Fig. 105, given by C. E. Fowler, *Eng. News-Rec.*, Feb. 13, 1919. Th formulas graphically represented are as follows:

Am. B.	Am. Bridge Co.	$19{,}000 - 100L/r$
A. R. E. A.	Am. Ry. Eng. Assn.	$16{,}000 - 70L/r$
A. R. E. A. 1919	Am. Ry. Eng. Assn. proposed	$13{,}000 - 0.25(L/r)^2$
E. I. C.	Eng. Inst. Canada	$12{,}000 - 0.3(L/r)^2$
F., 1893	Fowler's Spec. 1893	$12{,}500 - 41\frac{2}{3}L/r$
F., 1919 (Cl. A.)	Fowler's Spec. 1919	$15{,}000 - 60L/r$
F., 1919 (Cl. B.)	Fowler's Spec. 1919	$20{,}000 - 80L/r$
McK-F.	Fowler, mod. by McKibben	$12{,}500 - 50L/r$
N. Y. (Old)	New York Bldg. Code (Old)	$15{,}200 - 58L/r$
B.	Boston Bldg. Code	$16{,}000/1 + L^2/20{,}000r^2$
G.	Gordon Formula	$12{,}500/1 + L^2/36{,}000r^2$
P.	Philadelphia	$16{,}250/1 + L^2/11{,}000r^2$

The limitations of the formulas as to maximum unit stresses and maximum values of $L/$ are shown by the diagram. All of the formulas lie in a diagonal zone, the upper limit of whic is $18{,}000 - 60L/r$ and the lower limit of which is $12{,}000 - 60L/r$ with the exception of Fowler' 1919 (Cl.B.). The average of the zone would be $15{,}000 - 60L/r$, which is the formula that ha been adopted in a 1919 edition of "General Specifications for Steel Roofs and Buildings" b C. E. Fowler. The A. R .E. A. formula, $16{,}000 - 70L/r$, with a maximum stress of 14,000 lb per sq. in. and maximum limit of L/r at 120 has received very wide sanction in building code being found in the codes of New York, Detroit, Chicago, St. Louis, and Seattle.

Illustrative Problem.—Design a 25-ft. channel column for a total load of 300,000 lb. Lattice bars will conne the channels and prevent them from bending separately. Use the straight line formula

$$p = 16{,}000 - 70\frac{L}{r}$$

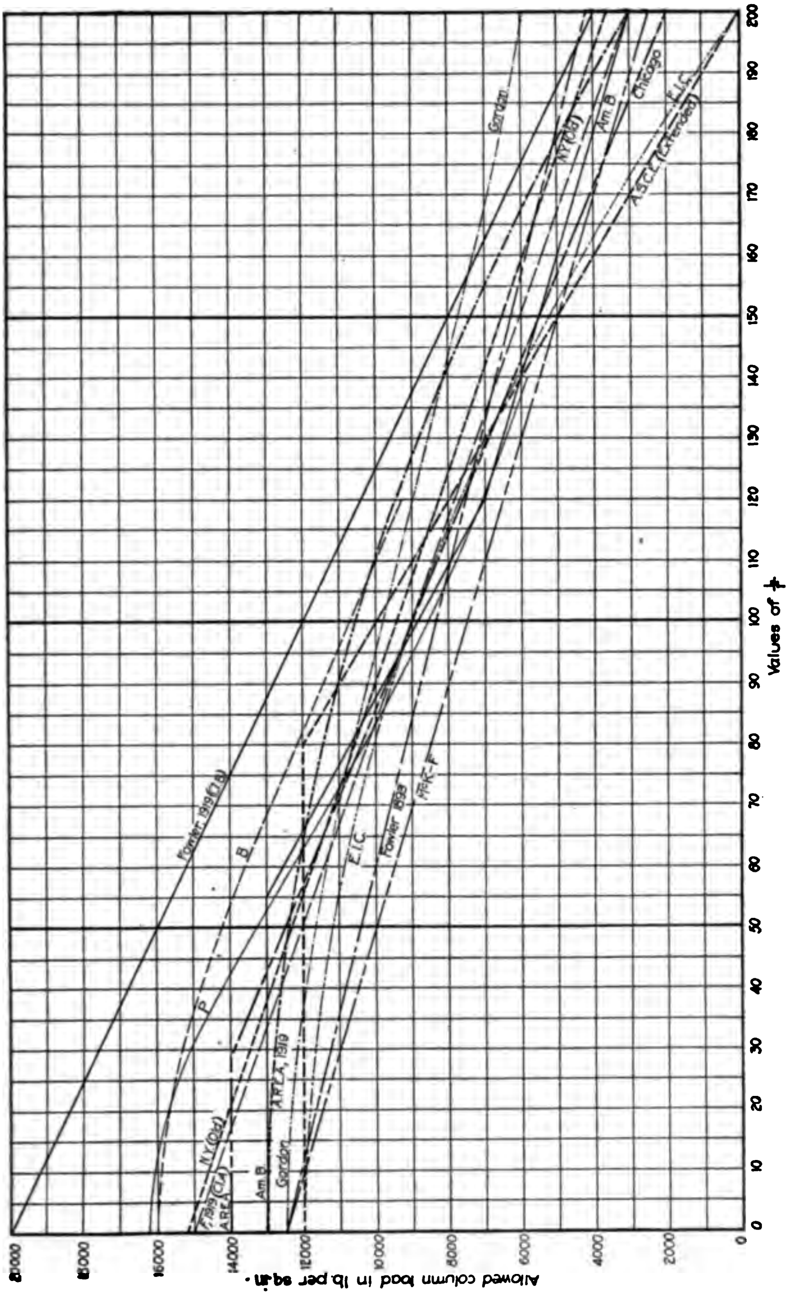

Gordon
N.Y.(Old)
Am.B.
Chicago
E.I.C.
A.S.C.E.(Extended)
Fowler 1919 (C.I.A.)
B
P
E.I.C.
Fowler 1893
P.R.R.-F
N.Y.(Old)
F.1919 (C.I.A.)
A.R.E.A.
Am.B.
Gordon
A.R.E.A. 1919
16000
14000
12000
10000
8000
6000
4000
2000
0
Allowed column load in lb. per sq. in.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Values of $\frac{l}{r}$

Fig. 105.

A trial section should first be determined by assuming $p = 12{,}000$ lb. This gives a trial area of $\frac{300{,}000}{12{,}000}$ 25 sq. in., which may be furnished by the use of two 15-in. channels at 45 lb. having a total area of 26.48 sq. i The radius of gyration for one channel about an axis perpendicular to the web is 5.32 in., hence the allowab value of

$$p = 16{,}000 - 70\,\frac{(25)(12)}{5.32} = 12{,}050 \text{ lb.}$$

The actual unit stress for this size of channel equals $\frac{300{,}000}{26.48} = 11{,}330$ lb. Thus the column would be well on tl safe side and may possibly be decreased in size. Try a 15-in. channel at 40 lb. The allowable value of

$$p = 16{,}000 - 70\,\frac{(25)(12)}{5.44} = 12{,}150 \text{ lb.}$$

The actual unit stress would be $\frac{300{,}000}{23.52} = 12{,}800$; hence, these channels are a little too small and the 15-i 45-lb. channels should be chosen. These should be placed to give the column equal strength in the tw directions—that is, by making the radius of gyration about one axis equal to that about the other axis.

98. Cast-iron Column Formulas.—The most commonly used formulas for allowabl stresses in cast-iron columns are of the straight-line type. The Chicago and Seattle buildin codes specify an allowable unit stress of $10{,}000 - 60L/r$ lb. per sq. in. with a maximum valu of L/r at 70. The New York and Boston building codes specify an allowable unit stress 11,300 − $30L/r$, with a maximum value of L/r at 70. The Philadelphia code specifies a allowable unit stress of $11{,}670/(1 + L^2/400d^2)$ lb. per sq. in.—in which d is the least dimensio in inches, and also specifies a maximum length of $20d$.

99. Timber Column Formulas.—The formulas of building codes of the principal cities fo timber columns vary for the same and for different kinds of timber. Some of the cities, notabl Philadephia, St. Paul, and Seattle, however, use the same formula for long leaf yellow pin white pine, Norway pine, spruce, oak, chestnut, hemlock, and locust. A comprehensive r view of these building code stresses revised to 1913 will be found in the "Cambria Steel" hand book. A safe formula for timber columns is $1000 - 12L/d$ which will give a safety factor about 6 for most kinds of timber. The formula specified in the Seattle Building code i $C\,(1 - L/70d)$, in which C = the allowable compressive stress in pounds per square inch, wit the grain, for the wood used, and d = least cross-sectional dimension of column in inches.

BENDING AND DIRECT STRESS—WOOD AND STEEL

By Clyde T. Morris

100. General.—Tension and compression members are frequently submitted to bendin stresses in addition to the axial stress. This bending may be due to transverse loads on th member or to the eccentricity of the longitudinal load, or to both.

The resulting maximum unit stress in the member may be said to be composed of thre parts, that due to the direct axial load, that due to the transverse bending moment, and tha due to the eccentricity of the axial load caused by the deflection of the member.

The deflection of the member in turn is caused both by th transverse load and by the eccentricity of the axial load due t this deflection. This is illustrated in Fig. 106.

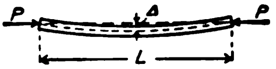


Fig. 106.

101. Bending Due to Transverse Loads Only.—An approx mate value for the maximum unit stress may be obtained b neglecting that part of the bending moment caused by the eccentricity of the axial load du to the deflection. In this case

$$f = \frac{P}{A} + \frac{Mc}{I} \qquad (1$$

in which M is the moment due to the transverse loads only. This gives sufficiently accurat results where the ratio of length to depth is small.

When a member is comparatively slender, a more accurate determination is desirabl

This may be obtained by adding to the bending moment, the effect of the deflection due to transverse load. The deflection due to transverse load is

$$\Delta = \frac{WL^2}{KEI} \quad I = \frac{Mc}{f_1} \text{ and } M = \frac{WL}{q}$$

in which f_1 is the fiber stress due to flexure only, and K and q are constants depending upon the fixity of the ends of the member and the character of the loading. From these we get

$$\Delta = \frac{qf_1L^2}{KEc}$$

The total bending moment $= M \pm P\Delta$ and $f_1 = \frac{(M \pm P\Delta)c}{I}$. Substituting in this the value for Δ and solving for f_1 we get

$$f_1 = \frac{Mc}{I \pm \frac{q}{K} \times \frac{PL^2}{E}}$$

Calling $\frac{q}{K} = C$ and adding the effect of the direct axial load, we get

$$f = \frac{P}{A} + \frac{Mc}{I \pm \frac{CPL^2}{E}} \tag{2}$$

In the denominator of the second term of eq. (2), the minus sign should be used for compression members and the plus sign for tension members. The moment of inertia used, should be calculated for an axis perpendicular to the plane of the bending. Values for the constant C are given below.

For pin ends, concentrated load	$C = \frac{4}{48}$	use $\frac{1}{12}$
For pin ends, uniform load	$C = \frac{40}{384}$	use $\frac{1}{10}$
For one pin and one fixed end, concentrated load	$C = \frac{2}{33.54}$ at center	use $\frac{1}{17}$
	$C = \frac{1}{20.12}$ at end	use $\frac{1}{20}$
For one pin and one fixed end, uniform load	$C = \frac{128}{1665}$ at center	use $\frac{1}{13}$
	$C = \frac{8}{185}$ at end	use $\frac{1}{23}$
For both ends fixed, concentrated load	$C = \frac{8}{192}$	use $\frac{1}{24}$
For both ends fixed, uniform load	$C = \frac{24}{384}$ at center	use $\frac{1}{16}$
	$C = \frac{12}{384}$ at end	use $\frac{1}{32}$

The fixed end condition is seldom realized in practice and this assumption should be made only after careful investigation of the actual end conditions. For this reason many engineers use $C = \frac{1}{10}$ for all cases of combined transverse bending and direct stress.

Illustrative Problem.—Fig. 107 shows a part of the top chord or rafter of a roof truss which carries purlin loads between the panel points in addition to its direct stress as a member of the truss.

The rafter is composed of 2 angles 6 × 3½ × ½, with the long legs vertical. Since the rafter is continuous over the panel points, there will be a negative moment at the panel points and a positive moment midway between under the purlin load. Each of these may be taken as equal to $\frac{8}{10}$ of the moment in a simple beam similarly loaded.

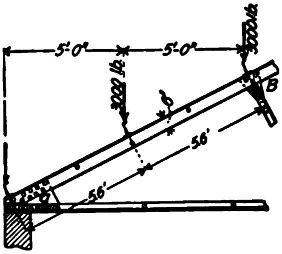


Fig. 107.

The direct compression as a member of the truss, $P = 47{,}000$ lb.

The weight of the member per horizontal foot, $w = 34.3$ lb.

The moments, considering the member as a simple beam, are:

$$\text{Moment due to weight} = \frac{(34.3)(10)^2}{8} = 430 \text{ ft.-lb.}$$

$$\text{Moment due to purlin load} = \frac{(3000)(10)}{4} = 7500 \text{ ft.-lb.}$$

$$\text{Total simple beam moment} = 7930 \text{ ft.-lb.}$$

5

Continuous beam moment = $(\frac{8}{10})(7930)$ = 6344 ft.-lb. = 76,130 in.-lb.

From equation (1)

$$\text{At the panel point, } f = \frac{47{,}000}{9.00} + \frac{(76{,}130)(3.92)}{33.18}$$
$$= 5220 + 8990 = 14{,}210 \text{ lb. per sq. in.}$$
$$\text{At the mid span, } f = \frac{47{,}000}{9.00} + \frac{(76{,}130)(2.08)}{33.18}$$
$$= 5220 + 4770 = 9990 \text{ lb. per sq. in.}$$

From equation (2)

$$\text{At the panel point, } f = \frac{47{,}000}{9.00} + \frac{(76{,}130)(3.92)}{33.18 - \frac{(47{,}000)(11.2)^2(12)^2}{(20)(30{,}000{,}000)}}$$
$$= 5220 + \frac{(76{,}130)(3.92)}{28.42}$$
$$= 5220 + 10{,}500 = 15{,}720 \text{ lb. per sq. in.}$$
$$\text{At the mid span, } f = \frac{47{,}000}{9.00} + \frac{(76{,}130)(2.08)}{33.18 - \frac{(47{,}000)(11.2)^2(12)}{(17)(30{,}000{,}000)}}$$
$$= 5220 + \frac{(76{,}130)(2.08)}{27.59}$$
$$= 5220 + 5740 = 10{,}960 \text{ lb. per sq. in.}$$

Note that those values of C in equation (2) have been used for a member with one pin end and one fixed end. This is probably on the safe side, but the connection at "a" is not sufficient to fix that end of the member. Due to the continuity of the member at "B," and the purlin load in the panel beyond, it is probably safe to consider the member as fixed there. Note that "c" in each case is the distance from the center of gravity of the section to the compression side of the member.

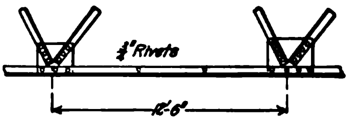


FIG. 108.

The maximum fiber stress "f" should not exceed that given by the column formula of the specifications being used.

Illustrative Problem.—Fig. 108 shows a tension member of a roof truss which is subject to bending due to its own weight. It is composed of 2 angles 3½ × 3½ × $\frac{5}{16}$.

The direct tension in the member, P = 36,000 lb.

The weight of the member per foot, w = 14.4 lb.

$$\text{The bending moment, } M = \frac{8}{10} \cdot \frac{(14.4)(12.5)^2}{8} = 225 \text{ ft.-lb.} = 2700 \text{ in.-lb.}$$

The net area of the member, $A = 4.18 - 2(\frac{7}{8})(\frac{5}{16}) = 3.63$ sq. in.

From equation (1)

$$\text{At the panel point, } f = \frac{36{,}000}{3.63} + \frac{(2700)(2.51)}{4.9}$$
$$= 9920 + 1380 = 11{,}300 \text{ lb. per sq. in.}$$
$$\text{At the mid span, } f = \frac{36{,}000}{3.63} + \frac{(2700)(0.99)}{4.9}$$
$$= 9920 + 540 = 10{,}460 \text{ lb. per sq. in.}$$

From equation (2)

$$\text{At the panel point, } f = \frac{36{,}000}{3.63} + \frac{(2700)(2.51)}{4.9 + \frac{(36{,}000)(12.5)^2(12)^2}{(32)(30{,}000{,}000)}}$$
$$= 9920 + 1180 = 11{,}100 \text{ lb. per sq. in.}$$
$$\text{At the mid span, } f = \frac{36{,}000}{3.63} + \frac{(2700)(0.99)}{4.9 + \frac{(36{,}000)(12.5)^2(12)^2}{(16)(30{,}000{,}000)}}$$
$$= 9920 + 410 = 10{,}330 \text{ lb. per sq. in.}$$

In case any load is suspended from the member between panel points, its moment should be added to that due to the weight of the member.

Illustrative Problem.—Fig. 109 shows a building column which is subject to bending stress under wind loads, due to the thrust of the knee brace.

The total direct load on the column, P = 62,000 lb.

The bending moment, M = 1,200,000 in.-lb.

A = 26.00 sq. in. I = 854

From equation (1)

$$f = \frac{62{,}000}{26.00} + \frac{(1{,}200{,}000)(7.12)}{854}$$
$$= 2390 + 10{,}000 = 12{,}390 \text{ lb. per sq. in.}$$

From equation (2)

$$f = \frac{62{,}000}{26.00} + \frac{(1{,}200{,}000)(7.12)}{854 - \frac{(62{,}000)(20)^2(12)^2}{(12)(30{,}000{,}000)}}$$
$$= 2390 + \frac{(1{,}200{,}000)(7.12)}{844}$$
$$= 2390 + 10{,}120 = 12{,}510 \text{ lb. per sq. in.}$$

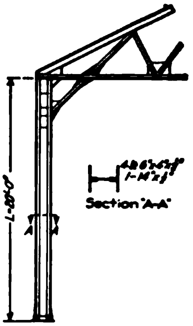


FIG.. 109

102. Eccentrically Loaded Columns.—When the bending moment on a column is caused by the column load, or a part of it, being applied away from the axis of the column, the column is said to be eccentrically loaded. This bending moment may be treated similar to that caused by transverse loads, and approximate results obtained by the use of eqs. (1) and (2).

If the entire bending moment is due to eccentric loading, theoretically exact results may be obtained by the use of the equation

$$f = \frac{P}{A} + \frac{Mc}{KI} \tag{3}$$

in which $K = \cos\left(28.65\,\frac{L}{r}\sqrt{\frac{P}{AE}}\right)$. Values of K for pin ends are given by the curves in the diagram, Fig. 110. If conditions are such as to warrant the assumption of fixed ends, ½L may be used in determining the value of $\frac{L}{r}$ to use in Fig. 110.

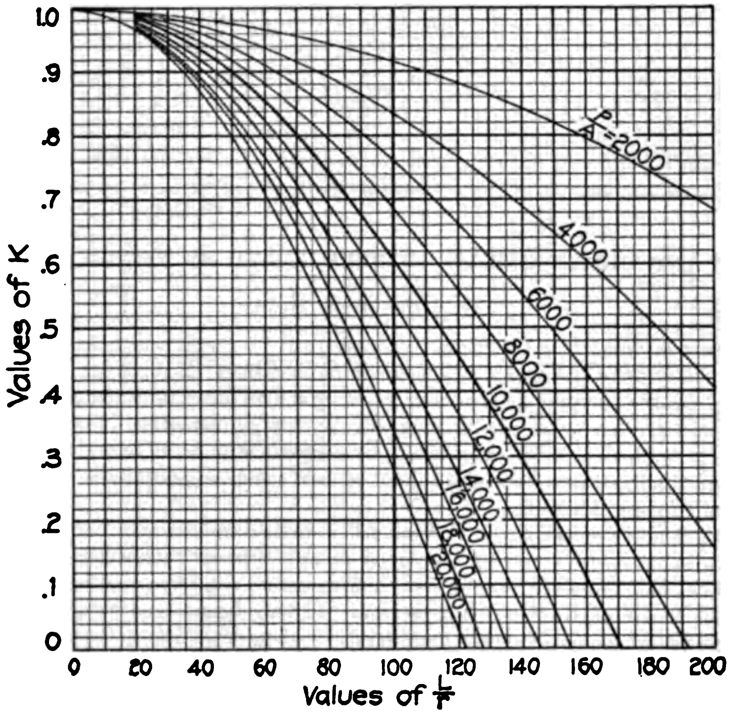


FIG. 110.—Use for eccentrically loaded columns with pin ends. For columns with fixed ends use ½L in determining $\frac{L}{r}$.

The radius of gyration should be taken about an axis normal to the plane of bending. This may not give the greatest value of $\frac{L}{r}$ which should be used in the column formula for determining the allowed unit stress.

Illustrative Problem.—Fig. 111 shows a building column to which floor beams are connected unsymmetrically, causing an eccentric load on the column. If the beams are riveted to the column in addition to resting on shelf angles, it is safe to assume that the load is applied at the face of the column. The deflection of the shelf angle would probably be sufficient to bring the center of pressure very near to the face of the column in any case.

The total load, $P = 90{,}000 + 32{,}000 + 32{,}000 + 40{,}000 = 194{,}000$ lb.

The bending moment, $M = (40{,}000)(5⅞) = 235{,}000$ in.-lb.

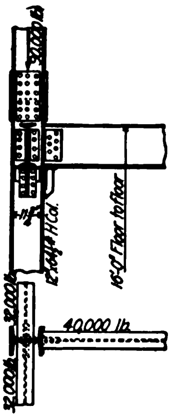


Fig. 111.

From equation (1)

$$f = \frac{194{,}000}{19.00} + \frac{(235{,}000)(5.875)}{499.0}$$

$$= 10{,}210 + 2760 = 12{,}970 \text{ lb. per sq. in.}$$

From equation (2)

$$f = \frac{194{,}000}{19.00} + \frac{(235{,}000)(5.875)}{499.0 - \frac{(194{,}000)(16)^2(12)^2}{(10)(30{,}000{,}000)}}$$

$$= 10{,}210 + 2900 = 13{,}110 \text{ lb. per sq. in.}$$

From equation (3)

$$\frac{L}{r} = \frac{192}{5.13} = 38$$

$$K = 0.935 \text{ (from Fig. 110)}$$

$$f = \frac{194{,}000}{19.00} + \frac{(235{,}000)(5.875)}{(0.935)(499.0)}$$

$$= 10{,}210 + 2960 = 13{,}170 \text{ lb. per sq. in.}$$

Illustrative Problem.—A wooden column 12 in. square supports a concentric load of 70,000 lb. and an eccentric load of 15,000 lb. acting at 4 in. from the face of the column. Compute the maximum stress on the column.

The total load, $P = 70{,}000 + 15{,}000 = 85{,}000$ lb.

The bending moment, $M = (15{,}000)(10) = 150{,}000$ in.-lb.

From equation (1)

$$f = \frac{85{,}000}{144} + \frac{(150{,}000)(6)}{1728} =$$

$$= 590 + 520 = 1110 \text{ lb. per sq. in.}$$

Since the value of $\frac{L}{d}$ is usually small for wooden columns, the value of f, if computed by eqs. (2) and (3), will be practically the same as obtained above. This indicates that the deflection is small.

BENDING AND DIRECT STRESS—CONCRETE AND REINFORCED CONCRETE

By George A. Hool

103. Theory in General.—If a beam is acted upon by forces which are all normal to its length, then the stresses resulting are due to simple bending. If, however, any of the forces acting throughout the length of a beam be inclined, or if additional forces be applied at the ends, then our beam formulas for simple bending will not apply. Likewise, in columns, if the load be eccentrically applied or if lateral pressure be exerted, both bending and direct stresses will result and the ordinary column formulas cannot be used except to give approximate results when the amount of bending is small.

The same combination of stresses occurs also in arch rings and may occur in special cases. The formulas to be derived can be employed in any type of reinforced-concrete structure provided the normal component of the resultant thrust on the given section acts with a lever arm about the center of gravity of the section. In long beams and columns, the deflection resulting from flexure should be given consideration when determining the eccentricity of the axial and inclined forces.

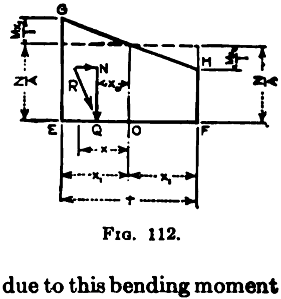


Fig. 112.

Let us first consider structures of plain concrete. The distribution of pressure on any section due to a resultant pressure acting at different points will be explained. Consider a section represented in projection by EF, Fig. 112. When the resultant R acts at the center of gravity O, the intensity of stress is uniform over the section and is equal to the vertical component of R divided by the area of section, or $\frac{N}{A}$. If R acts at any other point, as Q, and if the projection of the section is taken such that the distance x_0 represents the true lever arm of N about the center of gravity, then the force N is equivalent to an equal N at O and a couple whose moment is Nx_0. The intensity of the uniformly varying stress due to this bending moment at a distance x from O is (by the common flexure formula for homogeneous beams) $\frac{Nx_0x}{I}$. in which I is the moment of inertia of the section about an axis through O at right angles to the plane of the paper. At the edges E and F this intensity $= \frac{Nx_0x_1}{I}$. Regarding compressive and tensile stresses as positive and negative respectively, the intensity of stress at edge E is

$$f_c = \frac{N}{A} + \frac{Nx_0x_1}{I}$$

At edge F it is

$$f_c' = \frac{N}{A} - \frac{Nx_0x_1}{I}$$

If the stress f_c' comes out minus, the value obtained is the maximum tension as shown in Fig. 113. In plain concrete construction a greater tension than about 50 lb. per sq. in. should not be allowed.

When we come to reinforced concrete, which is composed of two materials (concrete and steel) with different values of E, then the steel area at any given cross section may be replaced by an area of concrete equal to n times the area of the steel, placed in the plane of the steel reinforcement. This section may be called the transformed section, or section of concrete theoretically equivalent in resistance to the actual section. Under this heading rectangular sections only will be considered and Fig. 114 represents a transformed section as referred to above.

Thus, if A_c is the area of the concrete, and A_0 is the area of the steel $= A_s + A'$; then the equivalent area

$$A = A_c + nA_0 = bt + n(A_s + A')$$

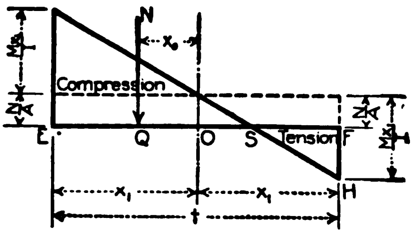


FIG. 113.

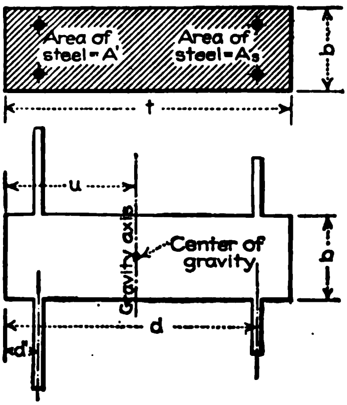


FIG. 114.

If I_c is the moment of inertia of the concrete about the gravity axis, and I_s is the moment of inertia of the steel about the same axis, then

$$I = I_c + nI_s$$

and

$$\frac{(f_c)}{(f_c')} = \frac{N}{A_c + nA_0} \begin{matrix}(+)\\(-)\end{matrix} \frac{Nx_0x_1}{I_c + nI_s}$$

If we denote p and p' by $\frac{A_s}{bt}$ and $\frac{A'}{bt}$ respectively, then the distance from the face most highly stressed to the center of gravity of the transformed section is (by moments)

$$u = \frac{bt\frac{(t)}{2} + nA_sd + nA'd'}{A} = \frac{\frac{bt^2}{2} + nA_sd + nA'd'}{bt + n(A_s + A')} = \frac{t/2 + npd + np'd'}{1 + np + np'}$$

$$I_c = \tfrac{1}{3}bu^3 + \tfrac{1}{3}b(t-u)^3 = \frac{b}{3}\left[u^3 + (t-u)^3\right]$$

$$I_s = A_s(d-u)^2 + A'(u-d')^2$$

$$I = I_c + nI_s = \frac{b}{3}\left[u^3 + (t-u)^3\right] + nA_s(d-u)^2 + nA'(u-d')^2$$

If the reinforcement is symmetrical, then $u = \frac{t}{2}$ and

$$I = \tfrac{1}{12}bt^3 + 2nA_s(\tfrac{1}{2}t - d')^2 = \tfrac{1}{12}bt^3 + 2npbt\,(\tfrac{1}{2}t - d')^2$$

Since, $A = bt + n(A_s + A') = bt + 2nbt(p + p')$

$$\begin{matrix}(f_c)\\(f_c')\end{matrix} = \frac{N}{bt + nbt(p + p')} \begin{matrix}(+)\\(-)\end{matrix} \frac{Nx_0\frac{t}{2}}{\frac{1}{12}bt^3 + 2npbt\left(\frac{1}{2}t - d'\right)^2}$$

104. Compression Over the Whole Section (Case I).—The formulas developed in perceding article apply when the stress is either compression over the entire section, or when there is compression over a portion of the section with a tension over the remainder not exceeding the allowable tensile stress in the concrete. The formulas we shall use will apply to rectangular sections with symmetrical reinforcement and are given in the following form for convenience, letting p_0 denote the quantity $p + p'$:

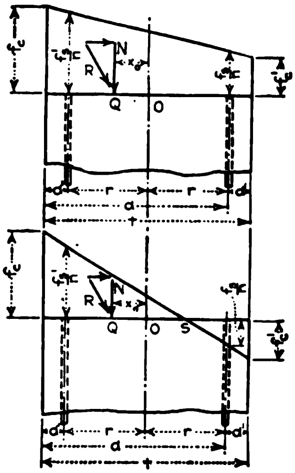


FIG. 115.

$$r = \frac{t}{2} - d'$$

$$\begin{matrix}(f_c)\\(f_c')\end{matrix} = \frac{N}{bt}\left[\frac{1}{1 + np_0} \begin{matrix}(+)\\(-)\end{matrix} \frac{6x_0t}{t^2 + 12np_0r^2}\right] \quad \begin{matrix}(1)\\(2)\end{matrix}$$

By referring to Fig. 115 it will be clear that the stress in the steel is always less than $n \times f_c$; thus, if f_c is kept within its allowable value, the steel is sure to be safely stressed.

Eq. 2 gives a means of determining the eccentricity of the resultant force, or x_0, for which there can be neither tension nor compression at the surface opposite to that near which the thrust acts. To obtain the value of x_0 which gives a zero value to f_c', equate the two terms within the brackets, and solve.

$$\frac{1}{1 + n(p + p')} = \frac{6x_0t}{t^2 + 12np_0r^2}$$

or

$$x_0 = \frac{t^2 + 24npr^2}{1 + n(p + p')} \cdot \frac{1}{6t} \quad (3)$$

If n is assumed to be 15, and, if the steel is embedded in the concrete one-tenth of the total depth from each surface so that $2r = \frac{4}{5}t$, eq. (3) becomes

$$\frac{x_0}{t} = \frac{1 + 28.8p_0}{6 + 90p_0} \quad (4)$$

If the values $n = 15$ and $2r = \frac{4}{5}t$ are substituted in eq. (1), this equation becomes

$$f_c = \frac{N}{bt}\left[\frac{1}{1 + 15p_0} + \frac{x_0}{t} \cdot \frac{6}{1 + 28.8p_0}\right] \quad (5)$$

or if the expression in the brackets is denoted by K,

$$f_c = \frac{NK}{bt} \quad (6)$$

Diagrams 1 to 3 inclusive give values of K for various values of p_0, $\frac{x_0}{t}$, and $\frac{d'}{t}$, and for $n = 15$. The termination of the curves are determined in Diagram 2 by eq. (4) and in the other diagrams by similar equations. For greater values of $\frac{x_0}{t}$, Case I does not apply; that is, there is tension in the concrete and Case II must be employed.

105. Tension Over Part of Section (Case II).—It will be on the safe side and convenient as regards the construction of working diagrams to consider that, when any tension exists in the concrete, the steel carries all tensile stresses. In this case there are three unit stresses to be determined: namely, maximum unit compression in concrete f_c, maximum unit compression in steel f_s', and maximum unit tension in steel f_s. The general formulas developed in Art. 103 are not applicable to this case and the following method may be used:

DIAGRAM 1.

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 15$ and $A' = A_s$.

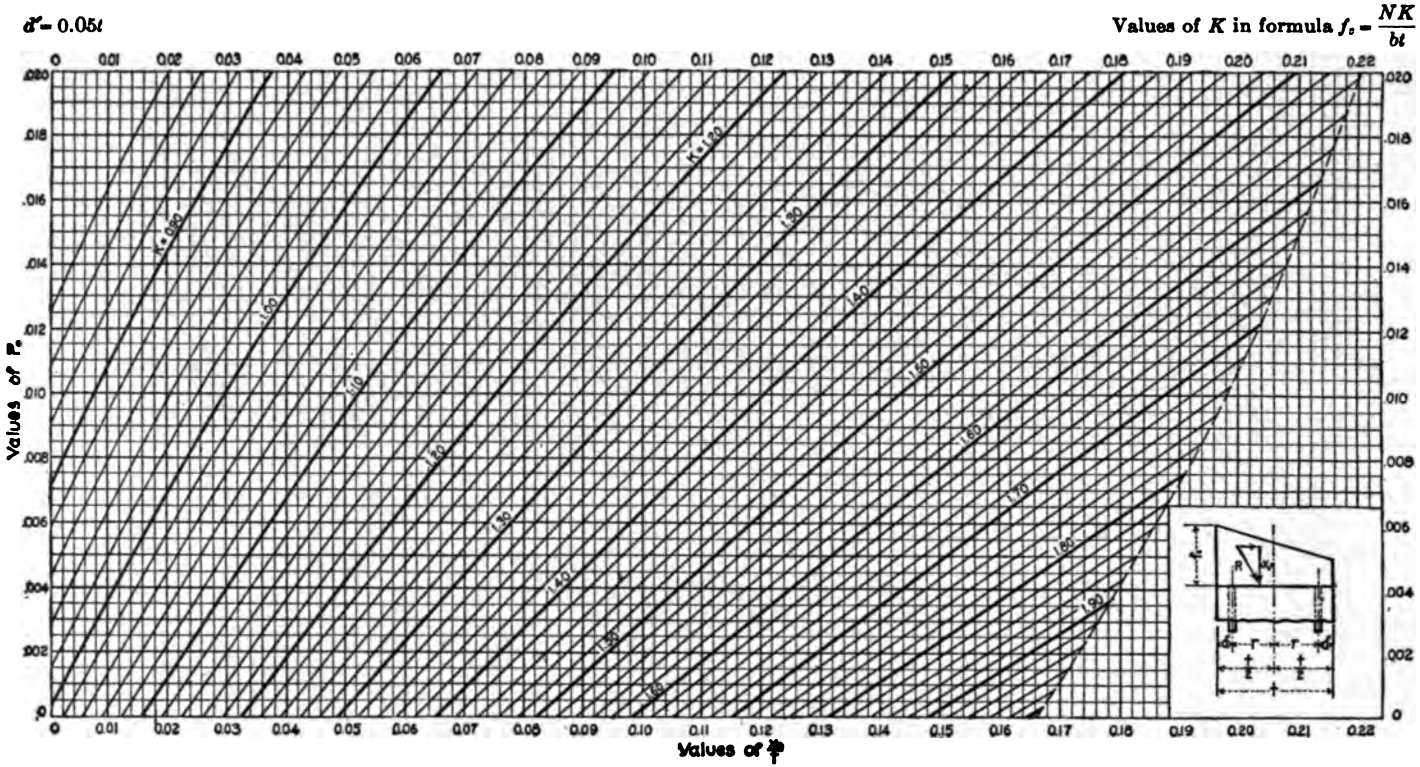


DIAGRAM 2

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 15$ and $A' = A_s$.

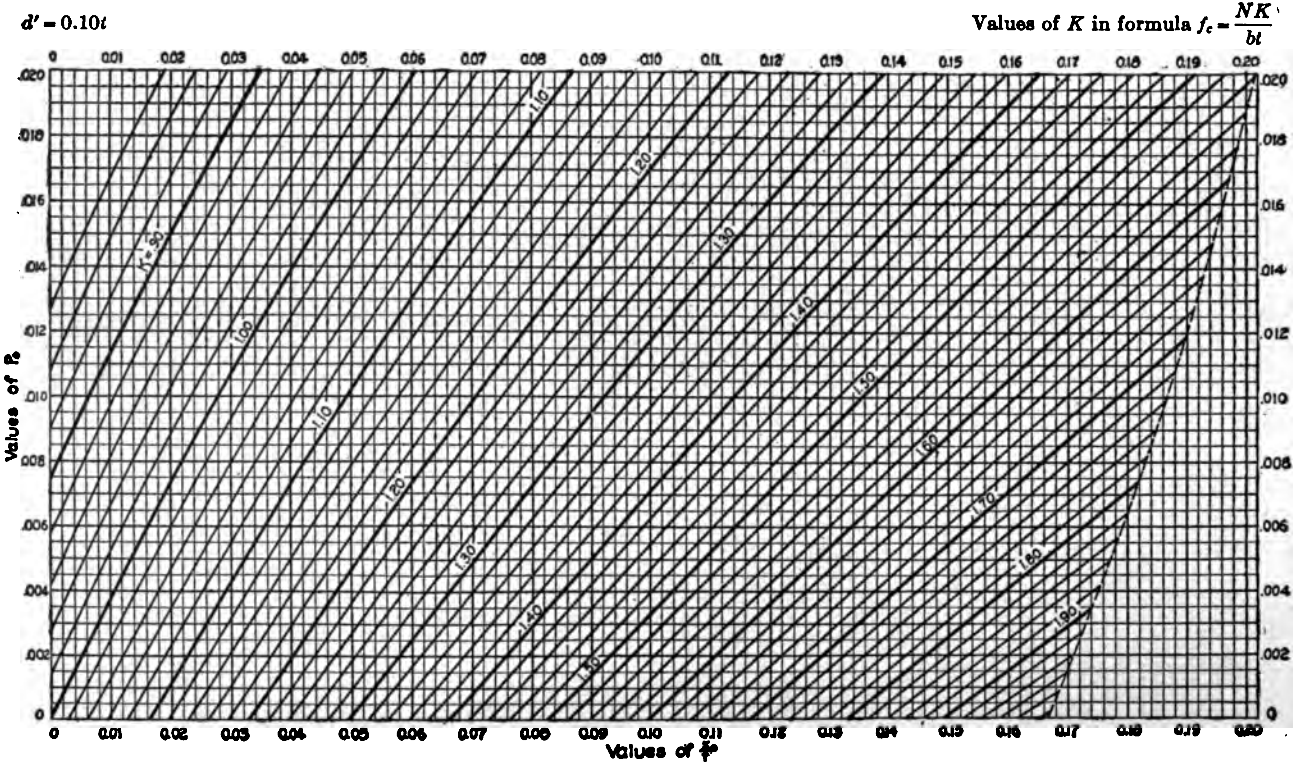


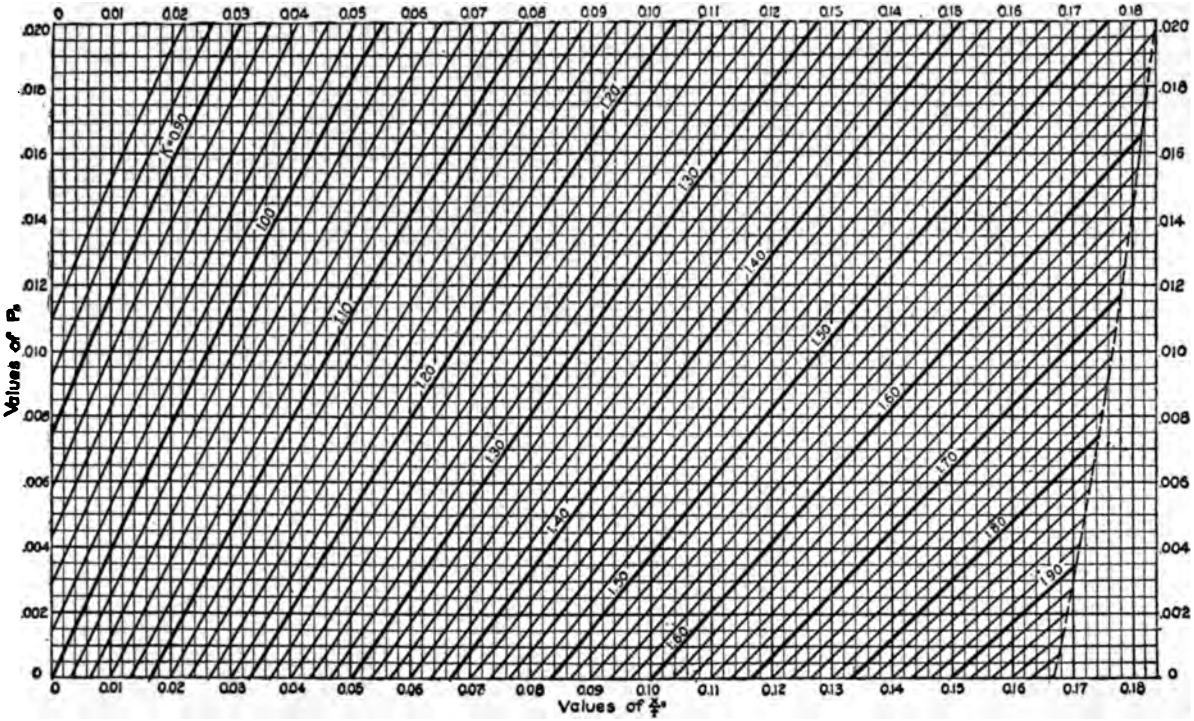
DIAGRAM 3

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 15$ and $A' = A_s$.

$d' = 0.15t$

Values of K in formula $f_c = \frac{NK}{bt}$



Referring to Fig. 116, it follows that

$$f_s' = nf_c\left(1 - \frac{d'}{kt}\right) \qquad (7)$$

and

$$f_s = nf_c\left(\frac{d}{kt} - 1\right) \qquad (8)$$

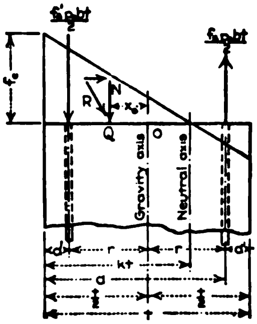


Fig. 116.

Since the resultant fiber stress equals N

$$N = \frac{f_s' p_0 bt}{2} + \frac{f_c bkt}{2} - \frac{f_s p_0 bt}{2}$$

Eliminating f_s' and f_s by means of eq. (7) and (8)

$$N = \frac{f_c bt}{2} \cdot \frac{k^2 + 2np_0k - np_0}{k}$$

$$= \frac{f_c bt}{2} \cdot \frac{k^2 + 2nkp_0 - np_0}{k} \qquad (9)$$

The moment of the stresses about the gravity axis, eliminating f_s' and f_s as before, is

$$M = f_c bt^2\left[\frac{np_0 r^2}{kt^2} + \frac{k}{12}(3 - 2k)\right] \qquad (10)$$

or, if the quantity within the brackets is designated by L, then

$$M = f_c bt^2 L, \text{ or } f_c = \frac{M}{Lbt^2} \qquad (11)$$

The position of the neutral axis must be determined before eq. (11) can be used. Since $Nx_0 = M$, we may multiply eq. (9) by x_0 and equate it to eq. (10). Proceeding in this manner the following equation results

$$k^3 - 3\left(\tfrac{1}{2} - \frac{x_0}{t}\right)k^2 + 6np_0k\frac{x_0}{t} = 3np_0\left(\frac{x_c}{t} + 2\frac{r^2}{t^2}\right) \qquad (12)$$

Diagrams 4, 5 and 6, based on eq. (12), give values of k for various values of p_0, $\frac{x_0}{t}$, and $\frac{d'}{t}$ and for $n = 15$. Diagram 7 gives values of L.

The method of procedure in solving problems under Case II is as follows: (1) Determine k from the proper diagram; (2) find L from Diagram 7; (3) solve eq. (11) for f_c; (4) find unit stresses in the steel from eqs. (7) and (8).

Illustrative Problem.—A beam is 9 in. wide and 20 in. deep. The reinforcement both above and below consists of one steel rod 1 in. in diameter embedded at a depth of 2 in. At a certain section, the normal component of the resultant force is 60,000 lb., acting at a distance of 3.4 in. from the gravity axis. Assume $n = 15$. Compute the maximum unit compressive stress in the concrete.

$$p_0 = \frac{A_s + A'}{bt} = \frac{(2)(0.7854)}{(9)(20)} = 0.0087$$

$$\frac{x_0}{t} = \frac{3.4}{20} = 0.17$$

For these values of p_0 and $\frac{x_0}{t}$, Diagram 2 gives $K = 1.70$ and shows that the problem falls under Case I. Then by eq. (6)

$$f_c = \frac{NK}{bt} = \frac{(60{,}000)(1.70)}{(9)(20)} = 567 \text{ lb. per sq. in.}$$

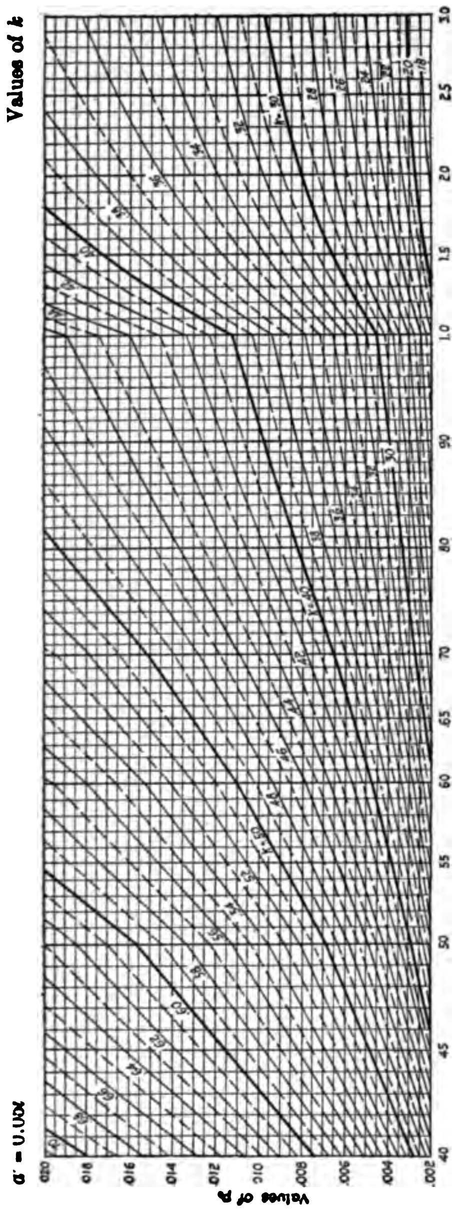
Illustrative Problem.—Change the eccentricity of the preceding problem to 6 in. and solve.

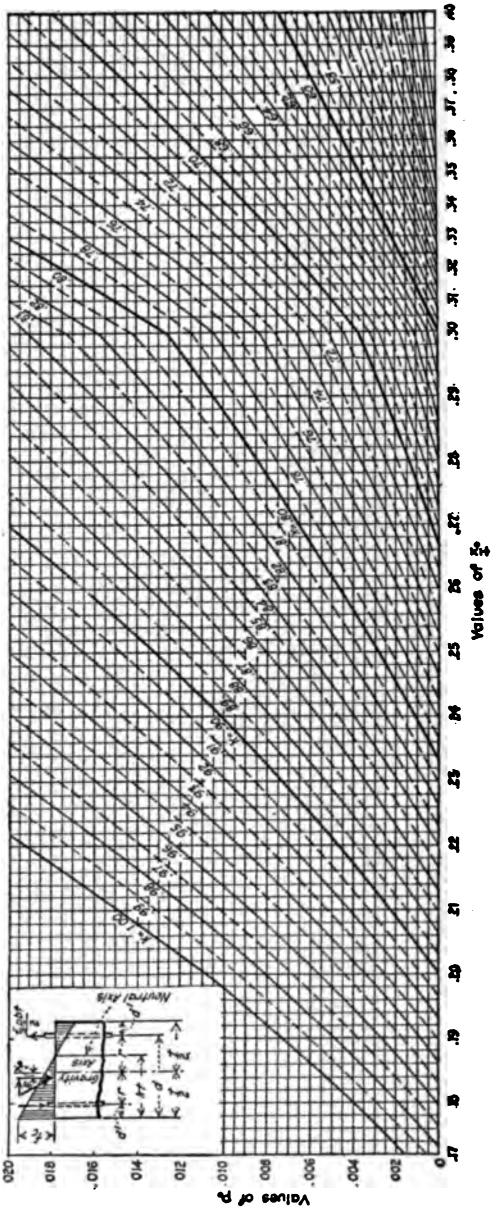
$$\frac{x_0}{t} = \frac{6}{20} = 0.30$$

For $p_0 = 0.0087$ and $\frac{x_0}{t} = 0.30$, Diagram 2 shows that $\frac{x_0}{t}$ is too great for the problem to come under Case I. The method of procedure for Case II must then be followed.

Diagram 5 gives $k = 0.73$ for the values of p_0 and $\frac{x_0}{t}$ given above. With $k = 0.73$ and $p_0 = 0.0087$, Diagram 7 shows L to be 0.123. Solving equation (11)

$$f_c = \frac{M}{Lbt^2} = \frac{(60{,}000)(6)}{(0.123)(9)(20)^2} = 815 \text{ lb. per sq. in.}$$


Values of k
d' = 0.10d
Values of p
Values of p'


Values of p
Values of $\frac{f_c}{f}$
Neutral Axis
Gravity Axis

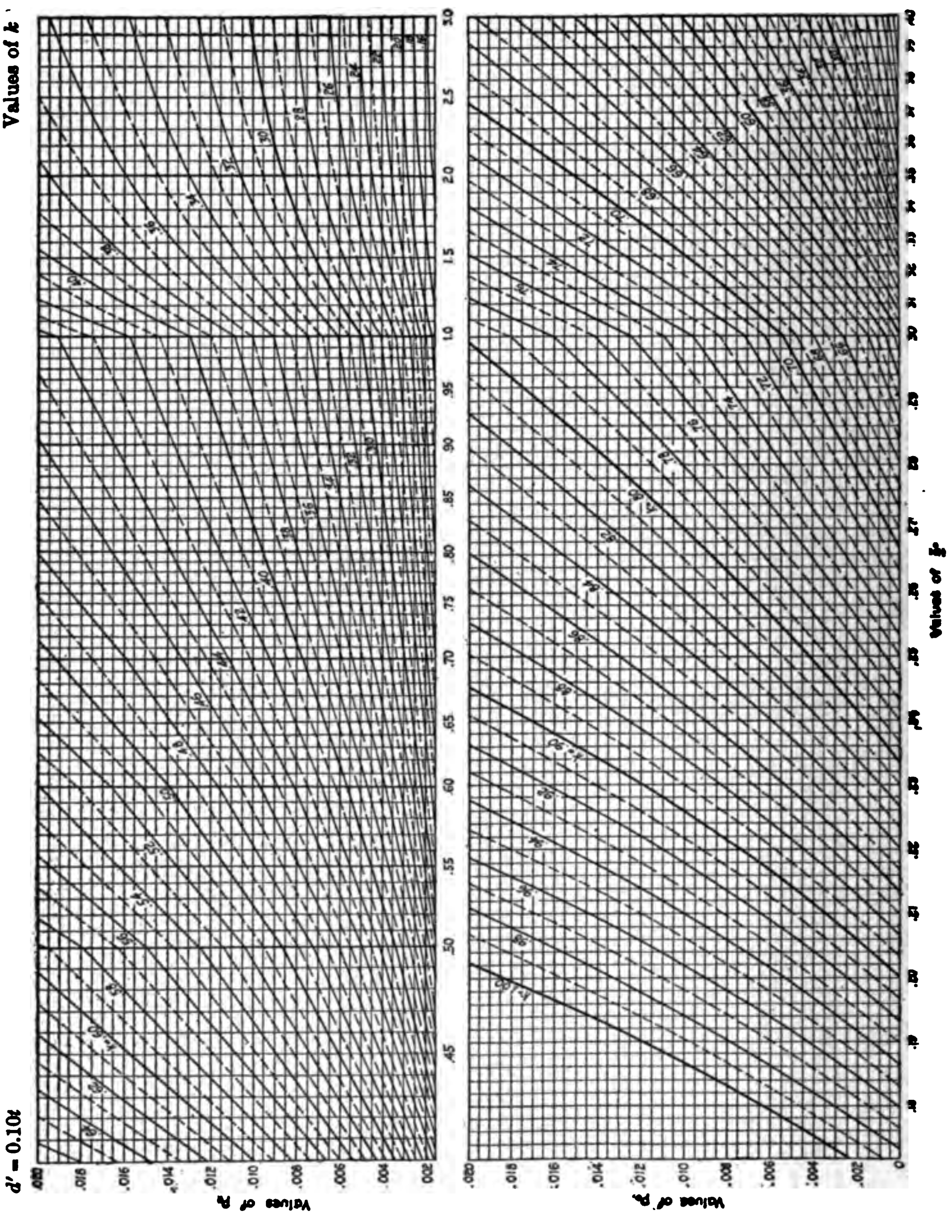


DIAGRAM 5

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 15$ and $A' = A_s$.

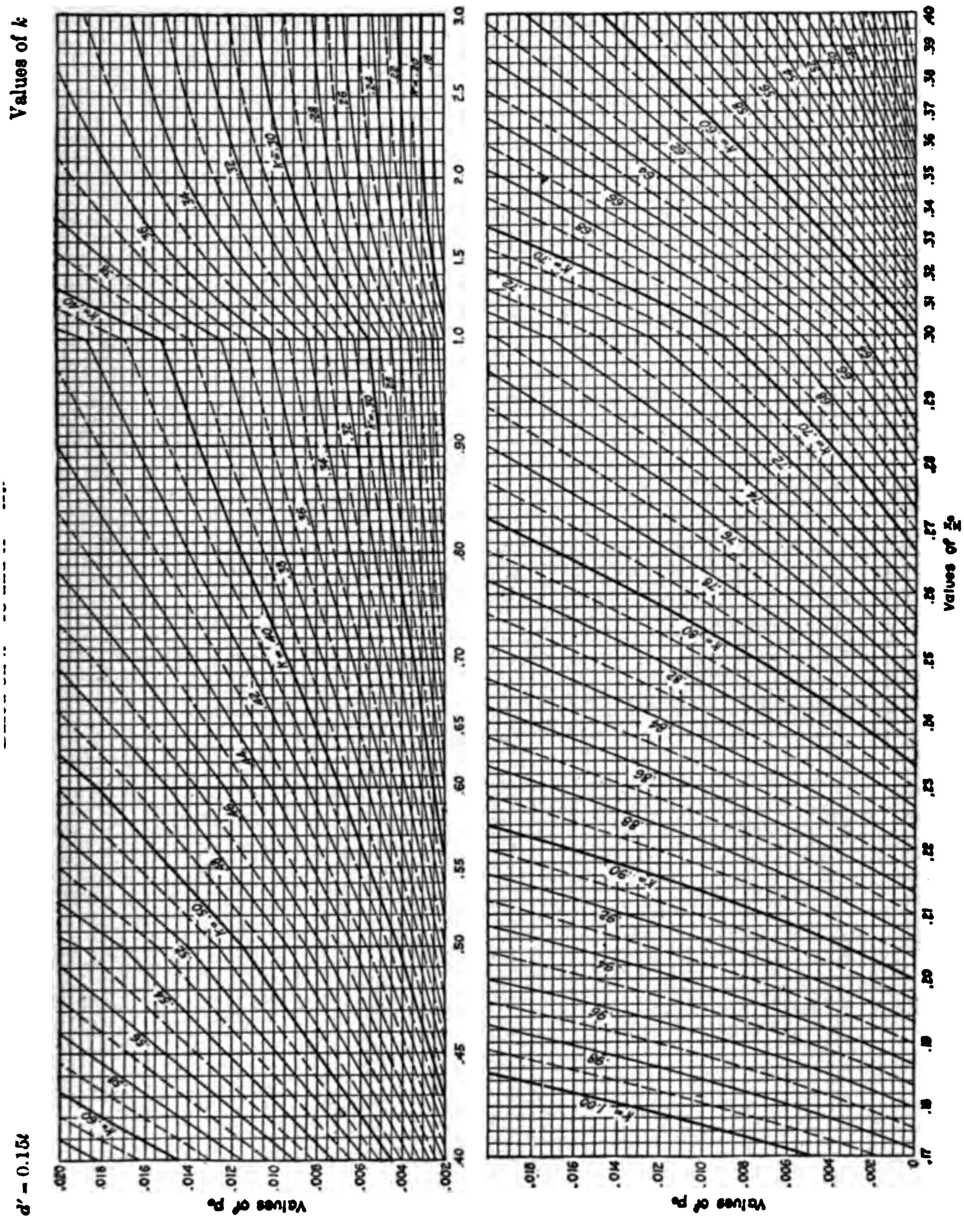
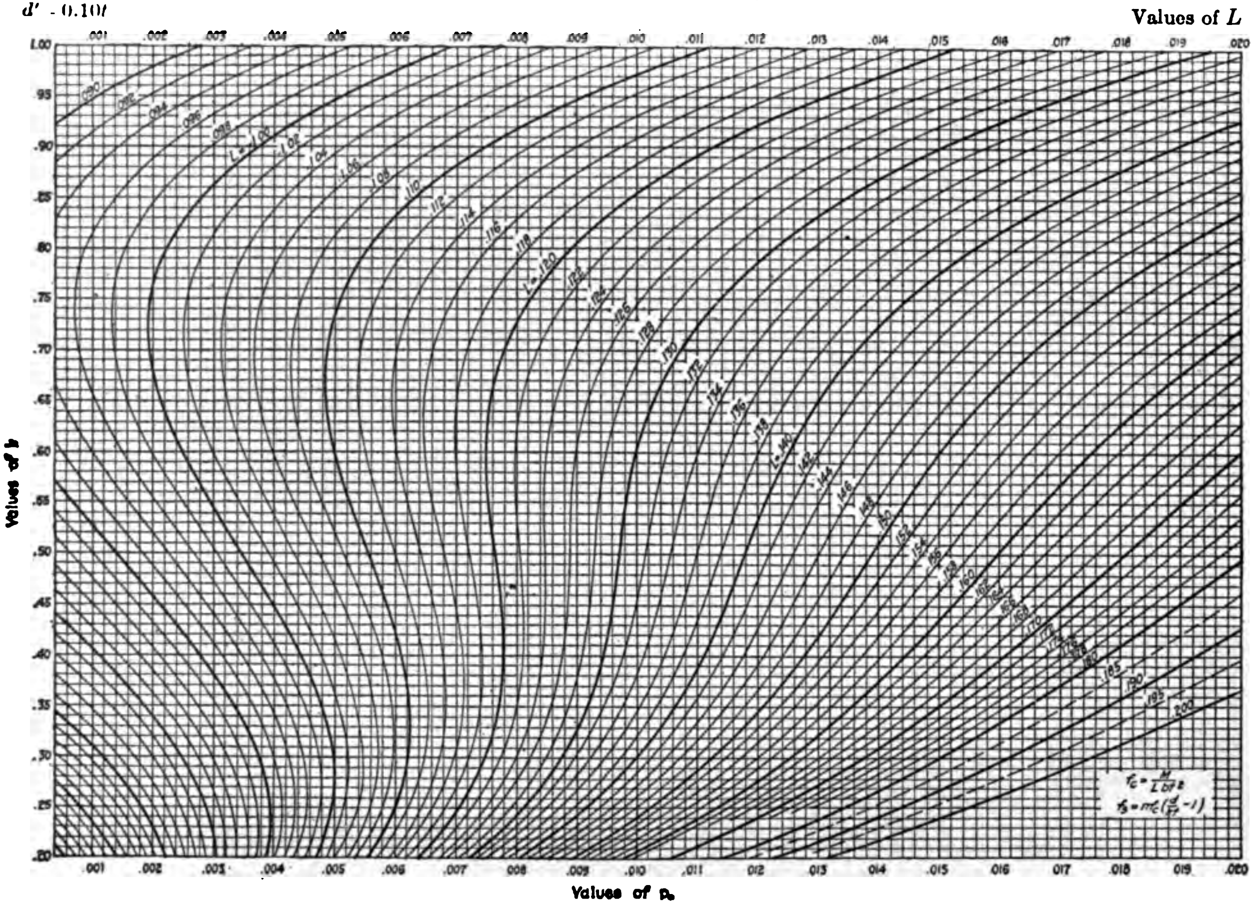

Values of k
d' = 0.15t
Values of p_e
.020
.018
.016
.014
.012
.010
.008
.006
.004
.002
.40
.45
.50
.55
.60
.65
.70
.80
.90
1.0
1.5
2.0
2.5
3.0
k = .60
.58
.56
.54
.52
k = .50
.48
.46
.44
.42
k = .40
.38
.36
.34
.32
k = .30
.28
.26
.24
.22
k = .20
.18
Values of p_e
.018
.016
.014
.012
.010
.008
.006
.004
.002
0
Values of e/t
.17
.18
.19
.20
.21
.22
.23
.24
.25
.26
.27
.28
.29
.30
.31
.32
.33
.34
.35
.36
.37
.38
.39
.40
k = 1.00
.98
.96
.94
.92
k = .90
.88
.86
.84
.82
k = .80
.78
.76
.74
.72
k = .70
.68
.66
.64
.62
k = .60

DIAGRAM 7

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 15$ and $A' = A_s$.



For $d' = 0.05t$ divide p_o by 0.700 and find value of L

Using eq. (8)

$$f_s = nf_c\left(\frac{d}{kt} - 1\right) = (15)(815)\left(\frac{18}{0.73 \times 20} - 1\right) = 2830 \text{ lb. per sq. in.}$$

The stress f_s' may be found by eq. (7) but is always less than $n \times f_c$.

UNSYMMETRICAL BENDING

By W. S. Kinne

In certain types of construction it is found necessary to place beam sections with their axes of symmetry at an angle to the plane of loading, as shown in Fig. 117. For the conditions shown, the principal axes of the section and the plane of loading do not coincide, as assumed in the cases considered in the preceding chapters. Bending of the nature shown in Fig. 117 is known as *unsymmetrical bending*. The brief treatment of the subject given in this chapter is confined to cases of pure bending only.

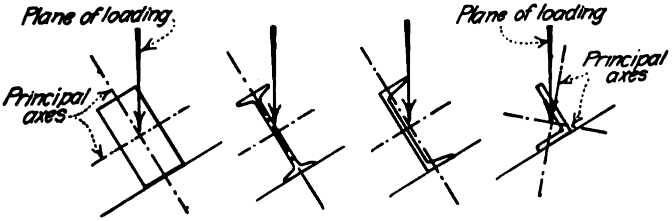


Fig. 117.

106. General Formulas for Fiber Stress and Position of Neutral Axis for Unsymmetrical Bending.—The full line rectangle of Fig. 118 shows a right section of a straight beam of uniform cross section subjected to a bending moment M acting in a plane which passes through the longitudinal axis of the beam, making an angle θ with OX, one of the principal axes of the section. In the work to follow, point O will be taken as the origin of coördinates, and the principal axes of the section, OX and OY of Fig. 118, will be taken as the coördinate axes. As the formulas are greatly simplified thereby, the properties of the section will be referred to the principal axes. These quantities are given directly or are easily calculated from data given in any of the structural steel handbooks.

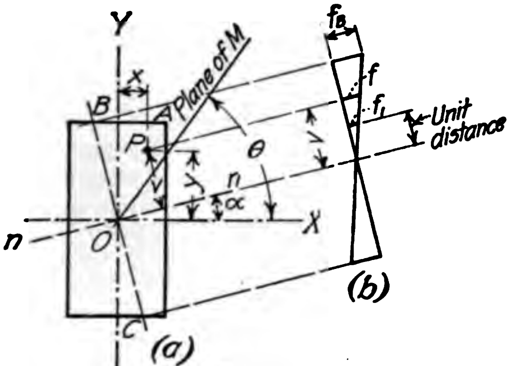


Fig. 118.

Let n-n of Fig. 118 (a) represent the position of the neutral axis of the assumed section for the given plane of loading, and let α be the angle which the neutral axis makes with OX. Angle α and also angle θ are to be considered as positive when measured in a counter clockwise direction. Fig. 118 (b) shows the fiber stress conditions on a line at right angles to the neutral axis, assuming linear distribution of stress.

Let P, Fig. 118 (a), be any fiber of infinitely small area a at a distance v from the neutral axis. Assuming positive (clockwise) moment, the intensity of fiber stress at P is $f = -f_1v$, where f_1 is the fiber stress intensity at unit distance from the neutral axis. The minus sign indicates compression, for, as shown in Fig. 118, the fiber under consideration is above the neutral axis.

The moment of resistance of the section, which is equal to the stress on each fiber multiplied by its distance from the neutral axis is $M_R = \Sigma f_1av^2$, where Σ represents the summation for the entire rectangle. But Σav^2 is the moment of inertia of the section about the neutral axis (see Art. 61*c*), which will be denoted by I_n. With this notation, $M_R = f_1 I_n$. Substituting for f_1 its value $-\frac{f}{v}$, we have

$$M_R = -\frac{f}{v} I_n$$

Since the beam is in equilibrium, the moments of internal and external forces at any section must be equal. Taking the neutral axis as the axis of moments, the external moment in a plane perpendicular to the neutral axis is $M \sin(\theta - \alpha)$. The moment of internal forces is the resisting moment of the section, which is given above as $M_R = -\frac{f}{v} I_n$. Equating these two expressions

$$f = -M \frac{v \sin(\theta - \alpha)}{I_n}$$

This expression can be placed in a more convenient form by referring both v and I_n to the principal axes of the section. From Fig. 118 (*a*), $v = y \cos \alpha - x \sin \alpha$. Values of x and y are positive when measured upward and to the right. In treatises on Mechanics it is shown that in terms of the principal moments of inertia of the section, I_x and I_y, the moment of inertia about the neutral axis is $I_n = I_x \cos^2\alpha + I_y \sin^2 \alpha$. Substituting these values in the general equation given above

$$f = -M \frac{(y \cos \alpha - x \sin \alpha) \sin(\theta - \alpha)}{(I_x \cos^2 \alpha + I_y \sin^2 \alpha)}$$

To determine the relation between the angles α and θ, a summation of external moments about any two axes will yield two independent equations from which the desired relation can be obtained. Two convenient axes are OX and OY, the principal axes of the section.

For axis OX, using the value of v given above,

$$M \sin \theta = \Sigma f_1 avy = \Sigma f_1 (y^2 \cos \alpha - xy \sin \alpha) a$$

But Σay^2 is the moment of inertia of the section about the axis OX, which is denoted by I_x, and Σaxy is the product of inertia of the section, which is zero for principal axes. Then,

$$M \sin \theta = f_1 I_x \cos \alpha$$

In the same way, for axis OY,

$$M \cos \theta = -f_1 I_y \sin \alpha$$

Solving these equations for α, we have

$$\tan \alpha = -\frac{I_x}{I_y} \cot \theta \qquad (1)$$

which is the general equation for direction of the neutral axis for bending in any given direction.

Substituting the value of α, as given by eq. (1), in the above expression for f, we have

$$f = -M\left(\frac{I_y y \sin \theta + I_x x \cos \theta}{I_x I_y}\right)$$

which is the general expression for fiber stress at any point in a section of a beam due to a moment M acting in a plane at an angle θ to the axis OX. This equation can be made to apply to any particular point, as A, Fig. 118 (*a*), an extreme point of the section, by substituting for x and y the coördinates of the point in question. Let these coördinates be x_A and y_A, and let f_A be the resulting fiber stress. Then

$$f_A = -M\left(\frac{I_y y_A \sin \theta + I_x x_A \cos \theta}{I_x I_y}\right) \qquad (2)$$

Since in eqs. (1) and (2), x_A, y_A, I_x, and I_y are constants for any given point in a given section, it follows that the direction of the neutral axis and the intensity of the stress are dependent upon the value of θ. For $\theta = 90$ deg., eq. (2) becomes $f_A = -My_A/I_x$, and eq. (1) becomes, $\tan \alpha = 0$, or, $\alpha = 0$ deg. Again, for $\theta = 0$ deg., eq. (2) becomes, $f_A = -Mx_A/I_y$, and eq. (1) becomes, $\tan \alpha =$ infinite, or, $\alpha = 90$ deg.

It will be noted that these special values of fiber stress are of the form given in Sect. 1, Art. 61*c*, that is, $f = M(c/I)$, where I/c is known as the *section modulus* of the section. Also, the neutral axis in each case is perpendicular to the plane of loading. This condition holds true only when the plane of loading coincides with one of the principal axes of the section, at which time the other principal axis is the neutral axis, a fact which can be verified by a study of the values of α given above.

Eq. (2) can also be written in the form

$$f_A = -\left[(M \sin \theta)\frac{y_A}{I_x} + (M \cos \theta)\frac{x_A}{I_y}\right] \tag{3}$$

As shown by the substitutions made above, this expression is the sum of two quantities obtained by resolving the bending moment into its components parallel to the principal axes of the section. Then by adding the fiber stresses due to these component moments, there is obtained an expression identical to eq. (3), and on transformation, to eq. (2). This offers a simple and easily remembered method for the calculation of fiber stresses due to unsymmetrical bending.

107. Flexural Modulus.—In Sect. 1, Art. 61c, it is shown that for bending in the plane of a principal axis, the fiber stress in a beam is given by an expression of the form

$$f = M(c/I) = \frac{M}{I/c}$$

where for any given section I/c is a constant quantity known as the *section modulus.*

In eq. (2), the reciprocal of the expression in parenthesis is seen to be a quantity of the same dimensions as the section modulus, but more general in nature, as it involves planes of loading other than the principal axes. Let S denote this quantity. Then

$$f = M/S \tag{4}$$

where

$$S = \frac{I_xI_y}{I_yy_A \sin \theta + I_xx_A \cos \theta} \tag{5}$$

The expression of eq. (5) is known as the *flexural modulus* of the section. For any given direction of loading and for any given point in a section, S is a constant. Having given the value of S for any given conditions, the resulting fiber stress is obtained by substitution in eq. (4).

108. The S-line.—For any point in a given section, the value of S as given by eq. (5), gives a measure of the strength of the section for bending in any direction.

From Analytical Geometry it can be shown that eq. (5) is in the form of the polar equation of a straight line. A convenient graphical representation of the variation in flexural modulus for various planes of bending is thus readily obtained. In Fig. 119, the line C–D shows the variation in flexural modulus for point A, one of the corners of a rectangular section. This is known as an S-line of the section. The vector OE shows the value of S_A for bending moment at an angle θ to OX, one of the principal axes of the section.

It will be found convenient to express the equation of the S-line in terms of rectangular coördinates. If $y = S \sin \theta$ and $x = S \cos \theta$ be placed in eq. (5), we have

$$y = -\frac{I_x}{I_y}\frac{x_A}{y_A} \cdot x + \frac{I_x}{y_A} \tag{6}$$

which is the slope form of the equation of the S-line for point A, Fig. 119.

109. S-polygons.—Every extreme point or corner of a section is liable to become, at some time, a point of maximum stress. In order to determine graphically which of several extreme points is the one having maximum stress, it is necessary to plot the S-lines for all such points. In this way the values of S for the several points can be compared.

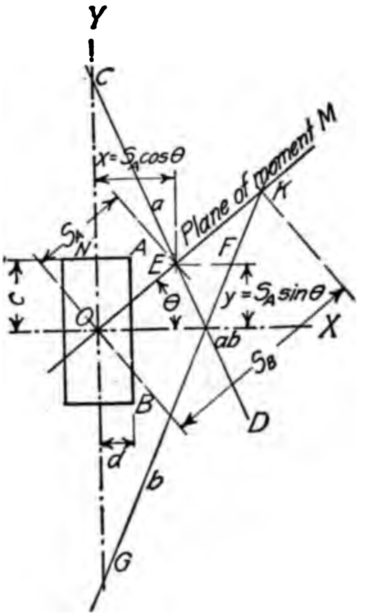


Fig. 119.

In Fig. 119, the line F–G represents the S-line for point B. The equation for this line is similar to that for point A, and can be obtained from eq. (6) by substituting x_B and y_B, the coördinates of B, in place of the corresponding values for A. Thus the required equation is

$$y = -\frac{I_x}{I_y}\frac{x_B}{y_B}\cdot x + \frac{I_x}{y_B} \tag{7}$$

As before, the vector OK represents the value of S_B for bending at an angle θ to OX. Eq. (4) shows that the point of greatest stress is the one with the least S. Since vector OE is smaller than OK, fiber A has a greater stress than fiber B for the given plane of bending.

Equations similar to eqs. (6) and (7) can be made up for each extreme point of the section. If all these S-lines are plotted in Fig. 119, they will enclose a figure known as an *S-polygon.* Examples of S-polygons are given in Art. 110.

S-polygons can be constructed by two different methods. One method of construction is carried out by plotting the S-lines, as given by equations similar to eqs. (6) and (7). The S-lines for adjacent points of the section are run to an intersection, and the resulting enclosed figure will form the desired S-polygon. Another and better method locates the coördinates of the points of intersection of adjacent S-lines by the methods of Analytical Geometry. This is done by solving simultaneously equations such as eqs. (6) and (7) for adjacent extreme points of the section. This process is repeated for each pair of adjacent points of the section. The resulting coördinates are plotted and connected up to form the complete S-polygon. This latter method, which is the one used in the work to follow, will now be explained in detail.

To determine the coördinates of the intersection of the S-lines for points A and B of Fig. 119, the equations for these lines, as given by eqs. (6) and (7), are to be solved simultaneously. Let x_{ab} and y_{ab} be the coördinates of the point of intersection—that is, the values of x and y common to the two equations. Then

$$x_{ab} = \frac{I_y\,(y_B - y_A)}{x_A y_B - x_B y_A} \tag{8}$$

$$y_{ab} = \frac{I_x(x_A - x_B)}{x_A y_B - x_B y_A} \tag{9}$$

Similar values for pairs of adjacent extreme points will differ only in the subscripts of x and y. The resulting values, when plotted and connected up, will form the desired S-polygon.

Eqs. (8) and (9) give general values for the coördinates of points of intersection of S-lines. Under certain conditions these equations take on a much simpler form. As shown in Fig. 119, extreme points A and B form an edge which is parallel to the axis OY, and $x_A = x_B = d$. If these values be placed in eqs. (8) and (9), the resulting equations are

$$x_{ab} = I_y/d \tag{10}$$

and

$$y_{ab} = 0 \tag{11}$$

For two adjacent points, as A and N of Fig. 119, which form a side parallel to the OX axis, $y_A = y_N = c$, and eqs. (8) and (9) become

$$x_{an} = 0 \tag{12}$$

and

$$y_{an} = I_x/c \tag{13}$$

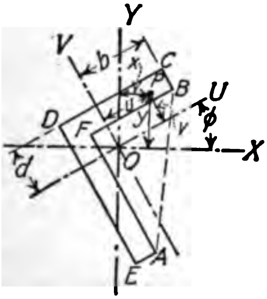


Fig. 120.

In cases where S-polygons are to be determined for sections which are irregular in outline, as shown in Fig. 120, where some of the sides of the section are not parallel to the principal axes, OX and OY, eqs. (8) and (9) must be used in the determination of the coördinates of the S-polygon. It is possible, however, to make use of certain short cuts which will greatly simplify the calculations. This is done by revolving the axes of reference for coördinates of extreme points through such an angle that the side in question and the axes of reference will be parallel.

Suppose that the coördinates of the intersection points of the S-lines for adjacent points B and C of Fig. 120 are required. Choose a set of coördinate axes OU and OV, such that OV is parallel to the side C–B. Let ϕ be the angle which OU makes with OX, a principal axis of the section. This angle is to be considered as positive when measured counter-clockwise. If x and y be the coördinates of any point P with respect to the OX and OY axes, and u and v be the coördinates of the same point with respect to the OU and OV axes, it can be shown from Fig. 120 that

$$y = v \cos \phi + u \sin \phi$$

and

$$x = u \cos \phi - v \sin \phi$$

In these equations u and v are considered positive when measured upward and to the right with respect to the axes OU and OV.

Substituting in eqs. (8) and (9) values of x and y as given by the above equations, using subscripts to correspond to the point in question, we have

$$x_{bc} = \frac{I_y[(u_B - u_c)\sin \phi + (v_B - v_c) \cos \phi]}{(u_c v_B - u_B v_c)}$$

and

$$y_{bc} = \frac{I_x[(v_B - v_c)\sin \phi + (u_c - u_B)\cos \phi]}{(u_c v_B - u_B v_c)}$$

Since the angle ϕ was so chosen that OV is parallel to side B–C, we have $u_B = u_c = b$, as shown in Fig. 120. Substituting these values in the above equations, we have

$$\left.\begin{aligned} x_{bc} &= \frac{I_y \cos \phi}{b} \\ y_{bc} &= \frac{I_x \sin \phi}{b} \end{aligned}\right\} \qquad (14)$$

In using eq. (14) it is to be noted that the coördinates x_{bc} and y_{bc} are referred to the principal axes of the section, for in deriving the equations given above, only the coördinates of the extreme points of the section were referred to the axes OU and OV.

In a like manner, the coördinates of the intersection point of the S-lines for points D and C of the edge D–C, Fig. 120, parallel to the OU axis, are

$$\left.\begin{aligned} x_{dc} &= -\frac{I_y \sin \phi}{d} \\ y_{dc} &= +\frac{I_x \cos \phi}{d} \end{aligned}\right\} \qquad (15)$$

where $d = v_D = v_c$.

In this discussion it has been assumed that C–B and C–D are perpendicular sides. If they are not perpendicular, it will be necessary to determine the proper value of ϕ for each side in order to obtain the desired results.

When a section has a re-entrant corner, such as F, Fig. 120, it is quite evident that for any given plane of bending the fiber stress at F is less than at D. This is due to the fact that F is nearer the neutral axis for the plane of bending than is D. Hence the S-line for point D lies inside that for point F, whose S-line will be located entirely outside the S-polygon for the section. It is therefore necessary to draw S-lines only for the outside points of the section, as these points will be farthest from the successive positions of the neutral axis, and therefore have the least values of flexural modulus.

A simple and definite test for the determination of the points for which S-lines need be drawn is given by rolling a right line around the perimeter of the section for which the S-polygon is to be drawn. Since the successive positions of this rolling line are parallel to successive positions of the neutral axis as the plane of bending varies through all possible angles, it is evident that the points touched by this rolling line are those farthest removed from the neutral axis, and that they are points of possible maximum stress. It is to be noted that in rolling around the section, the right line will not cut across the section, which at once eliminates re-entrant corners.

For the section of Fig. 120, a line rolling as described above will touch points A, B, C, D, and E. The polygon formed by connecting these points is known as the *circumscribing polygon* of the section.

110. Construction of S-polygons.—The S-polygons for a few of the standard sections used as beams will now be calculated and constructed in order to illustrate the principles set forth in the preceding articles.

110a. S-polygon for a Rectangle.—The S-polygon for a 2 × 12-in. rectangle will be computed and constructed. Fig. 121 shows the section with the principal axes OX and OY in position. The principal moments of inertia are $I_x = 288$ in.^{4}, and $I_y = 8$ in.^{4}; and the coördinates of the extreme points of the section, which in this case are also apices of the circumscribing polygon, are, $x_A = +1$, $y_A = +6$; $x_B = +1$, $y_B = -6$; $x_C = -1$, $y_C = -6$; and, $x_D = -1$, $y_D = +6$.

Since the sides of the rectangle are all parallel to the principal axes of the section, the coördinates of the apices of the S-polygon are given by eqs. (10) to (13). For sides A-B and C-D, which are parallel to the OY axis, eqs. (10) and (11) are to be used. With $I_y = 8$ in^4., and $a = x_A = x_B = +1$, eq. (10) gives, $x_{ab} = +8/1 = +8$ in.^{3}; and eq. (11) gives, $y_{ab} = 0$. This apex of the S-polygon is located on the OX axis, as shown in Fig. 121. For side D-C the substitutions are similar to those for A-B, differing only in the signs of the coördinates of the extreme points. It will be found from eqs. (10) and (11) that $x_{cd} = -8$ in.^{3}, and $y_{cd} = 0$.

Sides A-D and C-B, which are parallel to the OX axis, require the use of eqs. (12) and (13). For side A-D, with $I_x = 288$ in.^{4} and $c = y_A = y_D = +6$ in., eq. (12) gives $x_{ad} = 0$, and eq. (13) gives $y_{ad} = +288/6 = +48$ in.^{3} From the same equations we find for C-B, $x_{cb} = 0$, and $y_{cb} = -48$ in.^{3} These apices of the S-polygon are located on the OY axis, one above and the other below the OX axis, as shown in Fig. 121.

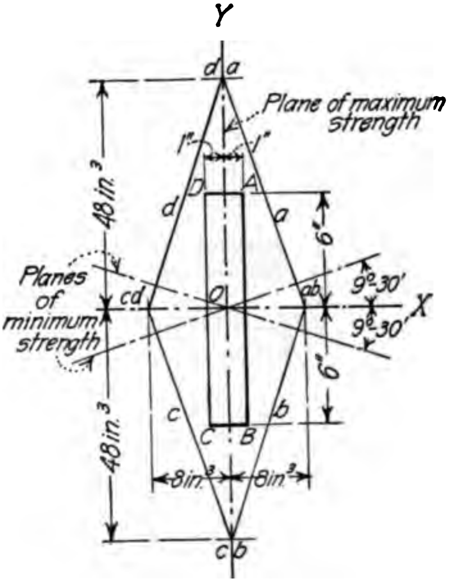


FIG. 121.—S-polygon for 2 × 12-in. rectangle.

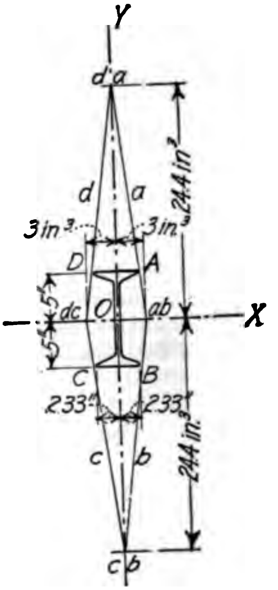


FIG. 122.—S-polygon for a 10-in. 25-lb. I.

The complete S-polygon is obtained by plotting the points determined above, and connecting by straight lines the points which have a common letter, as, for example, points da and ab are connected by a line denoted by a in Fig. 121; likewise, points ab and bc are connected by a line denoted by b. Following this procedure for all points, the complete S-polygon is obtained, as shown in Fig. 121.

It will be noted that the coördinates of the apices of the S-polygon, as y_{da}, x_{ab}, etc., are equal to the section moduli of the rectangle for axes OX and OY respectively. This offers a convenient method for constructing this polygon without the use of eqs. (10) to (13). The section moduli can be calculated or taken from the steel handbooks, plotted on the principal axes of the section, and the polygon drawn as described above.

110b. S-polygon for a 10-in. 25-lb. I-beam.—Fig. 122 shows the S-polygon for a 10-in. 25-lb. I-beam. As the circumscribing polygon for the I-beam is a rectangle, the methods of calculation are exactly the same as given above for the rectangular section. The detail calculations will not be given here. All data are shown on Fig. 122.

110c. S-polygon for a 10-in. 25-lb. Channel.—The circumscribing polygon for a channel is also a rectangle, but as the axis OY is not an axis of symmetry, the resulting S-polygon will not be symmetrical about the OY axis, as in the case of the rectangle and I-beam.

For a 10-in. 25-lb. channel, $I_x = 91.0$ in.^{4}, $I_y = 3.4$ in.^{4}; $x_A = +2.28$, $y_A = +5.0$; $x_B = +2.28$, $y_B = -5.0$; $x_C = -0.62$, $y_C = -5.0$; and, $x_D = -0.62$, $y_D = +5.0$. (All coördinates in inches.)

Substituting these values in eqs. (10) to (13), the coördinates of the apices of the S-polygon are found to be

$$x_{ab} = +3.4/2.28 = +1.49 \text{ in.}^3$$
$$y_{ab} = 0$$
$$x_{bc} = -91.0/5.0 = -18.2 \text{ in.}^3$$
$$y_{bc} = 0$$
$$x_{cd} = -3.4/0.62 = -5.48 \text{ in.}^3$$
$$y_{cd} = 0$$
$$x_{da} = +91.0/5.0 = +18.2 \text{ in.}^3$$
$$y_{da} = 0$$

These values when plotted give the S-polygon of Fig. 123, on which all data are shown.

110*d*. S-polygon for an Angle Section.—The S-polygon for a 5 × 3½ × ½-in. angle will be computed and constructed. In the case of angle sections, the steel handbooks do not give directly the principal moments of inertia of the section. The moments of inertia given are those for the gravity axes of the section (*OU* and *OV* of Fig. 124). By the application of a few well-known principles, the location of the principal axes and the values of the principal moments of inertia are readily determined.

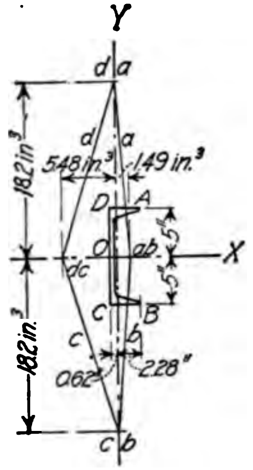


Fig. 123.—S-polygon for a 10-in., 25-lb. channel.

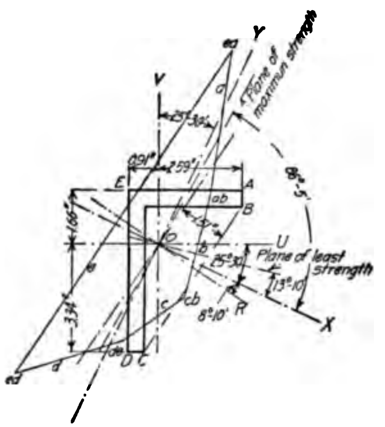


Fig. 124.—S-polygon for a 5 × 3½ × ½-in. angle.

Fig. 124 shows the angle section with the gravity axes *OU* and *OV* in position. The moments of inertia for these axes are $I_u = 10.0$ in.⁴, and $I_v = 4.0$ in.⁴ Moments of inertia for principal axes are not given directly. However, the minimum radius of gyration of the section is given; this is a property of the minor principal axis of the section. From Art. 92, $I = Ar^2$, where A = area of section, and r = radius of gyration. For the section in question, $A = 4.0$ sq. in., and $r_y = 0.75$ in. Then, $I_y = 4.0 \times (0.75)^2 = 2.25$ in.⁴

The value of I_x, the moment of inertia for *OX*, the major principal axis of the section, can be determined from the well-known relation connecting the moments of inertia for principal and other axes, which is: $I_x + I_y = I_u + I_v$. As I_x is the only unknown, we have: $I_x = I_u + I_v - I_y = 10{,}0 + 4.0 - 2.25 = 11.75$ in.⁴

The value of the angle between the principal and gravity axes, angle ϕ of Fig. 124, is given by the expression

$$\sin \phi = \left(\frac{I_v - I_y}{I_x - I_y}\right)^{\frac{1}{2}}$$

This expression is found in works on Mechanics.

For the values given above

$$\sin \phi = \left(\frac{4.0 - 2.25}{11.75 - 2.25}\right)^{\frac{1}{2}} = 0.430$$

or $\phi = 25$ deg. 30 min. The gravity and principal axes are shown in their relative positions in Fig. 124.

As shown in Fig. 124, the sides of the circumscribing polygon, *ABCDE*, are not parallel to either of the principal axes of the section. The coördinates of the apices of the S-polygon are to be calculated by eqs. (8) or (9); or, by rotating the axes of reference as explained by Fig. 120, eqs. (14) and (15) can be used. As the latter method is the simpler, it will be used here.

Axes *OU* and *OV* are parallel to sides *A–B*, *C–D*, *D–E*, and *E–A* of the circumscribing polygon, and will be used as the new axes of reference. The angle ϕ is seen from Fig. 124 to be 25 deg. 30 min.,

For side *A–B*, which is paralled to the *OV* axis, eq. (14) is to be used. With $\phi = 25$ deg. 30 min., $I_y = 2.25$ in.,⁴ and $u_A = u_B = 2.59$ in., we have,

$$x_{ab} = \frac{(+2.25)(0.903)}{2.59} = +0.785 \text{ in.}^3$$

$$y_{ab} = \frac{(+11.75)(0.431)}{2.59} = +2.00 \text{ in.}$$

In plotting these points it must be remembered that x_{ab} and y_{ab} are referred to axes OX and OY, the rotation of axes of reference having been made only with respect to the extreme points of the section.

Side D–E is also paralled to the OV axis, and eq. (14) is to be used, which gives

$$x_{de} = \frac{(+2.25)(0.903)}{-0.91} = -2.23 \text{ in.}^3$$
$$y_{de} = \frac{(+11.75)(0.431)}{-0.91} = -5.57 \text{ in.}^3$$

Sides A–E and D–C are parallel to axis OU. Substitution in eq. (15) gives

$$x_{ae} = \frac{(-2.25)(0.431)}{1.66} = -0.584 \text{ in.}^3$$
$$y_{ae} = \frac{(+11.75)(0.903)}{1.66} = +6.39 \text{ in.}^3$$

and

$$x_{dc} = \frac{(-2.25)(0.431)}{-3.34} = +0.290 \text{ in.}^3$$
$$y_{dc} = \frac{(+11.75)(0.903)}{-3.34} = -3.18 \text{ in.}^3$$

The side B–C of the circumscribing polygon is parallel to a pair of rectangular axes shown by OR and OT in Fig. 124. These axes make an angle of 33 deg. 40 min. with the gravity axes, or 8 deg. 10 min. with the principal axes of the section, as shown in Fig. 124. This angle can be calculated, or scaled with a protractor from a large layout of the section. Since the axis OR is in the fourth quadrant with respect to the axes OX and OY,

$$\phi = (360° - 8° \, 10') = 351 \text{ deg. } 50 \text{ min.}$$

Using eq. (14), with ϕ as above and $b = 1.51$ in., as shown on Fig. 124, we have

$$x_{bc} = \frac{(+2.25)(0.990)}{1.51} = +1.48 \text{ in.}^3$$
$$y_{bc} = \frac{(+11.75)(-0.142)}{1.51} = -1.11 \text{ in.}^3$$

Plotting these points with respect to the OX and OY axes, and connecting the proper points, the complete S-polygon is obtained as shown in Fig. 124.

110e. S-polygons for Z-bars and T-bars.—Two rolled sections which are used occasionally as beam sections are the Z and T-bars. S-polygons for these sections are shown in Fig. 125. The detail work of calculating these polygons will not be given, as the methods are similar to those used above.

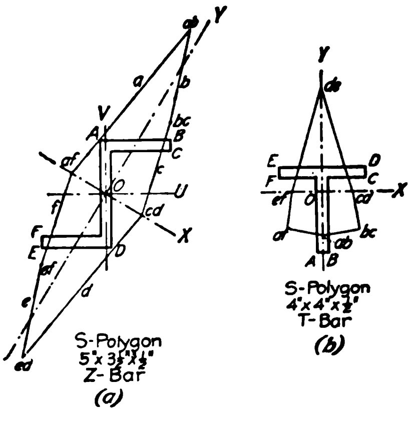


FIG. 125.

Fig. 125(a) shows the S-polygon for a 5 × 3¼ × ½-in. Z-bar. The coördinates of the apices of the S-polygon, referred to the principal axes of the section are:

$x_{ab} = -0.600$ in.^{3}, $y_{ab} = +8.56$ in.^{3}; $x_{bc} = +0.848$ in.^{3}, $y_{bc} = +4.38$ in.^{3};
$x_{cd} = +1.89$ in.^{3}, $y = 0$; $x_{af} = -1.89$ in.^{3}, $y_{af} = 0$;
$x_{ef} = -0.848$ in.^{3}, $y_{ef} = -4.38$ in.^{3}; $x_{de} = +0.600$ in.^{3}, $y_{de} = -8.56$ in.^{3}.

Fig. 125(b) shows the S-polygon for a 4 × 4 × ½-in. T-bar, for which the coördinates of the S-polygon are:

$x_{ab} = 0$, $y_{ab} = -2.02$ in.^{3}; $x_{de} = 0$, $y_{de} = +4.83$ in.^{3};
$x_{cd} = +1.40$ in.^{3}, $y_{cd} = 0$; $x_{ef} = -1.40$ in.^{3}, $y_{ef} = 0$;
$x_{bc} = +1.69$ in.^{3}, $y_{bc} = -1.71$ in.^{3}; $x_{af} = -1.69$ in.^{3}, $y_{af} = -1.71$ in.^{3}

111. Solution of Problems in Unsymmetrical Bending.—Problems in unsymmetrical bending can be solved algebraically by the use of eqs. (1) and (2), or by semi-graphical methods involving the use of S-polygons. A few simple problems will be worked out to show the general methods employed.

In problems involving the determination of fiber stress in a given beam section under bending in any direction. the desired result is generally the maximum fiber stress and the fiber on which it occurs. A complete solution of this problem can be obtained by two methods. In the first method, the stresses are computed for all extreme fibers of the section. On comparing these values, the maximum can readily be determined. By the second, and better method. the neutral axis of the section is located on a large scale layout of the section. From this sketch the fiber most remote from the neutral axis is determined by inspection, or by scaling if necessary, and a fiber stress calculation made only for this fiber, thus giving the required maximum stress intensity.

Illustrative Problem.—A 10-in. 25-lb. channel section is used as a beam to support a moment M acting in a vertical plane. Fig. 126 shows the position of the channel and the direction of the plane of bending with respect to OX and OY, the principal axes of the section. The solution will be carried out for both of the general methods outlined above.

Algebraic Solution.—The moments of inertia of the section, as given by the steel handbooks, are: $I_x = 91.0$ in.^{4}, and $I_y = 3.4$ in.^{4}. The coördinates of the extreme points of the section are: $x_A = +2.28$, $y_A = +5.0$; $x_B = +2.28$, $y_B = -5.0$; $x_C = -0.62$, $y_C = -5.0$; and, $x_D = -0.62$, $y_D = +5.0$. (All coördinates in inches.)

From eq. (2), with $\theta = 60$ deg., as shown in Fig. 126, and with the coördinates given above, we find for point A,

$$f_A = -M\left[\frac{(+3.4)(5.0)(0.866) + (0.91)(2.28)(0.50)}{(91)(3.4)}\right] = -\frac{+14.72 + 103.8}{309.5}M$$

$$f_A = -0.3835M$$

The minus sign indicates that the fiber stress is compressive.

For fiber B, substitution in eq. (2) involves the same quantities as for A, except that y_B is negative. The first term in the numerator of the above expression then becomes negative. Using the same form as given above, we have

$$f_B = -\frac{-14.72 + 103.8}{309.5}M = -0.2875\,M$$

In the same way, we have for points C and D

$$f_C = -M\left[\frac{(+3.4)(-5.0)(0.866) + (91.0)(-0.62)(0.5)}{(91)(3.4)}\right]$$

$$f_C = +\frac{+14.72 + 28.20}{309.5} = +0.1386M$$

and

$$f_D = +\frac{-14.72 + 28.20}{309.5} = +0.04355M$$

The plus signs indicate tensile stresses.

On comparing the calculated values, it will be found that fiber A has the maximum fiber stress, and that the stress intensity is $0.3835M$ lb per sq. in., compression.

Proceeding with the second method of solution outlined above, we find from eq. (1) that the angle between the axis OX and the neutral axis for the given plane of bending is

$$\tan\alpha = \frac{(-91.0)(\cot 60°)}{3.4} = \frac{(-91.0)(0.5774)}{3.4} = -15.46$$

from which, $\alpha = 93$ deg. 38 min. In Fig. 126 the neutral axis, as located by this angle, is shown in position. It is evident by inspection that fiber A is most remote from the neutral axis. A single substitution in eq. (2) for fiber A gives the desired result. The calculations are as given above for point A; they will not be repeated.

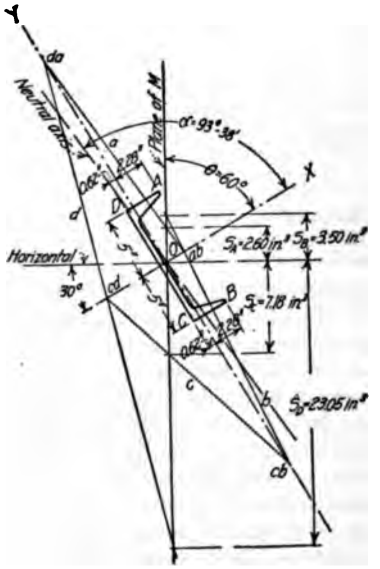


Fig. 126.

Solution by Means of an S-polygon.—On Fig. 126 there is given a solution of this problem by means of an S-polygon. The S-polygon is constructed from the calculations made in Art. 110 and shown on Fig. 123.

From eq. (4) of Art. 107, the fiber stress at any point is $f = M/S$, where S is the flexural modulus of the section. As explained in Art. 108, the value of S for any point is equal to the intercept on the plane of bending of the S. line produced and the origin of coördinates. These intercepts are shown on Fig. 126, each with a subscript corresponding to the point for which the value of S is given. Then from eq. (4), the fiber stresses are: $f_A = M/2.60 = 0.385M$, $f_B = M/3.50 = 0.286M$, $f_C = M/7.18 = 0.139M$, and $f_D = M/23.05 = 0.0435M$.

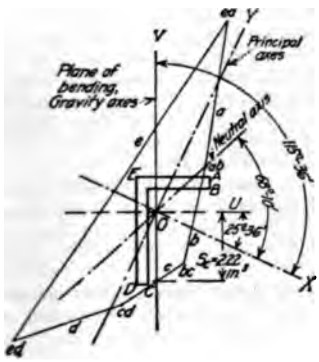


Fig. 127.

The character of fiber stress is not given directly by the S-polygon. To determine the character of the fiber stress, locate the position of the neutral axis, as shown in Fig. 126. For positive moment, all points below the neutral axis will be under tensile stress, and points above the neutral axis will be under compression. Thus in the case under consideration, points A and B are above the neutral axis and are under compression, while C and D are below the neutral axis and are under tension. These results are checked by the algebraic solution given above.

Illustrative Problem.—A 5 × 3½ × ½-in. angle with the longer leg vertical carries a moment M acting in a vertical plane, as shown in Fig. 127. Required the intensity of the maximum fiber stress and the fiber on which it occurs.

This is the angle section for which the S-polygon is calculated in Art. 110 and shown on Fig. 124. The principal moments of inertia of the section are: $I_x = 11.79$ in.^{4}, and $I_y = 2.25$ in.^{4}. In Fig. 127 the principal axes OX and OY are shown in position.

Algebraic Solution.—The fiber of maximum stress intensity will be determined by plotting the position of the neutral axis on the angle section. From eq. (1), with $\theta = 115$ deg. 36 min., as shown on Fig. 127, we have

$$\tan\alpha = \frac{(-11.79)(\cot 115°\,36')}{2.25} = +2.51, \text{ or, } \alpha = 68 \text{ deg. } 17 \text{ min.}$$

The position of the neutral axis is shown on Fig. 127. It will be found that fiber C is most remote from the neutral axis, and is therefore the fiber of maximum stress intensity.

The coördinates of point C must be referred to the principal axes of the section, OX and OY, in substituting in eq. (2). This information is not given in the steel handbooks. It can be obtained by scaling from a large scale drawing of the section, or it can be calculated by means of the formulas for rotation of the axes of reference given for the conditions shown in Fig. 120 of Art. 109. The values of u and v to be used in the formulas of Art. 109 can be found in the steel handbooks, for OU and OV are the gravity axes of the section. Then for $u_C = -0.41$, $v_C = -3.34$, and $\phi = 25$ deg. 36 min., we have, $y_C = (-3.34)(0.902) - (0.410)(0.432) = -3.19$, and, $x_C = (-0.410)(0.902) + (3.34)(0.432) = +1.07$, both values in inches. Calculated and scaled values were found to check.

Substituting in eq. (2) the values of x_C and y_C given above, and $\phi = 115$ deg. 36 min., the fiber stress at C is found to be

$$f_C = -M\left[\frac{(2.25)(-3.19)(\sin 115^\circ 36') + (11.79)(1.07)(115^\circ 36')}{(11.79)(2.25)}\right]$$

$$f_C = +0.449M$$

Fiber C is under tensile stress, as indicated by the positive sign of the result.

In calculating the tables of safe loads on angle sections given in the steel handbooks, it is usually assumed that the neutral axis is horizontal for all planes of bending. If the neutral axis be assumed to be parallel to the shorter leg of the angle of Fig. 127, the fiber stress at is found to be: $f_C = Mc/I = 3.34\ M/10 = 0.334M$, a result only about 75% of the true stress given above.

Solution by S-polygon.—The S-polygon solution of the preceding illustrative problem is shown on Fig. 127. This polygon is constructed from data calculated in Art. 110 and shown on Fig. 124. From an inspection of Fig. 127, it can be seen that for the given plane of bending, fiber C has the least S, and is therefore the desired fiber of maximum stress. By scale from Fig. 127 we find $S_C = 2.22$ in.^{3} Therefore, $f_C = M/2.22 = 0.450M$, which checks the result obtained by the algebraic method. As fiber C is located below the neutral axis, the fiber stress is tensile.

The design of beams subjected to unsymmetrical bending is greatly simplified by the use of S-polygons. Where several possible loading conditions are involved, the algebraic calculations are long and tedious, while the semi-graphical S-polygon offers a comparatively simple and easily understood method of solution.

In designing by the S-polygon method, the process consists in comparing graphically the flexural modulus required for any plane of bending with that furnished by the assumed section. From eq. (4), Art. 107, $S = M/f$. Having given the bending moment to be carried and the allowable working stress, the required flexural modulus is readily determined.

The required S is plotted to scale on a set of coördinate axes placed in the proper position in space. The S-polygons of the trial sections are then plotted to scale on the same set of axes. In order to answer the requirements of the design, the S furnished by the trial section must be equal to, or greater than, the required value.

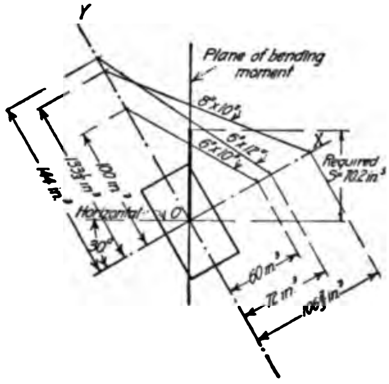


FIG. 128.

Illustrative Problem.—Design a wooden beam set with its faces at an angle of 30 deg. with the vertical, and subjected to an unsymmetrical bending moment acting in a vertical plane. The span of the beam is 12 ft., and the allowable working stress in the timber is 1000 lb. per sq. in. Determine the beam section required to support a net uniform load of 300 lb. per ft.

As the weight of the beam section is not known to begin with, it will be assumed to be 25 lb. per ft. The total load to be carried is then 325 lb. per ft.; the bending moment in a vertical plane is $M = \frac{1}{8}wl^2 = \frac{1}{8}(325)(12)^2(12) = 70{,}200$ in.-lb.; and the required flexural modulus is $S = M/f = 70{,}200/1000 = 70.2$ in. This is shown to scale in the proper position in Fig. 128.

From the S-polygon of a rectangle shown in Fig. 121, Art. 110, it can be seen that for bending at an angle of 60 deg. with the axis OX, fibers A and C have values of S which are equal and smaller than those for D and B. It is evident, then, that it is necessary to draw only the S-line for point A in order to determine the proper section.

In Fig. 128 the S-lines for several rectangular sections are shown. The 6 × 10-in. section is too small, for the S furnished by the section is not equal to that required by the moment. The 6 × 12-in. section is a little too large, but as beams usually come in even inch sizes, it will be adopted.

Before this section is finally adopted, the assumed weight must be checked up. At 4 lb. per ft. board measure, a 6 × 12-in. section will weigh $(12 \times \frac{6}{12})4 = 24$ lb. per ft. As the weight assumed in the calculations was 25 lb. per ft., a revision is not necessary.

In Sect. 2, Art. 64, there is given the design of a roof purlin for several combinations of dead, snow, and wind load. The solution is based on the principles used in the above problem.

112. Investigation of Beams.—An important problem in the investigation of the relative value of the various rolled sections when used as beams is their moment carrying capacity. By means of the S-polygons of the sections, a direct comparison can be made. Thus, if it be required to determine the relative moment carrying capacity of an I-beam and a channel of the same depth and weight per foot—as for example, a 10-in. 25-lb. I-beam—we can refer to the S-polygons for these sections. Fig. 122 gives the S-polygon for a 10-in. 25-lb. I-beam, and Fig. 123 gives the S-polygon for a 10-in. 25-lb. channel.

These polygons are drawn to the same scale so that the relative strength of the two sections is proportional to their sizes. It can be seen at once that the advantage is in favor of the I-beam section. In the same way, any sections can be compared by this method.

Another problem of considerable importance is the determination of the planes of greatest and least strength for any given section. In this way it is possible to place a section in such a position that its plane of greatest resisting moment coincides with the plane of the bending moment, and the section is used to its greatest advantage. It is also possible to avoid loading a beam in the plane of its least resisting moment.

From eq. (4) of Art. 107, it can be seen that the fiber stress varies inversely as the value of *S*. Therefore the plane of greatest strength is the one with the largest *S*, and the plane of least strength is the one with the smallest *S*. The values are measured as shown by the vector *OE* of Fig. 119.

The plane of greatest strength in bending of the rectangle, I-beam, and channel sections, as shown by their S-polygons, (see Figs. 121, 122, and 123) is in the plane of the *OY* axis. By an inspection of the S-polygons, it can be seen that the plane of least strength is perpendicular to the S-lines, for on these planes the values of *S* are a minimum. There will be four such planes for the rectangle and I-beam sections, one for each S-line. For the channel section there two planes of least strength, one perpendicular to the S-line *a* and another perpendicular to S-line *b*.

The angles which these planes make with the axis *OX* can be determined from a large scale drawing of the section by means of a protractor. The angles can also be determined by means of a proposition of Analytical Geometry which states that when a line is perpendicular to a given line, the slope of the perpendicular is the negative reciprocal of that of the given line. Thus from the equation of the S-line for fiber *A*, as given by eq. (6), Art. 108, the slope of the perpendicular is $+\frac{I_y}{I_x}\frac{y_A}{x_A}$. For the rectangle of Fig. 121, we find from the data given in Art. 110 (*a*), that the angle between the *OX* axis and the plane of least strength, as determined from the above equation, is

$$\text{tan of slope} = +\frac{8}{288} \times \frac{6}{1} = +0.167, \text{ or slope angle} = 9 \text{ deg. } 30 \text{ min.}$$

This plane is shown in position on Fig. 121.

The determination of the planes of greatest and least strength of the angle section, for which the S-polygon is shown in Fig. 124, is not as simple a matter as for sections of rectangular form due to the unsymmetrical form of the S-polygon. From an inspection of the S-polygon of Fig. 124, it is evident that the angle section has its greatest strength as a beam for the plane of loading for which the fiber stresses, and hence the values of *S*, for fibers *A* and *D* are equal. This plane can be located by trial by means of a straight edge and a pair of dividers. It can also be located by means of eq. (5) of Art. 107. If values of *S*, as given by eq. (5) for fibers *A* and *D*, be equated and the resulting expression be solved for θ, the result will be the desired plane of greatest strength. Performing the operation indicated above, we have

$$\tan \theta = -\frac{I_x}{I_y} \cdot \frac{x_A + x_D}{y_A + y_D}$$

For the angle section whose S-polygon is shown in Fig. 124, $x_A = +1.61$, $y_A = +2.60$; $x_D = +0.59$, $y_D = -3.40$; $I_x = 11.75$, and $I_y = 2.25$. From the above equation

$$\tan \theta = -\frac{11.75}{2.25} \cdot \frac{1.61 + 0.59}{2.60 - 3.40} = +14.35,$$

or, $\theta = 86$ deg. 5 min. This plane of loading is shown in position on Fig. 124. The plane of least strength is determined by methods similar to those used for the rectangle. It is shown on Fig. 124.

In the above discussion the planes of greatest strength have been located and are shown in position on a few of the sections in general use as beams. To secure the best results, it is evident that the section should be so placed that the plane of bending and the plane of greatest strength coincide. It is not possible, however, to realize these ideal conditions in all cases. This is due to the fact that the methods of attaching the beam section to its supports determines the position of the beam. Thus beams supported on a sloping surface must usually be set with their faces perpendicular to the supporting surface. In Sect. 3, Art. 127, details of purlin connections are shown which bring out this point.

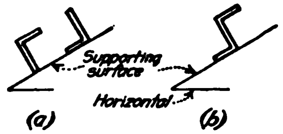


FIG. 129.

When an angle section is used as a beam, it should be placed as shown in Fig. 129(*a*), for as shown by the S-polygon, this position is very close to its position for greatest strength for bending in a plane which is vertical or nearly so. At the same time, attachment to the supporting structure is readily made.

Z-bars are seldom used as beam sections, as it is difficult to obtain them except in large quantities. From the S-polygon for this section, Fig. 125(*a*), it can be seen that for the position shown in Fig. 129(*b*), the section is advantageously placed for bending in a vertical plane.

The T-bar, as shown by its S-polygon, Fig. 125(*b*), does not form an ideal beam section, due to the fact that the fiber stresses on the extreme fiber of the stem are much greater than those on the flange. In any case it is desirable that the section be placed with the stem down. The upper, and wider face, is then in compression, which increases the lateral stiffness of the section.

In some types of roof covering, T-bars closely spaced, are used to support tile or short span slabs carried directly on the T-bars. The stem of the T is placed up, the bottom flange forming a support for the title. From the discussion given above, it can be seen that the T-bar is not well placed in this type of construction, for the narrow stem of the T is in compression, and is liable to fail due to insufficient lateral support, unless low working stresses are maintained. The material is then not used to as great advantage as in the other sections considered.

113. Tables of Fiber Stress Coefficients for Beams.—The variety of conditions encountered in problems in unsymmetrical bending renders it impractical to attempt any very extensive tabulation of fiber stresses in beams. Each case must be worked out by means of the general equations or the S-polygon methods given in the preceding articles. Where S-polygon methods are to be used to any great extent, it will save time if the S-polygons of standard sections be plotted on tracing cloth, or some transparent material. The required S can be plotted on a sheet of paper, as explained in the illustrative problem, p. 88. By laying the plotted S-polygons over the required S, and shifting to different sections, the desired section can readily be determined.

There is, however, one very important and frequently encountered condition of unsymmetrical loading for which tabulations of fiber stress can be made. The case referred to is that of loading in a vertical plane on sections inclined at an angle to the vertical.

Table 1 gives coefficients for I-beams; Table 2 gives values for channels; and Table 3 gives values for angles. The fiber stress in any case is obtained by multiplying the moment, M, by the coefficient given in the tables. The sketch shows the conditions for which the values are given. These tables were taken from articles by R. Fleming, which appeared in the *Eng. Rec.*, March 3, 1917, and in the *Eng. News-Rec.*, Feb. 27, 1919.

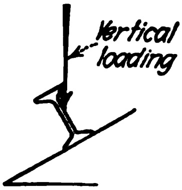


TABLE 1.—FIBER STRESS COEFFICIENTS, BENDING MOMENT DUE TO VERTICAL LOADING ON I-BEAMS

I-beam section	Pitch of roof in inches per foot								
	0	1	2	3	4	5	6	7	8
6-in. 12¼-lb.	0.138	0.212	0.284	0.352	0.415	0.473	0.526	0.573	0.614
7-in. 15 -lb.	0.097	0.153	0.208	0.260	0.308	0.353	0.393	0.430	0.461
8-in. 18 -lb.	0.070	0.114	0.157	0.196	0.234	0.268	0.300	0.328	0.352
9-in. 21 -lb.	0.053	0.088	0.121	0.153	0.183	0.210	0.235	0.257	0.277
10-in. 25 -lb.	0.041	0.069	0.096	0.122	0.146	0.168	0.188	0.206	0.222
12-in. 31½-lb.	0.028	0.050	0.071	0.091	0.110	0.127	0.143	0.157	0.170

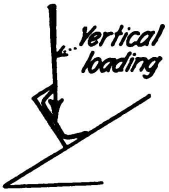


TABLE 2.—FIBER STRESS COEFFICIENTS, BENDING MOMENT DUE TO VERTICAL LOADING ON CHANNELS

Channel section	Pitch of roof in inches per foot								
	0	1	2	3	4	5	6	7	8
6-in. 8 -lb.	0.231	0.396	0.557	0.709	0.851	0.982	1.101	1.207	1.301
7-in. 9¾-lb.	0.166	0.296	0.422	0.542	0.655	0.758	0.852	0.935	1.010
8-in. 11¼-lb.	0.124	0.228	0.330	0.427	0.517	0.600	0.676	0.743	0.804
9-in. 13¼-lb.	0.095	0.180	0.263	0.342	0.415	0.483	0.545	0.600	0.650
10-in. 15 -lb.	0.075	0.145	0.214	0.279	0.340	0.397	0.448	0.494	0.535
12-in. 20½-lb.	0.047	0.094	0.141	0.184	0.225	0.263	0.298	0.329	0.357

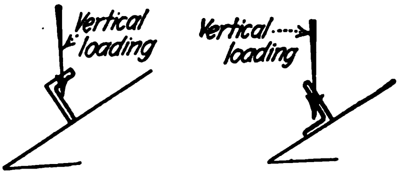


Table 3.—Fiber Stress Coefficients, Bending Moment Due to Vertical Loading on Angles

Angle section	Pitch of roof in inches per foot								
	0	1	2	3	4	5	6	7	8
2½ × 2 × ¼-in.....	3.49	3.30	3.11	2.88	2.68	2.46	2.30	2.14	2.01
2½ × 2 × 5/16-in.....	2.91	2.76	2.61	2.41	2.22	2.04	1.90	1.78	1.67
3 × 2½ × ¼-in.....	2.33	2.22	2.10	1.98	1.85	1.71	1.60	1.49	1.38
3 × 2½ × 5/16-in.....	1.89	1.83	1.73	1.63	1.51	1.41	1.30	1.24	1.15
3½ × 2½ × ¼-in.....	1.80	1.69	1.60	1.46	1.35	1.22	1.15	1.06	1.12
3½ × 2½ × 5/16-in.....	1.47	1.39	1.31	1.22	1.14	1.02	0.96	0.89	0.93
4 × 3 × 5/16-in.....	1.06	1.00	0.94	0.88	0.81	0.75	0.69	0.65	0.66
4 × 3 × ⅜-in.....	0.92	0.87	0.81	0.75	0.70	0.63	0.59	0.55	0.52
5 × 3½ × 5/16-in.....	0.68	0.65	0.61	0.56	0.51	0.47	0.43	0.41	0.48
5 × 3½ × ⅜-in.....	0.60	0.57	0.53	0.48	0.43	0.40	0.37	0.35	0.41
6 × 4 × ⅜-in.....	0.41	0.38	0.35	0.32	0.29	0.27	0.25	0.27	0.30
6 × 4 × 7/16-in.....	0.35	0.33	0.31	0.28	0.25	0.23	0.22	0.23	0.26

114. Variation in Fiber Stress Due to Changes in Position of the Plane of Bending.—The S-polygon shows in a striking manner that small changes in the position of the plane of loading cause relatively large changes in the fiber stress on a given point in the section. This variation in position of the plane of loading may be due to a variety of causes. The deflection of the beam under loading may tend to twist the section about its longitudinal axis, thus changing the position of the plane of bending from that assumed in the design. In the case of wooden beams, warping of the timber may have a similar effect. To counteract these effects, the beam should be held rigidly in line by some form of lateral support. Bridging in wooden floor construction is one method of providing this lateral support.

The effect of a small change in the position of the plane of loading will now be shown graphically by means of an S-polygon. Fig. 130 shows the S-polygon of a 10-in. 25-lb. I-beam, data for which are given in Art. 110(*b*). A comparison will be made of fiber stresses for bending in the plane of the *OY* axis, and for bending in another plane 1 deg. away from the first plane; that is, for $\theta = 90$ deg. and 89 deg. respectively. By scale from Fig. 130, we have $S_1 = 24.4$ in³. for $\theta = 90$ deg., and $S_2 = 21.3$ in³. for $\theta = 89$ deg. The resulting fiber stresses are: $f_1 = 0.04099\ M$, and $f_2 = 0.04795\ M$. These values differ by 14.6% of f_1. Values of S are also indicated for bending planes at 5 and 6 deg. from the axis *OY*. At this place the stresses differ by about 7.5%.

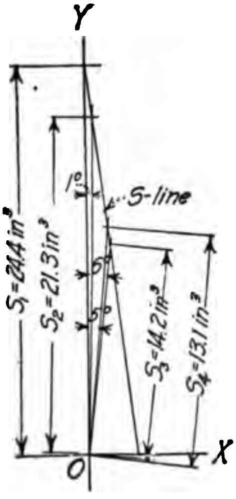


Fig.. 130.

It can be seen by comparing the calculated values given above, and also by inspection from Fig. 130, that this percentage is a maximum for planes of loading near the *OY* axis.

In narrow deep sections, the fiber stress increase is large for a relatively small change in the direction of the plane of loading. To avoid this effect, beam sections should be chosen from rolled shapes or rectangular sections which have considerable lateral rigidity. If narrow sections must be used, they should be thoroughly braced to prevent overturning.

It is also interesting to note the change in position of the neutral axis due to changes in the plane of bending. This effect is best studied by means of eq. (1), Art. 106. For the beam sec-considered above, suppose, as before, that the plane of bending is 1 deg. from the axis OY, or 89 deg. in eq. (1). Then

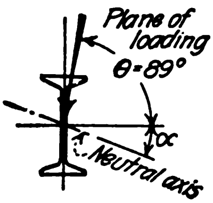
$$\tan \alpha = -(I_x/I_y)\,\text{cotan.}\,\theta = \frac{(-122.1)(0.01746)}{6.9}$$

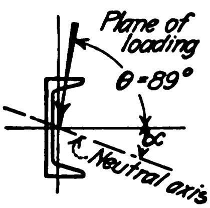
$$\tan \alpha = -0.309, \text{ or, } \alpha = 180° - 17°\ 10'$$

rill be noted that a 1 deg. change in the position of the plane of bending causes a 17-deg. age in the position of the neutral axis.

Table 4 gives the percentage change in fiber stress and the corresponding change in the tion of the neutral axis due to a 1-deg. change in the direction of the plane of bending from OY axis of standard I-beam and channel sections. These values were calculated by the meth- given above.

TABLE 4.—PERCENTAGE INCREASE IN FIBER STRESS AND CHANGE IN POSITION OF NEUTRAL AXIS FOR A ONE-DEGREE CHANGE IN DIRECTION OF PLANE OF BENDING.





Section	$\frac{I_x}{I_y}$	Increase in fiber stress (per cent.)	Change in slope of neutral axis α (degrees)
20-in. 65-lb. I-beam	41.8	22.8	36° 10′
18-in. 55-lb. I-beam	37.5	21.8	33° 15′
15-in. 42-lb. I-beam	30.2	19.3	27° 50′
12-in. 31½-lb. I-beam	22.7	16.5	21° 35′
10-in. 25-lb. I-beam	17.7	14.4	17° 10′
9-in. 21-lb. I-beam	16.4	13.8	16° 0′
8-in. 18-lb. I-beam	15.0	13.1	14° 40′
7-in. 15-lb. I-beam	13.5	12.3	13° 20′
6-in. 12¼-lb. I-beam	11 8	11.5	11° 40′
15-in. 33-lb. channel	38.1	23.2	33° 40′
12-in. 20½-lb. channel	32.8	21.4	29° 50′
10-in. 15-lb. channel	29.1	19.9	27° 0′
9-in. 13¼-lb. channel	26.3	18.5	24° 40′
8-in. 11¼-lb. channel	24.8	18.2	22° 55′
7-in. 9¾-lb. channel	21.5	16.5	20° 35′
6-in. 8-lb. channel	18 6	15.2	18° 0′

115. Deflection of Beams Under Unsymmetrical Bending.—The amount and direction ıe deflection of a beam subjected to unsymmetrical bending is often desired. To determine desired deflection, the bending moment can be resolved into its components parallel to the cipal axes of the section and the deflection determined for these component moments by ns of the usual formulas for the case in question. The required resultant deflection is equal be vector sum of the component deflections.

Suppose the rectangular section of Fig. 131 is subjected to bending in a plane at an angle axis OX due to a uniform load of w lb. per foot. Required the amount and direction of resulting deflection.

As the components of moment parallel to the axes OX and OY are proportional to the com- ents of the applied load for these same axes, the deflection parallel to the axes can be written a the deflection formula for uniform loading, which is, $d = \frac{5}{384}\frac{wl^4}{EI}$ (see Art. 66). For component of load parallel to the OX axis, we have from the above formula

$$d_x = \frac{5}{384}\cdot\frac{l^4}{E}\cdot\frac{w \cos \theta}{I_y}$$

for the load parallel to the OY axis, we have

$$d_y = \frac{5}{384}\cdot\frac{l^4}{E}\cdot\frac{w \sin \theta}{I_x}$$

re d_x and d_y are the components of deflection for the OX and OY axes respectively.

The vector sum of these deflections is

$$d = (d_x^2 + d_y^2)^{1/2}$$

where d is the desired deflection. Substituting the above values of d_x and d_y, we have

$$d = \frac{5}{384}\cdot\frac{wl^4}{E}\left(\frac{I_x^2\cos^2\theta + I_y^2\sin^2\theta}{I_x^2 I_y^2}\right)^{1/2} \qquad (16)$$

From Fig. 131(a) the angle which the resultant deflection makes with axis OX is

$$\tan\beta = \frac{d_y}{d_x} = \frac{I_y}{I_x}\tan\theta \qquad (17)$$

As this expression is the negative reciprocal of that given in eq. (1), Art. 106, it can be seen that the direction of deflection is perpendicular to the neutral axis for the given plane of bending.

If the loading conditions differ from those assumed in the above analysis, it is only necessary to change the value of the constant $\frac{5}{384}\frac{l^4}{E}$ of eq. (16) to meet the required conditions.

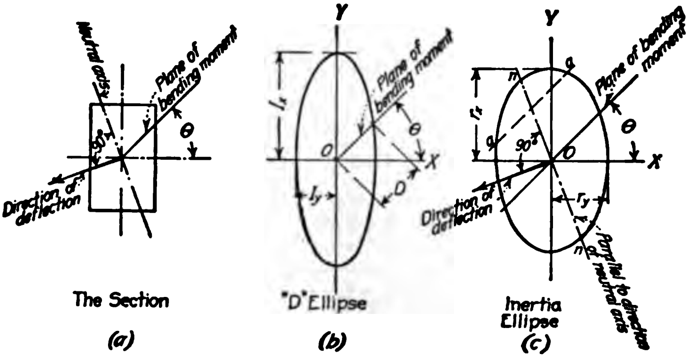


Fig. 131.

The amount and direction of deflection can also be determined by graphical methods which are based on certain properties of the ellipse. Eq. (16) can be written in the form

$$d = \frac{5}{384}\cdot\frac{wl^4}{E}\times\frac{1}{D}, \qquad \text{where } D = \left(\frac{I_x^2 I_y^2}{I_x^2\cos^2\theta + I_y^2\sin^2\theta}\right)$$

This value of D can be shown to be the equation of an ellipse with major and minor axes I_x and I_y. Fig. 131(b) shows the D-ellipse for a rectangular section. The vector D, measured as shown in Fig. 131(b), gives the denominator of the above equation for loading on the given plane.

As stated above, the direction of deflection is perpendicular to the neutral axis. The neutral axis can be located by means of the inertia ellipse of the section. A complete discussion of the inertia ellipse will be found in advanced works on Mechanics, to which the reader is referred.

Fig. 131(c) shows the inertia ellipse for a rectangular section. It is constructed with major and minor axes equal to the radii of gyration of the section for the axes OX and OY. To locate the neutral axis, draw through point O a line parallel to the plane of bending. Draw a–a, any chord of the ellipse parallel to the plane of bending. Bisect this chord, and through its center point draw a line n–n which passes through the point O. This line is parallel to the direction of the neutral axis for the given direction of bending. This construction is based on the fact that eq. (1) expresses the relation which exists between the conjugate diameters of an ellipse.

A line perpendicular to n–n gives the direction of the desired deflection, as shown in Fig. 131(c).

SECTION 2

DESIGNING AND DETAILING OF STRUCTURAL MEMBERS AND CONNECTIONS

STEEL SHAPES AND PROPERTIES OF SECTIONS

By Walter W. Clifford

1. Steel Shapes.—The steel used in structures is in the form of single pieces, or combinations of two or more pieces, to which the general term *shapes* is applied. The procedure in the manufacture of these shapes consists of the following operations: (1) smelting iron ore and producing pig iron; (2) converting the pig iron into rectangular prisms of steel, called *ingots;* and (3) rolling the ingots to the desired shapes. The shapes used in building construction are: square and round rods or bars, flat bars or *flats*, plates, angles, channels, I-beams, H-sections, zees and tees. Flat members 6 to 7 in. wide and less are usually designated as *bars* or *flats;* over 6 to 7 in. wide are designated as *plates.* Zees and tees are not now used to any great extent. Zees have been used extensively for columns but are rapidly becoming obsolete. H-sections are designed for use as columns.

The process of rolling I-beams, channels, angles, etc., is in general as follows: The ingots are heated to a uniform temperature in *soaking pits*, and then are taken out and passed several times through a set of rolls, called *blooming rolls.* These rolls give to a piece only the general shape (rectangular, flat, or square) of the finished product. The next step is to pass the steel through the *roughing rolls*, and then the piece is passed to the *finishing rolls* where the final shaping takes place. The pieces, still very hot, are then passed on by movable tables to circular saws where they are cut into required lengths.

The method of increasing sectional area of standard shapes is shown in Fig. 1. For example, suppose it is desired to roll channels or I-beams having the same depth, but different thicknesses of web. These sections are always rolled horizontally and the increase in thickness of web is accomplished by changing the distance between the rolls, the effect being to change the width of flange as well. Thus, two beams with the same height but different weights differ simply by a rectangle as shown. It will be seen, also, that for an angle with certain size of legs the effect of increasing weight is to change slightly the length of legs, and to increase the thickness.

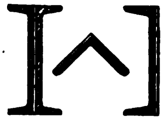


Fig. 1.

Bethlehem beam, girder and H-sections are shaped by four rolls instead of the two grooved rolls used for manufacturers' standard shapes. The use of so many rolls makes possible a variation of height as well as width, and both are increased with additional weight in H-sections.

Plates when rolled to exact width, the width being controlled by a pair of vertical rolls, are known as *universal mill* or *edged* plates. Plates rolled without the width being controlled have uneven edges and must be sheared to the correct width. Such plates are known as *sheared* plates.

The properties of the standard shapes manufactured by the different steel companies are the same. The *standard* shapes of the Assoc. of Am. Steel Mfrs., are rolled by all mills, but each company also has its own list of *special* shapes. These special shapes, which are different for the different mills, are not as likely to be in stock as the standard shapes.

Standard I-beams are rolled in depths from 3 to 24 in. and standard channels from 3 to 15 in. The different depths of standard I-beams are: 3 to 10 in. consecutively, then 12 in., 15 in., 18 in., 20 in., and 24 in. For channels, 3 to 10 in. consecutively, then 12 in. and 15 in. For each depth of I-beam and channel there are several standard weights.

Minimum sizes of steel shapes are more likely to be found in stock and are the most efficient for resisting bending considering the weight of material used. The rolls are made especially for these sections and the heavier sections for a given depth of beam are obtained by spreading the rolls as explained above.

I-beams and channels, 15 in. and under, and angles 6 in. and under, take the *base price*. Heavier sections are charged for at a higher rate, usually 10 c. per 100 lb., above base price.

2. Properties of Sections.—The fundamental properties of sections may be said to be: sectional dimensions, location of the center of gravity, and the moments of inertia about the various axes. The distance from the center of gravity to the most stressed fiber c; the section modulus S; and the radius of gyration r, follow from these.

The method of finding the center of gravity is explained in Sect. 1, Art. 44. The derivation and use of I and S are explained in the chapter on "Simple and Cantilever Beams" in Sect. 1. The use of r is considered in the chapter on "Columns" in Sect. 1.

To facilitate the work of the designer, certain so-called *properties of steel sections* are published. The facility with which a designer can find and use these properties, which are given in manufacturers' handbooks and elsewhere, has much to do with the amount of work which he can accomplish.

It is not intended to include in this handbook steel tables similar to those which are available in the steel manufacturers' handbooks or in Ketchum's "Structural Engineers' Handbook." Articles which follow, however, give the necessary general information concerning such tables and their use.

2a. Properties of Wood Sections.—Wood sections are commonly rectangular and therefore easily designed by the fundamental formulas. It should be remembered, however, that the actual sizes of dressed lumber are not the nominal sizes. This handbook gives all the tables commonly needed for the structural design of wooden members, but tables are also published by various lumber associations. The "Southern Pine Manual"[1] contains excellent tables. This manual gives I and S for various sections; tables of allowable uniform loads for plank and beams, considering moment, shear, and deflection; and tables of column loads. In addition there are tables of allowable loads for trussed beams and much miscellaneous information about yellow pine.

2b. Properties of Steel Sections—*Beams.*—The steel manufacturers' handbooks give very complete tables of properties of steel sections. Uniformly loaded I-beams, channels, and angles should be selected from the tables of safe or allowable uniform loads. These tables can also be adapted for other loadings, such as for a load concentrated at the center, in which case a beam should be selected which will carry twice the load, uniformly distributed. For a number of load concentrations, approximately equal in amount and spacing, the load may be considered as uniform.

For irregular loadings on I-beams and channels the moment and shear should be computed and the tables used which give the allowable resisting moment and shear of the various shapes. If desired, however, the beams may be designed by computing the section modulus and selecting the proper size of beam from the tables of properties. Angles, tees and other miscellaneous shapes used as beams must usually be designed by use of the section modulus, as few tables of safe loads or resisting moments and shears are given for these shapes.

Bethlehem beams and girders differ from the manufacturers' standard sections rolled by other manufacturers. The beams have heavier flanges, and, where moment is the consideration, they are lighter for the same strength than other sections. Their webs are lighter than in standard sections. Bethlehem girder sections are, for their depths, the strongest sections rolled. They have nearly twice the carrying capacity of the manufacturers' standard section for the same depth, but they are uneconomical where there is room for a deeper section. Tables of uniform loads for Bethlehem sections are given in Bethlehem Handbook. The common properties are also given.

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[1] Southern Pine Association, New Orleans, La.

Built-up steel beam properties usually have to be computed with the properties of the component parts as a basis. Some properties of the more common plate-girder sections are given in the principal steel handbooks.

To compute the moment of inertia, *I*, of a built-up girder section about the neutral axis of the net section—that is, when rivet areas on the tension side are to be deducted—the moment of inertia is first computed about an axis through the geometrical center of the section and then corrected so as to obtain the value about an axis through the center of gravity of the net section.

In regard to the position of the neutral axis in a plate girder section Lewis E. Moore has the following to say in his book on the "Design of Plate Girders."

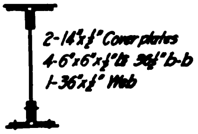

FIG. 2.

> Some authors claim that the neutral axis should be determined by considering the net section on the tension side and the gross section on the compression side. The net section exists only over a short proportion of the length of the beam and it seems very reasonable that the neutral axis should in general be nearer the position which is determined by using the gross area than that determined by using partly gross and partly net areas. It seems an entirely reasonable assumption that the axis does not shift violently up and down, but remains in substantially the same vertical position throughout the length of a properly designed beam. It seems reasonable that this position will be nearer to the neutral axis of the preponderating section, which is the gross section. The truth of the matter probably is that the neutral axis lies somewhere between the two extreme positions determined by the two methods mentioned above and probably nearer to that determined by using the gross section.

In keeping with Mr. Moore's discussion the resisting moment of a plate girder is usually determined by considering the neutral axis through the center of gravity of the gross area and then finding the moment of inertia about that axis deducting for the rivet holes in the tension flange.

The following example illustrates the method of computing *I* about the neutral axis of the gross section by the rules and methods given in Arts. 44 and 61*g*, Sect. 1. A girder is assumed as shown in Fig. 2 with three ¾-in. rivets in the tension side of the section.

Part	*A* (area)	*x* Dist. c of g. of part to c. of g. of whole	*Ax*	Ax^2	*I*	$I + Ax^2$
Web	18 sq. in.	0	0	0	1944 in.^{4}	1,944 in.^{4}
4 angles	23 sq. in.	16.57 in.	381 in.^{3}	6310 in.^{4}	80 in.^{4}	6,390 in.^{4}
2 cover plates	14 sq. in.	18.5 in.	259 in.^{3}	4800 in.^{4}	......	4,800 in.^{4}
	55 sq. in.					13,134 in.^{4}
Flange rivet holes	−1.75 sq. in.	18.25 in.	31.9 in.^{3}	583 in.^{4}		
Web rivet hole	−1.31 sq. in.	14.75 in.	19.3 in.^{3}	283 in.^{4}	....	866 in.^{4}
	3.06 sq. in.				Net *I* =	12,268 in.^{4}
Net area = 51.94 sq. in.						

The allowance made for a rivet hole is for a hole ⅛ in. more in diameter than the diameter of the rivet—that is, ⅞ in. for a ¾-in. rivet. The properties of the plates may be taken from tables in the steel handbook or may be easily computed. The area and *I* for the angles may be taken directly from the handbook (properties of angles). The *x* distance used for an angle is one-half the distance back to back of the angles, less the distance from the back of the angle to its center of gravity. Areas of rivet holes may be taken from the steel handbook or from table on p. 276 of this handbook. *I* for the cover plates and rivet holes is neglected.

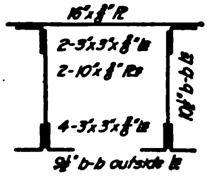

FIG. 3.

The same general form of computation may be used for built-up chord sections. In the following computations for radius of gyration, a chord section as shown in Fig. 3 is assumed.

Member	A	x (above bottom of section)	Ax	Ax^2	I	$I + Ax^2$
Top plate	6.0 in.^{2}	10.7 in.	64.2 in.^{3}	687 in.^{4}	......	687 in.^{4}
2 web plates	7.5 in.^{2}	5.25 in.	39.4 in.^{3}	207 in.^{4}	250 in.^{4}	457 in.^{4}
2 top angles	4.22 in.^{2}	9.61 in.	40.5 in.^{3}	389 in.^{4}	4 in.^{4}	393 in.^{4}
4 bottom angles	8.44 in.^{2}	0.89 in.	7.5 in.^{3}	7 in.^{4}	7 in.^{4}	14 in.^{4}
	26.16 in.^{2}		151.6 in.^{3}			1551 in.^{4}

$\frac{151.6}{26.16} = 5.8$ in. distance of center of gravity above bottom of section.

$1551 - (26.16)(5.8)^2 = 671$ in.^{4} $= I$ about center of gravity of entire section.

$$r = \sqrt{\frac{671}{26.16}} = 5.08 \text{ in.}$$

Columns.—I-beams are occasionally used as columns. Their properties will be found as noted under beams. The only rolled steel column section in common use is the H-section. The Carnegie Co. rolls 4, 5, and 6-in, H-sections; and the Bethlehem Co. rolls 8, 10, 12, and 14-in. H-sections in a large range of weights. The properties of various built-up columns of pairs of channels, both latticed and with cover plates, and of plate and angle sections are given in the steel handbooks. Ketchum also gives properties of built-up column sections. For general method of computing I and r for compound sections, see preceding article.

There are also patent columns such as Lally columns,[1] and cast-iron columns for second-class construction or light loads, whose properties are given in books issued by the manufacturers.

Struts and Ties.—In the design of struts and ties, it is found convenient to have tables giving the values of the radius of gyration r, and also tables giving net areas deducting rivet holes. The principal steel handbooks give values of r for pairs of different angles back to back, and also the net areas for angles. It should be noted that the minimum r for a single angle is not about an axis parallel to either leg. This minimum r is given in the tables of the properties of angles.

2c. Properties of Concrete Sections.—Various tables and curves for concrete design are published both in this handbook and in Hool and Johnson's "Concrete Engineers' Handbook," also in "Reinforced Concrete Design Tables" by Thomas and Nichols.

2d. Properties of Cast-iron and Miscellaneous Sections.—The shapes in which materials like cast-iron and masonry are used are not standard. There are therefore, in general, no available tables of properties. Recourse must be had to the general principles previously given. Sections in these materials can ordinarily be divided into geometric figures. The properties of the more common geometric forms are to be found in the steel handbooks.

WOODEN BEAMS

By Henry D. Dewell

Under this heading will be considered only timber beams and girders of solid and uniform section.

Wooden beams are used in building construction generally as joists or girders supporting vertical loads only. Certain exceptions to this general rule are cases in which timbers may be employed in wall framing, as girts or vertical beams, to resist the lateral force of wind.

3. Factors to be Considered in Design.—The factors determining the selection of the size of a wooden beam are:

(*a*) The maximum unit fiber stress in bending must not be excessive.

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[1] The Lally Column Co., New York and Chicago.

(*b*) The maximum unit stress due to horizontal shear must not be excessive.

(*c*) The deflection of the beam under maximum loading must be within the allowable limit.

(*d*) The depth must be within any limits of space between floor and ceiling, or in accordance with any restrictions as to clear story height.

(*e*) The cross-sectional dimensions should be of a size easy to obtain.

(*f*) The cross-sectional dimensions should be considered as to requirements of details of connection.

(*g*) One or both of the cross-sectional dimensions may be limited by the building, as in frame or mill construction.

The fundamental bending formula used in the design of beams, is treated in the chapter on "Simple and Cantilever Beams" in Sect. 1. Shear and deflection are also treated in the same chapter.

4. Allowable Unit Stresses.—Unit stresses for design of wooden beams are usually prescribed by building ordinances for the various kinds of timber. These allowable stresses vary widely in different cities, the older ordinances in general prescribing lower limits than the more recent ones. The tendency in revising ordinances is to increase the allowable unit stresses in timber, at least for timber in bending. This feature is due largely to the efforts of the lumber manufacturers' organizations in competition with the constantly widening use of reinforced concrete. At the same time these manufacturers, in conjunction with engineering organizations, are giving more attention to the grading rules and to furnishing timber of uniform high quality. In comparing the allowable unit stresses found in various building ordinances the prescribed live loading must also be taken into consideration. For example, a limit of 1500 lb. per sq. in. in bending with a 60-lb. live load will give the same size beam as a 40-lb. live load with a limiting fiber stress in bending of 1000 lb. per sq. in.

It is obvious that the allowable unit stresses are dependent on the quality of timber used. In this respect most of the newer building ordinances allow higher stresses for a select grade of lumber, whereas older ordinances make no distinction in grade, or, more accurately speaking, they prescribe for the grade of timber most likely to be used.

5. Kinds of Timber.—The timbers most commonly used for wooden beams in building construction are long-leaf yellow pine and Douglas fir, the first being employed almost exclusively throughout the Eastern states, and the latter having its widest use in the Pacific Coast states. Less extensively employed, may be mentioned short-leaf yellow pine, white pine, Norway pine, spruce, hemlock and redwood.

6. Quality of Timber.—The desired quality of timber is determined by specifications or by referring to grading rules established by the lumber manufacturers. Thus, the timber for joists or girders may be specified by the designer to be Select Structural, Dense Grade, Sound Grade, No. 1 Common, or Select No. 1 Common. In the Pacific Coast states, the two latter terms are generally used, very little structural timber entering into a building being above No. 1 Common. Both the Southern Pine Association and the West Coast Lumbermen's Association have established a Structural Grade for long-leaf yellow pine and Douglas fir, and in the larger cities lumber of this quality can probably be obtained. In many cases, however, the lumber may be purchased from the smaller yards, and, even if specified as No. 1 Common, may contain a considerable percentage of No. 2 Common, since it is a practice of some lumber yards to purchase No. 2 Common material and select the better pieces to be sold as No. 1 Common.

The designer may not control the construction of the building. If he does not, and suspects that his specifications may not be followed, he will be wise to use conservative stresses.

7. Holes and Notches for Pipes, Conduits, etc.—Plumbers, electricians, and gas fitters are no respecters of architects and engineers, and have no hesitation in boring a hole or cutting a notch in a joist or girder. This fact is an additional reason for using conservative stresses in the calculation of joists and girders, and especially the former.

8. Horizontal Shear.—In deep short beams the safe unit stress in horizontal shear may be the determining feature. This will seldom be the case in the design of joists, but may be a factor in the selection of the proper size for girders. In this connection the effect of possible checks at the ends of the beam, in or near the horizontal plane, should be considered. Such checks obviously decrease the section of beam for resisting shearing stresses.

9. Bearing at Ends of Beams.—Sufficient bearing must be provided at the ends of all beams, so that, with the maximum reaction at the support, the timber may not crush in side bearing. Most structural timbers are comparatively weak in cross bearing. The details at the ends of timber beams are often poor, insufficient bearing area being provided, so that the beams could never develop their safe loads as determined by bending strength. In general no beam should have a smaller bearing area than given by the product of the width of the beam by 4 in. Details of end connections of beams and girders are discussed in Arts. 122 and 123.

10. Deflection.—If a beam has insufficient depth for its span, it will deflect excessively. The result may be a cracked ceiling, if the latter is plastered, or, in an unplastered building, merely a floor that shakes when walked upon. The limit of deflection of a timber joist is generally placed at $\frac{1}{360}$ of the span.

Timber is different from the other building materials, such as steel or concrete, in that, if loaded excessively with a constant load, its deflection will continue to increase with no increase of load, even though the maximum unit stress in bending be within the elastic limit of the particular timber. For this reason, many specifications require that the modulus of elasticity for "dead," or constant, loads be taken as one-half the modulus of elasticity used for "live," or occasional, loading, the latter quantity being the value determined from a short-time loading test. For example, the Am. Ry. Eng. Assoc. through the committee on "Wooden Bridges and Trestles," recommends "To compute the deflection of a beam under long-continued loading instead of that when the load is first applied, only 50% of the corresponding modulus of elasticity given in the table[1] is to be employed." Tests by Tieneman[2] indicate that a beam may be loaded to within 20% of its elastic limit without danger of increase of deflection.

The recommendation is here made that for constant or "dead" loads the modulus of elasticity be taken at $\frac{3}{4}$ that given in the table in Sect. 7, Art. 10, while for occasional or "live" loading the full values of this table be used.

11. Lateral Support for Beams.—A timber beam needs to be supported laterally in the same manner as a beam of steel or concrete. Floor joists are braced by the flooring and also by the bridging, while the girders are held by the attachment of joists.

In the case of a beam unsupported laterally, the maximum unit fiber stress in flexure should not exceed the value

$$f = f_1\left(1 - \frac{1}{90} \cdot \frac{l}{b}\right)$$

where f_1 = basic unit flexural fiber stress, l = span of beam in inches, and b = breadth of beam in inches.[3]

12. Sized and Surfaced Timbers.—The fact must always be borne in mind by the designer of timber beams that a variation from the nominal size of timbers is allowed by all grading rules; also, that if timber beams are sized, the actual depth is less than the nominal depth. Further, if timber is bought from a local lumber yard, joists may come surfaced one side. In general, all-rail shipments of timbers are surfaced one side one edge (S1S1E) while all-water shipments are not surfaced. *The actual dimensions of the finished stick must be used in all calculations.* Tables 1, 2, 3, 6, 7, 8, and 9 show the relation between actual sizes and nominal sizes.

13. Joists.—Joists usually carry only a uniform load composed of the weight of the joists themselves plus the flooring plus superimposed loads of people, furniture, etc. The latter loads are commonly termed "live" loads in contrast with the constant loads due to the weight of the floor construction itself, called "dead" loads. The joists carry the flooring directly on their upper surfaces, and are in turn supported at their ends by girders, bearing partitions or

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[1] See table in Sect. 7, Art. 10.

[2] See *Eng. News*, vol. 62, pp. 216–217.

[3] Properly the factor $\frac{1}{90}$ holds only for the case of simple beams loaded uniformly and at the third points, and for cantilever beams with uniform loading. For a simple beam with single concentrated load at any point of span the factor is $\frac{1}{120}$, while for quarter point loading the factor is $\frac{1}{80}$.

bearing walls. Joists are always single sticks of timber. Joists may, and often do, carry concentrated loads in addition to the uniform loads mentioned above. Such concentrations may be caused by heavy pieces of furniture, safes, etc., by cross partitions resting on the floor, or by special floor framing as required by openings in the floor.

Many designs of joists or girders are faulty in that the designer has not considered such concentrated floor loads in addition to the uniform loading. In design, with the use of tables giving safe loads for timber, the beams selected thereby may not be sufficient for all cases of framing where loading has been assumed to be uniform. For such cases, the concentrations are sometimes reduced to equivalent uniform loads before entering the tables. A correct and satisfactory method, except for the simpler cases, is to compute the separate bending moments due to each load and combine these partial moments to get the amount and position of the maximum moment. The combination of the partial moments may be quickly accomplished by graphical methods, as illustrated in Art. 46. Having this, the required section is easily found (see chapter on "Simple and Cantilever Beams," Sect. 1).

Table 6 gives the resisting moments of rectangular beams, computed on the basis of the actual finished sections, for maximum unit fiber stresses varying from 1000 to 2000 lb. per sq. in.; also the factors by which the moments in the tables are to be multiplied to get the resisting moments of the rough sections.

14. Girders.—Girders may be single sticks or composite sections. Girders usually support joists, and in turn are supported by columns or bearing walls. When girders are carried otherwise than by columns, the fact must always be borne in mind that such girders deliver a concentrated load of some magnitude to the wall, or bearing partition, and care must be taken to see that such wall or partition is strong enough in column action to carry the load imposed upon it by the girders.

For ordinary building construction, where timber not better than No. 1 Common is likely to be used, it is recommended that the maximum unit fiber stress in bending for long-leaf yellow pine or Douglas fir be limited to 1500 lb. per sq. in., and the maximum unit longitudinal shearing stress be limited to 150 lb. per sq. in. For timber of the grade of Select Structural, or Select No. 1 Common, the unit flexural stress, computed always on the basis of actual finished sections, may be increased to 1800 lb. per sq. in., and the unit longitudinal shearing stress to 175 lb. per sq. in.

Tables 1, 2, and 3 give the safe loads, deflection, and maximum unit shearing stresses for 2, 3 and 4-in. joists, respectively. The maximum unit fiber stress in bending is 1500 lb. per sq. in., computed on the finished size of joist. The deflection is based on a modulus of elasticity of 1,643,000. The maximum intensity of horizontal shearing stress is given for the shortest span. To use this table for other unit flexural fiber stresses, the values in the tables must be multiplied by the factors of Tables 4 and 5.

Illustrative Problem.—Required to find proper size of joist to support a load of 5500 lb. on a 14-ft. span, with a fiber stress of 1200 lb. per sq. in.

From Table 5 we find factor of multiplication to be 1.250. The new load to use in entering Tables 1, 2, and 3 is therefore 5500 lb. × 1.250 = 6870 lb. From Table 1 it is seen that a 3 × 16-in. joist on a 14-ft. span has a safe carrying capacity of 7150 lb. (at 1500 lb. per sq. in.).

Illustrative Problem.—Given a 2 × 14-in. joist on a 16-ft. span. Required the safe load as limited by a maximum unit fiber stress of 1200 lb. per sq. in. in bending. From Table 1, the safe load for 1500 lb. per sq. in. in bending is seen to be 3085 lb. From Table 4, the factor of multiplication is seen to be 0.80, giving the safe load as 2468 lb.

Diagram on p. 102 gives a simple method for solving the strength of any timber beam as determined by maximum unit strength in bending, also for determining the proper size of any timber beam to support a given load in bending.

Illustrative Problem.—Given a total floor load, dead and live, of 174 lb. per sq. ft., span 13 ft. 1 in. What size joists, spaced 16 in. on centers, will support this load with a maximum unit fiber stress of 1800 lb. per sq. in.?

Lay a flexible straight edge, such as a card, on the diagram, p. 102, joining Point *A* (174 lb. per sq. ft.) with *B* (16-in. spacing), and mark intersection *C* on Working Line. Pivoting card about *C*, connect *C* with *D* (13 ft. 1 in.) and read 5000 ft.-lb. at *E*. Connect *E* with *F* (1800 lb. per sq. in.), crossing Working Line at *G*. Pivoting card about *G*, set card on 1⅝ in. (width of beam) at *H* and read 11½ in. (depth of beam) at *K*.

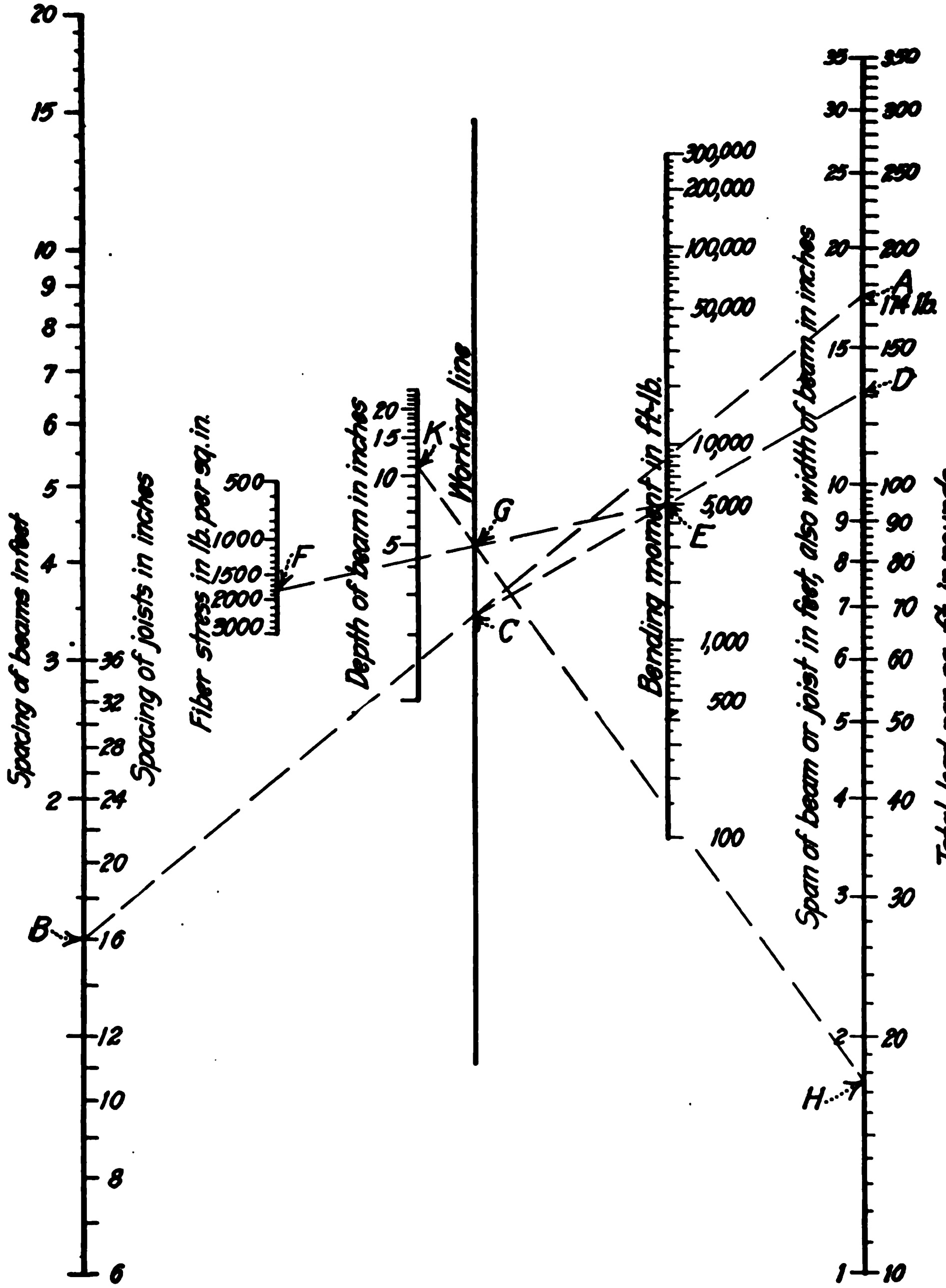

Diagram for capacities of timber beams as determined by bending strength.

15. Explanation of Tables.—In Tables 1, 2, and 3, the first line of figures in each group represents the safe load for the particular beam, including the weight of the beam itself. The second line of figures gives the deflection in inches for the beam at the maximum safe load, computed for a modulus of elasticity of 1,643,000 lb. per sq. in. The third figure, where such figure occurs, indicates the maximum unit horizontal shearing stress. The shearing stress is given, only in those cases in which such shear is in excess of 150 lb. per sq. in. All quantities in these tables are based upon the surfaced sizes of sticks. To obtain the safe loads for the rough or full nominal sizes of timber, the loads of Tables 1, 2, and 3 must be multiplied by the "multiply ing factors" of Table 6. These tables have been adapted from similar tables in the "Structural Timber Handbook on Pacific Coast Woods" published by the West Coast Lumbermen's Association.

Tables 7, 8 and 9 give for timber joists: (1) the safe loads corresponding to a maximum flexural stress of 1800 lb. per sq. in., indicated in the tables by the letter "*B*"; (2) the safe load, uniformly distributed, limited by a maximum intensity of horizontal shear of 175 lb. per sq. in., indicated in the tables by the letters "*HS*"; (3) the uniformly distributed load that produces a deflection of $\frac{1}{30}$ in. per foot of span, indicated in the tables by the letter "*D*"; and (4) the deflection in inches for a load of 1000 lb., uniformly distributed, indicated in the tables by "*D*1." All deflections are computed for a modulus of elasticity of 1,620,000 lb. per sq. in. All loads and deflections are computed on the finished or surfaced sizes of joists. For convenience, the section moduli of the various sizes of joists are given, based on finished sizes. These tables are taken from the "Southern Pine Manual" published by the Southern Pine Association.

Attention is called to the variation of sizes of finished joists in Tables 1, 2, 3 and Tables 7, 8, and 9, representing the difference between the standards of the West Coast Lumbermen's Association and the Southern Pine Association, the finished sections of the Southern Pine Association utilizing a greater percentage of the rough timber than the standards of the West Coast Lumbermen's Association. All sizes of joists in Tables, 1, 2, and 3 (West Coast Lumbermen's Association) are for joists surfaced one side and one edge, or surfaced four sides (S4S). All sizes in Tables 7, 8, and 9 (Southern Pine Association) are for joists surfaced one side and one edge (S1S1E).

TABLE 1.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 2 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL FIBER STRESS OF 1500 LB. PER SQ. IN.

Sizes	Rough size	2×4	2×6	2×8	2×10	2×12	2×14	2×16	×18
	Surfaced size S1S1E or S4S[1]	1⅝×3⅝	1⅝×5⅝	1⅝×7½	1⅝×9½	1⅝×11½	1⅝×13½	1⅝×15½	1⅝×17½
	Section modulus	3.56	8.57	15.23	24.44	35.82	49.36	65.07	82.94
Spans in feet	3	1187 0.0681 151							
	4	890 0.121	2142 0.0780 176						
	5	712 0.189	1714 0.122						
	6	593 0.272	1428 0.176	2538 0.131 156					
	7	509 0.370	1224 0.239	2176 0.179	3491 0.141 170				
	8		1071 0.312	1904 0.234	3055 0.185	4478 0.153 180			
	9		953 0.395	1692 0.296	2716 0.243	3980 0.193			
	10		857 0.487	1523 0.365	2444 0.289	3582 0.238	4936 0.203 169		
	11		779 0.589	1385 0.442	2222 0.349	3256 0.288	4487 0.245	5915 0.214 167	
	12			1269 0.526	2037 0.415	2985 0.343	4113 0.292	5423 0.254	6912 0.225 182
	13			1172 0.617	1880 0.487	2755 0.403	3797 0.343	5005 0.299	6380 0.265
	14			1088 0.716	1746 0.565	2559 0.467	3526 0.397	4648 0.347	5924 0.307
	15			1015 0.822	1629 0.649	2388 0.536	3291 0.456	4338 0.398	5529 0.352
	16				1528 0.738	2239 0.610	3085 0.519	4067 0.453	5184 0.401
	17				1438 0.834	2107 0.688	2904 0.586	3828 0.511	4879 0.452
	18				1358 0.935	1990 0.773	2742 0.657	3615 0.572	4608 0.504
	19					1885 0.860	2598 0.732	3426 0.637	4365 0.565
	20					1791 0.953	2468 0.811	3254 0.706	4147 0.626
	21						2350 0.895	3099 0.779	3949 0.690
	22						2244 0.981	2958 0.855	3770 0.758
	23							2829 0.935	3606 0.829
	24								3456 0.901
	25								3318 0.979

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[1] S1S1E = surfaced one side and one edge.
S4S = surfaced four sides.

ABLE 2.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 3 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL FIBER STRESS OF 1500 LB. PER SQ. IN.

izes	Rough size	3×6	3×8	3×10	3×12	3×14	3×16	3×18
	Surfaced size S1S1E or S4S[1]	2⅝×5⅝	2⅝×7⅝	2⅝×9⅝	2⅝×11⅝	2⅝×13⅝	2⅝×15⅝	2⅝×17⅝
	Section modulus	12.60	23.42	37.61	55.10	75.94	100.10	127.60
Spans in feet	4	3150 0.0797 172						
	5	2520 0.125						
	6	2100 0.179	3903 0.131 156					
	7	1800 0.244	3346 0.179	5373 0.141 170				
	8	1575 0.319	2928 0.234	4701 0.185	6888 0.153 180			
	9	1400 0.404	2602 0.296	4179 0.234	6122 0.193			
	10	1260 0.498	2342 0.365	3761 0.289	5510 0.238	7594 0.203 169		
	11	1145 0.603	2129 0.442	3419 0.349	5009 0.288	6904 0.245	9100 0.214 176	
	12		1952 0.526	3134 0.415	4592 0.343	6328 0.292	8342 0.254	10,633 0.225 182
	13		1802 0.617	2893 0.487	4239 0.403	5842 0.343	7700 0.299	9,815 0.265
	14		1673 0.716	2686 0.565	3936 0.467	5424 0.397	7150 0.347	9,114 0.307
	15		1561 0.822	2507 0.649	3673 0.536	5063 0.456	6673 0.398	8,507 0.352
	16			2351 0.738	3444 0.610	4746 0.519	6256 0.453	7,975 0.401
	17			2212 0.834	3241 0.688	4467 0.586	5889 0.511	7,506 0.452
	18			2089 0.935	3061 0.773	4219 0.657	5561 0.572	7,089 0.507
	19				2900 0.917	3997 0.732	5268 0.637	6,716 0.565
	20					3797 0.811	5005 0.706	6,380 0.626
	21					3616 0.895	4767 0.779	6,076 0.690
	22					3452 0.981	4550 0.855	5,800 0.758
	23						4352 0.935	5,548 0.829
	24							5,317 0.901
	25							5,104 0.979

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[1] S1S1E = surfaced one side and one edge.
S4S = surfaced four sides.

TABLE 3.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 4 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL FIBER STRESS OF 1500 LB. PER SQ. IN.

Sizes	Rough size	4×4	4×6	4×8	4×10	4×12	4×14	4×16	4×18
	Surfaced size S1S1E or S4S[1]	3½×3½	3½×5½	3½×7½	3½×9½	3½×11½	3½×13½	3½×15½	3½×17½
	Section modulus	7.15	17.64	32.81	52.65	77.15	106.31	140.15	178.65
Spans in feet	3	2383 0.0705 146							
	4	1788 0.125	4410 0.0797 172						
	5	1430 0.196	3528 0.125						
	6	1192 0.282	2940 0.179	5468 0.131 156					
	7	1021 0.384	2520 0.244	4687 0.179	7521 0.141 170				
	8		2205 0.319	4101 0.234	6581 0.185	9644 0.153 180			
	9		1960 0.404	3646 0.296	5850 0.234	8572 0.193			
	10		1764 0.498	3281 0.365	5265 0.289	7715 0.238	10,631 0.203 169		
	11		1604 0.603	2983 0.442	4786 0.349	7015 0.288	9,665 0.245	12,741 0.214 176	
	12			2734 0.526	4388 0.415	6429 0.343	8,861 0.292	11,679 0.254	14,888 0.225 182
	13			2524 0.617	4050 0.487	5935 0.403	8,178 0.343	10,781 0.299	13,742 0.265
	14			2344 0.715	3761 0.565	5511 0.467	7,594 0.397	10,011 0.347	12,761 0.307
	15			2187 0 822	3510 0.649	5143 0.536	7,087 0.456	9,343 0.398	11,910 0.352
	16				3291 0.738	4822 0.610	6,644 0.519	8,759 0.453	11,166 0.401
	17				3097 0.834	4538 0.688	6,254 0.586	8,244 0.511	10,508 0.452
	18				2924 0.935	4286 0.773	5,906 0.657	7,786 0.572	9,925 0.507
	19					4061 0.860	5,595 0.732	7,376 0.637	9,403 0.565
	20					3858 0.953	5,316 0.811	7,008 0.706	8,933 0.626
	21						5,063 0.895	6,674 0.779	8,507 0.690
	22						4,832 0.981	6,370 0.855	8,120 0.758
	23							6,093 0.935	7,767 0.829
	24								7,444 0.901
	25								7,146 0.979

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[1] **S1S1E = surfaced one side and one edge.**
S4S = surfaced four sides.

TABLE 4.—FACTORS BY WHICH SAFE LOADS IN TABLES 1, 2 AND 3 MUST BE MULTIPLIED TO FIND SAFE LOADS THAT GIVEN SIZE OF JOIST WILL SUPPORT AT A UNIT FLEXURAL STRESS OTHER THAN 1500 LB. PER SQ. IN.

TABLE 5.—FACTORS BY WHICH GIVEN LOAD MUST BE MULTIPLIED TO FIND EQUIVALENT LOAD TO BE USED IN ENTERING TABLES 1, 2, AND 3 TO FIND PROPER SIZE OF JOIST

TABLE 4

Desired unit fiber stress	Factor of multiplication
1000	0.667
1100	0.734
1200	0.800
1300	0.867
1400	0.933
1500	1.000
1600	1.067
1700	1.133
1800	1.200
2000	1.333

TABLE 5

Desired unit fiber stress	Factor of multiplication
1000	1.500
1100	1.363
1200	1.250
1300	1.153
1400	1.071
1500	1.000
1600	0.939
1700	0.883
1800	0.833
2000	0.750

TABLE 6.—MAXIMUM BENDING OR RESISTING MOMENTS IN FOOT-POUNDS FOR RECTANGULAR BEAMS

Values in this table are based on surfaced sizes.[1] To get values for rough sizes, multiply Resisting Moment for any given size by "multiplying factor" in dark type in same horizontal line

Size		"Multiplying factor"	Section modulus (in.³)	Moment of inertia (in.⁴)	Resisting moments in foot-pounds for safe fiber stresses in pounds per square inch, as indicated								
Nominal (inches)	Actual[1] (inches)				1000	1100	1200	1300	1400	1500	1600	1800	2000
2 × 4	1⅝× 3⅝	1.50	3.56	6.45	297	327	356	386	416	446	475	535	594
2 × 6	1⅝× 5⅝	1.40	8.57	24.10	714	785	857	928	1000	1071	1142	1285	1428
2½× 6	2¼× 5½	1.32	11.34	31.18	945	1040	1134	1228	1323	1417	1512	1701	1890
2 × 8	1⅝× 7½	1.40	15.23	57.13	1269	1396	1523	1650	1777	1904	2030	2284	2538
2½× 8	2¼× 7½	1.26	21.10	79.10	1758	1934	2110	2285	2462	2637	2813	3165	3518
2 ×10	1⅝× 9½	1.36	24.44	116.10	2037	2241	2444	2648	2852	3056	3259	3667	4074
2½×10	2¼× 9½	1.23	33.84	160.76	2820	3102	3384	3666	3948	4230	4512	5076	5640
2 ×12	1⅝×11½	1.34	35.82	205.95	2985	3284	3582	3881	4179	4478	4776	5373	5970
2½×12	2¼×11½	1.21	49.59	285.16	4133	4546	4958	5371	5786	6197	6612	7436	8265
2 ×14	1⅝×13½	1.32	49.36	333.18	4113	4524	4936	5347	5758	6170	6581	7403	8226
2 ×14	1¾×13½	1.23	53.16	358.80	4430	4873	5316	5759	6202	6645	7088	7974	8860
2½×14	2¼×13½	1.20	68.34	461.32	5695	6264	6834	7403	7973	8542	9112	10251	11390
2 ×16	1⅝×15½	1.31	65.07	504.28	5423	5965	6507	7050	7592	8135	8677	9761	10846
2 ×16	1¾×15½	1.22	70.10	543.06	5842	6426	7010	7594	8178	8762	9347	10515	11683
2½×16	2¼×15½	1.18	90.10	698.23	7508	8260	9010	9760	10512	11262	12013	13515	15018
2 ×18	1⅝×17½	1.30	82.94	725.75	6912	7603	8294	8986	9677	10368	11059	12442	13824
2 ×18	1¾×17½	1.21	89.32	781.57	7446	8188	8932	9676	10421	11165	11901	13398	14887
2½×18	2¼×17½	1.17	114.84	1004.88	9570	10527	11484	12441	13398	14355	15312	17226	19140
3 × 6	2½× 5½	1.43	12.60	34.66	1050	1155	1260	1365	1470	1575	1680	1890	2100
3 × 6	2¾× 5½	1.30	13.86	38.13	1155	1271	1386	1501	1617	1732	1848	2079	2310
3 × 8	2½× 7½	1.37	23.42	87.89	1952	2147	2342	2538	2733	2928	3123	3514	3904
3 × 8	2¾× 7½	1.24	25.78	96.68	2148	2363	2578	2793	3008	3222	3437	3867	4297
3 ×10	2½× 9½	1.33	37.61	178.62	3134	3447	3761	4074	4388	4701	5014	5641	6268
3 ×10	2¾× 9½	1.21	41.36	196.48	3447	3791	4136	4480	4825	5170	5515	6204	6893
3 ×12	2½×11½	1.31	55.10	316.85	4592	5051	5510	5970	6429	6888	7347	8266	9184
3 ×12	2¾×11½	1.19	60.61	348.53	5051	5556	6060	6565	7071	7575	8081	9090	10102
3 ×14	2½×13½	1.29	75.94	512.58	6328	6961	7594	8226	8859	9492	10125	11390	12656
3 ×14	2¾×13½	1.17	83.53	563.84	6961	7657	8353	9049	9745	10441	11137	12529	13922
3 ×16	2½×15½	1.28	100.10	775.81	8342	9176	10010	10845	11679	12513	13347	15016	16684
3 ×16	2¾×15½	1.16	110.11	853.39	9176	10093	11011	11928	12846	13763	14681	16516	18352
3 ×18	2½×17½	1.27	127.60	1116.54	10633	11696	12760	13823	14886	15950	17013	19139	21266
3 ×18	2¾×17½	1.15	140.36	1228.19	11696	12866	14036	15205	16375	17545	18715	21054	23393
4 × 4	3½× 3½	1.49	7.15	12.51	596	656	715	775	834	894	954	1073	1192
4 × 4	3⅝× 3⅝	1.34	7.94	14.39	662	728	794	860	926	992	1059	1190	1323
4 × 6	3½× 5½	1.36	17.64	48.53	1470	1617	1764	1911	2058	2205	2352	2646	2940
4 × 6	3⅝× 5⅝	1.26	19.12	53.76	1593	1753	1912	2071	2231	2390	2549	2868	3187
4 × 8	3½× 7½	1.30	32.81	123.05	2734	3007	3281	3554	3828	4101	4374	4921	5468
4 × 8	3¾× 7½	1.21	35.16	131.83	2930	3223	3516	3809	4102	4395	4688	5274	5860
4 ×10	3½× 9½	1.27	52.65	250.07	4388	4827	5265	5704	6143	6582	7021	7898	8776
4 ×10	3¾× 9½	1.18	56.41	267.93	4701	5171	5641	6110	6581	7050	7521	8461	9402

TABLE 6.—MAXIMUM BENDING OR RESISTING MOMENTS IN FOOT-POUNDS FOR RECTANGULAR BEAMS—(*Continued*)

Size		"Multiplying factor"	Section modulus (in.^{3})	Moment of inertia (in.^{4})	Resisting moments in foot-pounds for safe fiber stresses in pounds per square inch, as indicated								
Nominal (inches)	Actual[1] (inches)				1000	1100	1200	1300	1400	1500	1600	1800	2000
4 ×12	3½×11½	1.25	77.15	443.59	6429	7072	7715	8358	9001	9644	10286	11572	12858
4 ×12	3¾×11½	1.16	82.66	475.27	6888	7577	8266	8955	9644	10332	11021	12399	13777
4 ×14	3½×13½	1.23	106.31	717.61	8859	9745	10631	11517	12403	13289	14174	15946	17718
4 ×14	3¾×13½	1.15	113.91	768.87	9493	10442	11391	12340	13289	14238	15188	17086	18985
4 ×16	3½×15½	1.22	140.15	1086.13	11679	12847	14015	15183	16351	17519	18686	21022	23358
4 ×16	3¾×15½	1.14	150.16	1163.71	12513	13765	15016	16267	17519	18770	20021	22524	25027
4 ×18	3½×17½	1.21	178.65	1563.15	14888	16377	17865	19354	20843	22332	23821	26798	29776
4 ×18	3¾×17½	1.13	191.41	1674.80	15951	17546	19141	20736	22331	23926	25521	28711	31902
6 × 6	5½× 5½	1.30	27.73	76.26	2311	2542	2773	3004	3235	3467	3698	4160	4622
6 × 8	5½× 7½	1.24	51.56	193.36	4297	4727	5156	5586	6016	6446	6875	7735	8594
6 ×10	5½× 9½	1.21	82.73	392.96	6894	7583	8273	8962	9652	10341	11030	12409	13788
6 ×12	5½×11½	1.19	121.23	697.07	10103	11113	12123	13134	14144	15155	16165	18185	20206
6 ×14	5½×13½	1.17	167.06	1127.67	13922	15314	16706	18099	19491	20883	22275	25060	27844
6 ×16	5½×15½	1.16	220.23	1706.78	18353	20188	22023	23859	25694	27530	29365	33035	36706
6 ×18	5½×17½	1.16	280.73	2456.38	23394	25733	28073	30412	32752	35091	37430	42109	46788
6 ×20	5½×19½	1.15	348.56	3398.49	29047	31952	34856	37761	40666	43571	46475	52285	58094
8 × 8	7½× 7½	1.21	70.31	263.67	5859	6445	7031	7617	8203	8789	9374	10546	11718
8 ×10	7½× 9½	1.18	112.81	535.86	9401	10341	11281	12221	13161	14102	15042	16922	18802
8 ×12	7½×11½	1.16	165.31	950.55	13776	15154	16531	17909	19286	20664	22042	24797	27552
8 ×14	7½×13½	1.15	227.81	1537.74	18984	20882	22781	24679	26578	28476	30374	34171	37968
8 ×16	7½×15½	1.14	300.31	2327.43	25026	27529	30031	32534	35036	37539	40042	45047	50052
8 ×18	7½×17½	1.13	382.81	3349.61	31901	35091	38281	41471	44661	74852	51042	57422	63802
8 ×20	7½×19½	1.12	475.31	4634.30	39609	43570	47531	51492	55453	59414	63374	71296	79218
10 ×10	9½× 9½	1.17	142.89	678.76	11908	13099	14289	15480	16671	17862	19053	21434	23816
10 ×12	9½×11½	1.15	209.40	1204.03	17450	19195	20940	22685	24430	26175	27920	31410	34900
10 ×14	9½×13½	1.13	288.56	1947.80	24047	26452	28856	31261	33666	36071	38475	43285	48094
10 ×16	9½×15½	1.12	380.40	2948.07	31700	34870	38040	41210	44380	47550	50720	57060	63400
10 ×18	9½×17½	1.11	484.90	4242.84	40408	44449	48490	52530	56571	60612	64653	72734	80816
10 ×20	9½×19½	1.11	602.06	5870.11	50172	55189	60206	65224	70241	75258	80275	90310	100344
12 ×12	11½×11½	1.14	253.48	1457.51	21123	23235	25348	27460	29572	31685	33797	38021	42246
12 ×14	11½×13½	1.12	349.31	2357.86	29109	32020	34931	37842	40753	43664	46574	52396	58218
12 ×16	11½×15½	1.11	460.48	3568.72	38373	42210	46048	49885	53722	57560	61397	59071	76746
12 ×18	11½×17½	1.10	586.98	5136.07	48915	53807	58698	63590	68481	73373	78264	88047	97830
12 ×20	11½×19½	1.10	728.81	7105.93	60734	66807	72881	78954	85028	91101	97174	109321	121468
14 ×14	13½×13½	1.12	410.06	2767.93	34172	37589	41006	44424	47841	51258	54675	61510	68344
14 ×16	13½×15½	1.11	540.56	4189.37	45047	49552	54056	58561	63066	67571	72075	81085	90094
14 ×18	13½×17½	1.10	689.06	6029.30	57422	63164	68906	74649	80391	86133	91875	103360	114844
14 ×20	13½×19½	1.09	855.56	8341.74	71297	78427	85556	92686	99816	106946	114075	128335	142594
16 ×16	15½×15½	1.10	620.64	4810.01	51720	56892	62064	67236	72408	77580	82752	93096	103440
16 ×18	15½×17½	1.09	791.15	6922.53	65929	72522	79115	85708	92301	98894	105486	118672	131858
16 ×20	15½×19½	1.09	982.31	9577.55	81859	90045	98231	106417	114603	122789	130974	147346	163718
16 ×22	15½×21½	1.08	1194.15	12837.07	99513	109464	119415	129367	139318	149270	159221	179123	199026
16 ×24	15½×23½	1.08	1426.65	16763.10	118888	130777	142665	154554	166443	178332	190221	213998	237776
18 ×18	17½×17½	1.09	893.23	7815.76	74436	81880	89323	96767	104210	111654	119098	133985	148872
18 ×20	17½×19½	1.08	1109.06	10813.37	92422	101664	110906	120149	129391	138633	147875	166360	184844
18 ×22	17½×21½	1.08	1348.23	14493.47	112353	123588	134823	146059	157294	168530	179765	202235	224706
18 ×24	17½×23½	1.07	1610.73	18926.08	134228	147651	161073	174496	187919	201342	214765	241610	268456
18 ×26	17½×25½	1.07	1896.56	24181.18	158047	173852	189656	205461	221266	237071	252875	284485	316094
20 ×20	19½×19½	1.08	1235.81	12049.18	102984	113282	123581	133879	144178	154476	164774	185371	205968
20 ×22	19½×21½	1.07	1502.31	16149.87	125193	137712	150231	162751	175270	187790	200309	225347	250386
20 ×24	19½×23½	1.07	1794.81	21089.06	149568	164525	179481	194438	209395	224352	239309	269222	299136
20 ×26	19½×25½	1.07	2113.31	26944.74	176109	193720	211331	228942	246553	264164	281774	316996	352218
20 ×28	19½×27½	1.06	2457.81	33794.90	204818	225300	245781	266263	286745	307227	327709	368672	409636
20 ×30	19½×29½	1.06	2828.31	41717.62	235693	259262	282831	306401	329970	353540	377109	424247	471386

[1] This table is based on tables from the "Southern Pine Manual" of the Southern Pine Association, and the "Structural Timber Handbook on Pacific Coast Woods" of the West Coast Lumbermen's Association.

The standards of the latter association are as follows:

"*Dimension, Plank and Small Timbers.*—Sizes—S1S1E or S4S: 2 × 3 to 1⅝ × 2⅝; 2 × 4 to 1⅝ × 3⅝; 2 × 6 to 1⅝ × 5⅝; 2 × 8 to 1⅝ × 7½; 2 × 10 to 1⅝ × 9½; 2 × 12 to 1⅝ × 11½; 2 × 14 to 1⅝ × 13½; 2 × 16 to 1⅝ × 15½; etc.; 3 × 4 to 2½ × 3⅝; 3 × 6 to 2½ × 5½; 3 × 8 to 2½ × 7½; 3 × 10 to 2½ × 9½; 3 × 12 to 2½ × 11½; 3 × 14 to 2½ × 13½; 3 × 16 to 2½ × 15½; etc.; 4 × 4 to 3½ × 3½; 4 × 6 to 3½ × 5½, etc.; 5 × 5 to 4½ × 4½; etc.; 6 × 6 and 8, ½ in. off each way."

"*Timbers.*—Sizes—S1S, S1E, S1S1E, or S4S: 8 × 8 and larger ½ in. off each way. Standard lengths are multiples of 2 ft."

The standards of the Southern Pine Association for timber surfaced four sides are the same as those of the West Coast Lumbermen's Association, *i.e.*, ½ in. off the nominal width and depth. For material surfaced one side one edge (S1S1E) their standards are ¼ in. off the nominal width and ½ in. off the nominal depth.

able 7.—Table of Safe Loads and Deflections for Timber Joists With Nominal Width of 2 inches, Uniformly Loaded, Based on Maximum Flexural Stress of 1800 Lb. per Sq. In.

izes	Rough size	2×4	2×6	2×8	2×10	2×12	2×14	2×16	2×18
	Surfaced size S1S1E[1]	1⅝×3⅝	1⅝×5⅝	1⅝×7½	1⅝×9½	1⅝×11½	1¾×13½	1¾×15½	1¾×17⅝
	Section modulus	3.56	8.57	15.23	24.44	35.82	53.16	70.10	89.32
Spans in feet									
3	HS	1372							
	D1	0.0581							
4	B	1068							
	D	967							
	D1	0.1379	0.0369						
	HS	......	2135						
5	B	854	2056						
	D	619	......						
	D1	0.2693	0.0720						
6	B	712	1714						
	D	430	1607						
	D1	0.4651	0.1244	0.0525					
	HS	......	......	2843					
7	B	610	1469	2611					
	D	316	1180	......					
	D1	0.7384	0.1977	0.0834					
8	B	534	1285	2284					
	D	242	904	2142					
	D1	1.1020	0.2950	0.1245	0.0612				
	HS	......	......	......	3601				
9	B		1143	2031	3258				
	D		714	1693	......				
	D1		0.4202	0.1772	0.0872	0.0492			
	HS		......	......	......	4361			
10	B		1028	1828	2933	4298			
	D		578	1371	2786	......			
	D1		0.5767	0.2431	0.1196	0.0674			
11	B		935	1661	2666	3908			
	D		478	1133	2303	......			
	D1		0.7671	0.3236	0.1592	0.0897	0.0515		
	HS		......	......	......	......	5512		
12	B		857	1526	2444	3582	5316		
	D		402	952	1935	3432	......		
	D1		0.9950	0.4202	0.2067	0.1165	0.0669		
13	B			1406	2256	3306	4907		
	D			811	1648	2924	......		
	D1			0.5343	0.2630	0.1482	0.0850	0.0562	
	HS			......	......	......	......	6328	
14	B			1306	2095	3070	4556	6008	
	D			700	1422	2521	4393	......	
	D1			0 6667	0.3282	0.1851	0.1062	0.0702	
15	B			1218	1955	2865	4253	5608	
	D			609	1238	2196	3827	......	
	D1			0.8210	0.4040	0.2277	0.1306	0.0863	0.0599
	HS			......	......	......	......	......	7147
16	B			1142	1833	2686	3987	5257	6699
	D			536	1089	1931	3364	5091	......
	D1			0.9950	0.4898	0.2762	0.1585	0.1047	0.0728

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[1] S1S1E = surfaced one side and one edge.

Table 7.—Table of Safe Loads and Deflections for Timber Joists with Nominal Widths of 2 Inches, Uniformly Loaded, Based on Maximum Flexural Stress of 1800 Lb. per Sq. In.—(*Continued*)

Sizes	Rough size	2×4	2×6	2×8	2×10	2×12	2×14	2×16	2×18
	Surfaced size S1S1E[1]	1⅝×3⅝	1⅝×5⅝	1⅝×7½	1⅝×9½	1⅝×11½	1¾×13½	1¾×15½	1¾×17½
	Section modulus	3.56	8.57	15.23	24.44	35.82	53.16	70.10	89.32
Spans in feet 17	B				1725	2528	3753	4948	6305
	D				964	1710	2980	4510	
	D1				0.5878	0.3314	0.1902	0.1256	0.0873
18	B				1629	2388	3544	4673	5954
	D				860	1525	2658	4023	5790
	D1				0.6977	0.3934	0.2254	0.1492	0.1036
19	B				1544	2262	3358	4427	5641
	D				772	1369	2385	3610	5196
	D1				0.8204	0.4626	0.2655	0.1754	0.1219
20	B				1466	2149	3190	4206	5359
	D				697	1236	2153	3258	4690
	D1				0.9565	0.5395	0.3097	0.2046	0.1422
21	B					2047	3038	4006	5104
	D					1121	1953	2956	4254
	D1					0.6244	0.3585	0.2368	0.1646
22	B					1954	2900	3824	4872
	D					1021	1779	2693	3876
	D1					0.7183	0.4122	0.2723	0.1892
23	B					1869	2774	3657	4660
	D					934	1628	2464	3546
	D1					0.8208	0.4710	0.3112	0.2162
24	B					1791	2658	3505	4466
	D					858	1495	2263	3257
	D1					0.9324	0.5351	0.3535	0.2456
25	B						2552	3365	4287
	D						1378	2085	3001
	D1						0.6048	0.3996	0.2777
26	B						2454	3235	4122
	D						1274	1928	2775
	D1						0.6804	0.4495	0.3123
27	B						2363	3116	3970
	D						1181	1788	2573
	D1						0.7619	0.5034	0.3498
28	B						2278	3004	3828
	D						1098	1663	2392
	D1						0.8498	0.5614	0.3902
29	B							2901	3696
	D							1550	2230
	D1							0.6238	0.4334
30	B							2804	3573
	D							1448	2085
	D1							0.6905	0.4798
31	B								3457
	D								1952
	D1								0.5294
32	B								3349
	D								1832
	D1								0.5823

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[1] S1S1E = surfaced one side and one edge.

TABLE 8.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 3 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL STRESS OF 1800 LB. PER SQ. IN.

Sizes	Rough size	3×6	3×8	3×10	3×12	3×14	3×16	3×18
	Surfaced size S1S1E[1]	2¾×5½	2¾×7½	2¾×9½	2¾×11½	2¾×13½	2¾×15½	2¾×17½
	Section modulus	13.86	25.78	41.36	60.61	83.53	110.11	140.36
Spans in feet								
4	HS	3528						
	D1	0.0233						
5	B	3326						
	D1	0.0455						
6	B	2773						
	D	2542						
	D1	0.0787	0.0310					
	HS	......	4812					
7	B	2376	4419					
	D	1867	......					
	D1	0.1250	0.0493					
8	B	2079	3867					
	D	1429	3625					
	D1	0.1866	0.0735	0.0362				
	HS	......	......	6097				
9	B	1848	3437	5515				
	D	1129	2865	......				
	D1	0.2657	0.1047	0.0515	0.0291			
	HS	......	......	......	7378			
10	B	1663	3093	4963	7273			
	D	915	2320	4715	......			
	D1	0.3643	0.1437	0.0707	0.0398			
11	B	1512	2812	4512	6612			
	D	756	1918	3897	......			
	D1	0.4850	0.1912	0.0941	0.0530	0.0328		
	HS	......	......	......	......	8662		
12	B	1386	2578	4136	6061	8353		
	D	635	1612	3275	5808	......		
	D1	0.6299	0.2481	0.1221	0.0689	0.0426		
13	B		2380	3818	5595	7710		
	D		1373	2790	4949	......		
	D1		0.3156	0.1553	0.0875	0.0541	0.0357	
	HS		......	......	......	......	9947	
14	B		2210	3545	5195	7159	9438	
	D		1184	2406	4267	6904	......	
	D1		0.3941	0.1939	0.1094	0.0676	0.0446	
15	B		2065	3309	4849	6682	8809	
	D		1031	2096	3717	6014	......	
	D1		0.4850	0.2386	0.1345	0.0831	0.0549	0.0382
	HS		......	......	......	......	......	11,228
16	B		1934	3102	4546	6264	8258	10,527
	D		906	1842	3267	5286	8001	......
	D1		0.5887	0.2895	0.1632	0.1009	0.0666	0.0463
17	B			2920	4278	5896	7772	9,908
	D			1632	2894	4682	7087	......
	D1			0.3472	0.1958	0.1210	0.0799	0.0555

[1] S1S1E = surfaced one side and one edge.

TABLE 8.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 3 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL STRESS OF 1800 LB. PER SQ IN.—(*Continued*)

Sizes		Rough size	3×6	3×8	3×10	3×12	3×14	3×16	3×18
		Surfaced size S1S1E[1]	2¾×5½	2¾×7½	2¾×9½	2¾×11½	2¾×13½	2¾×15½	2¾×17½
		Section modulus	13.86	25.78	41.36	60.61	83.53	110.11	140.36
Spans in feet	18	B			2758	4041	5568	7341	9356
		D			1455	2582	4177	6322	9098
		D1			0.4124	0.2324	0.1437	0.0949	0.0659
	19	B			2612	3828	5275	6954	8865
		D			1306	2317	3748	5673	8165
		D1			0.4849	0.2733	0.1689	0.1116	0.0775
	20	B			2481	3636	5012	6606	8421
		D			1179	2091	3383	5120	7369
		D1			0.5655	0.3188	0.1971	0.1302	0.0904
	21	B				3463	4773	6292	8020
		D				1897	3069	4644	6684
		D1				0.3690	0.2281	0.1507	0.1047
	22	B				3306	4556	6006	7656
		D				1728	2796	4232	6090
		D1				0.4244	0.2623	0.1733	0.1204
	23	B				3162	4358	5745	7323
		D				1581	2558	3872	5572
		D1				0.4849	0.2997	0.1980	0.1376
	24	B				3031	4176	5505	7018
		D				1452	2350	3556	5118
		D1				0.5510	0.3405	0.2250	0.1563
	25	B					4009	5285	6737
		D					2165	3277	4716
		D1					0.3849	0.2543	0.1767
	26	B					3855	5082	6478
		D					2002	3030	4361
		D1					0.4329	0.2860	0.1987
	27	B					3712	4894	6238
		D					1856	2810	4043
		D1					0 4848	0.3203	0.2226
	28	B					3579	4719	6015
		D					1726	2612	3760
		D1					0.5407	0.3573	0.2482
	29	B						4556	5808
		D						2436	3505
		D1						0.3969	0.2758
	30	B						4404	5614
		D						2276	3275
		D1						0.4394	0.3053
	31	B							5433
		D							3067
		D1							0.3369
	32	B							5263
		D							2879
		D1							0.3708

[1] S1S1E = surfaced one side and one edge.

ABLE 9.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 4 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL STRESS OF 1800 LB. PER SQ. IN.

izes	Rough size		4×4	4×6	4×8	4×10	4×12	4×14	4×16	4×18
	Surfaced size S1S1E[1]		3⅝×3⅝	3⅝×5⅝	3¾×7½	3¾×9½	3¾×11½	3¾×13½	3¾×15½	3¾×17½
	Section modulus		7.94	19.12	35.16	56.41	82.66	113.91	150.16	191.41
Spans in feet	3	HS	3066							
		D1	0.0261							
	4	B	2382							
		D	2152							
		D1	0.0618	0.0165						
		HS	......	4760						
	5	B	1905	4588						
		D	1382	......						
		D1	0.1206	0.0323						
	6	B	1588	3824						
		D	960	3584						
		D1	0.2083	0.0558	0.0227					
		HS	......	......	6562					
	7	B	1361	3277	6027					
		D	705	2633	......					
		D1	0.3307	0.0886	0.0361					
	8	B	1191	2868	5274					
		D	540	2016	4944					
		D1	0.4938	0.1323	0.0539	0.0265				
		HS	......	......	......	8312				
	9	B		2549	4688	7521				
		D		1593	3906	......				
		D1		0.1883	0.0768	0.0378	0.0213			
		HS		......	......	......	10,062			
	10	B		2294	4219	6769	9919			
		D		1290	3164	6430	......			
		D1		0.2584	0.1053	0.0518	0.0292			
	11	B		2086	3835	6154	9017			
		D		1066	2615	5315	......			
		D1		0.3440	0.1402	0.0690	0.0389	0.0240		
		HS		......	......	......	......	11,812		
	12	B		1912	3516	5641	8266	11,391		
		D		896	2197	4466	7921	......		
		D1		0.4464	0.1821	0.0896	0.0505	0.0312		
	13	B			3246	5207	7630	10,515		
		D			1873	3805	6750	......		
		D1			0.2313	0.1139	0.0642	0.0397	0.0262	
		HS			......	......	......	......	13,562	
	14	B			3014	4835	7085	9764	12,870	
		D			1615	3281	5820	9415	......	
		D1			0.2890	0.1422	0.0802	0.0496	0.0327	
	15	B			2813	4513	6613	9113	12,013	
		D			1406	2858	5070	8201	......	
		D1			0.3556	0.1750	0.0986	0.0610	0.0403	0.0279
		HS			......	......	......	......	......	15,312
	16	B			2637	4230	6199	8543	11,262	14,356
		D			1236	2511	4456	7208	10,909	......
		D1			0.4316	0.2124	0.1197	0.0740	0.0489	0.0339

[1] S1S1E = surfaced one side and one edge.

TABLE 9.—TABLE OF SAFE LOADS AND DEFLECTIONS FOR TIMBER JOISTS WITH NOMINAL WIDTH OF 4 INCHES, UNIFORMLY LOADED, BASED ON MAXIMUM FLEXURAL STRESS OF 1800 LB. PER SQ. IN.—(*Continued*)

Sizes		Rough size	4×4	4×6	4×8	4×10	4×12	4×14	4×16	4×18
		Surface size S1S1E[1]	3⅝×3⅝	3⅝×5⅝	3¾×7½	3¾×9½	3¾×11½	3¾×13½	3¾×15½	3¾×17½
		Section modulus	7.94	19.12	35.16	56.41	82.66	113.91	150.16	191.41
Spans in feet	17	B				3982	5835	8041	10599	13511
		D				2225	3947	6385	9664	
		D1				0.2547	0.1436	0.0887	0.0586	0.0407
	18	B				3760	5510	7594	10010	12760
		D				1985	3521	5695	8620	12406
		D1				0.3023	0.1704	0.1053	0.0696	0.0483
	19	B				3563	5221	7194	9484	12089
		D				1782	3160	5112	7737	11134
		D1				0.3554	0.2004	0.1239	0.0818	0.0569
	20	B				3384	4959	6834	9009	11484
		D				1608	2852	4613	6982	10049
		D1				0.4146	0.2337	0.1445	0.0955	0.0663
	21	B					4724	6509	8580	10938
		D					2587	4184	6330	9115
		D1					0.2706	0.1673	0.1105	0.0768
	22	B					4509	6213	8190	10440
		D					2357	3813	5770	8305
		D1					0.3111	0.1923	0.1271	0.0883
	23	B					4313	5943	7834	9986
		D					2156	3488	5280	7598
		D1					0.3556	0.2198	0.1452	0.1009
	24	B					4133	5695	7508	9570
		D					1980	3204	4849	6978
		D1					0.4040	0.2497	0.1650	0.1146
	25	B						5468	7208	9188
		D						2952	4469	6431
		D1						0.2822	0.1865	0.1296
	26	B						5257	6930	8834
		D						2730	4132	5946
		D1						0.3175	0.2099	0.1457
	27	B						5063	6674	8507
		D						2531	3831	5514
		D1						0.3555	0.2349	0.1632
	28	B						4882	6435	8203
		D						2354	3562	5127
		D1						0.3965	0.2620	0.1820
	29	B							6214	7920
		D							3321	4780
		D1							0.2911	0.2022
	30	B							6006	7656
		D							3103	4466
		D1							0.3222	0.2239
	31	B								7409
		D								4183
		D1								0.2470
	32	B								7178
		D								3925
		D1								0.2717

[1] S1S1E = surfaced one side and one edge.

STEEL BEAMS AND GIRDERS

By Alfred Wheeler Roberts

Beams of I-section are the steel beams in most common use. In beams of this section the greater part of the material occurs in the upper and lower portions of the beam and where it is most effective in resisting bending. Channels, angles, and tees are used only to meet some special condition. Channels, for example, are not as economical as I-beams and require more lateral support to keep them from buckling, but they are especially suitable for use as lintels and around floor openings.

This chapter deals only with simple rolled sections. Plate and box girders are treated in another chapter. For the selection of sizes of steel beams see Art. 1. For properties of steel sections, see Art. 2*b*. For loads supported by lintels, see Art. 29.

16. Considerations in the Design of Steel Beams.—Steel beams must be designed to resist bending, shear, sidewise buckling of the web, lateral buckling of the compression flange, and excessive flexure or deflection. (For derivation of formulas and for terms used, see "Simple and Cantilever Beams," Sect. 1.)

16*a*. Bending.—The section modulus must be sufficient so that the external bending moment will be safely resisted. The section modulus required is found by dividing the bending moment in inch pounds by the allowable extreme fiber stress in pounds per square inch. The fiber stress usually allowed is 16,000 lb. per sq. in.

16*b*. Shear.—The web area, obtained by multiplying the depth of beam by the thickness of web, must be sufficient for the beam to resist the maximum shear (see Sect. 1, Art. 63*d*). The usual allowance for shear is 10,000 lb. per sq. in.

16*c*. Buckling of Web.—The tendency of the web to buckle or crush occurs over the supports and immediately under the points of application of concentrated loads. There is also the tendency to sidewise buckling near the ends of a beam due to the inclined compressive stress referred to in Sect. 1, Art. 64. With I-beams and channels, this inclined compressive stress need not be considered in any ordinary case if the beam is made amply strong over supports.

Usually if a beam has sufficient section modulus to take care of the bending moment, the web is sufficiently strong as regards shear and buckling. The exception occurs, however, where the span is short and the load heavy.

The Carnegie Pocket Companion gives the following formulas for safe end reaction and safe interior load:

$$R = pt\left(a + \frac{d}{4}\right)$$

$$W = pt\left(a_1 + \frac{d}{2}\right)$$

in which

R = end reaction.
W = concentrated load.
t = web thickness.
a = distance over which the end reaction is applied.
a_1 = distance over which the concentrated load is applied.
$p = 19{,}000 - 173\frac{d}{t}$, but never greater than 13,000 lb. per sq. in.

The first formula applies to any loading. Whenever the end reaction or concentrated loads are greater than determined by the above formulas, then either a beam must be chosen having a greater web area, or the web of the beam investigated must be reinforced by stiffener angles riveted to the web and milled top and bottom to bear against the flanges. It is usually more economical to use a beam with greater web area than to use stiffeners.

The formula for p given above is based on the column formula ($19{,}000 - 100\frac{L}{r}$, maximum

13,000 lb. per sq. in.) used by the American Bridge Company and Carnegie's Pocket Companion (see Sect. 1, Art. 97). The length of the column is taken as $\frac{d}{2}$ and $r = \sqrt{\frac{t^2}{12}}$. Any other column formula could be used, such as the formula $(16{,}000 - 70\frac{L}{r}$, maximum 14,000 lb. per sq. in.) of the Am. Ry. Eng. Assn. Substituting the proper values for L and r in this formula, we have

$$p = 16{,}000 - 121\frac{d}{t}$$

The formulas for R and W above given assume that the length of the web withstanding direct compression is greater than the distance over which the end reaction or a concentrated load is applied. Some authorities consider only the loaded length in direct compression which is obviously on the safe side.

To withstand crippling of the web due to inclined compressive stress, the intensity of the vertical shear which is equal to the intensity of this compressive stress, must be kept within a safe value, otherwise stiffeners must be used or the web thickness increased. A beam may be amply secure against a straight shear of 10,000 lb. per sq. in. and yet not have sufficient web area to be safe as regards web buckling. Assuming the inclined compressive stress to act at 45 deg. with the neutral axis throughout the entire depth of beam and using the American Bridge Company's column formula, the maximum safe unit value for the shear

$$\frac{V}{dt} = 19{,}000 - 488\frac{h}{t}$$

in which h = the distance between the flange fillets. Using the A. R. E. A. formula

$$\frac{V}{dt} = 16{,}000 - 342\frac{h}{t}$$

The Cambria Steel Handbook gives

$$\frac{V}{dt} = \frac{12{,}000}{1 + \frac{h^2}{1500t^2}}$$

based on the Gordon column formula.

16*d*. Deflection.—In some cases the deflection may be the governing feature in selecting a suitable section for a beam, instead of the load it carries. For example, a beam may deflect sufficiently to crack a plastered ceiling, or to crack a marble or mosaic floor, because the proportion of the depth of the beam to its span is not sufficient. It will be found that a good workable proportion of the depth of a beam to its span, where excessive deflection is to be avoided, is that the depth of the beam should not be less than $\frac{1}{20}$ of the span, and that the deflection should not exceed $\frac{1}{360}$ of the distance between supports. However, where the deflection is not serious, as in mills, shops, etc., it is good practice to make beams $\frac{1}{24}$ of the span in depth, and for roof purlins of mill buildings, $\frac{1}{40}$ of the span if the roofs are $\frac{1}{5}$th pitch or steeper.

16*e*. Lateral Support of Compression Flange.—The compression flange of a beam is really a column and may fail by buckling laterally. If beams are without lateral support for a distance exceeding about 20 times the flange width, their carrying capacity should be reduced in accordance with table to be found in most any steel handbook. Each table in common use is based on some one of the column formulas (Sect. 1, Art. 97) making due allowance for the strengthening action of the web.

A formula in common use is the following modified Gordon column formula used in Cambria:

$$p = \frac{18{,}000}{1 + \frac{l^2}{3000b^2}}$$

in which p = allowable stress in pounds per square inch, l = length between lateral supports in inches, and b = width of flange in inches. When $p = 16{,}000$, $\frac{l}{b} = 19.37$, showing that lateral bending must be considered in beams where the maximum length of unsupported compression flange is greater than about 20.

In most cases in floor framing a beam is braced laterally either by other beams framing into it or by the floor construction itself, but cases do arise where conditions leave a beam unbraced for an excessive distance.

17. Multiple Beam Girders.—Two or more beams placed side by side and connected by means of bolts and separators are used where a single beam would not be sufficient to carry the loads imposed, where there is not sufficient head room to use a deep member, or where a wide member is needed either to give sufficient lateral stiffness or to provide a suitable support for a wall. The separators should fit closely between the flanges of the beams and should be placed at the support, at points where concentrated loads occur and at regular intervals of 5 or 6 ft. along the beam in order to insure that the beams will act as a unit both vertically and laterally.

Gas-pipe separators should not be used in this type of girder, but may be used in grillage beams or girders which are to be filled in with concrete. The cast-iron separator is generally used in multiple beam girders, but owing to its uncertainty of being true and square, it is better construction to use built-up steel separators or diaphragms made up of plates and angles.

If the loads are not delivered equally to each member of a multiple girder, each member should be designed, as near as practicable, to take its specific load so as not to depend any more than possible upon the separators equalizing the load. A good example of this is a spandrel section made up of two members carrying a wall and a floor load. The outer member should be designed to carry one-half the wall load and the inner member one-half of the wall load plus all of the floor load. This will give less chance for secondary stresses due to torsion which are impossible to calculate.

18. Beams with Cover Plates.—It is sometimes found advantageous to reinforce I-beams and multiple beam girders by adding cover or flange plates top and bottom. Such members should be figured considering the moment of inertia of the total net section, deducting metal to allow for rivet holes in both flanges. If rivets are carefully staggered, only one-half of this number need be deducted. The plate should be riveted with sufficient rivets to develop the stress in the cover plate beyond the point where the plate is actually needed. For method of computing rivets connecting cover plates to flanges, see Art. 55. The length of flange plates may be determined in the same manner as for plate girders (see Illustrative Problem, p. 185). It is sometimes necessary and is good construction in the case of a girder carrying a wall, to run the top flange plate the full length of the girder, to make an even surface on which to build the wall.

19. Double-layer Beam Girder.—A type of beam girder constructed by placing one beam on top of the other and riveting the top flange of the lower beam to the bottom flange of the upper beam to take up the horizontal shear, will be found a very effective girder. Flange plates or channels can be riveted to the extreme flanges of the beams and a high amount of efficiency can be developed from this form of girder. It is important, however, to make certain that the horizontal shear between beams is properly taken care of by the rivets and that the web is sufficient to withstand buckling. Although not usually as economical in material as a plate girder or a very deep beam, it will prove advantageous to use when deep beams and plate girder web plates are not readily available. The cost of shop work on this type of girdeı is a great deal lower than on plate girders.

20. Tie-beams.—A tie or tension beam is one which takes transverse stress and direct tension at the same time. Probably the best example of such a beam is a bottom chord of a truss which is taking tension and at the same time acting as a beam—for instance, supporting a ceiling or a concentrated load between panel points.

In designing a member of this kind care should be taken that the extreme tension fibers are not over stressed. As the maximum fiber stress cannot be calculated directly, it may be necessary to make trials with several sections before the proper section can be determined. The method of procedure is as follows: (1) Calculate the bending moment in inch-pounds due to the beam action, (2) select a member for trial and divide the bending moment by the section modulus of the member selected—the result gives the stress per square inch on the extreme fiber due to bending, (3) divide the amount of tension by the area of the cross section of the member selected for trial—the result gives the stress per square inch due to tension, and (4) add these two stresses. If the sum of the stresses is not greater than the allowable stress per square inch, the member is acceptable. If the member does not fit requirements, another section should be selected and the calculations repeated.

21. Strut-beams.—A strut or compression beam is one which is subjected to combined compressive and transverse stresses. An illustration of a beam of this kind would be a top chord of a truss subjected to direct compression and also taking bending due to a concentrated load between panel points. Still another illustration would be a column carrying its load and taking bending due to wind or other forces.

A member of this type can be designed in a manner similar to that explained above for tie-beams. The extreme compression fibers should be investigated, however, instead of the tension fibers. The column formula should be used to determine the maximum allowable fiber stress.

Another analysis of this type of beam is the same as used on columns which take axial loads and bending. By this method an equivalent axial load is computed from the bending moment to add to the direct load and then the member is designed as a column.

The method of procedure is as follows: (1) Calculate the bending moment in inch-pounds due to the beam action; (2) select a member for trial; (3) multiply the bending moment by the distance from the neutral axis to the extreme fiber and divide by the square of the radius of gyration—the result gives the equivalent axial load, due to the bending on the compression fibers; (4) add the equivalent and direct axial loads; (5) design the member to take these combined loads using the column formula.

22. Grillage Beams.—Grillage beams are beams used under columns in foundations for the purpose of distributing the column loads over a wide foundation bed. Steel beam grillages are made up of one or more layers of beams, the layers being built up in the manner shown in Fig. 4.

The space between the flanges of the beams should not be less than 2½ in., so as to permit the proper tamping of the concrete in which all grillage foundations should be incased. The distance between the flanges should never exceed 3 times the flange width.

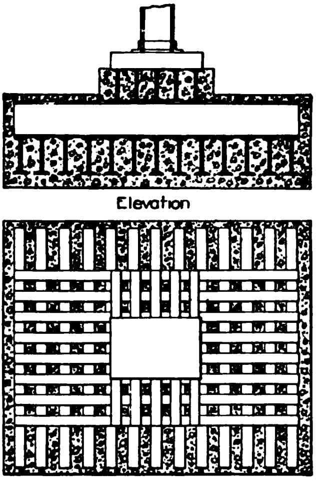


FIG. 4.

Beams should be provided with gas-pipe separators spaced near the ends and immediately under points where concentrated loads are applied in order to insure that the beams will act as a unit. A double line of separators should be provided for all members over 8 in. in depth. Cast-iron or built-up steel separators are not desirable, as they break up the continuity of the concrete.

Material for grillages should not be painted as the concrete is a preservative against rust and corrosion, and the concrete will bond more readily to an unpainted surface of steel.

The bearing area of a grillage is generally taken as the length multiplied by the out to out distance of the extreme flange edge providing beams are to be encased in concrete. Some specifications and building codes permit the above width plus the width of the upper outer flanges on both sides, on the basis that the concrete tamped under these flanges distributes the bearing to the concrete adjacent to the lower outer flanges.

The column base should be designed so that the load will be distributed in direct bearing to the webs of the beams, at the allowable unit bearing stress for steel on steel which usually means that stiffener angles must be used on the bottom of the columns or on the beam webs. This form of construction can be avoided by the use of a rolled steel slab of the proper thickness to distribute the loads over the grillage webs, or it is sometimes possible to place the grillage so that sufficient area of the column bears directly over the webs of the grillage beams to give the required bearing area.

The beams in a steel grillage should be figured for bending, shear, and buckling. The buckling due to direct compression in a lower layer is likely to occur where the web of the upper grillage bears on the web of the lower grillage. In the top layer the tendency to buckling comes from the direct application of the column load. Likelihood of the web buckling due to inclined compressive stresses should also be investigated in grillage beams.

Some engineers in designing grillages consider that inasmuch as the beams are incased in concrete and held together with separators, that the webs are not subject to buckling, as they are braced sideways and cannot buckle. With this assumption the webs are figured for bearing only, using the allowable unit bearing stress for steel on steel.

As channels make the best sections to resist shear and buckling, owing to their thick webs, 4 channels, placed back to back in pairs, which are capable of taking the shear and buckling, make an economical design for the upper layer of a grillage, where there is no restriction to the dimensions in either direction. These channels should be developed for their full length in bending.

23. Information Regarding Illustrative Problems.—Following are a number of illustrative problems pertaining to different kinds of beams and girders. (For methods of computing reactions, shear, and moment, see chapters in Sect. 1.) Some of the unit working stresses may not agree with those which are allowable for certain building codes or specifications, but they will tend to show the principles explained in the text of this chapter and other quantities may be substituted to suit the individual problem as it arises. In calculating the bending moment and section modulus of different problems, it will be found much more convenient to compute moments in thousands of foot-pounds and multiply by three-fourths (¾) to obtain the section modulus. The illustrative problems following, however, are worked out in inch-pounds for bending moments, but the aforesaid method will be found a big saver of time for the experienced engineer.

Illustrative Problem.—*Beam with a Uniformly Distributed Load.*—What size beam is required to carry a uniformly distributed load of 1000 lb. per lin. ft. over a span of 18 ft., assuming that the beam is sufficiently braced laterally?

$$\text{Total load} = (18)(1000) = 18{,}000 \text{ lb.}$$

$$R_1 = R_2 = \frac{18{,}000}{2} = 9000 \text{ lb.}$$

$$M = \frac{(18{,}000)(18)(12)}{8} = 486{,}000 \text{ in.-lb.}$$

$$S = \frac{486{,}000}{16{,}000} = 30.3$$

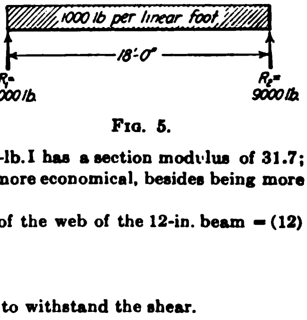


Fig. 5.

By referring to a table of properties of beams it will be seen that a 10-in. 40-lb. I has a section modulus of 31.7; but, as a 12-in. 31.5-lb. I has a section modulus of 36, the 12-in. beam is the more economical, besides being more readily obtained.

The beam should next be investigated for shear. Area of cross section of the web of the 12-in. beam = (12) (0.35) = 4.2 sq. in.

$$\frac{9000}{4.2} = 2142 \text{ lb. per sq. in.}$$

As the allowable shearing stress is 10,000 lb. per sq. in., this section is ample to withstand the shear.

This problem could readily be solved by using the tables of safe uniform loads for I-beams in the steel handbook.

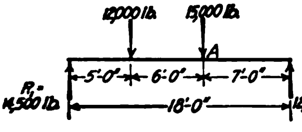


Fig. 6.

Illustrative Problem.—*Beam With Concentrated Loads.*—What size beam will be required to carry two concentrated loads over a span of 18 ft., with the loads spaced as shown in Fig. 6?

$$R_1 = \frac{(7)(15{,}000) + (13)(12{,}000)}{18} = 14{,}500 \text{ lb.}$$

$$R_2 = \frac{(5)(12{,}000) + (11)(15{,}000)}{18} = 12{,}500 \text{ lb.}$$

The point of maximum bending moment is at the point of no shear—that is, where the shear changes sign. The point of maximum bending in this particular case will be at the right-hand concentrated load, or at point "*A*" shown in the figure.

$$M = (12{,}500)(7)(12) = 1{,}050{,}000 \text{ in.-lb.}$$

$$S = \frac{1{,}050{,}000}{16{,}000} = 65.6$$

By referring to a table of properties of beams it will be seen that a 15-in. 60-lb. I has a section modulus of 81.2 and that an 18-in. 55-lb. I has a section modulus of 88.4. Since the 18-in. beam is of less weight besides developing more efficiency, it will be used.

Area of cross section of web of an 18-in. 55-lb.I = (18)(0.46) = 8.3 sq. in. Maximum shear = 14,500 lb. Therefore

$$\frac{14{,}500}{8.3} = 1746 \text{ lb. per sq. in.}$$

As the allowable shearing stress is 10,000 lb. per sq. in., this section is satisfactory for shear.

Illustrative Problem.—*Beam With Load Concentrated at Center.*—What size beam will be required to carry a center load of 20,000 lb. on an 18-ft. span?

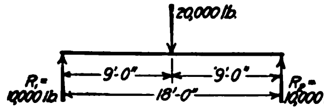


Fig. 7.

$$R_1 = R_2 = \frac{20{,}000}{2} = 10{,}000 \text{ lb.}$$

$$M = \frac{(20{,}000)(18)(12)}{4} = 1{,}080{,}000 \text{ in.-lb.}$$

$$S = \frac{1{,}080{,}000}{16{,}000} = 67.5$$

By referring to a table of properties of beams, it will be seen that a 15-in. 60-lb.I has a section modulus of 81.2, but since an 18-in. 55-lb. I develops a section modulus of 88.4, it is more economical to use the 18-in. section. Investigating for shear it will be found that the 18-in. beam has an area of web cross section of (18)(0.46) = 8.3 sq. in. The maximum shear = 10,000 lb. Therefore

$$\frac{10{,}000}{8.3} = 1204 \text{ lb. per sq. in.}$$

As the allowable shearing stress is 10,000 lb. per sq. in., this section is ample for shear.

This problem could be solved by using the tables of safe uniform loads for I-beams given in the steel handbook.

Illustrative Problem.—*Cantilever Beam.*—What size beam will be required to safely sustain the loads shown in Fig. 8?

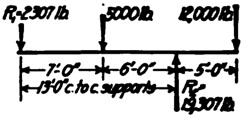


Fig. 8.

To ascertain R_2, take moments about R_1 as follows:

$$R_2 = \frac{(5000)(7) + (12{,}000)(18)}{13} = 19{,}307 \text{ lb.}$$

To find R_1, take moments about R_2, or

$$R_1 = \frac{(12{,}000)(5) - (5000)(6)}{13} = 2307 \text{ lb.}$$

As a beam must be in equilibrium, the sum of the loads must be equal to the algebraic sum of the reactions and it will be seen from the diagram that in order for the forces to balance there must be a downward force at R_1 of 2307 lb. to resist the uplift at that point.

The maximum bending moment occurs at support R_2, or

$$M = (12{,}000)\ (5)\ (12) = 720{,}000 \text{ in.-lb.}$$

$$S = \frac{720{,}000}{16{,}000} = 45$$

By referring to a table of properties of beams it will be seen that a 15-in. 42-lb.I has a section modulus of 58.9 and will satisfy the bending.

The maximum shear of 12,000 lb. occurs immediately beyond the support of the cantilever portion. A 15-in. 42-lb.I has a web area of (15) (0.41) = 6.15 sq. in. Therefore

$$\frac{12{,}000}{6.15} = 1951 \text{ lb. per sq. in.}$$

It is evident that the section is satisfactory as regards shear.

The web should be investigated for buckling to ascertain how much bearing it should have on the supporting column at R_2. Using the formula

$$p = 16{,}000 - 121\frac{d}{t}$$

from Art.16c, and assuming only the loaded length in direct compression

$$pta = R_2 = 19{,}307 \text{ lb.}$$

$$p = 16{,}000 - \frac{(121)(15)}{0.41} = 11{,}570$$

$$a = \frac{19{,}307}{(11{,}570)(0.41)} = 4.1 \text{ in.}$$

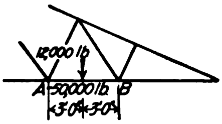


Fig. 9.

Illustrative Problem.—*Tie Beam.*—Design the member *AB* in Fig. 9 to carry a concentrated load of 12,000 lb. as shown, and to take simultaneously a tensile stress of 50,000 lb.

The bending moment due to the concentrated load

$$M = \frac{(12{,}000)(6)(12)}{4} = 216{,}000 \text{ in.-lb.}$$

For trial, select a section composed of two 10-in. 15-lb. channels which have a total *S* of 26.8. Then the stress on the extreme fiber due to bending will be

$$f_1 = \frac{216{,}000}{26.8} = 8059 \text{ lb. per sq. in.}$$

The stress per square inch due to tension will be the stress divided by the area of the section, or

$$f_2 = \frac{50,000}{8.92} = 5605 \text{ lb. per sq. in.}$$

Then the total stress on the extreme tension fiber will be

$$f_1 + f_2 = 13,664 \text{ lb. per sq. in.}$$

Therefore the member selected is satisfactory.

Care must be taken that there is no metal taken from the section due to punching at the center where the stress is a maximum sufficient to reduce the section to the point of overstressing the member. At the ends of *AB*, the bending moment is zero, so the net section at these points will only have the direct tensile stress to take care of.

Illustrative Problem.—*Strut Beam.*—What size member will be required to carry a concentrated load of 10,000 lb. at the center of a span of 8 ft. and take a direct compressive stress of 20,000 lb.?

$$M = \frac{(10,000)(8)(12)}{4} = 240,000 \text{ in.-lb.}$$

For trial select a section composed of two 9-in. 13¼-lb. channels each of which has a radius of gyration about the principal horizontal axis of 3.49 and an area of 7.78 sq. in.

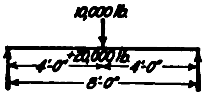


FIG. 10.

Using the A. R. E. A. column formula

$$p = 16,000 - 70\frac{L}{r}$$

the member is found to carry as a column 14,110 lb. per sq. in.—that is,

$$p = 16,000 - 70\frac{96}{3.49} = 14,110 \text{ lb. per sq. in.}$$

As the maximum compression in a column is limited by the formula used to 14,000 lb. per sq. in., the column will safely carry (7.78)(14,000) = 108,920 lb. The amount to be added to the direct compression due to bending is (see Art. 21).

$$\frac{(240,000)(4.5)}{(3.49)^2} = 88,669 \text{ lb.}$$

The sum of the direct and equivalent axial loads is

$$20,000 + 88,669 = 108,669 \text{ lb.}$$

Therefore the member selected is satisfactory.

Illustrative Problem.—*Single Layer Grillage.*—What size grillage will be required to carry a 10-in. H-column with a load of 200,000 lb. and an allowable bearing pressure on the foundation of 20,000 lb. per sq. ft.?

The area required to distribute the load over the foundation is

$$\frac{200,000}{20,000} = 10 \text{ sq. ft.}$$

Assuming that the grillage is properly incased in concrete, the webs will not be figured for buckling—only for shear and bearing. A grillage of this kind can be placed under an H-column so that the greater part of the column shaft bears directly on the webs of the grillage. The longitudinal distribution of the column load will be the width of the column flange plus twice the thickness of the base plate (10 + 2 = 12), assuming the load to be distributed at an angle of 45 deg. beyond the edge of the column shaft. Figuring bearing of steel on steel at 20,000 lb. per sq. in., the direct bearing area required is

$$\frac{200,000}{20,000} = 10 \text{ sq. in.}$$

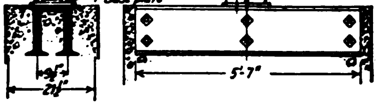


FIG. 11.

As the length is already determined as 12 in., the thickness required for each web is

$$\frac{10}{(12)(4)} = 0.208$$

assuming 4 channels. Considering the width of the grillage distributing to the foundation to be 9½ + (4)(3) = 21½ in. (see Art. 22) and as an area of 10 sq. ft. or 1440 sq. in. is needed, the length of the grillage will be $\frac{1440}{21.5} = 67$ in. Then

$$M = \left(\frac{200,000}{2}\right)\left(\frac{67}{4} - 3\right) = 1,375,000 \text{ in.-lb.}$$

$$S = \frac{1,375,000}{(16,000)(4)} = 21.4$$

As the point of maximum shear occurs at the edge of the base plate, the total maximum shear

$$V = \frac{200,000}{67}\,\frac{67 - 12}{2} = 82,088 \text{ lb.}$$

Then the amount of area required in the web of each member

$$td = \frac{82,088}{(4)(10,000)} = 2.05 \text{ sq. in.}$$

Therefore each of the 4 channels should have the following properties

Section modulus = 21.4
Web thickness = 0.208 in.
Web area = 2.05 sq. in.

By referring to a table of properties of channels, a 12-in. 20½-lb. channel is found to have a section modulus of 21.4, a web thickness of 0.28 in. and a web area of (12) (0.28) = 3.36 sq. in. Therefore this section will meet all requirements.

Illustrative Problem.—*Double Layer Grillage.*—What size grillage will be required to carry a 14-in. H-column with a load of 400,000 lb., the allowable bearing pressure on the foundation being 15,000 lb. per sq. ft.? As the assumption will be made that there are no limitations on the dimensions of this grillage, the first step is to select a section for the top layer as explained in the preceding problem. It is found that four 12-in. 25-lb. channels will safely resist the bearing and shear and will safely develop a length of 46 in.

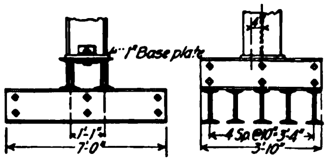


FIG. 12.

The length of the lower layer is determined as follows:

$$\frac{400,000}{(15,000)(3.83)} = 6.96, \text{ say 7 ft.}$$

Then the total bending moment on the lower grillage

$$M = \left(\frac{400,000}{2}\right)\left(\frac{84}{4} - 6.5\right) = 2,900,000 \text{ in.-lb.}$$

Assuming that the lower grillage is composed of 5 beams placed on 10-n. centers

$$S = \frac{2,900,000}{(16,000)(5)} = 36.25$$

By referring to a table of properties of beams, a 12-in. 40-lb. I is found to have a section modulus of 44.8 and therefore will be satisfactory for bending.

The shear on each beam

$$V = \frac{400,000}{(5)(84)} \cdot \frac{(84 - 19)}{2} = 30,940 \text{ lb.}$$

Since the section will develop (12) (0.46) (10,000) = 55,200 lb., it is satisfactory for shear.

The amount of bearing area required of steel on steel to take the load from the webs of the upper layer to the webs of the lower layer is

$$\frac{400,000}{20,000} = 20 \text{ sq. in.}$$

Therefore at each point of the ten intersections of the two layers there should be 2 sq. in. The webs of the upper layer have (2) (0.39) (5.25) = 4.09 sq. in. and the webs of the lower layer (0.46) (2) (3.05) = 2.80 sq. in.

As all conditions are satisfied, the five 12-in. 40-lb. I's will be satisfactory for the lower grillage.

Illustrative Problem.—*Beam Reinforced with Flange Plates.*—What load uniformly distributed will a 24-in. 80-lb. I-beam carry if the span is 40 ft. and a 10 × ½-in. cover plate is riveted to each flange?

The first thing to determine is the net moment of inertia about axis $X-X$ and from that the section modulus of the section in question. The allowance made for a rivet hole is for a hole ⅛ in. more in diameter than the diameter of rivet—that is, ⅞ in. for a ¾-in. rivet.

I of 24-in. 80-lb. I-beam $= 2087.9$

I of two 10 × ½-in. plates $= \frac{(2)(10)(0.5)^3}{12} = 0.208$

(Area of two 10 × ½-in. plates) $(12.25)^2 = 1500.625$

$ 3588.733$

Area of 1 rivet hole $= (0.875)(1.37) = 1.20$ sq. in.

I of 4 rivet holes $= \frac{(0.875)(1.37)^3(4)}{12}$ (see Sect. 1, Art. 61*c*) $= 0.748$

$(4)(1.20)(11.81)^2 = 668.444$

$ 669.192$

Net I $= 2919.541$

$$S = \frac{2919.541}{12.5} = 233.56$$

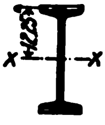


FIG. 13.

Then the safe load which this section is capable of supporting including the weight of the girder will be

$$\frac{(233.56)(16,000)(8)}{(40)(12)} = 62,283 \text{ lb.}$$

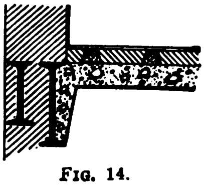


FIG. 14.

Illustrative Problem.—*A Spandrel or Wall Girder.*—What section of wall girder with span of 25 ft., will be required to carry a uniformly distributed floor load of 17,000 lb. applied from one side of the girder only and in addition to carry a wall load of 48,000 lb. equally distributed over both members (Fig. 14)?

The member on the side carrying the floor load should be designed to carry the floor load and one-half the wall load, or

$$M = \frac{(41,000)(25)(12)}{8} = 1,537,500 \text{ in.-lb.}$$

$$S = \frac{1,537,500}{16,000} = 96.09$$

A 20-in. 65-lb. I has a section modulus of 117 and is therefore selected.

The maximum shear is one-half the load or 20,500 lb. As the area of the web of a 20-in. 65-lb. I-beam = (20) (0.5) = 10 sq. in., the web is good for (10) (10,000) = 100,000 lb., which is ample.

The member carrying one-half of the wall only or 24,000 lb. will have a moment of

$$M = \frac{(24{,}000)(25)(12)}{8} = 900{,}000 \text{ in.-lb.}$$

$$S = \frac{900{,}000}{16{,}000} = 56.25$$

A 15-in. 42-lb. I has a section modulus of 58.9 and is the section selected.

The maximum shear equals one-half the load, or 12,000 lb. The web of a 15-in. 42-lb. I is good for (15) (0.41) (10,000) = 61,500 lb.

By proportioning members in a double-beam girder by this method, it will carry the loads applied most directly to the members in the most efficient manner. Separators should be provided as specified in Art. 17.

Illustrative Problem.—*A Double-layer Beam Girder.*—What load uniformly distributed will a double-layer beam girder carry which is composed of two 18-in. 55-lb. I-beams and has a span of 50 ft., assuming that the member is properly braced laterally?

The first step is to find the inertia of the combined section and from that the section modulus about axis x–x.

$$\begin{aligned} \text{I of the two beams} &= 1591.2 \\ (31.86)(9)^2 &= 2580.66 \\ \hline \text{Total I} &= 4171.86 \end{aligned}$$

$$S = \frac{4171.86}{18} = 231.77$$

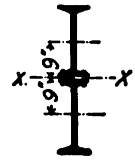


Fig. 15.

Then the safe carrying capacity is

$$\frac{(231.77)(16{,}000)(8)}{(12)(50)} = 49{,}444 \text{ lb.}$$

The web is capable of taking (36)(0.46)(10,000) = 165,600 lb. in shear. The maximum shear on the girder is but $\frac{49{,}444}{2} = 24{,}722$ lb.

The next consideration is the riveting of the two beams together. The maximum spacing at the ends of beam should be such that there would be sufficient rivets in a length equal to the depth of the girder to take the horizontal shear. The horizontal shear is equal in intensity to the vertical shear at any point and varies from a maximum at the ends to zero at the center of the span. Since the maximum shear = 24,722 lb., then the rivets at the ends should be spaced, assuming two lines of ¾-in. diam. rivets with an allowable shearing stress of 4420 lb. per rivet,

$$\frac{(36)(4420)(2)}{24{,}722} = 12.8 \text{ in. on centers.}$$

As this theoretical rivet spacing is not practical, the girder should have rivets spaced for a distance at the ends equal to about the depth of girder at not more than 3 in. on centers. The rivet spacing throughout the remainder of the girder should not be more than 6 in. on centers.

It should be noted that the section modulus of this girder (231.77) is an increase of 31 % over the same two beams if they were placed side by side.

CAST-IRON LINTELS

By Alfred Wheeler Roberts

Lintels made of cast iron are not extensively used in present-day construction, but can be used to good advantage on certain kinds of structures. For spanning openings where a flat soffit is desirable and no plastering is needed, and also for use over store fronts where cast-iron columns are employed, lintels of cast iron make a good practical form of construction and can be fluted on the outside face or otherwise ornamented.

On account of the many chances of imperfections in a casting, such as blow holes and cracks due to uneven cooling of the elementary portions of the lintel, cast iron is not the most dependable metal to be used in an important structural member. In any piece of cast iron there is always an internal initial stress produced during the process of cooling, and since this stress is an unknown quantity, it can only be assumed as being counteracted by the factor of safety allowed in choosing the working stresses.

Cast-iron lintels should be thoroughly inspected for cracks and blow holes before they are painted, as these defects can be easily hidden by filling in cracks and holes and painting over them.

24. General Proportions.—The width of the bottom flange should be made equal to the width of the wall that is to be carried, or if it is desirable or necessary to fireproof the lintel, it can be made several inches less than the wall width to allow for the fireproofing.

The web, or stem as it is sometimes called, should be made deep enough to prevent a deflection which would cause the wall to crack or open up joints in the brick courses.

When the bottom flange is sufficiently wide, it is desirable to cast brackets at the center of the lintel, as shown in Fig. 16, in order to give lateral stiffness to the lintel and brace the stem which is taking compression.

Lintels with two or three webs should have a vertical cross piece cast at each end connecting the webs. Where lintels are to be used over more than one span, the ends of abutting lintels should be bolted together.

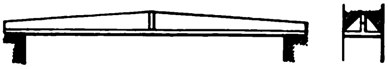


Fig. 16.

25. Working Stresses.—Cast iron to resist bending in compression should be figured at 16,000 lb. per sq. in. at the extreme fiber. To resist bending in tension it should be figured at 3000 lb. per sq. in. at the extreme fiber. The shearing stress should not exceed 3000 lb. per sq. in.

26. Form of Cross Section.—The cross sections commonly used for cast-iron lintels are shown in Figs. 17, 18, 19, and 20. The ideal condition in designing a cast-iron lintel from a strictly theoretical and economical standpoint is when the metal in compression is stressed up to the same proportion of the allowable stress as the metal in tension. This, however, is very seldom possible due to local conditions generally fixing the width of the flange and the span fixing the web or stem depth. The ideal condition, also, would make the thickness in the stem metal vary so much from the thickness of the flange metal, that there would be the tendency for the metal to crack in cooling at a point where they join together. It is therefore advisable to keep the metal thicknesses uniform throughout.

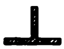


Fig. 17.

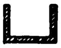


Fig. 18.

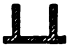


Fig. 19.

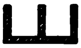


Fig. 20.

27. Shear.—In beveling the stem of a lintel, it should not be beveled so much that it will not allow sufficient web area at the edge of the end supports to take the shear. The outstanding legs of the bottom flange should not be considered as taking the end shear.

28. Bending.—The maximum depth of the lintel need only be maintained as far as it is needed to take the maximum bending moment. The stem can be beveled toward each end without impairing the strength of the lintel, as shown in Fig. 16. If the load is applied as a uniform load, the bending moment will vary as a parabola and to be theoretically correct the top of the stem of the lintel should vary as a parabolic curve; but as a straight bevel is more simple to cast, it can be made so, providing the stem does not become less at any point than is required to give the proper resistance to bending.

29. Loads Supported.—In determining the loads imposed on lintels, the floor loads, if any are carried on the wall supported, should be taken into account.

If the wall is solid with no window openings above the lintel, the wall will arch and carry a great deal of the load to the adjoining wall which supports the lintel without engaging the lintel. The portion for which the lintel should be designed would be a triangle whose base will be the span of the opening and whose height will be one-half of the span. This is only true when the adjoining wall is sufficient to take the resultant thrust due to the arch effect.

If the wall over the lintel has window openings with piers resting immediately over the lintel, the amount of wall and the manner in which it is delivered to the lintel, must be taken into account.

Each individual case must stand on its own merits and the lintel designed accordingly. If the loads are underestimated, it will cause a deflection sufficient to crack the walls and create a permanent damage to the building which would be hard to remedy.

Illustrative Problem.—What load will the lintel shown in Fig. 21 carry on a 12-ft. span?

The location of the neutral axis A–A through the center of gravity of the section should first be determined. To do this take moments of the areas of each elementary section about line B–B and divide by the total area of the section (see Sect. 1, Art. 44):

$$\begin{aligned}(7)(1)(3.5) &= 24.5\\ (12)(1)(7.5) &= \underline{90.0}\\ &\ 114.5\end{aligned}$$

$$\frac{114.5}{19} = 6.02 \text{ in. below line } B\text{–}B$$

or

$$1.98 \text{ in. above line } C\text{–}C$$

Having determined the location of the neutral axis, the next step is to determine the moment of inertia (see Sect. 1, Art. 61*c*):

$$\begin{aligned}\frac{(1)(7)^3}{12} &= 28.58\\ \frac{(12)(1)^3}{12} &= 1.00\\ (7)(2.52)^2 &= 44.45\\ (12)(1.48)^2 &= \underline{26.28}\\ I &= 100.31\end{aligned}$$

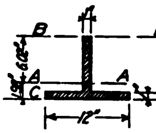


Fig. 21.

The section modulus or moment of resistance of the section

$$S = \frac{100.31}{1.98} = 50.66 = \frac{(W)(12)(12)}{(8)(3000)}$$

Then

$$W = \frac{(50.66)(3000)(8)}{(12)(12)} = 8443 \text{ lb.}$$

Therefore the section in question will carry 8443 lb. uniformly distributed over a span of 12 ft.

Illustrative Problem.—Determine the safe uniform load that the lintel shown in Fig. 22 is capable of carrying on a span of 10 ft.

The location of the neutral axis line A-A should first be determined:

$$\begin{aligned}(2)(7)(1)(3.5) &= 49\\ (16)(1)(7.5) &= \underline{120}\\ &\ 169\end{aligned}$$

$$\frac{169}{30} = 5.63 \text{ in. below line } B\text{-}B$$

or

$$2.37 \text{ in. above line } C\text{-}C$$

To find the moment of inertia:

$$\begin{aligned}\frac{(2)(1)(7)^3}{12} &= 57.16\\ \frac{(16)(1)^3}{12} &= 1.33\\ (2)(7)(2.13)^2 &= 63.42\\ (16)(1.87)^2 &= 55.84\\ I &= 177.75\end{aligned}$$

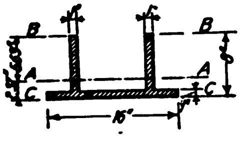


Fig. 22.

$$S = \frac{177.75}{2.37} = 75 = \frac{(W)(10)(12)}{(8)(3000)}$$

Then

$$W = \frac{(75)(3000)(8)}{(10)(12)} = 15{,}000 \text{ lb.}$$

Therefore the section in question will carry 15,000 lb. uniformly distributed over a span of 10 ft.

It should be noted that the least moment of resistance or section modulus is obtained by investigating the extreme tension fiber, or by dividing the moment of inertia by the distance from the neutral axis to the bottom.

The bending moments of lintels should be figured the same as any other beam and is dependent upon the way the load is applied to the lintel.

The section modulus required to resist a bending moment in tension is determined by dividing the moment in inch-pounds by 3000 lb. which is the allowable stress on the extreme fiber in tension.

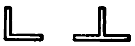
The compression side of an ordinary lintel section is generally much stronger than required and therefore does not usually have to be investigated. The question of shear, however, should be considered.

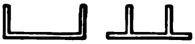
30. Table of Strength of Cast-iron Lintels.—The accompanying table gives the section modulus of various lintel sections and will cover most any requirement for the usual wall thicknesses. Some special widths may be determined by interpolation.

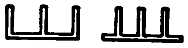
The position of the stem on a flange does not alter the resistance of a lintel to bending.

USE IN DESIGN OF CAST-IRON LINTELS

Moment of Resistance of Various Lintel Sections







Flange (inches)	Stem (inches)	Thickness of metal (inches)	Moment of resistance	Flange (inches)	Stem (inches)	Thickness of metal (inches)	Moment of resistance	**Flange (inches)**	**Stem (inches)**	**Thickness of metal (inches)**	**Moment of resistance**
6	6	¾	15.8	12	6	¾	31.6	24	6	¾	58.8
		1	18.9			1	37.8			1	70.2
		1¼	21.5			1¼	43.0			1¼	79.2
8	6	¾	19.6	12	8	¾	49.5	24	8	¾	91.8
		1	23.4			1	60.8			1	112.8
		1¼	26.4			1¼	69.8			1¼	130.2
8	7	¾	25.0	12	10	¾	58.0	24	10	¾	127.7
		1	30.3			1	72.2			1	159.5
		1¼	34.8			1¼	83.8			1¼	183.6
8	8	¾	30.6	12	12	¾	75.2	24	12	¾	166.5
		1	37.6			1	94.8			1	209.2
		1¼	43.4			1¼	111.4			1¼	247.6
12	6	¾	26.5	16	6	¾	39.2	28	6	¾	65.7
		1	31.6			1	46.8			1	78.4
		1¼	34.8			1¼	52.8			1¼	87.6
12	8	¾	41.7	16	8	¾	61.2	28	8	¾	102.9
		1	50.6			1	75.0			1	125.6
		1¼	58.6			1¼	86.8			1¼	145.4
12	10	¾	58.0	16	10	¾	83.3	28	10	¾	141.3
		1	72.2			1	105.1			1	177.3
		1¼	83.7			1¼	124.0			1¼	207.7
12	12	¾	75.2	16	12	¾	110.8	28	12	¾	186.0
		1	94.8			1	139.9			1	234.7
		1¼	111.4			1¼	166.5			1¼	277.9
				20	6	¾	47.2	32	6	¾	73.7
						1	55.0			1	86.6
						1¼	62.0			1¼	96.8
				20	8	¾	72.5	32	8	¾	114.2
						1	89.4			1	140.0
						1¼	102.4			1¼	161.0
				20	10	¾	100.4	32	10	¾	158.4
						1	125.3			1	197.5
						1¼	146.8			1¼	230.5
				20	12	¾	122.5	32	12	¾	197.7
						1	158.0			1	252.8
						1¼	189.4			1¼	300.8
				24	6	¾	53.0	36	6	¾	79.5
						1	63.2			1	94.8
						1¼	69.6			1¼	104.4
				24	8	¾	83.4	36	8	¾	125.1
						1	101.2			1	151.8
						1¼	117.2			1¼	175.8
				24	10	¾	116.0	36	10	¾	174.0
						1	144.4			1	216.6
						1¼	167.4			1¼	251.1
				24	12	¾	150.4	36	12	¾	225.6
						1	189.6			1	284.4
						1¼	222.8			1¼	334.2

REINFORCED CONCRETE BEAMS AND SLABS

By W. J. Knight

31. Flexure Formulas.—Assumptions as a basis for calculations:

1. Within the elastic limit of the steel, a plane section before bending remains a plane after bending.
2. The unit stresses in the concrete in compression vary as the ordinates to a straight line.
3. No tension exists in the concrete.
4. Within the elastic limit of the steel the adhesion between the concrete and steel is perfect.
5. No initial stresses exist in the concrete and steel due to the concrete setting and to contraction or expansion from temperature variations.
6. Modulus of elasticity of concrete is constant.
7. Calculations are made with reference to working stresses and safe loads.

Although the above assumptions are not in exact accordance with experimental data, they are sufficiently accurate and insure simplicity in making calculation. The formulas follow (see Fig. 23 and Notation in Appendix *A*):

Position of neutral axis

$$k = \sqrt{2pn + (pn)^2} - pn \qquad (1)$$

Arm of resisting couple

$$j = 1 - \frac{1}{3}k \qquad (2)$$

Balanced value for ratio k

$$k = \frac{1}{1 + \frac{f_s}{nf}} \qquad (3)$$

Steel ratio for balanced reinforcement

$$p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} \qquad (4)$$

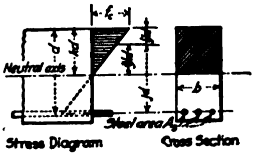


Fig. 23.

or

$$p = \frac{A_s}{bd} = \frac{f_ck}{2f_s} \qquad (4A)$$

When over-reinforced, the resisting moment depends on the concrete and its value, then, is

$$M_c = \tfrac{1}{2} f_ckj(bd^2) \qquad (5)$$

or

$$bd^2 = \frac{2M}{f_ckj}, \text{ or } f_c = \frac{2M}{kjbd^2} \qquad (5A)$$

When under-reinforced, the resisting moment depends on the steel and its value, then, is

$$M_s = pf_sj(bd^2) = f_sA_sjd \qquad (6)$$

or

$$bd^2 = \frac{M}{pf_sj}, \text{ or } f_s = \frac{M}{A_sjd} \qquad (6A)$$

Unit compressive stress in concrete

$$f_c = \frac{2M}{kjbd^2} = \frac{2pf_s}{k} = \frac{f_sk}{n(1-k)} \qquad (7)$$

Unit tensile stress in steel

$$f_s = \frac{M}{A_sjd} \qquad (8)$$

If $K = \frac{M}{bd^2}$, then the value of K in terms of steel stress is

$$K = \frac{M}{bd^2} = pf_sj = pf_s\left(1 - \frac{k}{3}\right) \qquad (9)$$

In terms of concrete stress, value of K is

$$K = \frac{M}{bd^2} = \tfrac{1}{2}f_ckj = \tfrac{1}{2}f_ck\left(1 - \frac{k}{3}\right) \qquad (10)$$

Illustrative Problem.—Find the values of p and k so that a beam or slab will be of equal strength in tension and compression. Assume $f_s = 16{,}000$, $f_c = 700$ lb. per sq. in. and $n = 12$.

Substituting values in (4)

$$p = \frac{\frac{1}{2}}{\frac{16{,}000}{700}\left(\frac{16{,}000}{(12)(700)} + 1\right)} = 0.00753$$

$$k = \sqrt{(2)(0.00753)(12) + (0.00753)^2(12)^2} - (0.00753)(12) = 0.344$$

With this combination of values for f_s and f_c and with n assumed at 12, the steel (or M_s) will control in any case when k is less than 0.344 and the concrete (or M_c) will control when k is greater than this value.

When M_s controls and is known for any combination of unit stresses, the resisting moment M_s can be found for any other combination of unit stresses (n and k remaining the same) by proportioning the two values of f_s and multiplying the known value of M_s by the proportional increase or decrease. This holds true when the steel controls in any two cases.

Illustrative Problem.—A 4½-in. slab with $d = 3½$ in., $A_s = 0.28$ sq. in. per foot width, $p = 0.0067$ and $k = 0.358$, has a moment $M_s = 13{,}810$ in.-lb., when $f_s = 16{,}000$, $f_c = 650$ and $n = 15$. Find the value of M_s by proportioning the two values for f_s for the same member when the limiting stresses for f_s and f_c are 18,000 and 750 respectively and $n = 15$. The proportion that $f_s = 18{,}000$ is greater than $f_s = 16{,}000$ is

$$\frac{18{,}000 - 16{,}000}{16{,}000} = 12.5\%$$

The resisting moment required is

$$M_s = 13{,}810 + (0.125)(13{,}810) = 15{,}540 \text{ in.-lb.}$$

The same condition applies in a similar manner when the concrete (or M_c) controls for any two combinations of unit stresses, the value of M_c for one being known.

Illustrative Problem.—A 4-in. slab with $d = 3$ in., $A_s = 0.29$ sq. in. per foot width, $p = 0.0081$, $k = 0.385$, $j = 0.872$, has a resisting moment $M_c = 11{,}780$, when $f_s = 16{,}000$, $f_c = 650$ and $n = 15$. Find the value of M_c for the same member when the limiting stresses for f_s and f_c are 18,000 and 750 respectively, and $n = 15$. The proportion that $f_c = 750$ is greater than $f_c = 650$ is

$$\frac{750 - 650}{650} = 15.4\ \%$$

The resisting moment required is

$$M_c = 11{,}780 + (11{,}780)(0.154) = 13{,}600 \text{ in.-lb.}$$

or

$$M_c = (\tfrac{1}{2})(750)(0.385)(0.872)(12)(3)^2 = 13{,}600 \text{ in.-lb.}$$

Illustrative Problem.—Determine whether M_s or M_c controls in a rectangular beam when $f_s = 16{,}000$, $f_c = 800$ and $n = 15$, assuming steel ratio $p = 0.0082$, from which $k = 0.387$.

Steel ratio for balanced reinforcement, Formula (4)

$$p = \frac{\tfrac{1}{2}}{\frac{16{,}000}{800}\left(\frac{16{,}000}{(15)(800)} + 1\right)} = 0.0107$$

$$k = \frac{1}{1 + \frac{16{,}000}{(15)(800)}} = 0.429 \qquad j = 1 - \frac{0.429}{3} = 0.857$$

Knowing k to have a value of 0.429 for equal strength in tension and compression, it follows that M_s controls for $k = 0.387$.

As the steel area A_s or steel ratio p increases, k increases and j decreases (though not in the same ratio), for the reason that as the percentage of steel gets larger, the neutral axis is lowered, resulting in a greater numerical value for k (thus lowering the neutral plane) and a lessening value for j since the centroid of compressive stress is lowered. This condition will be made clear by application of formulas and reference to stress diagram, Fig. 23.

The flexure formulas can be applied to any rectangular member in an existing structure for the purpose of finding the safe load capacity, or to any rectangular member in a proposed structure, where the structural sizes are to be established.

Illustrative Problem.—What will be the values of f_c and f_s in a beam 12 × 18 in. reinforced with three ¾-in. rounds, for a clear span of 15 ft. 0 in. non-continuous when sustaining a total load of 14,000 lb. $d = 16$ in. $n = 15$.

$$p = \frac{A_s'}{bd} = \frac{1.33}{(12)(16)} = 0.0069$$

$$k = \sqrt{(2)(15)(0.0069) + (15)^2(0.0069)^2} - (0.0069)(15) = 0.363$$

$$j = 1 - \frac{0.363}{3} = 0.879$$

$$M = \frac{(14{,}000)(15)(12)}{8} = 315{,}000 \text{ in.-lb.}$$

Substituting values in Formula (7)

$$f_c = \frac{(2)(315{,}000)}{(0.363)(0.879)(12)(16)^2} = 642 \text{ lb. per sq. in.}$$

Substituting values in Formula (8)

$$f_s = \frac{315{,}000}{(1.33)(0.879)(16)} = 16{,}840 \text{ lb. per sq. in.}$$

Illustrative Problem.—A rectangular beam 30 ft.–0 in. span, non-continuous, is required to support a brick wall 18 in. thick and 12 ft. 0 in. high. Find the depth d and steel area A_s, when $f_s = 18{,}000$ and $f_c = 750$, for equal strength in tension and compression. The width b is fixed to conform to thickness of brick wall. $b = 18$ in. $n = 15$.

$$\begin{aligned}\text{Brick wall load} &= (30)(12)(180) = 64{,}800\\ \text{Beam load assumed} &= (30)(780) = 23{,}400\\ \text{Total load} &\qquad\qquad 88{,}200\end{aligned}$$

$$M = \frac{(88{,}200)(30)(12)}{8} = 3{,}969{,}000 \text{ in.-lb.}$$

From Formula (4), for balanced reinforcement,

$$p = \frac{\frac{1}{2}}{\frac{18{,}000}{750}\left(\frac{18{,}000}{(15)(750)} + 1\right)} = 0.0080$$

From Formula (3)

$$k = \frac{1}{1 + \frac{18{,}000}{(15)(750)}} = 0.385, \quad j = 0.872$$

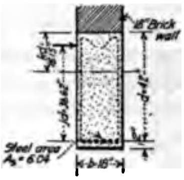


Fig. 24.

Since the values f_s and f_c are balanced, substitute in either Formula (5*A*) or (6*A*). From Formula (5*A*)

$$bd^2 = \frac{2M}{f_ckj}, \quad d^2 = \frac{(2)(3{,}969{,}000)}{(18)(750)(0.385)(0.872)}, \qquad d = 41.9 \text{ in.}$$

or from (6*A*)

$$bd^2 = \frac{M}{pf_sj}, \quad d^2 = \frac{3{,}969{,}000}{(18)(0.0080)(18{,}000)(0.872)}, \qquad d = 41.9 \text{ in.}$$

From Formula (4*A*)

$$A_s = (0.0080)(18)(41.9) = 6.04 \text{ sq. in.}$$

For practical reasons make $d = 42$ in. (see Fig. 24).

31*a*. Use of Tables and Diagrams.—After the application of formulas in the design of rectangular beams and solid slabs is thoroughly understood, the designer should resort to the use of tables and diagrams such as illustrated in subsequent pages. Tabular values are given for k and j for various percentages of steel, also diagrams giving the values $K = \frac{M}{bd^2}$ for the various steel and concrete stresses, and steel ratios p. Using these tables and diagrams will not only result in lessening the amount of work and time involved, but will reduce to a minimum the occasion for material errors when making calculations.

32. Lengths of Beams and Slabs Simply Supported.—As stated by the Joint Committee, the span length for beams and slabs simply supported should be taken as the distance from center to center of supports, but need not be taken to exceed the clear span plus the depth of beam or slab.

33. Shearing Stresses in Reinforced Concrete Beams.—The variation in shearing stresses in a reinforced beam differs from that in a homogeneous beam, due to the concentration of tensile stress in the steel. In Fig. 25 the opposing concrete forces acting through the centroid of compression are represented by C and C' in a short portion of a beam, where V represents the total vertical shear. T and T' indicate the opposing tensile stresses. v denotes the unit horizontal or vertical shearing stress at any point between the steel and the neutral axis, and b the width of the beam. It follows, then, since the tensile and compressive forces are in equilibrium, that $C' = T'$, and $C = T$. The total horizontal shearing stress upon any horizontal plane, immediately above the steel or between the steel and the neutral axis, is $T' - T$. Then

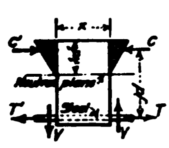


Fig. 25.

$$v = \frac{T' - T}{bx} \tag{1}$$

From equality of moments, or equilibrium produced by the various couples,

$$Vx = (T' - T)jd$$

Substituting the value of $T' - T = \frac{Vx}{jd}$ in equation (1), there follows:

Equation (2) gives the intensity of shearing stress for any point between the steel and the neutral axis. Since the value of j varies but slightly for various percentages of steel, the unit shearing value v will be only slightly affected if the average ratio $j = \frac{7}{8}$ is substituted in (2). Then

$$v = \frac{8}{7} \cdot \frac{V}{bd} \tag{3}$$

Fig. 26 represents the law of variation of shearing stress on a vertical crosssection. The intensity of shearing stress at any point between the steel and the neutral axis is the same whereas between the neutral axis and the extreme fiber of compressive face, the shear variation follows the parabolic law.

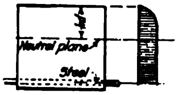


Fig. 26.

For all practical purposes the use of Formulas (2) or (3) can be relied upon to give results within the range of safety, although mathematical accuracy to a degree of nicety for all conditions of shear, is somewhat lacking. Like other designing formulas, experiments, theory, general practice and application have been given individual consideration in the determination of values and assumptions so as to avoid unnecessary complications and insure simplicity.

34. Web Reinforcement.

34a. Action of Web Reinforcement.—One of the most important and vital considerations in the design of rectangular or T-beam sections, consists in providing effective web reinforcement to resist diagonal tension.

The analytical treatment of diagonal tension in homogeneous beams is much less complex than in a composite structure. Owing to the complex nature of web stresses, and particularly diagonal tensile stresses, recourse is had to a more simplified or convenient method of stress determination, by assuming a vertical plane as a means of measuring the intensity of diagonal tension at any section of a member. This assumption reduces analytical treatment to its simplest form and hence its adoption is universal. A member subjected to the action of external forces, develops diagonal tension as a result of flexural action. After the concrete has reached its limit of resistance to diagonal tension, failure will inevitably occur unless vertical stirrups or bars bent up at approximately 45 deg. are introduced in the proper proportion and at intervals sufficient to develop their purpose. Unlike other formulas recommended for the designing of concrete members, the mere fact that the concrete must develop diagonal tension at the initial loading before the stirrups or bent rods have any material value, introduces an element in design heretofore entirely neglected in assumptions. The deformations in the concrete must first take place, which permits of little stress to be taken by the stirrups or bent rods.

Due to the many complications that arise from stresses produced by diagonal tension, which is measured in terms of shearing stress on a vertical plane, a complete analysis of the action of web reinforcement does not seem feasible, therefore more or less empirical formulas and methods have been adopted in general practice.

What is commonly termed "shear" is greatest at the support and is equal to the upward reaction or ½ the total load of the member, when uniformly loaded. This may be termed the critical section, though many experiments have demonstrated conclusively that failure from diagonal tension does not occur immediately at the support. The appearance of failure in the vicinity of the support and not directly at this point, in all probability is caused in part by the presence of vertical compressive stresses arising from the reaction of the support, which must be resisted, and no doubt serve to diminish or neutralize, to some extent, the principal stresses. Fig. 27 illustrates in a general way the conditions developed by diagonal tension. The cracks are more pronounced and inclined near points of support, and originate on the tension side of the beam. The function of the stirrups or bent rods is simply to prevent this condition and render the structure a more consistent unit of strength.

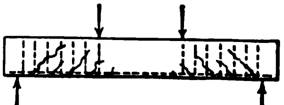


Fig. 27.

In simple beams it will be found most advantageous to have a low bond stress in the straight longitudinal bars at the ends extending into the supports, or else hooks should be provided to give efficient anchorage and thus obviate any chance of slipping or failure from this source.

The ends of all stirrup prongs extending into the upper face of beams should be given adequate anchorage, so they may fully develop the calculated tensile value.

In designing web members for any structure, the intimate relationship that should exist between theory and application should be constantly borne in mind. The form of the stirrup, and the logical means of holding the stirrups intact during the severe stages of disruption prior to and during concreting, should be given inseparable consideration. Such considerations are as vital to the construction as the knowledge of knowing how to proportion the design.

It has been shown by experiments that the combination of bent rods and stirrups gives the best results. It is good design to permit the stirrups to develop the required resistance to diagonal tension and allow the bent-up rods to act only as an additional safety factor, in reducing further the opportunity for failure. The spacing of stirrups has a decided influence on the function they are to perform. Referring to Fig. 28, it is reasonable to believe that since diagonal tension at critical sections occurs approximately at 45 deg. with the horizontal, stirrups should be spaced at such intervals as to effectually counteract this tendency. Experiments show that a spacing greater than ½ the depth of the member has little or no value.

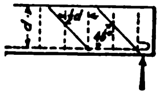


Fig. 28.

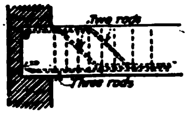


Fig. 29.

In considering the use of bent-up rods in conjunction with stirrups to resist diagonal tension, it will be well to note the limitations and difficulties in the arrangement of reinforcement that may arise. The case of a simple beam, or the end of a semi-continuous member bearing in a wall, exterior column or spandrel, offers a condition most favorable to the use of stirrups and bent rods in combination (Fig. 29). In any event one or more rods should be bent up into the top of the beam as shown, to prevent the appearance of cracks where tensile stress occurs due to deflection of the member and the restrained nature of bearing. The resisting moment will necessarily control the number and location of bends. The straight rods remaining in the bottom must also provide sufficient bond stress.

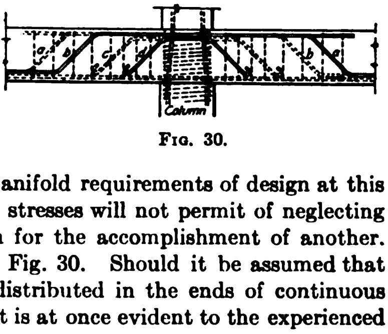


Fig. 30.

The difficulties in the case of continuous beams in this connection are numerous, demanding the closest study to obtain an arrangement that will fulfil the manifold requirements of design at this particular location, where the many important opposing stresses will not permit of neglecting one feature of design for the accomplishment of another. To illustrate, refer to Fig. 30. Should it be assumed that bent rods are to be distributed in the ends of continuous members as shown, it is at once evident to the experienced designer that complications naturally arise if consideration is entertained for the erector and the economic features of practical design. First, the design will probably require the same steel area A_s for the positive and negative moments, the negative stress varying from a maximum at the center of bearing, to zero at the point of inflection. This condition of negative stress demands a decreasing steel area proportionate with the negative moment at the various points, which fact will preclude the bending up of rods *a* and *b* at points too near the bearing. Additional rod units similar to *c* and *d* must be introduced to resist the diagonal tension, the ends of which should either be anchored by means of hooks or else the lower ends must be bent horizontal to lap the straight rods in the bottom. During erection, if spiral columns are employed, the use of additional rod units *c* and *d* will present great annoyance, for the rods must either be worked through the interval between spirals or the upper end of spiral unit must be forced down to allow adequate clearance

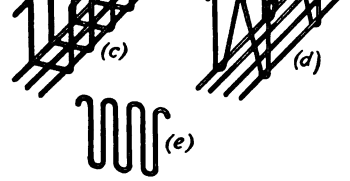


Fig.* 31.

between the two layers of rods. And finally the rods must be placed, spaced and held in their respective positions. The question of suitable stirrups and bent rods to resist diagonal tension necessarily resolves itself into the intelligent selection of units that can be installed with accuracy and speed, in order that the intention of the design may not be entirely defeated at the beginning of operations.

Fig. 31 shows the forms of stirrups mostly used in the average design. Types *d* and *e* are open to objection, for the reason they are most difficult to install in the case of continuous beams where top and bottom steel are required.

34*b*. Practical Consideration in Arrangement of Web Members.—In all structures for practical purposes, stirrups or bent rods should be used, whether or not theoretical calculations dictate their use. The exclusive use of bent rods to resist diagonal tension in continuous beams subjected to concentrated loads, and even for uniform loads, occasions many difficulties for the designer to solve, and when solutions are found merely from the standpoint of theory, the erector in the field has the option to execute the design as a whole or in part, depending entirely upon the character of supervision. The most effective way to avoid improper execution is to have constantly in mind the field superintendent or foreman's point of view, and adopt the design with common-sense intelligence, so that it can be carried out with the greatest degree of accuracy.

The most predominant disregard of accuracy, during the erection of the average reinforced concrete structure, is exercised in the placing of loose stirrups. There are many contributing causes. Foremost among them is the case in which the stirrups, having been placed and spaced with the average due care, are given the responsibility of remaining erect and spaced without any tangible tie, one with the other, to prevent subsequent displacement during concreting operations. A small rod ¼ in. or ⅜ in. in size, as illustrated in Fig. 31, type (*a*), extending from one stirrup to the other for the full length of member and tied to each hook by means of small wires, will obviate to a considerable extent the tendency of the stirrups to become disarranged.

There is certainly little consistency in design and practical execution when stirrups are shown spaced at 2, 3, 4, 5, or 6-in. intervals and then through the fault of construction methods specified, permit of a wide variation from this spacing. In this event, theoretical design in locating the stirrups is simply a matter of form and useless endeavor.

34*c*. Design of Web Reinforcement.—The variation in shear along the length of a uniformly loaded beam is shown in Fig. 32(*a*). The following simple graphical method may be used for determining the stresses and spacing of stirrups:

Let v, the total unit shearing stress, denote the height of triangle in Fig. 32(*a*), v_1 the unit shearing stress to be taken by the concrete, and $v-v_1$ the remaining shear to be taken by the steel. Also let x_1 denote the distance in feet from the support to the point beyond which no stirrups are required.

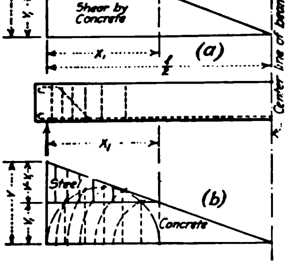


FIG. 32.

Now the total unit shearing stress is

$$v = \frac{V}{bjd} \qquad (1)$$

or, substituting ⅞ as the average value of j,

$$v = \frac{V}{b(\frac{7}{8}d)} \qquad (2)$$

The distance in feet from the support to the point beyond which no stirrups are needed is

$$x_1 = \frac{(v-v_1)l}{2v} \qquad (3)$$

or

$$x_1 = \frac{l}{2}\left(1 - \frac{v_1}{v}\right)$$

In Fig. 32(*a*), the total shear to be taken by all stirrups in one end of a beam is indicated by the triangle with the height $v-v_1$ and base x_1 and is equal to

$$V_1 = \frac{(v - v_1)bx_1}{2}(12) \qquad (4)$$

The diameter of a stirrup without any prong or hook should not exceed

$$i = \left(2.4\frac{u}{f_s}\right)d \qquad (5)$$

The minimum spacing of stirrups at the support will be

$$s = \frac{A_s f_s}{(v - v_1)b} \qquad (6)$$

Referring to Fig. 32(*b*), stirrups can be spaced by dividing the triangle with base x_1 and height $v-v_1$, into as many equal parts as there are stirrups required, such that no spacing will exceed $\frac{d}{2}$. The center of gravity of each subdivision will denote the location of stirrups, assuming the same size stirrup unit throughout. Equal areas can be easily obtained as shown, by projecting the points from the semi-circle with diameter equal to x_1.

In the average design of beams, ¼- or 5⁄16-in. stirrups with hooked ends are used for beams from 10 to 25 in. deep, 5⁄16- or ⅜-in. stirrups for beams 25 to 40 in. deep and ½-in. stirrups for beams 40 to 60 in. deep. The size of stirrup will, of course, depend on the unit stress f_s assumed and the spacing.

In the design of stirrups, various unit stresses are used in the steel ranging from 10,000 to 18,000 lb. per sq. in. A high unit stress is not recommended, when considering the function which stirrups must perform in a rigid member. The higher the stress, the more the elongation when the member is subjected to heavy loads, and the better should be the anchorage to prevent any possibility of slipping. A unit stress for steel stirrups of 10,000 to 12,000 lb. per sq. in. would be more consistent with good practice.

Illustrative Problem.—A simply supported beam 10 × 22 in. has a total uniform load of 2000 lb. per lin ft. The span is 20 ft. The tension reinforcement is 2 in. from the bottom. Find the web reinforcement to resist diagonal tension, using vertical U-stirrups, when the allowable $f_s = 12{,}000$ lb. and $v_1 = 40$ lb. Maximum bond stress allowed $u = 80$ lb. per sq. in.

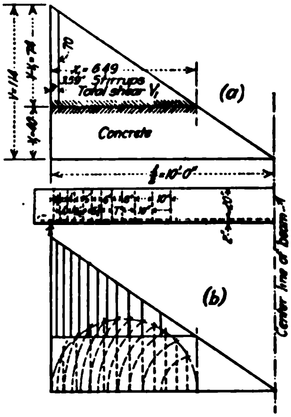


Fig. 33.

Substituting in (2)

$$v = \frac{(10)(2000)}{(10)(7/8)(20)} = \frac{20{,}000}{175} = 114 \text{ lb. per sq. in.}$$

Substituting in (3)

$$x_1 = \frac{(114-40)(20)}{(2)(114)} = 6.49 \text{ ft.}$$

The total shear denoted by triangle, Fig. 33(*a*), with height $v-v_1 = 74$ and base $x_1 = 6.49$ ft., will be

$$V_1 = \frac{(114-40)(10)(649)}{2}(12) = 28{,}810 \text{ lb.}$$

Assuming ⅜-in. round stirrups the area A_s for the 2 legs is (2) (0.1104) = 0.2208 sq. in. The value of each stirrup = (0.2208)(12,000) = 2650 lb. $\frac{28{,}810}{2650} = 10.87$ stirrups, or, say 11 stirrups required for each end. The closest spacing required at each end near support will be

$$s = \frac{(0.2208)(12{,}000)}{(114-40)(100} = 3.59 \text{ in. c. to c.}$$

Assuming this theoretical value 3.59 in. as the closest spacing, and checking back with diagram Fig. 33(*a*), it will be found that the total shear taken by first stirrup is equal to

$$\frac{(74+70)}{2}(3.59)(10) = 2585 \text{ lb.}$$

which is practically the same as the value assigned to each stirrup. The stirrups indicated in Fig. 33(*a*) have been projected from equal areas in diagram Fig. 33 (*b*) and spacing noted accordingly. One additional stirrup is used over requirements on account of spacing being limited to $\frac{d}{2}$ or 10 in.

The above method of finding the correct spacing of stirrups for a uniformly loaded member, as well as any other proposed or suggested method not mentioned, entails considerable work and delay when it is considered that some buildings require a hundred or more different designs of beams, and consequently is objectionable. In view of practical circumstances involving conditions that do not justify the spacing of stirrups to the exact inch, the following method will give satisfactory results on the side of safety:

First find the value of v by Formula (2) and then the distance x_1 beyond which stirrups are not needed by Formula (3). The total shear V_1 to be taken by stirrups, represented by the triangle of base x_1 and height $v-v_1$, can then be found by substituting in Formula (4). The total number of stirrups required for V_1 will be $\frac{V_1}{A_s f_s}$. The stirrup spacing at the critical point near bearing will be, assuming a given size of stirrup,

$$s = \frac{A_s f_s}{(v-v_1)b}$$

With the distance x_1, total number of stirrups required, and the minimum spacing known, it will be entirely safe and consistent to gradually increase the spacing over the distance x_1, from the smallest spacing to a maximum of one half the effective depth of the beam. On account of the minimum spacing of $\frac{d}{2}$ it may be necessary to add one or more stirrups to meet this limitation.

Illustrative Problem.—Assume the same conditions as in the preceding problem, when $v = 114$, $x_1 = 6.49$, $V_1 = 28{,}810$, the total number of stirrups 11, and the minimum spacing $s = 3.59$ in.

With the above conditions known, the approximate spacing can be ascertained at once, or 3 stirrups at 4 in., 2 at 5 in., 2 at 7 in., 3 at 9 in., and 2 at 10 in. The total of these spacings is 83 in. or slightly more than 78 in., the value of x_1, which will be satisfactory.

Illustrative Problem.—Assume the same beam in previous problem but with a concentrated load at the center of 40,000 lb. instead of a uniform load totalling 40,000 lb.

The reaction at each end will be 20,000 lb. The value of $v = 114$ lb. per sq. in. will be the same, but the intensity of shear is constant at all points between the center and the bearing, hence $x_1 = 10.00$ ft. and

$$V_1 = (114-40)(10)(10)(12) = 88{,}800 \text{ lb.}$$

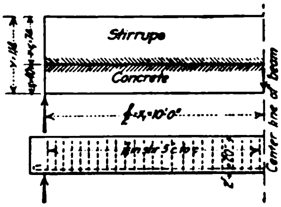


Fig. 34.

The value of one ⅜-in. U-stirrup at 12,000 was found to be 2650. Thus the number of these stirrups required equally spaced from the center to each bearing is

$$\frac{88{,}800}{2650} = 34$$

Since $l = 240$ in., the stirrup spacing required is

$$\frac{240}{68} = 3.6 \text{ in.}$$

This spacing is too close.

Assuming a $\frac{7}{16}$-in. stirrup, A_s will have a value equal to

$$(0.1503)(2)(12{,}000) = 3600 \text{ lb.}$$

or number required is

$$\frac{88{,}800}{3600} = 25$$

The spacing will then be $\frac{240}{50} = 5$ in. (approx.), which is satisfactory. Using a 5-in. spacing and referring to diagram Fig. 34, the stress in one stirrup will be $(5)(74)(10) = 3700$ lb. or slightly more than the tensile value assumed for each stirrup.

34d. Bent Bars for Web Reinforcement.—The following simple graphical method may be used in important cases for determining the stress or spacing of bent bars:

Assume a beam 10 × 20 in., 20-ft. span, uniformly loaded, with $v = 100$ lb. The bent-up rod nearest the support is assumed to be a ⅞-in. round, and the other bent rod a ¾-in. round, both rods being bent at 45 deg. Find the stress in each rod. Assume $v_1 = 40$ lb. The following method will make clear the principles involved: Referring to Fig. 35(*a*), project the axis *AB* upon an axis *AC* at 45-deg. inclination and lay off $v = 100$, $v_1 = 40$, and $v - v_1 = 60$. Then the ordinates between *BC* and *BD* will represent the shearing stress v along one-half of the beam. The area between any two ordinates like *DD'* and *EE'* multiplied by the width b of beam will equal the product of the total average shear over the length l', multiplied by the projection of this length on the inclined axis *BC*. In diagram Fig. 35(*a*), the stress taken by the ⅞-in. rod will be

$$\left(\frac{60+44}{2}\right)(14.5)(10) = 7540 \text{ lb.}$$

The area of a ⅞-in. round is 0.60 sq. in.

$$\frac{7540}{0.60} = 12{,}560 \text{ lb. per sq. in.}$$

This value is not too high if stirrups are also used, which in this case are neglected. The stress in the ¾-in. rod will be

$$\left(\frac{44+34}{2}\right)(9.5)(10) = 3700 \text{ lb.}$$

$$\frac{3700}{0.44} = 8410 \text{ lb. per sq. in.}$$

In Fig. 35(*b*), the stress taken by the ⅞-in. round will be

$$\frac{\left(\frac{60+44}{2}\right)(20.5)(10)}{\sqrt{2}} = \frac{10{,}660}{1.4142} = 7540 \text{ lb.}$$

or unit stress in one ⅞-in. round is

$$f_s = \frac{10{,}660}{(0.60)(\sqrt{2})} = 12{,}560 \text{ lb. per sq. in.}$$

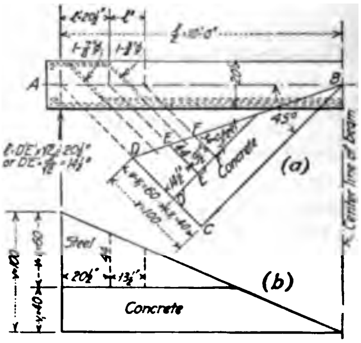


Fig. 35.

35. Bond Stress.—The development of proper bond stress between the steel and the concrete at all points in the design of a member, should receive careful attention. For simple beams with loads distributed as in Figs. 36, 37 and 38, positive moments are developed which begin immediately at the points of supports. This at once suggests a pull in the straight rods at the supports; the required intensity of which must be developed through adhesion of the concrete to the steel.

In the case of continuous beams, Fig. 39, the straight rods of end spans bearing in wall, spandrel or column, should be investigated to ascertain the pull in the rods at this point. In the case of continuous ends of beams the character of stress is compressive, by reason of cantilever action at this point, though the increment of stress is of the same sign. In the design of practical structures there are comparatively few designs executed in the past, which have given serious consideration to the development of the proper theoretical bond stress for the ends of rods in the compressive side of continuous beams at supports. Yet comparatively few failures have been recorded due to this source of seeming weakness.

If the safe adhesion or bond stress per square inch of bar surface exceeds that prescribed by the best practice, then the ends of rods in the case of pulling stress should be hooked as in Fig. 29. In designing a member it follows that the higher the unit stresses assigned to steel in tension, the smaller will be the rods or sectional area at this critical point and hence the surface of bars available for adhesion will be reduced. Deformed rods afford a suitable means of increasing bond resistance, but in many instances the resistance offered will not be sufficient to fully conform to requirements of design and prevent initial slip under working conditions. It has been noted that one of the fundamental assumptions in the theory of design consists in having perfect adhesion between the steel and concrete at all points within the elastic limit of the steel.

Theoretical results show that bond stress is a simple function of shear and varies with the shear. Figs. 36, 37, 38 and 39 show some of the conditions of moment and shear for different loadings. In Fig. 36 the value of bond stress is zero at the center and increases uniformly to a maximum at the supports. In Fig. 38 the bond stress is uniform from concentrated load to supports. Fig. 37 shows the same intensity of bond stress from points of loading to supports.

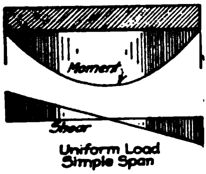


FIG. 36.

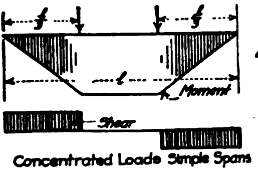


FIG. 37.

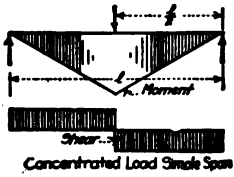


FIG. 38.

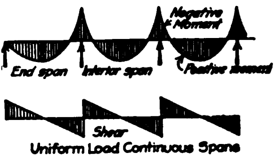


FIG. 39.

In proportioning members to resist bond stress it should be remembered that any slipping of the bars increases at once the deformation of the concrete and hence emphasizes the chance of failure by increasing the tension in the concrete.

Referring to Fig. 25, Art. 33, the shearing stress per linear inch over a distance x is

$$\frac{T' - T}{x}$$

But

$$Vx = (T' - T)jd$$

or the bond stress per linear inch is

$$\frac{T' - T}{x} = \frac{V}{jd}$$

The bond stress per square inch developed by the surface of steel bars is $\frac{V}{jd}$ divided by the sum in inches of all the perimeters of the bars at a given cross section. If Σo = the sum of perimeters of all bars in a member, and u the bond stress per square inch, then

$$u = \frac{V}{\Sigma o j d}$$

In other terms, the unit bond stress is simply the reaction in pounds divided by the sum of bar perimeters in inches multiplied by the lever arm. In the above formula, $j = \frac{7}{8}$ may be used as the average value.

The Joint Committee recommends in case of plain bars a unit bond stress between steel and concrete equal to 4% of the compressive strength of concrete and 5% in case of deformed bars. For a 1-2-4 gravel or hard limestone concrete with compressive value of 2000 lb. per sq. in., the working value of 80 lb. for plain and 100 lb. for deformed bars are the values recommended.

When the web reinforcement consists of a combination of bent bars and stirrups, tests of freely-supported rectangular and T-beam sections, indicate a greater reduction of bond stress than in the case of beams with stirrups, and beams with only straight longitudinal bars. Judging from the results of tests it will be conservative to assume a bond stress of 1½ times the above working values when members are thoroughly reinforced with stirrups and two or more bent rods, bent at intervals not to exceed the effective depth of the member and preferably less. The combination of bent bars and stirrups can be readily adapted at the ends of simple beams and end bearings of continuous beams, where all the tension bars are not required in the bottom.

Illustrative Problem.—A simply supported beam with span of 18 ft. requires a section 10 in. wide, effective depth $d = 18$ in., and reinforcement three ⅝-in. rounds straight and two ¾-in. rounds bent, to support a total uniform load of 890 lb. per lin. ft. when steel and concrete are of equal strength—the controlling values being $f_s = 16{,}000$, $f_c = 750$, $n = 15$, $u = 80$, $v_1 = 40$. Find the bond stress in the straight longitudinal rods.

The reaction is equal to

$$(890)(9) = 8010 \text{ lb.}$$

The perimeters of three ⅝-in. rounds will be

$$(3)(1.964) = 5.892 \text{ sq. in.}$$

Substituting in formula

$$u = \frac{V}{(\Sigma o)(7/8)(d)} = \frac{8010}{(5.892)(7/8)(18)} = 86 \text{ lb. per sq. in.}$$

$$v = \frac{8010}{(10)(7/8)(18)} = 51 \text{ lb. per sq. in.}$$

If the bent rods are not considered to resist diagonal tension, and since in any event stirrups are recommended, the value $u = 86$ lb. for plain or deformed bars is entirely conservative.

In comparing rectangular and T-beam sections it will be found that the investigation of bond stress for the latter will always be of greater importance than in the former case, for the reason that the required section for rectangular beams is proportioned for limiting values assigned to f_c, whereas for T-beams the necessary section for shear is of fundamental importance. Hence the shear in the former case will usually be much less per square inch than in the latter case. Bond stress being a function of shear, the member having the greatest shearing stress should be given especial attention.

36. Spacing of Reinforcement and Fire Protection.—The spacing of rods particularly in beams is a matter of great importance in the design of concrete structures. The location of beam and slab rods involves the following considerations:

1. The longitudinal bars should be spaced far enough apart to develop the required adhesion between concrete and steel.
2. A clear space between the bars should be allowed to permit the larger aggregates to pass between and around each bar.
3. A protective coating of concrete of adequate thickness should be provided for all bars, to insure fireproofness in the event of fire.

The bond stress determines the theoretical clear interval between beam bars, but under no circumstances should this interval be equal to or less than the size of aggregate used. It is advisable to use a clear spacing of not less than 1½ in. in any case as the larger sizes of gravel and limestone aggregate will range from ¾ in. to 1¼ in. It is good practice to use a clear spacing of 1½ to 3 times the diameter of bar used in the design, provided this spacing is not less than 1½ in. The clear spacing between the two layers of bars likewise should not be less than 1½ in. for practical reasons mentioned.

Concrete is incombustible and has a low rate of heat conductivity which makes the material highly efficient for fireproofing purposes. The fire resisting properties of concrete, however, are of little avail if the reinforcement is permitted to approach too near the exposed surfaces. The thickness of protective coating for ordinary purposes of design should be the greatest in the case of beams and girders which are in the event of fire, subjected to the most intense heat. Slabs or flat surfaces require less protection for the steel for obvious reasons.

It appears from past practice and fire tests, that a minimum protection of 2 in. for the steel in beams and girders, and 1 in. for the steel in slabs, are conservative allowances.

Another form of abuse practiced in the construction of fireproof buildings, in the majority of buildings constructed, is the total lack of proper care taken in the supporting and spacing of individual bars in beams and slabs. It is an illogical procedure to specify a certain spacing of bars and a minimum protective coating, and then expect the erector to execute the plans and details, without some specified means of accomplishing this purpose. It is hardly possible to maintain a given spacing for bars or to support the bars the required distance from the falsework without the use of some definite device made for the purpose. Formulas and details may be developed to a nicety but if the practical means of accomplishing the design are neglected, it is simply an invitation for poor workmanship, lax methods, and inefficient execution. As a consequence the advantages of correct design are overcome and the strength of the structure is impaired by materially reducing the factor of safety.

Rods in beams bunched together cannot possibly give the proper resistance to bond stress, and results in a source of weakness highly undesirable. If some mechanical device or devices could be generally employed by engineers, that would serve the purpose of minimizing the occurrence of improper workmanship, somewhat higher working stresses than now assumed could be consistently used with a greater degree of satisfaction.

37. Rectangular Beams Reinforced for Compression.—It is more economical to use rectangular beams without top reinforcement if the limitations of design will permit. Only in isolated cases does it become necessary to use beams of this character. Beams enclosing elevator openings, stair wells, or those deprived of T action with limited depth, by reason of openings at the section of greatest moment, sometimes require reinforcement in the top as well as in the bottom, to give equal tensile and compressive resistance.

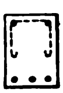


Fig. 40.

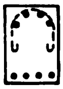


Fig. 41.

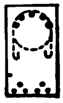


Fig. 42.

The action in the top of a beam reinforced for compression may be compared with that of a column. In the latter case the rods under stress are prevented from failure along the line of least resistance by the use of bands or hooping spaced at the proper intervals. The longitudinal rods of the column are placed in the corners or where the bands change direction and not at intermediate points where bending would be produced in the length of the band.

The same reasoning may be applied to that of compressive reinforcement in beams. Where only two rods are used, inverted U-stirrups will prove most effective in anchoring the rods into the body of the member, as shown in Fig. 40. Where three or more rods are required, this form of stirrup cannot be entirely effective, due to the fact that bending moment is produced in the straight portion of stirrup when the intermediate rods are in compression. A form of stirrup shown in Fig. 41 would no doubt give greater resistance to compressive stress, though the effective distance between the top and bottom steel will be slightly lessened. In important members spiral reinforcement has often been used in connection with compressive reinforcement with the most satisfactory results, Fig. 42.

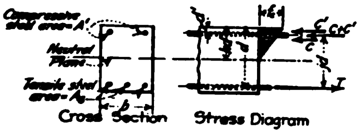


Fig. 43.

The Joint Committee recommends that top reinforcement for positive bending moment should not exceed 1% of the sectional area of the concrete.

The same fundamental principles given for beams reinforced for tension only apply to double reinforced beams. The tension in the concrete is neglected and the compression in the concrete is assumed to follow the linear law of variation. Hence the formulas apply to working conditions only.

Let p' = ratio of cross section of steel in compression to cross section of beam above the tensilesteel = $\frac{A'}{bd}$.

f'_s = compressive unit stress in steel.

Other notations are given in Fig. 43.

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \qquad (1)$$

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (1A)$$

$$j = \frac{k^2(1 - \tfrac{1}{3}k) + 2p'n\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)} \qquad (2)$$

$$f_s = \frac{M}{A_s jd} = \frac{M}{pjbd^2} \qquad (3)$$

$$f_c = \frac{f_s k}{n(1 - k)} \qquad (4)$$

$$f_s' = \frac{k - \frac{d'}{d}}{1 - k} f_s \qquad (5)$$

$$f_s = \frac{f_c n(1 - k)}{k} \qquad (6)$$

$$M_s = f_s pjbd^2 \qquad (7)$$

$$d = \frac{M_2}{A_s f_s j} \qquad (8)$$

The formulas given for rectangular beams reinforced for tension only, which determine the shear v, bond stress u, and web reinforcement, are the same for double reinforced beams. In finding these values j may be assumed to have an average value of 0.85.

37a. Formulas for Determining Percentages of Steel in Double Reinforced Rectangular Beams.[1]—For any given values of f_c and f_s, k has identically the same value, irrespective of shape or type of member. The formulas given below are based on this fundamental fact. The value of k for all beams is expressed by the formula

$$k = \frac{1}{1 + \frac{f_s}{nf_c}}$$

If the extreme fiber stresses are not changed by the addition of steel to the section, it follows that the added tensile and compressive steel must form a balanced couple, with unit stresses conforming to the stresses already in the section.

Let p_1 = steel ratio for the beam without compressive steel.
p_2 = steel ratio for the added tensional steel.
$p = p_1 + p_2$.
p' = steel ratio for compressive steel.
M_1 = moment of the beam without compressive steel.
M_2 = moment of the added steel couple.
$M = M_1 + M_2$.

Then

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (1)$$

$$p_1 = \frac{f_c k}{2f_s} \qquad (2)$$

$$M_1 = f_s p_1\left(1 - \frac{k}{3}\right)bd^2 \qquad (3)$$

$$M_2 = M - M_1 \qquad (4)$$

$$p_2 = \frac{M_2}{f_s\left(1 - \frac{d'}{d}\right)bd^2} \qquad (5)$$

$$p = p_1 + p_2 \qquad (6)$$

$$p' = p_2 \frac{1 - k}{k - \frac{d'}{d}} \qquad (7)$$

Illustrative Problem.—In a double reinforced beam the bending moment is 950,000 in.-lb. Practical conditions limit the size of the beam to b = 14 in. and d = 20 in. Find the required steel percentages for tension and compression. $\frac{d'}{d} = \frac{2}{20} = 0.10$. From Table 3, $k = 0.385$, $p_1 = 0.008$. $K = 125.74$, when $f_s = 18{,}000$, $f_c = 750$ and $n = 15$.

$$M_1 = (18{,}000)(0.008)\left(1 - \frac{0.385}{3}\right)(14)(20)^2 = 703{,}000 \text{ in.-lb.}$$

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[1] Taken from thesis by Robert S. Beard submitted to University of Kansas in partial fulfillment of the requirements for the Master's Degree.

or, M_1 may be obtained from formula $M_1 = Kbd^2$

$$M_2 = 950{,}000 - 703{,}000 = 247{,}000 \text{ in.-lb.}$$

$$p_2 = \frac{247{,}000}{18{,}000(1 - 0.10)(14)(20)^2} = 0.00272$$

$$p = 0.008 + 0.00272 = 0.01072$$

$$p' = (0.00272)\left(\frac{1 - 0.385}{0.385 - 0.10}\right) = 0.00587$$

Steel for compresson $A' = (0.00587)(14)(20) = 1.644$ sq. in.
Steel for tension $A_s = (0.01072)(14)(20) = 3.002$ sq. in.

For all practical purposes this problem can be solved by the following simple method of reasoning:

1. *To Find the Area A_s.*—The centroid of compressive area of the concrete from the top of the beam is

$$\frac{kd}{3} = \frac{(0.385)(20)}{3} = 2.57 \text{ in.}$$

Hence, if $d' = 2$ in., the average lever arm is

$$20 - \frac{(2 + 2.57)}{2} = 17.71 \text{ in.}$$

$$A_s = \frac{950{,}000}{(17.71)(18{,}000)} = 3.00 \text{ sq. in.}$$

2. *To Find the Area Required for Compressive Steel.*—The concrete in compression alone will sustain a moment of

$$M_1 = Kbd^2 = 703{,}000 \text{ in.-lb.}$$

The steel for compression must take the difference, or

$$950{,}000 - 703{,}000 = 247{,}000 \text{ in.-lb.}$$

$$kd = (0.385)(20) = 7.70 \text{ in.}$$

The extreme fiber stress in the concrete is 750. At 2 in. from the top the compressive stress is

$$750 - \left(\frac{750}{7.7}\right)(2) = 554 \text{ lb. per sq. in.}$$

Hence

$$A' = \frac{247{,}000}{(15)(554)(18)} = 1.65 \text{ sq. in.}$$

The analysis of the above problem illustrates that almost identical results may be obtained through simple reasoning and is done to show the value of adopting, when possible, methods of calculation which can be more thoroughly comprehended, and which may further elucidate the principles involved in the derivation of formulas.

38. Moments Assumed in the Design of Continuous Beams and Slabs.—The Joint Committee recommends the following rules for computing the positive and negative moments in beams and slabs continuous over several supports due to uniformly distributed loads:

1. For floor slabs, the bending moments at center and at support should be taken at $\frac{wl^2}{12}$ for both dead and live loads, where w represents the load per linear unit and l the span length.

2. For beams, the bending moment at center and at support for interior spans should be taken at $\frac{wl^2}{12}$ and for end spans it should be taken at $\frac{wl^2}{10}$ for center and interior support, for both dead and live loads.

3. In the case of beams and slabs continuous for two spans only, with their ends restrained, the bending moment both at the central support and near the middle of the span should be taken as $\frac{wl^2}{10}$.

4. At the ends of continuous beams, the amount of negative moment which will be developed in the beam will depend on the condition of restraint or fixedness, and this will depend on the form of construction used. In the ordinary cases a moment of $\frac{wl^2}{16}$ may be taken; for small beams running into heavy columns this should be increased, but not to exceed $\frac{wl^2}{12}$.

The above rules apply to beams uniformly loaded and do not apply to members when one span is considerably longer than the other.

Since a concentrated load at the center of a beam or girder will produce a moment twice as great as the same load uniformly distributed, the moment for such members continuous over several supports may be taken as $\frac{wl^2}{6}$ and for end spans with ends restrained and continuous over one support, the moment may be taken as $\frac{wl^2}{5}$.

In the design of complicated structures there will often arise occasion for a more accurate determination of negative and positive moment distribution, to insure a more intelligent proportion for the member.

39. Slabs.

39*a*. Slab Design.—Solid reinforced concrete slabs are designed for given loads by using the same formulas given for rectangular beams. A width of 12 in. is usually employed in proportioning the depth *d*, percentage *p*, etc. As a general rule it is more economical to use balancing values for f_c and f_s. After the point is reached beyond which the extreme fiber stress in the concrete controls in the design, it will be determined that the small increase in moment derived, will not justify the cost of additional steel, which is added only for the purpose of lowering the neutral plane to prevent exceeding the maximum working value assigned to f_c. Long span slabs of solid concrete are not only lacking in economy, but add to the cost of supporting beams, girders, columns and footings, by reason of their dead weight, in comparison with other types of floors that may be used. Floors consisting of concrete joists in combination with hollow tile, gypsum or metal domes, will give greater economy for long spans. Joist floors can be used for spans as great as 40 ft. or more if conditions demand such extremes.

It is good practice not to exceed 2½ times the effective depth of solid slabs, for the spacing of carrying bars.

For all solid slabs it is advisable to use temperature rods ¼ or ⅜ in. in size extending perpendicular to the carrying reinforcement, to lessen the chance of cracks from shrinkage and temperature stresses as well as to form ties to which carrying bars can be wired to preserve a given spacing. Roof slabs which are exposed to a greater variation in temperature require more attention in this respect than floors which are protected from the varying climatic conditions.

The investigation of shear in solid slabs is seldom necessary, except in the case of heavy concentrated loads, or loads that may effect the section beyond safe working assumptions.

39*b*. Negative Reinforcement in Continuous Slabs.—Continuous slabs should always be provided with sufficient steel extending over the supports to take negative moment. Even in short spans, unsightly cracks in tile or composition floors, so often seen in buildings, will be obviated by permitting part of the steel to be bent up into the top of slab over supports, thereby preventing cracks when the adjacent panels deflect.

It is customary practice to bend up one-half the bars from each opposite panel, at approximately the one-fourth point, which gives a steel section for negative moment equal to that of the positive moment requirements at the center of panel. Negative reinforcement should extend to the one-third or one-fourth point depending on the length of spans and the live loads to be supported. The point to which steel for negative moment should extend, will depend principally on the intensity of live load. The dead load is fixed, but the live load is a varying quantity as to intensity and position in important structures. The greater the live load the greater will be the tendency for the negative moment to approach the center of spans under the worst condition of loading.

39*c*. Two-way Reinforced Slabs Supported Along Four Sides.—A series of panels reinforced in two directions at right angles and supported along four bearings should be made continuous over supports. In oblong panels the greatest length should not exceed 1½ times the least width. As a panel becomes oblong the proportion of load carried by the longer span becomes rapidly less.

Let r = proportion of total load carried by shorter span.
l = length of longer span in feet.
b = breadth of panel or shorter span in feet.

Then

$$r = \frac{l}{b} - 0.50$$

For different ratios of $\frac{l}{b}$ the values for r are as given in the accompanying table. When a floor panel is square and uniformly loaded, one-half the dead and live loads are resisted by the moments in each direction.

$l:b$	r
1.00	0.50
1.10	0.60
1.20	0.70
1.30	0.80
1.40	0.90
1.50	1.00

The Joint Committee recommends that in placing reinforcement in such slabs, account may well be taken of the fact that the bending moment is greater near the center of the slab than near the edges. For this purpose two-thirds of the previously calculated moments may be assumed as carried by the center half of the slab and one-third by the outside quarters.

The distribution of loads to beams along the four edges of such slabs are often assumed incorrectly by proportioning the members for uniformly distributed loads. For more exact calculations the distribution of load may be expected to vary in accordance with the ordinates of a parabola, but for practical purposes it may be just as well to avoid unnecessary loss of time and assume this variation to be represented by a triangle, although the moment resulting from the former assumption will be less than in the latter case.

For practical purposes floor panels reinforced in two directions cannot well be termed economical in competition with other forms of panel construction.

40. T-Beams.

40*a*. T-Beams in Floor Construction.—In floor construction T-beams are by far the most generally used form of supporting member. The term T-beam expresses its shape. In calculating the strength of T-beams, advantage is taken of the floor slab, which in good design must act as the compression flange of the member, the same as the upper flange of a steel I-beam must act when subjected to bending. To properly perform its function, a T-beam must be poured simultaneously with the floor slab and the stem and flange securely tied together by means of bent rods, stirrups and cross reinforcement from the slab. Even with the presence of stirrups and bent rods, horizontal planes made during construction are most undesirable. The slab should be an integral part of the beam.

In important members of long spans, or short spans designed for heavy loads, a thin slab should be thoroughly investigated and mechanically bonded to the stem by means of stirrups along the center portion between bearings, as well as near the supports where the stirrups are designed primarily to resist diagonal tension for uniform loading. In special beams with thin flanges a small fillet or bevel at 45 deg. connecting the stem to the flange will prove effective in giving added strength. In very long spans other methods must be employed to give the required strength in compression.

When beginning the design of a T-beam, the thickness of the flange is fixed by the depth of slab, but the distance to either side of stem over which compression may be assumed to act is arbitrarily selected from the results of tests, which have established within safe limits the assumptions to be made.

The action of a continuous T-beam includes a complication of stresses, which in the main should be entirely comprehended by the designer before attempting the use of formulas for practical application.

In comparing T-beams with rectangular beams, the economy of the former is obvious.

40*b*. Flange Width of T-Beams.—The following rules are recommended by the Joint Committee for determining the flange width:

(*a*) It shall not exceed one-fourth of the span length of the beam.

(*b*) Its overhanging width on either side of the web shall not exceed 6 times the thickness of slab.

Beams in which the T-form is used only for the purpose of providing additional compression area of concrete should preferably have a width of flange not more than 3 times the width of the stem and a thickness of flange not less than one-third of the depth of the beam.

40*c*. T-Beam Flexure Formulas.—In the design of a T-beam it is necessary to distinguish two cases; namely, (1) The neutral axis in the flange, and (2) the neutral axis in the web.

Case I. The Neutral Axis in the Flange.—All formulas for "moment calculations" which apply to rectangular beams apply to this case. It should be remembered, however, that b of the formulas denotes flange width, not web width, and p (the steel ratio) is $\frac{A_s}{bd}$, not $\frac{A_s}{b'd}$ (Fig. 44).

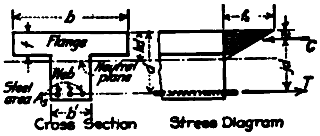

FIG. 44.

Case II. The Neutral Axis in the Web.—The amount of compression in the web is commonly small compared with that in the flange and in the analysis of this case is neglected. The formulas assume a straight line variation of stress and are:

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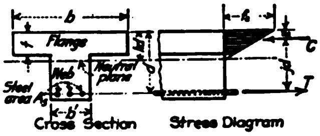

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Case II. The Neutral Axis in the Web.—The amount of compression in the web is commonly small compared with that in the flange and in the analysis of this case is neglected. The formulas assume a straight line variation of stress and are:

$$k = \frac{1}{1 + \dfrac{f_s}{nf_c}} \qquad (1)$$

$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt} \qquad (2)$$

$$k = \frac{pn + \frac{1}{2}\left(\frac{t}{d}\right)^2}{pn + \frac{t}{d}} \qquad (3)$$

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3} \qquad (4)$$

$$jd = d - z \qquad (5)$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3\left(\frac{1}{2pn}\right)}{6 - 3\frac{t}{d}} \qquad (6)$$

$$f_s = \frac{M}{A_sjd} = \frac{M}{pjbd^2} \qquad (7)$$

$$f_s = \frac{f_cn(1 - k)}{k} \qquad (8)$$

$$f_c = \frac{f_sk}{n(1 - k)} \qquad (9)$$

$$M_s = f_sA_sjd \qquad (10)$$

$$M_c = f_c\left(1 - \frac{t}{2kd}\right) bt \cdot jd \qquad (11)$$

Approximate formulas can also be established. From the stress diagram Fig. 44, it is evident that the arm of the resisting couple is never as small as $d - \frac{1}{2}t$, and that the average unit compressive stress is never as small as $\frac{1}{2}f_c$, except when the neutral axis is at the top of the web. Using these limiting values as approximations for the true ones

$$M_s = A_sf_s(d - \tfrac{1}{2}t), \quad \text{or} \quad A_s = \frac{M}{(f_s)(d - \frac{1}{2}t)} \qquad (a)$$

$$f_s = \frac{M}{A_s(d - \frac{1}{2}t)} \qquad (b)$$

$$M_c = \tfrac{1}{2}f_cbt(d - \tfrac{1}{2}t) \qquad (c)$$

$$f_c = \frac{M}{\frac{1}{2}bt(d - \frac{1}{2}t)} \qquad (d)$$

The errors involved in these approximations are on the side of safety.

Where the web is very large compared to the flange, formulas which take into account the compression in the web may be used.

$$kd = \sqrt{\frac{2ndA_s + (b - b')t^2}{b'} + \left(\frac{nA_s + (b - b')t}{b'}\right)^2} - \frac{nA_s + (b - b')t}{b'}$$

$$z = \frac{b(kdt^2 - \frac{2}{3}t^3) + [(kd - t)^2(t + \frac{1}{3}(kd - t))]b'}{t(2kd - t)b + (kd - t)^2b'}$$

$$jd = d - z$$

$$f_s = \frac{M}{A_sjd}$$

$$f_c = \frac{2Mkd}{[(2kd - t)bt + (kd - t)^2b']jd}$$

Formula (1) gives the balancing ratio k when the limiting stresses f_s and f_c are known. Formula (3) gives the ratio k for any steel percentage when t and d are known. It will be a simple operation to find j after z is obtained from Formula (4), if k is known, otherwise j should be obtained from Formula (6).

For ordinary cases the tensile stress in the steel will control, and hence M_s should be used in Formula (10). In special cases M_c will be the governing factor.

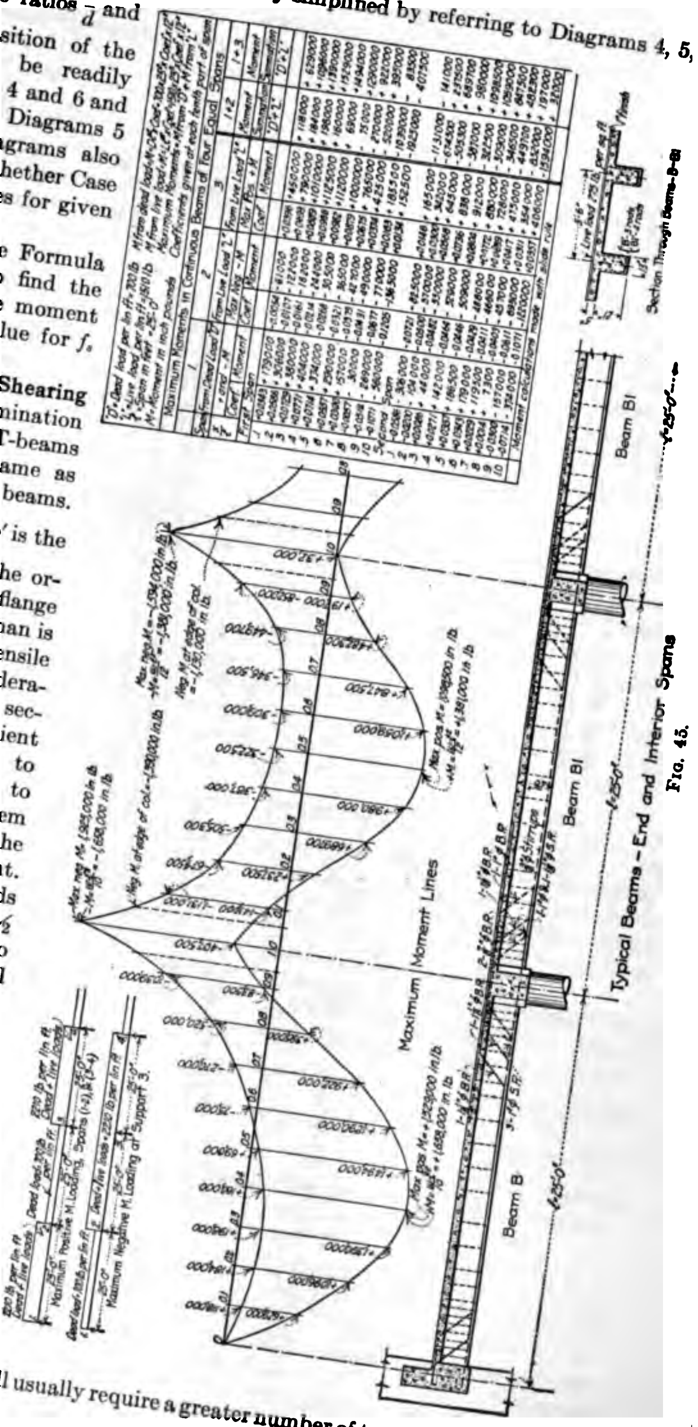
When $\frac{t}{d} = k$, the neutral axis will be at the junction of web and flange. When k is less than $\frac{t}{d}$, Case *I* applies, and when greater than $\frac{t}{d}$, Case *II* applies. For any combination of assigned values for f_s, f_c and n, it will be useful to obtain the "neutral" ratio k from Formula (1). This value of k being known, it can at once be determined whether M_s or M_c controls for any other value of k. In such a case M_s will control when any other ratio k is less than the neutral k, and M_c will control when any other k is greater than the neutral k.

Calculations for T-beams may be greatly simplified by referring to Diagrams 4, 5, p. 166. With the ratios $\frac{t}{d}$ and p known, the position of the neutral axis can be readily found in Diagrams 4 and 6 and the values of j in Diagrams 5 and 7. These diagrams also determine at once whether Case *I* or Case *II* applies for given conditions.

The approximate Formula (*a*) will be useful to find the steel area A, after the moment is found and unit value for f_s selected.

40*d*. Shearing Stresses.—The determination f shearing stresses in T-beams fundamentally the same as ven for rectangular beams. the formula $v = \frac{V}{b'jd}$. b' is the lth of the stem. In the or- ary T-beam design the flange rds greater strength than is ired to balance the tensile s, hence the first considera- should be to obtain a sec- that will give a sufficient nal area of concrete to shearing stresses and to a suitable width of stem proper spacing of the tudinal reinforcement. irrups and bent rods extend up to within 1½ from the top surface, to thorough mechanical bonding the slab and ther. As in the case of r beams, approximate shear and bond may d by assuming $j = \frac{7}{8}$.

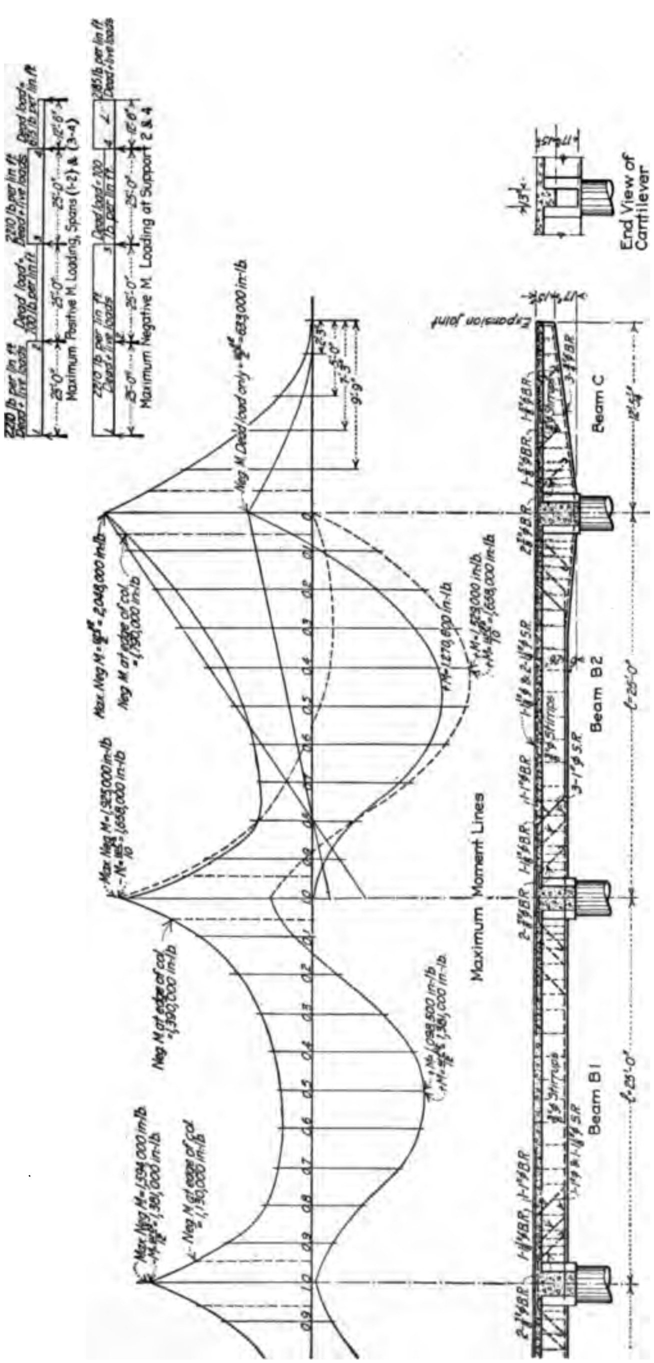
40*e*. Width of Depth.—In order for T-form to transmit web to flange, the m in proportion to d be chosen with onsidered good de- a width of web third to one-half eam. Large beams will usually require a greater number of tension rods, which will


Maximum Moments in Continuous Beams of Four Equal Spans
Maximum Moment Lines
Beam B
Beam B1
Section Through Beams B-B1
Maximum Positive M. Loading, Spans (1-2), & (3-4)
Maximum Negative M. Loading at Support 3.

Typical Beams—End and Interior Spans

FIG. 45.

control the width of stem to no little extent. The depth of T-beams is often limited on account of head room in buildings and frequently in extreme cases this depth may be as little as $\frac{1}{18}$th or $\frac{1}{20}$th of the span length. The design of such beams must be given special consideration, to develop rigidity and consistency in the strength of all contributing elements.


Maximum Positive M. Loading, Spans (1-2) & (3-4)
Maximum Negative M. Loading at Support 2 & 4
25'-0"
12'-6"
Expansion joint
Neg. M. Dead load only = 633,000 in-lb
Max. Neg. M = 2,048,000 in-lb
Neg. M at edge of col. = 1,780,000 in-lb
+M = 1,270,600 in-lb
Max. Neg. M = 1,925,000 in-lb
Neg. M at edge of col. = 1,390,000 in-lb
+M = 1,098,500 in-lb
Max. Neg. M = 1,534,000 in-lb
Neg. M at edge of col. = 1,130,000 in-lb
Maximum Moment Lines
Beam C
Beam B2
Beam B1
End View of Cantilever

Typical Beams Showing Cantilever at Expansion Joint

FIG. 46.

40f. Design of a Continuous T-beam at the Supports.—Figs. 45 and 46 illustrate the curve for negative moment, the maximum being over the center line of interior supports and decreases rather abruptly from this point. It is readily seen that this maximum point of negative moment is reached when the spans adjacent are fully loaded, producing bending in these members and consequently a pull in the top over the support. This tensile stress should have a counter balancing resistance in the bottom, and hence the compression in the bottom is equal in intensity to the corresponding negative moment in the top. A T-beam becomes a rectangular section at the supports on account of the reverse condition of bending, which changes from positive to negative at the zero point of inflection and varies in intensity to a maximum at the interior supports. The method of design clearly involves principles which govern the design of double-reinforced rectangular sections with the exception that the tensile and compressive stresses are reversed.

Negative moment at the center line of an interior support is generally greater than the corresponding stress at or near the center of span length, but with the presence of large columns or wide beams forming the supports, this negative bending is reduced appreciably at the face of bearings, which fact may be recognized in arriving at the proper proportion of stress for compression.

By reason of the general use of formulas $M = \frac{wl^2}{10}$ and $M = \frac{wl^2}{12}$ for both maximum positive and negative moments in continuous beams, one-half the steel required for positive stress from each adjoining member is usually bent up into the top over supports. This practice may be considered entirely applicable to the design of practical structures, when the consecutive spans are the same or nearly so, provided the compressive stress at or near the supports is proportioned for the same maximum assumed moment. When it is found advisable to reduce the compressive stress, this purpose may be accomplished either by adding a haunch to increase the effective depth and size of the section, or by the addition of compressive steel with effective anchorage; or by the use of the two methods in combination. For architectural reasons, beam haunches are often undesirable in hotels, apartments, office buildings and such structures, and for this reason occasion will often arise when additional strength for compression must be provided by adding compressive steel or by increasing the width or depth of the entire beam section for the sake of uniformity.

The bending up of steel bars at angles of 30 to 45 deg. to resist negative stresses is a question of importance. The points at which bends are made should be governed by the intensity of positive moment at the section. Figs. 45 and 46 show the maximum positive moment curve for an interior span when the member in question has its full live load with adjacent members not loaded. In this case, where the specified live load is 275 lb. per sq. ft., the positive moment approaches the supports. Diagram 8 shows with sufficient accuracy, the points at which bends may be made in continuous beams.

Bond stress along the horizontal tension rods in the top of continuous beams should be investigated. Formulas for tension rods at the ends of simply supported beams may be employed. These rods should extend to about the one-fourth point when small live loads are required and to the one-third point for heavy live loads.

To determine the maximum negative moment for continuous beams the formula $M = \frac{wl^2}{12}$ is generally recommended, but unfortunately is employed by many engineers more to determine the sectional area of steel in tension, than for the purpose of ascertaining a sufficient section for compression at the supports. It may be stated with more or less authority that the majority of designers neglect entirely the compressive stresses at the interior supports of continuous beams, which is a practice not to be recognized as commensurate with good design.

41. Comparing Accurate Moment Distribution in Continuous Beams with Ordinary Assumptions.—For the sake of simplicity in arriving at the moments in beams and slabs of reinforced concrete structures, it is now almost a universal practice to assume for members continuous over two supports, $M = \frac{wl^2}{12}$, and for members continuous over one support, or for end spans, $M = \frac{wl^2}{10}$. A practical illustration showing the relationship between the assumed conditions and the more accurate theory for determining the true moment distribution in continuous beams or slabs, should be a question of great significance to the designer. An intelligent understanding of positive and negative bending are vital considerations in the design of any continuous member, particularly when subject to heavy live loads, which influence to a marked degree the point of inflection or change from positive to negative bending.

In practice the true theorem of continuous moments cannot well be applied literally on account of practical complications that result in the arrangement of reinforcement, arising from the fact that the greatest positive moment in a continuous member is usually much less than the greatest negative moment. Literal adherence would require considerably more reinforcement over the supports than would be necessary at the center between supports. The disadvantages are quite obvious to the engineer accustomed to seeing his designs executed in the field. Again few building ordinances, if any, would permit of strict adherence to the exact theorem of moments, due no doubt to the variation in results from those obtained by the use of the established formulas ,$M = \frac{wl^2}{10}$ and $\frac{wl^2}{12}$. It may be understood from these standard moment assumptions that the general practice of resorting to the use of more complex methods of calculating moments, is not desirable in the solution of ordinary problems of design. However, this understanding should not prove the medium for evading the fundamental principles of continuity, so essential to the knowledge of the designer. A thorough understanding of continuous moments will not only familiarize the engineer with the maximum moment conditions resulting from the most unfavorable position of live loads, but will render a more intelligent and precise interpretation of the standard moment formulas established by practice.

Illustrative Problem.—The examples shown in Figs. 45 and 46 are selected from a number of beam calculations of a large structure completed in 1918. The coefficients given in the accompanying table are by Winkler and give the results of computations for a uniformly distributed load in the simplest form, from the ordinates of the maximum moment line for continuous beams. Beams B_1 continue for a large number of consecutive spans. The coefficients selected are for continuous beams of four spans. The loading required for maximum live load moments, Fig. 45, shows that the maximum positive moment is obtained for interior spans by loading alternate spans, and the maximum negative moment by loading the spans adjacent to the reaction in question. The moment lines are plotted from moment values in table for each point equal to one-tenth of the span. For comparative purposes moment values for $\frac{wl^2}{10}$ and $\frac{wl^2}{12}$ are given near maximum moment values obtained from coefficients.

It will be interesting to note that for interior spans the maximum positive moment is 1,098,500 in.-lb. whereas $M = \frac{wl^2}{12} = 1{,}381{,}000$ in.-lb. Keeping this latter moment value in mind it will be seen that the maximum negative moment at the first interior column face is 1,390,000 in.-lb. and at the second column, $M = 1{,}130{,}000$ in.-lb., which compares favorably with the moment value usually assumed. In the design of beams projected below, the tension rods for negative moment were not extended to meet fully the requirements of negative curve, for the reason that the sectional area of steel at the center of span was proportioned for $\frac{wl^2}{12}$ and not for the true moment which is about 21% less. This additional steel area reduces the unit stress in the steel and the deformation in the concrete in compression, which in combination serve to reduce the negative moment produced.

For the end span the maximum positive moment is 1,529,000 in.-lb., but $M = \frac{wl^2}{10} = 1{,}658{,}000$ in.-lb. The difference here is not so appreciable.

Fig. 46 includes the same members as shown in Fig. 45 with the exception that a cantilever beam is required for expansion joint. This cantilever beam changes the condition of moments in the adjacent span, as shown in moment diagram.

A close study of these examples will reveal many interesting stress conditions in continuous beams, and are given for the purpose of showing the relationship between the ordinary moment assumptions and the more accurate distribution of stress. An intimate knowledge of this relationship will be of inestimable value to any designer, and though not recommended for every day use, the knowledge of these conditions is fundamentally essential to the proper interpretation of the usual moment assumptions.

42. Designing Tables and Diagrams for Beams and Slabs.—It seems appropriate here to emphasize the importance of resorting to the use of tables and diagrams whenever it is possible to do so, since the tabulation of values in advance will minimize the time consumed in the preparation of designs. The measure of the time consumed in the development of a design, is a most essential factor in the determination of an engineer's worth and should not be subordinated to other conditions having a lesser value.

The engineer will often find it advantageous to adopt approximate formulas, and although the results obtained may vary slightly from those derived by the use of the more exact formulas recommended, it must be borne in mind that the divergence of practical conditions from the assumptions used in the formulas, does not justify too high a degree of mathematical precision in the design of practical structures, unless the particular problem in question demands such attention. The degree to which approximate formulas may be used will depend entirely upon the knowledge, training, initiative and experience of the engineer, which should be sufficient to justify a departure from the more accurate computations for shorter and simpler methods based on a clear conception of the fundamental principles embodied in theoretical design.

The number of designing tables and diagrams given on subsequent pages are necessarily

n account of the space allotted to this subject. The engineer will find it helpful to
ther tables of a similar character.

xplaining the solutions to problems, it is not the intention to advocate or recommend
f any particular combination of working stresses for f_s and f_c. Building ordinances in
ections of the United States show a great lack of consistency in the working stresses
for steel and concrete, which indicates that the differences of opinion prevailing at this
lude the immediate possibility of standardizing unit stresses to the entire approval of
ns concerned. The working values for f_s and f_c, now being used, vary from 500 to 800
. in., and in not a few instances even higher stresses for concrete are employed. The
king stresses in the steel vary from 16,000 to 20,000 lb. per sq. in., depending on
the steel is soft or hard grade. The many structures erected, judging from all avail-
rmation, have given a like degree of satisfaction, and in view of this fact it would
e consistent to condemn one practice or the other without some conclusive evidence
ld prove the custom to be a detriment to public safety and interests.

ative Problems.—The use of designing tables and diagrams can be explained to a greater advantage by
solutions of typical designing problems.

a beam of rectangular section to span 30 ft. Total uniformly distributed load is 1000 lb. per lin. ft.
mply supported. $f_s = 18{,}000$, $f_c = 750$, $n = 15$, $v_1 = 40$.

$$M = \frac{wl^2}{8} = \frac{(1000)(30)^2(12)}{8} = 1{,}350{,}000 \text{ in.-lb.}$$

e 3, for $n = 15$, $f_s = 18{,}000$, and $f_c = 750$

$$k = 0.385, \quad j = 0.872, \quad p = 0.00801, \quad K = 125.74.$$

$$K = \frac{M}{bd^2}, \text{ or } bd^2 = \frac{1{,}350{,}000}{125.74} = 10{,}730.$$

$b = 15$ in., then $d = 26\frac{3}{4}$ in. or say 27 in.

$$A_s = (0.00801)(15)(27) = 3.24 \text{ sq. in.}$$

ect three $\frac{7}{8}$-in. and two 1-in. rounds with total section of 3.38 sq. in.

$$v = \frac{15{,}000}{(15)(7/8)(27)} = 42 \text{ lb.}$$

42, provision for shear is unnecessary but for practical reasons it is advisable to use, say three $\frac{1}{4}$-in.
9 in. and two at 12 in. c. to c. at each end. All the tension steel is not needed near the supports so if
n. rounds are bent up at 45 deg. beginning at a point 2 ft. 6 in. from the supports, a better design will
ree $\frac{7}{8}$-in. rounds remain in the bottom to develop the safe bond stress.

$$\text{Bond stress } u = \frac{15{,}000}{(8.25)(\frac{7}{8})(27)} = 77 \text{ lb. per sq. in.}$$

s is within safe limits and will not require special anchorage.

alues K and p may be found from Diagram 2 where $n = 15$. Find the intersection of $f_s = 18{,}000$ and
750, and follow this point horizontally to the left or right hand margin where $K = 126$. Then follow
ction point to lower margin where $p = 0.0081$. The accuracy of these readings is sufficient for any
design.

ms 1 *and* 2.—These diagrams are very useful to find the relationship between any values for p, f_s, f_c,
any rectangular beam or solid concrete slab. For example (Diagram 2), if steel percentage $p = 0.0072$
niting steel stress is 16,000, the concrete stress f_c is found to be 625. If $f_c = 600$ and $p = 0.008$, f_s is
e about 14,300.

y rectangular beam of given section and reinforcement the safe load per linear foot may be readily
y means of these diagrams. Assume the steel percentage in the above problem to be $p = 0.007$. The
ing values for f_s, f_c and n prevail. Begin at lower margin of Diagram 2 at 0.7 % and follow vertically
tion with $f_c = 18{,}000$. From this intersection follow to left or right margin where $K = 110$ is found.

$$M = Kbd^2 = (110)(15)(27)^2 = 1{,}202{,}800 \text{ in.-lb.} = \frac{wl^2}{8}$$

$$w = \frac{8M}{(12)\ (l^2)} = \frac{(8)(1{,}202{,}800)}{(30)^2(12)} = 890 \text{ lb. per lin. ft.}$$

2.—Find the safe moment per 12-in. width for a 6-in. solid slab with $p = 0.006$, $d = 5$ in., $f_s = 20{,}000$,
$= 15$. Slab is freely supported.

$$p = 0.006, \quad k = 0.344, \quad j = 0.885.$$

$$bd^2 = \frac{M}{pf_sj}, \text{ or } M = bd^2pf_sj = (12)(5)^2(0.006)(20{,}000)(0.885)$$

$$M = 31{,}860 \text{ in.-lb.}$$

Table 3.—This table gives the values for k, j, p and K for balanced working stresses in rectangular beams and slabs, when f_s is 14,000, 16,000, 18,000 or 20,000, for various working stresses for f_c.

Tables 4 *and* 5.—These tables are for designing and estimating purposes. The area A_s per 12-in. width and net weight of steel per square foot are given for various spacings of merchantable bar sizes, which may be more readily obtained than odd sizes.

Tables 6, 7 *and* 8.—These slab tables have been prepared for balanced steel and concrete stresses when n = 15. Any thickness of slab from 3 to 10 in. and the reinforcement required may be obtained immediately for any given superimposed load per square foot. The distance from center of reinforcement to bottom surface is 1 in. in all cases and if a greater distance is required than this, ¼ or ½ in. may be added without effecting the effective depth d and table values may be adapted accordingly. All tables are prepared for $M = \frac{wl^2}{12}$ and may be adapted to $\frac{wl^2}{10}$ and $\frac{wl^2}{8}$ as per instructions given in tables.

Find thickness of slab and reinforcement required for a 12-ft. span when the superimposed load is 150 lb. per sq. ft.

$$M = \frac{wl^2}{12}, f_s = 16{,}000, f_c = 650, n = 15$$

In Table 6 find column for 12-ft. span and follow down to the 149 and then to left where a 6-in. slab is given, requiring ½-in. rounds, 5 in. c. to c., or a selection of other bar sizes with spacings as shown.

If the same example is assumed when $M = \frac{wl^2}{10}$, follow instructions given in Table 6. A 6½-in. slab with A_s = 0.508 has a superimposed load value of 189 lb. for a 12-ft. span. The dead load of this slab is 82 lb.

$$(189 + 82)(5/6) = 226 - 82 = 144 \text{ lb. per sq. ft. superimposed load.}$$

Table 9.—It is often necessary to retain the same thickness of slab for spans that vary within reasonable limits. This table gives the safe moment in inch pounds for slab thicknesses varying from 4 to 8½ in. with various steel percentages, for three combinations of allowable unit stresses, assuming n = 15.

For example, a 6-in. slab may be selected for moments varying from 20,070 to 33,510 in.-lb., when f_s = 16,000 and f_c = 650, or from 25,090 to 41,240 in.-lb. in case f_s = 20,000 and f_c = 800. It may be interesting to note that as the steel reaches its limit of safe working stress for any individual slab thickness, the increase in moment beyond this point is not very appreciable.

Table 10.—This table is for estimating purposes, and may also be employed to find the weight per linear foot of any beam size given. The instructions in table are self-explanatory.

Diagram 3.—The preparation of reinforced concrete shop drawings may be graelty facilitated by the use of this diagram to find the length of any bend which represents the hypothenuse of any triangle, when the length of two known sides are at right angles to one another. The diagram applies when bends are made at 30 deg., 45 deg. or any other angle.

For example, it is required to find the length of straight portion between the bends of a rod, when the vertical distance center to center of rod is 30 in. and the horizontal distance center to center of bends is 33 in. First find the designation 30 at the right hand margin and follow this line to the left until the vertical line from 33 on the lower margin intersects, then follow this point of intersection parallel to the nearest circular line to the lower margin where 44½ in. is read.

Diagrams 4, 5, 6 *and* 7.—Such diagrams are very useful in lessening the time consumed in the design of T-beams. When $\frac{t}{d}$ and p are known, either k or j may be found directly. With any given ratios for $\frac{t}{d}$ and p, or $\frac{t}{d}$ and k, it can at once be determined whether the neutral axis is in the flange or in the web.

Design the center cross section of a fully continuous beam of 20-ft. span to sustain a total load of 1500 lb. per lin. ft. f_s = 16,000, f_c = 650, n = 15. Maximum shear allowable v = 120. The slab having been previously designed, t = 5 in.

The first consideration in the design of a T-beam is to provide a sufficient section for shearing stresses and a width such that the bars can be properly spaced. The sectional area required for shear is

$$b'd = \frac{(1500)(10)}{(7/8)(120)} = 143 \text{ sq. in.}$$

$$M = \frac{(1500)(20)^2}{12}(12) = 600{,}000 \text{ in.-lb.}$$

If effective depth d = 16 in., then $b' = \frac{143}{16} = 9$ in.

Now the approximate steel area A_s may be obtained to find if the width b' = 9 in. is wide enough for the number of bars to be used.

$$A_s = \frac{600{,}000}{(0.87)(16)(16{,}000)} = 2.7 \text{ sq. in.}$$

This area will require say three ¾-in. rounds straight in the bottom and one ⅞-in. round and one 1-in. round bent in the top plane, or a total of 2.71 sq. in.

Assuming three diameters as the minimum distance center to center of the ¾-in. rods in the bottom, and a clear distance of 1½ in. from the sides, the minimum width b' is 8¼ in. Hence with the rods placed in two planes, the width 9 in. found above is satisfactory. The effective depth d = 16 in. will be measured from the top surface of slab or beam to the center between the two planes of rods in the bottom.

Now $\frac{t}{d} = \frac{5}{16} = 0.313$. Assuming a width of flange on either side of beam face equal to 4 times 5 or 20 in. which is within the allowable limit, the total width $b = 49$ in. The approximate percentage of steel is

$$p = \frac{2.71}{(49)(16)} = 0.00345$$

With these values for p and $\frac{t}{d}$, Diagram 6 determines at once that the neutral plane is in the flange, hence Case I applies (see Art. 40c).

$$K = \frac{M}{bd^2} = \frac{600{,}000}{(49)(16)^2} = 48$$

From Diagram 2 when $K = 48$ and $f_s = 16{,}000$, p is found to be 0.0033.

$$A_s = (0.0033)(49)(16) = 2.59 \text{ sq. in.}$$

The bar sizes selected above are sufficient and may be used. Since $p = 0.0033$ it is quite evident that the concrete stress is low, or from Diagram 2 not quite 400 lb. per sq. in. In this particular member it would not be necessary to investigate the compressive stress in concrete for positive moment unless the percentage p exceeded 0.00769 (Table 3), which is the controlling value for p when $f_s = 16{,}000$, $f_c = 650$ and $n = 15$.

Diagram 8.—To locate the points at which bends may be made in the bottom reinforcement of simple and continuous beams, consumes no little time, if a diagram showing these relationships is not available. To illustrate, assume a continuous beam has been designed for $M = \frac{wl^2}{12}$ and reinforced with three ¾-in. rounds straight in the bottom and two 1-in. rounds to be bent. It is desired to find the points at which rods may be bent. The total area of straight and bent rods is 2.89 sq. in. One 1-in. round bent rod represents 27 % of the total, and two 1-in. rounds 54 % of the total area. To find the point where one 1-in. round or 27 % of the steel may be bent up and leave sufficient area for positive moment, trace horizontally from the 27 % point at the right margin to the curve $M = \frac{wl^2}{12}$ and then vertically to the lower margin where $0.285l$ is read. By reading in the same manner two 1-in. rounds or 54 % of the steel may be bent up at $0.20l$.

TABLE 1.—AREAS, PERIMETERS AND WEIGHTS OF RODS

Round rods				Square rods		
Size (inches)	Area (square inches)	Perimeter (inches)	Weight per foot (pounds)	Area (square inches)	Perimeter (inches)	Weight per foot (pounds)
1/4	0.0491	0.7854	0.167	0.0625	1.00	0.212
5/16	0.0767	0.9817	0.261	0.0977	1.25	0.333
3/8	0.1104	1.1781	0.375	0.1406	1.50	0.478
7/16	0.1503	1.3744	0.511	0.1914	1.75	0.651
1/2	0.1963	1.5708	0.667	0.2500	2.00	0.850
9/16	0.2485	1.7671	0.845	0.3164	2.25	1.076
5/8	0.3068	1.9635	1.043	0.3906	2.50	1.328
11/16	0.3712	2.1598	1.262	0.4727	2.75	1.608
3/4	0.4418	2.3562	1.502	0.5625	3.00	1.913
13/16	0.5185	2.5525	1.763	0.6602	3.25	2.245
7/8	0.6013	2.7489	2.044	0.7656	3.50	2.603
15/16	0.6903	2.9452	2.347	0.8789	3.75	2.989
1	0.7854	3.1416	2.670	1.0000	4.00	3.400
1/16	0.8866	3.3379	3.014	1.1289	4.25	3.838
1/8	0.9940	3.5343	3.379	1.2656	4.50	4.303
3/16	1.1075	3.7306	3.766	1.4102	4.75	4.795
1/4	1.2272	3.9270	4.173	1.5625	5.00	5.312
5/16	1.3530	4.1233	4.600	1.7227	5.25	5.857
3/8	1.4849	4.3197	5.049	1.8906	5.50	6.428
7/16	1.6230	4.5160	5.518	2.0664	5.75	7.026
1/2	1.7671	4.7124	6.008	2.2500	6.00	7.650
9/16	1.9175	4.9087	6.520	2.4414	6.25	8.301
5/8	2.0739	5.1051	7.051	2.6406	6.50	8.978
11/16	2.2365	5.3014	7.604	2.8477	6.75	9.682
3/4	2.4053	5.4978	8.178	3.0625	7.00	10.410
13/16	2.5802	5.6941	8.773	3.2852	7.25	11.170
7/8	2.7612	5.8905	9.388	3.5156	7.50	11.950
15/16	2.9483	6.0868	10.020	3.7539	7.75	12.760
2	3.1416	6.2832	10.680	4.0000	8.00	13.600

TABLE 2.—VALUES OF *k* AND *j* FOR RECTANGULAR BEAMS AND SLABS

$k = \sqrt{2pn + (pn)^2} - pn$ $j = 1 - \frac{1}{3}k$

p	$n = 12$		$n = 15$		p	$n = 12$		$n = 15$	
	k	j	k	j		k	j	k	j
0.0010	0.145	0.952	0.158	0.947	0.0068	0.330	0.890	0.360	0.880
0.0012	0.155	0.948	0.169	0.944	0.0070	0.334	0.889	0.365	0.878
0.0014	0.166	0.945	0.181	0.940	0.0072	0.338	0.887	0.369	0.877
0.0016	0.177	0.941	0.192	0.936	0.0074	0.342	0.886	0.372	0.876
0.0018	0.186	0.938	0.202	0.933	0.0076	0.345	0.885	0.376	0.875
0.0020	0.196	0.935	0.217	0.928	0.0078	0.349	0.884	0.380	0.873
0.0022	0.201	0.932	0.222	0.926	0.0080	0.353	0.882	0.384	0.872
0.0024	0.212	0.929	0.231	0.923	0.0082	0.356	0.881	0.387	0.871
0.0026	0.220	0.927	0.240	0.920	0.0084	0.360	0.880	0.390	0.870
0.0028	0.227	0.924	0.248	0.917	0.0086	0.363	0.879	0.394	0.869
0.0030	0.235	0.922	0.258	0.914	0.0088	0.366	0.878	0.398	0.867
0.0032	0.241	0.920	0.263	0.912	0.0090	0.370	0.877	0.402	0.866
0.0034	0.248	0.917	0.271	0.910	0.0092	0.373	0.876	0.405	0.865
0.0036	0.254	0.915	0.277	0.908	0.0094	0.376	0.875	0.407	0.864
0.0038	0.260	0.913	0.284	0.905	0.0096	0.379	0.874	0.411	0.863
0.0040	0.266	0.911	0.292	0.903	0.0098	0.381	0.873	0.414	0.862
0.0042	0.270	0.910	0.297	0.901	0.0100	0.385	0.872	0.418	0.861
0.0044	0.276	0.908	0.303	0.899	0.0102	0.387	0.871	0.420	0.860
0.0046	0.281	0.906	0.309	0.897	0.0104	0.391	0.870	0.423	0.859
0.0048	0.286	0.904	0.315	0.895	0.0106	0.394	0.869	0.426	0.858
0.0050	0.291	0.903	0.320	0.893	0.0108	0.396	0.868	0.429	0.857
0.0052	0.295	0.901	0.324	0.892	0.0110	0.398	0.867	0.432	0.856
0.0054	0.300	0.900	0.329	0.891	0.0112	0.402	0.866	0.434	0.855
0.0056	0.304	0.899	0.333	0.889	0.0114	0.404	0.865	0.437	0.854
0.0058	0.309	0.897	0.337	0.888	0.0116	0.407	0.864	0.440	0.853
0.0060	0.314	0.895	0.344	0.885	0.0118	0.410	0.863	0.443	0.852
0.0062	0.317	0.894	0.348	0.884	0.0120	0.412	0.863	0.446	0.851
0.0064	0.322	0.893	0.352	0.883	0.0122	0.415	0.862	0.448	0.851
0.0066	0.325	0.892	0.356	0.881	0.0124	0.417	0.861	0.451	0.850

TABLE 3.—USE FOR RECTANGULAR BEAMS AND SLABS

$p = \dfrac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)}$ $k = \dfrac{1}{1+\frac{f_s}{nf_c}}$ $j = 1 - \dfrac{k}{3}$ $K = pf_sj \text{ or } \dfrac{f_ckj}{2}$

f_s	f	k	j	p	K	f_s	f_c	k	j	p	K
Ratio of Moduli $n = 12$											
14,000	500	0.300	0.900	0.00536	67.54	18,000	500	0.250	0.917	0.00347	57.30
	550	0.320	0.893	0.00630	78.58		550	0.268	0.911	0.00410	67.19
	600	0.340	0.887	0.00728	90.48		600	0.286	0.905	0.00476	77.56
	650	0.358	0.881	0.00831	102.46		650	0.302	0.899	0.00546	88.36
	700	0.375	0.875	0.00937	114.78		700	0.318	0.894	0.00619	99.55
	750	0.391	0.870	0.01048	127.59		750	0.333	0.889	0.00694	111.11
	800	0.407	0.864	0.01162	140.62		800	0.348	0.884	0.00773	123.00
16,000	500	0.273	0.909	0.00426	61.98	20,000	500	0.231	0.923	0.00289	53.26
	550	0.292	0.903	0.00502	72.50		550	0.248	0.917	0.00341	62.60
	600	0.310	0.897	0.00582	83.49		600	0.265	0.912	0.00397	72.50
	650	0.328	0.891	0.00666	94.88		650	0.281	0.907	0.00456	82.67
	700	0.344	0.885	0.00753	106.66		700	0.296	0.901	0.00518	93.31
	750	0.360	0.880	0.00844	118.79		750	0.310	0.897	0.00582	104.35
	800	0.375	0.875	0.00938	131.25		800	0.324	0.892	0.00649	115.60
Ratio of Moduli $n = 15$											
14,000	500	0.349	0.884	0.00623	77.06	18,000	500	0.294	0.90	0.00409	66.32
	550	0.371	0.876	0.00728	89.36		550	0.314	0.895	0.00480	77.38
	600	0.391	0.870	0.00839	102.08		600	0.333	0.889	0.00556	88.90
	650	0.411	0.863	0.00953	115.17		650	0.351	0.883	0.00634	100.82
	700	0.429	0.857	0.01071	128.56		700	0.368	0.877	0.00716	113.12
	750	0.446	0.852	0.01193	142.22		750	0.385	0.872	0.00801	125.74
	800	0.462	0.846	0.01319	156.26		800	0.400	0.867	0.00889	138.67
16,000	500	0.319	0.894	0.00499	71.30	20,000	500	0.273	0.909	0.00341	61.98
	550	0.340	0.887	0.00585	82.94		550	0.292	0.903	0.00402	72.51
	600	0.360	0.880	0.00680	95.04		600	0.310	0.897	0.00466	83.47
	650	0.379	0.874	0.00769	107.65		650	0.323	0.891	0.00533	94.89
	700	0.396	0.868	0.00867	120.37		700	0.344	0.885	0.00603	106.57
	750	0.413	0.862	0.00968	133.51		750	0.360	0.880	0.00675	118.80
	800	0.429	0.857	0.01071	146.87		800	0.375	0.875	0.00750	131.25

TABLE 4.—SPACING OF ROUND RODS IN SLABS

Spacing center to center (inches)	Sectional area of steel per foot of slab and weight per sq. ft. of slab, when spaced as follows																	
	¼"		⅜"		½"		⅝"		¾"		⅞"		1"		1⅛"		1¼"	
	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.
2	0.29	0.99	0.66	2.24	1.18	4.01	1.84	6.26	2.65	9.01	3.61	12.27	4.71	16.01	5.96	20.26	7.36	25.02
2½	0.24	0.78	0.53	1.80	0.94	3.20	1.47	5.00	2.12	7.21	2.88	9.79	3.77	12.82	4.77	16.22	5.89	20.03
3	0.20	0.68	0.44	1.50	0.78	2.65	1.23	4.18	1.77	6.02	2.40	8.16	3.14	10.68	3.98	13.53	4.91	16.69
3½	0.17	0.58	0.38	1.29	0.67	2.28	1.05	3.57	1.51	5.13	2.06	7.00	2.69	9.15	3.41	11.59	4.21	14.31
4	0.15	0.51	0.33	1.12	0.59	2.00	0.92	3.13	1.33	4.49	1.80	6.12	2.36	8.02	2.98	10.13	3.68	12.51
4½	0.13	0.44	0.29	0.99	0.52	1.77	0.82	2.79	1.18	4.01	1.60	5.44	2.09	7.11	2.65	9.01	3.27	11.12
5	0.12	0.41	0.26	0.88	0.47	1.60	0.74	2.52	1.06	3.60	1.44	4.90	1.88	6.39	2.39	8.13	2.95	10.03
5½	0.11	0.37	0.24	0.82	0.43	1.46	0.67	2.28	0.96	3.26	1.31	4.45	1.71	5.81	2.17	7.38	2.68	9.11
6	0.10	0.34	0.22	0.75	0.39	1.33	0.61	2.07	0.88	2.99	1.20	4.08	1.57	5.34	1.99	6.77	2.45	8.33
6½	0.09	0.31	0.20	0.68	0.36	1.22	0.57	1.94	0.82	2.79	1.11	3.77	1.45	4.93	1.84	6.26	2.27	7.72
7	0.08	0.27	0.19	0.65	0.34	1.16	0.53	1.80	0.76	2.58	1.03	3.50	1.35	4.59	1.70	5.78	2.10	7.14
7½	.....	....	0.18	0.61	0.31	1.05	0.49	1.67	0.71	2.41	0.96	3.26	1.26	4.28	1.59	5.41	1.96	6.66
8	.....	....	0.17	0.58	0.29	0.99	0.46	1.56	0.66	2.24	0.90	3.06	1.18	4.01	1.49	5.07	1.84	6.26
8½	.....	....	0.16	0.54	0.28	0.95	0.43	1.46	0.62	2.11	0.85	2.89	1.11	3.77	1.40	4.76	1.73	5.88
9	.....	....	0.15	0.51	0.26	0.88	0.41	1.39	0.59	2.01	0.80	2.72	1.05	3.57	1.33	4.52	1.64	5.58
9½	.....	....	0.14	0.48	0.25	0.85	0.39	1.33	0.56	1.90	0.76	2.58	0.99	3.37	1.26	4.28	1.55	5.27
10	.....	....	0.13	0.44	0.24	0.82	0.37	1.26	0.53	1.80	0.72	2.45	0.94	3.20	1.19	4.05	1.47	5.00
11	.....	....	0.12	0.41	0.21	0.71	0.33	1.12	0.48	1.63	0.66	2.24	0.86	2.92	1.08	3.67	1.34	4.56
12	.....	....	0.11	0.37	0.20	0.68	0.31	1.06	0.44	1.50	0.60	2.04	0.78	2.65	0.99	3.37	1.23	4.18
Net sect....	0.0491	....	0.1104	....	0.1963	....	0.3068	....	0.4418	....	0.6013	.....	0.7854	....	0.9940	.....	1.2272	

TABLE 5.—SPACING OF SQUARE RODS IN SLABS

Spacing center to center (inches)	Sectional area of steel per foot of slab and weight per sq. ft. of slab, when spaced as follows																	
	¼"		⅜"		½"		⅝"		¾"		⅞"		1"		1⅛"		1¼"	
	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.	A_s	Wt.
2	0.37	1.26	0.84	2.86	1.50	5.1	2.34	7.96	3.37	11.46	4.59	15.61	6.00	20.40	7.59	25.81	9.37	31.86
2½	0.30	1.02	0.67	2.28	1.20	4.08	1.87	6.36	2.70	9.18	3.67	12.48	4.80	16.32	6.08	20.67	7.50	25.50
3	0.25	0.85	0.56	1.90	1.00	3.4	1.56	5.30	2.25	7.65	3.06	10.40	4.00	13.60	5.06	17.20	6.25	21.25
3½	0.21	0.71	0.48	1.63	0.86	2.92	1.34	4.56	1.93	6.56	2.62	8.91	3.43	11.66	4.34	14.76	5.36	18.22
4	0.19	0.65	0.42	1.43	0.75	2.55	1.17	3.98	1.69	5.75	2.30	7.82	3.00	10.20	3.80	12.92	4.69	15.95
4½	0.17	0.58	0.37	1.26	0.67	2.28	1.04	3.54	1.50	5.10	2.04	6.94	2.67	9.08	3.37	11.46	4.17	14.18
5	0.15	0.51	0.34	1.16	0.60	2.04	0.94	3.20	1.35	4.59	1.84	6.26	2.40	8.16	3.04	10.34	3.75	12.75
5½	0.13	0.44	0.31	1.05	0.55	1.87	0.85	2.89	1.23	4.18	1.67	5.68	2.18	7.41	2.76	9.38	3.41	11.59
6	0.12	0.41	0.28	0.95	0.50	1.70	0.78	2.65	1.12	3.81	1.53	5.20	2.00	6.80	2.53	8.60	3.12	10.61
6½	0.12	0.41	0.26	0.88	0.46	1.56	0.72	2.45	1.04	3.54	1.42	4.83	1.86	6.32	2.34	7.96	2.88	9.79
7	0.11	0.37	0.24	0.82	0.43	1.46	0.67	2.28	0.96	3.26	1.31	4.45	1.71	5.81	2.17	7.38	2.68	9.11
7½	0.10	0.34	0.22	0.75	0.40	1.36	0.63	2.14	0.90	3.06	1.22	4.15	1.60	5.44	2.02	6.87	2.50	8.50
8	0.09	0.31	0.21	0.71	0.37	1.26	0.59	2.01	0.84	2.86	1.15	3.91	1.50	5.10	1.89	6.43	2.34	7.96
8½	0.09	0.31	0.20	0.68	0.35	1.19	0.55	1.87	0.80	2.72	1.08	3.67	1.42	4.83	1.79	6.09	2.20	7.48
9	0.08	0.27	0.19	0.65	0.33	1.12	0.52	1.77	0.75	2.55	1.02	3.47	1.33	4.52	1.69	5.75	2.08	7.07
9½	0.08	0.27	0.18	0.61	0.32	1.09	0.49	1.67	0.71	2.41	0.97	3.30	1.26	4.28	1.60	5.44	1.97	6.70
10	0.07	0.24	0.17	0.58	0.30	1.02	0.47	1.60	0.67	2.28	0.92	3.13	1.20	4.08	1.52	5.17	1.87	6.36
11	0.07	0.24	0.15	0.51	0.27	0.92	0.43	1.46	0.61	2.07	0.84	2.86	1.09	3.71	1.38	4.69	1.70	5.78
12	0.06	0.20	0.14	0.48	0.25	0.85	0.39	1.33	0.56	1.90	0.77	2.62	1.00	3.40	1.27	4.32	1.56	5.30
Net sect....	0.0625	....	0.1406	....	0.2500	....	0.3906	....	0.5625	.....	0.7656	.....	1.000	......	1.2656	.....	1.5625	

DIAGRAM 1.

USE FOR RECTANGULAR BEAMS AND SLABS.

Based on $n = 12$.

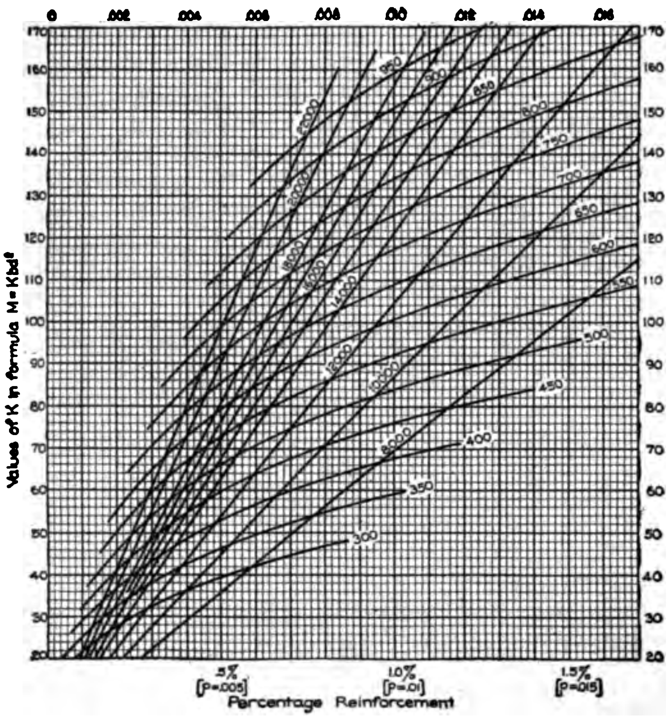


DIAGRAM 2.

USE FOR RECTANGULAR BEAMS AND SLABS.

Based on $n = 15$.

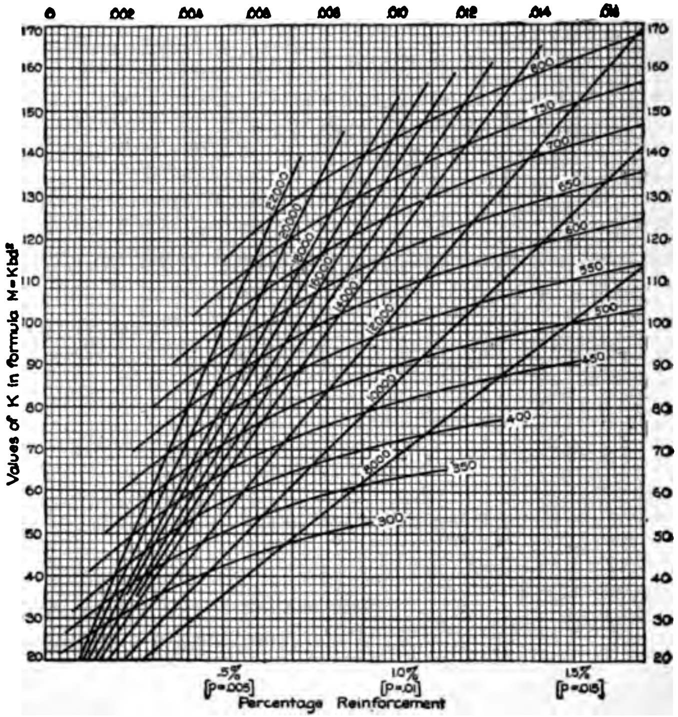


TABLE 6

SOLID CONCRETE SLABS

1-2-4 MIX

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT

Continuous Spans. Unit Stress Steel = 16,000 lb. per sq. in., Medium Steel

Extreme Fiber Stress Concrete = 650 lb. per sq. in.

$n = 15$

$w = \frac{M}{C} - D$

w = Safe superimposed loads in lb. per sq. ft.

M = moment in inch-pounds.

D = dead weight of slab per sq. ft.

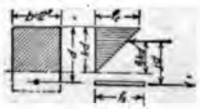
l = span in feet.

$C = \frac{l^2}{12} \times 12$

Min. elastic limit = 33,000 lb. per sq. in.

Min. ult. strength = 55,000 lb. per sq. in.

Moment in inch-pounds	Thickness of slab (inches)	Area of steel per 12 in. of width at 16,000 lb. per sq. in.	Corresponding size and spacing of round and square bars (inches)				M (in.-lb.) $= \frac{wl^2}{12} \times 12$																	Weight of slab per sq. ft.	Effective depth (in.)	Position of neutral axis (kd)
			Rounds		Squares		Span in feet																			
			ϕ	c.c.	ϕ	c.c.	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
5,160	3	0.185	¼ ⅜	3¼ 7	¼ ⅜	4 9	285	168	105	68	42	Values for p, k, and K when $f_c = 650$ lb. per sq. in. $f_s = 16{,}000$ lb. per sq. in. $n = 15$												38	2	0.757
8,060	3½	0.231	⅜	5¾	¼ ⅜	3¼ 7¼	460	279	180	121	82	56	36						$p = \frac{½}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = 0.00769$					44	2½	0.947
11,610	4	0.277	⅜ ½	4¾ 8½	⅜	6	676	414	273	187	131	93	66	46	30				$k = \sqrt{2pn + (pn)^2} - pn = 0.3786$					50	3	1.136
15,800	4½	0.323	⅜ ½	4 7¼	⅜ ½	5¼ 9¼	931	575	382	266	190	138	101	73	53	36			$k = 0.3786$ when $p = 0.00769$ and $n = 15$					57	3½	1.325
18,140	4¾	0.346	½	6¾	⅜ ½	4¾ 8½	1073	666	444	310	223	164	121	90	66	47	33		$K = \frac{M}{bd^2} = f_sp\left(1 - \frac{k}{3}\right) = 107.51$ by steel					60	3¾	1.420
20,610	5	0.369	½ ⅝	6½ 10	⅜ ½	4½ 8		763	510	358	260	192	143	107	80	59	43	29	$K = \frac{M}{bd^2} = ½f_ck\left(1 - \frac{k}{3}\right) = 107.51$ by concrete					63	4	1.514
23,300	5¼	0.392	½ ⅝	6 9½	⅜ ½	4¼ 7¾		866	581	410	298	221	167	129	96	72	53	37						66	4¼	1.609
26,120	5½	0.415	½ ⅝	5¾ 8¾	⅜ ½	4 7¼		976	657	464	339	254	192	147	112	86	64	47	33					69	4½	1.704
29,110	5¾	0.438	½ ⅝	5¼ 8½	½ ⅝	6¾ 10¾			736	522	383	288	219	169	130	100	76	57	42	29				72	4¾	1.798
32,250	6	0.461	½ ⅝	5 8	½ ⅝	6½ 10			821	583	429	322	248	191	149	116	90	68	51	36				75	5	1.893
35,560	6¼	0.484	½ ⅝	4¾ 7½	½ ⅝	6¼ 9¾			910	648	477	361	277	216	169	132	103	80	61	45	32			78	5¼	1.988
39,030	6½	0.508	⅝ ¾	7¼ 10½	½ ⅝	6 9¼			1002	714	528	400	308	241	189	149	117	92	70	53	38			82	5½	2.082



(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
42,650	6¾	0.531	⅝ ¾	7 10	½ ⅝	5½ 8¾				786	582	442	341	267	211	167	133	104	81	62	47	33		85	5¾	2.177
46,440	7	0.554	⅝ ¾	6½ 9½	½ ⅝	5½ 8½				860	638	485	376	296	235	187	149	118	93	73	55	41	28	88	6	2.272
54,510	7½	0.600	⅝ ¾	6 8¾	½ ⅝	5 7¾				1018	758	570	451	356	284	229	184	148	119	94	74	57	42	94	6½	2.461
63,220	8	0.646	⅝ ¾	5¾ 8¼	⅝ ¾	7¼ 10½					888	681	532	423	339	274	223	181	147	119	95	75	58	100	7	2.650
72,570	8½	0.692	⅝ ¾	5¼ 7½	⅝ ¾	6¾ 9¾					1026	789	619	493	397	322	263	216	176	144	117	94	74	107	7½	2.840
82,570	9	0.738	⅝ ¾	5 7¼	⅝ ¾	6½ 9¼						906	713	569	460	375	308	254	210	173	142	116	93	113	8	3.029
93,210	9½	0.784	¾ ⅞	6¾ 9¼	⅝ ¾	6 8½						1032	813	651	528	432	357	295	245	204	168	139	114	119	8½	3.218
104,500	10	0.831	¾ ⅞	6¼ 8¾	¾ ⅞	8 11							920	738	601	493	408	339	283	236	198	164	136	125	9	3.407
$\frac{l^2}{12} \times 12 = C =$							16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400			

NOTE.—This table is based on $M = \frac{wl^2}{12}$. Top reinforcement for negative M same area A_s as for positive M at center of span, top steel over supports extending to ¼ or ⅓ of span. For end spans, when $M = \frac{wl^2}{10}$, use ⁶⁄₅ of the combined superimposed and dead weight of floor given. For simple spans, when $M = \frac{wl^2}{8}$, use ³⁄₂ of the combined table values as for end spans.

TABLE 7

SOLID CONCRETE SLABS

1–2–4 MIX

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT

Continuous Spans. Unit Stress Steel = 18,000 lb. per Sq. In., Hard Grade Steel

Extreme Fiber Stress Concrete = 750 Lb. per Sq. In.

$n = 15$

$w = \frac{M}{C} - D$

w = safe superimposed loads in lb. per sq. ft.

M = moment in inch-pounds.

D = dead weight of slab per sq. ft.

l = span in feet.

$C = \frac{l^2}{12} \times 12.$

Min. elastic limit = 50,000 lb. per sq. in.

Min. ult. strength = 75,000 lb. per sq. in.

Moment in inch-pounds	Thickness of slab (inches)	Area of steel per 12 in. of width at 18,000 lb. per sq. in.	Corresponding size and spacing of round and square bars (inches)				M (in.-lb.) $= \frac{wl^2}{12} \times 12$																	Weight of slab per sq. ft.	Effective depth (in.)	Position of neutral axis (kp)
			Rounds		Squares		Span in feet																			
			ϕ	c.c.	ϕ	c.c.	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
6,030	3	0.192	1/4 3/8	3 7	1/4 3/8	4 8 3/4	339	203	130	86	56	37												38	2	0.77
9,430	3 1/2	0.240	3/8	5 1/2	1/4 3/8	3 7	545	333	218	149	104	73	50	34										44	2 1/2	0.962
13,580	4	0.288	3/8 1/2	4 1/2 8 1/4	3/8 1/2	5 3/4 10 1/2	799	493	327	227	162	118	86	62	44									50	3	1.154
18,450	4 1/2	0.337	3/8 1/2	4 7	3/8 1/2	5 9	1098	682	456	320	231	171	128	95	71	52	37							57	3 1/2	1.346
21,210	4 3/4	0.361	1/2	6 1/2	3/8 1/2	4 1/2 8 1/4	1268	789	529	373	271	202	152	115	88	65	48	34						60	3 3/4	1.442
24,140	5	0.385	1/2 5/8	6 9 1/2	3/8 1/2	4 1/4 7 3/4		903	608	429	314	235	178	136	105	80	61	44						63	4	1.538
27,250	5 1/4	0.409	1/2 5/8	5 3/4 9	1/2 5/8	7 1/4 11 1/2		1024	691	491	359	270	206	159	124	95	73	55	40					66	4 1/4	1.635
30,550	5 1/2	0.433	1/2 5/8	5 1/2 8 1/2	1/2 5/8	7 10 3/4		1153	780	555	408	308	237	184	143	112	87	67	51	36				69	4 1/2	1.731
34,040	5 3/4	0.457	1/2 5/8	5 1/4 8	1/2 5/8	6 1/2 10 1/4			874	623	460	349	268	210	165	129	101	79	61	45	33			72	4 3/4	1.827
37,720	6	0.481	1/2 5/8	5 7 1/2	1/2 5/8	6 1/4 9 3/4			973	695	514	391	302	236	187	148	118	93	73	55	42			75	5	1.923
41,580	6 1/4	0.505	1/2 5/8	4 3/4 7 1/4	1/2 5/8	6 9 1/4			1077	771	572	435	338	266	210	168	134	107	85	66	50	37		78	5 1/4	2.019
45,640	6 1/2	0.529	5/8 3/4	7 10	1/2 5/8	5 3/4 9			1185	849	631	481	374	295	235	188	151	121	96	76	59	44		82	5 1/2	2.115

Values for p, k, & K when $f_c = 750$ lb. per sq. in. $f_s = 18{,}000$ lb. per sq. in. $n = 15$

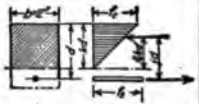
$$p = \frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = 0.0080138$$

$$k = \sqrt{2pn + (pn)^2} - pn = 0.3846$$

$k = 0.385$ when $p = 0.008013$ and $n = 15$

$$K = \frac{M}{bd^2} = f_s p\left(1 - \frac{k}{3}\right) = 125.74 \text{ by steel}$$

$$K = \frac{M}{bd^2} = \tfrac{1}{2} f_c k\left(1 - \frac{k}{3}\right) = 125.74 \text{ by concrete}$$



(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
49,880	6¾	0.553	⅝ ¾	6½ 9½	½ ⅝	5½ 8½				933	695	531	413	327	262	210	170	136	109	87	69	53	40	85	5¾	2.211
54,320	7	0.577	⅝ ¾	6½ 9¼	½ ⅝	5¼ 8				1020	761	583	455	361	289	234	189	153	124	100	80	62	48	88	6	2.308
63,750	7½	0.625	⅝ ¾	6 8½	⅝ ¾	7½ 10¾				1207	902	693	543	433	340	283	232	190	155	126	102	82	65	94	6½	2.500
73,930	8	0.673	⅝ ¾	5½ 7¾	⅝ ¾	7 10					1055	813	640	511	413	338	277	229	188	156	128	105	85	100	7	2.692
84,870	8½	0.721	⅝ ¾	5 7¼	⅝ ¾	6½ 9¼					1219	941	742	594	482	395	326	270	224	187	154	128	105	107	7½	2.885
96,570	9	0.769	¾ ⅞	7 9½	⅝ ¾	6 8¾						1079	853	685	558	458	379	316	264	221	185	154	128	113	8	3.077
109,010	9½	0.817	¾ ⅞	6½ 8¾	¾ ⅞	8¼ 11¼						1227	971	782	638	526	438	366	307	258	217	183	153	119	8½	3.269
122,220	10	0.865	¾ ⅞	6 8¼	¾ ⅞	7¾ 10½							1097	885	724	598	499	418	352	297	252	213	181	125	9	3.461
$\frac{l^2}{12} \times 12 = C =$							16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400			

NOTE.—This table is based on $M = \frac{wl^2}{12}$. Top reinforcement for negative M same area A_s as for positive M at center of span, top steel over supports extending to ¼ or ⅓ of span. For end spans, when $M = \frac{wl^2}{10}$, use ⅚ of the combined superimposed and dead weight of floor given. For simple spans, when $M = \frac{wl^2}{8}$, use ⅚ of the combined table values as for end spans.

TABLE 8

SOLID CONCRETE SLABS

1-2-4 MIX

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT

Continuous Spans. Unit Stress Steel = 20,000 lb. per Sq. In., Hard Grade Steel

Extreme Fiber Stress Concrete = 800 Lb. per Sq. In.

$n = 15$

$w = \frac{M}{C} - D$

w = safe superimposed loads in lb. per sq. ft.

M = moment in inch-pounds

D = dead weight of slab per sq. ft.

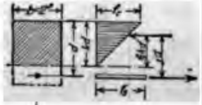
l = span in feet.

$C = \frac{l^2}{12} \times 12$

Min. elastic limit = 50,000 lb. per sq. in.

Min. ult. strength = 75,000 lb. per sq. in.

Moment in inch-pounds	Thickness of slab (inches)	Area of steel per 12 in. of width at 20,000 lb. per sq. in.	Corresponding size and spacing of round and square bars (inches)				M (in.-lb.) $= \frac{wl^2}{12} \times 12$																	Weight slab per sq. ft.	Effective depth (in.)	Position of neutral axis (kd)
			Rounds		Squares		Span in feet																			
			φ	c.c.	φ	c.c.	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	)24)	(25)	(26)	(27)
6,300	3	0.180	¼ ⅜	3¼ 7¼	¼ ⅜	4 9¼	356	214	137	91	60	40	Values for p, k, & K when f_c = 800 lb. per sq. in. f_s = 20,000 lb. per sq. in. $n = 15$											38	2	0.75
9,844	3½	0.225	⅜	5¾	¼ ⅜	3¼ 7½	571	350	229	157	110	78	54	37					$p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = 0.0075$					44	2½	0.938
14,175	4	0.270	⅜ ½	4¾ 8¾	⅜	6¼	836	517	344	239	171	125	92	67	48				$k = \sqrt{2pn + (pn)^2} - pn = 0.375$					50	3	1.125
19,294	4½	0.315	⅜ ½	4 7½	⅜ ½	5¼ 9½	1149	715	479	337	244	181	136	102	77	57	41		$k = 0.375$ when $p = 0.0075$ and $n = 15$					57	3½	1.313
22,145	4¾	0.338	½	7	⅜ ½	5 8¾	1324	826	555	392	286	213	161	123	94	71	53	38	$K = \frac{M}{bd^2} = f_s p\left(1 - \frac{k}{3}\right) = 131.25$ by steel					60	3¾	1.406
25,200	5	0.300	½ ⅝	6½ 10	⅜ ½	4½ 8¼		945	637	451	331	248	189	145	112	86	66	49	$K = \frac{M}{bd^2} = \frac{1}{2} f_c k\left(1 - \frac{k}{3}\right) = 131.25$ by concrete					63	4.	1.50
28,445	5¼	0.383	½ ⅝	6 9½	⅜ ½	4¼ 7¾		1072	724	515	378	285	218	169	132	102	79	60	45					66	4¼	1.594
31,894	5½	0.405	½ ⅝	5¾ 9	⅜ ½	4 7¼		1207	817	582	429	325	250	195	152	120	94	73	56	41				69	4½	1.688
35,535	5¾	0.428	½ ⅝	5½ 8½	½ ⅝	7 11			915	653	483	367	283	222	175	138	109	86	67	51	38			72	4¾	1.781
39,375	6	0.450	½ ⅝	5¼ 8	½ ⅝	6½ 10¼			1019	729	540	411	319	250	198	158	126	100	79	61	47			75	5	1.875
43,407	6¼	0.473	½ ⅝	5 7¾	½ ⅝	6¼ 9¾			1128	808	600	458	356	281	223	179	143	115	92	72	56	42		78	5¼	1.969
47,644	6½	0.495	½ ⅝	4¾ 7¼	½ ⅝	6 9½			1241	890	662	506	394	312	249	200	161	130	104	83	65	50		82	5½	2.062



(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
52,070	6¾	0.518	⅝ ¾	7 10¼	½ ⅝	5¾ 9				978	729	558	435	345	277	223	181	146	118	95	76	59	45	85	5¾	2.156
56,700	7	0.540	⅝ ¾	6¾ 9¾	½ ⅝	5½ 8½				1069	798	612	479	381	306	248	201	164	133	108	87	69	54	88	6	2.25
66,544	7½	0.585	⅝ ¾	6¼ 9	½ ⅝	5 8				1264	946	728	571	456	368	300	246	202	166	136	111	90	72	94	6½	2.438
77,175	8	0.630	⅝ ¾	5¾ 8¼	⅝ ¾	7¼ 10¾					1106	853	672	538	436	357	294	243	201	167	138	114	93	100	7	2.625
88,594	8½	0.675	⅝ ¾	5¼ 7¾	⅝ ¾	6¾ 10					1277	987	779	625	508	417	345	287	239	200	166	138	114	107	7½	2.813
100,800	9	0.720	⅝ ¾	5 7¼	⅝ ¾	6½ 9¼						1131	895	720	587	483	401	335	281	236	198	166	139	113	8	3.00
113,794	9½	0.765	¾ ⅞	6¾ 9¼	⅝ ¾	6 8¾						1286	1019	821	671	554	462	387	326	275	232	196	165	119	8½	3.188
127,575	10	0.810	¾ ⅞	6½ 8¾	¾ ⅞	8¼ 11¼							1151	929	761	630	526	442	373	316	269	228	194	125	9	3.375
$\frac{l^2}{12} \times 12 = C =$							16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400			

NOTE.—This table is based on $M = \frac{wl^2}{12}$. Top reinforcement for negative M same area A_s as for positive M at center of span, top steel over supports extending to ¼ or ⅕ of span. For end spans, when $M = \frac{wl^2}{10}$, use ⅚ of the combined superimposed and dead weight of floor given. For simple spans, when $M = \frac{wl^2}{8}$, use ⅔ of the combined table values as for end spans.

TABLE 9.—STRENGTH OF SOLID SLABS

For Various Percentages of Steel when (f_s = 16,000, f_c = 650), (f_s = 18,000, f_c = 750) and (f_s = 20,000, f_c = 800)

Ratio n = 15

Above heavy line M_s controls. Below heavy line M_c controls.

Slab thickness (inches)	Effective depth (inches)	Reinforcement (inches)		Sect'l area A_s (12 in. wide)	bd (sq. in.)	p	k	j	Moment (inch-pounds)		
		Round	Square						f_s = 16,000 f_c = 650 n = 15	f_s = 18,000 f_c = 750 n = 15	f_s = 20,000 f_c = 800 n = 15
		(Centers)	(Centers)								
4	3	½-10½ ⅜- 6	⅜- 7¾	0.22	36	0.0061	0.346	0.885	9,350	10,510	11,690
4	3	½-10 ⅜- 5½	⅜- 7	0.24	36	0.0067	0.358	0.881	10,150	11,420	12,690
4	3	½- 9½ ⅜- 5¼	⅜- 6¾ ½-12	0.25	36	0.0069	0.363	0.879	10,550	11,860	13,190
4	3	½- 9 ⅜- 5	⅜- 6½ ½-11½	0.26	36	0.0072	0.369	0.877	10,950	12,310	13,680
4	3	½- 8½ ⅜- 4¾	⅜- 6 ½-10¾	0.28	36	0.0078	0.380	0.873	11,650	13,200	14,330
4	3	½- 8 ⅜- 4½	⅜- 5¾ ½-10¼	0.29	36	0.0081	0.385	0.872	11,780	13,600	14,500
4	3	½- 7½ ⅜- 4¼	⅜- 5½ ½- 9¾	0.31	36	0.0086	0.394	0.869	12,020	13,870	14,790
4	3	½- 7 ⅜- 4	⅜- 5 ½- 8¾	0.34	36	0.0094	0.407	0.864	12,340	14,240	15,190
4½	3½	½- 9 ⅜- 5	⅜- 6½ ½-11½	0.26	42	0.0062	0.348	0.884	12,870	14,480	16,090
4½	3½	½- 8½ ⅜- 4¾	⅜- 6 ½-10¾	0.28	42	0.0067	0.358	0.881	13,810	15,540	17,270
4½	3½	½- 8 ⅜- 4½	⅜- 5¾ ½-10¼	0.29	42	0.0069	0.363	0.879	14,270	16,060	17,840
4½	3½	½- 7½ ⅜- 4¼	⅜- 5½ ½- 9¾	0.31	42	0.0074	0.372	0.876	15,210	17,110	19,010
4½	3½	½- 7 ⅜- 4	⅜- 5 ½- 8¾	0.34	42	0.0081	0.385	0.872	16,040	18,510	19,740
4½	3½	½- 6½ ⅝-10	⅜- 4¾ ½- 8¼	0.36	42	0.0086	0.394	0.869	16,360	18,870	20,130
4½	3½	½- 6 ⅝- 9½	⅜- 4¼ ½- 7¾	0.39	42	0.0093	0.406	0.865	16,780	19,360	20,650
5	4	½-10 ⅜- 5½	⅜- 7	0.24	48	0.0050	0.320	0.893	13,720	15,430	17,150
5	4	½- 9½ ⅜- 5¼	½-12 ⅜- 6¾	0.25	48	0.0052	0.324	0.892	14,270	16,060	17,840
5	4	½- 9 ⅜- 5	½-11½ ⅜- 6½	0.26	48	0.0054	0.329	0.891	14,830	16,680	18,530
5	4	½- 8½ ⅜- 4¾	½-10¾ ⅜- 6	0.28	48	0.0058	0.337	0.888	15,910	17,900	19,890
5	4	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	48	0.0060	0.344	0.885	16,430	18,480	20,530
5	4	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	48	0.0065	0.354	0.882	17,500	19,690	21,870
5	4	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	48	0.0071	0.367	0.878	19,100	21,490	23,880
5	4	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	48	0.0075	0.374	0.875	20,160	22,680	25,200
5	4	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	48	0.0081	0.385	0.872	20,950	24,170	25,780
5	4	½- 5½ ⅝- 8½	½- 7 ⅝- 11	0.43	48	0.0090	0.402	0.866	21,720	25,060	26,740

TABLE 9.—(*Continued*)

Slab thick-ness (inches)	Effect-ive depth (inches)	Reinforcement (inches)		Sect'l area A_s (12 in. wide)	bd (sq. in.)	p	k	j	Moment (inch-pounds)		
		Round	Square						f_s = 16,000 f_c = 650 n = 15	f_s = 18,000 f_c = 750 n = 15	f_s = 20,000 f_c = 800 n = 15
		(Centers)	(Centers)								
5½	4½	½- 9 ⅜- 5	½-11½ ⅜- 6½	0.26	54	0.0048	0.315	0.895	16,750	18,850	20,940
5½	4½	½- 8½ ⅜- 4¾	½-10¾ ⅜- 6	0.28	54	0.0052	0.324	0.892	17,980	20,230	22,480
5½	4½	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	54	0.0054	0.329	0.891	18,600	20,930	23,260
5½	4½	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	54	0.0057	0.335	0.889	19,840	22,320	24,800
5½	4½	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	54	0.0063	0.350	0.884	21,640	24,350	27,050
5½	4½	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	54	0.0067	0.358	0.881	22,840	25,690	28,540
5½	4½	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	54	0.0072	0.369	0.877	24,630	27,700	30,780
5½	4½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	54	0.0080	0.384	0.872	26,450	30,370	32,550
5½	4½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	54	0.0087	0.396	0.868	27,150	31,320	33,410
6	5	½- 8½ ⅜- 4¾	½-10¾ ⅜- 6	0.28	60	0.0047	0.312	0.896	20,070	22,580	25,090
6	5	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	60	0.0048	0.315	0.895	20,760	23,360	25,960
6	5	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	60	0.0052	0.324	0.892	22,120	24,890	27,650
6	5	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	60	0.0057	0.335	0.889	24,180	27,200	30,230
6	5	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	60	0.0060	0.344	0.885	25,490	28,670	31,860
6	5	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	60	0.0065	0.354	0.882	27,520	30,960	34,400
6	5	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	60	0.0072	0.369	0.877	30,170	33,940	37,710
6	5	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	60	0.0078	0.380	0.873	32,350	36,930	39,810
6	5	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	60	0.0087	0.396	0.868	33,510	38,670	41,240
6½	5½	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	66	0.0044	0.303	0.899	22,940	25,810	28,680
6½	5½	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	66	0.0047	0.312	0.896	24,440	27,500	30,550
6½	5½	½- 7c. ⅜- 4c.	½- 8¾ ⅜- 5	0.34	66	0.0052	0.324	0.892	26,690	30,020	33,360
6½	5½	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	66	0.0055	0.331	0.890	28,200	31,720	35,240
6½	5½	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	66	0.0059	0.340	0.887	30,440	34,250	38,050
6½	5½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	66	0.0065	0.354	0.882	33,370	37,550	41,720
6½	5½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	66	0.0071	0.367	0.878	36,310	40,850	45,390

TABLE 9.—(*Continued*)

Slab thickness (inches)	Effective depth (inches)	Reinforcement (inches)		Sect'l area A_s (12 in. wide)	bd (sq. in.)	p	k	j	Moment (inch-pounds)		
		Round	Square						$f_s = 16{,}000$ $f_c = 650$ $n = 15$	$f_s = 18{,}000$ $f_c = 750$ $n = 15$	$f_s = 20{,}000$ $f_c = 800$ $n = 15$
		(Centers)	(Centers)								
6½	5½	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	66	0.0079	0.382	0.873	39,340	44,940	48,420
6½	5½	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	66	0.0089	0.400	0.867	40,900	47,190	50,350
6½	5½	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	66	0.0102	0.418	0.861	42,460	48,990	52,260
7	6	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	72	0.0043	0.300	0.900	26,784	30,120	33,480
7	6	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	72	0.0047	0.312	0.896	29,240	32,900	36,560
7	6	½- 6½ ⅝-10	½- 8¼ ⅜- 4½	0.36	72	0.0050	0.320	0.893	30,860	34,720	38,580
7	6	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	72	0.0054	0.329	0.891	33,360	37,530	41,700
7	6	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	72	0.0060	0.344	0.885	36,530	41,100	45,670
7	6	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	72	0.0065	0.354	0.882	39,800	44,770	49,740
7	6	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	72	0.0072	0.369	0.877	43,780	49,250	54,720
7	6	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	72	0.0082	0.387	0.871	47,330	54,600	58,250
7	6	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	72	0.0093	0.406	0.865	49,310	56,890	60,690
7	6	⅝- 5 ¾- 7¼	½- 4 ⅝- 6¼	0.74	72	0.0103	0.421	0.860	50,830	58,650	62,560
7½	6½	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	78	0.0044	0.303	0.899	31,790	35,760	39,740
7½	6½	½- 6½ ⅝-10	½- 8¼ ⅜- 4½	0.36	78	0.0046	0.309	0.897	33,580	37,780	41,980
7½	6½	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	78	0.0050	0.320	0.893	36,220	40,750	45,270
7½	6½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	78	0.0055	0.331	0.890	39,800	44,780	49,750
7½	6½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	78	0.0060	0.344	0.885	43,260	48,670	54,070
7½	6½	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	78	0.0067	0.358	0.881	47,640	53,600	59,560
7½	6½	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	78	0.0076	0.376	0.875	53,690	60,400	66,720
7½	6½	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	78	0.0086	0.394	0.869	56,420	65,100	69,440
7½	6½	⅝- 5 ¾- 7¼	½- 4 ⅝- 6¼	0.74	78	0.0095	0.409	0.864	58,230	67,190	71,660
7½	6½	¾- 6¼ ⅞- 8½	⅝- 5½ ¾- 8	0.85	78	0.0109	0.430	0.857	60,720	70,060	74,730
8	7	½- 6½ ⅝-10	½- 8¼ ⅜- 4½	0.36	84	0.0043	0.300	0.900	36,290	40,820	45,360

TABLE 9.—(*Continued*)

Slab thickness (inches)	Effective depth (inches)	Reinforcement (inches)		Sect'l area A_s (12 in. wide)	bd (sq. in.)	p	k	j	Moment (inch-pounds)		
		Round	Square						$f_s = 16{,}000$ $f_c = 650$ $n = 15$	$f_s = 18{,}000$ $f_c = 750$ $n = 15$	$f_s = 20{,}000$ $f_c = 800$ $n = 15$
		(Centers)	(Centers)								
8	7	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	84	0.0046	0.309	0.897	39,180	44,080	48,980
8	7	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	84	0.0051	0.322	0.893	43,010	48,380	53,760
8	7	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	84	0.0056	0.333	0.889	46,800	52,650	58,500
8	7	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	84	0.0062	0.348	0.884	51,480	57,920	64,350
8	7	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	84	0.0070	0.365	0.878	58,020	65,270	72,520
8	7	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	84	0.0080	0.384	0.872	63,990	73,610	78,760
8	7	⅝- 5 ¾- 7¼	½- 4 ⅝- 6¼	0.74	84	0.0088	0.398	0.867	65,940	76,090	81,160
8	7	¾- 6¼ ⅞- 8½	⅝- 5½ ¾- 8	0.85	84	0.0101	0.419	0.860	68,860	79,450	84,750
8½	7½	½- 6c. ⅝- 9½	½- 7¾c. ⅜- 4¼	0.39	90	0.0043	0.300	0.900	42,120	47,390	52,650
8½	7½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	90	0.0048	0.315	0.895	46,180	51,950	57,730
8½	7½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	90	0.0052	0.324	0.892	50,310	56,600	62,890
8½	7½	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	90	0.0058	0.337	0.888	55,410	62,340	69,260
8½	7½	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	90	0.0066	0.356	0.881	62,370	70,170	77,970
8½	7½	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	90	0.0074	0.372	0.876	70,430	79,230	88,040
8½	7½	⅝- 5 ¾- 7¼	½- 4 ⅝- 6¼	0.74	90	0.0082	0.387	0.871	73,950	85,320	91,010
8½	7½	¾- 6¼ ⅞- 8½	⅝- 5½ ¾- 8	0.85	90	0.0094	0.407	0.864	77,140	89,010	94,940
8½	7½	¾- 5¾ ⅞- 7¾	⅝- 5¼ ¾- 7¼	0.92	90	0.0102	0.420	0.860	79,240	91,430	97,520

TABLE 10

Depth (inches)	Cubic feet in one linear foot of beam when beam widths are as follows:									
	6	8	9	10	11	12	13	14	15	16
8	0.33	0.44	0.50	0.56	0.61	0.67	0.72	0.78	0.83	0.89
9	0.38	0.50	0.56	0.62	0.69	0.75	0.81	0.88	0.94	1.00
10	0.42	0.56	0.62	0.69	0.76	0.83	0.90	0.97	1.04	1.11
11	0.46	0.61	0.69	0.76	0.84	0.92	0.99	1.07	1.15	1.22
12	0.50	0.67	0.75	0.83	0.92	1.00	1.08	1.17	1.25	1.34
13	0.54	0.72	0.81	0.90	0.99	1.08	1.17	1.26	1.36	1.45
14	0.58	0.78	0.88	0.97	1.07	1.17	1.26	1.36	1.46	1.56
15	0.63	0.83	0.94	1.04	1.15	1.25	1.36	1.46	1.56	1.67
16	0.67	0.89	1.00	1.11	1.22	1.33	1.45	1.56	1.67	1.78
17	0.71	0.94	1.06	1.18	1.30	1.42	1.54	1.65	1.77	1.89
18	0.75	1.00	1.12	1.25	1.38	1.50	1.62	1.75	1.88	2.00
19	0.79	1.06	1.19	1.32	1.45	1.58	1.72	1.85	1.98	2.11
20	0.83	1.11	1.25	1.39	1.53	1.67	1.81	1.94	2.08	2.22
21	....	1.17	1.31	1.46	1.60	1.75	1.90	2.04	2.19	2.34
22	....	1.22	1.37	1.53	1.68	1.83	1.99	2.14	2.29	2.44
23	....	1.28	1.44	1.60	1.76	1.92	2.08	2.24	2.40	2.56
24	....	1.33	1.50	1.67	1.83	2.00	2.17	2.33	2.50	2.67
25	....	1.39	1.56	1.74	1.91	2.08	2.26	2.43	2.60	2.78
26	....	1.44	1.62	1.80	1.99	2.16	2.35	2.53	2.71	2.89
27	....	1.50	1.69	1.87	2.06	2.25	2.44	2.62	2.81	3.00
28	....	1.55	1.75	1.94	2.14	2.33	2.53	2.72	2.92	3.11
29	....	1.61	1.81	2.01	2.22	2.42	2.62	2.82	3.02	3.22
30	....	1.67	1.87	2.08	2.29	2.50	2.71	2.92	3.12	3.34
31	....	1.72	1.94	2.15	2.37	2.58	2.80	3.01	3.23	3.44
32	....	1.78	2.00	2.22	2.44	2.67	2.89	3.11	3.33	3.56

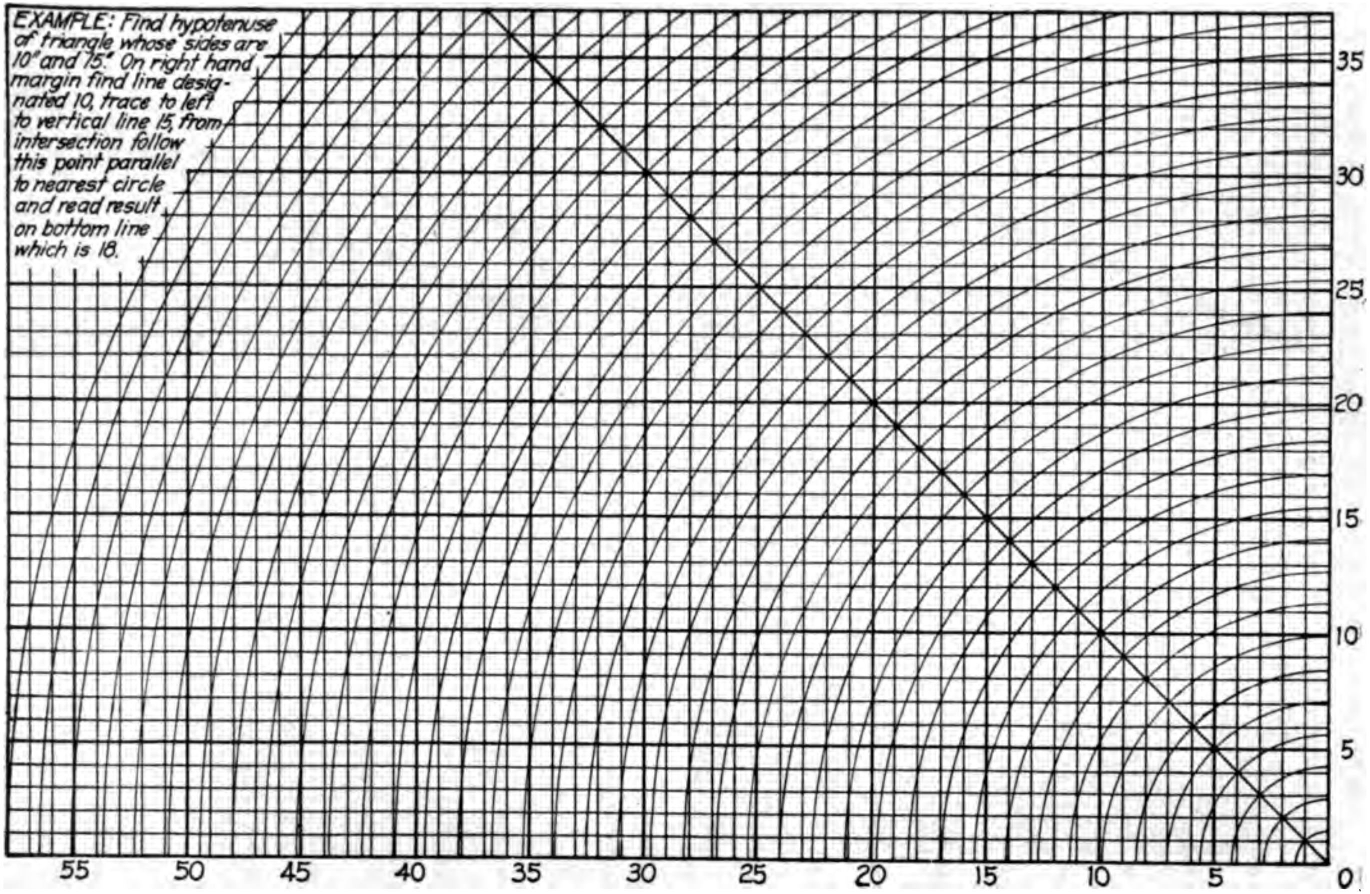
NOTE.—For concrete weighing 150 lb. per cu. ft. convert cu. ft. in table into lb. per lin. ft. by adding half of itself to any given quantity and shifting decimal point two places to the right.

EXAMPLE.—Beam 10 × 18 in. = 1.25 cu. ft. per. lin. ft.

1.25 + (½ × 1.25) = 1.88 = 188 lb. per lin. ft.

Diagram 3.

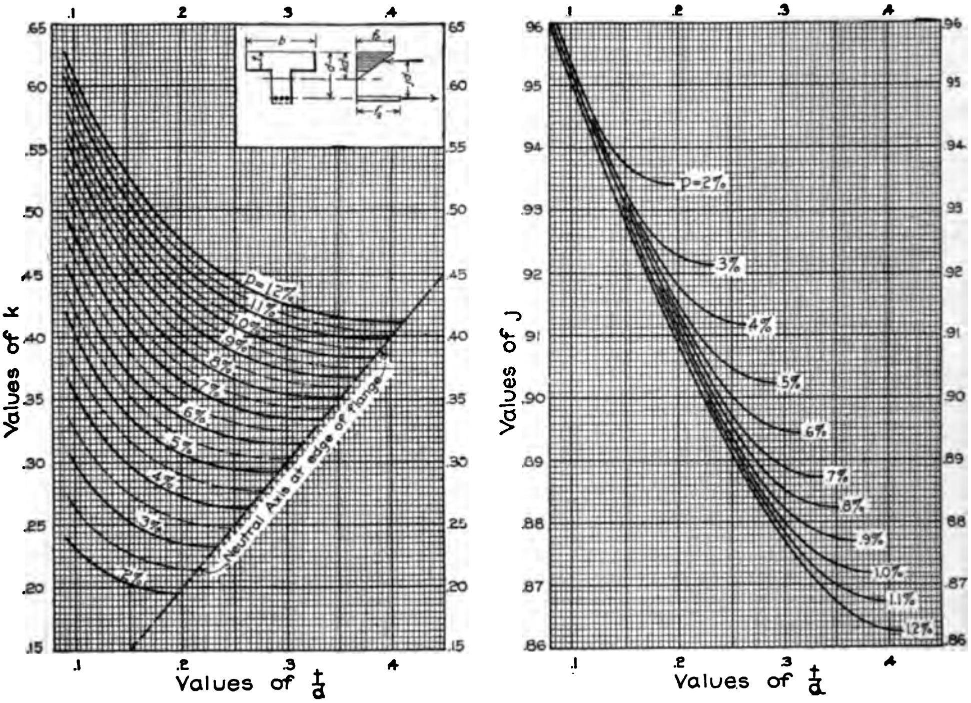
Diagram showing the hypotenuse of any right angle triangle. Use to find length of bent portion of beam or slab rods.



EXAMPLE: Find hypotenuse of triangle whose sides are 10" and 15." On right hand margin find line designated 10, trace to left to vertical line 15, from intersection follow this point parallel to nearest circle and read result on bottom line which is 18.

DIAGRAMS 4 AND 5.

Use for T-beams. Values of k and j for various percentages of steel. Based on $n = 12$.



DIAGRAMS 6 AND 7.

Use for T-beams. Values of k and j for various percentages of steel. Based on $n = 15$.

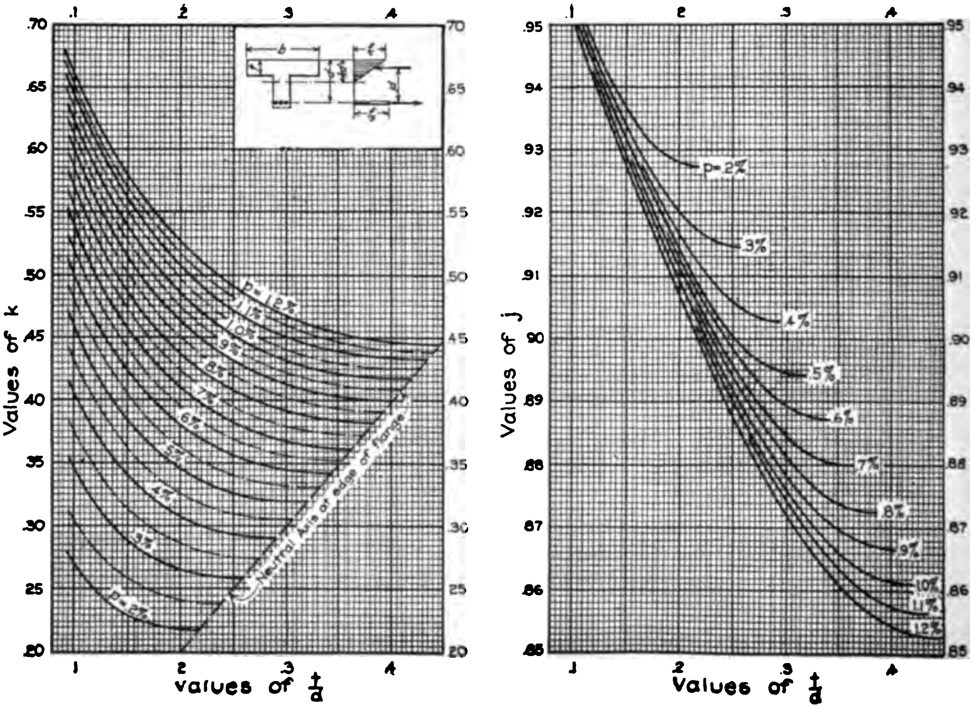
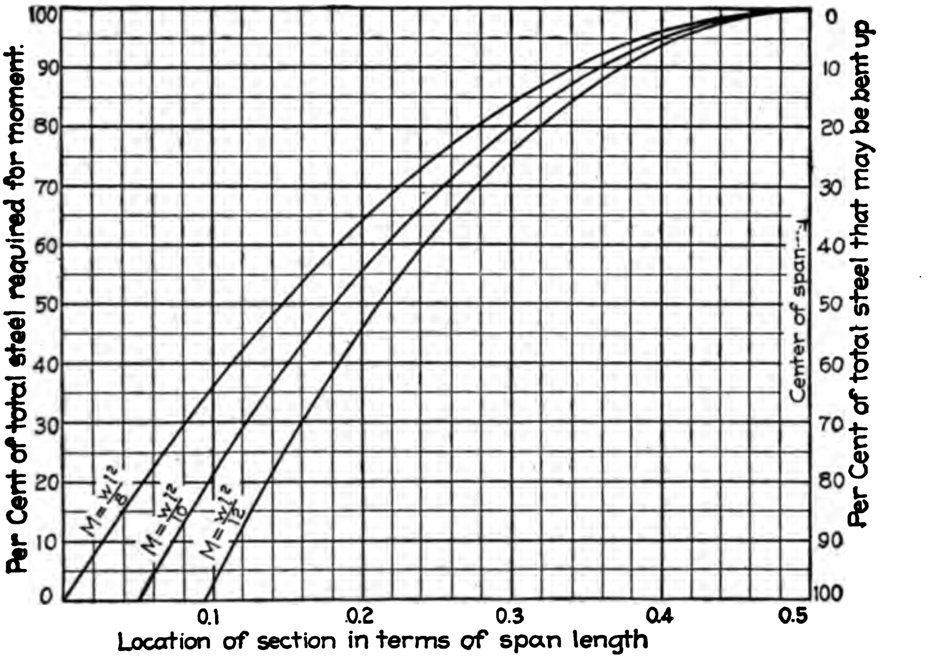


Diagram 8.
Use to Locate Points for Bending Reinforcement.



43. Reinforced Concrete Stairs.—Reinforced concrete on account of its fireproofness permanency and adaptability, has become a very common material for use in the construction of stairs and platforms, and has superseded to no little extent the use of steel and iron in stair construction in many types of buildings.

The most essential requirement of a stairway, with the exception of strength, is fireproofness, which will insure a safe and uninterrupted exit in the event of fire. Stairway shafts should be enclosed with fireproof partitions or walls having fire underwriters' labeled automatic firedoor entrances.

Stairways are usually designed with short straight flights, with one or two intermediate platforms. Long uninterrupted flights without platforms from one floor to that of another are objectionable and seldom employed.

43a. Design.—The design of a reinforced concrete stairway embodies the simplest form of non-continuous solid slab construction with span equal to the horizontal distance center to center of supports. The stairway consists simply of a solid slab with risers and treads formed upon its upper surface. The span of the slab usually includes the stairway slab and a platform between the supports. The stresses in the latter type of stairway slabs are more or less indeterminate, although the usual practice of computing such irregular ones as freely supported members, has given satisfactory results in every known instance.

The design of stairways often presents awkward problems of arrangement. The beginning of the stairway slab usually rests upon a beam girder or special member at the floor level, and the first platform is often supported by an intermediate spandrel beam or brick wall in case of a wall bearing building. When a platform occurs on the interior of a building (Figs. 47 *A* and 47 *B*), specially devised rod hangers are usually provided, suspending the edge of platform from a beam at the floor above. Such hangers should be encased preferably in concrete and concealed in partitions when the same enclose the stairway (see Figs. 47 *A* and 47 *B*). Occasionally it is required to design a stairway of unusual span without the opportunity of providing intermediate supports. In this case inclined concrete stringers or beams following the rake of the stairway and supporting one or both sides, as conditions may dictate, are employed to lessen the span of stair slab.

When a winding stairway consists of three stair slabs and two platforms, the intermediate stair slab is often supported directly by the two platforms (see Figs. 48 *A* and 48 *B*). In this case the upperand lower stair and platform slabs in combination are designed to support the concentrated load of intermediate stair slab, in addition to their own dead and live loads.

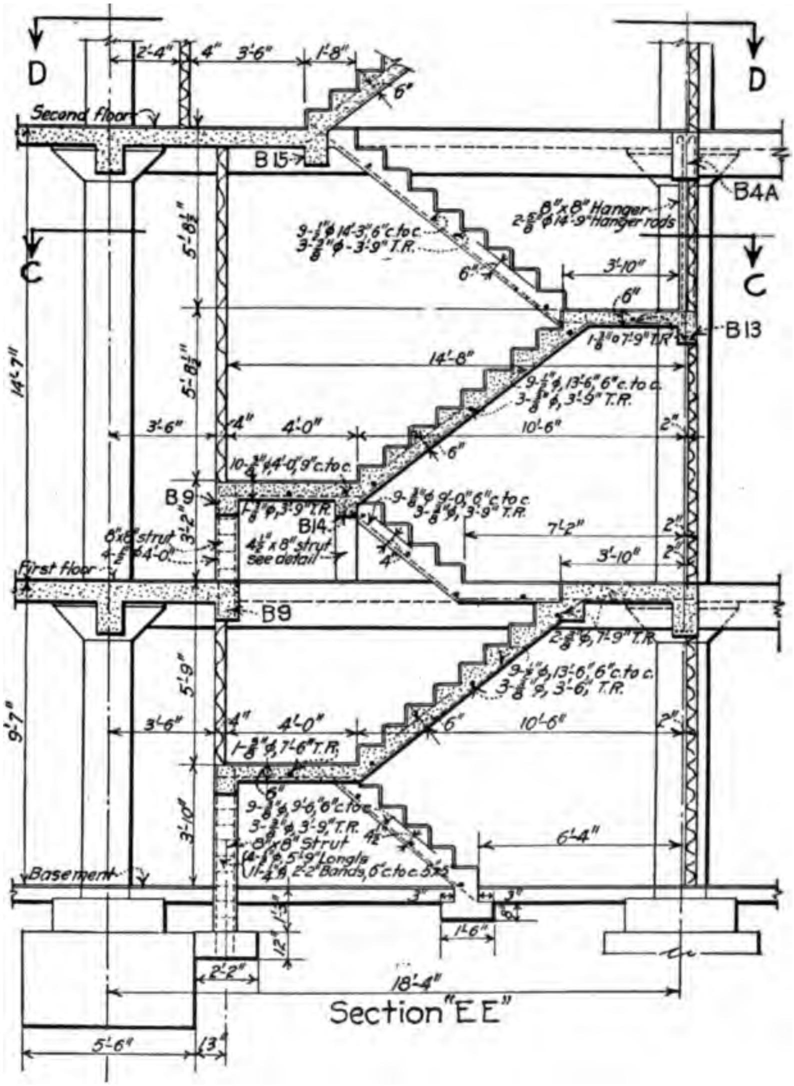
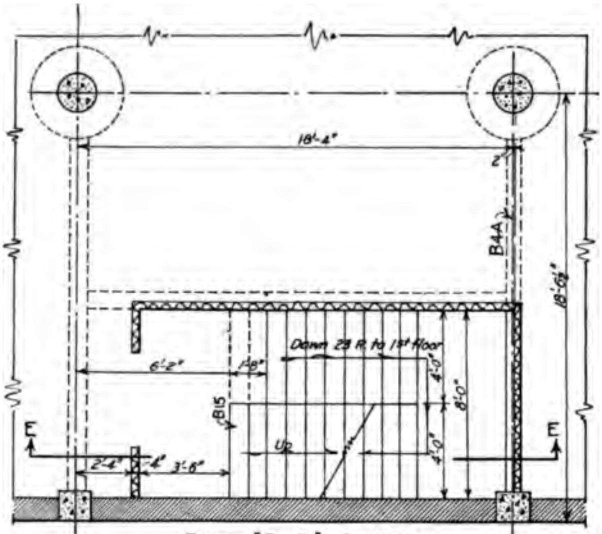


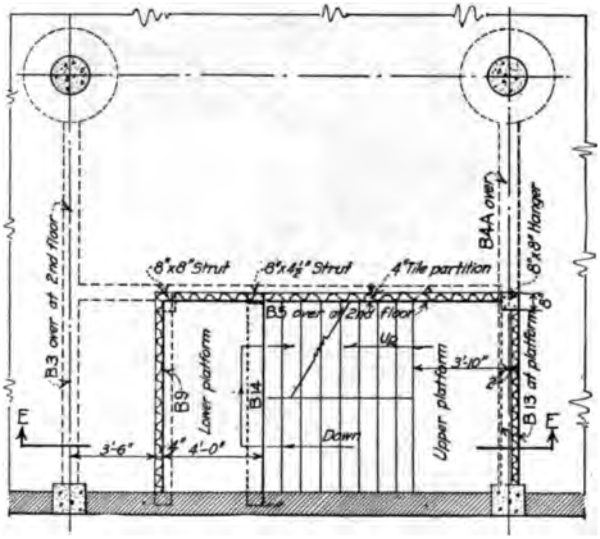
FIG. 47*A*.

Stairways are usually designed for a superimposed live load of from 40 to 100 lb. per horizontal square foot, depending upon the character of service desired. Theatres and public gathering places demand greater attention to the live loads assumed than stairways in office buildings, hotels, warehouses, etc., where frequent congestion is a remote possibility.

43b. Construction and Details.—Stairways are preferably poured at the same e as the supporting members. If constructed after the floors have been completed, it has n proved better construction to install the reinforcement, properly spaced, with ends of bars



Plan "D-D" 2nd Floor

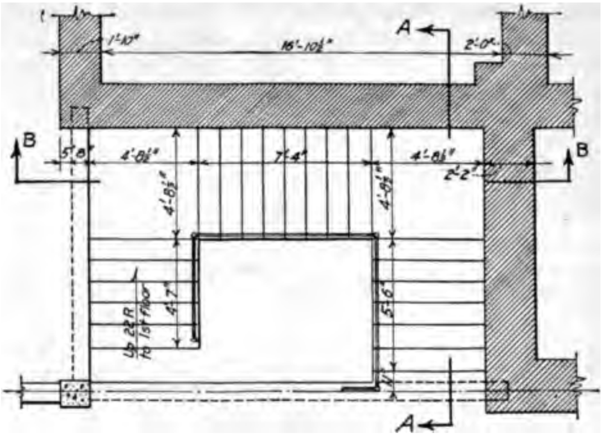


Plan "C-C"

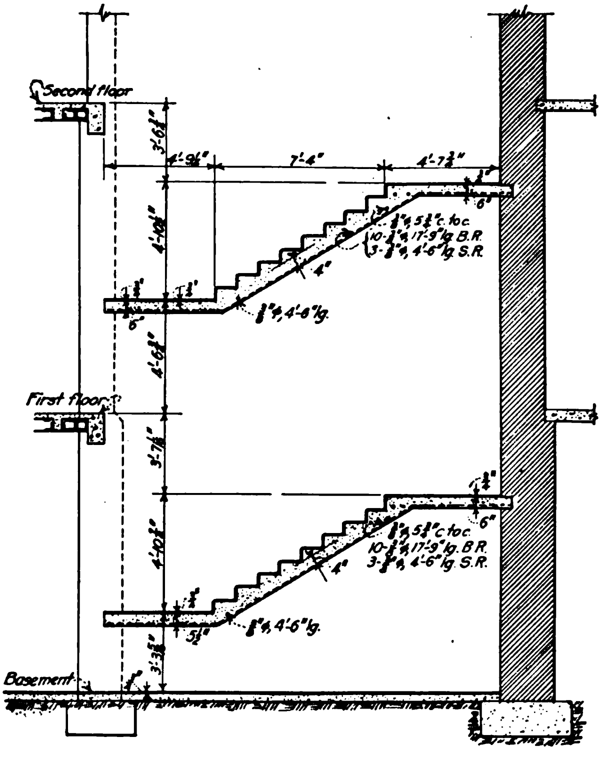
FIG. 47B.

ecting a sufficient distance into the supporting members at floors, prior to the pouring of rs, otherwise dowels at specified intervals should be inserted long enough to provide suitable for stair rods when placed. In addition to dowels, rabbets should be formed by means of a d strip secured to the side of beam form, to form a support for the future stair slab.

The method employed to finish the tread or run of a stairway is of considerable importance n considering durability and safety. The finish of tread, being subjected to the severest



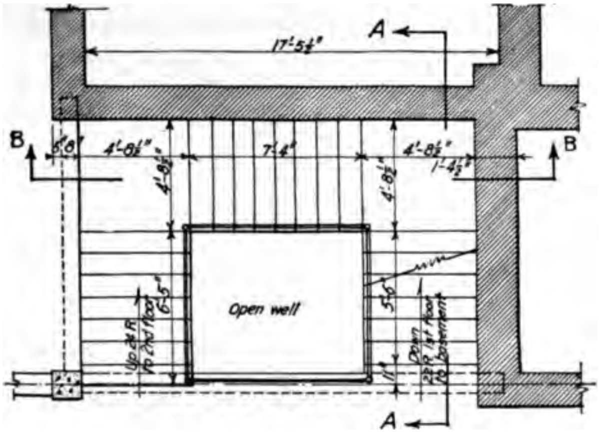
Basement to First Floor



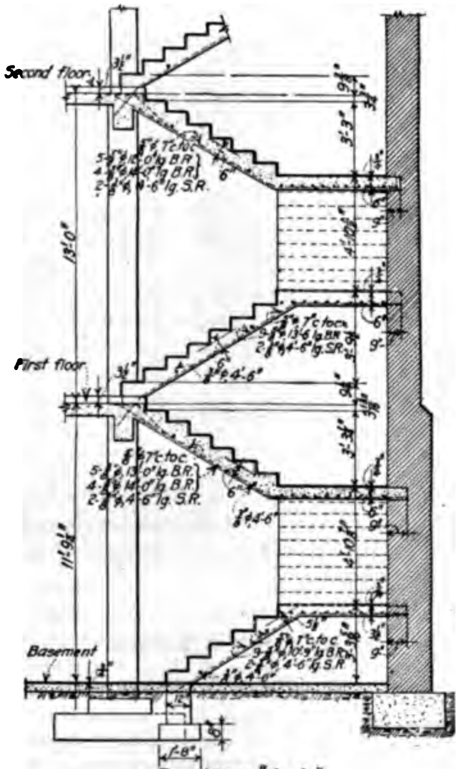
Section "B-B"

Fig. 48A.

wear, should be treated with one of the recognized chemical or metallic surface or integral floor hardeners or else safety treads of some desirable make should be employed to render the stairway permanent and safe (see Fig. 49).



First to Second Floor



Section "A-A"

Fig. 48B.

The rise of a stair represents the distance from the top of one step to the top of the next and **the run** the horizontal distance from the face of one riser to the face of the next. The customary rise employed varies from 6½ to 7¼ in. and the run from 10¼ to 11 in. A rise greater than 7½ is objectionable and results in making a stairway too steep for comfort and safety (see Fig. 50).

At the upper juncture of risers and treads, sharp or angular corners should be avoided in the case of cement finish. Rounded nosings of cement are more desirable in the absence of metallic treads, marble, etc. When cement finishes are used, the same should be applied soon after the stair is poured (see Fig. 51).

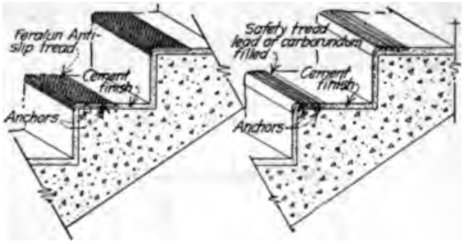


Fig. 49.

The railing most commonly used consists of a 2-in. gas pipe rail with stanchions at proper intervals to insure rigidity. The stanchions are usually secured in pockets provided by wood plugs placed prior to pouring of concrete, or by means of expansion bolts.

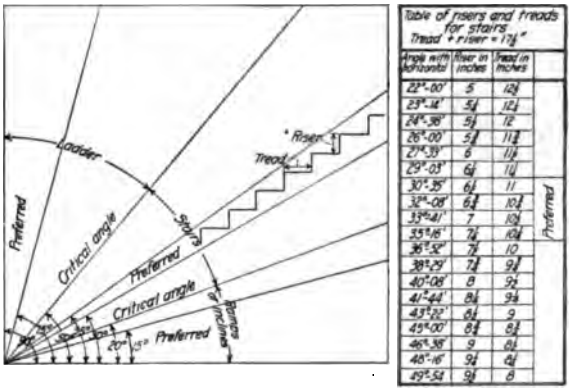


Table of risers and treads for stairs
Tread + riser = 17½"

Angle with horizontal	Riser in inches	Tread in inches	
22°-00'	5	12½	
23°-14'	5¼	12¼	
24°-38'	5½	12	
26°-00'	5¾	11¾	
27°-33'	6	11½	
29°-03'	6¼	11¼	
30°-35'	6½	11	Preferred
32°-08'	6¾	10¾	
33°-41'	7	10½	
35°-16'	7¼	10¼	
36°-52'	7½	10	
38°-29'	7¾	9¾	
40°-08'	8	9½	
41°-44'	8¼	9¼	
43°-22'	8½	9	
45°-00'	8¾	8¾	
46°-38'	9	8½	
48°-16'	9¼	8¼	
49°-54'	9½	8	

Fig. 50.

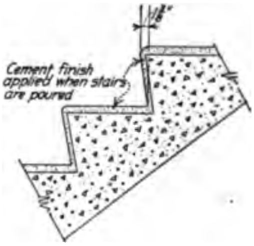


Fig. 51.

Concrete railings are often used where open railings are undesirable. This form of railing consists of a reinforced concrete slab 3 to 4 in. thick with provision for a wood hand rail secured to the top. The hand rail should be placed on an average of about 2 ft. 6 in. above the tread on a line vertical with the face of riser.

WOODEN GIRDERS

By Henry D. Dewell

The loads coming upon the girders of a floor system consist of the loads delivered by the floor joists, plus the weight of the girders themselves, plus any loads coming directly upon the girder, as distinguished from loads transmitted by the joists. Girders often carry partition loads directly.

In office buildings, dwelling houses, and certain areas of other buildings, exclusive of warehouses and storage buildings, where crowds of people cannot congregate, the live load coming upon the girders is reduced in intensity. The reduction factor is specified in building ordinances, and is usually taken as 20 %.

Horizontal shear at the ends of girders often governs the girder section, as in the case of short spans with heavy loading, and this stress should always be checked.

The end connections of girders are of much more importance than the end connections of joists, as the girders of a building, together with the posts, usually form the stiffening frame of the building against lateral forces. Particular attention also needs to be paid to the design of the support of wooden girders, as failure of a girder would mean the probable collapse of at least a whole floor bay.

Fig. 52.—Built-up girder—type (1).

Wooden girders, even if continuous over two spans, are generally computed as simple beams.

The detail of end connection of girders will depend on the type of building. If such building is of mill construction with heavy masonry walls, the wall ends of girders should be encased in wall boxes, the inner end connections designed to allow the girders to fall, in case of fire, without pulling the columns with them. In other types of buildings, as the mill type, stiff rigid connections of girders to posts may be desirable.

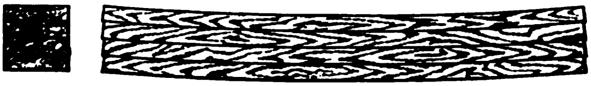


Fig. 53.—Built-up girder—type (2).

44. Girders of Solid Section.—The section of wooden girders composed of solid sticks of timber are to be designed exactly as treated under "Wooden Beams."

45. Built-up Wooden Girders.—Built-up wooden girders may be divided into the following types:

(1) Girders constructed of planking, set side by side, the width of plank vertical, as in Fig. 52.

(2) Girders constructed of two or more timbers set on top of one another, but not fastened together, as in Fig. 53.

(3) Girders constructed of two or more timbers set on top of one another, and diagonally sheathed with boards or planking, as in Fig. 54.

(4) Girders constructed of two or more timbers set on top of one another, and effectively fastened together by means of hard wood or metal keys or pins, combined with bolting, as in Fig. 55.

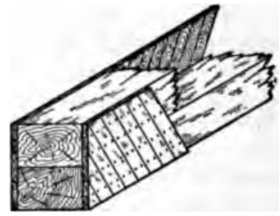


Fig. 54.—Built-up girder—type (3).

Type (1).—A girder, or beam, of this type, if all planking extends the full length of girder, is of full nominal thickness, and is well spiked and bolted together. It is generally given credit for being somewhat stronger than a girder or beam of solid section of the same dimensions, since the planking is assumed to be better seasoned and freer from defects, particularly checks, than the larger solid timber. A construction of this type is often observed in small buildings where planks are more easily obtained than heavy timbers, and where the solid section construction might incur purchase of additional material by the contractor. Insufficient spiking, lack of proper bolting, probability of planking under-running in thickness, thus giving an actual size of finished beam less than the solid section, possibility of some planks being spliced, and the probability of upper surface of girder being uneven—*i.e.*, one plank projecting higher than another, giving uneven bearings for the joists—are practical reasons for always advocating the beam of solid section. Incidentally, no building ordinance gives the built-up girder any advantage in strength. Solid sections should be insisted upon for important beams. When it is necessary to use this type of built-up girder, provide two bolts at each end, and pairs of bolts at intervals of 2 ft. along the length of beam, the size of bolts to be not less than 5/8 in., and preferably 3/4 in.

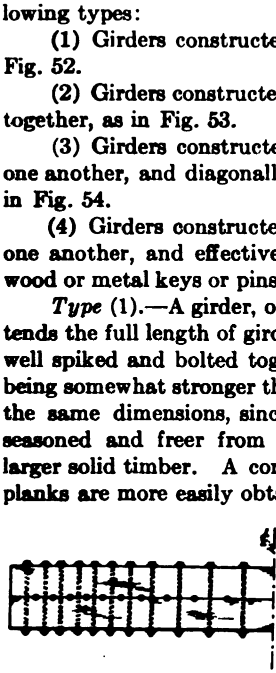


Fig. 55.—One-half typical built-up girder—type (4).

Type (2).—This type of girder should never be used. The strength of the combined section is practically no more than the sum of the strengths of the component sticks, each stick acting as a separate beam. Even if such a girder should be constructed of planking, well spiked together, the above statement of resulting strength would hold, as the nailing would be insufficient to prevent one plank from slipping on another.

Type (3).—In this type of built-up girder, as in the following type, the object of all connections between the component sticks (usually two) is to prevent relative motion along the plane of contact. If this condition of no-slip could be attained, the compound girder would have the strength of a single stick of timber of the same outside total dimensions. Type (3) is considerably less efficient than Type (4), both as regards ultimate strength and deflection under load. The diagonal sheathing is spiked to the timbers, and the sheathing should be at 45 deg. with the length of girder.

Tests made by Edgar Kidwell (see Trans. Am. Soc. Mining Engineers, 1897, vol. 27) showed an efficiency of approximately 70%., based on the ultimate strength, as compared to a beam of solid section, while the efficiency factor based on deflection was about 50%.

The sheathing for such girders should be not less than 1¼ in. and not over 2 in. in thickness. With such sheathing the nails should be 10 or 12-D for the smaller thickness, and 20 to 30-D for the 2-in. sheathing. For a girder supporting uniform load the diagonals near the ends require the most spikes. The spiking in each diagonal should be concentrated near the plane of junction of the timbers, and at the ends of the diagonals.

In designing a girder of this type, it must be remembered that the case is not similar to that of a truss. In a truss are two chords, in each of which, due to the small depth of chord as compared to the large depth of truss, the stress is practically uniform throughout the cross section of each chord, and the diagonals take either tension or compression. The side planking in the built-up girder under discussion is subjected to bending moments, and, consequently, the nails take unequal loading. Any slip of the nails under stress allows a corresponding slip in the plane of contact of the two main timbers, with a consequent deflection of the girder. By referring to p. 239 it will be found that nails under lateral or shearing strain slip at a small load.

Type (4).—In the girders of this class, the tendency of one timber to slip over the other is resisted by wedges, keys, or pins driven into the contact faces of the timbers. These wedges, whether rectangular, square, or round, perform their main function through bearing against the ends of the fibers of the timbers. A second action is pressure across the fibers of the timbers. The action of these wedges tends to separate the two timbers, resulting in tension in the bolts. The amount of such tension depends primarily upon the shape of wedge. For example, a square key will produce a greater bolt tension than a rectangular key with long axis parallel to the length of girder, while a circular key or pin will give the greatest tension in the bolts.

The number and size of keys is to be determined directly from consideration of horizontal shear in the girder, in accordance with the principles of Sect. 1, Art. 63, and illustrated in the typical example hereafter.

The bolts in such a girder are assumed to take only tension, although, due to their resistance to lateral forces, they add somewhat to the strength of the girder. However, it is always advisable, and on the safe side, to neglect such lateral resistance of the bolts.

Kidwell's series of tests on girders of this type showed a maximum efficiency of 75 to 80% of an equivalent girder of solid section, the former figure representing girders with white oak keys and the latter figure with keys of iron.

Any shrinkage in the timbers will allow the component parts of the girder to separate, with a consequent loss of efficiency, and an increased deflection. As fully seasoned timber is not always available, this type of girder should be avoided for cases in which the major portion of the load is a constant load. For situations in which the girder carries live load for the greater part, in which access may be had to tighten the bolts as the wood seasons, and when it is reasonably certain that such maintenance will be given, this girder may be used with confidence. Obviously, the keyed girder is particularly unsuited for such locations as will prohibit access for tightening the bolts, as in a floor system ceiled underneath.

46. Examples of Design of Solid and Built-up Girders.—The following typical examples will illustrate the method of design for the most common cases that will be encountered:

Conditions of Design:
Span: 26 ft.
Loading: Uniform load of 1500 lb. per linear foot.
One concentrated load of 6000 lb., 7 ft. from left support.
One concentrated load of 14,000 lb. at center of span.
One concentrated load of 2000 lb., 9 ft. from right support.
Timber: Long leaf yellow pine, Dense Structural Grade.

The reactions are given in Fig. 56 and the bending moment curves in Fig. 57. The parabola of moments for uniform load is plotted above the base line, and the polygon of moments for concentrated loads below this line.

The following unit stresses will be used:

Bending stress on outer fibers	1800 lb. per sq. in.
Longitudinal shear	175 lb. per sq. in.
Bearing across grain	400 lb. per sq. in.
Bearing against grain	1800 lb. per sq. in.

Solid Girder.—Maximum bending moment = 248,100 ft.-lb. From Table 6, p. 108, an 18 × 24-in. girder, surfaced to 17½ × 23½ in., has a resisting moment of 241,610 ft.-lb., which will be near enough to be used, or a double girder may be used. For example, 2 – 14 × 20-in. sticks would have a safe resisting moment of 256,670 ft.-lb. The required cross section for longitudinal shear is

$$\frac{\frac{3}{2}(31,600)}{175} = 271 \text{ sq. in.}$$

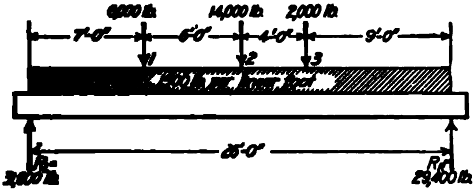


FIG. 56.—Loads and reactions for girder of Art. 56.

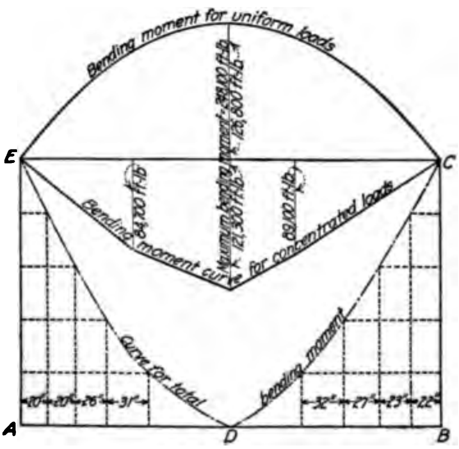

Bending moment for uniform loads
Maximum bending moment = 248,100 ft.-lb.
126,800 ft.-lb.
121,300 ft.-lb.
84,700 ft.-lb.
89,100 ft.-lb.
Bending moment curve for concentrated loads
Curve for total bending moment
E
C
A
D
B
20"
20"
26"
31"
32"
27"
23"
22"

FIG. 57.—Diagram for bending moments and spacing of shear keys for girder of Art. 46.

her of the above girders has an excess of timber for shear.

Built-up Girders.—Type (1) could not be considered, as no standard planking 20 or 24 in. is made.

2) would require 2 – 14×20-in. sticks, one on top of the other—an impractical consideration.

.—Maximum bending moment = 248,100 ft.-lb. Using an efficiency factor of 70% the moment to be 355,000 ft.-lb. Assume a width of 14 in. The required section modulus

$$S = \frac{(355,000)(12)}{1800} = 2370$$

$$d = \sqrt{\frac{(2370)(6)}{13.5}} = 32.4 \text{ in.}$$

inished section 13½ × 35 in.

ides, spiked with 40-D nails—detail similar to that of Fig. 54.

factor of 80%

$$\text{ning moment} = \frac{248,100}{80} = 310,000 \text{ ft.-lb}$$

$$S = \frac{(310,000)(12)}{1800} = 2070$$

depth is found to be 30.2 in. Use 2 – 14×16-in. sticks, S4S,* actual is 2160.

n in Fig. 58(*a*). Each ordinate of this diagram represents the is taken, and this total vertical shear is proportional to the ame point. Considering Point (1), directly under the concentrated just to the left of this point is 31,600 – (7)(1500) = 21,100 lb. The ordinate one foot to the left will have a value of 31,600 − (6)(1500) = 22,600 lb. The area of the trapezoid between these two ordinates is therefore $\frac{22{,}600 + 21{,}100}{2} = 21{,}850$ ft.-lb. The maximum intensity of horizontal shear at a point immediately to the right of Point (1), is

$$v = \tfrac{3}{2}\cdot\frac{V}{bd} = \tfrac{3}{2}\cdot\frac{21{,}100}{(13\tfrac{1}{2})(31)} = 76 \text{ lb. per sq. in.}$$

The next step is to find a means for determining the proper spacing of keys. Two methods will be explained.

Method 1.—For this purpose, the total vertical shear between the point of zero shear and each point of division of beam is computed by adding together the differential shears between these two points. The corresponding ordinates are drawn, giving the line ABC in Fig. 58(b). The summation of the vertical shears to the left of the point of zero shear is found to be 248,050 ft.-lb.; agreeing with the value of the bending moment, which furnishes a check on the work. Similarly, the summation of the vertical shears to the right of the point of zero shear will give the same value.

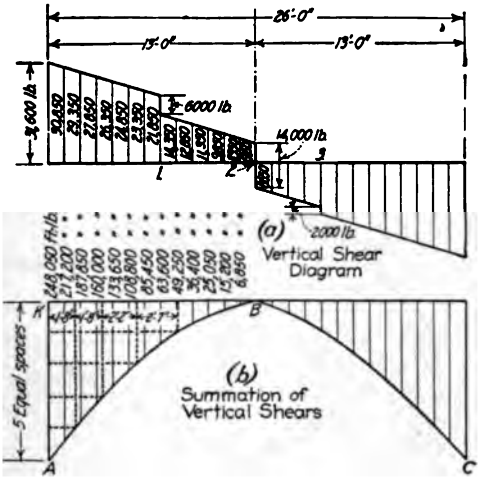


Fig. 58.

Since, for practical reasons, all keys will be of uniform size, and must therefore be stressed uniformly, the spacing of same must vary. The number of keys for the left half of girder will be taken at 5.

Method 2.—A much simpler method for constructing the total shear diagram will now be shown. In Fig. 57 the dot-dash line represents the curve of the total bending moment, the ordinates of this curve being the sums of the corresponding ordinates of the moment curves for the uniform and concentrated loadings.

If the horizontal line AB be drawn through the apex of this total moment curve, the latter curve referred to the line AB becomes the curve for the total vertical shears—in other words, the figure $ABCDE$ becomes the total shear diagram.

To find the proper spacing of the keys for the left half of beam, the vertical ordinate (248,100 ft.-lb.) of the total shear diagram is divided into 5 equal spaces, horizontals drawn from these division points to the curve of total shear, and vertical ordinates drawn from these intersection points to the base line. These ordinates divide the area ABK, (Fig. 58b) or ADE (Fig. 57) between the curve and base line, into 5 equal divisions. The points on the girder thus found determine the position of keys. Referring to either Fig. 58(b) or Fig. 57, the proper spacing of keys for the left half of the girder is found to be two spaces at 20 in., one at 26 in., and one at 31 in. The spacing of keys for the right of the center of girder may be found in the same manner.

Girder with Rectangular Keys.—In the above example the girder will first be designed for rectangular cast-iron keys. Assume 5 keys between the left support and the point of zero shear. Each key will therefore resist one-fifth of the total horizontal shear.

The required dimensions of each key will be determined from the following consideration. The forces acting upon the key are shown in Fig. 59.

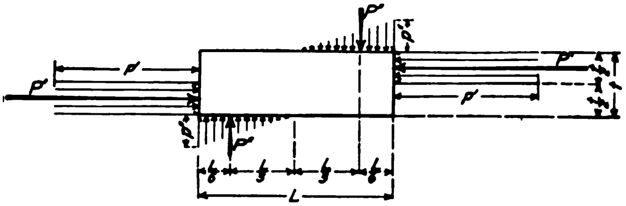


Fig. 59.—Diagram of distribution of pressures on rectangular key.

Let p' = maximum allowable intensity of pressure against ends of fibers.

p'' = maximum allowable intensity of pressure across fibers of timber.

t = thickness of key.

L = length of key.

P' = resultant pressure against fibers of timber for section of key one inch in width.

P'' = resultant pressure across fibers of timber for section of key one inch in width.

Then

$$P'\left(\frac{t}{2}\right) = P''\left(\frac{2}{3}L\right)$$

$$P' = p'\left(\frac{t}{2}\right)$$

$$P'' = \left(\frac{p''}{2}\right)\left(\frac{L}{2}\right)$$

Whence

$$p'\left(\frac{t}{2}\right)\left(\frac{t}{2}\right) = \frac{p''}{2}\left(\frac{L}{2}\right)\left(\frac{2}{3}L\right)$$

$$\frac{p't^2}{4} = \frac{p''L^2}{6}$$

$$L^2 = \frac{6p't^2}{4p''} = \frac{3}{2} \cdot \frac{p't^2}{p''}$$

$$L = 1.225\,t\sqrt{\frac{p'}{p''}}$$

For

$$p' = 1800,\text{ and } p'' = 400$$

$$\sqrt{\frac{p'}{p''}} = \sqrt{4.5} = 2.12$$

Whence

$$L = (1.225)(2.12)t = 2.6$$

The total horizontal shear is

$$(3/2)\left(\frac{248{,}100}{31}\right)(12) = 144{,}000 \text{ lb.}$$

Each key must therefore resist 28,800 lb. At 1800 lb. per sq. in. in bearing against the grain, and with a width of key of 13½ in., one-half the depth of key must be $\frac{28{,}800}{(13.5)(1800)} = 1.19$ in., or the total thickness of key must be 2.38 in. The minimum length of key must therefore be 2.38 × 2.6 = 6.19 in.

The minimum distance between keys, considering shear, must not exceed $\frac{28{,}800}{(175)(1.53)} = 12.2$; adding the width of key, the minimum spacing of keys, center to center, must not be less than 18¼ in., which is less than the smallest spacing found.

The bolts for each key should be spaced on each side of each key and equidistant from the center line. Assume four bolts for each key. The stress in each bolt will then be $\frac{1}{2} \cdot \frac{(P')(t/2)}{2/3L} = \frac{1}{2} \cdot \frac{(28{,}800)(2.375)}{(2/3)(6)(2)} = 4276$ lb., or four ¾-in. bolts are required. Washers 4 × 4 × ⁵⁄₁₆ in. will be used.

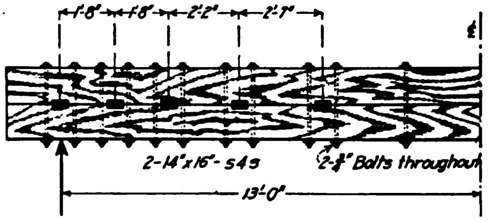


Fig. 60.—Detail of built-up girder with cast-iron keys.

The detail of the left half of the girder is shown in Fig. 60.

Girder with Circular Shear Pins.—For this design circular pins, 2-in. in diameter, of solid iron, extra heavy steel pipe, Australian Ironbark or Hawaiian Ohia will be used. Each pin will be considered capable of resisting a shear of 800 lb. per linear inch of pin. With a 13½-in. length of pin, therefore, one pin will have a resisting value of 13½ × 800 = 10,800 lb. Since the total horizontal shear is 144,000, the total number of pins required is $\frac{144{,}100}{10{,}800} = 14.4$. Dividing the end ordinate into 15 divisions and proceeding as before, it will be found that the minimum spacing of the pins near the end of the girder is 6 in. The spacings of all pins for the left half of girder, commencing at the center line of support of girder, are as follows: 2 spaces at 6 in.; 6 spaces at 7 in.; 1 space at 8 in.; 1 space at 9 in.; 1 space at 10 in.; 1 space at 13 in.; 1 space at 16 in.; and one space at 19 in. For each pin there will be required bolts sufficient in tension for 10,800 lb. Two ¾-in. bolts will be used, with 4 × 4 × ⁵⁄₁₆-in. washers. The detail of one-half of girder is shown in Fig. 61.

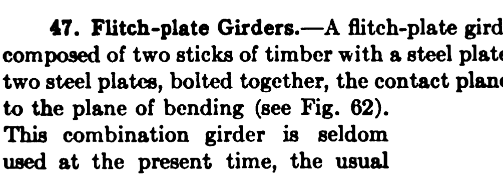


Fig. 61.—Details of built-up girder with circular shear pins.

47. Flitch-plate Girders.

47. Flitch-plate Girders.—A flitch-plate girder is a combination girder of timber and steel, composed of two sticks of timber with a steel plate between them, or three sticks of timber with two steel plates, bolted together, the contact planes between timber and steel plate being parallel to the plane of bending (see Fig. 62). This combination girder is seldom used at the present time, the usual availability of steel structural shapes making the flitch-plate girder practically obsolete. Situations may sometimes exist, however, when the use of this type of girder may be warranted.

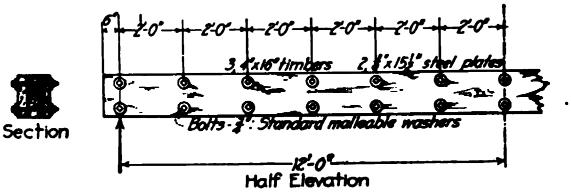


Fig. 62.—Detail of flitched-plate girder.

Consider any plane cross-section of such a combination girder: the deflection and also the deformation of all points in such section on a line normal to the plane of bending must be the same. Since the modulus of elasticity is the ratio of stress to deformation, it follows that the extreme fiber stresses of timber and steel will be in proportion to their moduli of elasticity, or

$$\frac{f_t}{f_s} = \frac{E_t}{E_s}$$

where the subscripts "*t*" and "*s*" represent timber and steel, respectively. This relation of extreme fiber stresses means practically that with the steel plate working efficiently (extreme unit fiber stress of 16,000 lb. per sq. in.) the limiting extreme unit fiber stress in the timbers is approximately $\frac{1}{18}$ to $\frac{1}{20}$ of the allowable working stress for steel. In the case of a flitch-plate girder of long-leaf yellow pine and steel, the timber would be stressed to approximately 900 lb. per sq. in. The timber is therefore working at an efficiency of about 50%, while that steel plate in the rectangular section is only approximately 55% efficient as compared to an I-beam of equal depth and weight.

As an illustration of the computation for the strength of a flitch-plate girder, assume a girder composed of 3 – 4 × 16-in. timbers of No. 1 Common Douglas fir (finished section 3½ × 15½ in.), with two ⅜ × 15½-in. steel plates between the timbers. With a span of 24 ft., it is desired to find the safe load, uniformly distributed, that the girder will support.

Maximum allowable unit fiber stress in timber = 1500 lb. per sq. in.
Maximum unit fiber stress for steel plate = 16,000 lb. per sq. in.
E for Douglas fir = 1,600,000
E for steel = 29,000,000

Therefore, for flitch-plate girder, the maximum unit fiber stress in bending can be only $\frac{1,600,000}{29,000,000}$ (16,000) = 880 lb. per sq. in.

The resisting moment of the three timbers in foot-pounds (see Sect. 1, Art. 61*d*) is

$$M = \frac{1}{6} fbd^2 \left(\frac{1}{12}\right) = \frac{(880)\,(10.5)\,(240)}{(6)(12)} = 30,800 \text{ ft.-lb.}$$

The resisting moment of the two steel plates is

$$M = \frac{1}{6} fbd^2 \left(\frac{1}{12}\right) = \frac{(16,000)(0.75)(240)}{(6)(12)} = 40,000 \text{ ft.-lb.}$$

The combined resisting moment is therefore

$$30,800 + 40,000 = 70,800 \text{ ft.-lb.}$$
$$M = \tfrac{1}{8}\,WL = 70,800 \text{ ft.-lb.}$$
$$W = \frac{(70,800)(8)}{24} = 23,600\text{-lb.}$$

The detail of this girder is shown in Fig. 62. The timbers and steel of the flitch-plate girder should be well bolted together; such bolting should consist of not less than two ¾-in. bolts, 2-ft. centers.

In designing a flitch-plate girder for a definite span and loading, the thickness of timber should be from 16 to 18 times the thickness of steel.

48. Trussed Girders.—For situations in which the span or loading, or both, are too great for a girder of single timber section, the trussed girder type is effective, if space limitations will allow its use. The trussed girder is preferable to either the built-up or deepened girder, or to the flitch-plate girder, principally on account of its efficiency and reliability of action. In the trussed girder no fear need be entertained as to decrease of initial efficiency or increase of deflection from initial conditions, due to shrinkage of timber, with consequent slip of fastenings.

Trussed girders may be divided into four types, as follows:

(1) King Post trussed girder.
(2) Queen Post trussed girder.
(3) Reversed King Post trussed girder.
(4) Reversed Queen Post trussed girder.

These types are illustrated in Figs. 63, 64, 65 and 66.

Trussed girders are adapted particularly for either uniform loading or concentrated loads situated symmetrically with respect to the center line of girder. Both the Queen Post girder and the Reversed Queen Post girder are unsuited for unsymmetrical loading. Since each contains a rectangular panel, loading unsymmetrical in distribution with respect to the center line of girder will cause bending stresses in the joints of the girder, which cannot take such stresses.

The determination of the stresses in a trussed girder is a problem in least work. For practical purposes the following approximate formulas are sufficient:

Uniformly Distributed Loading:

Figs. 63 and 65. (King Post and Reversed King Post types)

Tension in DB (Fig. 63) or compression in BD (Fig. 65) $= \frac{5}{8}W$

Tension in AB and BC (Fig. 63) or compression in AB and BC (Fig. 65) $= \frac{5}{32}\frac{Wl}{h}$

Compression in AD and DC (Fig. 63) or tension in AD and DC (Fig. 65) $= \frac{5}{16}\frac{Wa}{h}$

To the stresses thus found in members AB and BC, must be added the flexural stresses resulting from these members acting as beams carrying the uniform loading between A and B, and B and C.

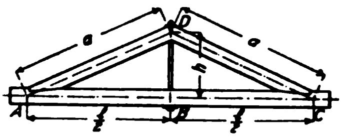


Fig. 63.—King post girder.

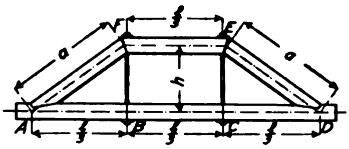


Fig. 64.—Queen post girder.

The bending moment in inch pounds in AB and BC is $M = (1/8)(W/2)(l/2)(12) = \frac{3}{8}Wl$; also $M = fS = f(\frac{1}{6}bd^2)$. The maximum unit flexural stress is, therefore,

$$f = \frac{2.25\,Wl}{bd^2}$$

Figs. 64 and 66. (Queen Post and Reversed Queen Post types)

Tension in FB and EC (Fig. 64) or compression in BF and CE (Fig. 66) $= \frac{11}{30}W$

Tension in AB, BC and CD (Fig. 64) or compression in AB, BC and CD (Fig. 66) $= \frac{11}{30}\frac{Wl}{h}$

Compression in FE (Fig. 64) or tension in FE (Fig. 66) $= \frac{11}{30}\frac{Wl}{h}$

Compression in AF and ED (Fig. 64) or tension in AF and DE (Fig. 66) $= \frac{11}{30}\frac{Wa}{h}$

As in the king post truss, to the unit stress in the members AD from the formula above must be added the flexural stress due to the timber acting as a beam. The extreme fiber stress due to this bending may be taken as

$$f = \frac{Wl}{bd^2}$$

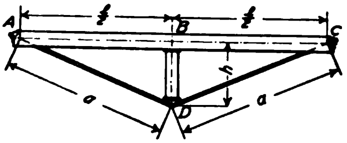


Fig. 65.—Reversed King post girder.

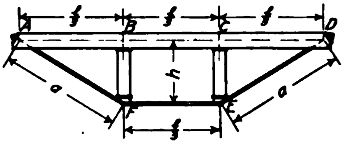


Fig. 66.—Reversed Queen post girder.

Concentrated Loading:

Figs. 63 and 65. (King Post and Reversed King Post types)

Concentrated load P at center of span.

Tension in DB (Fig. 63) or compression in BD (Fig. 65) $= P$

Tension in AB and BC (Fig. 63) or compression in AB and BC (Fig. 65) $= \frac{Pl}{4h}$

Compression in AD and DC (Fig. 63) or tension in AD and DC (Fig. 65) $= \frac{Pa}{2h}$

Obviously, there are no flexural stresses in this case to be added to the primary stresses found above.

Figs. 64 and 66. (Queen Post and Reversed Queen Post types)

Concentrated load P at B and C

Tension in FB and EC (Fig. 64) or compression in BF and CE (Fig. 66) $= P$

Tension in AB, BC and CD (Fig. 64) or compression in AB, BC and CD (Fig. 66) $= \frac{\frac{1}{3}Pl}{h}$

Compression in FE (Fig. 64) or tension in FE (Fig. 66) $= \frac{\frac{1}{3}Pl}{h}$

Compression in AF and ED (Fig. 64) or tension in AF or ED (Fig. 66) $= \frac{Pa}{h}$

The stresses resulting from these formulas are all that need to be considered.

48a. Details of Trussed Girders.—In the girders of Figs. 63 and 64, the vertical members only are of iron or steel, in the form of rods. Since such rods are short, plain rods—*i.e.*, without upset ends—should be used. Attention must be given to the washers, to the end that sufficient area be provided to avoid crushing the fibers of the timber. As great a depth as possible should be given to these girders, not alone to reduce the stresses and the deflection but in order that the stresses of the end connections may be kept within limits. With a small depth of girder, the inclination of the members *AD* and *DC* of Fig. 63, and *AF*, and *ED* of Fig. 64 will be so small that it may be found impossible to design connections at *A* and *C* of Fig. 63 and *A* and *D* of Fig. 64 that will hold. As a matter of fact, trussed girders of these types are seldom used.

The horizontal timbers of the girders of Figs. 65 and 66 may be single sticks or double or triple sticks of timber, spaced with a distance between sufficient to allow the diagonal rods to pass. One or two rods may be employed. The ends of the timbers are usually beveled off at the upper corners to provide a seat for the washers of the rods. The vertical struts may be of timber or of cast iron, and must be sufficient in section to take their stress acting as columns. The unit bearing stress between the upper end of the strut and the chord timber must be within the allowed limit for cross bearing. To accomplish this, the strut may be given the area required for bearing, or a smaller strut sufficient for column action may be employed, and a steel plate washer used. The strut should be designed with as wide a base as possible, as there is a tendency to pull the struts out of line, when the rods are tightened. Similarly, at the lower end of the struts, the bearing between rods and the strut must be examined. Cast-iron washers with grooves for the rods, are often used. To do away with the necessity for cast iron shoes, square bars are sometimes used instead of round rods, and a flat steel washer placed at the bottom of the strut, the bend in the bars being made just outside the strut.

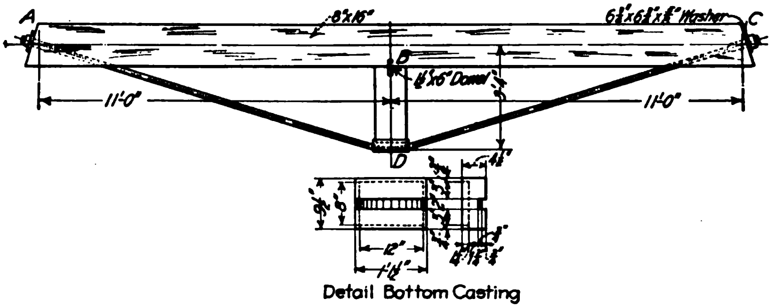


FIG. 67.—Detail of trussed girder.

Illustrative Problem.—Required to design a trussed girder, as shown in Fig. 67, for a building to be used for light storage; span 22 ft., depth on center lines 3 ft. 4 in., loading uniform 2000 lb. per lin. ft., material dense Southern yellow pine and steel.

The modulus of elasticity of the timber will be taken at 1,200,000,[1] the corresponding quantity for steel at 29,000,000. Assume dead weight of girder at 50 lb. per lin. ft. Then total load per lin. ft. = 2050 lb.

$$\text{Total load} = (22)(2050) = 45{,}000 \text{ lb.}$$

$$\text{Direct stress in beam } AB = BC = \frac{(5)(45{,}000)(22)}{(32)(3.33)} = 46{,}500 \text{ lb.}$$

$$\text{Stress in strut } BD = (\tfrac{5}{8})(45{,}000) = 28{,}100 \text{ lb.}$$

$$\text{Stress in rod } AD = DC = \frac{(5)(45{,}000)(11.5)}{(16)(3.33)} = 48{,}600 \text{ lb.}$$

$$\text{Length } a = \sqrt{(11)^2 + (3.33)^2} = 11.5 \text{ ft.}$$

Size of rod:

At 16,000 lb. per sq. in., the required area of rod is

$$\frac{48{,}600}{16{,}000} = 3.00 \text{ sq. in.}$$

A 1¾-in. square bar is required, upset at the ends to 2½ in.

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[1] This low value will be used in computing deflection, since its assumed load is largely constant or fixed.

Size of strut:

For bearing between the strut and beam the area required at 300 lb. per sq. in. is

$$\frac{28{,}100}{300} = 94 \text{ sq. in.}$$

For the column, the area required is

$$\frac{28{,}100}{1000} = 28 \text{ sq. in.}$$

Size of beam:

$$M = \frac{(\tfrac{1}{8})(45{,}000)(11)}{2} = 31{,}000 \text{ ft.-lb.}$$

Assume an 8 × 16-in. timber, S4S. The section modulus, from Table 6, p. 108, is 300.31. The maximum unit fiber stress is $\frac{(31{,}000)(12)}{300.3} = 1240$ lb. per sq. in.

Since the area of section is 116.25, the direct stress is

$$\frac{46{,}500}{116.25} = 400 \text{ lb. per sq. in.}$$

The maximum unit stress on the extreme fibers is therefore

$$1240 + 400 = 1640 \text{ lb. per sq. in.}$$

End washer:

Angle between the plane of the washer and direction of the fibers of wood is

$$\cot^{-1}\frac{11.00}{3.33} = 3.30 = 73 \text{ deg.}$$

Allowable unit pressure by Diagram 8, p. 249 = 1200 lb. per sq. in.

Area required is

$$\frac{48{,}600}{1200} = 40 \text{ sq. in.}$$

Add area hole, or 40 + 5.4 = 45.4 sq. in. = total gross area required.

Side of square washer = $\sqrt{45.4} = 6.75$ in.

The short diameter of a square nut for a 2½-in. rod is 3⅞ in.

The maximum bending moment is along the edge of nut when sides of nut and washer are at 45 degrees, and is in amount 9100 in.-lb. The full width of plate along line of edge of nut is 5.67 in. and, with this width and a flexural stress of 24,000 lb. per sq. in., the required thickness of plate is 0.64 in.

Washer will be made 6¾ × 6¾ × 11/16 in.

An 8 × 12-in. timber will be used for the strut, and top and bottom castings used as detailed in Fig. 67.

48b. Deflection.—The exact method for finding the deflection of a trussed girder is problem in least work. An approximate solution will be illustrated below. In the example of ig. 67, assume the average depth between center line of the 8 × 16-in. beam and the center ne of rod as ⅝th total depth, or 25 in. This dimension is the depth at the third point of the ngth of girder. Compute the equivalent moment of inertia of the girder at this point.

Area 8 × 16-in. timber = (7½)(15½) = 116 sq. in.

Equivalent area in steel = $(116)\left(\frac{1{,}200{,}000}{29{,}000{,}000}\right) = 4.81$ sq. in.

Area 1¾-in. square bar = 3.06 sq. in.

These equivalent areas are 25 in. on centers. Then center of gravity of combined sections is

$$25 - \frac{(4.81)(25)}{7.87} = 9.7 \text{ in.}$$

elow center line of the 8 × 16-in. beam.

Moment of inertia of combined section:

$$\begin{aligned} (4.81)(9.7)^2 &= 452.5 \\ (3.06)(25 - 9.7)^2 &= \underline{716.0} \\ & 1168.5 \end{aligned}$$

$$\text{Deflection} = \frac{5Wl^3}{384EI} = \frac{(5)(45{,}000)(18{,}399{,}744)}{(384)(29{,}000{,}000)(1168.5)} = 0.318 \text{ in., say } \tfrac{5}{16} \text{ in.}$$

It must be realized that this method is approximate only, the principal indeterminate fac-or being the assumed average depth. For the case of the reversed Queen Post type, the depth hould be taken as the distance between the center line of beam and the center line of the hori-ontal rods.

PLATE AND BOX GIRDERS

By Alfred Wheeler Roberts

For long spans and heavy loads, which are excessive for the rolled sections of beams and girders, plate or box girders, built up of plates and angles, are used. The most simple form of plate girder is composed of one web plate and four angles, as shown in Fig. 68. Another form of the plate girder is one with flange plates, as shown in Fig. 69.

For methods of determining reactions, moments, shears, and moment of inertia of sections, see chapters in Sect. 1. See also the chapter on "Steel Shapes and Properties of Sections" in Sect. 2. Steel beams and beam girders are treated in a preceding chapter. Riveting is treated in the chapter on "Connections Between Steel Members."

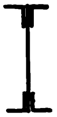


Fig. 68.

49. Determination of Resisting Moment.—There are two general methods used in determining the resisting moment of plate and box girders. The accurate method which is much to be preferred in all cases for heavy shallow girders, is called the *moment of inertia method.* In this method the procedure is the same as for determining the resisting moment of a simple rolled beam. The moment of inertia is figured for the total net section of the member and, from that, the moment of resistance or section modulus.

The approximate or *chord stress method* assumes that the tensile and compressive stresses are distributed uniformly over the entire area of the tensile and compressive flanges respectively. The moment arm of the couple, or "effective depth," then, is the distance between the centers of gravity of the flange sections.

There is absolutely no doubt but that the web takes some of the bending and relieves the flanges. Consequently, most specifications permit $\frac{1}{8}$ of the gross area of the web to be counted at the center of gravity of each flange section. For shallow girders, it is customary to design by the approximate method and then check the design by the moment of inertia method.

50. The Web.—The depth of a girder is governed by the width of the web plate and to produce the minimum deflection should not be less than $\frac{1}{12}$ of the span. Some authorities, however, permit $\frac{1}{15}$ to $\frac{1}{20}$ of the span for depth. If these ratios are used, care should be taken that there is sufficient metal in the flanges to reduce the deflection. The web should have sufficient sectional area to take all the vertical shear, which is maximum at the supports, and is generally figured at 10,000 lb. per sq. in. on the gross area of web. Many specifications give a value for shear based on the net section. The net area, which takes into account the holes caused by rivets in the end stiffeners, is sometimes assumed as $\frac{3}{4}$ the gross area. In the illustrative problems of this chapter, a shear of 10,000 lb. per sq. in. is allowed on the exact net section.

The thickness of web plates should be not less than $\frac{1}{60}$ of the unsupported distance between flange angles and not less than $\frac{5}{16}$ in. thick.

Since edges of the web plates are not likely to be straight unless planed, the back of the flange angles are usually set $\frac{1}{4}$ in. beyond the edge of the plate.

51. The Flanges.—The tension flange should be designed to have sufficient net section to take the tensile stress, allowing from 14,000 to 16,000 lb. per sq. in. in the extreme fiber. An allowable stress of 16,000 lb. is quite generally used in designing by both the moment of inertia and chord stress methods.

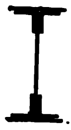


Fig. 69.

The compression flange for ordinary cases should not have less gross area than the tension flange and should not have an unsupported lateral length of more than 30 times its width (see Art. 16*e*).

If the A.R.E.A. column formula (see Sect. 1, Art. 97) is taken as a basis, and allowance made for the bracing effect of the web in a horizontal direction (see also Art. 16*e*), the maximum stress in compression flange should not be more than $16{,}000 - 200\frac{L}{b}$ and not to exceed 14,000 lb. per sq. in. for girders with angles only or with angles and flange plates. In the formula L = unsupported length and b = width of flange.

If the flange has a channel in place of a flange plate, or if it has reinforcing angles riveted to the general flange angles, thus congregating a mass of metal on the extreme edges of the section, it is permissible to stress it up to $16{,}000 - 150\frac{L}{b}$ but not to exceed 14,000 lb. per sq. in.

In proportioning members to make up flange sections, it is desirable, if possible, to put at least one-half the total flange area required in the chord angles. A flange should never be proportioned so that the center of gravity is outside the backs of the chord angles. As the required flange area varies with the bending moment, flange plates when required may be built up of several plates of different lengths, each one of which needs be only as long as theoretically needed plus a length at each end which will accommodate sufficient rivets to develop the stress carried by the plate.

If there is more than one cover plate in a flange section, it is good practice to run the plate next to the chord angles the full length, especially if the girder carries a wall or is used as a grillage girder to distribute the load over a foundation.

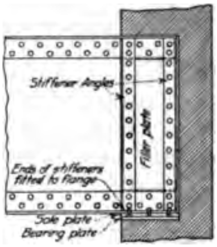


Fig. 70.

52. Stiffener Angles.—Stiffener angles should be placed at the ends of girders and at the inner edges of bearing plates and should be of sufficient section to take the end buckling (see Fig. 70). They should be riveted to the girder with a sufficient number of rivets to take the vertical shear.

To prevent buckling of the web between supports, stiffeners should also be placed at points of concentrated loads and at intermediate points when the thickness of the web is less than $\frac{1}{60}$ of the unsupported distance between flange angles (see Fig. 71). They should not, however, be spaced farther apart than the depth of the full web plate, with a maximum spacing of 5 ft. (In this connection, see Art. 16c.)

Stiffener angles at ends of girders and at points of concentrated loads should be designed as columns taking the shear or load as the case may be through sufficient rivets to transmit it to or from the web. In calculating these as columns, their length is to be considered as one-half the depth of the girder. In proportioning the sizes of these main stiffeners, the outstanding leg should not be less than $\frac{1}{30}$ of the depth of the girder plus 2 in. It is considered good practice and good construction to make the outstanding legs of stiffener angles 1 in. less than the outstanding leg of the chord angles.

In proportioning the size of intermediate stiffener angles, which are simply to prevent buckling, there is no accurate way to determine their size, but in common practice they are generally made the same size as the end stiffeners only of thinner metal, and the rivets are spaced twice as far apart as in the end stiffener angles. All stiffener angles should be milled to bear top and bottom against the chord angles and although they are sometimes crimped to avoid the use of fillers under them, it is considered by most authorities to be better construction to provide fillers under the stiffeners and avoid crimping.

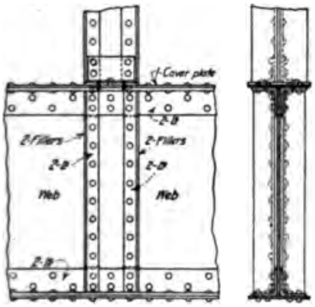


Fig. 71.

53. Web and Flange Splices.—It sometimes becomes necessary to splice the web of a girder, either on account of the excessive shipping length of the member or owing to the web plate being unobtainable in one piece. The maximum lengths at which wide plates are obtainable are given in the various steel manufacturers' handbooks. For design of web splices, see Art. 127. For design of flange splices, see Art. 128.

54. Web Riveting.—When a girder is loaded there is a tendency for the flange angles and plates to slide horizontally past the web, due to the horizontal shear. The horizontal shear at any point along the connection between flange and web per linear inch of girder is given by the general formula (see Sect. 1, Art. 63)

$$v_1 = \frac{VQ}{I}$$

in which v_1 = horizontal shear per linear inch of girder.
V = total vertical shear at section through point under consideration.
Q = statical moment of the two flanges about the neutral axis of girder at the section considered.
I = moment of inertia of entire cross section of girder about neutral axis of girder at the section considered.

The above formula gives the horizontal shear per linear inch. If a load is applied directly to the top flange at the section considered, under which no stiffener angles are used, the rivets at this point in the top flange would evidently have a vertical component of stress as well as a horizontal component. The vertical component to consider would be the load per inch of girder. The stress to use in determining the rivet pitch in such a case would be the resultant of these two components.

In especially heavy and shallow girders, where the girder is designed by the moment-of-inertia method, the rivet pitch in the web-legs of the flange angles should be determined as suggested above. For ordinary conditions, however, where the chord-stress or approximate method is used, the horizontal shear per linear inch is found by dividing the shear at the section considered by the effective depth at that section. The following simple formula may be employed for figuring rivet spacing at any point:

$$\text{Rivet pitch} = \frac{\text{effective depth} \times \text{rivet value}}{V}$$

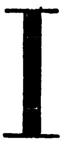


FIG. 72.

The rivet pitch at the end of a girder is usually assumed constant for a distance equal to the effective depth of the girder.

The number of rivets required in the end stiffener angles and the number of rivets required for a distance equal to the effective depth adjacent to the end is identical.

Rivets should not be spaced closer than 3 diameters of the rivets apart or a greater distance than 16 times the least metal thickness, with a maximum of 6 in. on centers.

In designing plate or box girders, the spacing of rivets should be investigated to make sure that the section can be developed for the shear, as in many cases girders are designed which cannot be properly riveted.

55. Flange Riveting.—Cover plates should be riveted at their ends with rivets spaced from 2½ to 3 in. on centers to develop the stress which the plate is taking. Some specifications call for the member to be fully developed in rivets. The rivets in the remainder of the plate should be spaced not over 16 times the least metal thickness and not over 6 in. on centers in a direction parallel with the line of stress. The maximum edge distance for any rivet should not be greater than 8 times the least thickness of metal and not over 6 in. The maximum distance apart in a direction at right angles to the line of stress, should not exceed 32 times the least metal thickness.

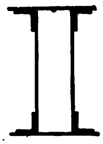


FIG. 73.

56. Web Reinforcement.—Web plates are reinforced against buckling with stiffener angles, as explained in Art. 52. If a girder has a heavy load concentrated near a support, thus producing a large amount of shear at the support, it is not economical to provide a web the entire length of the girder capable of withstanding the maximum shear. This can be overcome by reinforcing the web plate by the addition of reinforcing web plates, as shown in Fig. 72 and only extending this plate far enough beyond the point where it is needed to develop it with rivets.

57. Box Girders.—For a girder requiring a large amount of resistance to shear, or a wide flange for lateral stiffness and for distributing of loads either to or from the girder, the box girder is very effective. Two common types are shown in Figs. 73 and 74.

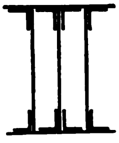


FIG. 74.

58. Combined Stresses.—Probably the best example of combined stresses due to compression and lateral bending is the top flange of a crane runway girder, which is taking compression due to the vertical load and is taking lateral bending due to the cross travel of a load on the crane. The extreme fibers should be designed to take the combined stress due to direct compression and compression produced by bending.

59. Information Regarding Illustrative Problems.—Following are illustrative problems in the design of plate and box girders for ordinary conditions. The working stresses used are taken for illustrative purposes only. Other working stresses may be substituted.

Illustrative Problem. *A Simple Plate Girder Analyzed by the Two Methods.*—What is the moment of resistance of a plate girder composed of 1 web plate 48 × ½ in. and 4 angles 6 × 6 × ⅜ in., as shown in Fig. 75?

Moment of Inertia Method.—The first step is to determine the moment of inertia of the entire section about the axis x-x, which is taken through the center of gravity of the gross section (see Art. 2*b*).

Then

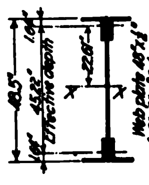


Fig. 75.

$$I \text{ of 4 angles } 6 \times 6 \times \tfrac{3}{8} \begin{cases} 4(15.4) & = 61.6 \\ 4(4.36)(22.61)^2 & = 8{,}911.18 \end{cases}$$

$$I \text{ of 1 plate } 48 \times \tfrac{1}{2} = \frac{(0.5)(48)^3}{12} = 4{,}608.0$$

$$\text{Total gross } I = 13{,}580.78$$

$$I \text{ of 2 holes } \left\{ \begin{array}{ll} \dfrac{(1.25)(0.87)^3(2)}{12} & = 0.137 \\ (1.25)(0.87)(2)(22)^2 & = 1053.7 \end{array} \right\} = 1{,}053.837$$

$$\text{Total net } I = 12{,}526.95$$

Then

$$M = \frac{fI}{c} = \frac{(16{,}000)(12{,}526\ 95)}{\tfrac{1}{2}(48.5)} = 8{,}265{,}204 \text{ in.-lb.}$$

Chord Stress Method.—One-eighth of the gross area of the web will be considered available for each flange section.

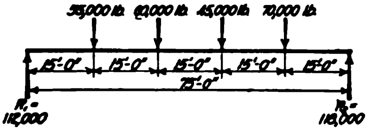


Fig. 76.

2 angles 6 × 6 × ⅜ = (2) (4.36) = 8.72
⅛ of the area of web plate = 3.00

Area in compression flange = 11.72
Area of hole in flange angles = (2) (0.87) (0.375) = 0.65

Net area in tension flange = 11.07 sq. in.

Then

$$M = (11.07)\ (16{,}000)\ (45.22) = 8{,}009{,}000 \text{ in.-lb.}$$

Illustrative Problem. *Plate Girder with Flange Plates.*—Make a general design of a plate girder to span 75 ft. and to support the concentrated loads shown in Fig. 76, with a depth limited to a 6-ft. web plate. Consider ⅛ of the gross area of the web plate as flange section and assume that allowance has been made in the loads given, to take care of the dead weight of the girder itself. Reactions are shown.

As mentioned in Art. 50, the web should not be less in thickness than 1/160 of the clear distance between flange angles. Therefore, assuming that the flange angles will have 6-in. legs against the webs, the least thickness that the web should be made is $\frac{60.5}{160} = 0.377$ in.—say ⅜ in. A 72 × ⅜-in. web will be investigated for shear. Assuming that the girder will frame into a column at the supports by means of the end stiffener angles, the number of ¾-in. rivets (5630, bearing value on ⅜-in. web) required at the end to take the maximum shear is

$$\frac{118{,}000}{5630} = 21 \text{ rivets.}$$

The net web area (allowing ⅞-in. hole for a ¾-in. rivet) is

(72) (0.375) = 27.0 sq. in.
minus (21) (0.375) (0.875) = 6.89 sq. in.

20.11 sq. in. net.

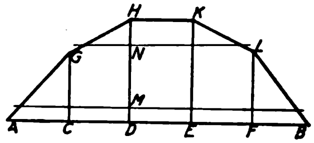


Fig. 77.

Then the web will be good for (20.11) (10,000) = 201,100 lb., and is therefore good for the shear.

As the point of maximum bending moment is at the point where the shear changes sign, M occurs at the 60,000-lb. load and equals 2,535,000 ft.-lb. Assuming the effective depth to be 5 ft. 9 in., the flange stress will be $\frac{2{,}535{,}000}{5.75} = 440{,}869$ lb. Then the flange area required will be $\frac{440{,}869}{16{,}000} = 27.55$ sq. in. net, and the flange can be composed as follows:

(⅛) (72) (0.375) = 3.375
2 angles 6 × 6 × ⅞ (minus 2 holes in each) = 16.400
1 Pl. 14 × 11/16 (minus 2 holes) = 8.421

28.196 sq. in.

The length of the cover plate can be determined either analytically or graphically. It can be found analytically by determining the point at each side of the section of maximum moment where the chord angles and portion of the web considered as flange area is sufficient to take the flange stress. The graphical method is commonly used however, where there are a number of concentrations. This method is also very convenient for a girder with a uniform load in which the bending moment varies in the form of a parabolic curve.

For the case in hand a diagram should be plotted, as in Fig. 77.

Lay off a line *A-B* to any convenient scale equivalent to the span of the girder. Lay off points to scale where the different concentrated loads occur, as *C*, *D*, *E* and *F*. Calculate the bending moments at each of these points and lay them off to some convenient scale at right angles to line *AB*, such as *CG*, *DH*, *EK* and *FL*. Draw a line connecting *A*, *G*, *H*, *K*, *L*, and *B* which will give the bending moment diagram.

At the maximum moment point *D* divide line *DH* into as many equal parts as there are square inches in the total flange area and lay off on this line the proportional part of the area contained in each portion of the flange section, such as *DM* = area of $\frac{1}{8}$ gross area of web plate, *MN* = net area of 2 angles 6 × 6 × $\frac{7}{8}$, and *NH* = net area of the 14 × $\frac{11}{16}$-in. cover plate.

Where the horizontal line passing through point *N* intersects the bending moment line each side, will be the extreme length for which cover plate is required. However, the plate should be extended each side a sufficient distance beyond these points to permit the plate to be developed with sufficient rivets before it is actually needed as a part of the girder. At each point where the concentrated loads occur there should be stiffener angles of sufficient size and with enough rivets to transmit into the web the loads applied.

The end stiffeners should be capable of taking the end web buckling and be riveted to the web with sufficient rivets to take the end shear.

Since the web is less in thickness than $\frac{1}{60}$ of the unsupported distance between flange angles, the girder must be provided with intermediate stiffener angles at a distance not over 5 ft. apart to prevent the buckling of the web.

Illustrative Problem. *Box Girder.*—Design a box girder supporting two 10-in. H-columns, each carrying a load of 176,000 lb. as shown in Fig. 78, assuming that an allowance is made in the loads given to include the dead weight of the girder. $R_1 = R_2 = 176{,}000$ lb. M(max.) = 3,520,000 ft.-lb.

As $\frac{1}{12}$ of the span is a good proportion for the depth of the web plate, assume that a 60-in. web plate will be used. On account of the manner in which the loads are delivered to the girder a double web or box girder will make the best section to use, placing the webs under the flanges of the column.

Consider the design of the web for shear. As the reaction is 176,000 lb. and the allowable shearing stress 10,000 lb. per sq. in., a net area of $\frac{176{,}000}{10{,}000} = 17.6$ sq. in., will be needed. The number of rivets required in the end stiffener angles, to take the end reaction, assuming $\frac{3}{4}$-in. rivets in single shear (4420 lb., shearing value of each rivet) will be $\frac{176{,}000}{(2)\ (4420)} = 20$ rivets in each web. Therefore the net width of the web plate, allowing $\frac{7}{8}$-in. hole for a $\frac{3}{4}$-in. rivet, will be 60 − (20)(0.875) = 42.5 in. Then the combined thickness of the two webs required will be

$$\frac{17.6}{42.5} = 0.41 \text{ in.}$$

As each web should not be of less thickness than $\frac{5}{16}$ in., each web will be made 60 × $\frac{5}{16}$ in.

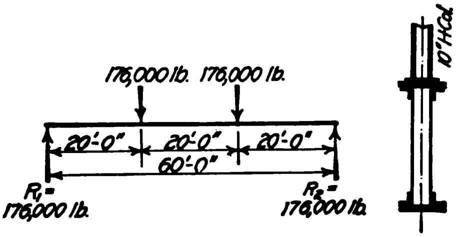


FIG. 78.

Assuming the effective depth to be 57 in. = 4.75 ft., the maximum flange stress will be $\frac{3{,}520{,}000}{4.75} = 741{,}052$ lb. Then the flange area required at this point will be $\frac{741{,}052}{16{,}000} = 46.31$ sq. in. net. Considering $\frac{1}{8}$ of the gross area of the web plate as flange area and assuming the cover plates to be 24 in. the flange may be composed of the following:

($\frac{1}{8}$) (2) (60) ($\frac{5}{16}$) = 4.68
2 angles 6 × 6 × $\frac{13}{16}$ (minus 2 holes in each) = 15.34
1 Pl. 24 × $\frac{5}{8}$ (minus 2 holes) = 13.90
1 Pl. 24 × $\frac{9}{16}$ (minus 2 holes) = 12.46
46.38 sq. in.

As the maximum flange section is only needed for a part of the length of the girder, there is a point where the 24 × $\frac{9}{16}$-in. cover plate can be omitted, but the 24 × $\frac{5}{8}$-in. plate should be continued the full length of the girder in order to hold the two webs together. It is not necessary to make the thickest plate the one to be extended, but it is considered good practice to place the thickest plate immediately on the chord angles.

In order to determine how long the upper cover should be, it can be determined graphically as explained in the preceding problem. The length, however, can be determined analytically as follows: The area of the members in the flange, excluding the 24 × $\frac{9}{16}$-in. plate, is 33.92 sq. in. net. This amount of area will develop a flange stress of (33.92)(16,000) = 542,720 lb., and a bending moment of (542,720)(4.75) = 2,577,920 ft.-lb. Then the point on the girder at each end where this flange area will be used to its limit, will be the point where the bending moment will be 2,577,920 ft.-lb. or a distance from the end of

$$\frac{2{,}577{,}920}{176{,}000} = 14.6472 \text{ ft. or 14 ft. } 7\tfrac{3}{4} \text{ in.}$$

Therefore the length of the cover plate will be 30 ft. $8\frac{1}{2}$ in. plus the distance at each end necessary to develop with rivets the stress carried by the plate.

The maximum pitch of the rivets connecting the web to the chord angles should be as follows: In the distance between the support and the nearest concentrated load the pitch should not exceed

$$\frac{(57)(8840)}{176{,}000} = 2.86 \text{ in.}$$

n the distance between the concentrated loads, where the shear is theoretically zero, the rivet pitch is theoretically indeterminate, but as the rivet pitch of any rivet in the girder should not exceed 16 times the least metal thickness in a line parallel to the line of stress, the maximum pitch in this case should not exceed 5 in.

The end stiffeners should be designed to take the end shearing stress and, assuming that the ends of the girder will frame into a supporting member, only two stiffener angles can be used, one on the outside of each web on account of stiffener angles on the inside of webs being inaccessible for field riveting.

As the $\frac{L}{r}$ for the ordinary girder stiffener is usually small, and since according to the A.R.E.A. column formula see Sect. 1, Art. 97) the maximum allowable stress per sq. in. should not exceed 14,000 lb., it will be found (except n special cases) that it is safe to figure stiffener angles at 14,000 lb. per sq. in. Therefore the end stiffener angles should have an area of

$$\frac{176,000}{14,000} = 12.57 \text{ sq. in.}$$

Two 5 × 5 × $\frac{11}{16}$-in. angles will be satisfactory.

At the two points of concentrated loads, there should be eight stiffener angles, two on each side of each web, and the four on the inside of the girder should be connected with a web plate, forming a diaphragm or separator between the two webs, all being milled to bear top and bottom and with sufficient rivets to take the load into the web.

As the concentrated load is the same as the end reaction, there will be needed in the eight stiffener angles a combined area of 12.57 sq. in., or 8 – $3\frac{1}{2}$ × 3 × $\frac{5}{16}$-in. angles will suffice.

As the thickness of the webs is less than $\frac{1}{60}$ of the distance between the flange angles, the girder should be provided with intermediate stiffener angles on both sides of both webs, not over the effective depth of the girder apart.

Illustrative Problem. *Distributing Grillage Girder.*—Design a girder distributing the load of two columns over foundation, as shown in Fig. 79, assuming the bearing pressure on the foundation at 30,000 lb. per sq. ft. and the distance "A" limited to 2 ft. by local conditions.

The center of gravity or point c.g. of the loads must first be determined.

$$\text{Distance } B = \frac{(800,000)(16.0)}{1,440,000} = 8.89 \text{ ft.}$$

$$\text{Distance C} = \frac{(640,000)(16.0)}{1,440,000} = 7.11 \text{ ft.}$$

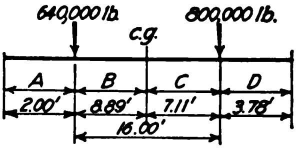


FIG. 79.

In order for the girder to equally distribute the loads over the foundation, the girder must be made symmetrical in length about the center of gravity of the loads. Knowing distance A to be 2 ft., the distance D is readily determined, making a total length of 21.78 ft. for the girder.

Since the total load = 1,440,000 lb., then the load per linear foot will be $\frac{1,440,000}{21.78}$ = 66,115 lb. If the allowable bearing capacity of the foundation is 30,000 lb. per sq. ft., then the width of the girder must be $\frac{66,115}{30,000}$ = 2.2038 ft., say 27 in. On account of the required width of the girder flange, a box girder as shown in Fig. 80 will be most adaptable. The center web will be figured to take one-half, and the side webs one-quarter each of the total load.

The next thing to consider is the number and the size of the stiffener angles required under each of the column loads, and also the number of rivets required in each stiffener angle, so that the net width of the web plates can be determined. At the point of the 800,000-lb. concentration, a combined area will be needed in the stiffener angles of $\frac{800,000}{14,000}$ = 57.15 sq. in. Assuming 16 stiffener angles at this point, 16 angles 5 × $3\frac{1}{2}$ × $\frac{1}{2}$ will give sufficient area. Assuming the rivets to be in single shear on the outer webs and in double shear for the middle web, $\frac{800,000}{(16)(4420)}$ = 12 rivets will be needed in each stiffener. As the maximum shear = (8)(66,115) = 528,920 lb., a total net web area will be needed of $\frac{528,920}{10,000}$ = 52.89 sq. in. Assuming a web 48 in. deep, the net width will be 48 – (12)(0.875) = 37.5 in. The total web thickness required will be $\frac{52.89}{37.5}$ = 1.40 in. Then the center web should be $\frac{1.40}{2}$ = 0.7in. thick, or say $\frac{3}{4}$ in. The side webs should be $\frac{1.40}{4}$ = 0.35 in. thick, or say $\frac{3}{8}$ in. The girder will be made $48\frac{1}{2}$ in. back to back of angles.

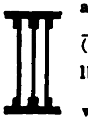


FIG. 80.

At the point of the 640,000-lb. concentration a combined area will be needed in the stiffener angles of

$$\frac{640,000}{14,000} = 45.72 \text{ sq. in.}$$

Assuming 16 stiffener angles at this point, 16 angles 5 × $3\frac{1}{2}$ × $\frac{3}{8}$ will give sufficient area. Taking the rivet values as before $\frac{640,000}{(16)\ (4420)}$ = 10 rivets will be needed in each stiffener angle. As the number of rivet holes to be deducted from the web plate at this point is less than at the other point and the maximum shear is the same, the webs selected are more than sufficient.

The maximum bending moment will occur midway between the concentrated loads and will equal $\frac{(66,115)(16)(16)}{8}$ = 2,115,680 ft.-lb. Assuming an effective depth of 45 in., the maximum flange stress will equal $\frac{2,115,680}{3.75}$ =

564,181 lb. The net flange area required will be $\frac{564{,}181}{16{,}000} = 35.26$ sq. in. By proportioning the flange area with one-half for the center portion and one-quarter each for the side members, the flange may be composed as follows:

Web – ⅛ × 48 × 1½	= 9.00
4 angles 6 × 6 × ½ (minus 2 holes each angle)	= 19.48
1 cover plate 27 × ⅜ (minus 4 holes)	= 8.81
	37.29 sq. in.

The cover plate both top and bottom should be extended the full length of the girder although it is not needed for strength. The rivet heads on the under side of the bottom cover plate should either be countersunk and chipped or the girder should be thoroughly grouted with a thin grout, to insure the girder bearing properly throughout its entire length and width.

As the side webs are less in thickness than 1/60 of the clear distance between the chord angles, these webs should be provided with intermediate stiffener angles to prevent buckling, at the ends and at a distance not greater than the effective depth of the girder apart.

Although there are no intermediate stiffeners required for the center web, the ends of these webs should have stiffeners. In designing the base of the columns resting on this girder, it should be seen that the load is distributed in a proper manner to the girder, so as to make each elementary portion of the girder take that portion of the load for which it is designed. This can be done by means of stiffener angles and by getting as much of the column in direct bearing over the girder stiffener angles as possible.

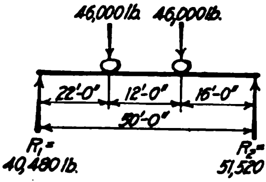


FIG. 81.

As the shear of this girder varies from zero, at the point between the two concentrations and at the extreme ends, to maximum at the points of concentrations, the web rivet spacing should be figured as explained in Art. 54, by dividing the girder into sections equal to the effective depth and using the maximum shear occurring in that division as a basis.

Rivets along the bottom flange will be subjected to vertical stress in addition to the horizontal stress due to longitudinal shear. The vertical stress is caused by the uniform load applied in distributing the load over the foundation. The rivets along this flange should be figured to take the resultant of the horizontal and vertical forces.

On very heavy work of this type, the web plates are chipped to bear directly against the cover plate which is good construction, but unless the shop work is exceptionally good it is apt to overstress the web rivets due to the web not bearing properly.

The above type of girder is also used to distribute the loads to a lower layer of grillage beams, where it would be impractical to make the girder wide enough to get sufficient bearing over the foundation.

Illustrative Problem. *Girder with Moving Loads.*—Design a crane runway girder of 50-ft. span, to support a 10-ton crane having two wheels on the truck 12-ft. on centers, with a load on each wheel including impact of 46,000 lb. as shown in Fig. 81. It will be assumed that an allowance is made in the loads given for the dead weight of the girder.

On a girder carrying moving loads, the bending moment throughout the girder varies for every different position of the loads. On a girder with two equal moving loads, the maximum moment will occur under one of the loads when the quarter point distance between the two loads is coincident with the center of the span of the girder (see Sect. 1, Art. 58e). The maximum moment is found to be 890,560 ft.-lb.

Assuming the web plate of the girder to be 48 in. deep and the chord angles 48½ in. back to back, the effective depth will be about 45 in., or 3.75 ft. Then the maximum flange stress due to vertical loads will be $\frac{890{,}560}{3.75} = 237{,}482$ lb. and the required flange area will be $\frac{237{,}482}{16{,}000} = 14.84$ sq. in. The flange area required is correct for the bottom flange only. Assuming a web plate 48 × 5/16 and taking ⅛ of the web-plate area as flange section, the bottom flange may be composed as follows:

Web ⅛ × 48 × 5/16	= 1.87
2 angles 6 × 6 × ⅝ (minus one hole in each)	= 13.14
	15.01 sq. in.

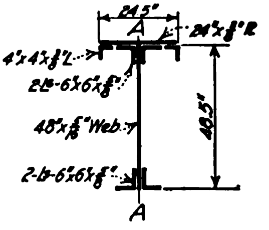


FIG. 82.

The top flange will get the same stress as the bottom flange due to the vertical loads and in addition will get a lateral stress due to bending caused by the cross travel or acceleration of the crane trolley, from which the load is suspended. The amount of this force is usually taken as 2/10 of the capacity of the crane, or 2 tons in this case, causing a force of 2000 lb. acting on each wheel. The position of the wheels causing the greatest lateral bending moment on the girder is the same position which causes the greatest vertical bending moment. Therefore the greatest lateral bending moment will be directly proportional to the maximum vertical bending moment, or $\frac{2000}{46{,}000}(890{,}560)(12) = 464{,}640$ in.-lb. Then the top flange must be designed to take a direct stress in compression of 237,482 lb. plus a cross-bending stress of 464,640 in.-lb.

Assuming a top flange section as shown in Fig. 82, the top flange should be designed within the following limitations (see Art. 51): The maximum combined compressive stress should not exceed $16{,}000 - 150 \dfrac{\text{length}}{\text{flange width}}$, with a maximum stress of 14,000 lb. per sq. in., figured about axis A-A.

By the method explained in Art. 21 the bending moment should be transposed to an equivalent direct compressive stress and added to the direct maximum compressive stress due to the vertical loads. The flange should then be designed for the sum of the two stresses. It will be found that a top flange composed of the following members will be of sufficient size:

1 plate—24 × 3/8
2 angles—6 × 6 × 5/8
2 angles—4 × 4 × 3/8

The next step in the design is to determine the maximum end shear so that the end stiffener angles and the web can be designed. The position of the loads which will give the greatest shear is when one wheel is at the end and the other 12 ft. away from the end. The maximum shear is found to be 80,960 lb.

The total area required in the end stiffener angles is $\dfrac{80{,}960}{14{,}000} = 5.78$ sq. in. Assuming 2 stiffener angles, it is found that 2 angles 5 × 3½ × 3/8 will be sufficient. Assuming rivets as bearing on a 5/16-in. web plate at 4690 lb. each, $\dfrac{80{,}960}{4690} = 18$ rivets are required in the stiffener angles.

The net area required in the web plate for shear will be

$$\frac{80{,}960}{10{,}000} = 8.09 \text{ sq. in.}$$

The net width of the web plate will be $48 - (18)(0.875) = 32.25$ in. Since 8.09 sq. in. are needed in the web, then the thickness should be $\dfrac{8.09}{32.25} = 0.25$ in. or ¼ in. As the web of a girder should not be less than 5/16 in. thick, a 48 × 5/16-in. web will be used.

Since the web is less in thickness than 1/60 of the unsupported distance between flange angles, intermediate stiffener angles should be provided to prevent web buckling at a distance apart not greater than the effective depth of the girder. The web rivet spacing for the first 12 ft., from each end should be the same, as the maximum shear will not change until the second wheel position is reached. As the top cover plate with its outside angles is acting as a flat girder taking lateral thrust, the rivets connecting the web and outer angles should be spaced the same as any girder using the shears produced by the horizontal forces.

DESIGN OF PURLINS FOR SLOPING ROOFS

By W. S. Kinne

60. Purlins Subjected to Unsymmetrical Bending.—A *purlin* is a member, generally a simple beam, which supports the roofing between adjacent trusses. Fig. 83 shows the position of a purlin with respect to the other parts of a roof. A complete discussion of choice of purlin sections, details of connections of purlins to trusses, and methods of fastening roof covering to purlins will be found in Sect. 3.

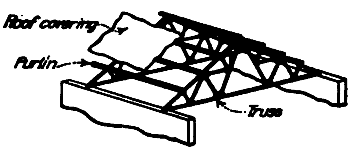


Fig. 83.

As shown in Fig. 147, p. 460 for steel roof trusses, and in Fig. 146, p. 459 for wooden roof trusses, purlins consisting of rolled shapes, or wooden beams, are usually placed with the webs, or sides, perpendicular to the top chord of the truss. Since in most cases the applied loads are vertical, or nearly so, it follows that the plane of loading and the principal axes of the section do not, in most cases, lie in the same plane. Problems in design and stress determination for such conditions can not be solved by the methods described in the chapter on "Simple and Cantilever Beams," Sect. 1, but require more general formulas which take into account the fact that the plane of bending and the principal axes of the section are not coincident. Bending of this nature is known as *unsymmetrical bending,* the formulas for which are given in the last chapter of Sect. 1.

61. Load Carried By a Purlin.—The amount and character of the load to be carried by a roof purlin depends to some extent upon the kind of roof covering, the slope of the roof, and the location of the structure. These points are considered in detail in Sect. 3, Arts. 133 to 136 incl., where tables of values are given for the different loads.

The load which a purlin must be designed to carry is a combination of the weight of the purlin and roof covering, the snow load, and the wind load. These loadings are to be combined so as to give the maximum possible stress on the beam section. In general three combinations of loading are used. They are:

(1) Dead load and snow load.
(2) Dead load and wind load.
(3) Dead load, wind load, and one-half snow load.

Under Case 3 only one-half of the snow load is considered. This is due to the fact that maximum wind and snow loadings are not likely to occur at the same time. If a high wind is blowing at the time snow is falling, the snow will be blown from the roof as fast as it falls. In the case of a wet snow or sleet, part of the snow will stay on the roof in spite of the wind. An allowance of one-half the maximum snow load seems to be reasonable for this condition.

The dead and snow loads are vertical forces, while the usual assumption regarding the wind load is that it acts perpendicular to the surface of the roof. For the combinations given above, (1) represents a vertical load, while (2) and (3) are inclined at an angle to the vertical.

62. Conditions of Design.—The conditions of the design are governed to some extent by the roof covering. Where the covering is very rigid, as in the case of wooden sheathing on common rafters, the loads can be resolved into components parallel and perpendicular to the roof. The component parallel to the roof is assumed as carried by the sheathing, and the component perpendicular to the roof is assumed as carried by the purlin. This is equivalent to assuming that the beam section is free to bend only in a plane perpendicular to the roof.

Where the roof covering consists of a material such as corrugated steel, which provides little or no lateral support for the purlin, the assumptions made above can not be used. It is then necessary to design the purlin as a beam which is free to bend in any direction, making use of the methods of unsymmetrical bending set forth in the last chapter of Sect. 1.

Purlins designed under this assumption are likely to require excessively large sections. To avoid this, the purlins are often partially supported laterally by means of tie rods. Smaller sections can then be used for the purlins.

The methods of design to be used in the cases mentioned above will be followed out for typical cases which will illustrate the methods to be used.

63. Design of Purlins for a Rigid Roof Covering.—Let it be required to design the sheathing, rafters, and purlins for a roof capable of withstanding the maximum combination of the dead load of its members and the wind and snow loads given in Sect. 3, Art. 137. The material is to be pine with a working stress of 1000 lb. per sq. in. Assume that the roof is covered with shingles; that the span of the rafters is 9 ft. (measured along the line of the roof surface, which makes an angle of 30 deg. with the horizontal), and that the trusses are 12 ft. apart. Fig. 84 (*a*) shows the general arrangement of members.

In making up the combinations of loads carried by the members it will be found convenient to determine the resultant load carried by a single square foot of roof. The resultants for the several combinations given above are as follows:

Case 1.—From the tables given in Sect. 3, Art. 133, shingles weigh about 3.0 lb. per sq. ft. of roof, and 1-in. sheathing weighs about 4.0 lb. per foot board measure. The dead load is then 7.0 lb. per sq. ft. of roof, a vertical load. From Table 8, p. 467, the snow load for a roof at an angle of 30 deg. to the horizontal is 15.0 lb. per sq. ft. of roof. The total vertical load is then 22.0 lb. per sq. ft. of roof, and the component perpendicular to the roof is 19.0 lb. per sq. ft., as determined by the force diagram of Fig. 84(*c*).

Cases 2 and 3.—It is quite evident that the resultant for Case 3 has a greater component perpendicular to the roof than Case 2. As the direction of bending is not in question under the assumed conditions, we can pass at once to Case 3.

The dead load for Case 3 is the same as for Case 1, and the snow load is one-half as large as for Case 1. The vertical component of loading is then, 4 + 3 + 7.5 = 14.5 lb. per sq. ft. of roof. From Table 7, p. 467, the wind pressure normal to the roof is 24.0 lb. per sq. ft. of roof. As these loads are not in the same direction, the resultant can be obtained by means of a force diagram. The component of load perpendicular to the roof can be determined by resolving forces parallel and perpendicular to the roof surface. The force diagram of Fig. 84(*e*) shows that the component perpendicular to the roof is 36.9 lb. per sq. ft. of roof. Similar calculations have been made for Case 2; the force diagram is shown in Fig. 84(*d*).

Design of Sheathing.—The sheathing is not usually designed, except where unusual conditions are encountered, such as heavy loads or rafter spacing greater than the normal, which is from 16 to 24 in. Under normal conditions, 1-in. sheathing will be found to provide sufficient strength.

In the case under consideration, assume that 1-in. sheathing is used and that the spacing of rafters is 24 in. The moment due to the normal component of Case 3 for a section of sheathing 1 ft. wide is, $M = \frac{1}{8}wl^2 = \frac{1}{8}(36.9)(2)^2(12) = 221.4$ in.-lb. This moment is resisted by a 1 × 12-in. section of sheathing, for which the section modulus is $I/c = \frac{1}{6}bd^2 = \frac{1}{6}(12) \times (1)^2 = 2.0$ in.³ The resulting fiber stress is then $f = Mc/I = 221.4/2.0 = 110.7$ lb. per sq. in. This stress is very low, indicating that for ordinary conditions the design need not be carried out.

Design of Common Rafters.—A 2 × 6-in. rafter will be assumed. At 4 lb. per ft. board measure, the dead weight per foot of rafter is $(2 \times \frac{6}{12})4 = 4$ lb. The roof area per foot of rafter is 2.0 sq. ft., and the normal load to be carried for Case 3 is $2 \times 36.9 = 73.8$ lb. per ft. of rafter. Adding the weight of the rafter, the total load to be carried by the rafter is a uniform load of 77.8 lb. per ft. The moment is $M = \frac{1}{8}wl^2 = \frac{1}{8}(77.8)(9)^2(12) = 9460$ in.-lb.

The section modulus of a 2 × 6-in. rectangle is $\frac{1}{6}bd^2 = \frac{1}{6}(2)(6)^2 = 12$ in.³, and the fiber stress is $f = Mc/I = 9{,}460/12 = 788.0$ lb. per sq. in. As the allowable fiber stress is 1,000 lb. per sq. in., the assumed section is sufficient. Rafter sections come in commercial sizes, which are 2 × 4, 2 × 6, 2 × 8, etc. It is therefore not possible to meet exactly the allowable fiber stress conditions with the available sections.

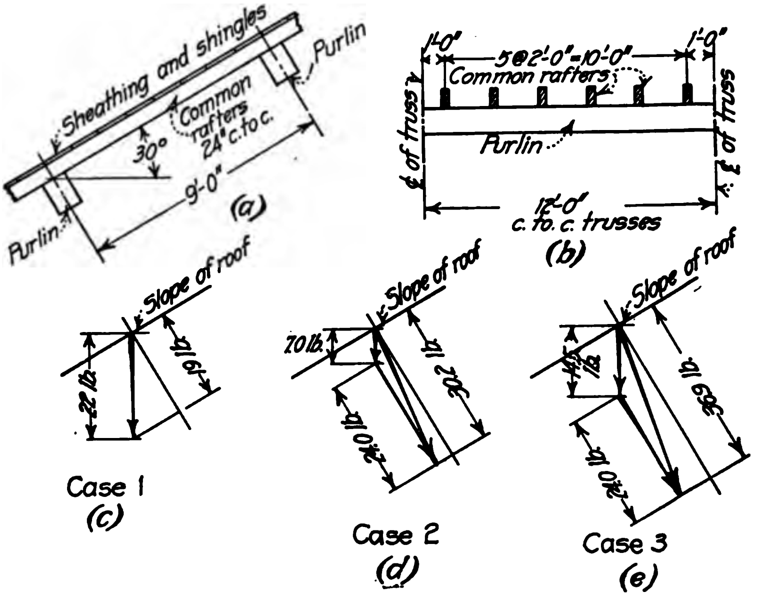


Fig. 84.

Design of Purlins.—As shown in Fig. 84(*a*), the purlin section is set at right angles to the rafter. It is then subjected to a normal load due to the rafters from adjacent panels. In some cases the applied loads are considered to be uniformly distributed along the purlin, and in other cases the loads are assumed as concentrated at each rafter. This latter assumption more nearly approximates the actual conditions; it will be used in this design.

As shown in Fig. 84(*a*), each purlin carries the ends of two rafters. Each rafter load is then due to the normal load on 9 ft. of rafter. Including the weight of the rafter, each load is $9 \times 77.8 = 700$ lb. Fig. 84(*b*) shows the position of the loads. It will be found that the maximum moment for the position shown is slightly less than for an arrangement which places a load directly at the center of the purlin. From Fig. 84(*b*), the moment at the beam center is, $M = [(2100)(6) - 700(1 + 3 + 5)]\,12 = 75{,}600$ in.-lb. Assuming a 6 × 10-in. purlin, whose weight is $(6 \times 10/12)4 = 20$ lb. per ft., the moment due to its weight is $M = \frac{1}{8}wl^4 = \frac{1}{8}(20)(12)^2(12) = 4{,}320$ in.-lb. The total moment is then $75{,}600 + 4320 = 79{,}920$ in.-lb.

For allowable $f = 1000$ lb. per sq. in., $I/c = M/f = 79{,}920/1{,}000 = 79.92$ in³. The section modulus of the assumed 6 × 10-in. purlin is $I/c = \frac{1}{6}bd^2 = \frac{1}{6}(6)(10)^2 = 100$ in.³ which is sufficient. This is as close an agreement between assumed and adopted sections as is possible, using commercial size.

64. Design of Purlins for a Roof with a Flexible Roof Covering.—In the preceding article the design is given for a purlin section for a roof which is so rigid that it is possible to assume that the purlin is supported laterally so that it is necessary to provide only for bending in a plane perpendicular to the roof surface. A case will now be considered where the roof covering is not rigid enough to provide this support. The purlin will have to be designed as if it were free to bend in any direction. This is a case of unsymmetrical bending. Two cases will be considered, one in which the purlin is free to bend in any direction, the other in which the purlin is partially supported by tie rods.

64a. Purlin Free to Bend in any Direction.—A purlin is to be designed to support a corrugated steel roof. The purlins are to be spaced 3 ft. apart, and the roof surface is inclined at an angle of 30 deg. to the horizontal; trusses are spaced 16 ft. apart.

The working loads will be taken the same as for the preceding design, and the working stress in the steel will be taken as 16,000 lb. per sq. in. Combinations of loading similar to those for the wooden purlin will be made, and a purlin section determined by the methods used in the illustrative problem, p. 88.

From Table 3, p. 459, 24-gage corrugated steel, weighing 1.3 lb. per sq. ft., can be used to span 3 ft. As stated in Sect. 3, Art. 185*b*, an anti-condensation lining, weighing 1.3 lb. per sq. ft. is to be used in connection with the corrugated steel. The total weight of covering is then 2.6 lb. per sq. ft. To this must be added the weight of the purlin. In the preliminary design, the purlin was assumed to weigh 4.0 lb. per sq. ft. of roof. After the purlin section was determined, its true weight was found and the calculations revised as given below.

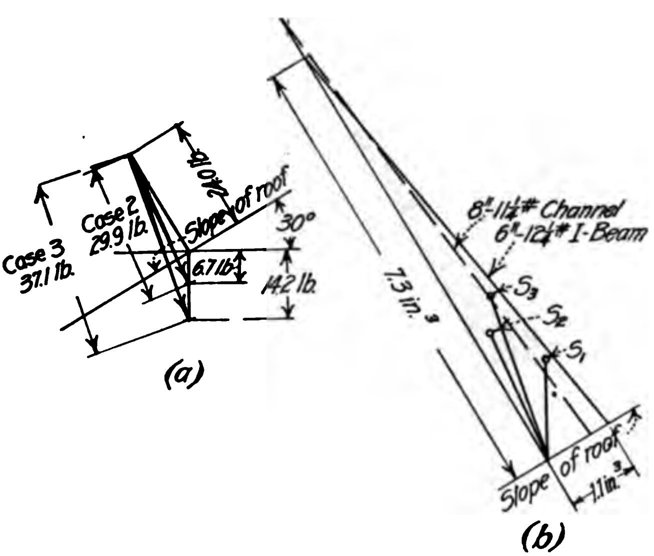


FIG. 85.

Case 1. *Dead Load and Snow Load.*—As given above, the weight of the roof covering is 2.6 lb. per sq. ft. of roof. The revised purlin weight is 4.1 lb. per sq. ft. of roof. As in the preceding design, the snow load is 15 lb. per sq. ft. of roof. The total vertical load is then, 2.6 + 4.1 + 15.0 = 21.7 lb. per sq. ft. As the purlins are 3 ft. apart, the load per ft. of purlin is 3 × 21.7 = 65.1 lb. Considering the purlin as a simple beam of span equal to the distance between trusses, 16 ft., the moment to be carried is, $M = \frac{1}{8}wl^2 = \frac{1}{8}(65.1)(16)^2(12) = 25{,}100$ in.-lb. For an allowable working stress of 16,000 lb. per sq. in., the required section modulus is $S = M/f = 25{,}100/16{,}000 = 1.57$ in.^{3} This value is shown in the proper position in Fig. 85(*b*), and is the *S* value denoted by 1.

Case 2. *Dead Load and Wind Load.*—The dead load is the same as for Case 1, and the wind load is a normal load of 24 lb. per sq. ft. of roof, as in the preceding design. In Fig. 85(*a*), the resultant of the dead and wind loads as determined graphically, is 29.9 lb. per sq. ft. The load per ft. of purlin is 3 × 29.9 = 89.7 lb.; the moment to be carried is $M = \frac{1}{8}wl^2 = \frac{1}{8}(89.7)(16)^2(12) = 34{,}500$ in.-lb.; and the required $S = M/f = 34{,}500/16{,}000 = 2.16$ in.^{3} This is shown in Fig. 85(*b*) in the direction determined by the force diagram of Fig. 85(*a*).

Case 3. *Dead Load, Wind Load, and One-half Snow Load.*—The dead load is the same as for Case 1, and the wind load is the same as for Case 2. One-half the snow load, as given by Case 1, is 7.5 lb. per sq. ft. of roof. The total vertical load is then 14.2 lb. per sq. ft. of roof, and the normal load is 24 lb. per sq. ft. The resultant of the loads, which is 37.1 lb. per sq. ft., is shown in amount and direction on Fig. 85(*a*).

The load per foot of purlin is 3 × 37.1 = 111.3 lb.; the moment to be carried is $M = \frac{1}{8}wl^2 = \frac{1}{8}(111.3)(16)^2(12) = 42{,}800$ in.-lb.; and $S = M/f = 42{,}800/16{,}000 = 2.67$ in.^{3} This is shown in position in Fig. 85(*b*).

Determination of Beam Section.—A purlin will be selected from I-beam and channel sections with the intention of keeping the weight as low as possible. It is usually specified that the depth of beam section shall be not less than $\frac{1}{60}$ of the span. This is done to avoid the use of sections for which the deflection would be excessive.

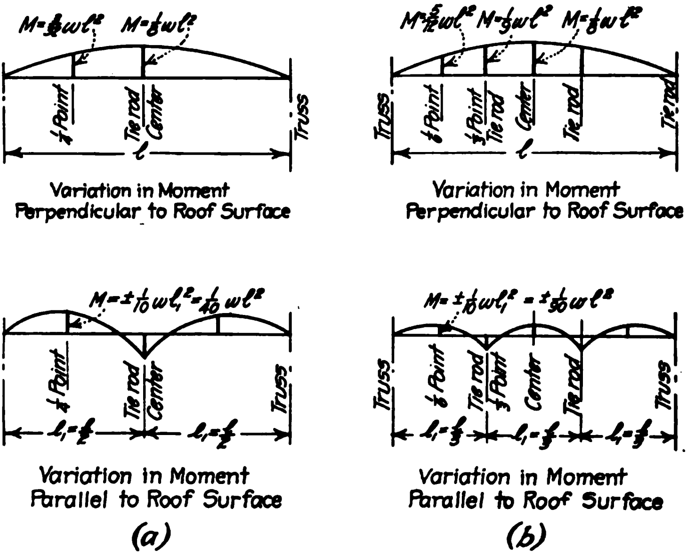
In Fig. 85(*b*), the S-polygon for a 6-in. 12¼-lb. I-beam is shown. This section is slightly larger than necessary, but it provides a closer fit than any other section of its weight. The true weight of the section is 12.25/3 = 4.1 lb., the value used in the revised calculations.

Fig. 85(*b*) also shows the S-Polygon for an 8-in. 11¼-lb. channel. This section does not provide sufficient strength, since S_1 projects beyond the S-Line. As other channels are heavier than the adopted I-beam, there is nothing to be gained by further trials.

64b. Purlin Supported Laterally by Tie Rods.—Lateral support for purlins is generally provided by means of tie rods where the roof covering, such as corrugated steel, is not rigid enough to provide the proper support. These tie rods consist of round rods fastened to the web of the purlin section in the manner shown in Fig. 88. The ties should extend over the ridge, forming a continuous line between the eaves. This must be done to avoid an excessive side pull on the ridge purlin. If the arrangement of purlins at the ridge is such that a continuous line can not be used, then the upper ties should be run diagonally to the truss.

The number of ties required for each purlin will depend upon the length of purlin to be supported and the load to be carried. Generally a single line of ties at the center of the purlin will be found sufficient. Tie rods will not be found necessary for lateral support in the case of roofs where the slope is less than about 3 in. to 1 ft. It is considered good practice to use tie rods in roofs with a rigid covering because of the lateral support provided for the purlins during the erection of the structure. The purlins are held in line without additional falsework until the roof covering is applied.

When a purlin is supported laterally by tie rods, the span of the beam, for components of load parallel to the roof surface, is equal to the distance between the tie rods, or between the tie rods and the truss. As far as these loads are concerned, the purlin is a continuous beam supported at its ends by the trusses and at intermediate points by the tie rods. For components of load perpendicular to the roof surface, the span of the purlin is equal to the distance between the trusses, as in the preceding design.



Variation in Moment Perpendicular to Roof Surface

Variation in Moment Perpendicular to Roof Surface

Variation in Moment Parallel to Roof Surface

(a)

Variation in Moment Parallel to Roof Surface

(b)

Fig. 86.

The applied loads are uniform per foot for both components of loading. They are determined by resolving the resultant forces, determined as for the preceding design, into components parallel and perpendicular to the roof surface. Moments at critical points can be determined by the methods given in Sect. 1 for simple and continuous beams.

In calculating the moments to be carried by a purlin, it will probably be best to assume that the purlins are only long enough to span the distance between adjacent trusses. The moment due to the component of loads perpendicular to the roof surface will then be given by the formula $M = \frac{1}{8}wl^2$. It will be found that if a purlin be assumed to span several trusses, and the moments calculated by continuous girder methods, the moment to be provided for will be only slightly less than for a simple beam.

For components of load parallel to the roof surface, the purlin can be considered as a continuous beam supported at its ends by the trusses, and at other points by the tie rods. The supports provided by the tie rods are not as rigid as those provided by the truss, so that the continuous girder coefficients given in Sect. 1, Art. 72(*d*), should be modified somewhat. Fig. 86(*a*) shows the values proposed for cases in which the purlin is assumed as divided into two parts by the tie rod, and Fig. 86(*b*) shows the values where the tie rods divide the purlin into three parts. It is assumed that the coefficient is $\frac{1}{10}$ instead of $\frac{1}{8}$, and that the span is equal to the distance from truss to tie rod.

In making use of the S-Polygon methods in the design of purlins for the assumed conditions, it will be necessary to determine the resultant moment at the tie rod and also at a point half way between the tie rod and the truss. These resultant moments are equal to the vector sum of the component moments parallel and perpendicular to the roof surface. The values of the flexural modulus, S, are determined from these resultant moments, and the required and provided S compared by the methods used in the preceding design.

A purlin will now be designed supported by tie rods. The conditions will be taken the same as for the preceding design, with the futher condition that the purlin is to be supported by a line of tie rods placed at the center of the purlin.

As the depth of the purlin is usually limited to $\frac{1}{30}$ of the span, a 6-in. section must be used. The 6-in. section of least weight is a 6-in. 8-lb. channel, which will be taken as the trial section. The weight of the assumed section per square foot of roof surface is $\frac{8}{3} = 2.7$ lb. Using other values as in the preceding design, the several combinations are as follows:

Case 1. *Dead Load and Snow Load.*—As before, the dead load due to corrugated steel and lining is 2.6 lb. per sq. ft. of roof, and the snow load is 15.0 lb. per sq. ft. The weight of the assumed purlin section as given above is 2.7 lb. per sq. ft. of roof. The total vertical load is then 20.3 lb. per sq. ft. of roof. From the force diagram of Fig. 87(a) the component of this load parallel to the roof surface is 10.2 lb. per sq. ft., and the component perpendicular to the roof is 17.6 lb. per sq. ft.

Using the oecfficients shown on Fig. 86(a), the component of moment parallel to the roof is $+\frac{1}{40}wl^2 = \frac{1}{40}(+10.2)(3)(12)(16)^2 = +2350$ in.-lb. at the quarter point, and -2350 in.-lb. at the tie rod. The component of moment perpendicular to the roof is $+\frac{3}{32}wl^2 = +\frac{3}{32}(17.6)(3)(12)(16)^2 = +15{,}200$ in.-lb. at the quarter point, and $+\frac{1}{8}wl^2 = +\frac{1}{8}(17.6)(3)(12)(16)^2 = +20{,}300$ in.-lb. at the tie rod.

The resultants of these moments, which are determined graphically by means of the force diagrams of Figs. 87 (c) and (d), are 15,350 in.-lb. at the quarter point, and 20,450 in.-lb. at the tie rod. It is to be noted that at the tie rod the component moment parallel to the roof surface is negative. In determining the resultant moment Fig. 87(d), this component is plotted to the left of the origin. The component of moment perpendicular to the roof surface is positive, and is plotted above the OX axis, as in the preceding cases.

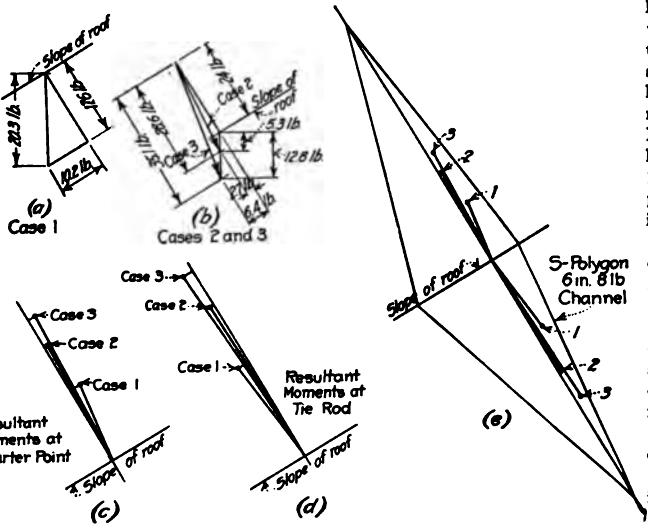


FIG. 87.

With allowable $f = 16{,}000$ lb. per sq. in., $S = M/f = 15{,}350/16{,}000 = 0.96$ in.^{3} at the quarter point, and $20{,}450/16{,}000 = 1.28$ in.^{3} at the tie rod. These values of S are shown in position on the S-Polygon of Fig. 87(e). The values of S for the section at the tie rod are plotted below the OX axis, for, as shown by the complete S-Polygon, the values of S for the given plane of bending are determined by the fourth quadrant S-Line.

Case 2. *Dead Load and Wind Load.*—The dead load due to the roof covering and the purlin is a vertical load of 5.3 lb. per sq. ft., as determined for Case 1, and the wind load is a normal load of 24 lb. per sq. ft., as determined for Case 2 of the preceding design. From the force diagram of Fig. 87(b), the component perpendicular to the roof is 28.6 lb. per sq. ft., and that parallel to the roof is 2.7 lb. per sq. ft. By the methods of Case 1, it will be found that at the quarter point the component of moment perpendicular to the roof is +24,700 in.-lb., and that parallel to the roof is +625 in.-lb.; the resultant moment, as determined graphically by Fig. 87(c), is 24,800 in.-lb.; and the required $S = 24{,}800/16{,}000 = 1.55$ in.^{3}

At the center point, the moment perpendicular to the roof is 32,900 in.-lb., and that parallel to the roof is -625 in.-lb.; the resultant moment, as determined by Fig. 87(d), is 33,000 in.-lb.; and the required $S = 33{,}000/16{,}000 = 2.06$ in.^{3}. These values are shown on Fig. 87(e).

Case 3. *Dead Load, Wind Load, and One-half Snow Load.*—With the half snow load as 7.5 lb. per sq. ft., the total vertical load is 12.8 lb. per sq. ft. As in the preceding cases, the normal wind load is 24.0 lb. per sq. ft. From Fig. 87(b), the component perpendicular to the roof is 35.1 lb. per sq. ft., and that parallel to the roof is

6.4 lb. per sq. ft. At the quarter point, the moment perpendicular to the roof is 30,300 in.-lb., and that parallel to the roof is +1480 in.-lb.; at the tie rod the corresponding values are: moment perpendicular to the roof = 40,500 in.-lb.; moment parallel to the roof = −1480 in.-lb. From Fig. 87(c), the resultant moment at the quarter point is 30,350 in.-lb.; the required S = 30,350/16,000 = 1.90 in.^{3} From Fig. 87(d), the resultant moment at the tie rod = 40,600 in.-lb.; the required S = 40,600/16,000 = 2.54 in.^{3}

Determination of Purlin Section.—Fig. 87(e) shows the S-Polygon of the assumed 6-in. channel section. It will be found that all of the plotted values of S lie inside of the polygon. The assumed section is therefore ample, and will be adopted.

The results of this design show that the use of tie rods makes it possible to use smaller sections for purlins than for the conditions assumed in the preceding design, where the purlins were assumed to be free to bend in any direction. Where the purlin was assumed to be free to bend in any direction, a 6-in. 12¼-lb. I-beam was required. Where tie rods were used, a 6-in. 8-lb. channel was found to answer. This represents a saving of 4¼ lb. per ft. of purlin.

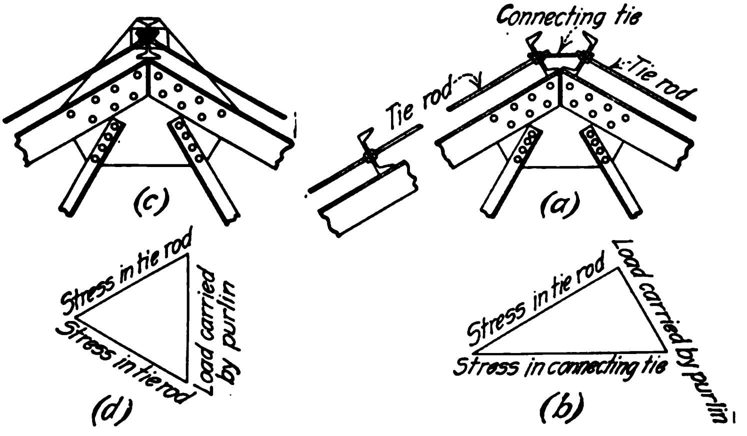


Fig. 88.

From an inspection of the S-Polygon of Fig. 87(e), it can be seen that the values of required S lie close to the OY axis. For all cases, except where the roof slope is very steep, it will probably be correct to assume that the tie rods offer complete lateral support for the purlin. The design can then be carried out by the methods used in the design of the purlins for rigid roof covering, as given in the first part of this article.

Design of Tie Rods.—Tie rods usually consist of round rods threaded at the ends to provide a means of fastening the tie to the purlin section. Fig. 88(a) shows the type of connection usually used.

As the tie rods form a continuous line from the eaves to the ridge, the stress in the rods increases to a maximum at the ridge. The area of the tie rod at the root of thread must be sufficient to carry a load caused by the component of loads parallel to the roof acting over the area tributary to the tie rod of maximum stress.

To illustrate the methods of design, assume that the slant height of the roof considered in the preceding design is 36 ft. As the trusses are 16 ft. apart, and there is a single line of tie rods at the center of the purlin, the area tributary to the tie rod of maximum stress is 36 × 8 = 288 sq. ft. From the force diagrams of Fig. 87, it will be found that the greatest component of load parallel to the roof is caused by the loading of Case 1, and that this component is 10.2 lb. per sq. ft. of roof. The load to be carried by the tie rod is then 288 × 10.2 = 2940 lb. With an allowable working stress of 16,000 lb. per sq. in., the area at the root of thread is 2940/16,000 = 0.184 sq. in. From the table of screw threads on p. 238, also given in the steel handbooks, it will be found that a ⅝-in. round rod will answer. If the load to be carried is too large for a single line of ⅝ or ¾-in. tie rods, the load can be reduced by adding another line of ties.

The method of attachment of tie rods at the ridge requires some consideration. Two methods of making the ridge connection are shown in Fig. 88. In Fig. 88(a), two purlins are provided at the ridge. The line of tie rods on either side of the ridge is joined by means of a short connecting tie. Fig. 88(b) shows the force diagram for the determination of the stresses in the rods and the load to be carried by the purlin due to the tie rods. It is probable that a larger section will have to be provided at the ridge in order to carry the heavy concentration brought to this point by the tie rod. Fig. 88(c) shows an arrangement in which a single I-beam forms the ridge support. The diagram of forces is shown in Fig. 88(d).

WOODEN COLUMNS

By Henry D. Dewell

Interior columns of buildings, supporting floors only, are normally square in cross section, while columns supporting roof trusses are usually made rectangular in order to attain greater stiffness in the plane of the roof truss than in the plane of the building wall. Columns supporting roof trusses may take bending stresses, due to wind, far in excess of the unit stresses produced by the weight of the roof and wall constructions.

Interior columns, when exposed, are usually surfaced four sides, and the corners champfered. Sometimes the columns are bored from end to end with a 1½-in. hole, and with ¾-in. holes at top and bottom extending from the face of column to the core hole. This is done in order to prevent dry rot, and to relieve the usual condition of rapid drying out of the exterior of the column, and slow seasoning of the interior timber.

Wooden columns with a ratio of $\frac{L}{d}$ greater than 20 will fail by lateral buckling. No wooden column should be designed with a greater $\frac{L}{d}$ than 60, and good practice will lower this limiting slenderness ratio to 40.

A general treatment pertaining to columns and column loads is given in the chapter on "Columns" in Sect. 1. For splicing wooden columns and for column connections, see Arts. 121 and 123. Bending and direct stress in columns is treated in Sect. 1.

65. Formulas for Wooden Columns.—All modern formulas for wooden columns assume the case of square-ended columns, and this condition of ends is the only condition recognized in practice. Practically all of the tests on wooden columns have been made with flat ends.

A number of formulas have been proposed and are in use for determining the safe working strength of wooden columns. With few exceptions these formulas are of the experimental type—that is, they are based on the results of tests. The straight-line formula is the type most favored by engineers. The two formulas of this type most generally used are (see also Sect. 1, Art. 99): (1) the formula of the American Railway Engineering Association

$$p = C\left(1 - \frac{1}{60}\frac{L}{d}\right)$$

and (2), the Winslow formula

$$p = C\left(1 - \frac{1}{80}\frac{L}{d}\right)$$

The second class of column formulas gives a curved graph. Of this type, the following formula of the U. S. Department of Agriculture is extensively employed

$$p = C\left(\frac{700 + 15c}{700 + 15c + c^2}\right)$$

In the above formulas, p = average unit compression (lb. per sq. in.).

C = compressive strength for short columns (lb. per sq. in.).

$c = \frac{L}{d}$

L = length of column in inches.

d = least cross-sectional dimension of column in inches.

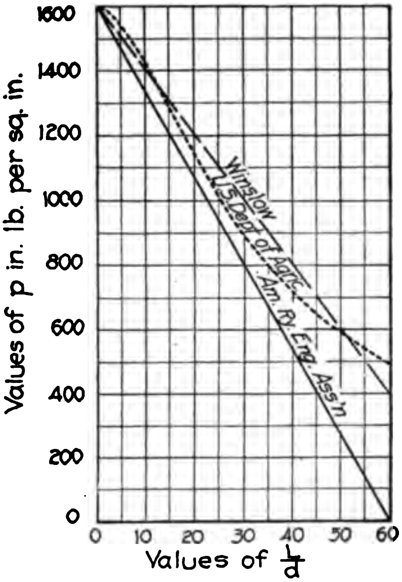


FIG. 89.—Curves of column formulas. (C = 1600)

For the range of values of $\frac{L}{d}$ occurring in ordinary building construction, the three preceding formulas will give approximately the same results. Fig. 89 shows the graphs of these formulas for working conditions, with C = 1600. For columns with a slenderness ratio $\left(\frac{L}{d}\right)$ less than 15, the unit stress to be used is that for $\frac{L}{d}$ = 15.

Table 1, p. 199, gives the unit stress for timber columns for various ratios of $\frac{L}{d}$, and values of C from 1000 to 1600 inclusive, corresponding to the formula of the U. S. Department of Agriculture. Table 2 gives similar quantities using the American Railway Engineering Association formula. Table 3 gives the safe loads in thousands of pounds for surfaced square timber columns, by the American Railway Engineering Association formula.

66. Ultimate Loads for Columns.—It is sometimes necessary to investigate the ultimate strength of wooden columns. Unfortunately, the ultimate strength of a timber column, especially of a long column, or a column with an $\frac{L}{d}$ of from 40 to 60, is indeterminate The tests which have been made on long columns of sections commensurate with those used in building construction are not sufficient in number to justify confidence in the values given by formulas.

From the results of tests made by the Watertown Arsenal, J. B. Johnson proposed for timber columns the following formulas:

Ultimate strength for partially seasoned yellow pine columns

$$p = 4500 - 1.0\left(\frac{L}{d}\right)^2$$

Ultimate strength for partially seasoned white pine column

$$p = 2500 - 0.5\left(\frac{L}{d}\right)^2$$

Ultimate strength for dry long leaf pine column

$$p = 6000 - 1.5\left(\frac{L}{d}\right)^2$$

Ultimate strength for dry white pine column

$$p = 3600 - 0.72\left(\frac{L}{d}\right)^2$$

W. H. Burr, from a study of the same tests, recommends the formulas:

For yellow pine

$$p = 5800 - 70\frac{L}{d}$$

For white pine

$$p = 3800 - 47\frac{L}{d}$$

One other column formula needs to be mentioned, since it has been used quite extensively in the past. This is the formula of C. Shaler Smith who made some 1200 tests on full-sized specimens of square and rectangular yellow pine columns for the Ordnance Department of the Confederate Government. For green, half-seasoned sticks of good merchantable lumber the formula of Smith is

$$p = \frac{5400}{1 + \frac{1}{250}\frac{L^2}{d^2}}$$

This formula gives much lower strength values for wooden columns than any of the preceding formulas.

All of the above formulas for ultimate strengths are based on short-time loadings. J. B. Johnson, in some 75 tests made to investigate the effect of time on continued uniform loading of timber in end compression, found that but little more than one-half the short-time ultimate load will cause a column to fail, if left on permanently. In other words, the ultimate strength of a timber column under permanent loads is approximately one-half the ultimate strength of the same column, as determined from the results of an actual test in a testing machine.

67. Built-up Columns.—The preceding discussion applies only to columns consisting of single sticks of timber. Built-up columns may be divided into two types: (1) those of solid section made up of thin planking and nailed, or nailed and bolted; and (2) columns of solid section bolted and keyed together, also latticed or trussed columns.

Type (1).—Columns of the first class are often used in cheap construction and, unfortunately, in situations where there is no excuse for not using a solid section. Carpenters, in order to make use of material available or handy, will often build up posts spiked together instead of using a solid section, in the belief that they are furnishing a stronger column than the larger timber of one piece. Tests have conclusively shown that a column of two or three pieces of timber blocked apart and bolted together at the ends and middle has no greater strength than the sum of the strengths of the component sticks, each acting as a single column, entirely independent of the other sticks.

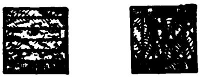

(*a*) (*b*)

Fig. 90.—Sections of built-up columns.

The strength of a built-up column of this class depends wholly upon the ability of the fastenings to resist initial deflection under loading. Such columns are usually designed with one of two typical sections: a column composed of a number of planks laid face to face and bolted or spiked together, as shown in Fig. 90(*a*); or a column composed of planks face to face with their edges tied together by cover-plates, as in Fig. 90(*b*). Of the two details, that of Fig. 90(*b*) is far superior to Fig. 90(*a*). When a column of the type shown in Fig. 90 (*b*) is thoroughly spiked, in addition to being bolted, the strength of column is undoubtedly greater than the sum of the strengths of the component planks acting as individual sticks. From tests made by the writer, it is recommended that the strength of a built-up column of the type of Fig. 90(*a*) be taken at 80% of the mean of the strength computed, (1) as a solid stick, and (2) as a summation of the strength of the individual sticks considered as individual columns. For columns of the type of Fig. 90(*b*), it is recommended that the strength be taken as 80% of that of a solid stick of equal cross section and length.

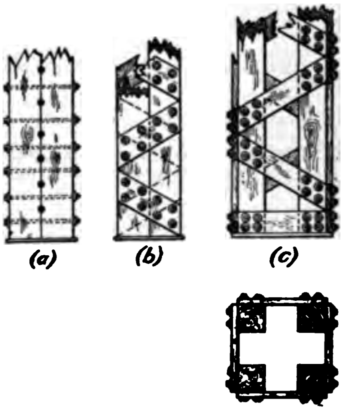

(*a*) (*b*) (*c*)

Fig. 91.—Heavy built-up columns.

The preceding recommendations are for built-up columns taking no appreciable bending stresses; in other words, for columns whose loads are balanced about the gravity center of the column section. Obviously, the resistance to bending of a built-up column of this class is low, as has been pointed out in the case of built-up girders (see Art. 45).

Type (2).—In framing for large timber buildings, as for expositions, wooden columns are sometimes constructed of two posts bolted and keyed together (Fig. 91*a*), two posts laced with diagonal sheathing (Fig. 91*b*), or four posts laced together (Fig. 91*c*). Such a construction may be necessary for the long story heights encountered in such buildings. The lacing shown in the detail of Fig. 91(*c*) may be spiked, bolted, or attached by means of lag screws, as determined usually by consideration of the stresses in the lacing due to wind shear. For dead loads, it is well to assume that the individual timbers act as separate columns, not held together by the fastenings. The lacing may be at 60 or at 45 deg. with the axis of the column, depending on the judgment of the designer. In general, the writer prefers the 60-deg. lacing.

ble 1.—Working Unit Stresses in Pounds per Square Inch for Timber Columns with Square Ends, Symmetrically Loaded

(Formula of U. S. Department of Agriculture)

L/d	Working unit stresses in pounds per square inch for values of "*C*" as indicated						
	1000	1100	1200	1300	1400	1500	1600
15	804	884	965	1046	1127	1206	1284
16	785	864	943	1022	1100	1179	1255
17	767	844	921	998	1075	1150	1226
18	749	823	899	974	1050	1124	1199
19	730	805	878	950	1025	1097	1170
20	712	786	857	928	1000	1071	1143
21	695	768	837	905	975	1046	1117
22	679	750	817	883	951	1020	1090
23	663	731	796	861	929	996	1063
24	647	714	778	841	906	971	1039
25	631	697	759	821	884	949	1013
26	617	681	741	802	864	927	989
27	601	664	724	784	844	905	965
28	587	648	707	766	824	883	942
29	573	632	690	748	805	862	920
30	559	617	674	730	787	841	899
31	547	601	659	713	768	821	878
32	534	587	643	696	750	801	856

ble 2.—Working Unit Stresses in Pounds per Square Inch for Timber Columns with Square Ends, Symmetrically Loaded

(Formula of American Railway Engineering Association)

L/d	Working unit stresses in pounds per square inch for values of "*C*" as indicated						
	1000	1100	1200	1300	1400	1500	1600
15	749	824	900	974	1049	1125	1200
16	732	806	879	952	1025	1100	1182
17	716	787	860	930	1002	1075	1145
18	700	769	840	909	979	1050	1119
19	683	750	819	887	955	1025	1092
20	666	732	800	866	932	1000	1065
21	649	714	779	843	909	975	1039
22	632	696	760	822	885	950	1012
23	616	677	739	801	862	925	985
24	600	659	720	779	839	900	959
25	582	640	699	757	815	875	932
26	566	622	680	735	792	850	906
27	549	604	659	714	769	825	879
28	533	585	639	692	746	800	852
29	516	567	620	670	722	775	825
30	500	548	599	649	699	750	799
31	483	530	580	627	675	725	772
32	466	512	559	606	651	700	745

TABLE 3.—TABLE OF SAFE BEARING LOADS IN 1000-POUND UNITS FOR TIMBER COLUMNS WITH SQUARE ENDS, SYMMETRICALLY LOADED

Values in this table are based on surfaced sizes. To get values for rough sizes, multiply bearing load by "multiplying factor" in dark type in same horizontal line. To get cross section of rough size, multiply area given by factor in dark type directly below.

(Based on Formula Adopted by American Railway Engineering Association)

Size		Area cross section	Length of column	L/d	"Multiplying factor"	Safe bearing loads in 1000-pound units for values of "C" as indicated						
Rough	Surfaced S1S1E or S4S¹											
Inches	Inches	Square inches	Feet			1000	1100	1200	1300	1400	1500	1600
6×6	5½× 5½	30.25	6	13.1	**1.19**	30.25	33.28	36.30	39.33	42.35	45.38	48.40
		1.19	8	17.5	**1.23**	21.39	23.53	25.67	27.81	29.95	32.09	34.22
			10	21.8	**1.25**	19.21	21.13	23.05	24.97	26.89	28.82	30.74
			12	26.2	**1.27**	17.00	18.70	20.40	22.10	23.80	25.50	27.20
			14	30.5	**1.29**	14.85	16.34	17.82	19.31	20.79	22.28	23.76
			8	12.8	**1.14**	56.25	61.88	67.50	73.13	78.75	84.38	90.00
			10	16.0	**1.16**	41.18	45.30	49.42	53.53	57.65	61.77	65.89
8×8	7½× 7½	56.25	12	19.2	**1.17**	38.19	42.01	45.83	49.65	53.47	57.29	61.10
		1.14	14	22.4	**1.18**	35.21	38.73	42.25	45.77	49.29	52.82	56.34
			16	25.6	**1.19**	32.18	35.40	38.62	41.83	45.05	48.27	51.49
			18	28.8	**1.20**	29.19	32.11	35.03	37.95	40.87	43.79	46.70
			8	10.1	**1.11**	90.25	99.28	108.30	117.33	126.35	135.38	144.40
			10	12.6	**1.11**	90.25	99.28	108.30	117.33	126.35	135.38	144.40
10×10	9½× 9½	90.25	12	15.2	**1.13**	67.24	73.96	80.69	87.41	94.14	100.86	107.58
		1.11	14	17.7	**1.13**	63.54	69.89	76.25	82.60	88.96	95.31	101.66
			16	20.2	**1.14**	59.75	65.73	71.70	77.68	83.65	89.63	95.60
			18	22.7	**1.14**	56.05	61.66	67.26	72.87	78.47	84.08	89.68
			20	25.3	**1.15**	52.16	57.38	62.59	67.81	73.02	78.24	83.46
			8 to	8.3	**1.09**	132.25	145.48	158.70	171.93	185.15	198.38	211.60
			14	14.6	**1.09**	132.25	145.48	158.70	171.93	185.15	198.38	211.60
12×12	11½×11½	132.25	16	16.7	**1.11**	95.35	104.89	114.42	123.96	133.49	143.03	152.56
		1.09	18	18.8	**1.11**	90.72	99.79	108.86	117.94	127.01	136.08	145.15
			20	20.9	**1.11**	86.09	94.70	103.31	111.92	120.53	129.14	137.74
			22	23.0	**1.12**	81.47	89.62	97.76	105.91	114.06	122.21	130.35
			24	25.0	**1.12**	76.97	84.67	92.36	100.06	107.76	115.46	123.15
			8 to	7.1	**1.08**	182.25	200.48	218.70	236.93	255.15	273.38	291.60
14×14	13½×13½	182.25	16	14.2	**1.08**	182.25	200.48	218.70	236.93	255.15	273.38	291.60
		1.08	18	16.0	**1.09**	133.41	146.75	160.09	173.43	186.77	200.12	213.46
			20	17.8	**1.09**	128.12	140.93	153.74	166.56	179.37	192.18	204.99
			22	19.6	**1.09**	122.65	134.92	147.18	159.45	171.71	183.97	196.24
			24	21.3	**1.10**	117.37	129.11	140.84	152.58	164.32	176.06	187.79

16×16	15½×15½	240.25 1.07	10 to	7.7	1.07	240.25	264.28	288.30	312.33	336.35	360.38	384.40
			18	14.0	1.07	240.25	264.28	288.30	312.33	336.35	360.38	384.40
			20	15.5	1.08	177.79	195.58	213.35	231.13	248.91	266.69	284.46
			22	17.0	1.08	172.02	189.22	206.42	223.63	240.83	258.08	275.23
			24	18.6	1.08	165.53	182.08	198.64	215.19	231.74	248.30	264.85
18×18	17½×17½	306.25 1.06	10 to	6.9	1.06	306.25	336.88	367.50	398.13	428.75	459.38	490.00
			20	13.7	1.06	306.25	336.88	367.50	398.13	428.75	459.38	490.00
			22	15.1	1.07	229.08	251.99	274.90	297.80	320.71	343.62	366.53
			24	16.5	1.07	221.73	243.90	266.08	288.25	310.42	332.60	354.77
20×20	19½×19½	380.25 1.05	10 to	6.2	1.05	380.25	418.28	456.30	494.33	532.35	570.38	608.40
			24	14.8	1.05	380.25	418.28	456.30	494.33	532.35	570.38	608.40
22×22	21½×21½	462.25 1.05	10 to	5.6	1.05	462.25	508.48	554.70	600.93	647.15	693.38	739.60
			24	13.4	1.05	462.25	508.48	554.70	600.93	647.15	693.38	739.60
24×24	23½×23½	552.25 1.04	10 to	5.1	1.04	552.25	607.48	662.70	717.93	773.15	828.38	883.60
			24	12.3	1.04	552.25	607.48	662.70	717.93	773.15	828.38	883.60
26×26	25½×25½	650.25 1.04	10 to	4.7	1.04	650.25	715.28	780.30	845.33	910.35	975.38	1040.40
			24	11.3	1.04	650.25	715.28	780.30	845.33	910.35	975.38	1040.40

[1] S1S1E means surfaced one side and one end.
S4S means surfaced all four sides.

68. Column Bases.—Except for temporary construction, building footings at the present time are constructed of concrete, reinforced concrete, or steel grillages incased in concrete. The statement may be made, therefore, that the first-story column of any building will rest on a concrete footing. A base plate between the bottom of post and top of footing is a necessity for two reasons: (1) to distribute the column pressure over the footing without exceeding the safe unit bearing pressure for concrete; and (2) to prevent moisture from entering the bottom of the column and causing rot. For this purpose a wooden plate, preferably of redwood or cedar, a standard metal column base, a cast-iron base, or a plain steel plate may be used. The latter is often found as satisfactory and more economical than the standard metal post base. If a single plate is used, the thickness must be sufficient to give strength to the plate, in flexure, to distribute the load uniformly over the footing, with a uniform distribution of pressure on the footing.

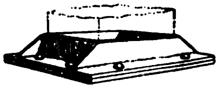


Fig. 92.

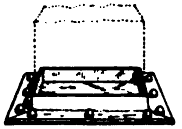


Fig. 93.—Duplex steel post base.

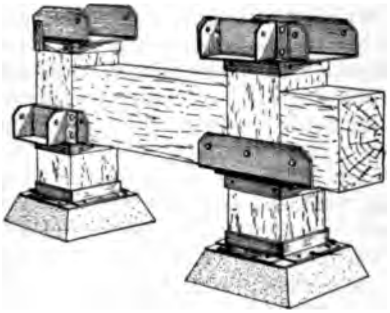


Fig. 94.—Typical details of construction with "Falls" post caps and bases.

Illustrative Problem.—Given a 12 × 12-in. column carrying a load of 130,000 lb. Using a working value of 400 lb. per sq. in. for bearing on the concrete, a base of 130,000/400 = 326 sq. in. is required, or 18 in. square. The plate will then project 3¼ in. from each face of column. The bending moment on the plate may be taken as $\left(\frac{130{,}000}{4}\right)\left(\frac{3}{8}\right)(9) - \left(\frac{130{,}000}{4}\right)\left(\frac{3}{8}\right)\left(5\frac{3}{4}\right) = (32{,}500)(2.17) = 70{,}500$ in.-lb. This moment is resisted by the full width of base. As the plate is in effect a short, thick beam, a maximum flexural fiber stress of 20,000 lb. per sq. in. for structural steel may be used, giving a required section modulus of 3.53. Therefore $S = (\frac{1}{6})(18)\,d^2) = 3.53$, or $d = \sqrt{1.18} = 1.08$, or a $1\frac{1}{16}$-in. plate.

In detailing the base of column, it is well to set a dowel into the concrete and let the same project into the bottom of post. The size of dowel is a matter of judgment. For a 12 × 12-in. post, the dowel should be not less than 1¼ × 6 in.

If the use of a standard column base is contemplated, the particular base should be examined to make sure its composition is sufficiently strong to distribute its load equally over the foundation.

It remains to be stated that all metal bases should be well painted. The bottoms of columns should be given two coats of a good wood preservative. The top of the concrete footing should be set a few inches above the floor to prevent moisture standing around the bottom of the column.

Figs. 92, 93 and 94 show standard post bases, taken from manufacturers' catalogs.

CAST-IRON COLUMNS

By H. S. Rogers

69. Use of Cast-iron Columns.—Cast-iron columns are suitable only for small buildings of non-fireproof construction. They offer somewhat greater resistance to fire than unprotected steel columns and occupy a minimum of space in the building, but cast iron is by no means as reliable as steel and the bolted connections of cast-iron columns allow more or less lateral movement which is serious in high buildings.

Columns of this material should not be used with fabricated steel in skeleton construction or under conditions which produce flexural stresses of any magnitude, other than those due to concentrically-loaded column action. The unreliability of cast-iron columns is due to the variation in quality of the material, defects likely to occur in casting, and the difficulty of thorough inspection.

70. Properties of Cast Iron.—Cast iron has a very high unit compressive strength—usually considered to be about 80,000 lb. per sq. in. This material, however, is not strong in shear or tension, the average ultimate shearing stress being 18,000 lb. per sq. in., and the average ultimate tensile stress 15,000 lb. per sq. in. The ultimate intensity of stress which can be developed in a piece of cast iron varies with its fineness of grain, and depends largely upon its thickness and the rate of cooling, as well as its composition. The high compressive stresses make it a very desirable material to use in compression, but because of the somewhat treacherous nature of cast iron, the high compressive stresses found are often misleading. Also, the low shearing and tensile values preclude its use under any condition other than that of direct compression. It does not rust as quickly as steel and resists fire somewhat better, but may, however, be subjected to serious strains because of sudden cooling with water from a fire stream. It is very hard and brittle, and fractures suddenly without warning. No riveted connections should be made to cast iron. All connections of girders to columns, or column to column, must therefore be made by bolts which impair the rigidity of a structure by the allowance for clearance.

71. Manufacture of Cast-iron Columns.—Cast-iron columns may be cast in sand molds either upon the side or on end. In either case a baked core molded to the dimensions of the inside of the column must be made of sand, flour, and water, and supported within the sand mold. There are practical conditions surrounding every part of the work which will determine the quality of the column produced. Many pronounced defects found in columns are due to the method of pouring used in their manufacture.

If the column is cast on its side, the core will be buoyed up within the mold because of the great difference in density between it and the molten metal. Provision must, therefore, be made to prevent the core from rising toward the top side of the mold, or from being sprung from line so that the mid-portion of the top side of the casting will be thinner than the desired thickness. This defect produced by "floating cores" is one which is frequently found in cast-iron columns. The molten metal rising in the mold carries dirt and air above, in which will form "honeycomb" and "blowholes" along the top side of the column, unless provision is made by vents for the escape of the air. This provision can be made by forcing a wire rod through the mold at intervals. When these difficulties have been overcome, there are still others which may arise due to unequal cooling produced by the manner or speed of pouring, by the condition of part of the mold, or by the unequal radiation in the molds. The last may be due to an unequal uncovering of the mold. Unequal cooling may produce stresses which will crack the column before any load is placed upon it.

The end method of casting avoids some of these difficulties if the molten metal is introduced at the bottom of the mold. The dirt, sand, and air that collect will thus be borne to the top of the mold so that they can be removed, but the pressure produced by the head of molten metal will often be greater than the mold can withstand, if the column is of any considerable length. The defects found in columns cast on end will not, however, be so numerous as those found in columns cast on the side. These defects can be eliminated to some extent by careful foundry work. If not eliminated, they should be caught at the time of inspection.

72. Inspection of Cast-iron Columns.—Cast-iron columns may have defects either in the surface, or within the metal, or may have insufficient strength due to variation in the section of the metal due to displacement of the core. Defects in the surface can be found by a careful examination of the column. Defects within the metal can be discovered by a careful tapping of the column with a hammer, as the honeycomb or sand spots will sound dead. In hollow square or round columns, variation in thickness of the metal can be determined by drilling two or three ¼-in. holes through the column. If this variation is more than ¼ in., the column should be rejected. The H-section affords easy access to the surface for inspection and painting, and opportunity to measure the section. Columns with brackets should be carefully inspected at these details, especially if the column has been poured on its side through the bracket.

73. Tests of Cast-iron Columns.—The Department of Buildings of New York City made a series of tests upon cast-iron columns some years ago at the works of the Phoenix Bridge Co. Nine columns were tested to destruction and a tenth to the capacity of the testing machine. Six of the ten columns had a diameter of 15 in., a length of 15 ft. 10 in., and a thickness of shell of 1 in.; two had a diameter of 8 in., a ratio of L/d equal to 20, and a shell thickness of 1 in.; two had a diameter of 6 in., a ratio of L/d equal to 20, and a shell thickness of 1 in.

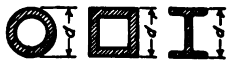


FIG. 95.—Cast-iron column sections.

The columns broke at loads varying from 22,700 lb. per sq. in. to over 40,400 lb. per sq. in., the latter being the intensity of stress in one of the 15-in. columns which withstood the total capacity of the machine. The other five 15-in. columns all exhibited foundry dirt, honeycomb, cinderpockets, or blowholes.

74. Design of Cast-iron Columns.—The sections of cast-iron columns in general use are shown in Fig. 95. The hollow cylindrical section gives the best distribution of metal in a column, but the connection details do not work as nicely as those for the hollow square section, which is almost as efficient in distribution of material. The hollow square section, on the other hand, has disadvantages which are not found in the hollow cylindrical section. The corners of the square section are very liable to crack, due to the cooling of the column; but this can be obviated by an outside curved corner and an inside fillet. The H-section, though not affording a distribution of material as efficient as the hollow cylindrical or hollow square column, has the advantages of being open to inspection, of being cast without a core, and of being easily built into a brick wall. It meets the greatest favor as a wall column.

The allowable unit stresses in the sections of cast-iron columns are determined as discussed in Sect. 1, Art. 98. The type of column is first selected and then tested for its total strength by the application of one of the column formulas for unit stresses. There are two types of formulas in general use for determining the unit stresses in cast-iron columns: the Gordon and the Straight Line. The Gordon type is specified by the building code of Philadelphia and the straight-line type by the codes of New York, Boston, Chicago, and Seattle. In the Gordon type the radius of gyration has been replaced by the value, d, which is the outside diameter of cylindrical section, or the outside dimension of the square. This can be done by changing the constant in the denominator of the factor, $a\frac{L^2}{r^2}$, since the radius of gyration for any particular value of thickness of the shell, bears a direct relation to the outside dimension, and since the radii of gyration for any outside dimension are practically the same for all the standard thicknesses of shell. The formulas adopted in several codes are given in Sect. 1, Art. 98.

The following specifications should be observed in the design of the shafts of cast-iron columns:

The minimum thickness of the shell should not be less than ¾ in.; the maximum thickness should not be greater than 1¾ to 2½ in.

The maximum diameter should not be greater than 16 in.; the minimum diameter should not be less than 5 or 6 in.

The slenderness ratio, L/r, should not exceed 70; the unsupported length of the column should not exceed 20 times the least diameter.

All corners should be filleted with a radius of ¼ or ⅜ in.

No inside offset nor any sudden change in the thickness of shaft should be made.

75. Column Caps and Bases.—Hollow cylindrical and square cast-iron columns are generally fastened together by a simple flanged base and cap as shown in Fig. 96 (*a*) and 96 (*b*). The flanges should not be thinner than the shaft of the column and should be at least 3 in. wide; which width will be sufficient for hexagonal nuts on ¾-in. bolts. These flanges should be faced at right angles to the axis of the column. The bolt holes in the flanges should be drilled to a templet so that the columns can be fitted together in proper alignment and the flanges should be spot-faced at bolt holes so that they will give a square firm bearing to bolts and nuts. If the ends of cast-iron columns must be left rough, sheets of lead or copper should be placed between flanges of columns bolted together, so that an even bearing will be obtained by the soft metal taking up the inequalities of the surface. In no case should shims be used to wedge up one side of a column.

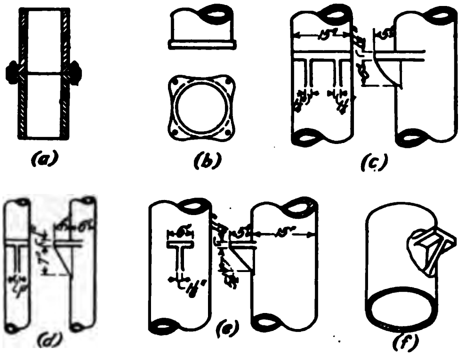


FIG. 96.—Cast-iron column details.

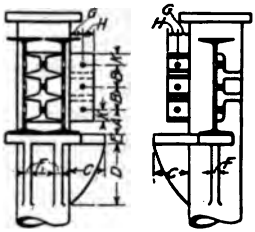
If it is desired to give any architectural pretentions to the caps or bases of cast-iron columns, the design of such should be made so as not to weaken the shaft section of the column by change of dimensions or offsets that will throw transverse stresses into the column. Ornamental caps or bases of large size should be cast separate from the column.

76. Bracket Connections.—The usual forms for the connections of beams and girders of cast-iron columns are shown in Fig. 96(*c*), 96(*d*), and 96 (*e*) and in the table of "Manufacturers' Standard Cast-iron Column Connections." The beam rests upon the bracket shelf and is bolted to the lug on the column through the web. The holes in the web of the beam for bolting to the lugs should be drilled in the field in order to match the cored holes of the lug.

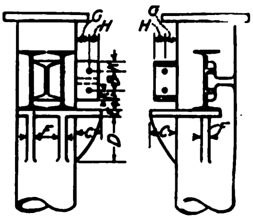
Connections should be designed with a bracket directly below the web of a single girder or below each web of a box girder so that no transverse bending strains will be thrown into the bracket shelf. The bracket shelf should be given a slope of ⅛ in. to the foot away from the column so that the load cannot be applied at the end of the shelf. A bracket will bear only about one-half as great a load applied eccentrically at the edge of the shelf as one distributed over the shelf. A bracket shelf may fail in one of three ways, (1) by shearing through shelf and bracket next to the column, (2) by transverse bending, or (3) by tearing out a section of the column as shown in Fig. 96(*f*).

Manufacturers' Standard Cast-iron Column Connections

(Dimensions in Inches)



Depth of beam	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	Thickness of lugs	Holes cored for ¾-in. bolts
20	5	5	6	10½	1½	1½	2	1½	2	1	
18	4	5	6	10½	1½	1½	2	1½	2	1	
15	4	3½	5½	9½	1½	1¼	2	1½	1¾	1	
12	3	3	4½	7¾	1¼	1¼	2	1½	1½	1	



Depth of beam	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	Thickness of lugs	Holes cored for ¾-in. bolts
10	3¼	3½	4	7	1¼	1	2	1½	1½	1	
9	3	3	4	7	1	1	2	1½	1½	1	
8	2½	3	4	7	1	1	2	1½	1½	¾	
7	2¼	2½	4	7	1	1	2	1½	1½	¾	

Tests by the Building Department of New York City have shown that brackets will not fail by shear or transverse bending on columns of more than 6-in. diameter if designed according to standard practice. Of 22 brackets tested, those on 8 or 15-in. columns failed by tearing holes in the body of the column, and 4 on 6-in. columns failed by shearing or transverse stress.

The design of bracket shelves by any rigorous analytical method is impossible. Some of the factors which complicate it are the rate of cooling, variations in the thickness of metal, and imperfections. The design should, however, be checked against failure due to shear or transverse bending.

STEEL COLUMNS

By Clyde T. Morris

77. Steel Column Formulas.—Practical column formulas that are in use in this country are of three types, the Rankine or Gordon type (Formula 1), the straight line (Formula 2), and the parabolic type (Formula 3).

$$p = \frac{f}{1 + \frac{L^2}{ar^2}} \quad \text{.......Rankine or Gordon formula.......} \quad (1)$$

$$p = f - m\frac{L}{r} \quad \text{.......Straight line formula.......} \quad (2)$$

$$p = f - n\frac{L^2}{r^2} \quad \text{.......Parabolic formula.......} \quad (3)$$

in which p = allowable intensity of stress over the column section.
f = maximum allowable intensity of stress in short blocks.
L = length.
r = radius of gyration.
$\frac{L}{r}$ is called the slenderness ratio.
a, m, and n are constants.

The constants in these formulas are determined from experiments. Many authorities give three values for the constant "a" in Formula (1), corresponding to two fixed ends, one fixed and one pin end, and two pin ends.

A general treatment pertaining to columns and column loads is given in the chapter on "Columns" in Sect. 1. Bending and direct stress in columns is treated in the chapter on "Bending and Direct Stress—Wood and Steel" in Sect. 1. For column connections, see Sect. 3, Art. 72*b*.

78. Slenderness Ratio.—The unsupported length of a compression member should never exceed 200 times its least radius of gyration. The following are usually recognized as the upper limits of the value of $\frac{L}{r}$ for the various classes of structures.

For lateral struts carrying wind stresses only, in buildings	150 to 200.
For lateral struts carrying wind stresses only, in bridges	120 to 150.
For columns in buildings with quiescent loads	120 to 150.
For compression members in bridges	100 to 120.

79. Forms of Cross Section.—For economy, the radius of gyration of the section should be as large as possible. This makes it desirable to place as much of the material as possible as far from the axis of the column as is consistent with good design. The hollow cylinder is theoretically the most economical form of column cross section, for in this form all of the material is at a maximum distance from the axis.

Steel pipe columns are frequently used for light loads where the loads are quiescent and there is no probability of a lateral component to the forces acting on the column. The caps and bases of these are usually cast iron and the use of this form of column has the same limitations as that of cast-iron columns.

Fig. 97 shows the more common forms of cross section for steel columns and struts.

Struts of 2 angles (Fig. 97*a*) are commonly used for light lateral bracing. The section is unsymmetrical and for this reason is undesirable for main compression members. Columns composed of 2 channels laced (Fig. 97*g*, *h*, and *k*) or 2 pairs of angles laced (Fig. 97*b*) are not as rigid in the plane of the lacing as those in which the parts are connected by plates. Care should be used in proportioning the lacing in such columns. Types *i* and *l* are forms which are commonly used for top chords and end posts of bridges. The lattice on the lower side permits access for cleaning and painting. The Bethlehem H-section (Fig. 97 *e* and *f*) is a form much used in building work. Type *e* without cover plates is very economical on account of the small amount rication necessary. Type *f* is much more expensive as it is necessary to drill the holes in ɜavy flanges of the H-section for riveting on the cover plates. These flanges are too thick ach. Z-bar columns (Fig. 97 *q* and *r*) are seldom used in modern structures. The Grey ın (Fig. 97*s*) and the 4-angle column (Fig. 97*t*) are frequently used in combined steel and ete columns.

0. Steel Column Details.—The component parts of a column must be so rigidly connected ıer that they cannot deform independently. The entire section must act as a unit. In the of columns which do not have lacing, the riveting necessary to hold the parts in contact ıake tight joints, will be sufficient to transmit the transverse shear and ensure the action : column as a unit.

80*a*. Lattice or Lacing.—When latır lacing is used to connect the parts of ımn, it must be proportioned to take the verse shear caused by the bending of the ın. Professors Talbot and Moore, in the ı. Am. Soc. C. E., Vol. LXV, p. 202, give count of experiments performed at the ɹrsity of Illinois to determine the stresses e bars. The following is quoted from eport:

> e measurements indicate stresses in the lattice hich would be produced by a transverse shear in amount to 1 to 3 % of the applied comn load, or to that produced by a concentrated erse load at the middle of the column length to 2 to 6 % of the compression load.

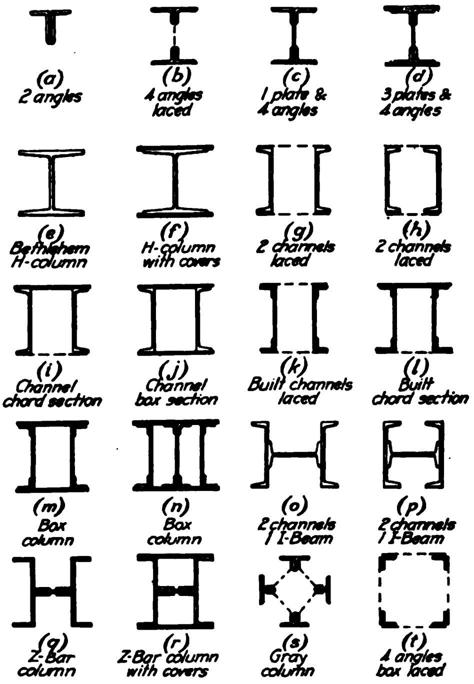


FIG. 97.

Two methods of proportioning lace bars ı common use: *First Method.*—Column ılas used in design give a reduced allowed stress which is the average over the sec- The maximum allowed fiber stress on oss section is usually included as a factor formula, and the difference between the num and the average is the fiber stress d by the bending due to column action. difference in fiber stress is assumed to be o a uniform transverse load applied to olumn, and from this the equivalent transverse shear may be calculated as follows: n Formulas (2), (3), or (4)

f = the maximum allowed fiber stress.
p = the average unit stress.

$$f - p = \frac{Mc}{I} = \frac{Mc}{Ar^2} \text{ and } M = \frac{(f - p)Ar^2}{c}$$

which

$$w = \frac{8(f - p)Ar^2}{L^2c} \quad \text{and shear} = \frac{wL}{2} = \frac{4(f - p)Ar^2}{Lc} \tag{4}$$

econd Method.[1]—A column under stress will deform into a curve with a point of contrae near each end, the distance from the end depending upon the degree of fixity of the end Fig. 103, Sect. 1, p. 59). At these points of contra-flexure the bending moment is zero onsequently the stress on the column cross section is uniform. Midway between these the maximum bending moment occurs, and the maximum unit stress in compression ocon the concave side. Therefore in a distance equal to one-half the length between the of contra-flexure, the unit stress in the concave side of the column must change from the ge to the maximum allowed.

rom "Steel Structures" by CLYDE T. MORRIS, p. 120.

Suppose a column to be made up of two leaves connected by lacing.

As before, f = the maximum allowed fiber stress.

p = the average unit stress.

Let F = the total change in stress in one leaf of the column in a distance l.

s = the total change in stress in one leaf per unit of length $= \frac{F}{l}$.

l = the least distance from a point of contra-flexure to a point of maximum bending moment.

L = the total length of the column.

A_1 = the area of cross section of one leaf.

Then

$$F = A_1(f - p) \text{ and } s = \frac{A_1(f - p)}{l}$$

For a pivoted end column, $L = 2l$, and for a fixed end column, $L = 4l$. Any column in practice will lie somewhere between these two limits. This theory assumes that the rate of change of stress in the leaf is uniform, which is not true, but in any case eccentricities of manufacture and loading may make l different than theory would indicate. Therefore, to be on the safe side, take $L = 4l$ in all cases; then

$$s = \frac{4A_1(f - p)}{L} \tag{5}$$

Formula (5) gives the longitudinal increment of stress in one leaf per unit of length of column, and sufficient connection must be provided between the leaves to transmit this stress.

In either the first or second method, if the column is subject to an external bending moment in the plane of the lacing, this must be included in getting the value of $(f - p)$. In all cases the lace bars must be proportioned to carry the calculated stress in either tension or compression.

The inclination of lace bars with the axis of the member should never be less than 45 deg., and their thickness should not be less than 1/40 of the distance between rivets for single lattice and 1/60 for double lattice.

The following minimum widths for lace bars are sanctioned by good practice.

For members 15 in. and over in depth	2½ in.
For members 9 to 12 in. in depth	2¼ in.
For members 7 to 9 in. in depth	2 in.
For members under 7 in. in depth	1¾ in.

Illustrative Problem.—A column 14 ft. long is composed of 4 angles 3½ × 3 × 5/16 laced, 12 in. back to back (see Fig. 97*b*). The straight-line formula, $p = 16{,}000 - 70\frac{L}{r}$, will be used.

$A = 7.76$ sq. in.

$r = 5.27$ in. in the plane of the lacing.

$f = 16{,}000$ lb. per sq. in.

$p = 13{,}770$ lb. per sq. in.

$- p = 2230.$

First Method.

$$\text{Shear} = \frac{(4)(2230)(7.76)(5.27)^2}{(14)(12)(6)} = 1905 \text{ lb.}$$

If the lacing makes an angle of 45 deg. with the axis of the member.

Stress in lace bar = (1905)(1.414) = 2690 lb.

Distance beween gage lines in the angles = 12 − (2)(1¾) = 8.5 in.

Distance beween end rivets in lace bar = (8.5)(1.414) = 12 in.

$$\text{Minimum thickness of lace bar} = \frac{12}{40} = 0.3 \text{ in.}$$

Try lace bars 2 × 5/16. $A = 0.62$ sq. in. $r = 0.09$ in.

$$\text{Allowed unit stress for lace bar} = 16{,}000 - \frac{(70)(12)}{0.09} = 6670 \text{ lb. per sq. in.}$$

$$\text{Required area} = \frac{2690}{6670} = 0.40 \text{ sq. in.}$$

Second Method.

$$s = \frac{(4)(3.88)(2230)}{(14)(12)} = 206 \text{ lb. per lin. in.}$$

If the lacing makes an angle of 45 deg. with the axis of the member, the length of the column which will be served by one lace bar will be 8.5 in. Longitudinal increment of stress in one leaf per lace bar = (8.5)(206) = 1750 lb.

$$\text{Stress in lace bar} = 1.414 \times 1750 = 2475 \text{ lb.}$$

$$\text{Required area} = \frac{2475}{6670} = 0.37 \text{ sq. in.}$$

At the ends of latticed compression members, *stay plates* must be provided to equalize the distribution of stress to the end connections. These stay plates should be not less in width than the width of the member, and preferably not less in length than 1½ times the width, and not less in thickness than 1/60 of the unsupported width. At the ends of large compression members (say over 24 in. in width) a diaphragm is desirable between the webs, with a length of about 1½ times the width of the member.

80*b*. Splices.—At all intermediate joints in columns, splice plates should be provided connecting the two sections (see Fig. 268, p. 317). If the ends of the sections are not faced so as to secure a good bearing of one section on the other, sufficient splicing material and rivets must be provided to take the entire stress at the point. If the joint is properly faced and a good bearing is ensured, only sufficient splice need be provided to take care of the bending moment at the point and to hold the parts in position. In case of a concentrically loaded column, the moment due to column action used in the derivation of Formula (4) should be provided for. If there is an external bending moment due to eccentric loads or to transverse forces, it should be added to the moment due to column action.

80*c*. Caps and Bases.—The use of column caps should be avoided. If columns composed of rolled shapes are used, such as are shown in Fig. 97, the beams or trusses connecting to them should generally be riveted to the webs or flanges with connection angles, and not be set on top of a cap plate. At intermediate floors the column shaft should never be interrupted, but the lower story column section should be run through the floor and be spliced to the upper section just above the floor line. In columns of one-story length, column caps may be used provided the beams or trusses resting on them are properly stayed.

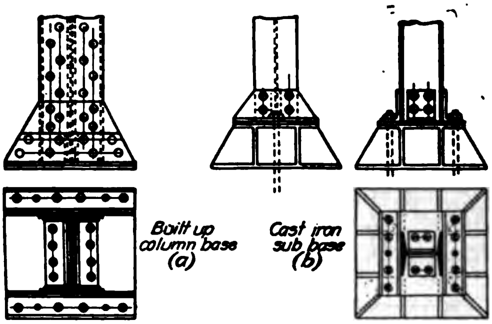


Fig. 98.

It is necessary to put a base on a column large enough to distribute the loads to the masonry footings so that the allowed bearing unit will not be exceeded. This may be built up entirely of rolled plates and shapes (Fig. 98*a*) or a cast-iron or cast-steel subbase may be interposed between the column base proper and the masonry (see Fig. 98*b*). In case a cast-iron subbase is used, the anchor bolts should run through it and connect directly to the column base proper. Gusset plates connecting the base to the column shaft should be large enough to properly distribute their proportion of the stress to the base.

81. Combined Steel and Concrete Columns.—In reinforced concrete buildings it is sometimes desirable to reduce the size of the columns below that which would be required for a reinforced concrete column of the usual type. This may be done by using a steel column filled in and cased in concrete.

Tests made by Professors Talbot and Lord at the University of Illinois, and published in the University of Illinois Bulletin No. 56, show that the strength of the combined column may be calculated on the assumption that the steel column, and the concrete core inside the steel act independently.

The Gray column (Fig. 97*s*) or some form of latticed angle column (Fig. 97*t*) is best adapted to this style of reinforcement. The steel column should be designed and detailed in all respects similar to a steel column without concrete casing. The concrete core enclosed within lines joining the toes of the angles, may be figured as a concrete column reinforced with vertical steel only. The steel column should be enclosed with light hooping to prevent the concrete casing from cleaving loose from the smooth faces of the steel.

CONCRETE COLUMNS

By W. Stuart Tait

82. Column Types.—Concrete columns are of five principal types:

(1) Plain concrete columns or piers.
(2) Concrete columns reinforced with vertical bars and hoops or ties.
(3) Concrete columns reinforced with spirals and vertical bars.
(4) Structural steel columns encased in concrete.
(5) Emperger columns.

83. Plain Concrete Columns or Piers.—The Joint Committee recommends that the height of a plain concrete pier or column be not allowed to exceed four times its least dimension. Many building codes, however, limit the height to six times the least dimension of the pier. Piers of greater height must be suitably reinforced. Where the load carried by a pier is applied in an eccentric manner the designer must apply the formula for combined bending and direct stress and increase the section of the pier if the unit pressure on outer fiber exceeds the allowable stress (see Sect. 1, Art. 103).

For working stress allowed by the Joint Committee, see *Appendix J.*

84. Columns with Vertical Bars and Ties.—The Joint Committee recommends that all concrete columns in which the length exceeds four times the least dimension be reinforced with a minimum of 1% of vertical steel. This vertical steel must be supported laterally by column ties made of ¼-in. round mild steel spaced a maximum distance of 12 in. apart. These ties serve several purposes. They are wired to the column bars, thus holding these bars in place while the concrete is being placed. They also prevent the column bars from buckling and causing the concrete covering to spall off.

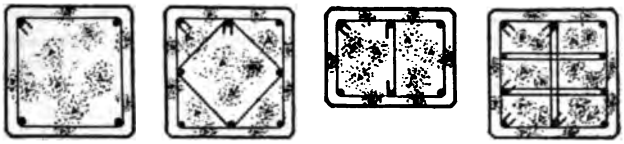

(a) (b) (c) (d)

Fig. 99.—Arrangement of ties in square and rectangular columns.

The ties further act in a manner similar to beam stirrups and tend to prevent column failure by shearing along a diagonal plane. In general, building codes may be satisfied by using ¼-in. round ties 12 in. on centers. In large columns of this kind, however, heavier ties should be used. A simple and satisfactory method of determining the size of tie to be used is to take 0.02% of the core area of the column as being the sectional area of the tie used in a height of 12 in. Thus, in a 22×22-in. column with a 19×19-in. core, the area of the tie would be $(0.0002)(361) = 0.072$ sq. in. in 1 ft. of height, and 5⁄16-in. round ties 12 in. on centers or ¼-in. round ties 8 in. on centers could be used.

Tests have shown that the strength of a tied column is materially reduced by the use of a number of ties in the same column crossing through the core. The most simple form of reinforcement for a square column is the use of 4 bars and 1 set of ties (Fig. 99*a*). If the column size is such that four 1¼-in. square bars do not provide sufficient steel, 8 bars may be used, tied as shown in Fig. 99(*b*). In rectangular columns it is sometimes desirable to use 6 bars and Fig. 99(*c*) shows how they should be tied. Round columns may have any number of vertical bars with one set of circular ties. Arrangements of ties similar to those shown in Fig. 99(*d*) should be avoided for the reason cited above.

Assuming that the bond is perfect between the steel and concrete in a column, it follows that the deformations of the concrete and steel when the column is loaded must be the same. In consequence, $f_s = nf_c$. The total load which a column will carry, therefore, is given by the formula $P = A_c f_c + A_s n f_c$, or, if A_c is taken as the total section of concrete without deducting the steel

$$P = A_c f_c + A_s(n-1)f_c$$

Thus, according to the Joint Committee,[1] a 20×20-in. column of 1-2-4 gravel concrete with four 1-in. round bars, deducting 1½ in. for fireproofing, would safely carry a load of

$$P = (17)\ (17)(450) + (4)(0.785)(14)(450) = 149{,}832 \text{ lb.}$$

The Joint Committee recommends that not more than 4% of vertical steel be used and, owing to the doubt as to the effectiveness of larger percentages of steel than this, due to lack of test data, it is well to be governed by this recommendation. Also, as a matter of economy, it is advisable to use a richer mixture of concrete rather than a high percentage of steel.

85. Columns with Vertical Steel and Spiral Reinforcement.—On account of its economy this type of column is now used more extensively than any other in concrete buildings. The use of spiral reinforcement tends to prevent lateral or radial deformation in a column subjected to vertical load and thereby increases the amount of vertical stress to which the concrete may be safely subjected. The spiral also acts to prevent failure by diagonal shearing in a manner similar to column ties. The Joint Committee, therefore, allows 55% more stress on the concrete core of a column containing 1% of spiral than it allows on the concrete core where ties are used. Thus the Joint Committee permits a stress in the concrete of 34.9 % of the 28-day strength.[1] The spiral column is then designed in the same manner as the tied column. A 20-in. round column of 1-2-4 gravel concrete having a 17-in. core with 1 % of spiral and eight ¾-in. round bars would carry a safe load of

$$P = (227)(698) + (8)(0.44)(14)(698) = 192{,}843 \text{ lb.}$$

Following upon the tests and analysis made by Considère, most building codes recognize any percentage of spiral between ½ and 1½ %. The American Concrete Institute recommends that between ½ and 2 % be used. All these rulings provide that a minimum percentage of vertical steel equal to the spiral must be used. A limit under the various rulings of from 4 to 8 % is given as the maximum amount of vertical steel. Considère's formula credits the spiral as being effective to the extent of 2.4 of its volume as vertical steel. The formula is as follows:

$$P = A_c f_c + A_s(n-1)f_c + (2.4n-1)A_s' f_c$$

The American Concrete Institute formula considers the spiral to be 4 times as effective as the same volume of vertical steel. The formula is as follows:

$$P = A_c f_c + A_s(n-1)f_c + (4n-1)A_s' f_c$$

In the above formula A_s' is the equivalent area of spiral. Thus $A_s' = p$ (the spiral percentage) $\times A_c$.

86. Structural Steel Columns Encased in Concrete.—The Joint Committee makes no recommendation as to the design of a steel column encased in concrete. The proposed American Concrete Institute ruling provides that where the steel is designed to take all the load that the allowable stress per square inch shall be determined by the formula $18{,}000 - 70\frac{L}{r}$, but shall not exceed 16,000 lb. per sq. in. In this formula L is the unsupported height and r the least radius of gyration, both in inches. The concrete shell is to be reinforced with mesh or hoops weighing at least 0.2 lb. per sq. ft. This formula gives credit to the stiffening action of the enclosing concrete by allowing a stress of 16,000 lb. per sq. in., where in columns otherwise protected a maximum stress of 14,000 is allowed. Figs. 100(*a*) to 100(*e*) show the most usual types of steel columns encased in concrete. The 4-angle column, Fig. 100(*a*), with latticing is well adapted for use in reinforced concrete buildings. The Gray column, Fig. 97(*s*) is also

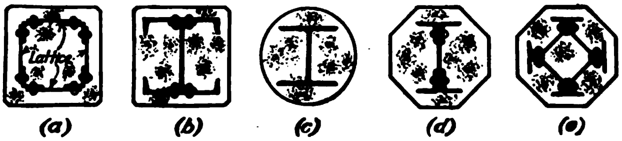


FIG. 100.—Types of steel columns encased in concrete.

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[1] See working stresses recommended by the Joint Committee in *Appendix J*.

often used. Where, however, steel work frames into these columns it is advisable on account of the connections to use one of the other types. In the case of the Gray column the mesh or hoops required on the other types may be omitted as the concrete holds itself securely in place. Steel columns encased in concrete are much more expensive than concrete columns and should only be used in concrete buildings where the size of the necessary concrete column is objectionable or to support the ends of an important steel truss or girder used in the framing. Where steel columns are used in a concrete building, suitable angle-iron brackets must be provided to carry the load from the concrete structure direct to the steel column.

87. Emperger Columns.—The Emperger column (Fig. 101) consists of a cast-iron column surrounded by concrete reinforced both spirally and vertically. Cast iron would be used for columns to a far greater extent but for the fact that it has no reliable strength in tension and therefore must be designed with low stresses to provide against an eccentric loading producing tension. The Emperger column, on which a patent has been applied for, is designed to overcome this objection and at the same time to furnish a fireproof protection on the cast iron. A number of tests have been made on these columns and have been published. The U. S. Bureau of Standards has proposed the following formula for determining the ultimate strength of this column

$$f_s = 5300(1 - p) + 63{,}000p - 240\frac{L}{d}$$

where f_s is the average stress per square inch on the area within the outside of the spiral, p is the percentage of cast iron used, and L and d the length and diameter of the column respectively. Several building departments have made rulings providing for the design of columns of this type. Tests indicate that safe columns will result from the use of a stress of 1120 lb. per sq. in. on the concrete deducting 2 in. of fireproofing, and 11,200 lb. on the cast iron. The spiral and vertical steel must each be not less than 1% of the effective area of the column but are not taken into account in the strength calculations. The concrete used to fill the cast-iron column and for encasing the same is 1-1-2. By comparing these stresses with the proposed formula of the Bureau of Standards it will be found that a factor of safety of about 4 is obtained. The strength of a 20-in. column containing a 7-in. cast-iron core with metal 1 in. thick would be calculated as follows:

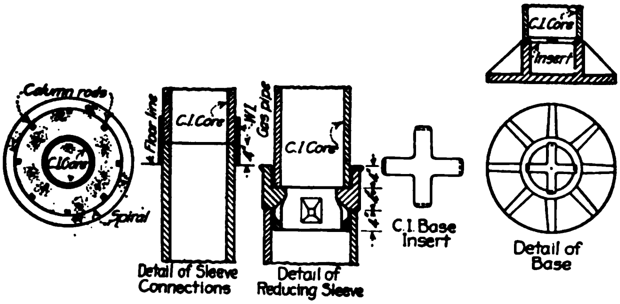

FIG. 101.—Emperger column.

$$\text{Section of cast-iron core} = 18.8 \text{ sq. in.}$$

$$\text{Effective concrete section} = \frac{(\pi)(16)^2}{4} - 18.8 = 182.2 \text{ sq. in.}$$

$$\text{Safe load} = (1120)(182.2) + (11{,}200)(18.8) = 415{,}000 \text{ lb.}$$

Table 1, p. 217, gives the safe loads for Emperger columns according to the above stresses. These columns can be used with economy where the size of an ordinary type of concrete column is objectionable. Designers should maintain the same outside diameter of cast-iron core for as many stories as possible in order to minimize the use of reducing sleeves. All cast-iron cores for use in these columns should be cast vertically. Cast bases as shown in Fig. 101 should be used. Suitable brackets to receive any beams framing into these columns must be provided. In some cases where concrete beams are used the brackets may be omitted as it is possible that sufficient bearing on the concrete encasing may be obtained. A minimum thickness of 5 in. for the encasing concrete should be maintained.

88. Long Columns.—Extensive tests on long columns of steel and cast iron are on record but there are practically no test data on record covering long columns of concrete. In practice it is seldom necessary to use a concrete column more slender than $1/15$ of its length and many rulings allow the normal stresses to apply on columns of this proportion. The Joint Committee recommends that the length be limited to 10 times the diameter of the hooped core for spiral columns and for tied columns to 15 times the least dimension of the column. Designers may, where not limited by city rulings to the contrary, use the usual unit stresses in all concrete columns whose least dimension is not less than $1/15$ the unsupported height. Where a designer must use a more slender column, the stresses may be reduced in accordance with the following formula of the Los Angeles Building Ordinance:

$$fac. = 1.6 \times 1/25\left(\frac{L}{d}\right)$$

where *fac.* is the factor by which the ordinary column stress is to be multiplied for columns in which $\frac{L}{d}$ exceeds 15. d = least dimension of effective section.

The Chicago Ordinance provides that in the case of brick, masonry or plain concrete piers exceeding 6 times their least dimension in height, the following formula shall be used:

$$f_c = C\left(1.25 - \frac{L}{20d}\right)$$

where f_c is the reduced unit stress to be used, C the normal stress used for short piers, and L and d the unsupported length and least dimension respectively.

89. Lap on Column Bars.—For convenience in construction, column bars should be laid out in single story heights. In order that there may be no stress at the lower end of a column bar, there must be a sufficient amount of steel projecting above the floor from the column below to absorb the stress from the upper bars by bond. For bond, the Joint Committee allows a stress of 4% of the concrete strength for plain bars, *i.e.*, 80 lb. per sq. in. The steel stress in a 1-2-4 spirally reinforced column is (698)(15) = 10,470 lb. per sq. in. If, therefore, the same number and size of bars are used in the upper and lower column, the lap would be

$$\text{lap} = (10{,}470)\left(\frac{\pi d^2}{4}\right)\left(\frac{1}{80\pi d}\right) = 33 \text{ diameters}$$

If more bars or bars of a greater diameter are used in the lower column, the length of lap may be reduced. A minimum lap of 2 ft., however, should be maintained. Pipe sleeve connections on column bars should not be used as it is an impossibility to obtain a tight and true bearing of the upper bar upon the lower.

90. Bending Column Bars.—Column bars should be perfectly straight in the shaft of the column. Where a change from one column diameter to another occurs, however, the bars must be bent. The bend should not be abrupt; a slope of 3 in. in 18 in. is a good maximum. Where a bend of more than 3 in. would be necessary, it is advisable to use straight bars, ending at the floor line, and insert straight dowels of sufficient length in the desired position.

91. Spiral Spacing Bars.—All spirals below 20 in. diameter may be fabricated with two spacing bars. Spirals over 20 and under 30 in. should have 3 spacers, and those over 30 in., 4 spacing bars.

92. Spiral Notes.—Spirals should extend to the underside of the floor slab where beam and slab construction is used and to the bottom of the depressed panel in flat slab construction. Where the end of a length of wire used in coiling a spiral occurs, the wires should be lapped half a turn round the spiral and hooked round the spacing bar and the ends left projecting 6 in. or more into the column.

93. Reinforcement at Base of Columns.—If the same mixture of concrete is used in the column as in the top of the foundation, it will usually be necessary to place only the same number of dowel bars in the foundation as there are column bars. These should be lapped as explained above. Where the column is of 1–1–2 concrete and the base of 1–2–4, an analysis of a section of the foundation immediately below the base of the column must be made and it will usually be found that it is necessary to insert a spiral about 12 in. long in the top of the foundation in addition to the dowel bars.

94. Provision for Adding Additional Stories.—Since pipe sleeves put on in the usual way cannot prove effective, the writer believes that, wherever possible, dowels of structural grade steel should be placed in the top of columns which are designed to carry additional stories. These bars may be bent down and protected by the fill and roofing and later straightened up when the additional stories are added. This can only be done, however, with satisfaction in the case of small bars. For bars over 7/8 in., pipe sleeves should be used. The column bars should project 6 in. above the original construction. The pipe sleeves may be placed when the extension is erected and should be large enough to leave at least 1/4 in. of space around the bar. The new column bars should be placed in the sleeves (about 12 in. long) and the space poured in with lead.

95. Columns Supporting Long-span Beams.—Few if any building codes make provision for the fixed end condition existing at the junction of a concrete beam and column. When the present codes were formulated, the use of long-span beams in concrete was unusual. Concrete beams are now frequently used, however, having spans of over 35 ft. Where such beams are supported on brick walls, their treatment as simple beams is entirely satisfactory. Where these beams are supported on concrete columns, designers should always investigate the bending moments occurring at the connection between the beam and column. For spans up to about 30 ft. this is usually unnecessary, but in spans of over 35 ft. it is essential. In recent years the analytical process for determining these moments, known as the Slope-Deflection Method, has been worked up. In the Concrete Engineers' Handbook by Hool and Johnson, Sect. 10, this method of analysis is treated.

On account of the restrictions imposed by existing building codes, designers may not take proper advantage of this method of analysis. For instance, most codes provide that single span beams shall be designed for a moment of $\frac{wl^2}{8}$, no matter what end conditions exist. The designer must therefore use this moment at the center of the beam but must also (even though the code may not require it) provide for a bending moment in the column as determined by the Slope-Deflection Method.

Most structures of this class are only one story in height, the long-span beams supporting roof loads only. In this case the writer's investigations indicate that a satisfactory structure will result if square exterior columns, about 1/18 of the beam span in size, are used. This is, of course, based on the use of a moment of $\frac{wl^2}{8}$ at the center of the beam and upon proper provision being made for the negative moment occurring at the supports.

96. Spiral Tables.—Tables 5 to 10 inclusive give the weights and equivalent areas of spirals for rods from 1/4 to 9/16 in. diameter varying by sixteenths. The weights are given in light type and the equivalent areas in dark type. The weights given do not include the weight of mechanical spacing bars as the various manufacturers have different standards. For estimating purposes it is safe to figure 2 lb. per lin. ft. of spiral to cover the spacers. There will be found upon these tables zigzag lines marked 1/2%., 1%., etc. The spiral immediately above or to the right of these lines is the size of commercial spiral nearest to these percentages. The equivalent areas referred to above are the cross sections of cylinders having a volume equal to the volume of the spiral hooping. To obtain the percentage of spiral in a column containing a given spiral it is necessary to read from the table the dark figure opposite the size of spiral and divide this figure by the area of the column core. Thus, if we have a 7/16-in. spiral, 2-in. pitch, 25-in. diameter, the equivalent area = 5.90 sq. in. and the percentage $= \frac{(5.90)(100)}{490.8} = 1.2\%$

If the size of spiral shown to the right or above the zigzag line for a given percentage does not comply with the code requirements as to pitch, a satisfactory spiral may be found from these tables by picking out one of the same diameter but with another size of rod having the same weight per linear foot. Thus, for 1½ per cent. spiral on a 19-in. core, it is found that a 7/16-in. rod at 2⅛-in. pitch would give the necessary area. The weight is 14.3 lb. per sq. ft. Now a 5/8-in. spiral, 19 in. diameter, at 1⅝-in. pitch, weighs 13.8 lb. and at 1½-in. pitch weighs 14.9 lb. A 5/8-in. spiral, 19 in. diameter, and 1 9/16-in. pitch would therefore give the same weight per foot.

97. Column Graphs.—Diagram 1 is a graph of the proposed column design stresses of the .merican Concrete Institute. Both spirally reinforced and tied column stresses are given. ines are shown on the graph for ½, 1, 1½, and 2 % of spiral used with 1–1–2, 1–1½–3 and -2–4 concrete. The designer may interpolate between these lines for other percentages of piral. It will be found that on each of the lines referred to is designated the mixture of conrete and percentage of spiral on which each line applies.

Diagram 2 is a graph of the Joint Committee column stresses. Under this code recognition nly is given to 1% of spiral, otherwise the same notes as above apply.

Table 11 gives the areas of circles and is convenient in designing spiral columns with the raphs referred to above.

Illustrative Problem.—Design a column for a load of 700,000 lb.

The most economical column to carry a given load is probably a spiral column containing ½ % each f spiral and vertical steel, using 1–1–2 concrete. Such a column would, however, be almost 1.3 as great in diamter as one containing 2% of both spiral and vertical, and for a minimum for this load it would be better olicy to use moderately high percentages of steel. Using a 24-in. core which has an area of 452.4 sq. in. gives an verage core stress of $\frac{700,000}{452.4} = 1550$ lb. per sq. in. Reading from Diagram 1 this stress would require with -1–2 concrete and 1½ % of spiral, 3.2 % of vertical steel. This is too high a percentage of vertical steel for reasnable economy and it would be wise to use 1¾ % of spiral. By interpolation it is found that 2.2 % of vertical ould be required.

This result may now be checked by computation. The core stress $= f_c + (n-1)f_s p + (4n-1)f_s p'$

$$= 750 + (11)(750)\left(\frac{2.20}{100}\right) + (47)(750)\left(\frac{1.75}{100}\right)$$
$$= 1548.3 \text{ lb. per sq. in.}$$

hich checks the value 1550 lb. very closely.

Now the equivalent area of 1¾ % of spiral on a 24-in. core $= \left(\frac{1.75}{100}\right)(452.4) = 7.9$ sq. in. Referring Table 9 it is found that a ½-in. spiral, 1⅞-in. pitch, gives this area on a 24-in. core.

Now 2.2% of vertical steel $= \left(\frac{2.2}{100}\right)(452.4) = 9.95$ sq. in. Referring to Table 4 it is found that irteen ⅞-in. square bars = 9.95 sq. in.

The column then would be 28 in. in diameter, of 1–1–2 concrete, containing a ½-in. spiral, at 1⅞-in. pitch 4 in. diameter, and thirteen ⅞-in. square bars.

98. Plotting Column Graphs.—The best paper to use for plotting column graphs has 20 ivisions to the inch in each direction. This can be obtained with green lines on heavy paper r with orange-red lines on transparent paper for blue printing.

After making some preliminary figures to obtain the range of stress to be covered in the graph, lay off on one de of the sheet the core stresses and on one edge the various percentages of vertical steel. Now plot two of the nes shown on Diagram 1. First take 1-2-4 concrete with ties. The A.C.I. allows 25% of the 28-day rength for concrete columns, so with gravel concrete we have 25% of 2000 = 500 lb. per sq. in. With ½ of vertical steel the average

$$\text{core stress} = f_c + (n-1)pf_c$$
$$= 500 + (14)\left(\frac{1}{200}\right)(500) = 535 \text{ lb. per sq. in.}$$

ith 4% of vertical steel, we have

$$\text{core stress} = 500 + (14)\left(\frac{4}{100}\right)(500) = 780 \text{ lb. per sq. in.}$$

ith these two values the line for 1-2-4 concrete with ties may be plotted.

For spirally reinforced columns the AC.I. also allows 25% of the 28-day strength for the concrete nd the spiral is considered equivalent to 4 times its volume of vertical steel. Hence the

$$\text{core stress} = f_c + (n-1)pf_c + (4n-1)p'f_c$$

or ½ % spiral, ½ % vertical and 1-2-4 concrete, we have

$$\text{core stress} = 500 + (14)\left(\frac{1}{200}\right)(500) + (59)\left(\frac{1}{200}\right)(500) = 682 \text{ lb. per sq. in.}$$

or ½ % spiral, 4% vertical and 1-2-4 concrete,

$$\text{core stress} = 500 + (14)\left(\frac{4}{100}\right)(500) + (59)\left(\frac{1}{200}\right)(500) = 928 \text{ lb. per sq. in.}$$

ith these two values the line for 1-2-4 concrete with ½ % spiral may be plotted.

Many column codes allow the spiral only to be considered as effective to the extent of 2.4 times its volume of ertical steel. The formula would then become

$$\text{core stress} = f_c + (n-1)pf_c + (2.4n-1)p'f_c$$

n this formula, unity is deducted from n since a unit volume of vertical steel displaces a unit volume of concrete. imilarly for the spiral steel. In this case, however, n must first be multiplied by the factor designating the efficiency of the spiral steel as compared with the vertical.

DIAGRAM 1.

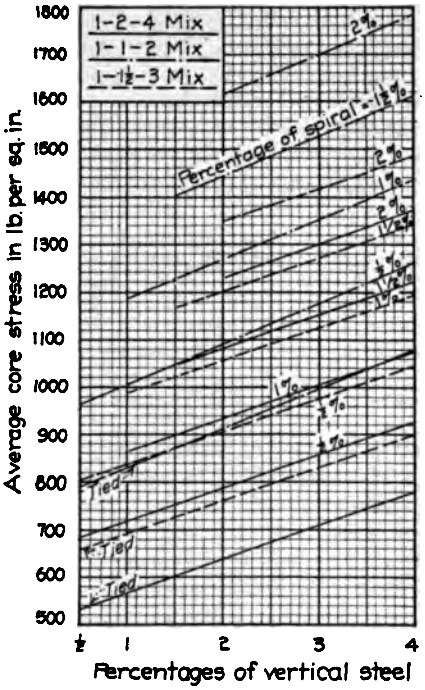

DIAGRAM 2.

JOINT COMMITTEE COLUMN GRAPH

Based on gravel concrete f_c for tied columns = 22½ % of 28-day strength and for spiral columns = 34.9 %.
For tied or spiral columns, $P = A_cf_c + A_s(n - 1)f_c$.
Diameter of effective core for tied and spiral columns, 3 in. less than size of column.
2-in. fireproofing for all column steel.

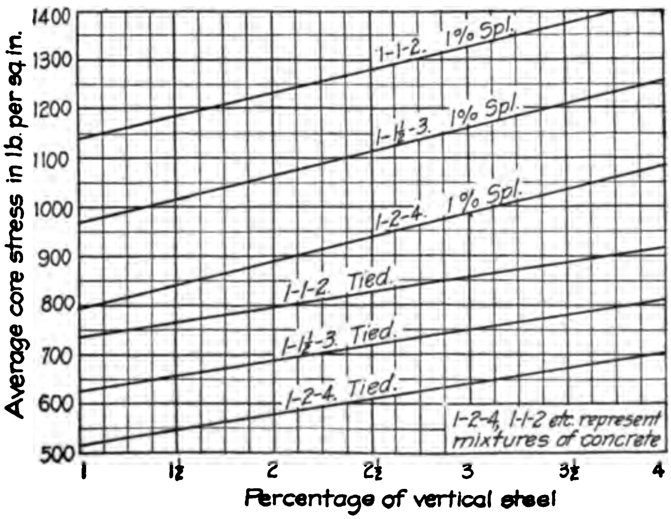

Table 1.—Table of Safe Loads in Thousands of Pounds for Emperger Columns

1-1-2 mix		Reinforcement				Cast-iron core																		
Column diam.	Core diam.	Spiral		Verticals		5-in diam. Metal thickness			7-in diam. Metal thickness				9¼-in diam. Metal thickness				11⅝-in diam. Metal thickness				13½-in diam. Metal thickness			
(in.)	(in.)	Diam. (in.)	Pitch (in.)	No.	Size (in.)	½"	¾"	1"	½"	¾"	1"	1¼"	½"	¾"	1"	1¼"	¾"	1"	1¼"	1½"	¾"	1"	1¼"	1½"
16	12	3/16	1	8	⅜□	197	226	253																
18	14	¼	1⅜	8	½ϕ	243	272	299	275	320	362	400												
20	16	¼	1¼	8	½□	296	325	352	328	373	415	453	363	426	486	542								
22	18	¼	1⅛	8	⅝ϕ	356	385	412	388	433	475	513	423	486	546	602	535	615	687	755				
24	20	¼	1	8	⅝□	423	452	479	455	500	542	580	490	553	613	669	602	682	754	822	657	750	840	925
26	22	5/16	1 7/16	8	¾ϕ	....	....	....	528	573	615	653	563	626	686	742	675	755	827	895	732	823	913	998
28	24	5/16	1 5/16	8	¾□	....	....	....	608	653	695	733	643	706	766	822	755	835	907	975	812	903	993	1078
30	26	5/16	1¼	8	⅞ϕ	....	....	....	696	741	783	821	731	794	854	910	843	923	995	1063	900	991	1081	1166
32	28	5/16	1⅛	10	⅞ϕ	....	....	....	791	836	878	916	826	889	949	1005	938	1018	1090	1158	995	1086	1176	1261
34	30	⅜	1½	10	⅞□	....	....	....	....	939	981	1019	929	992	1052	1108	1041	1121	1193	1261	1098	1189	1279	1364
36	32	⅜	1⅜	10	1ϕ	....	....	....	....	....	1090	1128	1038	1101	1161	1217	1150	1230	1302	1370	1207	1298	1388	1473

TABLE 2.—TABLE SHOWING VOLUME OF COLUMN IN CUBIC FEET PER FOOT OF LENGTH AND PERIMETER IN FEET FOR DIAMETER D

D (inches)	Square		Round		Octagonal		D (inches)	Square		Round		Octagonal	
	Vol.	Perim.	Vol.	Perim.	Vol.	Perim.		Vol.	Perim.	Vol.	Perim.	Vol.	Perim.
10	0.69	3.33	0.55	2.62	0.58	2.76	27	5.06	9.00	3.98	7.08	4.19	7.46
11	0.84	3.67	0.66	2.88	0.70	3.04	28	5.44	9.33	4.28	7.33	4.51	7.74
12	1.00	4.00	0.79	3.15	0.83	3.31	29	5.84	9.67	4.62	7.60	4.84	8.01
13	1.17	4.33	0.92	3.41	0.97	3.59	30	6.25	10.00	4.91	7.85	5.18	8.28
14	1.36	4.67	1.07	3.67	1.13	3.87	31	6.68	10.33	5.24	8.13	5.53	8.56
15	1.56	5.00	1.23	3.93	1.29	4.14	32	7.11	10.67	5.58	8.38	5.88	8.84
16	1.78	5.33	1.40	4.18	1.47	4.41	33	7.55	11.00	5.94	8.64	6.26	9.10
17	2.01	5.67	1.58	4.45	1.66	4.69	34	8.02	11.33	6.31	8.90	6.64	9.38
18	2.25	6.00	1.77	4.71	1.87	4.97	35	8.50	11.67	6.67	9.17	7.05	9.66
19	2.51	6.33	1.97	4.98	2.08	5.24	36	8.98	12.00	7.05	9.43	7.46	9.94
20	2.78	6.67	2.18	5.24	2.30	5.52	37	9.49	12.33	7.46	9.68	7.88	10.22
21	3.06	7.00	2.41	5.50	2.54	5.80	38	10.00	12.67	7.87	9.94	8.30	10.48
22	3.36	7.33	2.64	5.77	2.78	6.07	39	10.60	13.00	8.29	10.22	8.75	10.76
23	3.67	7.67	2.89	6.02	3.05	6.35	40	11.10	13.33	8.71	10.49	9.20	11.05
24	4.00	8.00	3.14	6.28	3.32	6.63	41	11.70	13.67	9.15	10.73	9.67	11.32
25	4.34	8.33	3.41	6.55	3.59	6.90	42	12.30	14.00	9.61	10.99	10.10	11.60
26	4.70	8.67	3.68	6.80	3.89	7.18							

TABLE 3.—TABLE OF AREAS AND WEIGHTS OF ROUND COLUMN RODS

Heavy figures = area | Number of rods | Light figures = weight per foot

Diameter (inches)	1	4	6	8	10	11	12	13	14	15	16	17	18	19	20
5/8	**0.3068**	**1.23**	**1.84**	**2.46**	**3.07**	**3.38**	**3.68**								
	1.043	4.08	6.26	8.35	10.4	11.5	12.5								
3/4	**0.4418**	**1.77**	**2.65**	**3.54**	**4.42**	**4.86**	**5.30**	**5.75**	**6.19**						
	1.502	6.01	9.01	12.0	15.0	16.6	18.1	19.6	21.1						
7/8	**0.6103**	**2.40**	**3.61**	**4.81**	**6.01**	**6.62**	**7.22**	**7.82**	**8.43**	**9.03**	**9.63**				
	2.044	8.18	12.3	16.4	20.4	22.5	24.6	26.6	28.7	30.7	32.8				
1	**0.7854**			**6.29**	**7.85**	**8.64**	**9.43**	**10.2**	**11.0**	**11.8**	**12.6**	**13.4**	**14.1**	**14.9**	
	2.670			21.4	26.7	29.4	32.0	34.7	37.4	40.0	42.7	45.4	48.0	50.7	
1 1/8	**0.9940**						**11.9**	**12.9**	**13.9**	**14.9**	**15.9**	**16.9**	**17.9**	**18.9**	**19.9**
	3.379						40.5	44.0	47.3	50.7	54.0	57.4	60.8	64.2	67.6
1 1/4	**1.227**										**19.6**	**20.8**	**22.1**	**23.3**	**24.5**
	4.173										66.8	71.0	75.1	79.4	83.5
1 3/8	**1.485**										**23.8**	**25.2**	**26.7**	**28.2**	**29.7**
	5.049										80.7	85.9	90.9	95.9	101.0
1 1/2	**1.767**										**28.3**	**30.0**	**31.8**	**33.7**	**35.3**
	6.008										96.1	102.0	108.1	115.0	120.1

Table 4.—Table of Areas and Weights of Square Column Rods

	Heavy figures = area						Number of rods						Light figures = weight per foot		
Size (inches)	1	4	6	8	10	11	12	13	14	15	16	17	18	19	20
5/8	**0.3906** 1.328	**1.56** 5.31	**2.34** 7.96	**3.12** 10.6	**3.91** 13.3	**4.30** 14.6	**4.69** 15.9								
3/4	**0.5625** 1.913	**2.25** 7.65	**3.38** 11.5	**4.50** 15.3	**5.62** 19.1	**6.19** 21.0	**6.75** 23.0	**7.31** 24.9	**7.78** 26.8						
7/8	**0.7656** 2.603	**3.06** 10.4	**4.60** 15.6	**6.13** 20.8	**7.66** 26.0	**8.42** 28.6	**9.19** 31.2	**9.95** 33.8	**10.7** 36.5	**11.5** 39.1	**12.2** 41.7				
1	**1.000** 3.400			**8.00** 27.2	**10.0** 34.0	**11.0** 37.4	**12.0** 40.8	**13.0** 44.2	**14.0** 47.6	**15.0** 51.0	**16.0** 54.4	**17.0** 57.8	**18.0** 61.2	**19.0** 64.6	
1 1/8	**1.266** 4.303						**15.2** 51.6	**16.4** 56.0	**17.7** 60.3	**19.0** 64.5	**20.2** 68.9	**21.5** 73.1	**22.8** 77.5	**24.0** 81.8	**25.3** 86.1
1 1/4	**1.563** 5.312										**25.0** 85.0	**26.6** 90.3	**28.1** 95.6	**29.7** 101.0	**31.2** 106.2
1 3/8	**1.891** 6.428										**30.3** 102.8	**32.2** 109.1	**34.0** 115.7	**35.9** 122.0	**36.8** 128.6
1 1/2	**2.250** 7.650										**36.0** 122.4	**38.2** 130.0	**40.5** 137.8	**42.8** 145.4	**45.0** 153.0

TABLE 5

AREAS AND WEIGHTS OF ¼-IN. WIRE SPIRALS
Areas of Equivalent Cylinders in Square inches Given in Heavy Figures
Weights of Wire in Pounds per Foot Length of Spiral in Light Figures
Pitch of Spiral in Inches

ore diam. (inches)	3"	2⅞"	2¾"	2⅝"	2½"	2⅜"	2¼"	2⅛"	2"	1⅞"	1¾"	1⅝"	1½"
10	**0.515**	**0.537**	**0.562**	**0.588**	**0.618**	**0.651**	**0.686**	**0.726**	**0.771**	**0.824**	**0.882**	**0.950**	**1.03**
	1.75	1.83	1.91	2.00	2.10	2.21	2.33	2.47	2.62	2.80	3.00	3.23	3.50
11	**0.566**	**0.591**	**0.618**	**0.646**	**0.679**	**0.715**	**0.754**	**0.798**	**0.848**	**0.905**	**0.971**	**1.04**	**1.13**
	1.92	2.01	2.10	2.20	2.31	2.43	2.56	2.72	2.88	3.08	3.30	3.55	3.85
12	**0.617**	**0.645**	**0.674**	**0.705**	**0.741**	**0.781**	**0.823**	**0.871**	**0.925**	**0.987**	**1.06**	**1.14**	**1.23**
	2.10	2.19	2.29	2.40	2.52	2.65	2.80	2.96	3.15	3.36	3.60	3.88	4.19
13	**0.668**	**0.699**	**0.730**	**0.764**	**0.803**	**0.846**	**0.892**	**0.944**	**1.00**	**1.07**	**1.15**	**1.23**	**1.33**
	2.27	2.38	2.48	2.60	2.73	2.88	3.03	3.21	3.41	3.64	3.90	4.20	4.54
14	**0.720**	**0.752**	**0.786**	**0.823**	**0.865**	**0.911**	**0.961**	**1.02**	**1.08**	**1.15**	**1.23**	**1.33**	**1.44** 1 %
	2.45	2.56	2.67	2.80	2.94	3.10	3.26	3.46	3.67	3.92	4.20	4.52	4.90
15	**0.771**	**0.807**	**0.843**	**0.882**	**0.926**	**0.976**	**1.03**	**1.09**	**1.16**	**1.23**	**1.32**	**1.42**	**1.54**
	2.62	2.74	2.87	3.00	3.15	3.32	3.50	3.71	3.94	4.20	4.50	4.85	5.24
16	**0.823**	**0.860**	**0.898**	**0.940**	**0.987**	**1.04**	**1.10**	**1.16**	**1.23**	**1.31**	**1.41**	**1.52**	**1.64**
	2.80	2.92	3.05	3.20	3.36	3.54	3.73	3.95	4.20	4.48	4.80	5.17	5.60
17	**0.874**	**0.913**	**0.954**	**1.00**	**1.05**	**1.11**	**1.17**	**1.23**	**1.31**	**1.40**	**1.50**	**1.61**	**1.74**
	2.97	3.10	3.24	3.40	3.56	3.75	3.96	4.19	4.46	4.76	5.10	5.49	5.94
18	**0.926**	**0.966**	**1.01**	**1.06**	**1.11**	**1.17**	**1.24**	**1.31**	**1.39**	**1.48**	**1.59**	**1.71**	**1.85**
	3.15	3.29	3.44	3.60	3.78	3.98	4.20	4.45	4.73	5.05	5.40	5.82	6.28
19	**0.978**	**1.02**	**1.07**	**1.12**	**1.17**	**1.24**	**1.30**	**1.38**	**1.46**	**1.56**	**1.67**	**1.80**	**1.95**
	3.32	3.47	3.62	3.80	3.98	4.20	4.43	4.69	4.98	5.32	5.70	6.14	6.64
20	**1.03**	**1.07**	**1.12**	**1.17**	**1.23**	**1.30**	**1.37**	**1.45**	**1.54**	**1.64**	**1.76**	**1.90**	**2.06**
	3.50	3.65	3.82	4.00	4.20	4.42	4.66	4.93	5.25	5.60	6.00	6.46	7.00
21	**1.08**	**1.13**	**1.18**	**1.23**	**1.30**	**1.37**	**1.44**	**1.53**	**1.62**	**1.73**	**1.85**	**1.99**	**2.16**
	3.67	3.84	4.01	4.20	4.40	4.64	4.90	5.18	5.51	5.88	6.30	6.78	7.34
22	**1.13**	**1.18**	**1.23**	**1.29**	**1.36**	**1.43**	**1.51**	**1.69**	**1.70**	**1.81**	**1.94**	**2.09**	**2.27**
	3.85	4.02	4.20	4.40	4.62	4.86	5.13	5.43	5.77	6.16	6.60	7.11	7.70
23	**1.18**	**1.23**	**1.29**	**1.35**	**1.42**	**1.49**	**1.58**	**1.67**	**1.77**	**1.89**	**2.03**	**2.18**	**2.37**
	4.02	4.20	4.38	4.60	4.83	5.08	5.36	5.67	6.03	6.44	6.90	7.43	8.05
24	**1.24**	**1.29**	**1.35**	**1.41**	**1.48**	**1.56**	**1.65**	**1.74**	**1.85**	**1.97**	**2.12**	**2.28**	**2.47**
	4.20	4.38	4.57	4.80	5.04	5.30	5.60	5.92	6.30	6.72	7.20	7.75	8.40
25	**1.29**	**1.34**	**1.40**	**1.47**	**1.54**	**1.63**	**1.71**	**1.82**	**1.93**	**2.06**	**2.20**	**2.37**	**2.57**
	4.37	4.57	4.78	5.00	5.25	5.53	5.83	6.17	6.56	7.09	7.50	8.07	8.75
26	**1.34**	**1.40**	**1.46**	**1.53**	**1.61**	**1.69**	**1.78**	**1.89**	**2.00**	**2.14**	**2.29**	**2.47**	**2.67**
	4.55	4.75	4.96	5.20	5.46	5.75	6.06	6.42	6.82	7.28	7.80	8.39	9.10
27	**1.39**	**1.45**	**1.52**	**1.59**	**1.67**	**1.76**	**1.85**	**1.96**	**2.08**	**2.22**	**2.38**	**2.56**	**2.78** ½ %
	4.72	4.94	5.15	5.40	5.67	5.96	6.30	6.66	7.08	7.56	8.10	8.72	9.45
28	**1.44**	**1.50**	**1.57**	**1.64**	**1.73**	**1.82**	**1.92**	**2.03**	**2.16**	**2.30**	**2.47**	**2.66**	**2.88**
	4.90	5.11	5.34	5.60	5.88	6.19	6.53	6.91	7.34	7.84	8.40	9.04	9.80
29	**1.49**	**1.56**	**1.63**	**1.70**	**1.79**	**1.89**	**1.99**	**2.11**	**2.24**	**2.38**	**2.56**	**2.75**	**2.98**
	5.07	5.30	5.53	5.80	6.09	6.41	6.77	7.16	7.60	8.12	8.70	9.36	10.15

pirals above and to right of zigzag lines are nearest commercial size fully equal to percentage of core area indi-
at end of line.

Veights in above table include the wire only. To this must be added the weights of the spacing bars and the
t of extra turn at ends of spiral.

Table 6

Core diam. (inches)	Areas and Weights of 5/16-in. Wire Spirals Areas of Equivalent Cylinders in Square Inches Given in Heavy Figures Weights of Wire in Pounds per Foot Length of Spiral in Light Figures Pitch of Spiral in Inches													
	3"	2⅞"	2¾"	2⅝"	2½"	2⅜"	2¼"	2⅛"	2"	1⅞"	1¾"	1⅝"	1½"	
10	**0.803**	**0.838**	**0.876**	**0.918**	**0.964**	**1.01**	**1.07**	**1.13**	**1.20**	**1.28**	**1.38**	**1.48**	**1.61**	
	2.73	2.86	2.98	3.13	3.28	3.46	3.65	3.86	4.10	4.38	4.69	5.05	5.48	
11	**0.884**	**0.922**	**0.963**	**1.01**	**1.06**	**1.12**	**1.18**	**1.25**	**1.33**	**1.41**	**1.51**	**1.63**	**1.77**	
	3.01	3.14	3.28	3.44	3.61	3.80	4.01	4.25	4.52	4.82	5.16	5.56	6.02	
12	**0.965**	**1.01**	**1.05**	**1.10**	**1.16**	**1.22**	**1.28**	**1.36**	**1.44**	**1.54**	**1.65**	**1.78**	**1.93**	
	3.28	3.43	3.58	3.75	3.93	4.15	4.39	4.64	4.93	5.26	5.63	6.06	6.57	
13	**1.04**	**1.09**	**1.14**	**1.19**	**1.25**	**1.32**	**1.39**	**1.47**	**1.57**	**1.67**	**1.79**	**1.93**	**2.09**	
	3.55	3.72	3.88	4.06	4.27	4.50	4.75	5.02	5.34	5.70	6.10	6.57	7.11	1½%
14	**1.12**	**1.17**	**1.23**	**1.28**	**1.35**	**1.42**	**1.50**	**1.59**	**1.69**	**1.80**	**1.93**	**2.08**	**2.25**	
	3.83	4.00	4.18	4.38	4.60	4.84	5.11	5.41	5.75	6.13	6.56	7.08	7.67	
15	**1.20**	**1.26**	**1.31**	**1.38**	**1.45**	**1.52**	**1.61**	**1.70**	**1.81**	**1.93**	**2.07**	**2.22**	**2.41**	
	4.10	4.29	4.48	4.69	4.93	5.19	5.48	5.79	6.16	6.57	7.04	7.59	8.21	
16	**1.29**	**1.34**	**1.40**	**1.47**	**1.54**	**1.62**	**1.71**	**1.81**	**1.93**	**2.06**	**2.20**	**2.38**	**2.57**	
	4.38	4.57	4.78	5.01	5.26	5.53	5.84	6.18	6.57	7.01	7.51	8.08	8.76	
17	**1.37**	**1.42**	**1.49**	**1.56**	**1.64**	**1.72**	**1.82**	**1.93**	**2.05**	**2.19**	**2.34**	**2.52**	**2.73**	
	4.65	4.86	5.08	5.32	5.58	5.88	6.21	6.56	6.98	7.45	7.98	8.59	9.31	
18	**1.45**	**1.51**	**1.58**	**1.65**	**1.73**	**1.83**	**1.93**	**2.04**	**2.17**	**2.31**	**2.48**	**2.67**	**2.89**	
	4.93	5.14	5.37	5.63	5.91	6.22	6.57	6.95	7.39	7.88	8.44	9.10	9.86	
19	**1.53**	**1.59**	**1.66**	**1.74**	**1.83**	**1.93**	**2.03**	**2.16**	**2.29**	**2.44**	**2.62**	**2.82**	**3.05**	
	5.20	5.43	5.68	5.94	6.24	6.57	6.94	7.34	7.80	8.32	8.92	9.60	10.4	
20	**1.61**	**1.68**	**1.75**	**1.83**	**1.93**	**2.03**	**2.14**	**2.27**	**2.41**	**2.57**	**2.75**	**2.97**	**3.21**	
	5.47	5.71	5.97	6.26	6.57	6.91	7.30	7.72	8.21	8.76	9.39	10.1	11.0	1%
21	**1.69**	**1.76**	**1.84**	**1.93**	**2.02**	**2.13**	**2.25**	**2.38**	**2.53**	**2.70**	**2.89**	**3.12**	**3.37**	
	5.74	6.00	6.27	6.56	6.89	7.26	7.67	8.11	8.62	9.20	9.85	10.6	11.5	
22	**1.77**	**1.84**	**1.93**	**2.02**	**2.12**	**2.23**	**2.35**	**2.50**	**2.65**	**2.83**	**3.03**	**3.26**	**3.53**	
	6.02	6.29	6.57	6.88	7.22	7.61	8.03	8.49	9.04	9.65	10.3	11.1	12.0	
23	**1.85**	**1.93**	**2.02**	**2.11**	**2.22**	**2.33**	**2.46**	**2.61**	**2.77**	**2.95**	**3.17**	**3.41**	**3.70**	
	6.29	6.57	6.87	7.19	7.55	7.95	8.40	8.88	9.45	10.1	10.8	11.6	12.6	
24	**1.93**	**2.01**	**2.10**	**2.20**	**2.31**	**2.43**	**2.57**	**2.72**	**2.89**	**3.08**	**3.31**	**3.56**	**3.86**	
	6.56	6.85	7.17	7.51	7.88	8.30	8.76	9.27	9.86	10.5	11.3	12.1	13.1	
25	**2.01**	**2.09**	**2.19**	**2.29**	**2.41**	**2.54**	**2.68**	**2.83**	**3.01**	**3.21**	**3.44**	**3.71**	**4.01**	
	6.83	7.14	7.46	7.82	8.21	8.65	9.13	9.66	10.3	11.0	11.7	12.6	13.7	
26	**2.09**	**2.18**	**2.28**	**2.38**	**2.51**	**2.64**	**2.78**	**2.95**	**3.13**	**3.34**	**3.58**	**3.86**	**4.18**	
	7.11	7.43	7.77	8.13	8.54	9.00	9.50	10.0	10.7	11.4	12.2	13.1	14.2	
27	**2.17**	**2.26**	**2.37**	**2.48**	**2.60**	**2.74**	**2.89**	**3.06**	**3.25**	**3.47**	**3.72**	**4.01**	**4.34**	
	7.38	7.72	8.06	8.45	8.87	9.35	9.87	10.4	11.1	11.8	12.7	13.6	14.8	
28	**2.25**	**2.35**	**2.45**	**2.57**	**2.70**	**2.84**	**3.00**	**3.17**	**3.37**	**3.60**	**3.86**	**4.16**	**4.50**	
	7.66	8.00	8.36	8.75	9.20	9.69	10.2	10.8	11.5	12.3	13.1	14.1	15.3	
29	**2.33**	**2.43**	**2.54**	**2.66**	**2.80**	**2.94**	**3.11**	**3.29**	**3.49**	**3.73**	**3.99**	**4.30**	**4.66**	
	7.93	8.29	8.66	9.08	9.53	10.0	10.6	11.2	11.9	12.7	13.6	14.6	15.9	
30	**2.41**	**2.52**	**2.63**	**2.75**	**2.89**	**3.04**	**3.21**	**3.40**	**3.62**	**3.86**	**4.13**	**4.45**	**4.82**	
	8.21	8.58	8.96	9.39	9.86	10.4	11.0	11.6	12.3	13.1	14.1	15.2	16.4	
31	**2.49**	**2.60**	**2.72**	**2.84**	**2.99**	**3.14**	**3.32**	**3.52**	**3.74**	**3.99**	**4.27**	**4.60**	**4.98**	
	8.48	8.86	9.26	9.70	10.2	10.7	11.3	12.0	12.7	13.6	14.5	15.7	17.0	
32	**2.57**	**2.68**	**2.81**	**2.94**	**3.08**	**3.25**	**3.43**	**3.63**	**3.86**	**4.11**	**4.41**	**4.75**	**5.14**	
	8.75	9.15	9.56	10.0	10.5	11.1	11.7	12.4	13.1	14.0	15.0	16.2	17.5	
33	**2.65**	**2.77**	**2.89**	**3.03**	**3.18**	**3.35**	**3.53**	**3.74**	**3.98**	**4.24**	**4.55**	**4.90**	**5.31**	
	9.03	9.44	9.86	10.3	10.8	11.4	12.1	12.7	13.5	14.5	15.5	16.7	18.1	
34	**2.73**	**2.85**	**2.98**	**3.12**	**3.28**	**3.45**	**3.64**	**3.86**	**4.10**	**4.37**	**4.69**	**5.05**	**5.47**	
	9.30	9.72	10.2	10.6	11.2	11.8	12.4	13.1	14.0	14.9	16.0	17.2	18.6	
												½%		

TABLE 7

re diam. (inches)	AREAS AND WEIGHTS OF ⅜-IN. WIRE SPIRALS Areas of Equivalent Cylinders in Square Inches Given in Heavy Figures Weights of Wire in Pounds per Foot Length of Spiral in Light Figures Pitch of Spiral in Inches												
	3"	2⅞"	2¾"	2⅝"	2½"	2⅜"	2¼"	2⅛"	2"	1⅞"	1¾"	1⅝"	1½"	
15	**1.74**	**1.81**	**1.89**	**1.98**	**2.08**	**2.19**	**2.32**	**2.46**	**2.60**	**2.78**	**2.97**	**3.21**	**3.47**	
	5.91	6.16	6.44	6.73	7.18	7.46	7.87	8.32	8.85	9.45	10.1	10.9	11.8	
16	**1.85**	**1.93**	**2.02**	**2.12**	**2.22**	**2.34**	**2.47**	**2.62**	**2.78**	**2.96**	**3.17**	**3.42**	**3.70**	
	6.30	6.57	6.87	7.18	7.55	7.95	8.39	8.88	9.44	10.1	10.8	11.6	12.6	
17	**1.97**	**2.05**	**2.14**	**2.25**	**2.36**	**2.48**	**2.62**	**2.78**	**2.95**	**3.15**	**3.37**	**3.63**	**3.94**	
	6.69	6.98	7.30	7.63	8.02	8.45	8.92	9.44	10.0	10.7	11.5	12.3	13.4	
18	**2.08**	**2.17**	**2.27**	**2.38**	**2.50**	**2.64**	**2.78**	**2.94**	**3.12**	**3.33**	**3.57**	**3.84**	**4.16**	
	7.08	7.39	7.73	80.8	8.50	8.94	9.44	10.0	10.6	11.3	12.1	13.1	14.2	
19	**2.20**	**2.29**	**2.40**	**2.51**	**2.64**	**2.78**	**2.93**	**3.10**	**3.30**	**3.52**	**3.77**	**4.06**	**4.40**	
	7.48	7.81	8.15	8.53	8.97	9.45	9.97	10.5	11.2	12.0	12.8	13.8	14.9	1½ %
20	**2.32**	**2.41**	**2.52**	**2.64**	**2.78**	**2.92**	**3.09**	**3.27**	**3.47**	**3.70**	**3.97**	**4.27**	**4.63**	
	7.87	8.21	8.59	8.98	9.44	9.94	10.5	11.1	11.8	12.6	13.5	14.5	15.7	
21	**2.43**	**2.53**	**2.65**	**2.78**	**2.92**	**3.07**	**3.24**	**3.43**	**3.67**	**3.89**	**4.16**	**4.49**	**4.86**	
	8.26	8.62	9.02	9.43	9.92	10.4	11.0	11.6	12.4	13.2	14.2	15.2	16.5	
22	**2.55**	**2.65**	**2.77**	**2.91**	**3.05**	**3.22**	**3.39**	**3.59**	**3.82**	**4.07**	**4.36**	**4.70**	**5.09**	
	8.66	9.04	9.45	9.88	10.4	10.9	11.5	12.2	13.0	13.8	14.8	16.0	17.3	
23	**2.66**	**2.78**	**2.90**	**3.04**	**3.19**	**3.36**	**3.55**	**3.75**	**3.99**	**4.26**	**4.56**	**4.91**	**5.32**	
	9.05	9.45	9.88	10.3	10.9	11.4	12.1	12.8	13.6	14.5	15.5	16.7	18.1	
24	**2.78**	**2.90**	**3.03**	**3.17**	**3.33**	**3.51**	**3.70**	**3.92**	**4.17**	**4.44**	**4.76**	**5.12**	**5.55**	
	9.45	9.86	10.3	10.8	11.3	11.9	12.6	13.3	14.2	15.1	16.2	17.4	18.9	
25	**2.89**	**3.02**	**3.15**	**3.30**	**3.47**	**3.65**	**3.86**	**4.08**	**4.34**	**4.63**	**4.96**	**5.34**	**5.79**	
	9.84	10.3	10.7	11.2	11.8	12.4	13.1	13.9	14.7	15.7	16.9	18.2	19.7	
26	**3.01**	**3.14**	**3.28**	**3.44**	**3.61**	**3.80**	**4.01**	**4.25**	**4.51**	**4.81**	**5.16**	**5.55**	**6.02**	
	10.2	10.7	11.2	11.7	12.3	12.9	13.6	14.4	15.4	16.4	17.5	18.9	20.4	
27	**3.12**	**3.26**	**3.41**	**3.57**	**3.75**	**3.95**	**4.17**	**4.41**	**4.69**	**5.00**	**5.36**	**5.76**	**6.25**	
	10.6	11.1	11.6	12.1	12.7	13.4	14.2	15.0	15.9	17.0	18.2	19.6	21.2	
28	**3.24**	**3.38**	**3.53**	**3.70**	**3.89**	**4.09**	**4.32**	**4.57**	**4.86**	**5.19**	**5.55**	**5.98**	**6.48**	
	11.0	11.5	12.0	12.6	13.2	13.9	14.7	15.5	16.5	17.6	18.9	20.4	22.0	
29	**3.36**	**3.50**	**3.66**	**3.84**	**4.03**	**4.24**	**4.47**	**4.74**	**5.04**	**5.37**	**5.75**	**6.20**	**6.71**	
	11.4	11.9	12.5	13.0	13.7	14.4	15.2	16.1	17.1	18.3	19.6	21.1	22.8	1 %
30	**3.47**	**3.62**	**3.79**	**3.97**	**4.17**	**4.39**	**4.63**	**4.90**	**5.21**	**5.55**	**5.95**	**6.41**	**6.94**	
	11.8	12.3	12.9	13.5	14.2	14.9	15.7	16.6	17.7	18.9	20.2	21.8	23.6	
31	**3.59**	**3.74**	**3.91**	**4.10**	**4.31**	**4.53**	**4.78**	**5.06**	**5.39**	**5.74**	**6.15**	**6.62**	**7.18**	
	12.2	12.7	13.3	13.9	14.6	15.4	16.3	16.2	18.3	19.5	20.9	22.5	24.4	
32	**3.70**	**3.86**	**4.04**	**4.23**	**4.44**	**4.68**	**4.94**	**5.23**	**5.56**	**5.93**	**6.35**	**6.84**	**7.41**	
	12.6	13.1	13.7	14.4	15.1	15.9	16.8	17.8	18.9	20.1	21.6	23.3	25.2	
33	**3.82**	**3.98**	**4.17**	**4.36**	**4.59**	**4.82**	**5.09**	**5.39**	**5.73**	**6.11**	**6.55**	**7.05**	**7.64**	
	13.0	13.5	14.2	14.8	15.6	16.4	17.3	18.3	19.5	20.8	22.3	24.0	26.0	
34	**3.94**	**4.11**	**4.29**	**4.50**	**4.72**	**4.97**	**5.25**	**5.56**	**5.91**	**6.30**	**6.75**	**7.27**	**7.87**	
	13.4	14.0	14.6	15.3	16.1	16.9	17.8	18.9	20.1	21.4	23.0	24.7	26.8	
35	**4.05**	**4.23**	**4.42**	**4.63**	**4.86**	**5.12**	**5.40**	**5.72**	**6.08**	**6.48**	**6.95**	**7.48**	**8.10**	
	13.8	14.4	15.0	15.7	16.5	17.4	18.4	19.4	20.7	22.0	23.6	25.4	27.5	
36	**4.18**	**4.34**	**4.54**	**4.76**	**5.00**	**5.26**	**5.56**	**5.88**	**6.25**	**6.66**	**7.14**	**7.69**	**8.33**	
	14.2	14.8	15.4	16.2	17.0	17.9	18.9	20.0	21.3	22.7	24.3	26.2	28.3	
37	**4.28**	**4.47**	**4.67**	**4.89**	**5.14**	**5.41**	**5.71**	**6.05**	**6.43**	**6.85**	**7.34**	**7.91**	**8.56**	
	14.6	15.2	15.9	16.6	17.5	18.4	19.4	20.6	21.9	23.3	25.0	26.9	29.1	
38	**4.40**	**4.59**	**4.80**	**5.03**	**5.28**	**5.56**	**5.87**	**6.21**	**6.60**	**7.04**	**7.54**	**8.12**	**8.79**	
	15.0	15.6	16.3	17.1	18.0	18.9	19.9	21.1	22.4	23.9	25.6	27.6	29.9	
39	**4.51**	**4.71**	**4.92**	**5.16**	**5.42**	**5.70**	**6.02**	**6.37**	**6.77**	**7.22**	**7.74**	**8.33**	**9.03**	
	15.4	16.0	16.7	17.5	18.4	19.4	20.5	21.6	23.0	24.6	26.3	28.3	30.7	

TABLE 8

Core diam. (inches)	AREAS AND WEIGHTS OF 7/16-IN. WIRE SPIRALS Areas of Equivalent Cylinders in Square Inches Given in Heavy Figures Weights of Wire in Pounds per Foot Length of Spiral in Light Figures Pitch of Spiral in Inches												
	3"	2⅞"	2¾"	2⅝"	2½"	2⅜"	2¼"	2⅛"	2"	1⅞"	1¾"	1⅝"	1½"	
15	**2.36**	**2.47**	**2.58**	**2.70**	**2.84**	**2.98**	**3.15**	**3.34**	**3.54**					
	8.03	8.38	8.76	9.21	9.65	10.1	10.7	11.3	12.0					
16	**2.52**	**2.63**	**2.75**	**2.88**	**3.02**	**3.18**	**3.36**	**3.56**	**3.78**	**4.03**				
	8.56	8.94	9.35	9.80	10.3	10.8	11.4	12.1	12.8	13.7				
17	**2.68**	**2.80**	**2.92**	**3.06**	**3.22**	**3.38**	**3.57**	**3.78**	**4.01**	**4.28**	**4.59**			
	9.10	9.50	9.93	10.4	10.9	11.5	12.1	12.8	13.7	14.6	15.6			
18	**2.83**	**2.96**	**3.09**	**3.24**	**3.40**	**3.58**	**3.78**	**3.99**	**4.25**	**4.53**	**4.86**	**5.23**		
	9.64	10.1	10.5	11.0	11.6	12.2	12.9	13.6	14.5	15.4	16.5	17.8		
19	**2.99**	**3.12**	**3.26**	**3.42**	**3.59**	**3.78**	**3.99**	**4.22**	**4.49**	**4.79**	**5.13**	**5.53**	**5.98**	
	10.2	10.6	11.1	11.6	12.2	12.8	13.6	14.3	15.3	16.3	17.4	18.8	20.3	
20	**3.15**	**3.28**	**3.44**	**3.60**	**3.78**	**3.98**	**4.20**	**4.44**	**4.72**	**5.04**	**5.40**	**5.81**	**6.29**	
	10.7	11.2	11.7	12.2	12.8	13.5	14.3	15.1	16.1	17.1	18.3	19.7	21.4	2 %
21	**3.30**	**3.45**	**3.61**	**3.78**	**3.97**	**4.18**	**4.41**	**4.66**	**4.96**	**5.29**	**5.67**	**6.10**	**6.61**	
	11.2	11.7	12.3	12.8	13.5	14.2	15.0	15.8	16.9	18.0	19.3	20.7	22.5	
22	**3.46**	**3.62**	**3.78**	**3.96**	**4.16**	**4.38**	**4.62**	**4.89**	**5.19**	**5.54**	**5.94**	**6.39**	**6.92**	
	11.8	12.3	12.8	13.5	14.1	14.9	15.7	16.6	17.7	18.8	20.2	21.7	23.7	
23	**3.62**	**3.78**	**3.95**	**4.14**	**4.35**	**4.58**	**4.83**	**5.11**	**5.43**	**5.79**	**6.21**	**6.68**	**7.24**	
	12.3	12.9	13.4	14.1	14.8	15.6	16.4	17.4	18.5	19.7	21.1	22.7	24.6	
24	**3.78**	**3.94**	**4.12**	**4.32**	**4.54**	**4.78**	**5.04**	**5.33**	**5.67**	**6.04**	**6.48**	**6.97**	**7.56**	
	12.8	13.4	14.0	14.7	15.4	16.2	17.1	18.1	19.3	20.6	22.0	23.7	25.7	
25	**3.93**	**4.11**	**4.29**	**4.50**	**4.73**	**4.98**	**5.25**	**5.55**	**5.90**	**6.30**	**6.75**	**7.26**	**7.89**	
	13.4	14.0	14.6	15.3	16.1	16.9	17.9	18.9	20.1	21.4	22.9	24.7	26.8	
26	**4.09**	**4.28**	**4.47**	**4.68**	**4.92**	**5.18**	**5.46**	**5.77**	**6.14**	**6.55**	**7.02**	**7.56**	**8.18**	
	13.9	14.5	15.2	15.9	16.7	17.6	18.6	19.6	20.9	22.3	23.8	25.7	27.8	1½ %
27	**4.25**	**4.44**	**4.64**	**4.86**	**5.11**	**5.38**	**5.67**	**6.00**	**6.38**	**6.80**	**7.29**	**7.84**	**8.50**	
	14.4	15.1	15.8	16.5	17.3	18.3	19.3	20.4	21.7	23.1	24.8	26.7	28.9	
28	**4.41**	**4.60**	**4.81**	**5.04**	**5.29**	**5.57**	**5.88**	**6.21**	**6.61**	**7.05**	**7.56**	**8.13**	**8.81**	
	15.0	15.6	16.4	17.1	18.0	18.9	20.0	21.1	22.5	24.0	25.7	27.6	30.0	
29	**4.57**	**4.76**	**4.99**	**5.22**	**5.48**	**5.77**	**6.09**	**6.44**	**6.85**	**7.31**	**7.83**	**8.43**	**9.13**	
	15.5	16.2	16.9	17.7	18.6	19.6	20.7	21.9	23.3	24.8	26.6	28.6	31.1	
30	**4.72**	**4.93**	**5.16**	**5.40**	**5.67**	**5.97**	**6.31**	**6.66**	**7.09**	**7.56**	**8.10**	**8.72**	**9.45**	
	16.0	16.8	17.5	18.3	19.3	20.3	21.5	22.6	24.1	25.7	27.5	29.6	32.1	
31	**4.88**	**5.09**	**5.33**	**5.58**	**5.86**	**6.17**	**6.51**	**6.88**	**7.32**	**7.81**	**8.37**	**9.01**	**9.76**	
	16.6	17.3	18.1	19.0	19.9	20.9	22.2	23.4	24.9	26.6	28.4	30.6	33.2	
32	**5.04**	**5.26**	**5.50**	**5.76**	**6.05**	**6.37**	**6.72**	**7.10**	**7.56**	**8.06**	**8.64**	**9.30**	**10.1**	
	17.1	17.9	18.7	19.6	20.6	21.6	22.9	24.2	25.7	27.4	29.4	31.6	34.3	
33	**5.19**	**5.42**	**5.67**	**5.94**	**6.24**	**6.57**	**6.93**	**7.33**	**7.79**	**8.31**	**8.92**	**9.59**	**10.4**	
	17.7	18.4	19.3	20.2	21.2	22.3	23.6	24.9	26.5	28.3	30.3	32.6	35.3	
34	**5.35**	**5.58**	**5.84**	**6.12**	**6.43**	**6.77**	**7.15**	**7.55**	**8.03**	**8.56**	**9.19**	**9.89**	**10.7**	
	18.2	19.0	19.9	20.8	21.9	23.0	24.3	25.7	27.3	29.1	31.2	33.6	36.4	
35	**5.51**	**5.75**	**6.02**	**6.30**	**6.62**	**6.97**	**7.35**	**7.77**	**8.26**	**8.82**	**9.45**	**10.2**	**11.0**	
	18.7	19.5	20.4	21.4	22.5	23.7	25.0	26.4	28.1	30.0	32.1	34.6	37.5	
36	**5.66**	**5.91**	**6.18**	**6.48**	**6.80**	**7.17**	**7.56**	**7.99**	**8.50**	**9.07**	**9.72**	**10.5**	**11.3**	
	19.3	20.1	21.0	22.0	23.1	24.4	25.7	27.2	28.9	30.8	33.0	35.5	38.5	
37	**5.82**	**6.08**	**6.35**	**6.66**	**6.99**	**7.37**	**7.79**	**8.22**	**8.74**	**9.32**	**10.0**	**10.7**	**11.6**	
	19.8	20.7	21.6	22.6	23.8	25.0	26.4	27.9	29.7	31.7	33.9	36.5	39.6	
38	**5.98**	**6.24**	**6.52**	**6.84**	**7.18**	**7.57**	**7.98**	**8.44**	**8.98**	**9.58**	**10.3**	**11.0**	**12.0**	
	20.4	21.2	22.2	23.3	24.4	25.7	27.2	28.7	30.5	32.6	34.9	37.5	40.7	
39	**6.13**	**6.40**	**6.69**	**7.02**	**7.37**	**7.77**	**8.19**	**8.66**	**9.22**	**9.83**	**10.5**	**11.3**	**12.3**	
	20.9	21.8	22.8	23.9	25.1	26.4	27.9	29.5	31.3	33.4	35.6	38.5	41.7	1 %

Table 9

Core diam. (inches)	Areas and Weights of ½-in. Wire Spirals Areas of Equivalent Cylinders in Square Inches Given in Heavy Figures Weights of Wire in Pounds per Foot Length of Spiral in Light Figures Pitch of Spiral in Inches												
	3"	2⅞"	2¾"	2⅝"	2½"	2⅜"	2¼"	2⅛"	2"	1⅞"	1¾"	1⅝"	1½"	
15	**3.09** 10.5	**3.22** 11.0	**3.36** 11.4	**3.52** 12.0	**3.70** 12.6	**3.90** 13.2								
16	**3.29** 11.2	**3.43** 11.7	**3.59** 12.2	**3.76** 12.8	**3.94** 13.4	**4.15** 14.1								
17	**3.49** 11.9	**3.65** 12.4	**3.82** 13.0	**3.99** 13.6	**4.19** 14.3	**4.42** 15.0	**4.66** 15.9							
18	**3.70** 12.6	**3.86** 13.1	**4.04** 13.7	**4.23** 14.4	**4.44** 15.1	**4.68** 15.9	**4.94** 16.8	**5.23** 17.8						
19	**3.91** 13.3	**4.08** 13.9	**4.27** 14.5	**4.46** 15.2	**4.69** 16.0	**4.94** 16.8	**5.22** 17.7	**5.52** 18.8	**5.86** 20.0					
20	**4.11** 14.0	**4.29** 14.6	**4.49** 15.3	**4.70** 16.0	**4.93** 16.8	**5.19** 17.7	**5.49** 18.6	**5.80** 19.8	**6.17** 21.0	**6.59** 22.4				
21	**4.32** 14.7	**4.50** 15.3	**4.71** 16.0	**4.94** 16.8	**5.18** 17.6	**5.45** 18.5	**5.76** 19.6	**6.09** 20.8	**6.48** 22.0	**6.90** 23.5				
22	**4.52** 15.4	**4.72** 16.1	**4.94** 16.8	**5.16** 17.6	**5.42** 18.5	**5.71** 19.4	**6.03** 20.5	**6.38** 21.7	**6.79** 23.1	**7.24** 24.6	**7.76** 26.4			
23	**4.73** 16.1	**4.94** 16.8	**5.16** 17.6	**5.40** 18.4	**5.68** 19.3	**5.98** 20.3	**6.31** 21.4	**6.67** 22.7	**7.10** 24.1	**7.57** 25.8	**8.10** 27.6	**8.73** 29.9		
24	**4.93** 16.8	**5.15** 17.5	**5.39** 18.3	**5.64** 19.2	**5.93** 20.2	**6.24** 21.2	**6.58** 22.4	**6.96** 23.7	**7.40** 25.2	**7.90** 26.9	**8.46** 28.8	**9.11** 31.0		
25	**5.15** 17.5	**5.36** 18.3	**5.61** 19.1	**5.87** 20.0	**6.17** 21.0	**6.49** 22.1	**6.85** 23.3	**7.25** 24.7	**7.71** 26.2	**8.22** 28.0	**8.82** 30.0	**9.50** 32.3	**10.3** 35.0	
26	**5.35** 18.2	**5.59** 19.0	**5.84** 19.9	**6.11** 20.8	**6.42** 21.8	**6.76** 23.0	**7.13** 24.3	**7.55** 25.7	**8.02** 27.2	**8.56** 29.1	**9.18** 31.2	**9.87** 33.6	**10.7** 36.4	2%
27	**5.55** 18.9	**5.80** 19.7	**6.06** 20.6	**6.34** 21.6	**6.66** 22.7	**7.01** 23.9	**7.40** 25.2	**7.83** 26.7	**8.33** 28.3	**8.88** 30.2	**9.52** 32.4	**10.2** 34.9	**11.1** 37.8	
28	**5.75** 19.6	**6.01** 20.4	**6.29** 21.4	**6.58** 22.4	**6.90** 23.5	**7.28** 24.7	**7.68** 26.1	**8.12** 27.6	**8.64** 29.4	**9.21** 31.3	**9.87** 33.6	**10.6** 36.2	**11.5** 39.2	
29	**5.97** 20.3	**6.22** 21.2	**6.51** 22.2	**6.81** 23.2	**7.15** 24.4	**7.54** 25.6	**7.96** 27.1	**8.41** 28.7	**8.95** 30.4	**9.55** 32.5	**10.2** 34.8	**11.0** 37.5	**11.9** 40.6	
30	**6.17** 21.0	**6.44** 21.9	**6.74** 22.9	**7.05** 24.0	**7.40** 25.2	**7.79** 26.5	**8.23** 28.0	**8.70** 29.6	**9.25** 31.5	**9.87** 33.6	**10.6** 36.0	**11.4** 38.8	**12.3** 42.0	
31	**6.37** 21.7	**6.66** 22.6	**6.95** 23.6	**7.28** 24.8	**7.64** 26.0	**8.05** 27.4	**8.50** 28.9	**8.99** 30.6	**9.56** 32.5	**10.2** 34.7	**10.9** 37.2	**11.8** 40.0	**12.7** 43.3	
32	**6.58** 22.4	**6.87** 23.4	**7.18** 24.4	**7.52** 25.6	**7.89** 26.8	**8.31** 28.3	**8.77** 29.8	**9.28** 31.6	**9.87** 33.6	**10.5** 35.8	**11.3** 38.4	**12.1** 41.3	**13.2** 44.7	
33	**6.78** 23.1	**7.09** 24.1	**7.40** 25.2	**7.75** 26.4	**8.14** 27.7	**8.58** 29.2	**9.05** 30.8	**9.57** 32.6	**10.2** 34.6	**10.9** 36.9	**11.6** 39.5	**12.5** 42.6	**13.6** 46.1	
34	**6.99** 23.8	**7.30** 24.8	**7.62** 25.9	**7.98** 27.2	**8.38** 28.5	**8.83** 30.0	**9.32** 31.7	**9.86** 33.5	**10.5** 35.6	**11.2** 38.0	**12.0** 40.7	**12.9** 43.9	**14.0** 47.6	1½%
35	**7.19** 24.5	**7.51** 25.6	**7.85** 26.7	**8.22** 28.0	**8.63** 29.4	**9.09** 30.9	**9.60** 32.6	**10.1** 34.5	**10.8** 36.7	**11.5** 39.2	**12.3** 41.9	**13.3** 45.2	**14.4** 48.9	
36	**7.40** 25.2	**7.74** 26.3	**8.08** 27.5	**8.46** 28.8	**8.89** 30.2	**9.36** 31.8	**9.88** 33.6	**10.4** 35.5	**11.1** 37.8	**11.8** 40.3	**12.7** 43.2	**13.7** 46.5	**14.8** 50.4	
37	**7.60** 25.9	**7.95** 27.0	**8.30** 28.2	**8.69** 29.6	**9.13** 31.1	**9.60** 32.7	**10.1** 34.5	**10.7** 36.5	**11.4** 38.8	**12.2** 41.4	**13.0** 44.4	**14.0** 47.8	**15.2** 51.8	
38	**7.82** 26.6	**8.16** 27.8	**8.53** 29.1	**8.93** 30.4	**9.38** 31.9	**9.88** 33.6	**10.4** 35.5	**11.0** 37.5	**11.7** 39.9	**12.5** 42.5	**13.4** 45.6	**14.4** 49.1	**15.6** 53.2	
39	**8.02** 27.3	**8.37** 28.5	**8.75** 29.7	**9.16** 31.2	**9.63** 32.7	**10.1** 34.5	**10.7** 36.4	**11.3** 38.5	**12.0** 40.9	**12.8** 43.6	**13.7** 46.8	**14.8** 50.4	**16.0** 54.6	
										1%				

TABLE 10

Core diam. (inches)	AREAS AND WEIGHTS OF 5/16-IN. WIRE SPIRALS Areas of Equivalent Cylinders in Square Inches Given in Heavy Figures Weights of Wire in Pounds per Foot Length of Spiral in Light Figures Pitch of Spiral in Inches													
	3"	2⅞"	2¾"	2⅝"	2½"	2⅜"	2¼"	2⅛"	2"	1⅞"	1¾"	1⅝"	1½"	
20	**5.21** 17.7	**5.43** 18.5	**5.68** 19.3	**5.95** 20.2	**6.24** 21.2									
21	**5.46** 18.6	**5.70** 19.4	**5.96** 20.3	**6.24** 21.2	**6.55** 22.3	**6.90** 23.5								
22	**5.73** 19.5	**5.97** 20.3	**6.25** 21.3	**6.55** 22.2	**6.86** 23.4	**7.23** 24.6	**7.65** 26.0							
23	**5.99** 20.4	**6.25** 21.2	**6.53** 22.2	**6.84** 23.2	**7.18** 24.4	**7.56** 25.7	**7.99** 27.2	**8.45** 28.7						
24	**6.25** 21.2	**6.52** 22.2	**6.81** 23.2	**7.14** 24.2	**7.50** 25.5	**7.89** 26.8	**8.34** 28.3	**8.83** 29.9	**9.37** 31.9					
25	**6.51** 22.1	**6.79** 23.1	**7.10** 24.2	**7.44** 25.2	**7.81** 26.6	**8.22** 28.0	**8.69** 29.5	**9.20** 31.2	**9.76** 33.2	**10.4** 35.4				
26	**6.77** 23.0	**7.06** 24.0	**7.39** 25.1	**7.74** 26.3	**8.12** 27.6	**8.56** 29.1	**9.05** 30.7	**9.57** 32.4	**10.1** 34.6	**10.8** 36.8				
27	**7.03** 23.9	**7.33** 24.4	**7.67** 26.1	**8.03** 27.3	**8.43** 28.6	**8.88** 30.2	**9.39** 31.9	**9.93** 33.7	**10.5** 35.8	**11.2** 38.2	**12.0** 40.9			
28	**7.29** 24.8	**7.60** 25.8	**7.95** 27.0	**8.33** 28.3	**8.74** 29.7	**9.20** 31.3	**9.73** 33.0	**10.3** 34.9	**10.9** 37.2	**11.7** 39.6	**12.5** 42.5			
29	**7.55** 25.7	**7.88** 26.8	**8.24** 28.0	**8.62** 29.3	**9.06** 30.8	**9.55** 32.4	**10.1** 34.3	**10.6** 36.2	**11.3** 38.5	**12.1** 41.1	**12.9** 44.0	**13.9** 47.4		
30	**7.81** 26.6	**8.14** 27.7	**8.52** 28.9	**8.93** 30.3	**9.37** 31.8	**9.88** 33.5	**10.4** 35.4	**11.0** 37.4	**11.7** 39.8	**12.5** 42.4	**13.4** 45.5	**14.4** 49.0		
31	**8.06** 27.4	**8.41** 28.6	**8.80** 29.9	**9.22** 31.3	**9.68** 32.7	**10.2** 34.6	**10.8** 36.6	**11.4** 38.7	**12.1** 41.2	**12.9** 43.9	**13.8** 47.0	**14.9** 50.5	**16.1** 54.9	
32	**8.32** 28.3	**8.69** 29.5	**9.09** 30.9	**9.52** 32.3	**10.0** 33.9	**10.5** 35.8	**11.1** 37.8	**11.8** 39.9	**12.5** 42.5	**13.3** 45.3	**14.3** 48.5	**15.4** 52.2	**16.6** 56.6	2%
33	**8.58** 29.2	**8.97** 30.4	**9.37** 31.9	**9.82** 33.3	**10.3** 35.0	**10.8** 36.9	**11.4** 39.0	**12.1** 41.1	**12.9** 43.8	**13.7** 46.7	**14.7** 50.0	**15.8** 53.8	**17.2** 58.4	
34	**8.84** 30.1	**9.23** 31.3	**9.65** 32.8	**10.1** 34.3	**10.6** 36.0	**11.2** 38.0	**11.8** 40.2	**12.5** 42.4	**13.3** 45.2	**14.1** 48.2	**15.2** 51.6	**16.3** 55.4	**17.7** 60.2	
35	**9.10** 30.9	**9.50** 32.3	**9.95** 33.8	**10.4** 35.3	**10.9** 37.1	**11.5** 39.1	**12.1** 41.4	**12.8** 43.6	**13.7** 46.5	**14.5** 49.6	**15.6** 53.1	**16.8** 57.1	**18.2** 61.9	
36	**9.38** 31.9	**9.79** 33.2	**10.2** 34.8	**10.7** 36.4	**11.2** 38.2	**11.8** 40.3	**12.5** 42.5	**13.2** 44.9	**14.1** 47.8	**15.0** 51.0	**16.1** 54.7	**17.3** 58.8	**18.7** 63.7	
37	**9.63** 32.7	**10.0** 34.1	**10.5** 35.7	**11.0** 37.4	**11.5** 39.2	**12.2** 41.4	**12.8** 43.7	**13.6** 46.1	**14.4** 49.2	**15.4** 52.4	**16.5** 56.2	**17.8** 60.4	**19.2** 65.5	
38	**9.90** 33.6	**10.3** 35.1	**10.8** 36.7	**11.3** 38.4	**11.9** 40.3	**12.5** 42.5	**13.2** 44.9	**14.0** 47.4	**14.8** 50.5	**15.8** 53.8	**16.9** 57.7	**18.3** 62.0	**19.8** 67.3	
39	**10.2** 34.5	**10.6** 36.0	**11.1** 37.7	**11.6** 39.4	**12.2** 41.4	**12.8** 43.6	**13.5** 46.0	**14.3** 48.6	**15.2** 51.8	**16.2** 55.2	**17.4** 59.2	**18.7** 63.6	**20.3** 69.0	
						1%							1½%	

Spirals above and to right of zigzag lines are nearest commercial size fully equal to percentage of core area indicated at end of line.

Weights in above table include the wire only. To this must be added the weights of the spacing bars and the weight of the extra turn at ends of spiral.

Table 11.—Areas of Circles

Diam.	Area	Diam.	Area	Diam.	Area	Diam.	Area	Diam.	Area	Diam.	Area
10	78.5	15	176.7	20	314.2	25	490.8	30	706.9	35	962.1
11	95.0	16	201.1	21	346.4	26	530.9	31	754.8	36	1017.9
12	113.1	17	227.0	22	380.1	27	572.6	32	804.2	37	1075.2
13	132.7	18	254.5	23	415.5	28	615.8	33	855.3	38	1134.1
14	153.9	19	283.5	24	452.4	29	660.5	34	907.9	39	1194.6

BEARING PLATES AND BASES FOR BEAMS, GIRDERS, AND COLUMNS

By Clyde T. Morris

99. Allowable Bearing Pressures.—Where beams, girders, or columns rest on masonry walls or footings, the bearing area must be made sufficient so that the masonry will not be overstressed. The following table gives safe bearing values in pounds per square inch for different kinds of masonry:

First-class granite masonry	600
First-class concrete, 1-2-4 mix	600
First-class limestone masonry	500
First-class sandstone masonry	400
Concrete, 1-3-6 mix	400
Hard-burned brick work, cement mortar	300
Common brick work, cement mortar	250
Common brick work, lime mortar	125

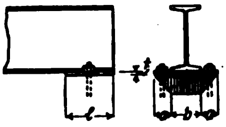

Fig. 102.

100. Simple Bearing Plates.—For ordinary loads, sufficient bearing can usually be secured by placing a plate from ½ to 1 in. thick under the end of the beam or girder, as shown in Fig. 102. The portion "a" of the plate which projects beyond the edge of the beam, will deflect upward under the load so that the pressure on the masonry will decrease from the edge of the beam outward as shown by the shaded area. For steel plates with the usual mortar bearing, the distance "a" beyond which there will be little of no pressure on the masonry, will not exceed 3 or 4 times the thickness of the plate. (This may be readily calculated from the deflection formula and the modulus of elasticity of the masonry.) Assuming $a = 4t$ as the effective projection, the maximum unit pressure on the masonry will be

$$p = \frac{R}{(b + 4t)l} \qquad (1)$$

in which

p = the maximum unit pressure.
R = the maximum end reaction of the beam.
b = the width of the flange of the beam.
t = the thickness of the plate.
l = the length of the bearing.

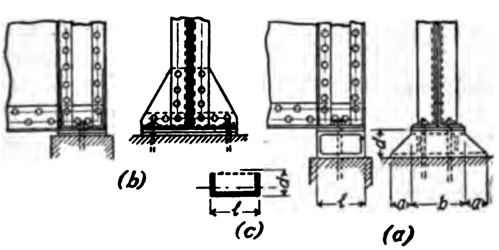

Fig. 103.

If the width of the bearing plate is less than $(b + 8t)$, the denominator of equation (1) must be reduced accordingly. If the maximum allowable pressure on the masonry is not exceeded, the fiber stress in the steel plate will be well within allowable limits.

If the length of bearing "l" is restricted and a greater width than $(b + 8t)$ is necessary, stiffening brackets must be placed on the end of the girder, or a cast-iron subbase may be used. If the bearing plate is stiffened, as shown in Fig. 103(b), or a cast base having stiffening webs is used, the pressure on the masonry may be assumed to be uniform over the entire bearing area. The stiffening brackets should have enough rivets to carry the entire load on the portions of the bearing plate projecting beyond the edges of the flange.

Bearing plates for columns are calculated in an exactly similar manner and may be stiffened as shown in Figs. 98 and 103. The thickness "t" in equation (1) may be taken as the thickness of the base plate plus the thickness of the shoe angle. Bases for wooden columns are treated in Art. 68.

101. Cast Bases.—If a cast base is used (Figs. 103*a* and 98*a*), the weak section will be at the edge of the upper bearing plate of the casting, and the vertical webs and lower plate must be strong enough to carry the load on the projecting portions "a" (see Fig. 103*a*). The maximum extreme fiber stress on the cast iron should not exceed about 2500 to 3000 lb. per sq. in. in tension, or 10,000 to 12,000 lb. per sq. in. in compression.

Illustrative Problem.—Design a cast-iron base to support the end of a girder whose reaction is 120,000 lb. and the length of the bearing "l" is limited to 12 in. Assume the bearing to be on a concrete wall having an allowable bearing value of 400 lb. per sq. in.

$$\text{Required bearing area} = \frac{120{,}000}{400} = 300 \text{ sq. in.}$$

$$\text{Required width of casting} = \frac{300}{12} = 25 \text{ in.}$$

If $b = 13$ in., $a = 6$ in., and the load on the portion "a" will be,

$$(12)(6)(400) = 28{,}800 \text{ lb.}$$

The moment at the edge of the upper bearing plate of the casting will be

$$M = (28{,}800)(3) = 86{,}400 \text{ in.-lb.}$$

The section at this point is shown in Fig. 103(*c*).

Assuming the metal to be ⅜ in. thick, the required depth "d" may be found by trial as follows.

Try $d = 4\frac{3}{4}$ in., then $I = 17.36$. $c = 1.08$ in. to bottom (tens. side)

$c = 2.92$ in. to top (comp. side).

$$f = \frac{Mc}{I} = \frac{(86{,}400)(1.08)}{17.36} = 5370 \text{ lb. per sq. in., tension}$$

$$= \frac{(86{,}400)(2.92)}{17.36} = 14{,}530 \text{ lb. per sq. in., compression.}$$

As these unit stresses are excessive, either the metal must be made thicker or the depth "d" greater.

Try $d = 6\frac{3}{4}$ in., then $I = 56.30$. $c = 1.775$ in. to bottom (tens. side)

$c = 4.225$ in. to top (comp. side)

$$f = \frac{Mc}{I} = \frac{(86{,}400)(1.775)}{56.30} = 2720 \text{ lb. per sq. in., tension}$$

$$= \frac{(86{,}400)(4.225)}{56.30} = 6480 \text{ lb. per sq. in., compression.}$$

These fiber stresses are within safe limits, so the depth of the casting may be made 6¾ in.

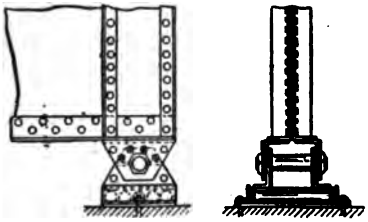

FIG. 104.

102. Expansion Bearings.—For steel girders and trusses over 30 ft. in length, provision must be made for expansion and contraction due to changes in temperature. For spans less than 30 ft. there will usually be sufficient play in the anchorages to allow for the movement.

For spans between 30 to 100 ft., provision for expansion should be made by providing two bearing plates at one end of the girder, as shown in Fig. 103(*b*), one riveted to the girder and the other one anchored to the masonry. The anchor bolt holes in the upper plate which is riveted to the girder should be slotted to provide for the necessary movement due to temperature changes. The extreme movement will be about 1 in. for each 80 ft. of span. If the bearing area exceeds about 120 sq. in., the sliding surfaces should be planed.

For spans exceeding 100 ft., nests of turned rollers should be placed between the bearing plates at the movable end of the span. These roller bearings should be so arranged that they can be readily cleaned and so that they will not collect dirt and moisture. The bearing pressure on the rollers should not exceed $600D$ per lin. in. of roller, where D = diameter of roller in inches. Fig. 104 shows a design for a roller bearing.

103. Hinged Bolsters.—For spans exceeding 100 ft., hinged bolsters should be provided at each end. These bolsters may be either cast or built up of plates and shapes.

The pin should be turned and the pin hole bored to a diameter not more than 1/32 in. greater than the diameter of the pin. The bearing area on the pin (diam. of pin × thickness of bearing) should be sufficient so that the unit pressure does not exceed 24,000 lb. per sq. in., and the maximum fiber stress on the pin due to bending should not exceed 24,000 lb. per sq. in. The unit shearing stress should not exceed 10,000 lb. per sq. in. Fig. 104 shows such a hinged bolster.

104. Anchors.—The ends of beams and girders should be anchored to their support with bolts securely fastened into the masonry. Anchor bolts for columns should be designed to resist 1½ times the bending moment at the base of the column and should engage a sufficent weight of masonry to withstand this moment and also 1½ times the calculated uplift (if any) on the column due to wind. Such an anchorage is shown in Fig. 105.

For simple I-beams built into walls, the anchor bolts are frequently put through the web of the beam, or small angles are riveted to the end of the web to provide the necessary anchorage. Fig. 105 shows several forms of anchor bolts. The position of anchor bolts is also shown in Figs. 98, 102, 103, and 104.

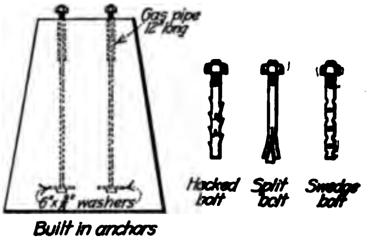

Fig. 105.

TENSION MEMBERS

By Clyde T. Morris

105. Rods and Bars.—The simplest form of tension member is the round or square rod with threads and nuts on the ends. Fig. 106 shows details of the end connections of several such members.

In designing such a member the required area is obtained by dividing the total stress by the allowable unit stress. The least area of cross section of the member must be equal to or exceed this required area.

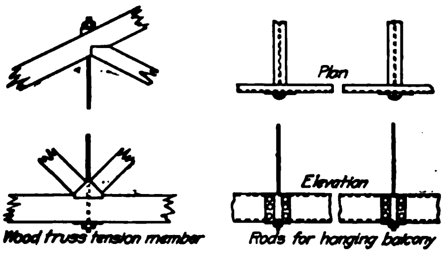

Fig. 106.

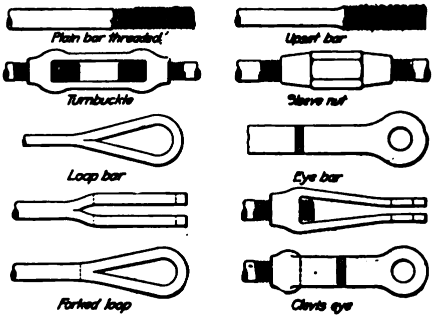

Fig. 107.

The least sectional area of a plain round rod with threads cut on the ends will be at the root of the threads. If the rod is long, the ends should be upset, that is, increased in diameter, so that the area at the root of the threads will be greater than the area of the body of the bar; but if the member is short, the cost of upsetting may be greater than the saving in material, in which case the bar may be made of sufficient size for the entire length to allow for the cutting of the threads.

Tables of standard upsets and areas at the root of threads are given in the steel handbooks (see also Table 15*b*, p. 238).

Plain loops for connection to pins are made by bending the rod around a pin of the required diameter and welding the end to the main bar. Forked loops are also sometimes used The forked loop is welded to the main bar and should have a total cross section through the eye at least 50% in excess of that of the main bar. The forked loop is not so reliable as a plain loop because it depends entirely upon the weld for itsconnection. Tables of standard loop bars are given in the handbooks of the various steel companies.

Fig. 107 shows various end connections for tension members composed of rods and bars.

106. Riveted Tension Members.—In riveted structures the tension members are usually made of rolled shapes built into forms which have considerable stiffness. Although theoretically there may be no compressive or bending stresses in these members, the structure will be stiffened and vibrations considerably reduced if the tension members are made of a form capable of resisting compression.

Fig. 108 shows cross sections of various forms of riveted tension members. The cost of fabrication of these types will vary roughly with the number of lines of holes that have to be punched and the number of lines of rivets that have to be driven.

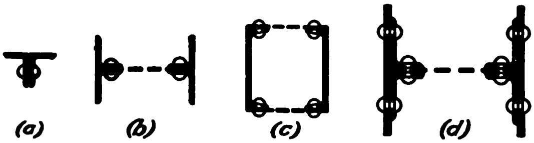

Fig. 108.

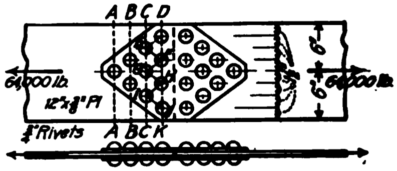

Fig. 109.

Single angles are sometimes used for tension members of light riveted trusses, but this practice is not good as it forms an unsymmetrical member and eccentric end connections are unavoidable.

Unless absolutely necessary, unsymmetrical cross sections should not be used. When unsymmetrical sections are used, the eccentric moments should be calculated and the resultant unit stresses, figured as shown in Sect. 1, Art. 101, should not exceed the allowable units specified.

It is impossible to so design a riveted tension member that the entire cross section of the body of the member is available for tension area, on account of the necessity of punching holes for the rivets. This of course reduces the effective net area. This may be illustrated by the solution of a problem.

Illustrative Problem.—Fig. 109 shows a splice in a plate carrying tension, so designed that a maximum of the gross section of the plate is available for net tension area. Assume the following data:

Allowed tension unit stress	16,000 lb. per sq. in.
Allowed shear on rivets	12,000 lb. per sq. in.
Allowed bearing on rivets	24,000 lb. per sq. in.
Total stress to be carried	64,000 lb.

The number of rivets required will in this case be determined by the bearing value of a rivet on the 12 × ⅝-in. plate. This is 6750 lb. The total number of rivets required is $\frac{64,000}{6750} = 10.$ The required net area of the 12 × ⅝-in. plate at the first line of rivets (*AA*), is $\frac{64,000}{16,000} = 4.12$ sq. in. The available net area on line *AA* is (12)(⅝) − (¾ + ⅛)(⅝) = 4.17 sq. in. (The diameter of the hole is assumed to be ⅛ in. larger than the rivet.)

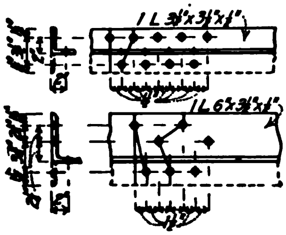

Fig. 110.

At the second line of rivets (*BB*), the stress in the main plate has been reduced by the portion carried by the first rivet, therefore the stress to be carried is only $\frac{9}{10}$(64,000) = 57,600 lb. The required net area of the 12 × ⅝-in. plate on the line $BB = \frac{57,600}{16,000} = 3.60$ sq. in. The available net area = (12)(⅝) − 2(¾ + ⅛)(⅝) = 3.84 sq. in.

In like manner the available net section at each line of rivets will be found to be in excess of that required.

The splice plates must be made thick enough so that the net section on the last line of rivets *DK*, is sufficient to carry the entire stress (= 4.12 sq. in.). The net width of the splice plates at this point is 12 − (4)(⅞) = 8.5 in. Therefore the required thickness of the two splice plates is $\frac{4.12}{8.5} = 0.49$ in. Use two splice plates 12 × ¼. Total thickness = 0.5 in.

The distance between the successive rows of rivets must be sufficient so that the net section on a zig-zag line, such as *DEFGHIJK*, is greater than the square section *KD*.

In members composed of shapes the net section is figured by considering the shape to be straightened out like a plate and calculating the net areas on the various possible rupture sections to find the least net area. Fig. 110 shows two angles so developed and the possible rupture sections.

In designing the end connections of riveted tension members the rivets should be so arranged that the maximum possible net section is available at the beginning of the connection where the stress is carried entirely by the main section. This was illustrated in the design of the splice in Fig. 109.

A riveted tension member in a horizontal or inclined position should have sufficient stiffness to prevent sagging between connections. The unit stresses in such a member caused by bending due to its own weight, are calculated in Sect. 1, Art. 101.

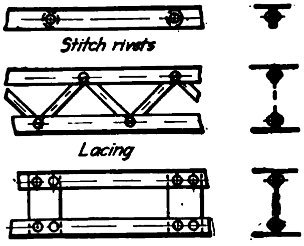

Fig. 111

When a tension member is composed of two or more parallel elements as shown in Fig. 111, these should be connected together throughout their length to form a unit, similar to a compression member. The distance between such successive stays should not be great enough so that the ratio of unsupported length to least width of the individual parts is as great as that of the member as a whole.

107. Wooden Tension Members.—Wooden tension members are not extensively used except for the bottom chords of wooden trusses. On account of the low shearing resistance of wood along the grain, the greatest difficulty is encountered in transmitting the stresses from the other truss members to the bottom chord near its end, and in splicing the chord where the span is too great to make it possible to get the timbers in full lengths.

These bottom chords are frequently made up of several leaves from 2 to 6 in. thick and 8 to 14 in. deep. Due to the necessity of notching into the timbers to obtain bearing for the ends of other members and for splice plates, and to the large number of holes necessary for bolting the pieces together, the effective net section cannot be a very large proportion of the gross area of cross section.

For design of tension splices, see Art. 119.

SPLICES AND CONNECTIONS—WOODEN MEMBERS

By Henry D. Dewell

108. Nails.—*Wire nails* are usually of steel, of circular cross section without taper, and with a head and point. In size they are designated as 8-D (8 penny), 10-D (10 penny), etc., and, in class, as common, finishing, casing, barbed roofing, shingle, fine, cement coated, etc.

Cut nails are of steel or iron, with a rectangular cross section, and taper from head to point, the latter being cut square, *i.e.*, not pointed. The sizes are designated as for wire nails.

Spikes designate the larger sizes of nails.

The sizes of nails and spikes are given in Tables 1 to 9 inclusive. For quantity of nails required in timber construction, see Table 10.

Boat spikes are employed in heavy timber construction. They are made from square bars of steel or wrought iron, have a forged head and a wedge-shaped point. The common sizes and weights are given in Table 11.

109. Screws.—Screws may be classified as (1) *common wood screws*, and (2) *lag*, or *coach screws*.

Wood screws have slotted heads; the shank is smooth for a portion of its length adjacent to the head, the remainder of the length being threaded, and tapering to a point. Wood screws are usually of steel, but are made also of bronze and brass. The ordinary wood screw has a flat head, but screws are also made with round heads. Wood screws are designated by gage and length. Given the gage number, the diameter of the smooth shank may be found from the formula

$$d = 0.0578 + 0.01316G$$

where d = diameter in inches, and G = gage number of screw. Table 12 gives the length and gage numbers of wood screws, flat head, bright steel.

Lag screws are of heavier stock than the common wood screws and have a square head without slot. Table 13 gives the sizes, lengths, and weights of lag screws.

110. Bolts.—Bolts, in timber construction, may be divided into two classes, (1) *common, ordinary*, or *machine bolts*, and (2) *drift bolts*.

Machine bolts are of steel or wrought iron, of circular cross section without taper, having a square head upset on one end, and the other end threaded to receive a nut. The length of a bolt is the length from underside or inside of head to end of thread. Nuts are usually square unless otherwise ordered, but hexagonal nuts may be obtained where desired. Table 14 gives the weights of 100 machine bolts with square heads and nuts. Table 15*a* gives the values in tension of bolts at various stresses, based on the areas of the bolts at the root of thread. Table 15*b* gives the strength of round rods with upset ends.

111. Lateral Resistance of Nails, Screws and Bolts.—When spikes, screws and bolts are subjected to lateral forces in a timber joint, shearing and bending stresses are produced in the spikes, screws, or bolts, and the timber in contact with the metal is subjected to pressure. In timber construction, joints of this nature are of common occurrence, and it is necessary to have safe working values for such details. The factors entering into a theoretical consideration of the stresses produced in such a joint are many and complex, and in the determination of safe working values, recourse must be had to the results of tests.

In the case of nails and screws a theoretical analysis of the stresses is not practical. Tests[1] have established fairly definitely the ultimate strength and elastic limits of such joints.

TABLE 1.—WIRE NAILS—COMMON

Size	Length (inches)	Gage (number)	Diameter (inches)	Approximate number to pound
2d	1	15	0.072	876
3d	1¼	14	0.083	568
4d	1½	12½	0.102	316
5d	1¾	12½	0.102	271
6d	2	11½	0.115	181
7d	2¼	11½	0.115	161
8d	2½	10¼	0.124	106
10d	3	9	0.148	69
12d	3¼	9	0.148	63
16d	3½	8	0.165	49
20d	4	6	0.203	31
30d	4½	5	0.220	24
40d	5	4	0.238	18
50d	5½	3	0.259	14
60d	6	2	0.284	11

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[1] Tests for nails: Walker and Cross, *Jour.* Assn. Eng. Soc., vol. 19, Dec. 1897; Darrow and Buchanan, *Proc.* Ind. Eng. Soc., 1900; Morgan and Marish, Eng. Exp. Sta., Iowa State College, *Bul.* No. 2; Tests made for Bureau of Buildings, Portland, Ore., *Eng. News-Rec.*, vol. 79, No. 19, Nov. 8, 1917, also vol. 79, No. 26, Dec. 27, 1917; also "*The Timberman*," Portland, Ore., vol. 18, No. 12, Oct., 1917; "Tests Made to Determine Lateral Resistance of Wire Nails," Thomas R. C. Wilson, *Eng. News-Rec.*, vol. 75, No. 8, Feb. 14, 1917; Jacoby's "Structural Details;" Dewell's "Timber Framing."

Table 2.—Wire Nails—Finishing

Size	Length (inches)	Gage (number)	Approximate number to pound
2d	1	16½	1351
3d	1¼	15½	807
4d	1½	15	584
5d	1¾	15	500
6d	2	13½	309
7d	2¼	13	238
8d	2½	12½	189
10d	3	11½	121
12d	3¼	11½	113
16d	3½	11	90
20d	4	10	62

Table 3.—Wire Nails—Casing

Size	Length (inches)	Gage (number)	Approximate number to pound
2d	1	15½	1010
3d	1¼	14½	635
4d	1½	14	473
5d	1¾	14	407
6d	2	12½	236
7d	2¼	12½	210
8d	2½	11½	145
10d	3	10½	94
12d	3¼	10½	87
16d	3½	10	71
20d	4	9	52
30d	4½	9	46
40d	5	8	35

Table 4.—Wire Nails—Fine

Size	Length (inches)	Gage (number)	Approximate number to pound
2d	1	16½	1351
3d	1⅛	15	778

Table 5.—Wire Nails—Shingle

Size	Length (inches)	Gage (number)	Approximate number to pound
3d	1¼	13	429
4d	1½	12	274

TABLE 6.—WIRE NAILS—BARBED ROOFING

Length (inches)	Gage (number)	Approximate number to pound
¾	12	548
⅞	12	469
⅞	13	613
⅞	14	811
1	12	411
1	13	536
1	14	710
2	9	103

TABLE 7.—WIRE NAILS—FELT ROOFING (GALVANIZED)

Length (inches)	Gage (number)	Diameter of head (inches)	Approximate number to pound
⅞	12	⅝	215
1	12	⅝	198

TABLE 8.—WIRE SPIKES

Length (inches)	Diameter	Approximate number to pound
6	1 gage	8
7	5/16 in.	7
8	⅜ in.	6
9	⅜ in.	5
10	⅜ in.	4
12	⅜ in.	3

TABLE 9.—CUT NAILS

Size	Length (inches)	Size	Length (inches)
3d	1¼	12d	3¼
4d	1½	16d	3½
5d	1¾	20d	4
6d	2	30d	4½
7d	2¼	40d	5
8d	2½	50d	5½
10d	3	60d	6

TABLE 10.—QUANTITY OF NAILS REQUIRED FOR TIMBER CONSTRUCTION

		Size nail	Nails in pounds for various spacing of joists and studding					
			12 in.	16 in.	20 in.	36 in.	48 in.	60 in.
1000 M.B.M.	Joists, frame building	20d	20	16	14			
	Joists, brick building	20d	12	10	8			
1000 pcs.	Bridging, 1 × 4	8d	..	....	35			
	Bridging, 2 × 4	10d	..	....	50			
1000 M.B.M.	Studding	20d	15	12				
	Studding	10d	5	4				
	Sheathing, 1 × 8	8d	26	20	17			
	Flooring, 1 × 4	8d	26	22				
	Flooring, 1 × 4	10d	40	32				
	Flooring, 1 × 6	8d	17	13	11			
	Flooring, 1 × 6	10d	26	20	17			
	Planking, 3 × 6, 2 nailings	60d	..	....	..	51	40	34
	Planking, 3 × 8, 2 nailings	60d	..	....	..	39	30	26
	Planking, 3 × 10, 2 nailings	60d	..	....	..	31	24	20
	Planking, 3 × 12, 3 nailings	60d	..	....	..	39	30	26
	Planking, 2 × 6, 2 nailings	20d	..	51	42	27	21	18
	Planking, 2 × 10, 2 nailings	20d	..	30	25	16	13	11
	Finishing	8d	..	20				
100 lin. ft.	Base	8 × 6d	..	1				
1	Door	8 × 6d	..	1/2				
1	Window	8 × 6d	..	3/8				

TABLE 11.—BOAT SPIKES—(WROUGHT IRON)

Size	Average number 100 lb.	Size	Average number in 100 lb.
1/4 × 3	1500	3/8 × 7	325
3 1/2	1350	8	300
4	1187	9	263
4 1/2	1110	10	238
5	1025	7/16 × 6	300
5 1/2	975	7	295
6	913	8	255
5/16 × 4	680	9	200
4 1/2	650	10	180
5	615	1/2 × 6	225
5 1/2	605	7	188
6	588	8	168
3/8 × 5	470	9	150
6	400	10	138
		12	120

TABLE 12.—WOOD SCREWS

(Flat Head, Bright Steel)

Length (inches)	Gage numbers															
¼	0	1	2	3	4											
⅜	*0	1	2	3	4	5	6	7	8	9						
½	*1	2	3	4	5	6	7	8	9	10	*11	*12				
⅝	*1	2	3	4	5	6	7	8	9	10	11	12				
¾	*2	3	4	5	6	7	8	9	10	11	12	*13	14	*15	*16	
⅞	*2	*3	4	5	6	7	8	9	10	11	12	*13	14	*15	*16	
1	*3	4	5	6	7	8	9	10	11	12	13	14	15	16	*17	*1
1¼	*4	5	6	7	8	9	10	11	12	13	14	15	16	*17	*18	*2
1½	*5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	*2
1¾	6	7	8	9	10	11	12	13	14	15	16	17	18	20	*22	*2
2	6	7	8	9	10	11	12	13	14	15	16	*17	18	20	*22	*2
2¼	6	7	8	9	10	11	12	13	14	*15	16	*17	18	*20	*22	*2
2½	8	9	10	11	12	13	14	*15	16	*17	18	20	22	24		
*2¾	*10	*11	*12	*13	*14	*15	*16	*17	*18	*20	*22	*24				
3	*10	11	12	*13	14	15	16	*17	18	20	22	*24	*26			
3½	*10	*11	*12	*13	14	*15	16	*17	18	20	*22	24	*26			
4	12	14	16	18	20	22	24	*26								
4½	*16	*18	20	22	24	26										
5	*18	20	*22	*24	*26	*28										
*6	*20	*22	*24	*26	*28	*30										

[1] Sizes not usually carried in stock.

TABLE 13.—LAG SCREWS

(Gimlet Point. Square Head)

Length (inches)	Diameter (inches)						
	¼	5/16	⅜	½	⅝	¾	⅞
	Weight in pounds of 100 screws						
1¼	2.6	3.9	5.1	10.4			
1½	2.7	4.0	6.0	11.0			
1¾	2.8	4.4	5.8	11.7			
2	3.1	4.8	6.7	13.0	24.0		
2½	3.7	5.6	8.4	15.6	27.2	39.0	
3	4.2	6.5	9.1	18.2	30.5	45.0	66.0
3½	4.8	7.3	10.6	20.6	33.7	51.0	72.0
4	5.4	8.2	12.0	22.9	37.0	57.0	78.0
4½	6.0	9.0	13.0	25.8	40.2	62.0	85.0
5	6.6	9.9	14.0	27.5	43.5	67.0	92.0
5½	....	10.8	15.0	30.3	47.0	72.0	100.0
6	....	11.7	16.0	32.0	50.6	77.0	107.0
7	....	.....	....	36.5	57.8	87.0	122.0
8	....	....	....	41.0	64.7	97.0	137.0
9	....	....	....	45.5	72.0	107.0	152.0
10	....	....	....	50.0	79.2	117.0	167.0
11	....	....	....	54.5	86.5	127.0	180.0
12	......	....	....	59.0	94.0	137.0	191.0

TABLE 14.—MACHINE BOLTS[1]

Length (Inches)	Diameter (inches)								
	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1
	Threads per inch								
	20	18	16	14	13	11	10	9	8
	Weight in pounds of 100 bolts with square heads and nuts								
3/4	2.4	4.4	6.9	10.4					
1	2.8	4.9	7.6	11.5	16.3				
1 1/4	3.1	5.5	8.4	12.5	17.7	31.7	52.2		
1 1/2	3.4	6.0	9.2	13.6	19.1	33.8	55.3	83.4	
2	4.1	7.1	10.8	15.7	21.8	38.1	61.5	91.8	129.0
2 1/2	4.8	8.2	12.3	17.8	24.6	42.4	67.7	99.7	140.1
3	5.5	9.2	13.8	19.9	27.4	46.7	73.9	108.1	151.1
3 1/2	6.2	10.3	15.3	21.8	29.8	51.0	80.1	116.6	162.2
4	6.9	11.4	16.9	24.0	32.6	55.4	86.3	125.0	173.2
4 1/2	7.5	12.4	18.4	26.1	35.4	59.3	92.1	132.9	182.7
5	8.2	13.5	19.9	28.2	38.1	63.6	98.3	141.3	193.7
5 1/2	8.9	14.6	21.5	30.3	40.9	67.9	104.5	149.8	204.8
6	9.6	15.6	23.0	32.4	43.7	72.3	110.7	158.2	215.8
6 1/2	10.3	16.7	24.6	34.5	46.4	76.6	116.9	166.7	226.9
7	11.0	17.8	26.1	36.6	49.2	80.9	123.1	175.1	237.9
7 1/2	11.7	18.9	27.7	38.8	51.9	85.2	129.4	183.6	248.9
8	12.4	20.0	29.2	40.9	54.7	89.5	135.6	192.0	260.0
9	13.7	22.1	32.4	44.9	60.0	97.8	147.5	208.8	281.3
10	15.1	24.3	35.5	49.1	65.5	106.4	160.0	225.2	303.3
11	16.5	26.4	38.6	53.4	71.0	115.1	172.4	242.2	325.5
12	17.9	28.6	41.7	57.6	76.5	123.7	184.8	259.1	347.6
13	19.3	30.7	44.8	61.8	82.0	132.0	197.2	276.0	369.6
14	20.6	32.9	47.9	66.0	87.6	140.6	209.7	292.9	391.7
15	22.0	35.1	51.0	70.3	93.1	149.2	222.1	309.8	413.8
16	23.4	37.2	54.1	74.5	98.6	157.9	234.5	326.7	435.9
17	24.8	39.4	57.2	78.7	104.1	166.5	246.9	343.6	458.0
18	26.2	41.5	60.3	82.9	109.7	175.1	259.4	360.5	480.1
19	27.5	43.7	63.4	87.2	115.2	183.7	271.8	377.5	502.2
20	28.9	45.8	66.5	91.4	120.7	192.4	284.2	394.4	524.3
21	30.3	48.0	69.6	95.6	126.2	201.0	296.6	411.3	546.4
22	31.7	50.2	72.7	99.9	131.7	209.6	309.1	428.2	568.4
23	33.1	52.3	75.8	104.1	137.3	218.3	321.5	445.1	590.5
24	34.4	54.5	78.9	108.3	142.8	226.9	333.9	462.0	612.6
25	35.8	56.6	82.1	112.5	148.3	235.5	346.3	478.9	634.7
26	37.2	58.8	85.2	116.8	153.8	244.1	358.8	495.8	656.8
27	38.6	60.9	88.3	121.0	159.4	252.8	371.2	512.7	678.9
28	40.0	63.1	91.4	125.2	164.9	261.4	383.6	529.7	701.0
29	41.3	65.3	94.5	129.5	170.4	270.0	396.0	546.6	723.1
30	42.7	67.4	97.6	133.7	175.9	278.7	408.5	563.5	745.2

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[1] See also table in Carnegie Pocket Companion.

TABLE 15a.—TENSILE STRENGTH OF BOLTS AND ROUND RODS WITHOUT UPSET ENDS

Diameter of rod	Diameter of root of thread	Weight per lin. ft.	Strength of rod			
			At 12,500 lb. per sq. in.	At 15,000 lb. per sq. in.	At 16,000 lb. per sq. in.	At 20,000 lb. per sq. in.
3/8	0.294	0.376	848	1,018	1,088	1,360
7/16	0.344	0.511	1,160	1,393	1,489	1,860
1/2	0.400	0.668	1,570	1,884	2,018	2,520
9/16	0.454	0.845	2,022	2,427	2,590	3,240
5/8	0.507	1.043	2,524	3,030	3,230	4,040
3/4	0.620	1.502	3,780	4,530	4,830	6,040
7/8	0.731	2.044	5,250	6,300	6,720	8,400
1	0.837	2.670	6,880	8,240	8,800	11,000
1 1/8	0.940	3.380	8,670	10,420	11,100	13,880
1 1/4	1.065	4.170	11,170	13,420	14,280	17,860
1 3/8	1.160	5.050	13,220	15,860	16,900	21,140
1 1/2	1.284	6.010	16,190	19,420	20,700	25,900
1 5/8	1.389	7.050	18,930	22,720	24,200	30,300
1 3/4	1.490	8.180	21,880	26,170	27,900	34,880
1 7/8	1.615	9.390	25,600	30,720	32,800	40,960
2	1.712	10.680	28,800	34,550	36,800	46,040
2 1/4	1.962	13.520	37,800	45,350	48,400	60,460
2 1/2	2.175	16.690	46,450	55,700	59,400	74,300
2 3/4	2.425	20.200	57,750	69,200	73,800	92,380
3	2.629	24.030	67,800	81,400	86,900	108,560

TABLE 15b.—STRENGTH OF ROUND RODS WITH UPSET ENDS

Diameter of rod	Diameter of upset	Weight per lin. ft.	Strength of rod			
			At 12,500 lb. per sq. in.	At 15,000 lb. per sq. in.	At 16,000 lb. per sq. in.	At 20,000 lb. per sq. in.
1/2	3/4	0.668	2,453	2,944	3,135	3,920
9/16	3/4	0.845	3,106	3,727	3,980	4,980
5/8	7/8	1.043	3,835	4,600	4,910	6,140
11/16	1	1.262	4,640	5,560	6,940	7,420
3/4	1	1.502	5,520	6,627	7,080	8,840
13/16	1 1/8	1.763	6,490	7,790	8,310	10,380
7/8	1 1/4	2.044	7,516	9,020	9,630	12,020
15/16	1 1/4	2.347	8,630	10,340	11,040	13,800
1	1 3/8	2.670	9,815	11,780	12,560	15,700
1 1/8	1 1/2	3.379	12,425	14,900	15,910	19,880
1 1/4	1 5/8	4.173	15,330	18,400	19,650	24,540
1 3/8	1 3/4	5.049	18,550	22,260	23,750	29,700
1 1/2	2	6.008	22,080	26,500	28,300	35,340
1 5/8	2 1/8	7.051	25,910	31,090	33,200	41,480
1 3/4	2 1/4	8.178	30,060	36,070	38,500	48,100
1 7/8	2 3/8	9.388	34,600	41,400	44,200	55,220
2	2 1/2	10.680	39,270	47,130	50,300	62,840
2 1/8	2 5/8	12.060	44,320	53,190	56,700	70,940
2 1/4	2 7/8	13.520	49,700	59,680	63,600	79,520
2 3/8	3	15.070	55,370	66,450	70,900	88,600
2 1/2	3 1/8	16.690	61,350	73,620	78,500	98,180
2 5/8	3 1/4	18.400	67,600	81,200	86,600	108,240
2 3/4	3 3/8	20.200	74,230	89,080	95,100	118,800

The safe working value for common wire nails or spikes for resistance to lateral forces in timber joints of yellow pine or Douglas fir may be taken at

$$p = 4000d^2$$

where p = safe lateral resistance of one nail, and d = diameter of nail in inches.

The working values for the common sizes of nails in accordance with this formula are given in Table 16.

TABLE 16.—SAFE WORKING VALUE FOR LATERAL RESISTANCE OF ONE NAIL IN YELLOW PINE OR DOUGLAS FIR

Size of nail	6d	8d	10d	12d	16d	20d	30d	40d	50d	60d	80d
Strength in pounds	53	62	88	88	110	165	194	226	268	322	364

All tests made on nailed joints indicate that the strength of the joint is approximately the same whether the nail be driven so that the compression on the timber is against or across the grain. The resistance of the joint is, however, decreased from 25 to 33⅓% if the nails are driven parallel to the fibers of the timber—for example, driving the nails into the ends of a stick of timber. A joint in which this condition exists is a header joint, frequently used in light joist construction.

When one piece of timber is spiked to another, the penetration of the nail into the second timber should not be less than one-half the length of the nail, and should preferably be in excess of this.

The slip of a nailed joint occurs at a comparatively small load, as may be seen from an inspection of the curve of Fig. 112, which is plotted from the published results of tests made by the Portland Bureau of Buildings.

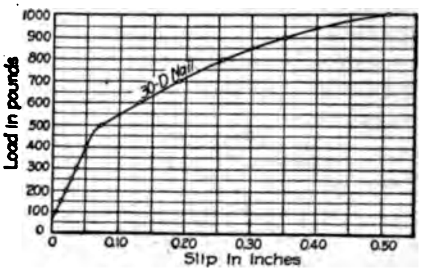

FIG. 112.—Typical load-slip curve of nailed joint, Bureau of Buildings, City of Portland.

The elastic limit of a nail in lateral resistance in air-dry long leaf yellow pine occurs at a value of approximately $C = 7000$ in the formula, $p = Cd^2$, and at an average slip of 0.028 in., as found by Wilson in the tests of the Forest Service (see reference in footnote, p. 232). The Portland tests show higher values for both elastic limit and slip at elastic limit.

112. Lateral Resistance of Wood Screws.—The lateral resistance of common wood screws was investigated as thesis work by Kolbirk and Birnbaum at Cornell University,[1] using timbers of cypress, yellow pine and red oak. From the results of these tests, the following formula for the safe lateral resistance may be used for yellow pine and Douglas fir:

$$p = 4375d^2$$

Table 17 gives the safe working values in terms of gage numbers. In giving these values the assumption is made that the screw is imbedded in the second or main piece of timber approximately 9/10 the length of the screw.

TABLE 17.—SAFE LATERAL RESISTANCE OF COMMON WOOD SCREWS WITH YELLOW PINE AND DOUGLAS FIR

Gage of screw	Diameter (inches)	Safe lateral resistance (pounds)
6	0.137	82
8	0.163	116
10	0.189	156
12	0.216	204
14	0.242	256
16	0.268	314
18	0.295	381
20	0.321	451
22	0.347	527
24	0.374	512
26	0.400	700

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[1] Abstract of results published in Cornell Civil Engineer, vol. 22, No. 2, Nov., 1913.

113. Lateral Resistance of Lag Screws.—Two typical cases of joints may be m boards or planks screwed to a timber block, and (2) a metal plate screwed to a block o The writer made a series of tests on both types of joint.[1] From the results of these te also from a theoretical consideration of the probable distribution of pressures of l against timber and resultant bending moments in the lag screw, the following value screws in lateral shear and bending are recommended:

SAFE LATERAL RESISTANCE OF ONE LAG SCREW

Metal plate lagged to timber	¾ × 4½-in. lag screw	10
	⅞ × 5 -in. lag screw	12
Timber planking lagged to timber	¾ × 4½-in. lag screw	9
	⅞ × 5 -in. lag screw	10

114. Lateral Resistance of Bolts.—In a typical detail of wooden joint, such as is ill in Fig. 113, a number of assumptions may be made as to the distribution of the bearing of the bolt against the timber. Since as many different bending moments will obta sumptions of distribution of pressure are made, the resultant computed resistance o resist relative moment of the timbers will vary accordingly. Two assumptions will b ered here: (1) a uniform distribution of pressures, and (2) triangular distribı bearing pressures.

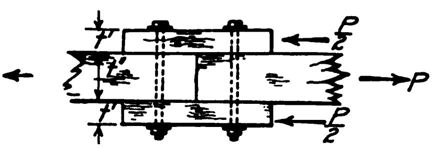

FIG. 113.—Typical bolted joint—bolts in "double shear."

(1) *Uniform Distribution of Beari sures.*—With this assumption, the bend ment in the bolt will be

$$M = \frac{1}{2}P(t'/2 + t''/4)$$

where t' = thickness of splice pad, and t'' = thickness of main timber. Under this assı the greater the thickness of side pieces t' (see Fig. 113), the larger diameter of bolt ı Table 18 gives the resisting moments of one bolt in flexure at various fiber stresses, from 12,000 to 24,000 lb. per sq. in.

The working values of bolts for typical timber joints, as found by this method are especially for joints with thick splice pads. Hundreds of such joints are giving service the bolts are working at more than the ultimate stresses as computed by this method.

Bolts are usually driven with a tight fit in the holes and when such a condition e: pressure of the bolt on the timber is not uniform along the length of bolt, as has been de by tests, and therefore the preceding value of bending moment on the bolt is incorrect

TABLE 18.—RESISTING MOMENTS OF BOLTS

Size of bolt	Section modulus	Fiber stresses (pounds per square inch)			
		12,000	16,000	20,000	22,500
⅝	0.0239	285	380	480	540
¾	0.0414	495	660	830	930
⅞	0.0656	785	1,050	1,310	1,475
1	0.0982	1180	1,570	1,960	2,205
1⅛	0.140	1680	2,240	2,800	3,150
1¼	0.191	2290	3,055	3,820	4,300
1⅜	0.255	3060	4,080	5,100	5,735
1½	0.331	3970	5,295	6,620	7,445
1⅝	0.421	5050	6,735	8,420	9,470
1¾	0.525	6300	8,400	10,500	11,810
1⅞	0.646	7750	10,335	12,920	14,535
2	0.785	9420	12,560	15,700	17,660

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[1] *Eng. News*, vol. 76, No. 3, July 20; No. 4, July 27, and No. 17, Oct. 26, 1916.

The following method is proposed as offering a satisfactory method of computing the strength of such bolt joints:

(2) *Triangular Distribution of Bearing Pressure on Bolts.*—The assumptions of this article are illustrated in Fig. 114 and are the result of a study of a series of tests of bolted joints made by the writer.[1] The theory of bearing pressures may be stated thus: It is assumed that the distribution of load on the bolt is triangular in shape; that the unit pressure (pounds per linear inch of bolt) is a maximum at the contact faces of the timbers, in amount equal to the strength of the timber in bearing,[2] and of approximately the distribution for the typical case, as shown in Fig. 114. It is also assumed that in the joint of Fig. 114, there is a definite minimum length "m," such that the moment resulting from the load on this length of bolt will just equal the flexural strength of the bolt. Further, it is assumed that in joints where the thickness of side timber is less than the limiting value "m" the pressure distribution diagram, while maintaining the general triangular shape, is modified in respect to the relative dimensions "a" and "b" (Fig. 114) within the limits $a = o$ and $a = t'/3$, and that the ratio a/t' remains such that the resulting bending moment in the bolt bears the same relation to the flexural strength of the bolt as the maximum intensity of pressure on the timber bears to the unit strength of the timber in compression. The above theory assumes that the ratio of thickness of timber to diameter of bolts is comparatively large. As the ratio of diameter of bolt to thickness of splice pad increases, the pressure distribution diagram on the length of bolt within the splice pad is assumed to change from a triangular shape (Fig. 114) through a trapezoidal shape (Fig. 115) until the limiting case is reached, with a short thick bolt of uniform distribution of pressure along the length of bolt (Fig. 116).

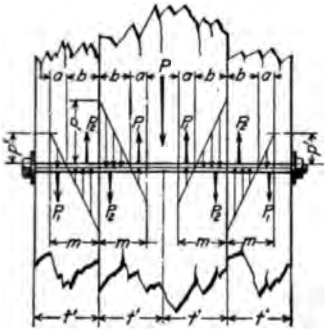

Fig. 114.

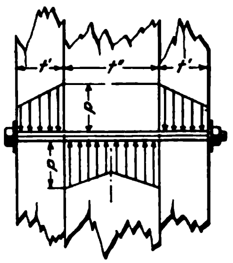

Fig. 115.

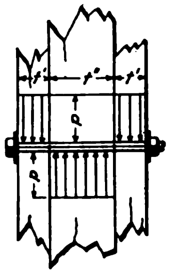

Fig. 116.

For the case illustrated in Fig. 114 there are two equal maximum bending moments in the bolt, occurring at points of zero shear. With the assumption that beyond a minimum value of t' or width of splice pad, the strength of joint is independent of the length of bolt, the length, for which the strength of the bolt in flexure is equal to the safe load on the bolt as determined from the compression on the timber, may be determined by equating the bending moment resulting from such load to the resisting moment of the bolt.

$$M = \frac{4}{9}P_1 m \qquad P_1 = \frac{pmd}{12}$$

whence

$$M = \frac{4}{9}\left(\frac{pmd}{12}\right)(m) = \frac{1}{27}(dpm^2) = \frac{\pi d^3 f}{32}$$

and

$$m = d\left(\frac{f\pi 27}{32p}\right)^{1/2}$$

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[1] See footnote, p. 240.

[2] By strength is meant working strength.

where M = bending moment on bolt in inch pounds.
p = maximum allowable unit bearing stress of bolt against timber.
f = maximum allowable flexural unit stress in bolt.
t' = thickness of splice pad.
d = diameter of bolt in inches.
m = length of portion of bolt on which pressure exists.

Using the same notation, when m is less than t', the theory assumes that the ratio of the dimensions a and b changes, within the limits $a = o$ and $a = t'/3$, to the end that the greatest strength of joint is obtained with the provision that the capacity of the bolt in bending and the timber in compression is maintained simultaneously. For these cases the bending moment may be expressed by the general formula $M = Ct'^2$, and the total load on the joint by the general formula $P = Kt'$. In these formulas, M = moment on bolt in inch pounds, t' = width of splice pad in inches, and C and K are factors to be obtained from Diagram 1.

Table 19 shows the relation of C and K to varying ratios of a/t; for a bolt of 1-in. diameter, for the case of a triangular pressure diagram.

TABLE 19

Ratio a/t'	C	K
0	433	1300
⅙	266	1040
¼	163	866
⅓	48	650

DIAGRAM 1.

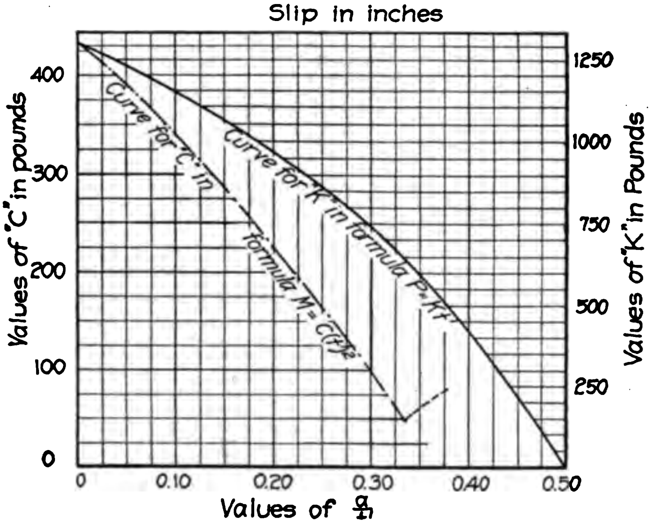


Diagram 1 shows the above variation of C and K with the ratios a/t', for a 1-in bolt. By means of this diagram, the safe strength of a bolt in double shear for any thickness of splice pad may be found. The diagram is based on the values, p = 1300 lb. per sq. in. for the safe pressure in end bearing of the diametral section of the bolt in timber, and f = 16,000 lb. per sq. in. for bolts.

Illustrative Problem.—Given a joint with 6-in. center timber, and two 3-in. splice pads, bolted with ⅞-in. bolts. What is the safe strength of one bolt, allowing a maximum unit compression against ends of fibers of timber and a maximum flexural stress of 16,000 lb. per sq. in. in the bolt?

From Table 18 the safe resisting moment of a ⅞-in. bolt at 16,000 lb. per sq. in. is 1050 in.-lb. Since Diagram 1 is for a bolt of 1-in. diameter, the equivalent moment for entering the diagram is $\frac{1050}{0.875}$ = 1200 in.-lb.

From the equation $M = Ct'^2$, $C = \frac{1200}{9} = 133.3$.

Entering the diagram, a vertical line through the point on the dash and dot "C" curve for the value $C = 133.3$, intersects the full line "K" curve at a point giving $K = 810$ lb. Remembering that this value is for the case of a 1-in. bolt, the safe load for a ⅞-in. bolt is

$$P = \frac{7}{8}Kt' = \left(\frac{7}{8}\right)(810)(3) = 2130 \text{ lb.}$$

For the cases in which the pressure distribution on the bolt is trapezoidal, as in Fig. 115, Table 20 gives the values of C and K, in the formulas $M = C(t')^2$ and $P = Kt'$, respectively, for various ratios of the minimum unit pressure to the maximum unit pressures, all for a bolt of 1-in. diameter.

TABLE 20

Ratio p'/p	C	K
0	433	650
¼	650	812
½	867	975
¾	1084	1138
1	1300	1300

DIAGRAM 2.

DIAGRAM FOR FINDING SAFE LOADS ON A BOLTED JOINT—BOLT IN "DOUBLE SHEAR." DIAGRAM DRAWN FOR 1-IN. BOLT.

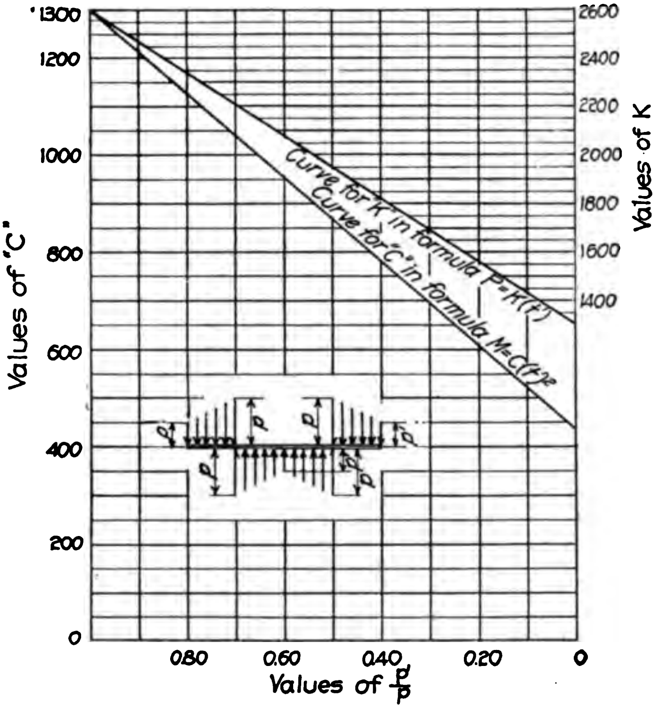


Diagram 2 gives the curves of these formulas for the trapezoidal distribution of pressure for a bolt 1 in. in diameter. These curves are to be used exactly as those of Diagram 1.

Illustrative Problem.—Given a joint of yellow pine timber with 5½-in. center, and two 2½-in. spliced pads, bolted with 1½-in. bolts. What is the safe strength of one bolt in lateral resistance?

From Table 18, the safe resisting moment of a 1½-in. bolt at 16,000 lb. per sq. in. is 5295 in.-lb. To enter Diagram 2, which is drawn for a 1-in. bolt, the value of 5295 must be divided by 1½. The equivalent moment is

$5295 \times \frac{2}{3} = 3530$. From the equation $M = C(t')^2$, $C = \frac{3530}{6.25} = 565$. From Diagram 2 the value of K in the curve $P = Kt'$, corresponding to $C = 565$, is 1500 lb. This value is for a 1-in. bolt. Therefore, the safe load for a 1½-in. bolt is

$$P = (1500)(2\tfrac{1}{2})(1\tfrac{1}{2}) = 5625 \text{ lb.}$$

The values of Table 21 have been worked from the preceeding theory by means of Diagrams 1 and 2.

TABLE 21.—VALUE OF ONE BOLT IN DOUBLE SHEAR

Bolt	Thickness side timbers (inches)				
	2	3	4	5	6
	Thickness center timbers (inches)				
	4	6	8	10	12
⅝	1060	1295	1465	1465	1465
¾	1440	1685	1990	2100	2100
⅞	1925	2135	2475	2845	2850
1	2480	2655	3000	3380	3700
1⅛	3120	3235	3580	4025	4520
1¼	3915	3940	4240	4680	5170
1⅜	4840	4690	4955	5415	5970
1½	5890	5570	5790	6245	6700

FIG. 117.—O.G. cast-iron washer.

Maximum fiber stress in bolt in bending, 16,000 lb. per sq. in.
Maximum intensity of bearing pressure on wood, 1950 lb. per sq. in.
Bearing on wood, average on diametral section of bolt, 1300 lb. per sq. in.

Bolts in Single Shear.—The safe values of bolts acting in "single shear" may be taken at one-half the values of Table 21.

Bolts Bearing Across the Grain of Timber.—For "double shear" joints in which the bolts bear across the grain of the timber, the safe values may be taken at five-eighths the values of Table 21.

Metal Plates Bolted to Timber.—The values of Table 21 may be used for joints in which steel plates are bolted to timber; in other words, a steel fish plate joint, provided that the values of this table do not exceed the safe loads as determined by bearing of the plate on the bolt, or shear in the bolts.

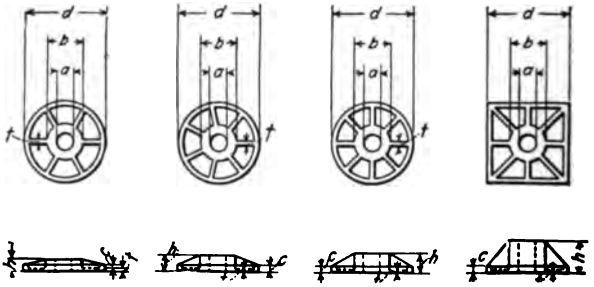


FIG. 118.—Cast-iron ribbed washers.

115. Resistance to Withdrawal of Nails, Spikes, Screws, and Drift Bolts.—The resistance of nails, spikes, screws and drift bolts to withdrawal from timber is a function of the surface area of contact between metal and timber, and the unit resistance to withdrawal. Expressed algebraically,

$$P = AC$$

in which

P = total pounds required to move the spike, screw, or drift bolt.
A = surface of contact between metal and wood.
C = unit resistance to withdrawal.

FIG. 119.—Malleable iron washer.

The value of C depends upon the kind, quality, and condition of timber, condition of surface of nail, screw, or drift bolt, size of hole in which nail, screw, or bolt may have been driven or screwed, and direction of fibers of timber with reference to length of nail, spike, screw, or drift bolt. For practical purposes, C is a quantity determined solely by experiment. Ultimate values for C for wire and cut nails, boat spikes, and drift bolts are given in Table 22. These values are taken from a study of the numerous tests that have been made. The values for resistance to withdrawal as found by the tests vary so widely that, for safe working values, a safety factor of four should be used.

116. Washers.—For the more common timbers employed in building construction, the resistance to crushing across the grain of the timber is much smaller than resistance to end crushing. For this reason it is necessary to use washers under heads and nuts of bolts in timber construction to prevent the nuts and head from crushing into the timber when the nuts are tightened, and also when the bolts take their assumed stresses.

There are five types of washers used in timber construction: (1) cast-iron O.G. washers, (2) cast-iron ribbed washers, (3) malleable iron washers, (4) circular pressed steel washers, and (5) square plate washers.

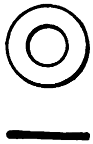


Fig. 120.—Circular pressed steel washer.

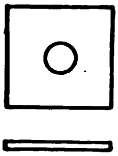


Fig. 121.—Square steel plate washer.

Table 22.—Ultimate Resistance to Withdrawal of Wire and Cut Nails, Wood Screws, Lag Screws, Boat Spikes and Drift Bolts

(All Quantities Expressed in Pounds per Square Inch of Contact Between Metal and Timber)

	Yellow pine	Douglas fir	White pine	White oak	Redwood
Cut nails[1]	500	500	300	1200	300
Cut nails[2]	300	300	275	1000	150
Wire nails[1]	300	300	170	900	300
Wire nails[2]	250	250	100	800	200
Wood screws	1500	1500	900	2200	900
Lag screws	800	800	500	1200	
Boat spikes[3]	500	500	270	1000	
Boat spikes[4]	370	370	200	750	
Drift bolts[5]	400	400	240	600	
Drift bolts[5]	200	200	120	300	

[1] Driven perpendicular to grain of timber.
[2] Driven parallel to grain of timber.
[3] Edge of point parallel to grain of timber.
[4] Edge of point across grain of timber.
[5] Driven in holes $\frac{1}{16}$ to $\frac{1}{8}$ in. less in diameter than drift bolt.

For cases in which the axis of bolt is inclined to the bearing surface of the timber, bevelled cast-iron washers may be employed (see Fig. 122 and Table 28). The five types of washers mentioned are illustrated in Figs. 117 to 121 inclusive and Tables 23 to 27 inclusive give detailed dimensions.

In the case of bolts acting wholly in tension there can be no question of the necessity of washers. Washers should be properly designed, both for strength and stiffness, and of proper size to limit the bearing pressure on the timber to the safe working value. For Douglas fir or yellow pine either the square plate washers, ribbed cast-iron, or cast-iron O.G. washers of equivalent area should be used. Attention is called to the fact that in the malleable washer, the full area of the base of washer is not available for bearing. For example, the $\frac{3}{4}$-in. malleable washer has an actual bearing area of about 4 sq. in., or an actual efficiency of approximately 60% of its nominal area. Even the cast-iron O.G. washers of Table 23 stress the timber to approximately 750 lb. per sq. in., for a unit stress of 16,000 lb. per sq. in. in the rod.

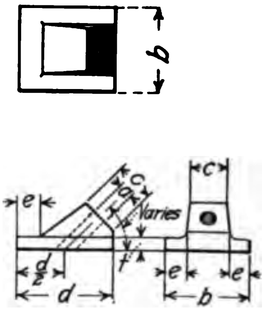


Fig. 122.—Bevelled cast-iron washer.

When the bolt acts wholly in shear and bending, smaller washers, such as the malleable washers, are permissible, though not necessarily advisable. In such instances it is often practically certain that the timber will shrink, and that the washers will never be tightened, and for this reason the use of malleable washers may be justified, in order to save expense. On the other hand, when there is a chance that some maintenance work may be counted upon in the shape of washer tightening, good construction will prescribe either a special cast-iron washer or a square plate washer, sufficient in size to meet the capacity of the bolt in tension.

In order to avoid special washers, malleable washers of larger size than the nominal size for the bolt used are sometimes specified. Such a procedure is unwise for two reasons: (1) the holes in the larger washer are of such diameter with respect to the diameter of the head and nut of the bolt, that a poor bearing between head or nut and washer results; and (2) the carpenter will invariably put stock sizes of washers and bolts together if there is a chance to do so.

The circular cut or pressed steel washer should never be used in timber construction, except between metal and metal.

The selection of a washer as between a special size O.G., ribbed cast-iron, or a square steel plate washer, will depend on the relative prices of cast iron and steel, availability of foundry and steel shops, and size of jobs. When large size washers are required and the job is a small one, the square plate washer will usually be found cheapest.

No square plate washer should have a thickness less than one-half the diameter of bolt. A good rule is to add 1/16 in. to the thickness thus found.

When the center line of bolt or rod is not normal to the bearing face of the timber, the timber must be notched, or a bevelled washer used. If the section of timber is ample, a notch

TABLE 23.—WASHERS—O.G. CAST-IRON

Size of bolt (inches)	Weight per 100 lb.	Diameter (inches)	Thickness (inches)	Area (square inches)
1/2	35	2 1/4	1/2	3.78
5/8	75	3	5/8	6.76
3/4	100	3 1/4	3/4	7.86
7/8	145	3 1/2	13/16	9.02
1	185	4	7/8	11.79
1 1/8	285	4 1/2	1 1/8	14.91
1 1/4	375	5	1 1/4	18.41
1 1/2	600	6	1 1/2	26.50

TABLE 24.—WASHERS—CAST-IRON RIBBED

(See Fig. 118)

Size bolt	Size upset	*a*	*b*	*c*	*d*	*h*	*t*	Shape base	No. ribs	Weight
5/8	Not upset	3/4	1 9/16	1/8	3 1/8	3/4	1/8	C	6	0.56
3/4	Not upset	7/8	1 3/4	3/16	4	1	1/4	C	6	1.10
7/8	Not upset	1	2 1/4	3/16	4 1/2	1 1/8	1/4	C	6	1.80
3/4	1	1 1/8	2 3/8	1/4	5 1/4	1 1/4	1/4	C	6	2.79
13/16	1 1/8	1 1/4	2 5/8	1/4	5 5/8	1 5/16	1/4	C	6	3.29
7/8	1 1/4	1 3/8	3	5/16	6 1/2	1 7/16	5/16	C	7	5.30
1	1 3/8	1 1/2	3 1/4	5/16	7	1 9/16	5/16	C	7	6.34
1 1/8	1 1/2	1 5/8	3 1/2	3/8	7 7/8	1 11/16	3/8	C	7	9.04
1 1/4	1 5/8	1 3/4	3 3/4	3/8	8 5/8	1 15/16	3/8	C	7	11.30
1 3/8	1 3/4	1 7/8	4	3/8	9 1/8	2 1/4	3/8	C	7	13.59
1 7/16	1 7/8	2	4 3/8	7/16	10	2 3/8	7/16	C	8	18.66
1 1/2	2	2 1/8	4 5/8	7/16	10 3/8	2 7/16	7/16	C	8	20.39
1 5/8	2 1/8	2 1/4	4 3/4	1/2	11 1/4	2 5/8	1/2	C	8	25.99
1 3/4	2 1/4	2 3/8	5 1/8	1/2	12 1/8	2 3/4	1/2	C	8	30.62
1 7/8	2 3/8	2 1/2	5 1/8	5/8	11 1/4	4 1/4	5/8	Sq.	8	48.23
2	2 1/2	2 5/8	5 5/8	3/4	12	5 1/8	3/4	Sq.	8	69.32

is the cheapest detail. The pressure of the washer against the timber is then inclined to the direction of fibers, and, consequently, a higher unit bearing pressure may be used, in accordance with the formula and values of Art. 118.

For the larger size of bolts and rods, notching the timber sufficiently to provide the required area for bearing may cut the stick beyond the safe limit. In such a case, either a combination of a flat washer with a smaller cast-iron bevelled washer may be used, or a special cast-iron bevelled washer may be designed. The latter solution is much the better of the two. If this washer be made square or rectangular, the component of the stress in the rod parallel to the face of the timber may be taken care of by setting the washer into the timber. In the former case, this component will produce bending in the rod or bolt.

TABLE 25.—WASHERS—MALLEABLE IRON

Size of bolt (inches)	Weight per 100 washers	Diameter (inches)	Thickness (inches)
1/2	15	2 1/2	1/4
5/8	22	2 3/4	5/16
3/4	33	3	7/16
7/8	50	3 1/2	7/16
1	68	4	1/2
1 1/8	87	4 1/2	1/2
1 1/4	150	5	5/8
1 1/2	190	6	3/4
2	420	7 1/2	3/4

TABLE 26.—WASHERS—WROUGHT-IRON

Size of bolt (inches)	No. in 100 lb.	Diameter (Inches)	Size of hole (inches)	Gage	Thickness (inches)
3/16	39,400	9/16	1/4	18	0.05
1/4	15,600	3/4	5/16	16	0.063
5/16	11,250	7/8	3/8	16	0.063
3/8	6,800	1	7/16	14	0.078
7/16	4,300	1 1/4	1/2	14	0.078
1/2	2,600	1 3/8	9/16	12	0.125
9/16	2,250	1 1/2	5/8	12	0.125
5/8	1,300	1 3/4	11/16	10	0.125
3/4	970	2	13/16	9	0.156
7/8	828	2 1/4	15/16	8	0.172
1	600	2 1/2	1 1/16	8	0.172
1 1/8	500	2 3/4	1 1/4	8	0.172
1 1/4	384	3	1 3/8	8	0.172
1 3/8	288	3 1/4	1 1/2	7	0.189
1 1/2	267	3 1/2	1 5/8	7	0.189
1 5/8	230	3 3/4	1 3/4	7	0.189
1 3/4	206	4	1 7/8	7	0.189
1 7/8	182	4 1/4	2	7	0.189
2	168	4 1/2	2 1/8	7	0.189
2 1/4	122	4 3/4	2 3/8	5	0.219
2 1/2	106	5	2 5/8	4	0.234

TABLE 27.—WASHERS—SQUARE STEEL PLATE

Unit Bearing Pressure—350 lb. per sq. in.
Unit Tension in Bolt or Rod—16,000 lb. per sq. in.

Diameter of bolt or rod	Diameter of upset	Side of square washer	Thickness of washer
5/8	Not upset	3 1/4	5/8
3/4	Not upset	4	7/16
7/8	Not upset	4 1/2	1/2
3/4	1 in.	4 3/4	9/16
13/16	1 1/8	5	5/8
7/8	1 1/4	5 1/2	11/16
1	1 3/8	6 1/4	3/4
1 1/8	1 1/2	7	13/16
1 1/4	1 5/8	7 3/4	7/8
1 3/8	1 3/4	8 1/2	15/16
1 1/2	2	9 1/4	1 1/16

TABLE 28.—WASHERS—CAST-IRON BEVELED

Size rod	*a*	*b*	*c*	*d*	*t*	*e*
3/4	7/8	3 1/2	1 3/4	4	5/8	3/4
7/8	1	4 1/4	2	4 1/2	3/4	1
1	1 1/8	4 3/4	2 1/4	5 1/4	7/8	1 1/8
1 1/8	1 1/4	5 1/4	2 1/2	6	1	1 1/4
1 1/4	1 3/8	6 1/4	2 3/4	6 1/2	1	1 1/2

117. Resistance of Timber to Pressure from a Cylindrical Metal Pin.—When a pin, bolt, etc. of circular cross-section bears against the ends of the fibers, the load on the pin is resisted by pressure of the timber against the metal, and such differential pressures are always normal to the surface of the pin. The differential pressures may be supposed to be replaced, for practical purposes, by two resultant reactions, one parallel and the other perpendicular to the line of action of the applied force. The second of these resultant reactions tends to split the timber, since it produces tension across the fibers of the timber. Consequently, for the case in hand, the usual permissible unit bearing pressure against the ends of the fibers must be reduced. Also the particular detail must be investigated to make sure that the tension across the fibers due to the cross pressure is within the safe unit stress for the timber in question.

Tests and theoretical considerations indicate that for a round pin or bolt bearing against the ends of timber, the safe average unit bearing pressure to be applied to the diametral plane of the pin may be taken at 2/3 the usual allowable compression against the ends of timber. The resultant secondary pressure across the fibers may be taken at 1/10 the applied load. When the direction of the applied load is perpendicular to the direction of the fibers, the safe average diametral pressure may be taken at 4/10 of the permissible unit compression across the fibers.

For the case of pins and bolts in tight fitting holes in dense Southern pine and Douglas fir, the values of 1300 lb. per sq. in. for end bearing and 800 lb. per sq. in. in cross bearing may be used.

Illustrative Problem.—What is the safe load on a 1 1/4-in. bolt, bearing against the ends of the fibers of a 6 × 6-in. block of Douglas fir, and what is the force tending to split the block of timber?

The safe load is 1 1/4 × 6 × 1300 = 1950 in.-lb. The force tending to split the timber is 1950 × 0.1 = 195 lb.

118. Compression on Surfaces Inclined to the Direction of Fibers.—The allowable intensity of pressure on timber, when the direction of pressure is neither parallel nor perpendicular to the direction of fibers, was investigated by Prof. M. A. Howe on specimens of yellow pine, white pine, cypress, white oak, and redwood.[1] On the basis of these tests, Prof. Howe recommends the formula:

$$r = q + (p - q)(\theta/90°)^2$$

where

r = allowable normal unit stress on inclined surface.
p = allowable unit stress against ends of fibers.
q = allowable unit stress normal to direction of fibers.

Using the same notation, Prof. Jacoby in "Structural Details" develops the formula:

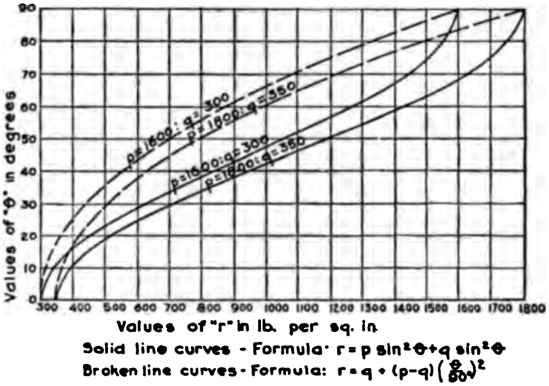
$$r = p \sin^2 \theta + q \cos^2 \theta.$$

Mr. Russell Simpson of the University of California, has recently made a series of tests, as thesis work, on the bearing values for inclined surfaces of Douglas fir and California white pine. He finds that Jacoby's formula gives results closely approximating the test values at the elastic limit, while Howe's formula holds for a constant indentation of 0.03 in. Diagram 3 gives the curves of the formulas of Howe and Jacoby for values of p = 1800 lb. per sq. in., q = 350 lb. per sq. in.; and p = 1600 lb. per sq. in., q = 300 lb. per sq. in.

Working values for actual design of timber joints involving bearing on surfaces inclined to the direction of fibers should be based on the elastic limit. The full line curves of Jacoby's formula are therefore recommended for design.

DIAGRAM 3.

DIAGRAM FOR SAFE BEARING PRESSURE ON TIMBER SURFACES INCLINED TO DIRECTION OF FIBER.



119. Tension Splices.—The tension splice in timber building construction occurs usually in the lower chord of a roof truss. This detail is probably the most troublesome to design and frame efficiently of all timber joints. A detail that is efficient on paper is often very unsatisfactory when viewed in the field. Any detail that depends for its action on the simultaneous bearing of more than two contact faces is to be avoided if possible, although it is often impracticable to so limit the design. Again, that detail which is so designed that the bearing faces of splicing members and the bearing faces of the spliced or main timbers may be pulled together in the field after the joint is framed, has a very decided advantage over any other type of tension splice. The ideal splice, just described, will be found to give a low efficiency when measured in terms of effective area of main timbers for resisting tension. However, in many cases, such inefficiency may well be allowed, in order to secure certain definite action of splice joint. Importance of the connection, cost of materials, quality of workmanship to be anticipated, possibility of only occasional or no inspection after completion, are all factors that should be carefully considered before deciding upon the particular type of tension splice to be adopted.

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[1] *Eng. News*, vol. 68, No. 5. and vol. 68, No. 10.

The following types of tension splices will be considered and a detail joint of each type developed for a typical example:

(1) Bolted wooden fish plate splice, (2) Modified wooden fish plate splice, (3) Bolted steel fish plate splice, (4) Tabled fish plate splice, (5) Steel tabled fish plate splice, (6) Tenon bar splice, and (7) Shear pin splice.

It will be assumed that a 6 × 8-in. Douglas fir stick must be spliced to safely stand a total stress of 40,000 lb. Specifications of steel structures often call for the detail of splice to be of sufficient strength to develop the strength of the members. The same specification may be applied to the timber joint, although it is customary to design the splice for the computed stress in the member.

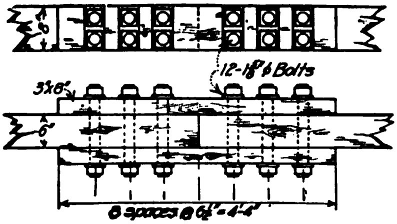

Fig. 123.—Bolted wooden fish plate splice.

For the case under discussion the safe working stress in the timber for tension will be taken at 1500 lb. per sq. in. The required net area for tension is therefore $\frac{40,000}{1500} = 26.7$ sq. in.

119a. Bolted Fish Plate Splice.—The bolted fish plate splice is shown in Fig. 123. The size of bolts will be computed in accordance with the formula

$$M = \tfrac{1}{2}P(t'/2 + t''/4)$$

where P is the total load on one bolt; t' is the thickness of splice pad, or fish plate; and t'' is the thickness of main timber (see Art. 114). This formula assumes the load on each bolt to be uniformly distributed along its length.

Assume 1¾-in. bolts, and splice plates 3 × 8 in. With bolts spaced in pairs, the net width of splice plate will then be 8 − (2) (1¾) = 4½ in. The required thickness of one plate is then $\frac{26.7}{9} = 2.97$, showing that a 3-in. thickness is sufficient. Assume 6 bolts required. The load on one bolt is then 40,000/6 = 6667 lb. The bending moment on one bolt is (6667/2)(½ × 3 + ¼ × 6) = 10,000 in.-lb. With a flexural stress of 24,000 lb. per sq. in., the required section modulus of one bolt = 10,000/24,000 = 0.416 in., and the required diameter of bolt = $\sqrt[3]{0.416/0.098} = \sqrt[3]{4.26} = 1.62$ in.

The unit bearing pressure on the diametral section of bolt = $\frac{6667}{(1.625)(6)}$ = 685 lb. per sq. in., which is about one-half the amount allowed. The minimum distance between bolts must next be computed. This distance will be taken as the sum of (a) computed distance necessary for shearing along the grain of the timber; (b) computed distance giving required area for transverse tension, and (c) diameter of bolt.

Total shearing area required .... $= \frac{6667}{150} = 44.44$ sq. in.

or distance (a) .... $= \frac{44.44}{12} =$ .... 3.7 in.

Area required for transverse tension .... $= \frac{(6667)(0.1)}{150} = 4.44$ sq. in.

or distance (b) .... $= \frac{4.44}{6} =$ .... 0.74 in.

Diameter of bolt (c) .... 1.63 in.

Minimum spacing of bolts .... 6.07 in.

The spacing of bolts will be made 6½ in.

119b. Modified Wooden Fish Plate Splice.—In the modified wooden fish plate splice, the size of bolts will be reduced to 1 in., and the value of each bolt taken at 2655 lb., in accordance with the values of Table 21, p. 244.

The number of bolts required is $\frac{40,000}{2655} = 15$.

14 1-in. bolts will be used, giving a load of 2857 lb. per bolt.

Spacing of bolts:

(a) Distance required for shear $\frac{2857}{(150)(12)} =$ 1.58 in.

(b) Distance required for transverse tension $= \frac{(2857)(0.1)}{(150)(6)} =$ 0.32 in.

(c) Distance of bolt .... = 1.00 in.

2.90 in.

Spacing of bolts will be made 3 in. The detail is shown in Fig. 124.

119c. Bolted Steel Fish Plate Splice.—Fig. 125 shows a bolted steel fish plate splice. The bending in the bolts is reduced from that in the first type, due to the smaller lever arm. The section of steel plate must be sufficient for tension, and for bearing on the bolts. Otherwise, the computations are similar to those of the bolted fish plate splice.

Net section of steel plate $= \frac{40{,}000}{15{,}000} = 2.67$ sq. in.

Assume two 1½-in. bolts in pairs. Then net width $= (2)(1\frac{9}{16}) = 4.875$ in., and required thickness is $\frac{2.67}{(2)(4.875)} = 0.28$ in., requiring a ⁵⁄₁₆-in. plate. Assume six bolts. As before, each bolt must take 6667 lb. The minimum diameter of bolt required with a ⁵⁄₁₆-in. plate at 15,000 lb. per sq. in. in bearing is ¾ in. Assuming a uniform distribution of pressure along the length of bolt, the bending on bolt $= \left(\frac{6667}{2}\right)(\frac{1}{2} \times \frac{5}{16} + \frac{1}{4} \times 6) =$ 5520 in.-lb. At 24,000 lb. per sq. in., the required diameter of bolt from Table 18 is seen to be 1⅜ in.

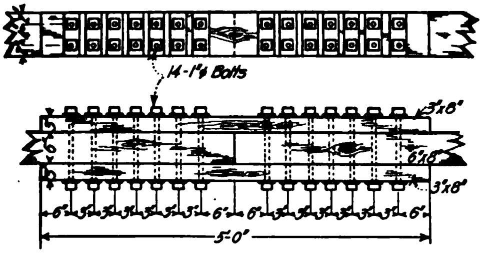

FIG. 124.—Modified wooden fish plate splice.

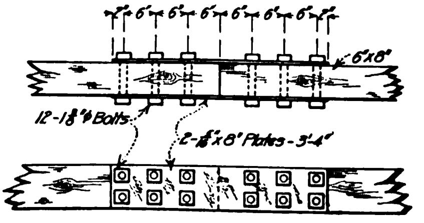

FIG. 125.—Bolted steel fish plate splice.

The unit pressure of the bolt on the ends of the fibers is $\frac{6667}{(1.375)(6)} = 810$ lb. per sq. in. The spacing of bolts may be figured as before, and will be less than that computed in the detail of the bolted fish plate splice by the difference in diameter of the bolts. The spacing will be made 6 in.

119d. Tabled Wooden Fish Plate Splice.—The detail of a tabled wooden fish plate splice is shown in Fig. 126. The points to be investigated in this detail are: (1) net section of main timber and splice pad; (2) bearing between splice pad and main timber; (3) length of table of fish plate for shear; (4) tension in bolts; and (5) possibility of bending on splice pads if bolts become loose because of shrinkage of timbers.

Net section of main timber required, as before, 26.7 sq. in.

Net section of fish plate required, as before, $\frac{40{,}000}{(2)(1500)} = 13.4$ sq. in.

Allowing for two ¾-in. bolts, net depth of fish plate $= \frac{13.4}{(8 - 1\frac{1}{2})} = 2.06$ in.

Total bearing area required between fish plate and main timber $= \frac{40{,}000}{1600} = 25$ sq. in.

Depth of cut into main timber $= \frac{25}{(8)(2)}$ 1.57 in. Depth will be made 1¾ in. It will be necessary to use an 8 × 8-in. timber, instead of a 6 × 8-in. stick, with 4 × 8-in. fish plates.

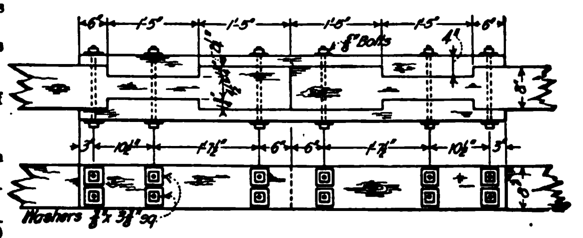

FIG. 126.—Tabled wooden fish plate splice.

Total net depth of fish plate 2¼ in.

Shearing area required for table of fish plate $= \frac{40{,}000}{(2)(150)} = 133$ sq. in. Length of table $= \frac{133}{8} = 17$ in.

The action of this joint produces a bending moment in the fish plate which must be resisted by the bolts. The resultant stress in the fish plate acts at the center of the uncut portion, while the resultant of the pressure between fish plate and main timber is at the center of the table. This couple produces a moment, in this case, of

$$(20{,}000)(\tfrac{1}{2})(2\tfrac{1}{4} + 1\tfrac{3}{4}) = 40{,}000 \text{ in.-lb.}$$

The lever arm of the bolts in the center of the table about the end of table is 8½ in. Using two bolts, the stress in each bolt is $\frac{40{,}000}{(2)(8\frac{1}{2})} = 2353$ lb. A ½-in. bolt is sufficient for this stress, but bolts less than ⅝-in. diameter are not advisable in a timber joint. The required area of washers is $\frac{2353}{350} = 6.72$ sq. in., which area would be supplied by a 3-in. circular washer. The washers shown are square steel ⅜ × 3⅜ × 3⅜ in.

If the timber should shrink and the bolts remain loose, each fish plate would be subjected to the full bending of 40,000 in.-lb., except as the friction of the ends of the table against the main timber might reduce such bending. The section modulus of the net section of fish plate is $(\frac{1}{6})(8)(2\frac{1}{4})^2 = 6.75$ (correct for two bolts). The extreme fiber stress due to bending would then be $\frac{40,000}{6.75} = 5926$ lb. per sq. in. To this stress must be added the uniform tensile stress, which is $\frac{20,000}{(8)(2\frac{1}{4})} = 1110$ lb. The maximum fiber stress would therefore be 7036 lb. per sq. in., an amount nearly equal to the ultimate strength of the timber. For this reason, the joint should be well spiked together, and in particular the fish plate should extend at either end beyond the table, to allow a number of spikes to be driven here. If the cut at the ends of the tables be made with a bevel towards the center of the joint, the same result will be obtained.

119e. Steel-tabled Fish Plate Splice.—The most economical and practical detail of the steel-tabled fish plate splice consists of steel splice plates with steel tables riveted to the plates, as shown in Fig. 127. The points to be investigated are: (1) necessary net area of plate to resist tension; (2) required thickness of tables to keep the bearing of tables against the ends of the fibers of the timber within the safe working stresses; (3) number of rivets between tables and fish plate; (4) distance between table, limited by longitudinal shear in the timber; and (5) bolts required to hold tables in the notches in the timber.

The 6 × 8-in. main timber will be sufficient for this type of splice.

Net area of steel plates $= \frac{40,000}{15,000} = 2.67$ sq. in.

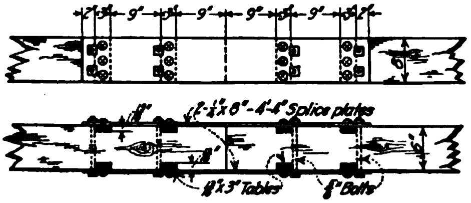

FIG. 127.—Steel-tabled fish plate splice.

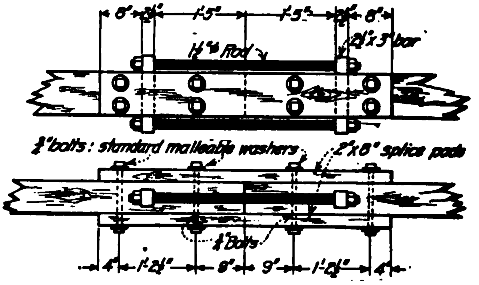

FIG. 128.—Tenon-bar splice.

Assume 3 rivets in one row. Then net width of plate is $8 - (3)(\frac{3}{4}) = 5.75$ in., and required thickness of plate is $\frac{2.67}{(2)(5.75)} = 0.23$ in. A ¼-in. plate will be sufficient for tensile strength. Bearing area required for tables $= \frac{40,000}{1600} = 25$ sq. in.

Assume 4 tables on each fish plate. Required total thickness of tables is $\frac{25}{(4)(8)} = 0.78$ in. Make the depth $\frac{13}{16}$ in. = 0.815 in.

Rivets required in each table, limiting value of one ¾-in. rivet in bearing at 20,000 lb. per sq. in. on ¼-in plate being 3750 lb. $= \frac{40,000}{(4)(3750)} = 2.67$.

Use three rivets and make table $\frac{13}{16} \times 3$ in.

The distance between end of main timber and first table, and the distance between tables, must be sufficient for longitudinal shear in the timber. Total shearing area required $= \frac{40,000}{150} = 267$ sq. in. Distance between tables $= \frac{267}{(4)(8)} = 8.35$ in. Call this distance 9 in., making the distance center to center of tables 12 in.

As in the case of the wooden fish plate splice, the bending moment to be resisted by bolts is the load transmitted by one table times one-half the combined thickness of fish plate and table, or

$$M = (10,000)(\tfrac{1}{2})(\tfrac{13}{16} + \tfrac{1}{4}) = 5300 \text{ in.-lb.}$$

Two bolts will be placed against the outer edge of table, making the lever arm of the bolts 3½ in. The stress in one bolt is then $\frac{5300}{(3\frac{1}{2})(2)} = 760$ lb. Two ⅝-in. bolts will be used for each table.

119f. Tenon Bar Splice.—The tenon bar splice is one of the oldest splices uesd, though not seen so frequently today as formerly. It is probably the simplest and most effective tension splice that can be made. The detail is shown in Fig. 128. The points to be computed are (1) size of rod for tension; (2) width of bar for proper bearing against the timber, and also for the hole for the rod passing through the ends; (3) depth of bar for bending; (4) distance of bar from end of timber to provide sufficient bearing area; and (5) net section of timber. To give general stiffness to this joint, Fig. 128 shows the addition of two 2 × 8-in. splice pads bolted with ¾-in. bolts.

An 8 × 8-in. main timber will be assumed. Size of rod area required $= \frac{40,000}{(2)(16,000)} = 1.25$ sq. in. A 1½-in. rod has an area of 1.295 sq. in. at the root of thread, and this size rod will be used. Since the rod must be placed at such a distance from the timber that the nuts may be tightened, and since it is desirable to keep the length of the bar as small as possible, hexagonal nuts will be used. (It is obvious that the bending moment on the bar increases with the distance between center lines of rods.) The long diameter of a 1½-in. hexagonal nut is 2¾ in., hence the distance from the side of timber to the center line of rod will be made 1½ in.

Size of bar required: The pressure of the timber against the bar will be assumed to be uniform. Hence the bending moment on the bar will be $(20,000)(1\frac{1}{2} + \frac{1}{4} \times 8) = (20,000)(3\frac{1}{2}) = 70,000$ in.-lb. Using a fiber stress of 24,000 lb. per sq. in. in bending, since the bar is a short beam, the required section modulus is $\frac{70,000}{24,000} = 2.92$ in.

The bearing area required is $\frac{40,000}{1600} = 25$ sq. in. The required width of bar is therefore $\frac{25}{8} = 3.13$ in. Since a 3-in. bar is a stock size, a width of 3 in. will be used. This width will give a full bearing for the hexagonal nut, and will allow $\frac{11}{16}$ in. of metal on each side of the hole. If a 6 × 8-in. timber were used, the required width of bar would be 4¼ in., which would reduce the section of timber below the allowable.

The depth of bar must now be computed. The section modulus $\frac{1}{6}bd^2 = 2.92$ in., when $d = \sqrt{\frac{(2.92)(6)}{b}} = \sqrt{\frac{(2.92)(6)}{3}} = \sqrt{5.84} = 2.4$ in. The bar size will be taken at 2½ × 3 × 14 in.

The shearing area required between the bar and end of timber is $\frac{40,000}{150} = 267$ sq. in. The distance required between the bar and end of timber is therefore $\frac{267}{(2)(8)} = 16.8$ in., say 17 in.

119*g*. Shear Pin Splice.—In the shear pin splice, the 6 × 8-in. main timber will be sufficient. This splice is shown in Fig. 129. The stress is transmitted across the joint by means of the circular pins of hardwood or steel. These pins are driven in a bored hole with a driving fit for the pins. The joint is a comparatively easy one to frame. The bolts take some tension, due to the couple of the forces acting on the pins. The working values for the pins are taken from Art. 117.

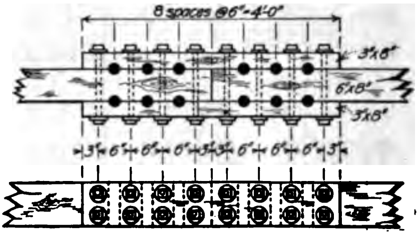

Fig. 129.—Shear pin splice.

The splice pads in this detail are 3 × 8-in. timbers. The pins are 2 in. in diameter, of extra heavy steel pipe. The total net section of splice pads is then $4 \times 8 = 32$ sq. in., giving a unit stress in tension of $\frac{40,000}{32} = 1250$ lb. Using the working value of 800 lb. per lin. in. of pin, the safe value of a 2 × 8-in. pin is 6400 lb. The number of pins required is then $\frac{40,000}{6400} = 6.25$. Six pins will be used.

The tension in. the bolts will be taken at one-half the total tensile stress, or 20,000 lb. Eight ⅝-in. bolts will be used, giving a working value of 2500 lb. per bolt. The bolts will be placed in pairs, endways between the pins. The pins will be placed 6-in. centers

120. General Comparison of Tension Splices.—The tenon bar splice, when it can be used, is to be recommended. It is direct in its action; shrinkage of the timber cannot destroy its effectiveness; there being but one bearing surface, the splice will surely act as designed; the two sections of timber can be drawn tightly together in the field; and the splice is almost fool-proof.

The wooden tabled fish-plate splice is also effective where there is but one table in each splice pad either side of the joint. In those joints where more tables are necessary, however, there enters at once the possibility, and even the probability, that all the contact faces will not act simultaneously. In other words, the effectiveness of the splice in such a case depends wholly on the skill and care in workmanship. In this detail, also, shrinkage of the timber adds an uncertainty as to the strength of the joint.

The bolted steel fish plate splice makes a neat appearing splice for exposed work, and is much in favor on that account. For a moderate stress in the timber to be spliced, it is fairly economical.

The steel tabled fish plate splice is open to the same objection as the wooden tabled splice. The bearing surfaces of the steel tables are very likely to be uneven, making a close fit between steel and timber almost impossible. On paper, the joint is neat and effective and adaptable to almost any case. Unless rigid inspection in the shop and field is maintained, the actual joint is likely to be disappointing. The bearing edges of all tables should be milled; the holes in the tables should be drilled, and tight riveting secured. Careless and inferior workmanship in the steel shop on the metal splice plates is to be expected.

The shear pin splice is effective and simple; its greatest drawback is the effect of shrinkage in the timber which will allow the pins to become loosened. This splice should not be used with unseasoned or partially seasoned timber, unless it is absolutely certain that the bolts will be kept tight as the timber seasons.

The bolted wooden splice is effective, but cumbersome, and unsuited for large stresses, due to the unusual size of bolts.

The modified wooden bolted splice is satisfactory for comparatively small stresses and when rigid inspection can be counted upon to see that the bolts are driven in close fitting holes. For large stresses, the required number of bolts will be excessive.

Architectural appearances may prohibit certain types of splices as being unsightly. The bolted steel fish plate splice and the tabled steel fish plate splice are the neatest in appearance, and for this reason are extensively used in exposed work.

121. Compression Splices.—Compression splices naturally divide into two divisions: (1) those joints which take only uniform compression at all times, and (2) those joints which, while compression is the principal stress, may be called upon at some time to take either flexure, or tension, or a combination of both.

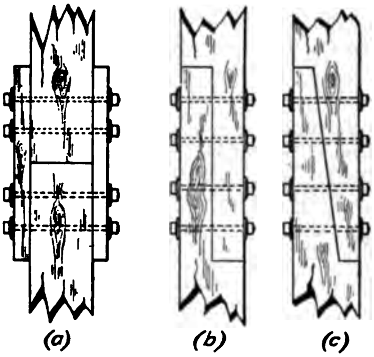

Fig. 130.—Compression splices.

Some of the compression splices used in construction are shown in Fig. 130. These joints, in the order lettered, are (*a*) the butt joint, (*b*) the half lap, and (*c*) the oblique scarf.

The butt joint differs from all the other joints in that it has but one surface of contact. For this reason, it is superior to all the others, where uniform compression alone is to be transmitted. The efficiency of all the other joints depends wholly upon the skill and care of the carpenter who frames the joint. In other words, the butt joint for the condition named is the simplest, and therefore the best. Indeed, the splice plates, if bolted, or bolted and keyed, may make the butt joint suitable for carrying both tension and flexure.

The oblique scarfed splice is stronger in flexure than the half lap. In the half lap joint, however, there is more timber in straight end bearing than in the oblique scarf.

In constructing compression joints in timbers which are vertical in position, the bolts through one end of the splice pads, if such exist, should be placed after the upper timber has come to a bearing on the lower timber; otherwise the bolts may receive a heavy load before the timbers come to a full bearing.

122. Connections Between Joists and Girders.—When possible, joists should rest upon the tops of girders, and not frame into the sides of the girders. The former construction, however, involves a loss in head room in a building, increased height of building walls and columns. It also involves more shrinkage, since the shrinkage is directly proportional to the depth of timber. In the case of a building with masonry walls and timber interior, the construction of joists resting upon the girders will, with green or unseasoned timber, result in unequal settlement of the floors. The inner ends of the outer floor bays will settle the amount of shrinkage of joist plus girder, while the outer ends will settle only the amount of shrinkage of the joists, since the joists frame directly into the masonry. The considerations of equal settlement and gain in building height will usually dictate the use of joist hangers in a building with heavy masonry walls.

In a building of the mill-building type with wall posts and girders, and corrugated steel or wooden sheathed walls, the increased height due to framing the joists on top of the girders will be offset by the saving in the cost of joist hangers.

The joists should extend over the full width of girder, and be toenailed into the girders. When the joists break over the girders they should lap at least 12 in. and be well spiked together. Solid bridging of a depth equal to the depth of the joists, and of a width not less than 2 in., is usually placed between the joists, and directly over the center of girder. Such bridging holds the joists firmly in position, and also acts as a fire stop. This construction is shown in Fig. 131.

122a. Joists Framed into Girders.—In very light construction the joists, when framed into the sides of a girder, are sometimes only toenailed. In other cases, especially when the joists frame into only one side of the girder, such girder built up of several vertical pieces, the outer piece is spiked into the ends of the joists, as in Fig. 132. All such joints are makeshifts, and extremely unreliable. As has been pointed out in a previous article (see Art. 111), nails driven into the ends of timbers—*i.e.*, parallel to the direction of fibers—have a low strength. Further, there is always the danger of the nails thus driven causing the joists to split.

Sometimes a strip is nailed or bolted to the sides of the girder, upon which the joists rest, as in Fig. 133. If properly designed, such strips will be not less than 4 in. wide and 4 in. deep, bolted, not nailed to the girder. The bolts should be sufficient in number to take the reaction of the joists, and should be not less than 2½ in. from the bottom of girder.

Illustrative Problem.—Given a floor bay 14 × 16 ft.; live load of 60 lb. per sq. ft.; girders spanning the shorter side of the floor bay. Assume double thickness of flooring 1-in. *T* and *G* finished floor over 1-in. rough floor. Working fiber stress is flexure 1600 lb. per sq. in.; working unit stress in longitudinal shear 150 lb. per sq. in.; working unit stress in cross bearing 300 lb. per sq. in.

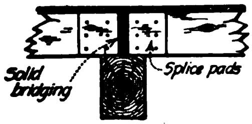

FIG. 131.

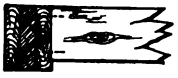

FIG. 132.

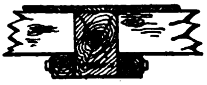

FIG. 133.

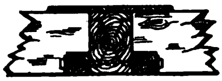

FIG. 134.

Weight of floor construction, exclusive of girders:

Flooring	6
Joists	5
Bridging	1
Total dead load	12
Live load	60
Total load	72 lb. per sq. ft.

With joists 16-in. centers, and counting the clear span for joists as 15 ft., the following figures result:

Total load on one joist $= (15)(1\frac{1}{3})(72) = 1440$ lb.

Bending moment $= (\frac{1}{8})(1440)(15)(12) = 32{,}400$ in.-lb.

Required section modulus $= \frac{32{,}400}{1600} = 20.$

Assume joist 2 × 10 in., actual section 1⅝ × 9½, actual section modulus 24.44.

For a 15-ft. span, this size is the minimum for deflection. In the computation for girder size, the live load may be reduced 20 %, making total load 60 lb. per sq. ft.

Load $= (14)(16)(60) = 13{,}440$ lb. $M = (\frac{1}{8})(13{,}440)(14)(12) = 282{,}000$ in.-lb.

Required section modulus $= S = \frac{282{,}000}{1600} = 176.$

An 8 × 14-in., finished section 7½ × 13½, has a section modulus of 227.8. An 8 × 12-in. girder, finished size 7½ × 11½, would have a section modulus of 165 under the required amount. The reaction of one joist is 720 lb., requiring a bearing area of $\frac{720}{300} = 2.4$ sq. in. The bolting strip will be 4 × 4 in. ⅝-in. bolts will be used, and the working load per bolt will be taken at 900 lb.[1] Since the load per linear foot of girder is $16 \times 60 = 960$ lb., the bolts must be spaced $\frac{900}{960}(12) = 11$ in. centers, or 13 bolts per girder.

In the above illustrative problem, the depth of joist plus the depth of bolting strip just equals the depth of girder. This relation does not always hold, as girder depth is often but little more than the depth of joist. To avoid having the bottom of joists lower than the girder, joists are often notched as shown in Fig. 134. Such construction is not good, since the strength of the joists is greatly reduced by notching. The joists tend to split in the corner of the notch, due to the difference in stiffness on either side of the vertical cut.

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[1] From Table 21, p. 244, ⅝-in. bolt "double shear" with 4 and 8-in. timbers, good for 1465 lb. in end bearing. For side bearing, safe load $= \frac{5}{8} \times 1465 = 915$ lb.

In some cases, the ends of the joists are framed with tenons fitting into sockets or recesses cut into the girder. This type of framing is to be condemned on account of the serious weakening of both joist and girder.

122b. Joist Hangers.—The most satisfactory manner of framing joists into the sides of girders is by the use of joist hangers. There are many stock types of these, among which may be named the Duplex, Van Dorn, Ideal, Lane, National, and Falls. Some of these different types are shown in Figs. 135 to 138 inclusive. A stock joist hanger should not be used without investigating carefully its strength and the amount of bearing given to the joist. Referring to the figures illustrating the different types, the fact should be noted that the Duplex hanger will result in less settlement of floor than any of the other types, since the connection of this hanger, unlike all the others, is on the side of the girder, and, hence, is affected by the shrinkage of one-half instead of the whole depth of girder. The published tests of joist hangers, as given in the various manufacturers' catalogs, will bear close scrutiny. Often in the effort to prove the merits of the particular hanger, the exact loads carried by one hanger are not always clear. Sometimes, also, hardwood is employed in the tests, in order to avoid failure of the joist by crushing of the fibers. The Duplex hanger unquestionably has many advantages over other hangers. It is practically certain that all the other hangers will fail by the hooks over the girder crushing the fibers of the timber on the corner of the girder and then straightening out.

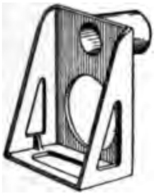


FIG. 135.—Duplex joist hanger.

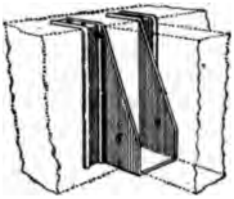


FIG. 136.—Van Dorn patented steel joist hanger.

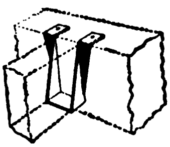


FIG. 137.—"Ideal" single hanger.

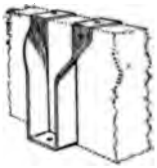


FIG. 138.—"Falls" joist hanger.

122c. Connection of Joist to Steel Girder.—When steel girders are used with timber floor joists, the types of connection are similar to those discussed for wooden girders, *i.e.*, the joists may frame on top of the steel girder (usually an I-beam) or into the side of the girder.

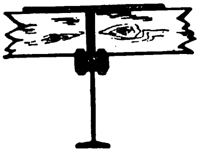


FIG. 139.

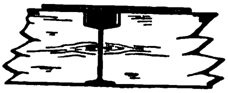


FIG. 140.

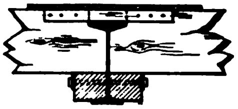


FIG. 141.

Buildings with this combination construction, in which the joists simply rest on top of the I-beams, without any attachment whatever, are sometimes seen. In such cases, the I-beam is supported laterally only by friction between the timber and steel. This practice is to be avoided. To secure a definite connection between the joists and girder, a wooden strip may be bolted to the top flange of the I-beam, and the joists toenailed to this wooden strip, as in Fig. 139. The principal objection to this construction is the weakening of the I-beams from the holes punched through the flange.

When the joists frame into the sides of the I-beams, they are often, for light loads, supported by the lower flanges of the I-beam, as in Fig. 140. Obviously the weak point of this detail is the small bearing of the joist on the steel. To overcome the difficulty, timbers may be cut to rest snugly against the flange and web, and bolted through the web. The joists may then be nailed into these timber strips, as illustrated in Fig. 141. The supporting timber should be of sufficient width to extend under and beyond the vertical cut of the notch in the joist for the upper flange.

A serious difficulty in constructions of this nature is the problem of supporting the flooring over the upper flange of the I-beam. If such flooring rests on the joists and the upper flange of the I-beam, the shrinkage of the joists will produce a high place in the floor over all the steel beams. To overcome this difficulty small strips, say of 1½ × 2-in. timber, may be spiked to the sides of the joists to carry the floor over the girder.

Joist hangers, notably the Duplex and Van Dorn hangers, may be obtained for connection between timber joists and steel girders (see Figs. 142, 143, and 144). The method of support shown in Fig. 141, however, will be found very satisfactory and generally cheaper than the joist hangers.

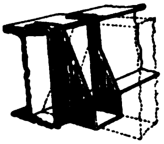


Fig. 142.—Van Dorn I-beam hanger.

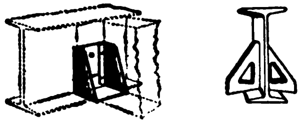


Fig 143.—Duplex I-beam hanger.

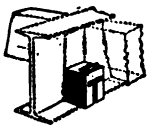


Fig. 144.—Duplex I-beam box.

123. Connections Between Columns and Girders.—The connection between timber columns and girders involves consideration, not only of strength of columns and of supports for the girders, but also of general stiffness of the building, since the posts and girders are generally counted upon to form the structural frames for resisting lateral forces, as wind and vibration of machinery. Columns always splice at or near the floor lines, hence the connection of girder to column includes the consideration of column splice. Continuity of the columns is always to be sought, both from the standpoint of stiffness and reduction of shrinkage. In total, the objects to be gained in the connection of girders and post are: (1) continuity of column for stiffness and reduction of shrinkage; (2) reduction of column area from a lower story to an upper story as determined by floor load; (3) sufficient bearing area for girders on the supports; (4) continuity of girders at the column for stiffness; and (5) provision for girders releasing from column, in event of a serious fire, without pulling the column down. All these provisions are not attainable in every case, and the nature of the building may not warrant the expense of securing all these objects.

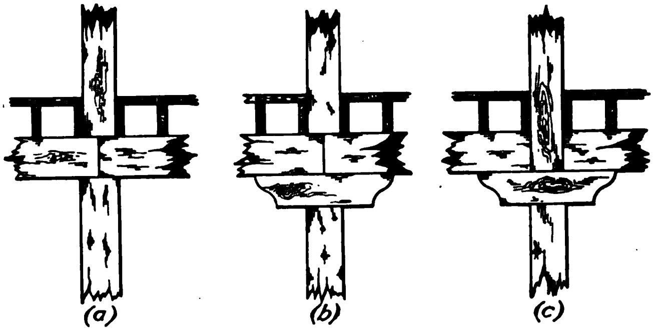


Fig. 145.—Defective details of column and girder connections.

In the discussion of this subject, a distinction must be made between the ordinary building, including both frame buildings and buildings with masonry walls, or corrugated steel walls, and the special type of building known as "mill construction" or "slow-burning construction" (see chapter on "Slow-Burning Mill Construction" in Sect. 3). The first class consists of those buildings which have the ordinary joist and girder construction, either with or without plastered ceilings and interior columns encased with lath and plaster. This class will be treated in the following paragraphs; the details for the special type of "mill construction" are discussed in Sect. 3.

For the purpose of illustrating these principles, some details of connection of columns and girders will be briefly discussed. Fig. 145 shows three defective details, which, nevertheless, are often seen. It is almost certain that in Fig. 145 (*a*) the girders have not sufficient bearing across the fibers, and that with full load, crushing will result. In (*b*) the bottom of the upper post will crush the fibers of the upper side of the girder, and a worse condition will prevail under the bolster, unless the latter is hardwood. Even then, if the posts are not working at a very low unit stress, crushing of the bolster will result. The shrinkage in both (*a*) and (*b*) will be considerable, and nearly double in (*b*) what it will be in (*a*). The detail of (*c*) with the upper post resting on a hardwood bolster is the best of the three details, although shrinkage has not been eliminated.

For many buildings, the details shown in Fig. 146 will provide satisfactory connections. All of the desirable conditions enumerated previously are fulfilled, with the exception of release of girders in case of fire. The vertical bolster blocks are set into the lower post and bolted, or bolted and keyed to the sides of the column with circular pins or with rectangular iron keys. In each of the three details, the girders may be given sufficient end bearing by properly proportioning the thickness of bolster block; the bolster has end bearing on the post, and no timber in cross bearing intervenes between the two sections of post. Partial continuity of post, sufficient for general stiffness of building, is secured by means of timber splice pads in detail (*c*), without sacrificing the girder ties. The splice plates of the girder across column may be of steel. This will avoid the use of wooden fillers under the girder splice pads. A further modification of these details to allow the girders to release in case of fire may be made by using dog-irons instead of the girder splice pads.

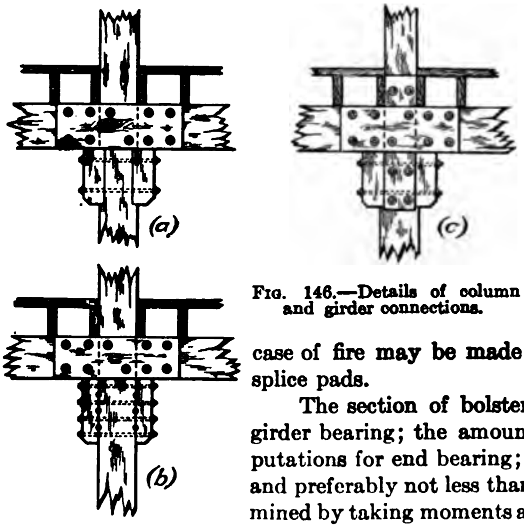


Fig. 146.—Details of column and girder connections.

The section of bolster is to be determined by requirements of girder bearing; the amount the bolster is set into the post by computations for end bearing; its length should be not less than 12 in., and preferably not less than 16 in. The size of bolts may be determined by taking moments about the center of the bearing on the post. The keyed and bolted bolster is proportioned as for the shear-pin tension splice.

Illustrative Problem.—Assume the problem of Art. 122*a*. Floor bay 14 × 16 ft., girders 8 × 14 in., joists 2 × 10 in., first story height 16 ft. Assume the detail to occur at the second floor of a four story building. The load in the upper column will be taken at 30,500 lb., the first story column will then take 30,500 lb. plus the second floor load. The live load will be 60% of 60 = 36 lb. per sq. ft., which, with a dead load of 12 lb. per sq. ft. will give a total unit load of 48 lb. per sq. ft., and a total increment of column load for the second floor of 10,800 lb. The first story column load will then be 41,300 lb. The upper column section will be made an 8 × 8-in., and the lower section a 10 × 10 in. The girder reaction is 6720 lb. (For design of girder and its connections, live load is 80 per cent. (60) = 48 lb. per sq. ft.) At 300 lb. per sq. in. the required bearing and thickness of bolster must be 22.5/7.5 = 3 in. The bolster size will be made 5½ × 9½ × 1 ft. 4 in.

The required area in end bearing is $\frac{6720}{1600} = 4.2$, or with a width of 9½ in. the bolster must be set into the post 4.2/9.5 = 0.44 in. Actually the dap will be made ¾ in. The upper bolts will be placed 3 in. below bottom of girder. Taking moments about the center of bearing of the bolster on the dap, and neglecting the lower bolts, $M = (6720)(2\frac{3}{4}) = 18{,}500$ in.-lb. This overturning moment will be resisted by compression of the lower portion of the bolster against the post, and tension in the two upper bolts. This pair of bolts is 13 in. above the seat of the bolster in the post, and the effective lever arm of these bolts may be taken at ¾ of their height above the bolster seat. The tension in either of the two bolts is then

$$p = \frac{18{,}500}{(2)(13)(\frac{3}{4})} = 950 \text{ lb.}$$

The maximum intensity of pressure between the bolster and post need not be investigated, as it will be very small with the length of bolster used.

Attention is called to the details of Fig. 146, in that the normal spacing of the joists has been modified at the posts, to bring a joist either side of the post. When these joists are either spiked or bolted to the post, and in addition a short piece of joist is spliced across the butt joint of the joists where such joint occurs at the post, a simple and inexpensive construction is secured which gives considerable stiffness to the building frame.

123a. Post and Girder Cap Connections.—The bolster connections above discussed are usually impractical to employ, if ceilings exist, as the bolster will project beneath the ceiling line. In such cases, and in other cases where the above construction may be deemed unsightly, metal post-caps of cast iron, wrought iron, or steel are used. Standard post-caps, usually of pressed steel, are made by the manufacturers of joist hangers, and may be purchased in stock sizes. Typical details of girder and post connections, using standard post-caps, are given in Figs. 147, 148, and 149 taken from manufacturers' catalogs. The prices of these caps based on the unit cost per pound of steel are rather high, and it may often be possible to build up structural post-caps that will give satisfaction at a lower cost. Sometimes short pieces of I-beams or heavy channels, unsuited on account of length for any other purpose, may be purchased cheaply, and used for post-caps for cases in which it is only necessary to frame girders into two opposite sides of the posts; in other words, in the case of a two-way connection.

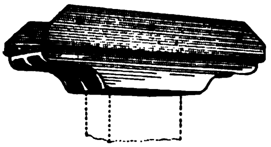


Fig. 147.—Duplex malleable iron and steel combination cap.

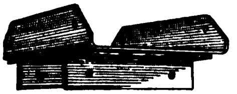


Fig. 148.—Ideal steel post cap, No. 3.

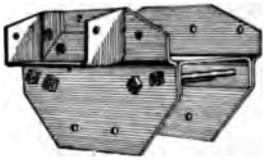


Fig. 149.—Duplex steel post cap.

A four-way post-cap is one which provides for beams on four sides of the posts. Four-way post-caps with joist and girder construction always result in unequal settlement of the floor. The joists, being supported on or by the girders, will settle an amount equal to the shrinkage in the depth of the girder, while the joists framing into the post and resting on the post-cap will not settle. The use of joist hangers between joist and girder will not do away with this settlement, although the use of that type of hanger which connects into the approximate center of the girder will reduce the settlement to that due to the shrinkage of one-half the depth of girder.

Cast-iron post-caps must be carefully designed to take care of the flexural stresses. A typical cast-iron post-cap is shown in Fig. 150.

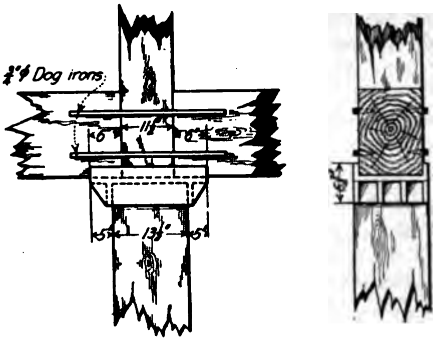


Fig. 150.—Details of column and girder connection with special cast-iron post cap.

Illustrative Problem.—Assume girder 12 × 16-in. on a 14-ft. span, upper story post 12 × 12 in. and lower story post 14 × 14 in. The actual section of sized girder will be 11½ × 15½. Using a working stress of 1800 lb. per sq. in., the safe load is 39,469 lb., say 40,000. The reaction is then 20,000 lb. At 300 lb. per sq. in., the required bearing area is $\frac{20,000}{300} = 67$ sq. in. With a width of 11½ in., the cap must have a seat $\frac{67}{11.5} = 5.8$ in. long, say 6 in., and will project 5 in. over the face of the 14 × 14-in. post. The moment on the post-cap may be assumed to be a maximum at the edge of the upper story post, with a value $M = (20,000)(3) = 60,000$ in.-lb. For cast iron, the working unit stress in flexure will be taken at 4000 lb. per sq. in. The required section modulus of cap must therefore be $\frac{60,000}{4000} = 15$. The sides of cap form two beams of rectangular section resisting this moment. Assuming a thickness of metal of 1 in., the depth of side must be $d = \sqrt{(7\frac{1}{2})(6)} = 6\frac{3}{4}$ in. The thickness of seat must now be computed. With a uniform bearing, the seat may be computed as a beam with fixed ends, or $M = (\frac{1}{12})(Wl)$; the projecting width of plate is 5 in. The load on this portion is $\frac{5}{6} \times 20,000 = 16,667$ lb. The length will be taken at 12½ in., or between the centers of sides. Therefore $M = (\frac{1}{12})(16,667)(12\frac{1}{2}) = 17,360$ in.-lb. The section modulus required is $\frac{16,667}{4000} = 4.34$. The width being 5 in., the depth must be $d = \sqrt{\frac{6}{5}(4.34)}$

= 2.28 in. The base must therefore be supported by ribs. Two ribs will be introduced. The bearing plate will now be assumed to take only one-half of the bending, one-half the load being transmitted by the ribs to the vertical collar around the post. The thickness of base and collar must then be sufficient for each to sustain 6650 in.-lb. Since both the projecting seat and the collar are fixed along one edge, the allowable unit stress in bending will be increased 50%. The required section modulus is then $\frac{8333}{6000} = 1.39$, or with a width of 5 in., the required thickness is 1.29. A thickness of 1¼ in. will be used.

SPLICES AND CONNECTIONS—STEEL MEMBERS

By Wm. J. Fuller

124. Rivets and Bolts.—A rivet is a short piece of cylindrical rod (usually soft steel) with one end, called the head, larger than the body or shank (see Fig. 151). Rivets are made by feeding rods, that have been heated to the proper temperature, into a rivet machine. The machine forms the head and cuts the rod off to the desired length. Different kinds of rivets may be made in the same machine by using the proper header and dies. To produce satisfactory rivets the dies used must be kept in perfect condition, and the bars must be heated to the proper temperature. If the dies become worn, the rivet is apt to have a shoulder where the head and shank meet (see Fig. 152). Also, if the inner edges of the dies do not meet, the rivet will have what is known as a fin on each side (see Fig. 153). Rivets having these defects are not satisfactory when driven, as the heads will not fit tight against the member.

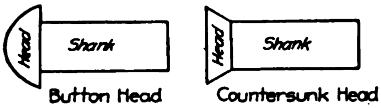


FIG. 151.

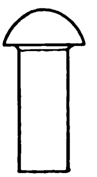


FIG. 152.

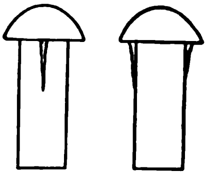


FIG. 153.

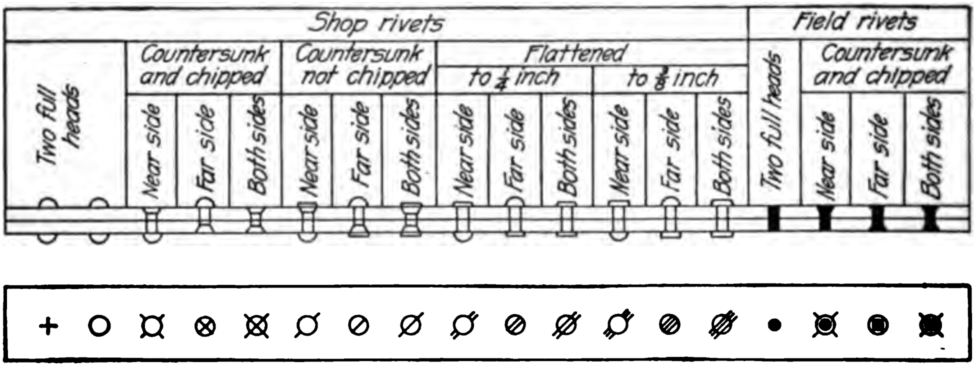


FIG. 154.—Conventional rivet signs.

Rivets are used not only to connect the different parts of built-up steel sections, such as columns and girders, but also for making the connections between different structural members.

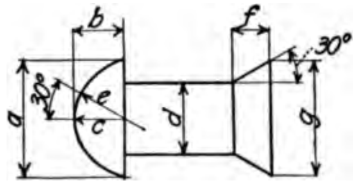
124a. Kinds, Dimensions, and Sizes of Rivets. *Kinds.*—Two classes of rivets are used in structural steel work: namely, the *button head* and the *countersunk head* (see Fig. 151). The button head rivet, which is used almost entirely for all structural work, has a head which is hemispherical. The countersunk head is flat and is made to fit a countersunk hole. It should not be used except when a flat surface is desired or when a button head would interfere with some member. When the desired clearance cannot be obtained because of a full button head on a rivet, the head of the rivet may be flattened. Sufficient clearance, of course, cannot always be provided in this way, but where the flattening of a button head is all that is necessary, the riveting is usually more efficient and less expensive than if a countersunk rivet were used. In case a flat surface is desired, it is necessary to chip the head of a countersunk rivet, since after driving, this kind of a head extends about 1/8 in. above the surface.

In order to show on a drawing whether a full button head, a flattened head, or a countersunk head is to be used, certain conventional signs have been adopted. Fig. 154 shows the Osborne system which is used almost entirely in this country.

Dimensions.—There is no standard shape for rivet heads, but the shapes found on the market do not differ greatly. Rivets are sometimes made with special shaped heads such that when driven with the proper die the tendency will be to first upset the shank. This is desirable as the hole should be completely filled even though somewhat irregular. Table 1 gives dimensions for finished rivet heads.

TABLE 1.[1]—GENERAL FORMULAS FOR PROPORTIONS OF RIVETS, IN INCHES

Full driven head, diameter a = $1.5d + 1/8$ in.
depth b = $0.425a$
radius c = b
radius e = $1.5b$
Countersunk head, depth f = $0.5d$
diameter g = $1.577d$.



All Dimensions in Inches

d	a	$b=c$	e	f	g
3/8	11/16	19/64	7/16	3/16	19/32
1/2	7/8	3/8	9/16	1/4	25/32
5/8	1 1/16	29/64	43/64	5/16	1
3/4	1 1/4	17/32	51/64	3/8	1 3/16
7/8	1 7/16	39/64	59/64	7/16	1 3/8
1	1 5/8	11/16	1 1/32	1/2	1 9/16
1 1/8	1 13/16	49/64	1 5/32	9/16	1 3/4
1 1/4	2	27/32	1 9/32	5/8	1 15/16

Sizes.—Rivets vary from 3/8 to 1 1/4 in. in diameter and, except in special cases, are made from soft steel. Most structural work requires either 3/4 or 7/8-in. rivets. Smaller sizes are used in light work while larger sizes are used only in very heavy construction.

As a general rule rivets should not be of less diameter than the thickness of the thickest plate through which they pass.

The diameter of a rivet should not be greater than 1/4 of the width of member connected.

Rivets as large as 7/8 in. should not be used if they are to be driven by hand, as they cannot be driven tight. (All shops do not have the required power to drive the larger rivets properly.)

The diameter of a rivet should not be less than 1/4 of its grip as tests show that the strength of a joint decreases when the total thickness of metal increases beyond four diameters of the rivet used. In such cases specifications usually require the number of rivets to be increased 1 % for each 1/16 in. of metal greater than four diameters.

The size of rivet that should be used in any given case depends on the sizes of the members to be connected. As a general rule, a 5/8-in. rivet is the maximum that should be used in the flanges of 6 and 7-in. channels and I-beams, and in 2-in. angles; 3/4-in. rivets may be used in all larger sized channels and I-beams and in all angles over 2 1/2 in. In all I-beams over 15 in., all channels over 10 in., and in all angles over 3 in., 7/8-in. rivets may be used. In unimportant connections, 3/4-in. rivets may be used in 2 1/2-in. angles, and 7/8-in. rivets may be used in 3-in. angles.

Not more than one size of rivet should be used in the same structure in order to avoid making changes in the punching and riveting machines and also to make unnecessary the rehandling of the different members.

Channels and I-beams, however, have to be rehandled when holes are punched in both the flange and web because a special die is required in punching the flange on account of the slope. In cases of this kind, when the holes in the web are larger than are permitted in the flange, a smaller punch may be used for the flange without causing extra handling.

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

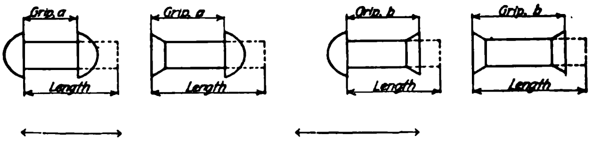
124b. Grip of Rivets and Bolts.—The grip of a rivet is the total thickness of metal through which it passes (see Fig. 155). In computing the length of shank required, the roughness of the parts connected should be considered and the grip increased accordingly. The amount to be added varies in different shops and is from 1/32 in. for each joint between members to 1/16 in. for each member. Thus, the total length of shank is the thickness of material plus the amount assumed for roughness of members plus the length of shank necessary to form a head. The grip should be taken to the nearest 1/8 in. Table 2 gives the required length of shank for different grips and sizes of rivets.

TABLE 2.[1]—STRUCTURAL RIVETS

American Bridge Company Standard

LENGTHS OF FIELD RIVETS FOR VARIOUS GRIPS

(Dimensions in Inches)



Grip	Diameter					Grip	Diameter				
a	½	⅝	¾	⅞	1	b	½	⅝	¾	⅞	1
½	1½	1¾	1⅞	2	2⅛	½	1⅛	1¼	1¼	1⅜	1⅜
⅝	1⅝	1⅞	2	2⅛	2¼	⅝	1¼	1⅜	1⅜	1½	1½
¾	1¾	2	2⅛	2¼	2⅜	¾	1⅜	1½	1½	1⅝	1⅝
⅞	1⅞	2⅛	2¼	2⅜	2½	⅞	1½	1⅝	1⅝	1¾	1¾
1	2	2¼	2⅜	2½	2⅝	1	1⅝	1¾	1¾	1⅞	1⅞
⅛	2⅛	2⅜	2½	2⅝	2¾	⅛	1¾	1⅞	1⅞	2	2
¼	2¼	2½	2⅝	2¾	2⅞	¼	1⅞	2	2	2⅛	2⅛
⅜	2⅜	2⅝	2¾	2⅞	3	⅜	2	2⅛	2⅛	2¼	2¼
½	2⅝	2⅞	3	3⅛	3¼	½	2⅛	2¼	2⅜	2⅜	2½
⅝	2¾	3	3⅛	3¼	3⅜	⅝	2¼	2⅜	2½	2½	2⅝
¾	3	3¼	3⅜	3½	3⅝	¾	2½	2⅝	2¾	2¾	2⅞
⅞	3⅛	3⅜	3½	3⅝	3¾	⅞	2⅝	2¾	2⅞	2⅞	3
2	3¼	3½	3⅝	3¾	3⅞	2	2¾	2⅞	3	3	3⅛
⅛	3⅜	3⅝	3¾	3⅞	4	⅛	2⅞	3	3⅛	3⅛	3¼
¼	3½	3¾	3⅞	4	4⅛	¼	3	3⅛	3¼	3¼	3⅜
⅜	3⅝	3⅞	4	4⅛	4¼	⅜	3⅛	3¼	3⅜	3⅜	3½
½	3¾	4	4⅛	4¼	4⅜	½	3¼	3⅜	3½	3½	3⅝
⅝	3⅞	4⅛	4¼	4⅜	4½	⅝	3⅜	3½	3⅝	3⅝	3¾
¾	4	4¼	4⅜	4½	4⅝	¾	3½	3⅝	3¾	3¾	3⅞
⅞	4⅛	4⅜	4½	4⅝	4¾	⅞	3⅝	3¾	3⅞	3⅞	4
3	4⅜	4⅝	4¾	4⅞	5	3	3⅞	4	4	4⅛	4¼
⅛	4½	4¾	4⅞	5	5⅛	⅛	4	4⅛	4⅛	4¼	4⅜
¼	4⅝	4⅞	5	5⅛	5¼	¼	4⅛	4¼	4¼	4⅜	4½
⅜	4¾	5	5⅛	5¼	5⅜	⅜	4¼	4⅜	4⅜	4½	4⅝
½	4⅞	5⅛	5¼	5⅜	5½	½	4⅜	4½	4½	4⅝	4¾
⅝	5	5¼	5⅜	5½	5⅝	⅝	4½	4⅝	4⅝	4¾	4⅞
¾	5⅛	5⅜	5½	5⅝	5¾	¾	4⅝	4¾	4¾	4⅞	5
⅞	5¼	5½	5⅝	5¾	5⅞	⅞	4¾	4⅞	4⅞	5	5⅛
4	5⅜	5⅝	5¾	5⅞	6	4	4⅞	5	5	5⅛	5¼
⅛	5⅝	5⅞	6	6⅛	6¼	⅛	5⅛	5¼	5¼	5⅜	5½
¼	5¾	6	6⅛	6¼	6⅜	¼	5¼	5⅜	5⅜	5½	5⅝
⅜	6	6¼	6⅜	6½	6⅝	⅜	5½	5⅝	5⅝	5⅝	5¾
½	6⅛	6⅜	6½	6⅝	6¾	½	5⅝	5¾	5¾	5¾	5⅞
⅝	6¼	6½	6⅝	6¾	6⅞	⅝	5¾	5⅞	5⅞	5⅞	6
¾	6⅜	6⅝	6¾	6⅞	7	¾	5⅞	6	6	6	6⅛
⅞	6½	6¾	6⅞	7	7⅛	⅞	6	6⅛	6⅛	6⅛	6¼
5	6⅝	6⅞	7	7⅛	7¼	5	6⅛	6¼	6¼	6¼	6⅜
⅛			7⅛	7¼	7⅜	⅛			6⅜	6⅜	6½
¼			7¼	7⅜	7½	¼			6½	6½	6⅝
⅜			7⅜	7½	7⅝	⅜			6⅝	6⅝	6¾
½			7⅝	7¾	7⅞	½			6⅞	6⅞	7
⅝			7¾	7⅞	8	⅝			7	7	7⅛
¾			7⅞	8	8⅛	¾			7⅛	7⅛	7¼
⅞			8	8⅛	8¼	⅞			7¼	7¼	7⅜

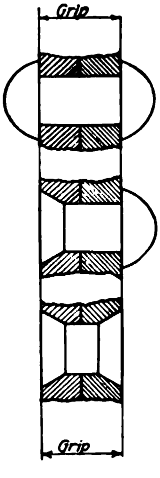


FIG. 155

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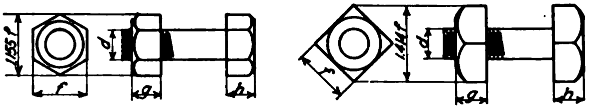
[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

In case bolts are used, the length is the grip, plus ¼ in., plus the thickness of nut, plus the thickness of washers. Table 3 gives the dimensions for bolt heads and nuts.

TABLE 3.[1]—BOLT HEADS AND NUTS
American Bridge Company Standard



Rough nut		Finished nut		Rough head		Finished head	
f	g	f	g	f	h	f	h
$1.5d + \frac{1}{8}$ in.	d	$1.5d + \frac{1}{16}$ in.	$d - \frac{1}{16}$ in.	$1.5d + \frac{1}{8}$	$0.5f$	$1.5d + \frac{1}{16}$ in.	$0.5f - \frac{1}{16}$ in.

124c. Rivet Holes.—Rivet holes may be punched to size, sub-punched, and reamed, or drilled from the solid. For all ordinary work satisfactory results can be obtained if a reasonable amount of care is taken in laying out and punching the holes. All holes should be $\frac{1}{16}$ in. larger in diameter than the nominal size of rivet used; that is, $\frac{1}{16}$ in. larger than the diameter of the rivet shank before heating. This will allow the heated rivet to enter the hole.

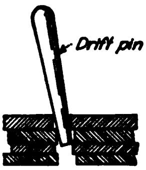


FIG. 156.

When metal ¾ in. thick or more, is used, or when the thickness of metal is greater than the diameter of the rivet, the holes should be drilled (1) because punches often break when the thickness of metal is greater than the diameter of the punch, and (2) because the punching of the holes injures the metal more or less around the edge of the hole, the thicker and harder the metal, the greater the injury. It is on account of this injury that holes are specified on important work to be sub-punched ⅛ in. less than the diameter of the rivet and reamed to $\frac{1}{16}$ in. larger, or to be drilled from the solid. When holes are sub-punched and reamed, the reaming is usually specified to be done after the structure is assembled, thus insuring well matched holes.

Punched holes do not always match and in such cases a reamer should be used to line them up instead of using a drift pin (see Fig. 156) and a sledge hammer as is often done. Although drift pins (which are tapering circular steel tempered rods) are necessary in assembling, yet their use in lining up holes, which do not match, should not be allowed because of the injurious effect on the metal around the holes. Reaming out holes which do not match should not be considered as reamed work because only part of the metal in part of the holes is removed.

Holes for countersunk rivets are punched or drilled in the same way as for button head rivets; the hole is then countersunk—that is, reamed out on a bevel to the required depth.

124d. Location of Rivets—*Gage.*—A gage line is a line parallel to the length of a member on which open holes or rivets are located. Gage is the distance between gage lines or the distance of a gage line from some surface, such as the back of an angle or channel. Fig. 157 shows both the gage and gage lines on an angle. Tables 4, 5, and 6 give the standard gages for I-beams, angles, and channels, respectively. The dimensions of channels and I-beams as manufactured by the different companies vary slightly; also the gages as given in the different manufacturers' handbooks.

Pitch.—Pitch is the distance center to center of holes on a gage line, and is indicated by p on Fig. 157.

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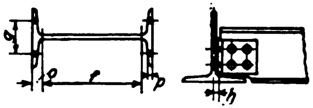
[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

TABLE 4.[1]—STANDARD GAGES AND DIMENSIONS FOR BEAMS



(Nominal dimensions are: flange width and "o" in eighths, web thickness in sixteenths. Gages for connection angles are determined by ½ web thickness. Standard gages may be varied if conditions require.)

Depth of beam	Weight per foot	Flange width	Web thickness	½ web thickness	Gage g	Grip p	Distance f	Distance o	Distance h	Max. rivet in flange
In.	Lbs.	In.	In.	In.	In.	In.	In.	In.	In.	In.
27	90.0	9	1/2	1/4	4	3/4	22 1/2	2 1/4	5/16	7/8
24	115.0	8	3/4	3/8	4	1 1/8	20 1/4	1 7/8	7/16	7/8
	110.0	8	11/16	3/8	4	1 1/8	20 1/4	1 7/8	7/16	
	105.0	7 7/8	5/8	5/16	4	1 1/8	20 1/4	1 7/8	3/8	
24	100.0	7 1/4	3/4	3/8	4	7/8	20 3/4	1 5/8	7/16	7/8
	95.0	7 1/4	11/16	3/8	4	7/8	20 3/4	1 5/8	7/16	
	90.0	7 1/8	5/8	5/16	4	7/8	20 3/4	1 5/8	3/8	
	85.0	7 1/8	9/16	5/16	4	7/8	20 3/4	1 5/8	3/8	
	80.0	7	1/2	1/4	4	7/8	20 3/4	1 5/8	5/16	
24	74.0	9	1/2	1/4	4	5/8	20	2	5/16	7/8
21	60.5	8 1/4	7/16	3/16	4	9/16	17 1/2	1 3/4	1/4	7/8
20	100.0	7 1/4	7/8	7/16	4	1	16 1/2	1 3/4	1/2	7/8
	95.0	7 1/4	13/16	7/16	4	1	16 1/2	1 3/4	1/2	
	90.0	7 1/8	3/4	3/8	4	1	16 1/2	1 3/4	7/16	
	85.0	7 1/8	11/16	3/8	4	1	16 1/2	1 3/4	3/8	
	80.0	7	5/8	5/16	4	1	16 1/2	1 3/4	3/8	
20	75.0	6 3/8	11/16	5/16	4	3/4	17	1 1/2	3/8	7/8
	70.0	6 3/8	9/16	5/16	4	3/4	17	1 1/2	3/8	
	65.0	6 1/4	1/2	1/4	4	3/4	17	1 1/2	5/16	
18	90.0	7 1/4	13/16	7/16	4	1	14 1/2	1 3/4	1/2	7/8
	85.0	7 1/8	3/4	3/8	4	1	14 1/2	1 3/4	7/16	
	80.0	7 1/16	5/8	5/16	4	1	14 1/2	1 3/4	3/8	
	75.0	7	9/16	5/16	4	1	14 1/2	1 3/4	3/8	
18	70.0	6 1/4	3/4	3/8	3 3/4	3/4	15 1/4	1 3/8	7/16	7/8
	65.0	6 1/4	5/8	5/16	3 3/4	3/4	15 1/4	1 3/8	3/8	
	60.0	6 1/8	9/16	5/16	3 3/4	3/4	15 1/4	1 3/8	3/8	
	55.0	6	1/2	1/4	3 3/4	3/4	15 1/4	1 3/8	5/16	
12	55.0	5 5/8	13/16	7/16	3 1/2	3/4	9 1/4	1 3/8	1/2	3/4
	50.0	5 1/2	11/16	3/8	3 1/2	5/8	9 1/4	1 3/8	7/16	
	45.0	5 3/8	9/16	5/16	3	3/4	9 1/4	1 3/8	3/8	
	40.0	5 1/4	1/2	1/4	3	3/4	9 1/4	1 3/8	5/16	
12	35.0	5 1/8	7/16	1/4	3	9/16	9 3/4	1 1/8	5/16	3/4
	31.5	5	3/8	3/16	3	9/16	9 3/4	1 1/8	1/4	
12	28.0	6	5/16	1/8	3	7/16	9 1/2	1 1/4	3/16	3/4
10	40.0	5 1/8	3/4	3/8	2 3/4	1/2	8	1	7/16	3/4
	35.0	5	5/8	5/16	2 3/4	1/2	8	1	3/8	
	30.0	4 7/8	1/2	1/4	2 3/4	1/2	8	1	5/16	
	25.0	4 5/8	5/16	3/16	2 3/4	1/2	8	1	1/4	
10	22.25	5 1/2	1/4	1/8	2 3/4	3/8	7 3/4	1 1/8	3/16	3/4
9	35.0	4 3/4	3/4	3/8	2 1/2	1/2	7	1	7/16	3/4
	30.0	4 5/8	9/16	5/16	2 1/2	1/2	7	1	3/8	
	25.0	4 1/2	7/16	3/16	2 1/2	1/2	7	1	1/4	
	21.0	4 3/8	5/16	1/8	2 1/2	1/2	7	1	3/16	
8	25.5	4 1/4	9/16	5/16	2 1/4	1/2	6 1/4	7/8	5/16	3/4
	23.0	4 1/4	7/16	1/4	2 1/4	7/16	6 1/4	7/8	5/16	
	20.5	4 1/8	3/8	3/16	2 1/4	7/16	6 1/4	7/8	1/4	
	18.0	4	1/4	1/8	2 1/4	7/16	6 1/4	7/8	3/16	
8	17.5	5	1/4	1/8	2 1/4	3/8	6	1	3/16	3/4
7	20.0	3 7/8	1/2	1/4	2 1/4	3/8	5 1/4	7/8	5/16	5/8
	17.5	3 3/4	3/8	3/16	2 1/4	3/8	5 1/4	7/8	1/4	
	15.0	3 5/8	1/4	1/8	2 1/4	3/8	5 1/4	7/8	3/16	
6	17.25	3 5/8	1/2	1/4	2	3/8	4 1/2	3/4	5/16	5/8
	14.75	3 1/2	3/8	3/16	2	3/8	4 1/2	3/4	1/4	
	12.25	3 3/8	1/4	1/8	2	3/8	4 1/2	3/4	3/16	

18	48.0	7⅛	⅝	3/16	3¾	½	14¾	1⅝	¼	⅞
15	75.0	6⅜	⅞	7/16	3½	⅞	11¾	1⅝	½	¾
	70.0	6¼	13/16	⅜	3½	⅞	11¾	1⅝	7/16	
	65.0	6⅛	11/16	⅜	3½	⅞	11¾	1⅝	7/16	
	60.0	6	⅝	5/16	3½	⅞	11¾	1⅝	⅜	
15	55.0	5¾	11/16	5/16	3½	⅝	12½	1¼	⅜	¾
	50.0	5⅝	9/16	5/16	3½	⅝	12½	1¼	⅜	
	45.0	5⅝	½	¼	3½	⅝	12½	1¼	5/16	
	42.0	5½	7/16	3/16	3½	⅝	12½	1¼	¼	
15	37.5	6¾	5/16	3/16	3½	7/16	12¼	1⅜	¼	¾

5	14.75	3⅜	½	¼	1¾	⅜	3½	¾	5/16	½
	12.25	3⅛	⅜	3/16	1¾	⅜	3½	¾	¼	
	9.75	3	¼	⅛	1¾	⅜	3½	¾	3/16	
4	10.5	2⅞	7/16	¼	1½	5/16	2¾	⅝	¼	½
	9.5	2⅞	⅜	3/16	1½	5/16	2¾	⅝	¼	
	8.5	2¾	¼	⅛	1½	5/16	2¾	⅝	3/16	
	7.5	2⅝	3/16	⅛	1½	5/16	2¾	⅝	3/16	
3	7.5	2½	⅜	3/16	1½	5/16	1¾	⅝	¼	⅜
	6.5	2½	¼	⅛	1½	5/16	1¾	⅝	3/16	
	5.5	2⅜	3/16	⅛	1½	5/16	1¾	⅝	⅛	

Edge Distance.—The distance from a hole or rivet to the edge of a member is called the edge distance (see Fig. 157).

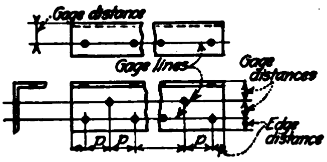
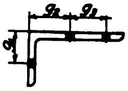


FIG. 157.

TABLE 5.[1]—GAGES FOR ANGLES



Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1	¾
g_1	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝	½
g_2	3	2½	2½	2											
g_3	3	3	2¼	1¾											
Max. rivet	1⅛	1	⅞	⅞	⅞	⅞	⅞	¾	⅝	½	⅜	⅜	⅜	¼	¼

For column details, 6-in. leg (½ in. thick or less) against column shaft, g_2 = 1¾ in., g_3 = 3 in.

For diagonal angles, etc., gage in middle, where riveted leg equals or exceeds 3 in. for ¾-in. rivets, 3½ in. for ⅞-in. rivets.

Use special gages to adapt work to multiple punch, or to secure desirable details.

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co, Pittsburgh, Pa.

[1] From Pocket Companion, 20th edition, Carnegie Steel Co, Pittsburgh, Pa.

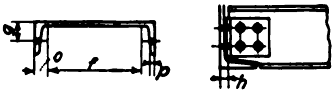

TABLE 6.[1]—STANDARD GAGES AND DIMENSIONS FOR CHANNELS

Nominal dimensions are: flange width and "o" in eighths, web thickness in sixteenths. Gages for connection angles are determined by ½ web thickness.

Standard gages may be varied if conditions require.

Depth of channel (inches)	Weight per foot (pounds)	Flange width (inches)	Web thickness (inches)	½ web thickness (inches)	Gage g (inches)	Grip p (inches)	Distance f (in.)	Distance o (in.)	Distance h (in.)	Max. rivet in flange (inches)
15	55.0	3 7/8	13/16	7/16	2 1/2	11/16	12 1/4	1 3/8	7/8	7/8
	50.0	3 3/4	3/4	3/8	2 1/2	11/16	12 1/4	1 3/8	13/16	
	45.0	3 5/8	5/8	5/16	2	5/8	12 1/4	1 3/8	11/16	
	40.0	3 1/2	1/2	1/4	2	5/8	12 1/4	1 3/8	9/16	
	35.0	3 1/2	7/16	1/4	2	5/8	12 1/4	1 3/8	1/2	
	33.0	3 3/8	7/16	3/16	2	5/8	12 1/4	1 3/8	1/2	
13	50.0	4 3/8	13/16	3/8	3	9/16	10 1/4	1 3/8	7/16	7/8
	45.0	4 1/4	11/16	5/16	2 3/4	9/16	10 1/4	1 3/8	3/8	
	40.0	4 1/4	9/16	5/16	2 3/4	9/16	10 1/4	1 3/8	3/8	
	37.0	4 1/8	1/2	1/4	2 1/2	9/16	10 1/4	1 3/8	5/16	
	35.0	4 1/8	7/16	1/4	2 1/2	9/16	10 1/4	1 3/8	5/16	
	32.0	4	3/8	3/16	2 1/2	9/16	10 1/4	1 3/8	1/4	
12	40.0	3 1/2	3/4	3/8	2	5/8	10	1	13/16	7/8
	35.0	3 3/8	5/8	5/16	2	5/8	10	1	11/16	
	30.0	3 1/4	1/2	1/4	1 3/4	1/2	10	1	9/16	
	25.0	3 1/8	3/8	3/16	1 3/4	1/2	10	1	7/16	
	20.5	3	5/16	1/8	1 3/4	1/2	10	1	3/8	
10	35.0	3 1/4	13/16	7/16	1 3/4	1/2	8 1/4	7/8	7/8	3/4
	30.0	3 1/8	11/16	3/8	1 3/4	1/2	8 1/4	7/8	3/4	
	25.0	2 7/8	1/2	1/4	1 3/4	1/2	8 1/4	7/8	9/16	
	20.0	2 3/4	3/8	3/16	1 1/2	7/16	8 1/4	7/8	7/16	
	15.0	2 5/8	1/4	1/8	1 1/2	7/16	8 1/4	7/8	5/16	
9	25.0	2 7/8	5/8	5/16	1 1/2	1/2	7 1/4	7/8	11/16	3/4
	20.0	2 5/8	7/16	1/4	1 1/2	1/2	7 1/4	7/8	1/2	
	15.0	2 1/2	5/16	3/16	1 3/8	7/16	7 1/4	7/8	3/8	
	13.25	2 1/2	1/4	1/8	1 3/8	7/16	7 1/4	7/8	5/16	
8	21.25	2 5/8	5/8	5/16	1 1/2	7/16	6 1/4	7/8	11/16	3/4
	18.75	2 1/2	1/2	1/4	1 1/2	7/16	6 1/4	7/8	9/16	
	16.25	2 1/2	7/16	3/16	1 1/2	7/16	6 1/4	7/8	1/2	
	13.75	2 3/8	5/16	3/16	1 3/8	3/8	6 1/4	7/8	3/8	
	11.25	2 1/4	1/4	1/8	1 3/8	3/8	6 1/4	7/8	5/16	
7	19.75	2 1/2	5/8	5/16	1 1/2	7/16	5 1/2	3/4	11/16	5/8
	17.25	2 1/2	1/2	1/4	1 1/2	7/16	5 1/2	3/4	9/16	
	14.75	2 3/8	7/16	1/4	1 1/4	7/16	5 1/2	3/4	1/2	
	12.25	2 1/4	5/16	3/16	1 1/4	3/8	5 1/2	3/4	3/8	
	9.75	2 1/8	1/4	1/8	1 1/4	3/8	5 1/2	3/4	5/16	
6	15.5	2 1/4	9/16	5/16	1 3/8	3/8	4 1/2	3/4	5/8	5/8
	13.0	2 1/4	7/16	1/4	1 3/8	3/8	4 1/2	3/4	1/2	
	10.5	2 1/8	5/16	3/16	1 1/8	3/8	4 1/2	3/4	3/8	
	8.0	2	3/16	1/8	1 1/8	5/16	4 1/2	3/4	1/4	
5	11.5	2 1/8	1/2	1/4	1 1/8	5/16	3 3/4	5/8	9/16	1/2
	9.0	1 7/8	5/16	3/16	1 1/8	5/16	3 3/4	5/8	3/8	
	6.5	1 3/4	3/16	1/8	1 1/8	5/16	3 3/4	5/8	1/4	
4	7.25	1 3/4	5/16	3/16	1	5/16	2 3/4	5/8	3/8	1/2
	6.25	1 5/8	1/4	1/8	1	5/16	2 3/4	5/8	5/16	
	5.25	1 5/8	3/16	1/8	1	5/16	2 3/4	5/8	1/4	
3	6.0	1 5/8	3/8	3/16	7/8	1/4	1 3/4	5/8	7/16	1/2
	5.0	1 1/2	1/4	1/8	7/8	1/4	1 3/4	5/8	5/16	
	4.0	1 1/2	3/16	1/8	7/8	1/4	1 3/4	5/8	1/4	

Rivet Spacing.—Rivets are spaced according to rules which have been derived from experience and the following may be considered as standard:

The minimum distance between centers of rivet holes is usually specified to be not less than three diameters of the rivet; but the distance shall preferably be not less than 3 in. for $\frac{7}{8}$-in. rivets, $2\frac{1}{2}$ in. for $\frac{3}{4}$-in. rivets, 2 in. for $\frac{5}{8}$-in. rivets, and $1\frac{3}{4}$ in. for $\frac{1}{2}$-in. rivets (see Table 7). The maximum pitch in the line of stress for members composed of plates and shapes

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

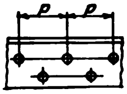
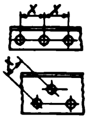


Fig. 158.

is sometimes specified to be 16 times the thickness of the thinnest outside plate with a maximum of 6 in. The following spacing is preferable: 6 in. for ⅞-in. rivets, 5 in. for ¾-in. rivets, 4½ in. for ⅝-in. rivets, and 4 in. for ½-in. rivets.

Table 7.—Minimum Rivet Spacing—All Dimensions in Inches



Diameter of rivet	½	⅝	¾	⅞
"x" minimum	1¼	1½	1⅞	2¼
"x" preferable	1¾	2	2½	3

For angles, in built sections, with two gage lines, with rivets staggered, the maximum pitch p (see Fig. 158) in each line may be twice as great as given above. Table 8 may be used in spacing rivets on two gage lines. The accompanying diagram[1] (Fig. 159) by Louis Metzger, C. E., may be used for the same purpose.

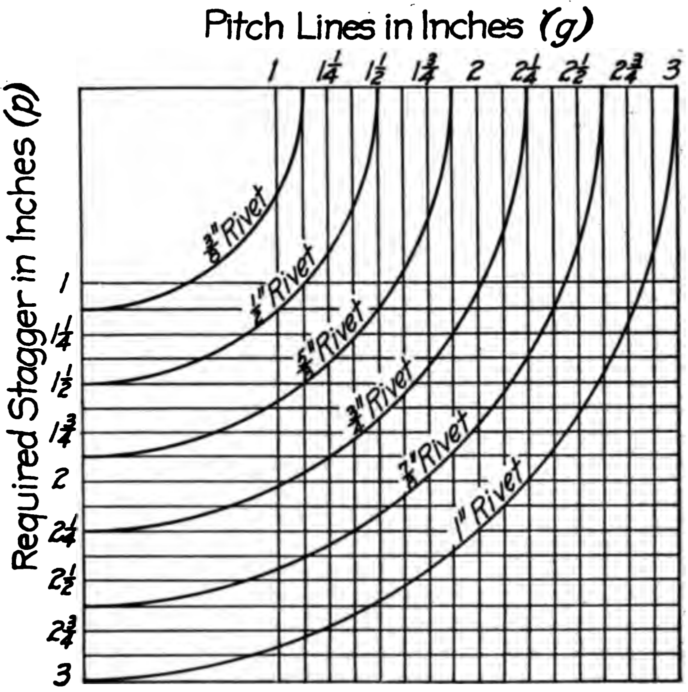


Fig. 159.

Illustrative Problem.—Suppose that g in Fig. 160 is 2 in., what is the minimum value of p that can be used for ⅞-in. rivets?

Table 8 shows that for $g = 2$ in., p should be 1¾ in.

Fig. 159 shows that for $g = 2$ in., p should be 1^{11}⁄$_{16}$ in. A value of 1¾ in. would be used, as rivet spacing should not be given in 16ths when it is possible to avoid it.

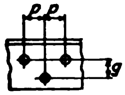


Fig. 160.

When two or more plates are in contact, rivets not more than 12 in. apart in either direction should be used to hold the plates together.

The minimum distance from the center of any rivet hole to a sheared edge should not be less than 1½ in. for ⅞-in. rivets, 1¼ in. for ¾-in. rivets, 1⅛ in. for ⅝-in. rivets,

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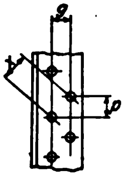
[1] *Eng. Rec.*, Jan. 11, 1913.

and 1 in. for ½-in. rivets; and to a rolled edge 1¼, 1⅛, 1 and ⅞ in. respectively. The maximum distance from any edge should not be greater than eight times the thickness of the plate.

The pitch of rivets at the ends of built compression members should not exceed four diameters of the rivets for a distance equal to one and one-half times the maximum width of the member.

TABLE 8.[1]—DISTANCE CENTER TO CENTER OF STAGGERED RIVETS

(Values of x for varying values of g and p)



p (in.)	g (inches)													
	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4	2 3/8	2 1/2
1 1/8	1 7/16	1 1/2	1 9/16	1 11/16	1 3/4	1 7/8	2	2 1/16	2 3/16	2 5/16	2 3/8	2 1/2	2 5/8	2 3/4
1 1/4	1 9/16	1 5/8	1 11/16	1 3/4	1 7/8	1 15/16	2 1/16	2 1/8	2 1/4	2 3/8	2 7/16	2 9/16	2 11/16	2 13/16
1 3/8	1 5/8	1 11/16	1 3/4	1 7/8	1 15/16	2	2 1/8	2 3/16	2 5/16	2 7/16	2 1/2	2 5/8	2 3/4	2 7/8
1 1/2	1 3/4	1 13/16	1 7/8	1 15/16	2	2 1/8	2 3/16	2 5/16	2 3/8	2 1/2	2 5/8	2 11/16	2 13/16	2 15/16
1 5/8	1 7/8	1 7/8	2	2 1/16	2 1/8	2 3/16	2 5/16	2 3/8	2 1/2	2 9/16	2 11/16	2 3/4	2 7/8	3
1 3/4	1 15/16	2	2 1/16	2 1/8	2 3/16	2 5/16	2 3/8	2 7/16	2 9/16	2 5/8	2 3/4	2 7/8	2 15/16	3 1/16
1 7/8	2 1/16	2 1/8	2 3/16	2 1/4	2 5/16	2 3/8	2 1/2	2 9/16	2 5/8	2 3/4	2 13/16	2 15/16	3	3 1/8
2	2 3/16	2 1/4	2 5/16	2 3/8	2 7/16	2 1/2	2 9/16	2 5/8	2 3/4	2 13/16	2 15/16	3	3 1/8	3 3/16
2 1/8	2 5/16	2 5/16	2 3/8	2 7/16	2 1/2	2 5/8	2 11/16	2 3/4	2 13/16	2 15/16	3	3 1/16	3 3/16	3 1/4
2 1/4	2 7/16	2 7/16	2 1/2	2 9/16	2 5/8	2 11/16	2 3/4	2 7/8	2 15/16	3	3 1/16	3 3/16	3 1/4	3 3/8
2 3/8	2 1/2	2 9/16	2 5/8	2 11/16	2 3/4	2 13/16	2 7/8	2 15/16	3	3 1/8	3 3/16	3 1/4	3 3/8	3 7/16
2 1/2	2 5/8	2 11/16	2 3/4	2 13/16	2 7/8	2 15/16	3	3 1/16	3 1/8	3 3/16	3 1/4	3 3/8	3 7/16	3 9/16

Values below and to right of upper zigzag line are large enough for ¾-in. rivets.
Values below and to right of lower zigzag line are large enough for ⅞-in. rivets.

124e. Driving of Rivets—Field and Shop.—Rivets driven in the shop are called shop rivets and those driven in the field are known as field rivets.

Rivets may be driven by machines or by hand. Hand riveting is resorted to only when a rivet is so located that it cannot be driven by a machine; also on small erection jobs where the expense of providing power would be too great; and in shops when a few rivets have to be driven after the member has been removed from the riveter.

The process of driving a rivet is as follows: The rivet is heated to the proper temperature, inserted in the rivet hole and while the head is held tight against the member, a head is formed on the end of the shank extending out to the hole (see Fig. 161).

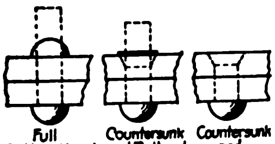


FIG. 161.

In hand riveting the end of the shank is hammered down in the shape of a head, then a hammer, called a snap, the head of which is cup shaped, is placed over the rough head and hammered until the head is of the proper shape. A dolly bar, which has a cup shaped face, is held against one head of the rivet while the other head is formed.

Machine riveters may be operated by compressed air, steam, or by hydraulic power. Compressed air riveters are portable, while steam and most hydraulic riveters are stationary. Power riveters may be either direct or indirect acting; by means of a direct acting riveter it is possible to keep the full pressure on the rivet as long as desired. Very satisfactory work can be perfomed by the pneumatic riveting hammer which delivers very rapid but comparatively light blows.

Loose Rivets.—Rivets are not always tight, as they should be, after driving. When a loose rivet is found it should be removed, if possible, and another driven in its place. Of course, if a rivet takes no definite stress and is so located that it is difficult to get at, judgment should

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

be used as to whether or not it should be removed. Loose rivets can be detected by tapping the rivet head with a hammer.

Clearance.—It is not possible to drive a rivet unless there is ample clearance for the die on the riveter. The required clearance varies with the size of the rivet (see Figs. 162 and 163). Tables 9 and 10 give the rivet spacing necessary for driving different sizes of rivets.

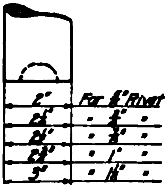


FIG. 162.

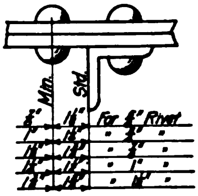


FIG. 163.

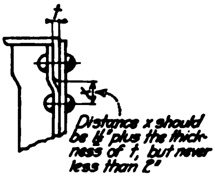

t
x
Distance x should be ½" plus the thickness of t, but never less than 2"

FIG. 164.

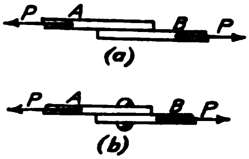


FIG. 165.

When an angle is crimped over a member the spacing used should not be less than that given in Fig. 164.

124f. Rivet Failures.—If in Fig. 165, the forces P are assumed to act in the directions indicated by the arrows, bar A will move to the left and bar B to the right. Suppose that before the forces P are applied, the bars are riveted together. Now if forces P are made large enough, the bars will move as indicated in Fig. 166 and the rivet is said to have sheared off in single shear. If three bars are used, as shown in Fig. 167, and the forces are made large enough, the rivet will shear off again, but this time on two planes (see Fig. 168), and the rivet is said to have failed in double shear.

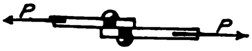


FIG. 166.

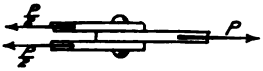


FIG. 167.

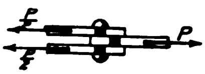


FIG. 168.

Failures as shown in Fig. 166 and 168 will occur providing the bars are wide and thick enough and the rivet is far enough from the ends of the bars. Suppose that bar A in Fig. 165 is not as thick as bar B; then instead of the rivet shearing off, the failure might occur as shown in Fig. 169. In this case the rivet has crushed through the top bar. This is called a failure in bearing. If the bar is harder than the rivet, which is usually the case, the rivet will be crushed by the bar.

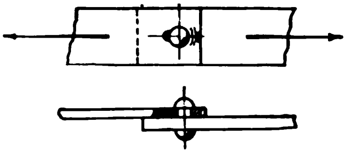


FIG. 169.

124g. Shearing and Bearing Values.—Practically all rivets used in structural work have to resist stresses caused by shear, bearing, and bending.

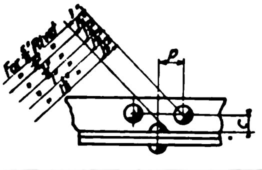


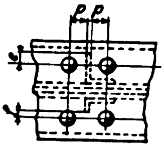
TABLE 9.[1]—RIVET SPACING
American Bridge Company Standard
Minimum Stagger for Rivets
(All Dimensions in Inches)

Dia. of rivet	Minimum stagger, d																
	1⅛	1 3/16	1¼	1 5/16	1⅜	1 7/16	1½	1 9/16	1⅝	1 11/16	1¾	1 13/16	1⅞	1 15/16	2 1/16	2 3/16	2 5/16
⅝	15/16	⅞	13/16	11/16	½	5/16	0										
¾	1¼	1 3/16	1⅛	1 1/16	15/16	⅞	¾	9/16	⅜	0							
⅞	1½	1 7/16	1⅜	1 5/16	1¼	1 3/16	1⅛	1	15/16	13/16	⅝	7/16	0				
1	1 13/16	1¾	1 11/16	1⅝	1 9/16	1½	1 7/16	1⅜	1 5/16	1 3/16	1⅛	1	⅞	¾	0		
1⅛	2 1/16	2	1 15/16	1 15/16	1⅞	1 13/16	1¾	1 11/16	1⅝	1 9/16	1½	1⅜	1 5/16	1¼	1	11/16	0

[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

TABLE 10.[1]—CLEARANCE FOR COVER PLATE RIVETING

(Dimensions in Inches)



e	½	1	1½	2	2½	3	3½	4	4½	5	5½	6
p	2½	2⅝	2¾	2¾	2⅞	2⅞	3	3⅛	3⅛	3¼	3¼	3⅜
f	0	½	1	1½	2	2½						
p	2½	2¼	2⅛	2	1½	0						

The allowable unit stresses on rivets are not at all uniform throughout the country. Values for shear on shop rivets vary from 9000 to 12,000 lb. per sq. in. and the corresponding unit bearing values are usually twice those for shear. Values for field driven rivets vary from ⅔ to ⅚ of those for shop driven rivets.

The value of a rivet in single shear is the area of the rivet times the allowable unit stress in shear and the double shearing value is just twice as great.

Illustrative Problem.—What are the values for a ¾-in. rivet in single and double shear when the allowable unit shearing stress is 10,000 lb. per sq. in.?

The area of a ¾-in. rivet is 0.442 sq. in., so the value in single shear is

$$(0.442)(10{,}000) = 4420 \text{ lb.}$$

and in double shear it is just twice as much, or

$$(4420)(2) = 8840 \text{ lb.}$$

The bearing value of a rivet is the diameter of the rivet, times the thickness of plate, times the allowable unit stress in bearing.

Illustrative Problem.—What is the bearing value of a ¾-in. rivet on a ½-in. plate if the allowable unit bearing stress is 20,000 lb. per sq. in.?

The value is

$$(\tfrac{3}{4})(\tfrac{1}{2})(20{,}000) = 7500 \text{ lb.}$$

Stresses caused by bending are usually considered only in case of long rivets or when loose fillers are used. For long rivets a certain per cent. of the number of rivets required is added (see Art. 124a). When rivets carrying stress pass through loose fillers, the number of rivets should be increased 50 % and when possible, the extra rivets should be outside of the connected member (see Fig. 218, p. 288).

Some specifications allow one-half the value of a button head rivet for a countersunk rivet if shop driven, and no allowance is made if the countersunk rivet is hand driven. A general rule is to allow half value for countersunk rivets in a plate ⅜ in. thick and over, and nothing when the plate is less than ⅜ in. thick.

R. Fleming recommends the following rules:[2]

> Rivets with countersunk heads shall be assumed to have ¾ the value of corresponding rivets with full heads, but no value shall be allowed for countersunk rivets in plates of a thickness less than one-half the diameter of the rivet.
>
> Rivets with flattened heads of height not less than three-eighths of an inch, or one-half the diameter of the rivet for ⅝-in. rivets and less, shall be assumed to have 9/10 the strength of rivets that have full heads.
>
> When heads are flattened to less than these heights, they shall be assumed to have the strength of countersunk rivets.

The allowable unit stresses on turned bolts in reamed holes are usually the same as on field rivets. The value for machine bolts is considered to be three-quarters of those for turned bolts.

124h. Rivets vs. Bolts in Direct Tension.—Direct tension on rivet heads should not be allowed except possibly in unimportant connections. If rivets are used in direct tension the connection should be compact, the material amply thick, and the groups of rivets should be symmetrically arranged about the line of action of the pull on the connection. Not less than

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

[2] See *Eng. News*, Sept. 14, 1916.

4 rivets should be used in a connection of this kind. The amount of stress on a rivet head after the rivet is driven is uncertain; also the rivet may have been burned in heating or it may not have been driven properly. Rivet heads may sometimes snap off (1) on cooling after driving, (2) in extreme cold weather, or (3) when struck with a hammer. Instead of using rivets in direct tension, it is better to ream out the holes and use bolts which have been turned to a driving fit.

In case rivets are used, the value of a rivet should not be greater than one-half its single shear value. In using turned bolts, a value of 10,000 lb. per sq. in. on the net area at the root of the thread should not be exceeded. Also, the bearing area under both the head and nut should be at right angles to the axis of the bolt.

124*i*. Use of Bolts.—Bolts are often used in place of rivets and for certain classes of work are preferable because they have proven to be satisfactory and are more economical.

The American Bridge Company allows the following unit stresses on bolts in building construction.

9000 lb. per sq. in. in shear
18,000 lb. per sq. in. in bearing

The above values are for ordinary bolts in holes punched ¹⁄₁₆ in. larger than the size of the bolt.

A washer under the nut will allow ample threading to tighten the nut properly. If a bolt is threaded too much, the bearing area will be reduced. After a nut is tightened up, some method of locking the nut should be used to prevent it from working off.

R. Fleming[1] makes the following suggestions for the uses of bolts:

> It is believed that bolted connections are permissible for the following:
>
> Buildings of one story, not of great height and acting mainly as shelters. Such buildings carry no shafting or electric traveling cranes and unless exposed to unusual winds there is little reason why field connections may not be bolted throughout.
>
> Buildings for temporary use.
>
> Subordinate framing such as that required for stairs, doors, windows, partitions, ceilings, monitors, pent houses, curbs and railing. It is often desirable, if not necessary, to have framing around windows, doors, skylights, and similar work bolted in order to secure proper adjustment for the work of other contractors.
>
> Purlins and girts, except where they form an integral part of a system of bracing. There is little reason why the clips to which purlins and girts are attached should not be shop-bolted, instead of shop-riveted, to main members. The same is true of many connections for subordinate framing.
>
> Platform and floor plates. If there are trucks moving on the floor, or if there is shoveling of coal or material, countersunk-head bolts should be used. An indentation in the head is convenient to hold a bolt while the nut is being turned. In other cases bolts with button heads not over ¼ or ⁵⁄₁₆ in. high may be used.
>
> Connections of beams to beams and beams to girders in floors that do not support machinery, shafting or rolling loads. This is an important item in a many-storied office building or hotel. If the connections of floor members to columns are riveted the structure is stiff transversely and longitudinally. Little is gained in stiffness and much is added to expense by riveting connections of filling-in members. Moreover, in fireproof construction the bolts are embedded in concrete, a fact which should assure any doubter that there is no chance of nuts becoming loose. The specification for a 12-story apartment house in New York City has the clause: "All connections within 3 ft. of the column centers must be riveted. All tank and sheave beam supports must be riveted. Other connections may be bolted." In this particular building the beams upon which some columns depend for lateral stiffness do not connect directly to the columns, but frame a foot or two away into other connecting beams. Is not this a commendable clause for similar cases?
>
> Bracing connections not subject to direct stress. This refers particularly to the intersection of bracing angles midway between trusses and columns. An over-zealous inspector will sometimes insist upon specifications being carried out to the letter and that rivets be used. This necessitates riveting from a special rigging at a cost of a dollar or two per rivet. The cost would not be a valid objection provided anything were gained by it.
>
> Connections not subject to shearing stress at points where members rest upon other members.

125. Lap and Butt Joints.—Joints in structural work may be divided into two kinds—viz., the lap joint and the butt joint (see Fig. 170). A lap joint is a joint in which the members joined extend over or lap on each other. A butt joint is one in which the ends of the members joined come together or butt against each other.

The joints shown in Figs. 170(*a*) and 170(*b*) are eccentric and are acted on by the moment

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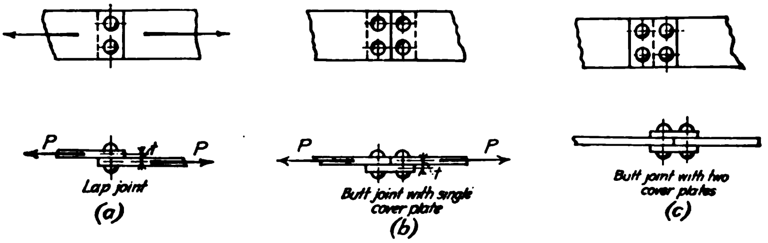
[1] *Eng. News-Rec.*, Aug. 14, 1919.

Pt. The joints however, deform and the bars tend to take the position shown in Figs. 171 and 172. This reduces the moment but causes some direct tension on the rivet heads.

Rivets may be arranged in different ways. Fig. 173(*a*) shows what is called chain riveting and the rivets in Fig. 173(*b*) are said to be staggered.

The butt joint with two cover plates makes the most satisfactory splice for bars and plates. It is also used for splicing both tension and compression members in a structure. Connections between the different members of a structure may be in the form of a lap or butt joint and very often take the form of what may be called a double lap joint (see Fig. 174).



Lap joint
(*a*)

Butt joint with single cover plate
(*b*)

Butt joint with two cover plates
(*c*)

Fig. 170.

125*a*. Failure of Joints.—A joint may fail (1) by shearing off the rivets (see Figs. 166 and 168), (2) by crushing the rivets or plate (see Fig. 169), (3) by tearing across a line of rivets (see Fig. 175), (4) by breaking through a hole (see Fig. 176), or (5) by the rivets shearing out the plate (see Fig. 177).

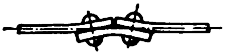


Fig. 171.

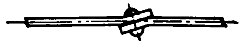
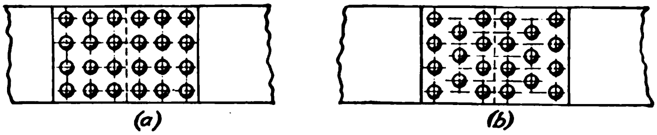


Fig. 172.



(*a*) (*b*)

Fig. 173.

The first failure may be prevented by using more or larger rivets; the second, by increasing the thickness of plates, or by increasing the number or size of rivets; the third, by making the plates wider, that is, increasing the edge distance; the fourth and fifth, by increasing the end distance.

125*b*. Distribution of Stress in Joints.—In a riveted joint or connection, it is not possible to determine just how the stress is distributed either through the members joined or the rivets joining them. The following assumptions are made: (1) that the stress in tension members is uniformly distributed over the net section; (2) that the rivets in compression members completely fill the holes, and that the stress is uniformly distributed over the gross area; and (3) that each rivet takes an equal part of the stress. (For eccentric connections, see Art. 130.)

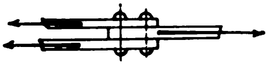


Fig. 174.

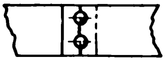


Fig. 175.

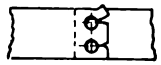


Fig. 176.

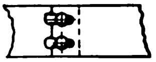


Fig. 177.

125*c*. Friction in Joints.—The stress on rivet heads due to shrinkage exerts great pressure on the members joined and causes friction between them. Tests[1] on riveted

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[1] Tests on riveted joints. *Proceedings* of The Am. Ry. Eng. and Maint. of Way Asso., vol. 6, 1905, p. 272.

joints have shown that the frictional resistance amounts to several thousand pounds per square inch of rivet area. In these tests there was practically no movement in the joint until considerable load had been applied. For the next few thousand pounds increase in the load, there was a slight slip evidently due to an adjustment of the joint after the frictional resistance had been overcome. After this adjustment, the rate of increase in slip was less until permanent distortion began.

Frictional resistance is not considered in computing the strength of a joint.

125*d*. Joint Computations.—The stresses on rivets in a joint are usually computed only for shear and bearing. Whether the strength of a joint is governed by shear or bearing depends on which gives the lesser value. The following problems are solved to show the method of procedure in computing the strength of a joint. In each case a ¾-in. rivet is used and the allowable unit stresses in shear and bearing are 10,000 and 20,000 lb. per sq. in.

Illustrative Problem.—Assume a lap joint composed of two ½-in. bars (see Fig. 178). Compute the strength of the joint.

The rivet is in single shear and bearing on a ½-in. bar. The area of the rivet is 0.442 sq. in. and the single shear value is

$$(0.442)(10{,}000) = 4420 \text{ lb.}$$

The bearing value is

$$(\tfrac{3}{4})(\tfrac{1}{2})(20{,}000) = 7500 \text{ lb.}$$

Since the value in bearing is the larger, the strength is governed by the shearing value and is 4420 lb.

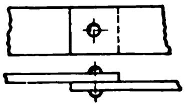

FIG. 178.

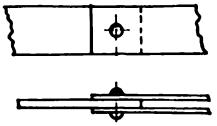

FIG. 179.

Illustrative Problem.—Assume one of the bars in Fig. 178 to be ¼ in. thick. Compute the strength of the joint.

The shearing value remains the same as in the preceding problem and is 4420 lb. The bearing value is

$$(\tfrac{3}{4})(\tfrac{1}{4})(20{,}000) = 3750 \text{ lb.}$$

The bearing value governs since it is less than the shearing value, and the strength of the joint is 3750 lb.

Illustrative Problem.—Assume a double lap joint composed of two ¼-in. bars and one ½-in. bar (see Fig. 179). Compute the strength of the joint.

In this case the rivet is in double shear and (since the sum of the thicknesses of the two outside bars is ½ in.) bearing on a ½-in. bar. The value in double shear is

$$(2)(4420) = 8840 \text{ lb.}$$

The bearing value on a ½-in. bar is 7500 lb. The strength of the joint is, therefore, 7500 lb.

Illustrative Problem.—Assume the ½-in. bar in Fig. 179 to be changed to a ⅜-in. bar. What is the strength of the joint?

The shearing value is the same as in the preceding problem, or 8840 lb. The sum of the two ¼-in. bars is greater than ⅜ in., so the ⅜-in. bar governs for bearing. The bearing value on the ⅜-in. bar is

$$(\tfrac{3}{4})(\tfrac{3}{8})(20{,}000) = 5625 \text{ lb.}$$

Since this value is less than the shearing value, the strength of the joint is 5625 lb.

For members carrying stress, not less than two rivets should be used in a connection. This does not hold for lacing bars.

Table 11 will save considerable work in computing the shearing and bearing values on rivets. The values computed in the above problems may be found directly from the table. At 10,000 lb. per sq. in., the shearing values in the table for a ¾-in. rivet are: single shear, 4420 lb.; double shear, 8840 lb. At 20,000 lb. per sq. in., the bearing values are as follows: bearing on a ½-in. plate, 7500 lb.; on a ¼-in. plate, 3750 lb.; and on a ⅜-in. plate, 5625 lb.

TABLE 11[1]

RIVETS

SHEARING AND BEARING VALUES

(Values in Pounds, Dimensions in Inches)

3/8-INCH RIVETS—Area 0.1104 sq. in.

Shear	Unit, lb. per sq. in.		7000	8000	9000	10000	11000	12000
	Single shear per rivet		770	880	990	1100	1210	1320
	Double shear per rivet		1540	1760	1980	2200	2420	2640
Bearing	Unit, lb. per sq. in.		14000	16000	18000	20000	22000	24000
	Thickness in Inches	1/8	660	750	840	940	1030	1130
		3/16	980	1130	1270	1410	1550	1690
		1/4	1310	1500	1690	1880	2060	2250
		5/16	1640	1880	2110	2340	2580	2810
		3/8	1910	2250	2530	2810	3090	3380

1/2-INCH RIVETS—Area 0 963 sq. in

Shear	Unit, lb. per sq. in.		7000	8000	9000	10000	11000	12000
	Single shear per rivet		1370	1570	1770	1960	2160	2360
	Double shear per rivet		2750	3140	3530	3930	4320	4710
Bearing	Unit, lb. per sq. in.		14000	16000	18000	20000	22000	24000
	Thickness in Inches	3/16	1310	1500	1690	1880	2060	2250
		1/4	1750	2000	2250	2500	2750	3000
		5/16	2190	2500	2810	3130	3440	3750
		3/8	2630	3000	3380	3750	4130	4500
		7/16	3060	3500	3940	4380	4810	5250
		1/2	3500	4000	4500	5000	5500	6000

5/8-INCH RIVETS—Area 0 3068 sq. in.

Shear	Unit, lb. per sq. in.		7000	8000	9000	10000	11000	12000
	Single shear per rivet		2150	2450	2760	3070	3370	3680
	Double shear per rivet		4300	4910	5520	6140	6750	7360
Bearing	Unit, lbs. per sq. in.		14000	16000	18000	20000	22000	24000
	Thickness in Inches	3/16	1640	1880	2110	2340	2580	2810
		1/4	2190	2500	2810	3130	3440	3750
		5/16	2730	3130	3520	3910	4300	4690
		3/8	3280	3750	4220	4690	5160	5630
		7/16	3830	4380	4920	5470	6020	4560
		1/2	4380	5000	5630	6250	6880	7500
		9/16	4920	5630	6330	7030	7730	8440
		5/8	5470	6250	7040	7810	8590	9380

3/4-INCH RIVETS—Area 0.4418 sq. in.

Unit, lb. per sq. in.	7000	8000	9000	10000	11000	12000
Single shear per rivet	3090	3530	3980	4420	4860	5300
Double shear per rivet	6190	7070	7950	8840	9720	10600
Unit, lb. per sq. in.	14000	16000	18000	20000	22000	24000
1/4	2630	3000	3380	3750	4130	4500
5/16	3280	3750	4220	4690	5160	5630
3/8	3940	4500	5060	5630	6190	6750
7/16	4590	5250	5910	6560	7220	7880
1/2	5250	6000	6750	7500	8250	9000
9/16	5910	6750	7590	8440	9280	10130
5/8	6560	7500	8440	9380	10310	11250

7/8-INCH RIVETS—Area 0 6013 sq. in.

Unit, lb. per sq. in.	7000	8000	9000	10000	11000	12000
Single shear per rivet	4210	4810	5410	6010	6610	7220
Double shear per rivet	8420	9620	10820	12030	13230	14430
Unit, lb. per sq. in.	14000	16000	18000	20000	22000	24000
1/4	3060	3500	3940	4380	4810	5250
5/16	3830	4380	4920	5470	6020	6560
3/8	4590	5250	5910	6560	7220	7880
7/16	5360	6130	6890	7660	8420	9190
1/2	6130	7000	7880	8750	9630	10500
9/16	6890	7880	8860	9840	10830	11810
5/8	7660	8750	9840	10940	12030	13130
11/16	8420	9630	10830	12030	13230	14430

1-INCH RIVETS—Area 0 7854 sq. in.

Unit, lb. per sq. in.	7000	8000	9000	10000	11000	12000
Single shear per rivet	5500	6280	7070	7850	8640	9420
Double shear per rivet	11000	12570	14140	15710	17280	18850
Unit, lb. per sq. in.	14000	16000	18000	20000	22000	24000
1/4	3500	4000	4500	5000	5500	6000
5/16	4380	5000	5630	6250	6880	7500
3/8	5250	6000	6750	7500	8250	9000
7/16	6130	7000	7880	8750	9630	10500
1/2	7000	8000	9000	10000	11000	12000
9/16	7880	9000	10130	11250	12380	13500
5/8	8750	10000	11250	12500	13750	15000
11/16	9630	11000	12380	13750	15130	16500
3/4	10500	12000	13500	15000	16500	18000
13/16	11380	13000	14630	16250	17880	19500

Values below dotted lines are greater than double shear.
Values above upper dotted lines are less than single shear.

Illustrative Problem.—Using the table for rivet values, determine the number of ¾-in. rivets required to connect the plates shown in Fig. 180. The unit values in shear and bearing are 10,000 and 20,000 lb. per sq. in.

The shear between plates 1 and 2 is 50,000 lb.; between 2 and 3 is 60,000 lb.; between 3 and 4 is 40,000 lb.; and between 4 and 5 is 70,000 lb.

The maximum shear occurs between plates 4 and 5, and is 70,000 lb. From the table the allowable shear on a ¾-in. rivet is 4420 lb. and the number of rivets required for shear is

$$\frac{70{,}000}{4420} = 16 \text{ rivets}$$

The bearing value of a ¾-in. rivet on a ½-in. plate is 7500 lb. and the number of rivets required for plates 2 or 4 is

$$\frac{110{,}000}{7500} = 15 \text{ rivets}$$

For plate 3 $\frac{100{,}000}{6560} = 16$ rivets

For plate 1 $\frac{50{,}000}{4690} = 10$ rivets

For plate 5 $\frac{70{,}000}{5625} = 15$ rivets

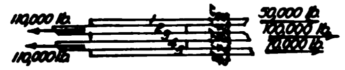

Fig. 180.

From the above it is seen that if 16 rivets are used, all the shearing and bearing stresses will be taken care of.

It will be noted that in this connection the tendency is to shear each rivet at four different sections. If plate 1 is placed between plates 2 and 3, the tendency will be to shear each rivet at three sections and the maximum shear will then be 110,000 lb. The rivets will be in triple shear. Thus it is seen that by properly arranging the plates the minimum shear on the rivets may be obtained. This consideration can very often be made use of in designing connections in which a number of plates are used.

The shearing and bearing values for unit stress not given in the table may be found from the table as explained in the following illustrative problem.

Illustrative Problem.—Suppose the allowable unit shearing stress is 7500 lb. per sq. in. and the unit bearing stress is 15,000 lb. per sq. in. Find the shearing value of a ¾-in. rivet and also the bearing value of a 7⁄16-in. plate.

At 7000 lb. per sq. in. the shearing value is 3090 lb. and at 8000 lb. per sq. in., it is 3530 lb.

Then at 7500 lb. per sq. in., the value is $\frac{3090 + 3530}{2} = 3310$ lb.

In the same way the bearing value is found to be $\frac{4590 + 5250}{2} = 4920$ lb.

The same results may be obtained by another method as follows: At 7000 lb. per sq. in. the shearing value is 3090 lb. Then at 7500 lb. per sq. in., it is $3090\left(\frac{7500}{7000}\right) = 3310$ lb., and the bearing value is $4590\left(\frac{15{,}000}{14{,}000}\right) = 4920$ lb.

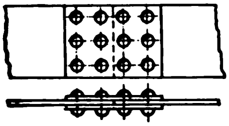

Fig. 181.

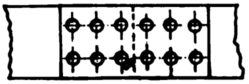

Fig. 182.

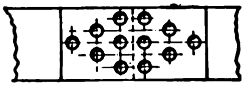

Fig. 183.

125e. Net Sections.—As the strength of a tension member depends on its net area, care should be taken in the arrangement of rivets so that the area will not be reduced more than necessary by the rivet holes. Consider the splice shown in Fig. 181. The area of the plate is reduced by three holes. By lengthening the splice plates (see Fig. 182) the rivets can be arranged so that the area of the plates will be reduced by only two holes. A better arrangement is shown in Fig. 183. Here the area of the plates is reduced by only one hole. In this case the area of the splice plates is reduced by three holes but it is much more economical to increase the area of the splice plates which are short, than the area of the main plates which may be of considerable length. Of course, there are cases in which a more economical splice may be designed if the rivets are so arranged that the area of the splice plates is not reduced too much (see Fig. 198, p. 280).

In computing the net area of a member, the diameter of the hole is considered to be ⅛ in. greater than the diameter of the rivet used. For countersunk rivets the diameter of the holes is usually considered to be ¼ in. greater than the diameter of the rivet when the thickness of the member is ⅝ in. or less. Table 12 gives the areas in sq. in. to be deducted for different sizes of holes through different thicknesses of metal.

TABLE 12.[1]—REDUCTION OF AREA FOR RIVET HOLES
(Area in Square Inches = Diameter of Hole × Thickness of Metal)

Thickness of metal (inches)	Diameter of hole (inches)											
	¼	½	9/16	⅝	11/16	¾	13/16	⅞	15/16	1	1 1/16	1⅛
3/16	0.05	0.09	0.11	0.12	0.13	0.14	0.15	0.16	0.18	0.19	0.20	0.21
¼	0.06	0.13	0.14	0.16	0.17	0.19	0.20	0.22	0.23	0.25	0.27	0.28
5/16	0.08	0.16	0.18	0.20	0.21	0.23	0.25	0.27	0.29	0.31	0.33	0.35
⅜	0.09	0.19	0.21	0.23	0.26	0.28	0.30	0.33	0.35	0.38	0.40	0.42
7/16	0.11	0.22	0.25	0.27	0.30	0.33	0.36	0.38	0.41	0.44	0.46	0.49
½	0.13	0.25	0.28	0.31	0.34	0.38	0.41	0.44	0.47	0.50	0.53	0.56
9/16	0.14	0.28	0.32	0.35	0.39	0.42	0.46	0.49	0.53	0.56	0.60	0.63
⅝	0.16	0.31	0.35	0.39	0.43	0.47	0.51	0.55	0.59	0.63	0.66	0.70
11/16	0.17	0.34	0.39	0.43	0.47	0.52	0.56	0.60	0.64	0.69	0.73	0.77
¾	0.19	0.38	0.42	0.47	0.52	0.56	0.61	0.66	0.70	0.75	0.80	0.84
13/16	0.20	0.41	0.46	0.51	0.56	0.61	0.66	0.71	0.76	0.81	0.86	0.91
⅞	0.22	0.44	0.49	0.55	0.60	0.66	0.71	0.77	0.82	0.88	0.93	0.98
15/16	0.23	0.47	0.53	0.59	0.64	0.70	0.76	0.82	0.88	0.94	1.00	1.05
1	0.25	0.50	0.56	0.63	0.69	0.75	0.81	0.88	0.94	1.00	1.06	1.13
1 1/16	0.27	0.53	0.60	0.66	0.73	0.80	0.86	0.93	1.00	1.06	1.13	1.20
1⅛	0.28	0.56	0.63	0.70	0.77	0.84	0.91	0.98	1.05	1.13	1.20	1.27
1 3/16	0.30	0.59	0.67	0.74	0.82	0.89	0.96	1.04	1.11	1.19	1.26	1.34
1¼	0.31	0.63	0.70	0.78	0.86	0.94	1.02	1.09	1.17	1.25	1.33	1.41
1 5/16	0.33	0.66	0.74	0.82	0.90	0.98	1.07	1.15	1.23	1.31	1.39	1.48
1⅜	0.34	0.69	0.77	0.86	0.95	1.03	1.12	1.20	1.29	1.38	1.46	1.55
1 7/16	0.36	0.72	0.81	0.90	0.99	1.08	1.17	1.25	1.35	1.44	1.53	1.62
1½	0.38	0.75	0.84	0.94	1.03	1.13	1.22	1.31	1.41	1.50	1.59	1.69

Illustrative Problem.—What is the net area of a bar 4 in. wide and ½ in. thick, with one hole for a ¾-in. rivet?

The diameter of the hole to be deducted is ¾ + ⅛ = ⅞ in. From Table 12 the area to be deducted is 0.44 sq. in. The net area, therefore, is (4)(½) − 0.44 = 1.56 in.

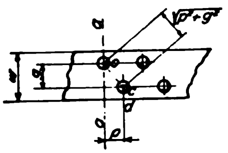

FIG. 184.

The proper design of a tension member requires that the net area should be computed on diagonal as well as on transverse lines. That is, the net area should be computed not only on line *aa* (see Fig. 184) but also on line *abcd*. Some specifications require that the net area should be considered on line *abcd* unless it exceeds that on *aa* by 30%. The usual method, however, is to make the net area on line *abcd* equal to that on line *aa*. When this method is used it is desirable to find the pitch *p* (see Fig. 184) which will give equal areas on sections *aa* and *abcd*.

Let w be the width of the member; g, the distance between gage lines; and d the diameter of the hole to be deducted. The net width on aa will then be $w - d$. On section $abcd$, the net width will be $w - g + \sqrt{g^2 + p^2} - 2d$. Equating these two widths,

$$w - d = w - g + \sqrt{g^2 + p^2} - 2d$$

or

$$\sqrt{g^2 + p^2} = w - d - w + g + 2d = g + d$$

Squaring

$$g^2 + p^2 = g^2 + 2gd + d^2$$

and

$$p^2 = 2gd + d^2$$

or

$$p = \sqrt{2gd + d^2}$$

Table 13 gives different values of p for corresponding values of g for ¾- and ⅞-in. rivets.

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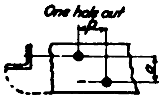
[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

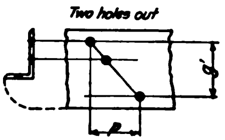
If the rivets are arranged as shown in Fig. 185, the value of p will be one-half as large and the formula will be

$$p = \frac{1}{2}\sqrt{2gd + d^2}$$

Table 13.[1]—Stagger of Rivets to Maintain Net Section

(American Bridge Company Standard)





d = diameter of rivet + $\frac{1}{8}$ in.

$g - d = \sqrt{g^2 + p^2} - 2d$ $\qquad g' - 2d = \sqrt{(g')^2 + p^2} - 3d$

$p = \sqrt{2gd + d^2}$ $\qquad p = \sqrt{2g'd + d^2}$

g = sum of gages minus thickness of angle.

$\frac{5}{8}$-in. rivets, can be taken at $\frac{1}{8}$ in. less than for $\frac{3}{4}$-in. rivets.

1-in. rivets, can be taken at $\frac{1}{8}$ in. more than for $\frac{7}{8}$-in. rivets.

Dimensions in inches

g	$\frac{3}{4}$-in. rivet	$\frac{7}{8}$-in. rivet	g'	$\frac{3}{4}$-in. rivet	$\frac{7}{8}$-in. rivet
	p	p		p	p
1	$1\frac{5}{8}$	$1\frac{3}{4}$	5	$3\frac{1}{16}$	$3\frac{5}{16}$
$1\frac{1}{2}$	$1\frac{7}{8}$	2	$5\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{2}$
2	$2\frac{1}{16}$	$2\frac{1}{4}$	6	$3\frac{3}{8}$	$3\frac{5}{8}$
$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{7}{16}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{3}{4}$
3	$2\frac{7}{16}$	$2\frac{5}{8}$	7	$3\frac{5}{8}$	$3\frac{7}{8}$
$3\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{13}{16}$	$7\frac{1}{2}$	$3\frac{3}{4}$	4
4	$2\frac{13}{16}$	3	8	$3\frac{7}{8}$	$4\frac{1}{8}$
$4\frac{1}{2}$	$2\frac{15}{16}$	$3\frac{3}{16}$	$8\frac{1}{2}$	4	$4\frac{1}{4}$

The following method takes into consideration the stress, on a diagonal section, caused by a combination of the shear (parallel to the section) and the tension normal to the section. From the formulas for maximum stress on a diagonal section as worked out by V. H. Cochrane[2], the following formula has been derived by T. A. Smith:[3]

$$X = \frac{g}{d} - \frac{2(g^2 + p^2 - d\sqrt{g^2 + p^2})}{d(g + \sqrt{g^2 + 4p^2})}$$

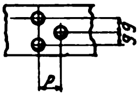


Fig. 185.

in which g is the gage (see Fig. 186), d is the diameter of the hole (diam. of rivet + $\frac{1}{8}$ in.), p is the pitch, and X is the amount of rivet hole to be deducted between the gage lines. Values of X in the diagram (Fig. 187) were worked out using the above formula. This diagram is for $\frac{7}{8}$-in. rivets and d was taken as 1 in.

In computing the net width of a tension member by this method, the number of rivets n, to be deducted, is as follows (see Fig. 188); considering $\frac{7}{8}$-in. rivets

$$n = 1 + X_1 + X_2 + X_3$$

where X_1, X_2, and X_3 are obtained from the diagram by using the corresponding values of p and g for each diagonal distance. The value 1 is for the outside halves of the two outside rivets and the values X_1, X_2, and X_3 are the values to be deducted from the gages g_1, g_2, and g_3. The net width, then, would be

$$w - (1 + X_1 + X_2 + X_3)$$

A larger value of n might be obtained by omitting rivet 2 and considering section 1-3-4. The gage for 1-3 would then be $g_1 + g_2$ and the corresponding value of p would be the horizontal distance between 1 and 3. In any case, the net area to be used will be for the section giving the largest value of n.

Consider the values for p_1, p_2, p_3, g_1, g_2, and g_3 as given on Fig. 188. Compute the net section assuming the plate to be $\frac{1}{2}$ in. thick, and the holes to be for $\frac{7}{8}$-in. rivets.

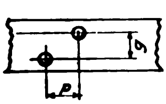


Fig. 180.

Considering all the holes

$$n = 1 + 0.4 + 0.93 + 0.4 = 2.73$$

Considering 1-3-4

$$n = 1 + 0.974 + 0.4 = 2.374$$

Since the larger value of n is obtained by considering all the holes, the net section will be through all the holes, and is

$$(10 - 2.73)\tfrac{1}{2} = 3.64 \text{ sq. in.}$$

For two lines of rivets (see Fig. 186), the value of p, such that only one hole must be deducted, is found where the gage line intersects the horizontal line AA in Fig. 187. Suppose g = 3 in., then in order that only one hole must be deducted, p would have to equal 3.32 in. or $3\frac{5}{16}$ in.

For three lines of rivets (see Fig. 189) the value of p, such that only two holes must be deducted, is found where the gage line intersects the line BB in Fig. 187. If g = 2 in., then p would have to equal 1.82 in. or $1\frac{13}{16}$ in.

For three lines of rivets (see Fig. 190) the value of p, such that only two holes must be deducted, is found from the location of a vertical line cutting gage lines g_1 and g_2 at an equal distance above and below the line BB in Fig. 187. If g_1 = 2 in. and g_2 = 3 in., the value of p from the diagram is found to be 2.05 in. or $2\frac{1}{16}$ in. This result may be checked as follows:

$$\begin{array}{ll} \text{For } p = 2.05 \text{ and } g = 3 & X = 0.63 \\ \text{For } p = 2.05 \text{ and } g = 2 & X = \underline{0.37} \\ & \ 1.00 \end{array}$$

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

[2] See *Eng. News*, April 23, 1908.

[3] See *Eng. News*, May 6, 1915.

The diagram (Fig. 187) may be used for any other size of rivet by dividing both p and g by the size of rivet plus ⅛ in and by multiplying the value of 1 by the same number.

Suppose the holes in Fig. 186 are for ¾-in. rivets. Find n for $p = 3.5$ in. and $g = 7$ in.

$$3.5 + \left(\frac{3}{4} + \frac{1}{8}\right) = 4 \text{ in.} \qquad 7 + \frac{7}{8} = 8 \text{ in.}$$

The diagram shows that $X = 0.644$. Then

$$n = (1)\left(\frac{7}{8}\right) + 0.644$$

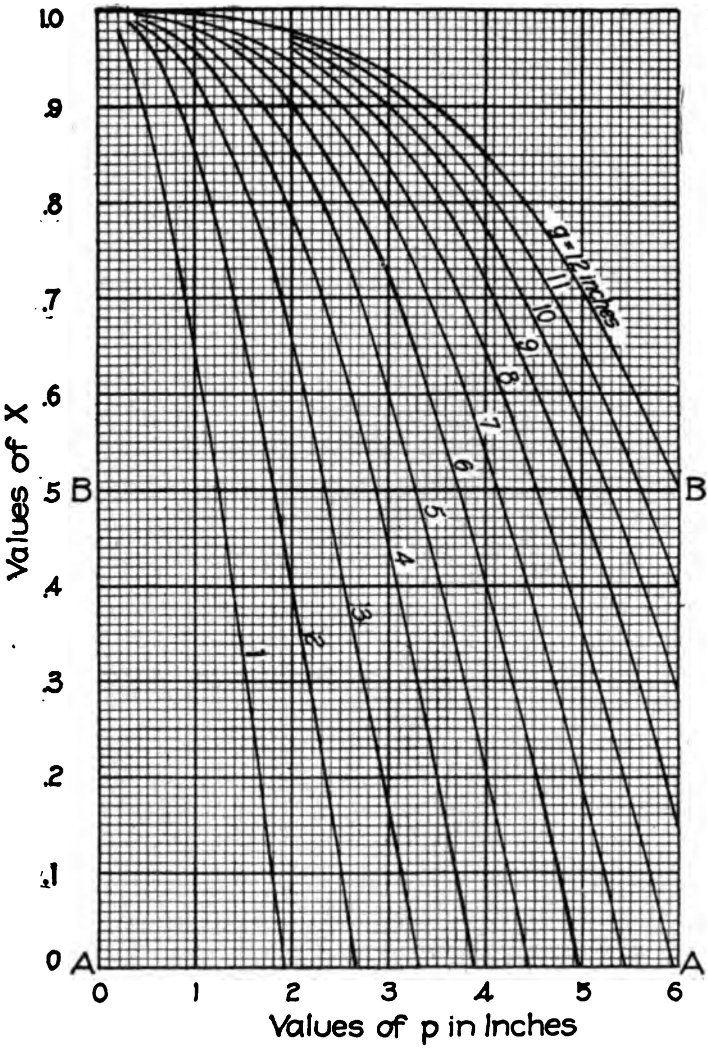


FIG. 187.—Diagram for values of X to be deducted for ⅞-in. rivets (d = 1 in.) in computing net sections.

125f. Design of Joints.—The joints at points where members are spliced or at points where the stress in one member is transferred to another, should be very carefully designed. A joint should be strong enough to develop the member joined even though the computed stress in the member may be less.

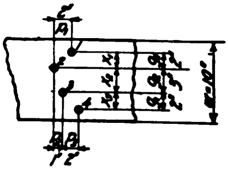


FIG. 188.

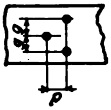


FIG. 189.

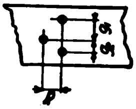


FIG. 190.

The solutions of the following problems show how the different tables may be used in the design of joints.

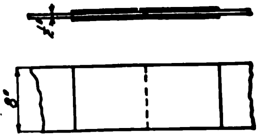


FIG. 191.

Illustrative Problem.—A plate 8 × ½ in. carrying 55,500 lb. is to be spliced. Assuming the allowable unit tensile value of the plate at 16,000 lb. and the unit values for rivets at 12,000 in shear and 25,000 in bearing, design a butt joint with two cover plates (see Fig. 191). Use ¾-in. rivets.

The best possible arrangement of rivets will reduce the area of the plate by one hole. Table 12 shows that the area to be deducted for one hole is 0.44 sq. in. The net area, therefore, is (8)(½) − 0.44 = 3.56 sq. in., and is satisfactory since the required area is $\frac{55,500}{16,000} = 3.47$ sq. in. Since the area of the splice plates will be reduced by more than one hole, a thickness of $\frac{5}{16}$ in. for each plate will be assumed. This gives a total thickness of $\frac{5}{8}$ in., which is greater than that of the plates spaced. Table 11 shows that the value of a $\frac{3}{4}$-in. rivet in bearing on a $\frac{1}{2}$-in. plate is 9000 lb. for a bearing value of 24,000 lb. per sq. in. At 25,000 lb. per sq. in. the corresponding bearing value is $9000\left(\frac{25,000}{24,000}\right) = 9375$ lb. The shearing value is found directly from the table and is 10,600 lb. since the rivets are in double shear. The number of rivets required is, therefore,

$$\frac{55,500}{9375} = 6$$

The rivets will be arranged as shown in Fig. 192, which shows that the area of each splice plate is reduced by three holes. Table 12 shows that the area to be deducted for one hole on a $\frac{5}{16}$-in. plate is 0.27 sq. in. Since there are two plates and three holes in each plate, the total area to be deducted is

$$(3)(0.27)(2) = 1.42 \text{ sq. in.}$$

and the net area of the cover plates is

$$(2)(8)(\tfrac{5}{16}) - 1.42 = 3.58 \text{ sq. in.}$$

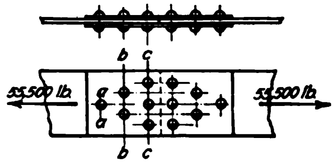


FIG. 192.

which is satisfactory. The net area of the 8 × $\frac{1}{2}$-in. plate on section *bb* is 4 − (2)(0.44) = 3.12 sq. in. Since the stress transmitted to the splice plates by each rivet is $\frac{55,000}{6} = 9250$ lb. (assuming each rivet to take the same amount of stress), the stress in the plate at section *bb* is 55,500 − 9250 = 46,350 lb. The required area is $\frac{46,350}{16,000} =$ 2.9 sq. in., and the area is satisfactory. On section *cc*, the net area is 2.68 sq. in. and the required area is 1.73 sq. in.

Illustrative Problem.—Using the same size rivets and the same unit stresses, design a lap joint for the above plates.

In this joint the rivets will be either in bearing on a $\frac{1}{2}$-in. plate, or in single shear. The bearing value is 9375 lb. and the shearing value is 5300 lb. so the latter value governs and the number of rivets required is

$$\frac{55,500}{5300} = 10.5, \text{ or } 11 \text{ rivets}$$

The rivets should be arranged as shown in Fig. 193. The net area on section *bb* is 3.12 sq. in. and the required area is

$$\frac{55,500 - 5300}{16,000} = 3.14 \text{ sq. in.}$$

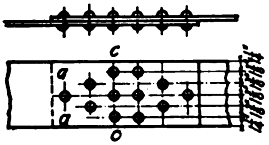


FIG. 193.

which is close enough.

Illustrative Problem.—The rivet pitch and spacing are shown on Fig. 193. What should be the pitch so that only one hole will have to be deducted on section *aa*?

$$p = \tfrac{1}{2}\sqrt{2gd + d^2} = \tfrac{1}{2}\sqrt{2(1\tfrac{3}{8})(\tfrac{7}{8}) + (\tfrac{7}{8})^2} = 0.90 \text{ in.}$$

This value checks with Table 13 which gives 0.91 in. ($\frac{1}{2}$ of the interpolated value for g equals $1\frac{3}{8}$.) Table 8 shows that p could not be less than $1\frac{7}{8}$ in. for a $\frac{3}{4}$-in. rivet.

If the other method is used, will more than three holes have to be deducted on section *cc*?

Fig. 187 shows that only three holes would have to be deducted if $\frac{7}{8}$-in. rivets were used so no more will have to be deducted for $\frac{3}{4}$-in. rivets.

125g. Efficiency of a Joint.—The ratio of the strength of a joint connecting two members to the strength of either member, is called the efficiency of the joint.

126. Splices in Trusses.

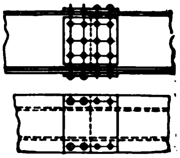


FIG. 194.

126a. Compression Members.—The usual method of splicing a compression member is to mill the ends of both members and to use splice plates with a couple of rows of rivets on each side of the splice to hold the members in line (see Fig. 194). A splice of this kind should be made at or near a joint, preferably far enough from the joint so that the splice connections will not interfere with the joint details. This method of splicing is entirely satisfactory for direct stress providing the ends of both members are milled properly. When the ends are not milled, the splice plates and number of rivets should be sufficient to transfer all the stress across the splice as no reliance should be allowed on the abutting ends. If only a part of a member is spliced, the splice should be made strong enough to develop the part spliced even though the ends may be milled. To illustrate, suppose only the web plate in Fig. 195 is to be spliced; then even though the ends of the web plate are milled, no allowance should be made for the milling. The splice plates and number of rivets should be sufficient to develop the plate spliced. This applies particularly to splices in plate girder flanges where the different parts of the flange are spliced at different points.

If the member is subjected to bending, the resultant stress on the section should be computed by the method given in Sect. 1, Art. 102. If there is tension on any part of the splice due to bending, the splice and number of rivets should be sufficient to properly transfer the stress across the splice. The method used in a case of this kind is to assume a splice and then to compute the fiber stress. Two or more trials may be necessary to obtain a satisfactory splice.

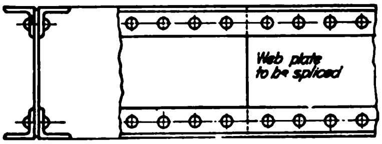


Fig. 195.

126b. Tension Members.—In light roof trusses the bottom chord splices are usually located so the gusset plate can be used as a splice plate (see Figs. 196 and 197). Splices may be made at points outside of the joint and no part of the gusset plate used (see Fig. 198). This simplifies the computations, especially when the members spliced carry a large total stress.

When the splice is made as shown in Fig. 196, a strip of gusset plate equal to the depth of the member spliced may be considered as splice plate. A splice plate should be used on the bottom of the members spliced (see Figs. 196 and 197). Of course, there are splices where a bottom plate would not be worth much (see Fig. 199). Better increase the thickness of the gusset plate, if necessary, and cut the plate as shown by dotted line.

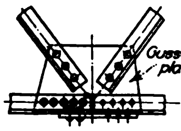


Fig. 196.

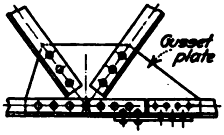


Fig. 197.

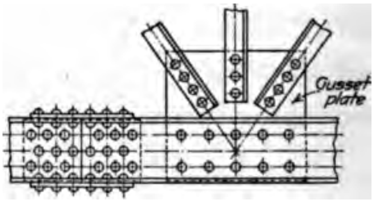


Fig. 198.

If part of the gusset plate is used as splice plate, it is well to investigate the stress at the bottom of the plate. This may be done as follows (see Fig. 200):

Taking moments about o on axis aa through the center of gravity of the plate

$$M_o = S_1y - Sx$$

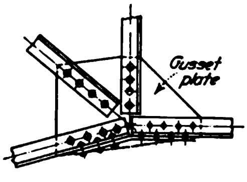


Fig. 199.

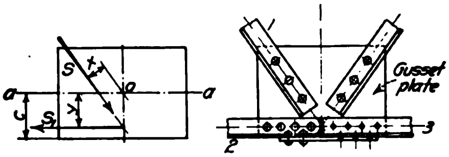


Fig. 200.

where S is the stress in member 1, and S_1 is the total value of the rivets connecting member 2 to the gusset plate. Then fiber stress due to bending is

$$f = \frac{Mc}{I}$$

n which c is the distance shown on Fig. 200 and I is the moment of inertia of the plate about axis aa through the center of gravity of the plate.

To this value of f add the unit stress due to direct tension on the part of the gusset plate considered as splice plate. This stress is the total value of the connection between member 3 and the gusset plate divided by the area of that portion of the gusset plate considered as splice plate.

In designing splices for built-up members, great care should be taken to arrange the splice material and rivets so each part of the member will be amply spliced. This applies to both tension and compression splices.

127. Plate Girder Web Splices.—Plate girder webs may be spliced in a number of different ways (see Figs. 201 to 205 inclusive). The kind of splice to be used in any given case depends somewhat on the assumptions made in the design of the girder.

The splice shown in Fig. 201 consists of a plate on each side of the web. When no part of the web is considered as flange area, this splice is designed for shear only. It may be designed for the maximum shear the web is capable of carrying, or for the maximum shear at the splice. More than enough rivets should be used on each side of the splice to carry the total shear considered in the design; usually not less than two rows of rivets on each side of the splice are used. Unless the splice is made at a point where there is considerable excess flange area, a few extra rivets should be used. Even though no part of the web is considered as flange area in designing the girder, the web will resist some of the stresses caused by bending. For this reason the rivets in the splice plates will be over-stressed if just enough are used to provide for shear. This splice is also used when a part of the web is considered as flange area, especially when the splice is made at a point where there is an excess of flange area. If the splice is made at a point where the shear is small, the design is usually made for the maximum moment the web is capable of carrying. At other points the shear should be considered in the design and the corresponding moment used.

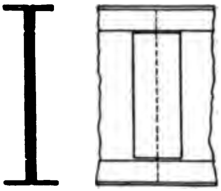


Fig. 201.

The splices shown in Figs. 202 and 203 are used when a part of the web is considered as flange area. The splice in each case consists of six plates, four plates marked *A* and two plates marked *B*. In Fig. 202, plates *A* are usually designed for moment and plates *B* for shear. In Fig. 203, plates *B* are designed for shear and moment and plates *A* for moment. In this design the splice is supposed to be equivalent to the web at all points.[1]

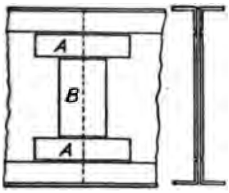


Fig. 202.

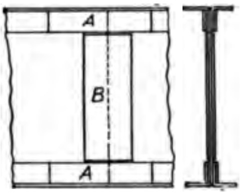


Fig. 203.

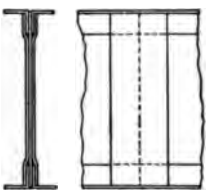


Fig. 204.

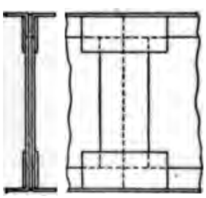


Fig. 205.

The splices shown in Figs. 204 and 205 are sometimes used by designers who claim that the other splices do not provide for horizontal shear in the web at the edge of the flange angles.

When a splice is made near the end of a cover plate, the cover plate may be extended and used in place of plates *A* in Figs. 202 and 203 (see Fig. 206). When this is done, plate *B* in Fig. 202 should be the full depth between flange angles. In Fig. 203 the splice will not be equivalent to the web at all points when the cover plate is used in place of plate *A*.

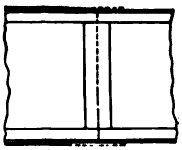


Fig. 206.

The following problems are worked out to show the computations in designing the kind of splices shown in Figs. 201 and 202. These splices will be stronger than necessary because they are designed to develop the web in bending and in addition to carry shear. In actual design the moment caused by the loading which gives the shear should be used or the maximum moment at the section and the corresponding shear. To illustrate, consider a girder carrying a fixed uniform load. If the splice is made at the center (which is not usually done) where the shear is zero, the splice should be designed for moment only. The usual method is to make the splice as strong in resisting bending stresses as the web would be if it were not spliced. If, on the other hand, the splice is made at say the quarter point, both shear and moment should be considered in designing the splice. The values used should be those computed at the point where the splice is made. In this case, neither the shear nor moment will be a maximum;

---

[1] See vol. 3 of Modern Framed Structures by Johnson, Bryan and Turneaure for a treatment of this splice.

the shear will be ½ of the maximum on the girder and the moment ¾. The design, however, brings out all the necessary computations in the design of web splices.

Illustrative Problem.—Assume a plate grider 68½ in. back to back of flange angles. Web plate, 68 × ⅜ in. Flange angles, 6 × 6 × ⁹⁄₁₆ in. with one cover plate 14 × ⁷⁄₁₆ in. and one 14 × ⅜ in. ⅞-in. rivets.

Rivet values, shear	12,000 lb. per sq. in.
bearing	24,000 lb. per sq. in.
Shear on web (gross area)	10,000 lb. per sq. in.
Tension extreme fiber	16,000 lb. per sq. in.
Shear at point of splice	100,000 lb.

The splice plates are assumed to be 56¼ in. deep (see Fig. 207). The area of the web considered as part of the flange area is ⅛ of the gross web area. One-eighth of web area is ⅛ × 68 × ⅜ = 3.19 sq. in. and is assumed to act at the center of gravity of the flange area which is 67.08 in. (see Fig. 207) between center of gravity of top and bottom flanges.

The splice will be designed assuming that ⅛ of the web area carries its full moment.

$$(3.19)(67.08)(16{,}000)\left(\frac{67.08}{70.13}\right) = 3{,}270{,}000 \text{ in.-lb.}$$

The stress on the extreme fiber is assumed to be 16,000 lb. per sq. in. (Some designers compute the stress on the girder flange and use the computed stress in designing the splice.) The stress at the center of gravity of the flange would then be

$$(16{,}000)\left(\frac{67.08}{70.13}\right) = 15{,}300 \text{ lb. per sq. in.}$$

Web splices of this kind may be designed to take the same moment as the gross web plate does. The moment would then be

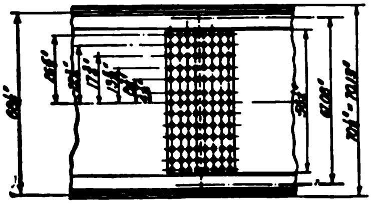


Fig. 207.

$$M = \frac{If}{c} = \frac{bd^2f}{6}$$

in which

$$f = \frac{(16{,}000)(68)}{70.13} = 15{,}510 \text{ lb. per sq. in., net area.}$$

For the gross area this stress would be

$$\frac{(15{,}510)(23.54)}{28.51} = 12{,}810 \text{ lb. per sq. in.}$$

Then

$$M = \frac{(12{,}810)(\tfrac{3}{8})(68)(68)}{6} = 3{,}700{,}000 \text{ in.-lb.}$$

The above method of computing M assumes that there are no holes in the web. If holes 4 in. apart are allowed for, the value of M will be (assuming ⅞-in. rivets are used)

$$M = \frac{bd^2f}{8}$$

in which f = 15,510 lb. per sq. in. and

$$M = \frac{(15{,}510)(\tfrac{3}{8})(68)(68)}{8} = 3{,}360{,}000 \text{ in.-lb.}$$

Either one of these methods of computing M would give a stronger splice than the one designed.

Rivet spacing in the splice plate will be assumed to be 4⁷⁄₁₆ in. center to center. The stress on the rivets will be found by the method given under eccentric connections (see Art. 130). The distance from the neutral axis only will be considered and the stress on the extreme rivet found for one row of rivets from which the number of rows required can be determined. When the distance back to back of flange angles is small, the horizontal distance between rivets and the center of gravity of the group should be considered, because in such cases a considerable difference will be found in the value of Σr^2.

$$\begin{array}{rl}
 & \text{From Table 15} \\
(4\tfrac{7}{16})^2 = & 19.70 \\
(8\tfrac{7}{8}\ )^2 = & 78.77 \\
(13\tfrac{5}{16})^2 = & 177.22 \\
(17\tfrac{3}{4}\ )^2 = & 315.06 \\
(22\tfrac{3}{16})^2 = & 492.28 \\
(26\tfrac{5}{8}\ )^2 = & \underline{708.90} \\
 & 1791.93 \\
 & \underline{\qquad 2} \\
 & 3583.86 = \Sigma r^2 \text{ for one row.}
\end{array}$$

The stress on a rivet at a unit distance from the neutral axis is

$$\frac{3{,}270{,}000}{3584} = 913 \text{ lb.}$$

Stress on extreme rivet is

$$(913)(26\tfrac{5}{8}) = 24{,}310 \text{ lb.}$$

If four rows of rivets are used, the maximum stress due to moment will be

$$\frac{24{,}310}{4} = 6080 \text{ lb.}$$

The total number of rivets on each side of the splice will be (4) (13) = 52 rivets. The stress on each rivet due to shear is assumed to be equal and is

$$\frac{100{,}000}{52} = 1920 \text{ lb.}$$

The resultant stress on the extreme rivet is

$$\sqrt{(6080)^2 + (1920)^2} = 6380 \text{ lb.}$$

Table 11 shows the shearing value of a ⅞-in. rivet to be 7220 lb. in single shear, and 14,440 lb. in double shear. The bearing value on the ⅜-in. web is 7880 lb.

The bearing value governs and the value at the extreme rivet is

$$\frac{(7880)(53.25)}{70.13} = 6000 \text{ lb.}$$

This value is less than the stress on the extreme rivet so the spacing will be arranged as shown in Fig. 208.

From Table 15

$$\begin{aligned}
(2\tfrac{1}{16})^2 &= 4.25\\
(6\tfrac{3}{16})^2 &= 38.29\\
(10\tfrac{5}{16})^2 &= 106.35\\
(14\tfrac{7}{16})^2 &= 208.44\\
(18\tfrac{9}{16})^2 &= 344.56\\
(22\tfrac{11}{16})^2 &= 514.73\\
(26\tfrac{13}{16})^2 &= 718.91\\
\hline
&\ 1935.53\\
&\ 2\\
\hline
&\ 5871.06 = \Sigma r^2 \text{ for one row.}
\end{aligned}$$

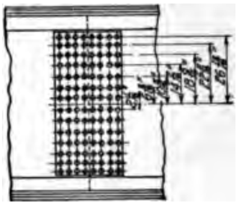


Fig. 208.

Then

$$\frac{3{,}270{,}000}{3871} = 845. \text{ lb.}$$

and

$$(845)(26\tfrac{13}{16}) = 22{,}660 \text{ lb. stress on extreme rivet}$$

Assuming four rows, the maximum stress due to moment will be

$$\frac{22{,}600}{4} = 5665 \text{ lb.}$$

If the horizontal distances between the center of gravity of the group of rivets and each rivet is considered, this value will be 5430 lb. The total number of rivets on each side of the splice is 4 × 14 = 56 rivets. Stress on each rivet due to shear is

$$\frac{100{,}000}{56} = 1790 \text{ lb.}$$

The resultant stress is

$$\sqrt{(1790)^2 + (5665)^2} = 5920 \text{ lb.}$$

If the horizontal distances are considered as noted above, this value will be 5660 lb. The allowable stress is

$$\frac{(7880)(53.625)}{70.13} = 6040 \text{ lb.}$$

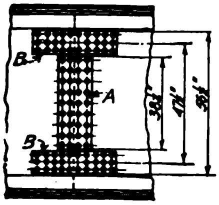


Fig. 209.

which is satisfactory.

The moment of inertia about the neutral axis of the splice plates should be equal to or greater than the moment of inertia of web plate or

$$\frac{(t)(56.25)^3}{12} = \frac{(\tfrac{3}{8})(68)^3}{12}$$

or

$$t = \frac{(\tfrac{3}{8})(68)^3}{56.25^3} = 0.662 \text{ in.}$$

Each plate should be

$$\frac{0.662}{2} = 0.331 \text{ in. thick.}$$

This is a very little over 5/16 in., so ⅜-in. plates will be used.

Illustrative Problem.—Using the data given in the proceeding problem, a splice similar to Fig. 202 will be designed.

The web area (3.19 sq. in.) considered as flange area is assumed to act at the center of gravity of the flanges Plates *B* (see Fig. 209) are assumed to be 9 in. wide and their distance center to center will be 47.5 in. The area of plate *B* should be

$$(3.19)\left(\frac{67.08^2}{47.5^2}\right) = 6.36 \text{ sq. in.}$$

or

$$I_1 + a_1x_1^2 = I_2 + a_2x_2^2$$

in which I_1 and I_2 represent the moment of inertia of the plate *B* and the web area (considered as flange area) about horizontal axes through their respective centers of gravity. These values are considered equal, hence

$$a_1x_1^2 = a_2x_2^2$$

or

$$a_1 = \frac{a_2x_2^2}{x_1^2}$$

a_1 represents the area of plate *B* and a_2 the area of the web considered as flange area. x_1 and x_2 are the distances of the center of gravity of each area from the neutral axis of the girder. In solving for the area of plate *B* above, the values of x_1 and x_2 used are the distances center to center of each set of areas. The result is the same as would be obtained by using the distances from the neutral axis to the center of each area because both numerator and denominator are just two times as great.

Two plates, 9 × ½ in., will be used. This gives a net area of 7 × ½ × 2 = 7 sq. in., which is satisfactory.

Assuming 16,000 lb. per sq. in. on extreme fiber, the allowable stress at the center of plate *B* is

$$\frac{(16{,}000)(47.5)}{70.13} = 10{,}830 \text{ lb. per sq. in.}$$

and the rivet value at the same point is

$$\frac{(7880)(47.5)}{70.13} = 5350 \text{ lb}$$

The number of rivets required on each side of the splice is

$$\frac{(6.36)(10{,}830)}{5350} = 12.9, \text{ or 13 rivets}$$

The number of rivets required in plate *A* on each side of the splice is

$$\frac{100{,}000}{7880} = 12.7, \text{ or 13 rivets.}$$

Plate *A* will be made ⅝ in. thick, which will give ample area for shear.

The plates are 38¼ in. deep, and the shearing value is

$$(38.25)(\tfrac{5}{8})(10{,}000) = 239{,}000 \text{ lb.}$$

Use two rows of rivets spaced 4⅜ in. center to center on each side of the splice.

Rivets are sometimes spaced closer near the top and bottom of splice plates designed for bending stresses. The spacing should be uniform because both the stress on the plates and rivets decrease in the same ratio from the flanges towards the neutral axis. It will be found that the rivet pitch will be the same whether computed for points near the flange or neutral axis.

In designing web splices, care should be taken to make the rivet spacing such that the area of the web is not reduced more than assumed in the design of the girder. If ⅛ of the web is considered as flange area, then the spacing of rivets in a vertical row should not be less than 4 in. c. to c. for ⅞-in. rivets.

128. Plate Girder Flange Splices.—When it is necessary to splice the flange of a plate girder, the splice should be arranged so that not more than one part of the flange is spliced at any point. Also, no part of the flange should be spliced at a point where the web is spliced. The different parts of the flange should be spliced at points where there is an excess of flange area. All flange splices should be designed to fully develop the member spliced, and enough rivets should be used to transfer all stress across the splice. No allowance should be made in the compression flange for abutting ends. Specifications usually require the splice to be somewhat stronger than the member spliced.

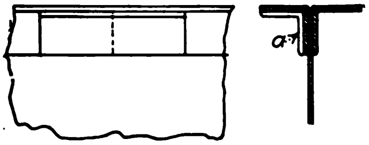


FIG. 210.

128a. Splicing Flange Angles.—The usual method of splicing flange angles is to splice one angle at some point between the center and left support and the other angle at a corresponding point at the right of the center. A splice angle should be used (see Fig. 210) and if possible, the net area should be equal to or greater than the net area of the flange angle. Enough rivets should be used to develop the splice angle, and the spacing should be close in order to reduce the length of the splice angles and to transfer the stress in a short distance. When the flange angle legs are equal, the splice angle legs should be equal and each leg assumed to take one-half of the stress. The same number of rivets should then be used in each leg. If the legs are unequal, the number of rivets in each leg should be in proportion to the area of each leg.

The number of rivets required through the splice angles on each side of the splice can be determined as follows:

$$N = \frac{fA_n}{r}$$

in which f is the allowable fiber stress, A_n the net area of the splice member, and r the rivet value (single shear in this case). The rivets required for shear in the flange can also be used as spliced rivets. When the net area of the splice angle is less than the net area of the flange angle, a splice plate should be used on the vertical leg of the other flange angle (see Fig. 211).

The stress may be considered as distributed between the splice angle and the splice plate in proportion to the area of each. If the splice is made near the end of a cover plate, the cover plate may be extended and used as a part of the splice.

When the splice member is in contact with the member spliced (as the splice angle a in Figs. 210 and 211) no increase in the computed number of rivets is necessary. On the other hand, the computed number on each side of the joint for the splice plate b should be increased by one-third for each intervening plate.

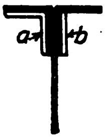


FIG. 211.

128b. Splicing Cover Plates.—A cover plate may be spliced by using a splice plate of the same net area and long enough to provide for the required number of rivets in single shear (see Fig. 212).

When a cover plate is spliced near the end of another cover plate (see Fig. 213), the cover plate may be extended as shown by dotted lines. If the extended cover plate is of the same size as the plate spliced, the splice will be satisfactory if enough rivets are used. The formula given for rivets through the splice angle may be used to determine the number of rivets required. When the splice plate is not in contact with the plate spliced, then the required number of rivets on each side of the splice should be increased by one-third for each intervening plate.[1]

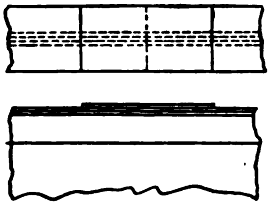


FIG. 212.

129. Connection Angles.—Beam and girder connections are usually made by means of angles (see Figs. 214 and 215). The method of computing the strength of the connections shown in Figs. 214 and 215 is the same except that the number of rivets in 215 will be increased according to Art. 124g.

Consider the connection shown in Fig. 216; the strength will depend on

1. Four shop rivets bearing on web of beam *A*.
2. Four shop rivets in double shear.
3. Eight field rivets in single shear.
4. Eight field rivets bearing on the web of beam *B* or on the $\frac{7}{16}$-in. angles.

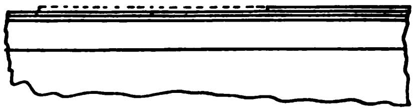


FIG. 213.

Illustrative Problem.—Assume beam *A* to be a 15-in. 42-lb. I, and beam *B* a 24-in. 80-lb. I. What is the strength of the connection if $\frac{3}{4}$-in. rivets with the following values are used?

Shear	shop	10,000 lb. per sq. in.
	field	7,000 lb. per sq. in.
Bearing	shop	20,000 lb. per sq. in.
	field	14,000 lb. per sq. in.

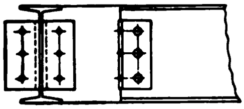


FIG. 214.

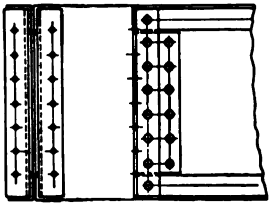


FIG. 215.

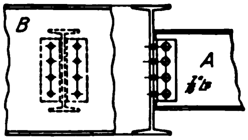


FIG. 216.

The web thickness of the 15-in. I is $\frac{7}{16}$ in. and of the 24-in. I is $\frac{1}{2}$ in. (see Table 5). From Table 11 the following values are obtained:

---

[1] For a more complete treatment of flange splices, see vol. 3 of Modern Framed Structures by Johnson, Bryan, and Turneaure.

Single shear	shop	4420 lb.
	field	3090 lb.
Bearing on 7/16-in.	shop	6560 lb.
	field	4590 lb.
Bearing on web of *A*		4 × 6560 = 26,240 lb.
Double shear through *A*		2 × 4 × 4420 = 35,360 lb.
Bearing on 7/16-in. angles		8 × 4590 = 36,720 lb.
Single shear		8 × 3090 = 24,720 lb.

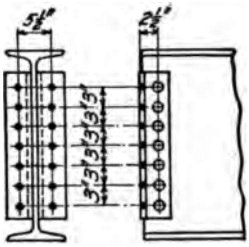
The strength of the connection, therefore, is 24,720 lb.

Connections of this kind may be divided into two classes—viz., standard and special.

129a. Standard Connections.—The end connections for beams may be made the same for different sizes of beams under certain limiting conditions of loading and span length. Many structural shops have their own standards for these connections. Table 14 gives the standard beam connections and limiting values. Standard connections should be used when possible.

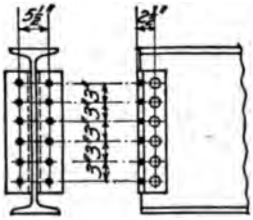
TABLE 14.[1]—BEAM CONNECTIONS

27"



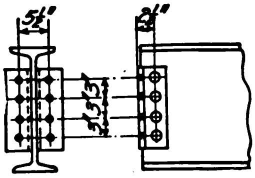
2∠s 4" × 4" × ½" × 1'-8½"
Weight 46 lb.

24"



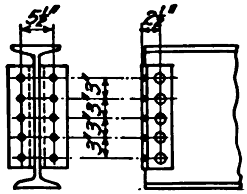
2∠s 4" × 4" × ½" × 1'-5½"
Weight 39 lb.

21"



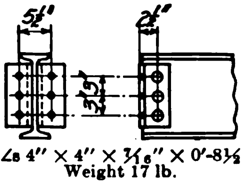
2∠s 4" × 4" × ½" × 1'-2½"
Weight 33 lb.

20", 18", 15"



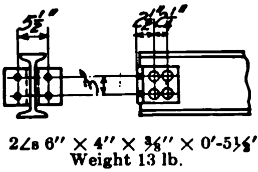
2∠s 4" × 4" × 7/16" × 0'-11½"
Weight 23 lb.

12"



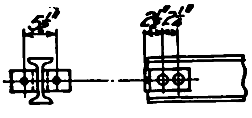
2∠s 4" × 4" × 7/16" × 0'-8½"
Weight 17 lb.

10", 9", 8"



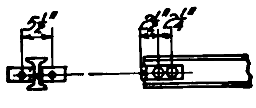
2∠s 6" × 4" × ⅜" × 0'-5½"
Weight 13 lb.

7", 6", 5"



2∠s 6" × 4" × ⅜" × 0'-3"
Weight 7 lb.

4", 3"



2∠s 6" × 4" × ⅜" × 0'-2"
Weight 5 lb.

Rivets and bolts ¾ in. diameter.

Weights given are for ¾-in. shop rivets and angle connections; about 20% should be added for field rivets or bolts.

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

LIMITING VALUES OF BEAM CONNECTIONS

I-beams		Value of web connection	Values of outstanding legs of connection angles					
			Field rivets			Field bolts		
(Inches depth)	Weight (pounds per foot)	Shop rivets in enclosed bearing, (pounds)	¾-in. rivets or turned bolts, single shear, (pounds)	Minimum allowable span in feet, uniform load	t, (inches)	¾-in. rough bolts, single shear, (pounds)	Minimum allowable span in feet, uniform load	t, (inches)
27	90	82,530	61,900	18.9	⅝	49,500	23.6	⅝
24	80	67,500	53,000	17.5	⅝	42,400	21.9	⅝
	74	64,260	53,000	16.4	⅝	42,400	20.4	⅝
21	60½	48,150	44,200	14.2	⅝	35,300	17.8	⅝
20	65	45,000	35,300	17.6	⅝	28,300	22.1	⅝
18	55	41,400	35,300	13.3	⅝	28,300	16.7	⅝
	48	34,200	35,300	12.8	9/16	28,300	15.4	⅝
15	42	36,900	35,300	8.9	⅝	28,300	11.1	⅝
	37½	29,880	35,300	9.7	½	28,300	10.2	9/16
12	31½	23,600	26,500	8.1	9/16	21,200	9.0	⅝
	28	19,170	26,500	9.2	7/16	21,200	9.2	½
10	25	27,900	17,700	7.4	⅝	14,100	9.2	⅝
	22¼	22,680	17,700	6.8	⅝	14,100	8.6	⅝
9	21	26,100	17,700	5.7	⅝	14,100	7.1	⅝
8	18	24,300	17,700	4.3	⅝	14,100	5.4	⅝
	17½	19,800	17,700	4.4	⅝	14,100	5.5	⅝
7	15	11,300	8,800	6.2	⅝	7,100	7.8	⅝
6	12¼	10,400	8,800	4.4	⅝	7,100	5.5	⅝
5	9¾	9,500	8,800	2.9	⅝	7,100	3.6	⅝
4	7½	8,600	8,800	2.2	9/16	7,100	2.7	⅝
3	5½	7,700	8,800	1.3	½	7,100	1.4	⅝

ALLOWABLE UNIT STRESS IN POUNDS PER SQUARE INCH

Single shear	Rivets....................Shop 12,000 Rivets and Turned bolts....Field 10,000 Rough bolts..............Field 8,000	Bearing	Rivets—enclosed..........Shop 30,000 Rivets—one side..........Shop 24,000 Rivets and turned bolts....Field 20,000 Rough bolts..............Field 16,000

t = Web thickness, in bearing, to develop max. allowable reactions, when beams frame opposite.

Connections are figured for bearing and shear (no moment considered).

The above values agree with tests made on beams under ordinary conditions of use.

Where web is enclosed between connection angles (enclosed bearing), values are greater because of the increased efficiency due to friction and grip.

Special connections shall be used when any of the limiting conditions given above are exceeded—such as end reaction from loaded beam being greater than value of connection; shorter span with beam fully loaded; or a less thickness of web when maximum allowable reactions are used.

129b. Special Connections.—When standard connections cannot be used, it is necessary to design special connections for each particular case. The following conditions may require special connections: (1) short spans heavily loaded, (2) spans with load near one end, (3) when two beams connect on opposite sides of the same web and use the same rivets, and (4) when two beams connect on opposite sides of the same web and only a part of the rivets are used in each connection.

For conditions 1 and 2, the reactions should be computed and enough rivets used to safely transfer the load from one member to the other.

For condition 3, standard connections may be satisfactory providing the thickness *t* (see Fig. 217) is such that ample bearing on the rivets is developed. Otherwise the web plate may be reinforced (see Fig. 218) or special connections used. Special connections will undoubtedly be necessary if the loads on the beams are applied near the ends to which the connections are made. In any case, the end reactions should be computed and the rivets proportioned accordingly.

For condition 4, the beams may not be at the same elevation (see Fig. 219) or may not be on the same line (see Fig. 220). Standard connections may be used in these cases if ample bearing is provided for the rivets and the spacing of the holes can be made standard.

When two beams are near each other (see Fig. 221), it is not possible to use more than one connection angle on each beam. Special connections should be designed for such cases.

When beams do not frame into each other at right angles, special connections may be necessary (see Fig. 222). When t is ⅝ in. or less and b is 3 in. or less, standard connections may be used providing the angles are bent to the proper bevel. When b is greater than 3 in., bent plates should be used in place of angles. For bevels in which b is greater than 3 in., care should be taken to see that rivet holes are not located where it is impossible to drive the rivets.

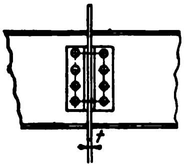

Fig. 217.

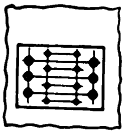

Fig. 218.

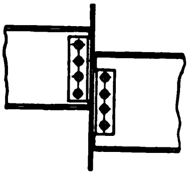

Fig. 219.

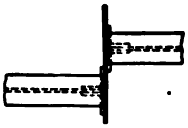

Fig. 220.

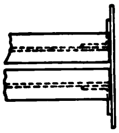

Fig. 221.

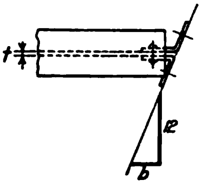

Fig. 222.

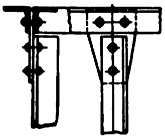

Fig. 223.

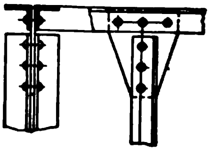

Fig. 224.

See Sect. 3, Art. 72*a* for illustrations of beam connections. See also Sect. 3, Art. 72*b* for beam connections to columns.

Connections between members carrying direct stress usually take the form of a lap joint. Consider the connection shown in Figs. 223 and 224. In Fig. 223 the connection of the angle to the plate is an ordinary lap joint and the rivets are in single shear or bearing. In Fig. 224 the connection can be considered as a double lap joint and the rivets are in double shear or bearing.

129*c*. Lug or Clip Angles in Connections.—Specifications usually require that an angle be connected by both legs (see Fig. 225). The allowable value of an angle connected by one leg varies somewhat. Some specifications allow only the value of the leg connected. Others allow from 75 to 80% of the net area of the angle. When an angle is connected by both legs, 90% of the net area is usually allowed. Tests show that an angle is stronger when connected by both legs.

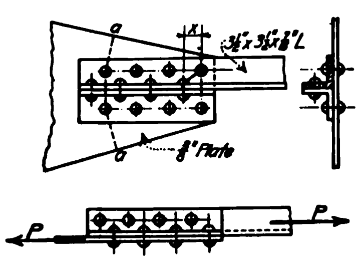

Fig. 225.

When a lug angle is used to connect an angle carrying tensile stress, the distance X (see Fig. 225) should be such that the area of the angle will not be reduced by more than one hole.

The net area of the gusset plate on line *aa* (see Fig. 225) should be such that the net area is equal to or greater than the net area of the member connected. If the connection is eccentric, both bending and direct stress should be considered in determining the area of the plate at section *aa* (see bottom chord splice).

The computations for the connection shown in Fig. 225 will be illustrated by the following problem.

Illustrative Problem.—Determine the strength of the connection shown in Fig. 226. The allowable tensile stress on the angle is 16,000 lb. per sq. in. Assume ¾-in. rivets with the following values:

Shear, 10,000 lb. per sq. in.
Bearing, 20,000 lb. per sq. in.

The lug angle is assumed to transmit one-half of the total stress to the plate. Also the stress is assumed to be divided equally among the rivets.

Table 11 shows that a ¾-in. rivet is good for 4420 lb. in single shear, and 5625 lb. in bearing on a ⅜-in. plate. The rivets are therefore good for (4420) (8) = 35,360 lb.

The 3½ × 3½ × 5⁄16-in. angle has a gross area of 2.87 sq. in.

Table 12 shows that the area to be deducted for a ¾-in. rivet (⅞-in. hole) through 5⁄16-in. metal is 0.38 sq. in. The net area of the angle therefore is 2.87 − 0.38 = 2.49 sq. in. At 16,000 lb. per sq. in. the angle is good for

(16,000)(2.49) = 39,840 lb.

But Art. 129*c* allows only 90 % of the net area of the angle for a connection of this kind. This value is

(0.9)(39,840) = 35,860 lb.

Since this is greater than the value of the rivets, the strength of the connection is 35,360 lb.

For a properly designed joint the strength should not depend on the rivets. The joint should be strong enough so that if a failure occurs it will be in the member rather than in the joint.

The number of rivets connecting the lug angle to the main angle should be the same as used in connecting the lug angle to the plate because, in this case, the rivets in both connections are in single shear. If the thickness of the plate were such that the rivets connecting the lug angle to the plate were governed by the bearing value, then in one case bearing would govern and in the other the single shear value. Conditions might be reversed, however, and the rivets connecting the lug angle to the main angle might be governed by their strength in bearing.

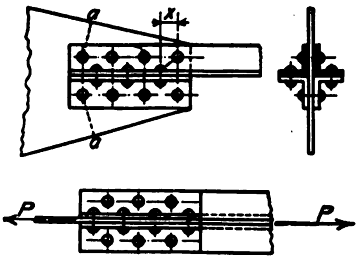

Fig. 226.

In order that the area of the angle will be reduced by not more than one rivet hole at a point of maximum stress, the first rivet connecting the main angle to the plate must be spaced far enough from the first rivet connecting the lug angle to the main angle so that the area through these holes will not be less than the net area considering one hole out. Table 13 shows that this distance should be 2⅝ in. (gage 2 in. on a 3½-in. angle, see Table 5).

Diagram 16 may also be used as follows: The value of *X* should be zero and the value of *g* is 3 9⁄16 in. If the rivets used were ⅞ in. in diameter, the value of *p* could be taken from the diagram at the point where *g* = 3 9⁄16 in. cuts the *A-A* line. As the rivets are ¾ in., the value of the gage *g* should be multiplied by (¾ + ⅛). The value of *p* will then be found where the new value of *g* cuts line *A-A*, or

(3 9⁄16)(¾ + ⅛) = 3.12 in.

Where this value of *g* cuts line *A—A*, a value of *p* equal to 3.38 is found.

The value of *X* in Fig. 225 then should be 3⅜ in. if this method of computing net areas is used.

The computations for the connection shown in Fig. 226 are similar to those just given except that the rivets connecting both the lug angle and the main angle are in bearing or double shear.

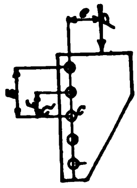

Fig. 227.

130. Eccentric Connections.—When the line of action of a force *P* does not pass through the center of gravity of the group of rivets (see Fig. 227), the joint should be designed to resist both the load *P* and the moment *Pe*. The moment *Pe* tends to revolve the plate about a center *c'*. The stress on any rivet, caused by the moment *Pe*, depends on the distance of the rivet from the center of gravity "*c*" of the group of rivets. The sum of the moments about "*c*" of the stresses on each rivet should equal *Pe*.

Assume that a rivet at a unit distance from *c* takes stress *s*, then at any distance *r*, the stress taken by a rivet will be *rs*; and for a distance r_2, it will be r_2s. Since the center of gravity (in this case) of the group of rivets is at the center of the rivet at *c*, this rivet will not be stressed by the moment *Pe*. The sum of the moments about *c* of the stresses taken by the rivets, is $2[(r_1s \times r_1) + (r_2s \times r_2)]$. The quantity inside the brackets is multiplied by 2 in order to include the rivets below *c*. Then

$$2(r_1^2s + r_2^2s) = Pe$$

or

$$s = \frac{Pe}{2(r_1^2 + r_2^2)}$$

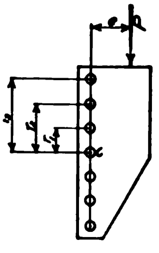

Fig. 228.

If two more rivets are added, as shown in Fig. 228, the value of *s* would be

$$s = \frac{Pe}{2(r_1^2 + r_2^2 + r_3^2)}$$

Consider Fig. 229

$$s = \frac{Pe}{2(r^2 + r^2 + g^2)}$$

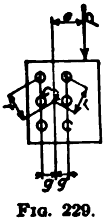

Fig. 229.

Expressing these equations in words: To find the stress s, caused by a moment Pe on a rivet at a unit distance from the center of gravity of the group of rivets, divide the moment Pe by the sum of the squares of the distance of each rivet from the center of gravity of the group.

Considering the values shown on Fig. 230

$$s = \frac{(4)(20{,}000)}{2(16 + 64)} = 500 \text{ lb.}$$

A moment of (4)(20,000) = 80,000 in.-lb. would cause a stress of 500 lb. on a rivet at 1 in. from c; at 4 in. from c the stress would be (4)(500) = 2000 lb.; and at 8 in. it would be (8)(500) = 4000 lb. In addition, each rivet takes a stress of $\frac{P}{n}$ lb. (n equals the number of rivets in the connection). For this connection the stress is $\frac{20{,}000}{5}$ = 4000 lb. per rivet and acts parallel to the direction of P. The stress on each rivet, caused by the moment Pe, acts perpendicular to a straight line between c and the center of the rivet in question. In this case, the direction is horizontal for each rivet (see Fig. 233a). The stress on a rivet is the resultant of the stress caused by the moment Pe and the stress $\frac{P}{n}$. The stress on the rivet at c is 4000 lb.; at 4 in. from c, the stress is the resultant of 2000 and 4000 lb. or

$$\sqrt{2000^2 + 4000^2} = 4470 \text{ lb.}$$

and at 8 in. from c

$$\sqrt{4000^2 + 4000^2} = 5650 \text{ lb.}$$

These results may be obtained graphically as shown on Fig. 233a. The only difference in Figs. 230, 231, and 232 is the location and direction of the force P. The stresses on the rivets, however, will vary and are as shown in Figs. 233(b) and 233(c).

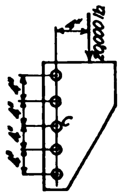

FIG. 230.

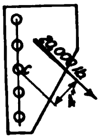

FIG. 231.

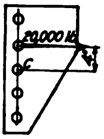

FIG. 232.

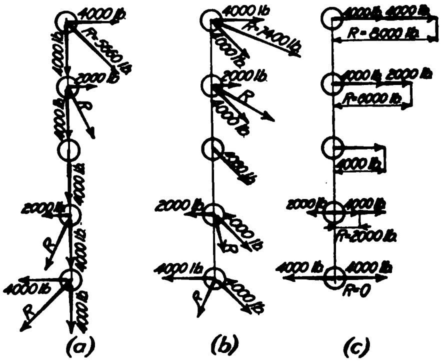

FIG. 233.

In computing the stresses on rivets in connections of this kind, it is necessary to know the square of the distance of each rivet from the center of gravity of the group of rivets. Table 15 gives the square of numbers varying by $\frac{1}{16}$ from 1 to 42 in. and will save a great deal of time in finding these values. This table may also be used in designing web splices for plate girders (see Art. 127).

To illustrate the use of the table, the stress s on a rivet at a unit distance from c (see Fig. 234) will be computed. Since the rivets are symmetrically arranged about aa and bb, it is necessary to find the square of the distance of each rivet from c for one-quarter and then multiply the result by 4.

From Table 15

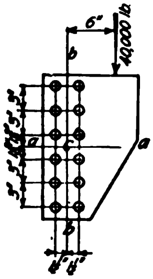

FIG. 234.

$(1\frac{1}{2})^2 = 2.25$
$(1\frac{3}{8})^2 = 1.89$

$4.14 = r_1^2$
$(1\frac{1}{2})^2 = 2.25$
$(4\frac{3}{8})^2 = 19.14$

$21.39 = r_2^2$
$(1\frac{1}{2})^2 = 2.25$
$(7\frac{3}{8})^2 = 54.39$

$56.64 = r_3^2$

The sum of the r squares is (4.14 + 21.39 + 56.64)4 = 328.68, and

$$s = \frac{(6)(40{,}000)}{328.68} = 730 \text{ lb.}$$

Since $1\frac{1}{2}$ in. enters the computations 3 times, the following method can be used:

$(1\frac{1}{2})^2 = 2.25$	(2.25)(3) = 6.75
$(1\frac{3}{8})^2 =$	1.89
$(4\frac{3}{8})^2 =$	19.14
$(7\frac{3}{8})^2 =$	54.39
	82.17

82.17 × 4 = 328.68, the same as above.

TABLE 15

	0	1/16 = 0.0625	1/8 = 0.1250	3/16 = 0.1875	1/4 = 0.2500	5/16 = 0.3125	3/8 = 0.3750	7/16 = 0.4375	1/2 = 0.5000	9/16 = 0.5625	5/8 = 0.6250	11/16 = 0.6875	3/4 = 0.7500	13/16 = 0.8125	7/8 = 0.8750	15/16 = 0.9375
1	1.00	1.13	1.27	1.41	1.56	1.72	1.89	2.07	2.25	2.44	2.64	2.85	3.06	3.29	3.52	3.75
2	4.00	4.25	4.52	4.79	5.06	5.35	5.64	5.94	6.25	6.57	6.89	7.22	7.56	7.91	8.27	8.63
3	9.00	9.38	9.77	10.16	10.56	10.97	11.39	11.82	12.25	12.69	13.14	13.60	14.06	14.54	15.02	15.50
4	16.00	16.50	17.02	17.54	18.06	18.60	19.14	19.70	20.25	20.82	21.39	21.97	22.56	23.16	23.77	24.38
5	25.00	25.63	26.27	26.91	27.56	28.22	28.89	29.57	30.25	30.94	31.65	32.35	33.06	33.79	34.52	35.25
6	36.00	36.75	37.52	38.29	39.06	39.85	40.64	41.44	42.25	43.07	43.89	44.72	45.56	46.41	47.27	48.13
7	49.00	49.88	50.77	51.66	52.56	53.47	54.39	55.32	56.25	57.19	58.14	59.10	60.06	61.04	62.02	63.00
8	64.00	65.00	66.02	67.04	68.06	69.10	70.14	71.19	72.25	73.32	74.39	75.47	76.56	77.66	78.77	79.88
9	81.00	82.13	83.27	84.41	85.56	86.72	87.89	89.07	90.25	91.44	92.64	93.85	95.06	96.29	97.52	98.75
10	100.00	101.25	102.52	103.79	105.06	106.35	107.64	108.94	110.25	111.57	112.89	114.22	115.56	116.91	118.26	119.63
11	121.00	122.38	123.77	125.16	126.56	127.97	129.39	130.82	132.25	133.69	135.14	136.60	138.06	139.54	141.02	142.50
12	144.00	145.50	147.01	148.54	150.06	151.60	153.16	154.69	156.25	157.81	159.39	160.98	162.56	164.16	165.76	167.39
13	169.00	170.63	172.27	173.91	175.57	177.22	178.89	180.56	182.25	183.95	185.65	187.34	189.06	190.79	192.51	194.26
14	196.00	197.76	199.51	201.28	203.07	204.85	206.64	208.44	210.25	212.07	213.90	215.73	217.57	219.41	221.27	223.13
15	225.00	226.87	228.77	230.66	232.56	234.48	236.39	238.32	240.25	242.19	244.14	246.10	248.07	250.04	252.01	254.00
16	256.00	258.01	260.02	262.04	264.07	266.10	268.14	270.19	275.25	274.32	276.39	278.47	280.58	282.66	284.76	286.88
17	289.00	291.13	293.27	295.42	297.56	299.72	301.90	304.07	306.25	308.45	310.64	312.85	315.06	317.29	319.52	321.75
18	324.00	326.26	328.52	330.78	333.06	335.35	337.64	339.94	342.25	344.56	346.90	349.23	351.56	353.91	356.27	358.63
19	361.00	363.38	365.76	368.17	370.56	372.97	375.39	377.81	380.25	382.69	385.14	387.61	390.07	392.53	395.02	397.51
20	400.00	402.51	405.01	407.53	410.07	412.60	415.14	417.69	420.25	422.81	425.39	427.97	430.56	433.17	435.77	438.38
21	441.00	443.64	446.27	448.91	451.57	454.22	456.90	459.56	462.25	464.95	467.64	470.35	473.07	475.79	478.51	481.25
22	484.00	486.75	489.51	492.28	495.06	497.85	500.65	503.44	506.25	509.07	511.89	514.73	517.57	520.42	523.27	526.13
23	529.00	531.88	524.77	587.67	540.56	543.47	546.39	549.32	552.25	555.19	558.14	561.10	564.06	567.03	570.01	573.01
24	576.00	579.01	582.02	585.03	588.07	591.09	594.14	597.20	600.25	603.02	606.38	609.47	612.56	615.66	618.77	621.88
25	625.00	628.13	631.27	634.41	637.56	640.73	643.90	647.06	650.25	653.44	656.64	659.85	663.06	666.29	669.51	672.75
26	676.00	679.25	682.52	685.79	689.07	692.35	695.64	698.95	702.25	705.57	708.90	712.22	715.57	718.91	722.26	725.63
27	729.00	732.38	735.77	739.17	742.57	745.98	749.39	752.82	756.25	759.69	763.14	766.60	777.07	773.04	777.01	780.51
28	784.00	787.51	791.02	794.53	798.06	801.61	805.15	808.69	812.55	815.82	819.39	822.97	826.56	830.16	833.76	837.37
29	841.00	844.63	848.26	851.90	855.56	859.22	826.89	866.56	870.25	873.94	877.64	881.35	885.07	888.78	892.51	896.26
30	900.00	903.76	907.52	911.29	915.06	918.85	922.64	926.44	930.25	934.07	937.89	941.72	945.56	949.91	953.27	957.13
31	961.00	964.87	968.76	972.66	976.57	980.47	984.38	988.38	992.25	996.19	1000.1	1004.1	1008.1	1012.0	1016.0	1020.0
32	1024.0	1028.0	1032.0	1036.0	1040.1	1044.1	1048.2	1052.2	1056.3	1060.3	1064.4	1068.5	1072.6	1076.7	1080.8	1084.9
33	1089.0	1093.0	1097.3	1101.4	1105.6	1109.7	1113.9	1118.1	1122.3	1126.4	1130.7	1134.9	1139.1	1143.3	1147.5	1151.8
34	1156.0	1160.3	1164.5	1168.8	1173.1	1177.3	1181.6	1185.9	1190.3	1194.6	1198.9	1203.2	1207.6	1211.9	1216.3	1220.6
35	1225.0	1229.4	1233.8	1238.2	1242.6	1247.0	1251.4	1255.8	1260.3	1264.7	1269.1	1273.6	1278.1	1282.5	1287.0	1291.5
36	1296.0	1300.5	1305.0	1309.5	1314.1	1318.6	1323.1	1327.7	1332.3	1336.8	1341.4	1346.0	1350.6	1355.2	1359.8	1364.4
37	1369.0	1373.6	1378.3	1382.9	1387.8	1392.2	1396.0	1401.6	1406.3	1410.9	1415.6	1420.3	1425.1	1429.8	1434.5	1439.3
38	1444.0	1448.8	1453.5	1458.3	1463.1	1467.9	1472.6	1477.4	1482.3	1487.1	1491.9	1496.7	1501.6	1506.4	1511.3	1516.1
39	1521.0	1525.9	1530.8	1535.7	1540.6	1545.5	1550.4	1555.3	1560.3	1565.2	1570.2	1575.1	1580.0	1585.0	1590.0	1595.0
40	1600.0	1605.0	1610.0	1615.0	1620.1	1625.1	1630.1	1635.2	1640.3	1645.3	1650.4	1655.5	1660.6	1665.7	1670.8	1675.9
41	1681.0	1686.1	1691.2	1696.2	1701.6	1706.7	1711.9	1717.1	1722.8	1727.4	1732.6	1737.9	1743.1	1748.3	1753.5	1758.8
42	1764.0	1769.3	1774.5	1779.8	1785.1	1790.4	1795.6	1800.9	1806.3	1811.6	1816.9	1822.2	1827.6	1832.3	1838.3	1843.6

The resultant stress on a rivet may be found as follows without finding the components in two directions[1] as in the method just given: Draw a line *aa* (see Fig. 235) through the center of gravity *c* of the group of rivets and perpendicular to the line of action of *P*. The stress $\frac{P}{n}$ on each rivet is equal and acts downward (parallel to the line of action of *P*). The stress on any rivet on line *aa* due to the moment *Pe* acts perpendicular to line *aa*. Between *c* and the line of action of *P*, this stress will be downward; on the left of *c*, the action will be upward. On the right of *c* the resultant stress on a rivet on line *aa* will be the sum of the stress due to *P* and that due to *Pe*; on the left of *c*, the resultant stress will be the difference. At some point to the left of *c*, on line *aa*, the upward stress will equal the downward and there will be a point of zero stress. This is the point about which the plate would revolve. This point may be determined by the following formula

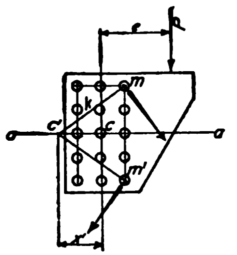

FIG. 235.

$$X' = \frac{\Sigma r^2}{ne}$$

in which X' is the distance from *c* to the point, Σr^2 is the sum of the squares of the distance of each rivet from the center of gravity of the group of rivets, *n* is the number of rivets in the group, and *e* is the distance from the center of gravity of the group of rivets to the line of action of *P*.

The stress *s* on a rivet at a unit distance from *c* is found as in the previous method. Then the stress on rivets *m* and *m'* (see Fig. 235) is *ks*, and acts perpendicular to lines *k*.

Consider the same connection as shown in Fig. 230. The distance to *c'* (see Fig. 236*a*) is $X' = \frac{\Sigma r^2}{ne}$. From the previous problem, Σr^2 is 160, *n* is 5, and *e* is 4 in.

$$X' = \frac{160}{(5)(4)} = 8 \text{ in.}$$

The distance *k* from *c'* to the most stressed rivet is

$$\sqrt{8^2 + 8^2} = 11.31 \text{ in.}$$

and the stress taken by this rivet is (since *s* is 500 lb. from previous problem) 11.31 × 500 = 5655 lb. and acts perpendicular to line *k*. Since θ is 45 deg., *R* makes an angle of 45 deg. with the vertical. These values check with those in the previous problem. Considering the connection shown in Fig. 236(*b*)

$$x = 8 \cos 45 \text{ deg.} = (8)(0.707) = 5.66 \text{ in.}$$
$$y = 8 \sin 45 \text{ deg.} = (8)(0.707) = 5.66 \text{ in.}$$
$$k = \sqrt{5.66^2 + 1.366^2}$$

Table 15 shows that 0.66 in. is about halfway between $\frac{5}{8}$ and $\frac{11}{16}$ in. Then from Table 15

$$(5.66)^2 = 32$$
$$(13.66)^2 = 186.6$$

and from the same table

$$k = 14\tfrac{3}{4} \text{ in. or } 14.75 \text{ in.}$$

The top rivet receives the maximum stress, which is

$$(14.75)(500) = 7375 \text{ lb.}$$
$$\text{Tan } \alpha = \frac{5.66}{13.66} = 0.4144 = 22 \text{ deg. } 30 \text{ min.}$$

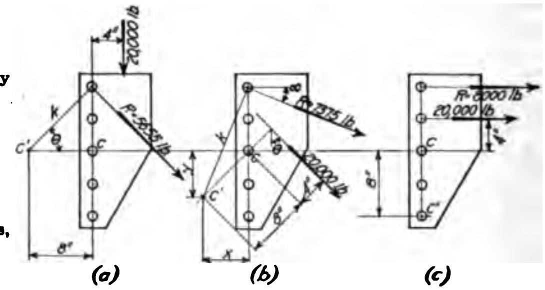

FIG. 236.

These values check with those obtained by the other method.

Consider the connection shown in Fig. 236(*c*). In this connection *c'* falls at the center of the bottom rivet and the rivet at the top receives the maximum stress. The value of *k* is 16 in. and the stress taken by the top rivet is

$$(16)(500) = 8000 \text{ lb.}$$

and acts parallel to the direction of *P*. Since *c'* is at the center of the bottom rivet, there will be no stress in this rivet. These values check with those obtained by the other method.

131. Avoiding Eccentric Connections.—Eccentric connections should be avoided if possible because they not only put additional stress on the rivets but also cause bending in the members connected. The stresses due to this bending may in some cases be very high. Eccentric connections, of course, have to be used in many cases; on the other hand, eccentric

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[1] See p. 518, *Eng. Rec.*, Nov. 7, 1914.

connections are often used where they can be avoided. The following figures illustrate a few of these connections:

The connections shown in Figs. 237(*a*) and 237(*b*) are both eccentric. In Fig. 237(*c*) the line of action of P_1, P_2, and R meet in a point at the center of the group of rivets in the bottom chord connection thus causing no bending in the joint. When there is a moment in the joint due either to eccentricity as in Figs. 237(*a*) and 237(*b*), or due to the top chord acting as a beam plate *a* should be made thicker than for the joint in Fig. 237(*c*). Usually a ½-in. plate is used and a few extra rivets added.

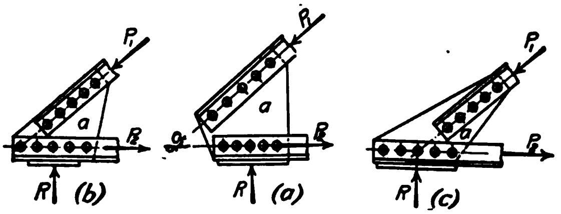

Fig. 237.

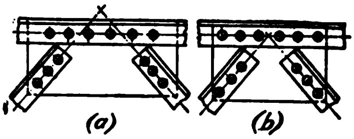

Fig. 238.

The connection in Fig. 238(*a*) should be made as shown in Fig. 238(*b*), that in Fig. 239(*a*) as shown in Fig. 239(*b*), and that in Fig. 240(*a*) as shown in Fig. 240(*b*).

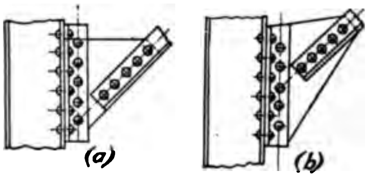

Fig. 239.

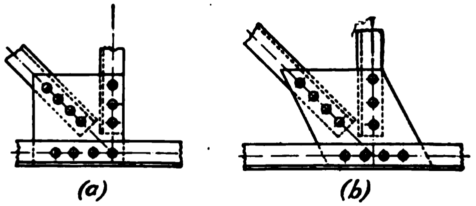

Fig. 240.

132. Requirements for a Good Joint.—(1) The rivet holes should match; the rivets should be properly heated and well driven.

(2) The line of thrust should pass through the center of gravity of the group of rivets and the rivets should be symmetrically arranged about this line.

(3) Direct tension on rivet heads should not be allowed.

(4) For a tension member, the rivets should be so arranged that the area of the member joined is not reduced more than necessary.

(5) The number and size of rivets should be sufficient to develop the member joined.

(6) The total thickness of metal should not exceed four diameters of the rivet used.

(7) No loose fillers should be used.

(8) Members should be straight and bolts used to draw them together before the rivets are driven.

133. Pin Connections.

133*a*. Bearing, Bending, and Shearing Stresses.—In building construction, pins are sometimes used to connect members meeting at a joint (see Fig. 241). Pins are subjected to bearing, bending, and shearing stresses; the latter, however, may usually be neglected except possibly for small pins. Shear and bearing values are computed in the same way as for rivets. Tables 16 and 17 give the bearing and bending moment values for different sizes of pins for various unit stresses.

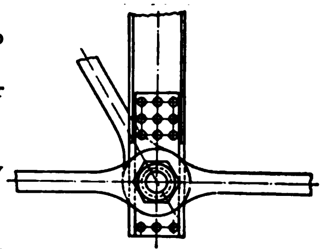

Fig. 241.

In computing the bending moment on a pin, the stresses from the different members are usually considered to be concentrated at the center of the bearing area of each member (see Fig. 242).

Illustrative Problem.—Compute the maximum bending moment on the pin shown in Fig. 242.

The bending moment is uniform between the centers of plates *a*, so the maximum moment is at the center of plate *a*, and is

$$(75{,}000)(1\tfrac{1}{2}) = 112{,}500 \text{ in.-lb.}$$

The moment at the center of the pin would be the same or

$$(75{,}000)(2\tfrac{1}{2}) - (75{,}000)(1) = (75{,}000)(1\tfrac{1}{2}) = 112{,}500 \text{ in.-lb.}$$

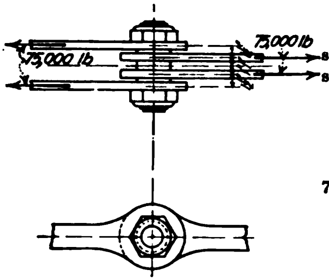

Fig. 242.

Illustrative Problem.—Consider the pin to be 4 in. in diameter. What should be the thickness of each of the members if the allowable unit bearing stress is 20,000 lb. per sq. in.?

$$(t)(20{,}000)(4) = 75{,}000$$

$$t = \frac{75{,}000}{(4)(20{,}000)} = \frac{15}{16} \text{ in.}$$

Table 16 shows that a 1-in. plate is good for 80,000 lb. Then for 75,000 lb., the thickness should be

$$\frac{75{,}000}{80{,}000} = \frac{15}{16} \text{ in.}$$

When members connected at a joint act in different directions (see Fig. 243), the stresses should be resolved into two planes at right angles to each other (usually horizontal and vertical). In Fig. 243 the stress in the diagonal member 3 should be resolved into its horizontal and vertical components. Then all the loads acting on the pin should be indicated as shown in Fig. 244, where *a* represents the horizontal forces and *b* the vertical forces.

To find the moment on the pin, the moments due to horizontal loads should first be computed at the different points; then the moments due to the vertical loads. The moment at any point, then, would be the resultant of the horizontal and vertical moments at that point, or

$$M = \sqrt{M_H{}^2 + M_V{}^2}$$

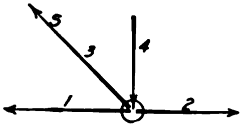

Fig. 243.

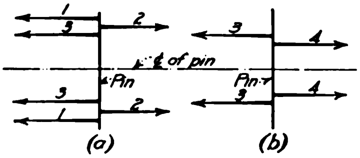

Fig. 244.

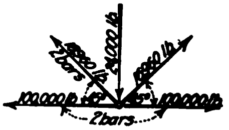

Fig. 245.

in which M_H and M_V are the horizontal and vertical moments at the same point on the pin. The maximum value for M then would be at a point where the resultant of M_H and M_V is a maximum.

The maximum shear will be the maximum resultant obtained from Figs. 244(*a*) and (244(*b*)

$$V \text{ max.} = \sqrt{V_H{}^2 + V_2{}^2}$$

The required bearing area should be computed for the stress in each member.

When the members are placed symmetrically about the center line (see Fig. 244) as they should be, only one-half of the pin needs to be considered.

Illustrative Problem.—Compute the maximum moment on the pin in the joint shown in Fig. 245. The horizontal and vertical components of 8480 lb. are

$$8480 \times \sin 45 \text{ deg.} = 8480 \times \cos 45 \text{ deg.} = 6000 \text{ lb.}$$

Fig. 246 shows the stresses in their assumed positions with the distance of each from the center line of the pin.

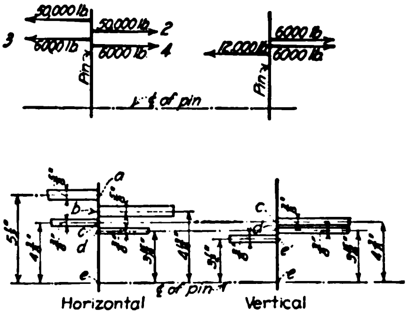

Fig. 246.

Hor. mom. about $b = (50{,}000)(\tfrac{11}{16}) = 34{,}380$ in.-lb.
Hor. mom. about $c = (50{,}000)(1\tfrac{1}{8}) - (50{,}000)(\tfrac{7}{16}) = 34{,}380$ in.-lb.
Hor. mom. about $d = (50{,}000)(1\tfrac{9}{16}) - (50{,}000)(\tfrac{7}{8}) + (6000)(\tfrac{7}{16}) = 37{,}000$ in.-lb.
Hor. mom. about $e = (50{,}000)(5\tfrac{1}{2}) - (50{,}000)(4\tfrac{13}{16}) + (6000)(4\tfrac{3}{8}) - (6000)(3\tfrac{15}{16}) = 37{,}000$ in.-lb.
Vert. mom. about $c = 0$
Vert. mom. about $d = (6000)(\tfrac{7}{16}) = 2630$ in.-lb.
Vert. mom. about $e' = (6000)(\tfrac{7}{8}) + (6000)(\tfrac{7}{16}) = 7880$ in.-lb.
Vert. mom. about $e = (6000)(4\tfrac{3}{8}) + (6000)(3\tfrac{15}{16}) - (12{,}000)(3\tfrac{1}{2}) = 7880$ in.-lb.

TABLE 16.[1]—PINS—BEARING VALUES IN POUNDS ON METAL ONE INCH THICK

Bearing Value = Diameter of Pin × Bearing Stress per Square Inch

Pin		Bearing stresses in pounds per square inch				
Diameter (inches)	Area (sq. in.)	12000	15000	20000	22000	24000
1	0.785	12000	15000	20000	22000	24000
1¼	1.227	15000	18800	25000	27500	30000
1½	1.767	18000	22500	30000	33000	36000
1¾	2.405	21000	26300	35000	38500	32000
2	3.142	24000	30000	40000	44000	48000
2¼	3.976	27000	33800	45000	49500	54000
2½	4.909	30000	37500	50000	55000	60000
2¾	5.940	33000	41300	55000	60500	66000
3	7.069	36000	45000	60000	66000	72000
3¼	8.296	39000	48800	65000	71500	78000
3½	9.261	42000	52500	70000	77000	84000
3¾	11.045	45000	56300	75000	82500	90000
4	12.566	48000	60000	80000	88000	96200
4¼	14.186	51000	63800	85000	93500	10200
4½	15.904	54000	67500	90000	99000	108000
4¾	17.721	57000	71300	95000	10450	114000
5	19.635	60000	75000	100000	110000	120000
5¼	21.648	63000	78800	105000	115500	126000
5½	23.758	66000	82500	110000	121000	132000
5¾	25.967	69000	86300	115000	126500	138000
6	28.274	72000	90000	120000	132000	144000
6¼	30.680	75000	93800	125000	137500	150000
6½	33.183	78000	97500	130000	143000	156000
6¾	35.785	81000	101300	135000	148500	162000
7	38.485	84000	105000	140000	154000	168000
7¼	41.282	87000	108800	145000	159500	174000
7½	44.179	90000	112500	150000	165000	180000
7¾	47.173	93000	116300	155000	170000	186000
8	50.265	96000	120000	160000	176000	192000
8¼	53.456	99000	123800	165000	181500	198000
8½	56.745	102000	127500	170000	187000	204000
8¾	60.132	105000	131300	175000	192500	210000
9	63.617	108000	135000	180000	198000	216000
9¼	67.201	111000	138800	185000	203500	222000
9½	70.882	114000	142500	190000	209000	228000
9¾	74.662	117000	146300	195000	214500	234000
10	78.540	120000	150000	200000	220000	240000
10¼	82.516	123000	153800	205000	225500	246000
10½	86.590	126000	157500	210000	231000	252000
10¾	90.763	129000	161300	215000	236500	258000
11	95.033	132000	165000	220000	242000	264000
11¼	99.402	135000	168800	225000	247500	270000
11½	103.869	138000	172500	230000	253000	276000
11¾	108.434	141000	176300	235000	298500	282000
12	113.097	144000	180000	240000	264000	288000

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

TABLE 17.[1]—PINS—BENDING MOMENTS IN INCH POUNDS

Bending Moment = (Diameter of Pin)^3 × 0.098175 × Stress per Square Inch

Pin		Fiber stress in pounds per square inch						
Diameter (inches)	Area (sq. in.)	15000	18000	20000	22000	22500	24000	25000
1	0.785	1500	1800	2000	2200	2200	2400	2500
1¼	1.227	2900	3500	3800	4200	4300	4600	4800
1½	1.767	5000	6000	6600	7300	7500	8000	8300
1¾	2.405	7900	9500	10500	11600	11800	12600	13200
2	3.142	11800	14100	15700	17300	17700	18800	19600
2¼	3.976	16800	20100	22400	24600	25200	26800	28000
2½	4.909	23000	27600	30700	33700	34500	36800	38300
2¾	5.940	30600	36800	40800	44900	45900	49000	51000
3	7.069	39800	47700	53000	58300	59600	63600	66300
3¼	8.296	50600	60700	67400	74100	75800	80900	84300
3½	9.621	63100	75800	84200	92600	94700	101000	105200
3¾	11.045	77700	93200	103500	113900	116500	124300	129400
4	12.566	94200	113100	125700	138200	141400	150800	157100
4¼	14.186	113000	135700	155700	165800	169600	180900	188400
4½	15.904	134200	161000	178900	196800	201300	214700	223700
4¾	17.721	157800	189400	210400	231500	236700	252500	263000
5	19.635	184100	220900	245400	270000	276100	294500	306800
5¼	21.648	213100	255700	284100	312500	319600	340900	355200
5½	23.758	245000	294000	326700	359300	367500	392000	408300
5¾	25.967	280000	336000	373300	410600	419900	447900	466600
6	28.274	318100	381700	424100	466500	477100	508900	530100
6¼	30.680	359500	431400	479400	527300	539300	575200	599200
6½	33.183	404400	485300	539200	593100	606600	647100	674000
6¾	35.785	452900	543500	603900	664300	679400	724600	754800
7	38.485	505100	606100	673500	740800	757700	808200	841800
7¼	41.282	561200	673400	748200	823100	841800	897900	935300
7½	44.179	621300	745500	828400	911200	931900	994000	1035400
7¾	47.173	685500	822600	914000	1005400	1028200	1096800	1142500
8	50.265	754000	904800	1005300	1105800	1131000	1206400	1256600
8¼	53.456	826900	992300	1102500	1212800	1240400	1323000	1378200
8½	56.745	904400	1085300	1205800	1326400	1356600	1447000	1507300
8¾	60.132	986500	1183900	1315400	1446900	1479800	1578500	1644200
9	63.617	1073500	1288300	1431400	1574500	1610300	1717700	1789200
9¼	67.201	1165500	1398600	1554000	1709400	1748300	1864800	1942500
9½	70.882	1262600	1515100	1683500	1851800	1893900	2020100	2104300
9¾	74.662	1364900	1637900	1819900	2001900	2047400	2183900	2274900
10	78.540	1472600	1767100	1963500	2159800	2208900	2356200	2454400
10¼	82.516	1585900	1903000	2114500	2325900	2378800	2537400	2643100
10½	86.590	1704700	2045700	2273000	2500300	2557100	2727600	2841200
10¾	90.763	1829400	2195300	2439200	2683200	2744100	2927100	3049100
11	95.033	1960100	2352100	2613400	2874800	2940100	3136100	3266800
11¼	99.402	2096800	2516100	2795700	3075200	3145100	3354800	3494600
11½	103.869	2239700	2687600	2986200	3284900	3359500	3583500	3732800
11¾	108.434	2388900	2866700	3185300	3503800	3583400	3822300	3981600
12	113.097	2544700	3053600	3392900	3732200	3817000	4071500	4241200

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co. Pittsburgh, Pa.

Max. mom., then, is at *e* and is

$$\sqrt{(37{,}000)^2 + (7880)^2} = 37{,}800 \text{ in.-lb.}$$

Illustrative Problem.—Assume an allowable unit stress of 24,000 lb. per sq. in. in bearing and a unit fiber stress of 24,000 lb. per sq. in. Determine the size of pin necessary for the joint in Fig. 245. The width of members shown in Fig. 246 are to be used.

The maximum bending moment is 37,800 in.-lb. Table 17 shows a pin 2¾ in. in diameter to be satisfactory for moment. By inspection it is seen that the ⅝-in. plate governs for bearing. The required diameter is

$$(\tfrac{5}{8})(24{,}000)(d) = 50{,}000$$

$$d = \frac{50{,}000}{(\tfrac{5}{8})(24{,}000)} = 3.33 \text{ in.}$$

A 3½-in. pin should be used.

Table 16 shows that a 3½-in. pin is good for 84,000 × ⅝ = 52,500 lb. in bearing, which is satisfactory. The maximum shear is 50,000 lb.; and the required area for shear at a unit stress of 12,000 lb. per sq. in. is

$$\frac{50{,}000}{12{,}000} = 4.16 \text{ sq. in.}$$

A pin 2½ in. in diameter would therefore be satisfactory for shear as its area is 4.91 sq. in.

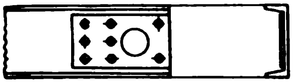


FIG. 247.

133b. Pin Plates.—Usually the webs of members, connected by pins, are not thick enough to transfer the stress between the pin and the member. Plates are riveted to the web (see Fig. 247) to increase the bearing area and enough rivets are used to transfer the stress taken by the pin plates to the web. The stress in bearing taken by the web and by the pin plate is in proportion to the thickness of each.

Illustrative Problem.—Consider the thickness of the channel web to be ¼ in. and that of the plate ⅜ in Compute the number of ¾-in. rivets necessary to connect the plate to the channel. Assume a 3-in. pin.

Bearing value on pin	24,000 lb. per sq. in.
Bearing value of rivets	24,000 lb. per sq. in.
Shearing value of rivets	12,000 lb. per sq. in.

The stress taken by the pin plate is

$$(\tfrac{3}{8})(24{,}000)(3) = 27{,}000 \text{ lb.}$$

The value of a ¾-in. rivet in single shear is (from Table 11) 5300 lb. and the bearing value is 4500 lb. The number of rivets required is, therefore

$$\frac{27{,}000}{4500} = 6 \text{ rivets}$$

The value of the pin connection is

$$(\tfrac{5}{8})(24{,}000)(3) = 45{,}000 \text{ lb.}$$

Illustrative Problem.—Suppose a ¼-in. plate is used on the back of the channel and the ⅜-in. plate is made ½ in. Determine the number of rivets required to develop the value of the pin in bearing.

The total thickness of metal is

$$\tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{2} = 1 \text{ in.}$$

and the bearing value is

$$(1)(24{,}000)(3) = 72{,}000 \text{ lb.} \qquad \text{(see Table 16)}$$

The bearing on the ½-in. plate is

$$(\tfrac{1}{2})(72{,}000) = 36{,}000 \text{ lb.}$$

The bearing on the ¼-in. plate is

$$(\tfrac{1}{4})(72{,}000) = 18{,}000 \text{ lb.}$$

One rivet is good for 4500 lb. If one-half of the rivet value, or 2250 lb., be allowed in each plate, the number of rivets required for the ¼-in. plate is

$$\frac{18{,}000}{2250} = 8 \text{ rivets}$$

Then, in the ⅜-in. plate, additional rivets at a value of 4500 will be necessary, or

$$\frac{36{,}000 - 18{,}000}{4500} = 4 \text{ rivets}$$

If the value of a rivet is assumed to be divided between the plates in proportion to their thicknesses, the values will be

For the $\frac{1}{4}$-in. plate .... 1500 lb.
For the $\frac{1}{2}$-in. plate .... 3000 lb.

Then the number of rivets required will be the same in each plate, or

$$\frac{18,000}{1500} = 12 \text{ rivets}$$

$$\frac{36,000}{3000} = 12 \text{ rivets}$$

The total number of rivets required to carry the stress in the plates is

$$\frac{36,000 + 18,000}{4500} = 12 \text{ rivets}$$

In designing tension members the net area through the pin hole and also at the back of the pin, should be such that failure will not occur at these points. Some specifications require that the net area on line *xx*(see Fig. 248) be 25% greater than the net area of the pin plate on *aa*, and that the net area on *yy* be 75% of the area on *xx*. Other specifications require that the net area on line *xx* be 25% greater than the net area of the pin plate on *aa* and that on *yy* be equal to the net area on *aa*. The net area of the plate on section *aa* should be equal to or greater than the net section of the member to which it is riveted. The method outlined under rivets should be used.

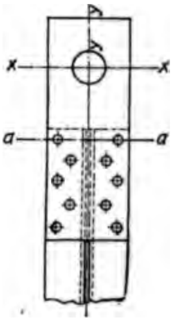


Fig. 248.

133c. Pin Packing.—A sketch showing the arrangement of the members connected by a pin should always be made in order that the different members will be placed properly when the structure is erected. Suppose in Fig. 246, members 2 and 3 are interchanged; the moments would then be (see Fig. 249).

Hor. mom. about a = $(1\frac{1}{8})(50,000) + (\frac{9}{16})(6000)$ = 59,625 in.-lb.

Hor. mom. about b = $(1\frac{11}{16})(50,000) + (1\frac{1}{8})(6000) - (\frac{9}{16})(50,000)$ = 63,000 in.-lb.

Vert. mom. about b = $(6000)(1\frac{1}{8})$ = 6750 in.-lb.

$$M_b = \sqrt{(63,000)^2 + (6750)^2} = 63,360 \text{ in.-lb.}$$

which is almost two times the maximum moment found for the other arrangement of members. When there is a space between two members, fillers should be used to keep them in position.

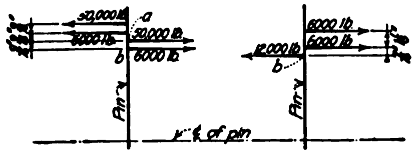


Fig. 249.

133d. Clearance.—In designing a pin-connected joint, usually $\frac{1}{16}$ in. is allowed between eyebars; $\frac{1}{8}$ in. between an eyebar and a built-up member; and $\frac{1}{4}$ in. between built-up members. Rivet heads or any projection should be considered and the above clearances allowed in addition to the height of the projection.

133e. Grip.—The length of a pin is computed allowing the above clearances. Then to this length $\frac{1}{4}$ to $\frac{3}{4}$ in. is added to obtain the grip. Tables 18 and 19 give the dimensions for standard pins. Cotter pins are not used a great deal except in lateral connections and when used the bars should be arranged so the pin will be in double shear.

133f. Pin Holes.—Specifications usually require that the diameter of a pin hole shall not exceed the diameter of the pin by more than $\frac{1}{50}$ in. for pins up to 5 in. in diameter; for larger pins, $\frac{1}{32}$ in. may be allowed. The distance center to center of pin holes is usually required to be correct to $\frac{1}{32}$ in.

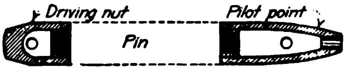
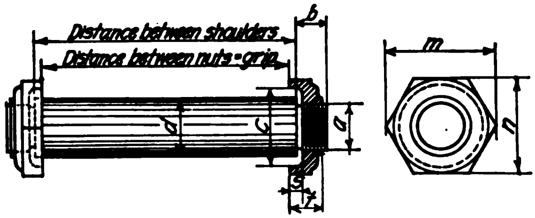


Fig. 250.

133g. Pilot Point and Driving Nut.—To prevent the threads on the ends of the pin from being injured when the pin is driven, a pilot point and driving nut are used (see Fig. 250). These are threaded the same as the pin nuts and after driving the pin, they are unscrewed and the nuts put on.

Table 18.[1]—Recessed Pin Nuts—American Bridge Company Standard

(All Dimensions in Inches)

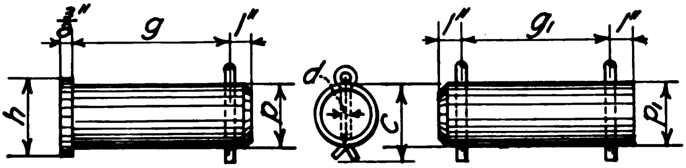


Diameter of pin, d	Pin: Thread a	Pin: Thread b	Pin: Add to grip	Nut: Thickness t	Nut: Diameter n	Nut: Diameter m	Nut: Diameter c	Nut: Depth s	Nut: Diameter rough hole	Nut: Weight (pounds)	Nut: Pattern No.
2, 2¼	1½		¼	⅞	2 15/16	3⅜	2⅝	¼	1 5/16	1.1	PN 21
2½, 2¾	2	1⅛	¼	1	3 9/16	4⅛	3⅛	¼	1 13/16	1.7	PN 22
3, *3¼, 3½	2½	1¼	¼	1⅛	4 5/16	5	3⅞	⅜	2 5/16	2.5	PN 23
*3¾, 4	3	⅜	½	1¼	4 ⅞	5⅝	4⅜	⅜	2 13/16	3.7	PN 24
*4¼, 4½, *4¾	3½	½	½	1⅜	5 ¾	6⅝	5¼	½	3 5/16	4.6	PN 25
5, *5¼	4	⅝	½	1½	6 ¼	7 3/16	5¾	½	3 13/16	6.2	PN 26
5½, *5¾, 6	4½	¾	½	1⅝	7	8⅛	6½	⅝	4 5/16	7.8	PN 27
*6¼, *6½	5	⅞	¾	1¾	7 ⅝	8⅞	7	⅝	4 13/16	9.9	PN 28
*6¾, 7	5½	2	¾	1⅞	8 ⅛	9⅜	7½	¾	5 5/16	11.8	PN 29
*7¼, 7½	5½	2	¾	1⅞	8 ⅝	10	8	¾	5 5/16	14.3	PN 30
*7¾, 8, *8¼	6	2¼	¾	2⅛	9 ⅜	10⅞	8¼	¾	5 13/16	18.6	PN 31
*8½, 9	6	2¼	¾	2⅛	10 ¼	11⅞	9⅝	¾	5 13/16	23.8	PN 32
*9½, 10	6	2⅜	¾	2¼	11 ¼	13	10⅝	¾	5 13/16	31.1	PN 33

Pins marked * are special.

Table 19.[1]—Cotter Pins—American Bridge Company Standard

(All Dimensions in Inches)



Pin p	Head h	g	Cotter c	Cotter d	Pins p₁	g₁	Cotter c	Cotter d
	1½	Net Grip + ½″	2	¼	1¼	Net Grip + ¾″	2	¼
1¼	1¾		2½	¼	1½		2½	¼
1½	2		2¾	¼	1¾		2¾	¼
1¾	2⅜		3	⅜	2		3	⅜
2	2⅝		3¼	⅜	2¼		3¼	⅜
2¼	2⅞		3¾	⅜	2½		3¾	⅜
2½	3⅛		4	⅜	2¾		4	⅜
2¾	3½		5	½	3		5	½
3	3¾		5	½	3¼		5	½
3¼	4		6	½	3½		6	½
3½	4¼		6	½	3¾		6	½
3¾								

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[1] From Pocket Companion, 20th edition, Carnegie Steel Co., Pittsburgh, Pa.

MASONRY ARCHES

By Alfred Wheeler Roberts

Flat arches are common in the walls of ordinary buildings for spanning over window or door openings, but in buildings which call for a great deal of architectural adornment, the curved arch is used as it adds a great deal to the appearance. The exact form of arch to be used in any given case depends upon the style of the building and the amount of space available.

An arch over an opening in a building does the work of a lintel by supporting the wall over the opening and any superimposed load.[1] Thus an arch answers the same purpose as an ordinary beam, but the action is quite different, inasmuch as a beam produces vertical reactions only, while an arch produces an outward thrust upon its supports as well as a vertical pressure. In designing arches, special care should be taken that the supporting abutments are capable of taking this outward thrust.

In plain buildings where the window openings form no particular adornment to the structure, it is usually a great deal cheaper to carry brick work on lintels over an opening. These lintels usually consist of several pieces of plain angle irons, the outer one of which is set a trifle below the ones supporting the back courses of brick work, to hold the window box in position and to act as a weather guard.

In the construction of masonry arches, forms are built usually of wood, the top of these forms coinciding with the line of the intrados of the arch. The forms serve as a support for the different arch sections until the keystone is placed and the masonry has had sufficient time to set.

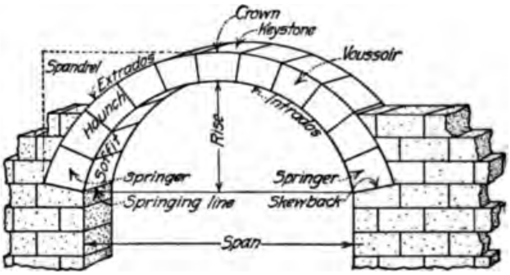

Fig. 251.

134. Definitions.—The *intrados* is the inner curve of the arch (Fig. 251). The outer curve is termed the *extrados.* The *soffit* is the concave surface of the arch. *Voussoirs* or *ringstones* are the pieces composing the arch. The highest or center stone is called the *keystone* or *key block.* The *crown* is the highest part of the arch. The first courses at each side are called *springers.* In a segmental arch, the inclined surface or joint upon which the end of an arch rests is called a *skewback.* The *springing line* is the inner edge of the skewback. The voussoirs between the keystone and the springers are called collectively the *haunch* of the arch, and the portion of the wall above the haunches and below a horizontal line through the crown is termed the *spandrel.* The sides of the arch which are seen are called *faces.* The *span* is the horizontal distance between springing lines measured parallel to the faces. The *rise* is the height of intrados at crown above level of springing lines.

The keystone is sometimes made to project several inches above the extrados line, but this portion so projected adds nothing to the strength of the arch and is usually elevated for appearances only.

135. Depth of Keystone.—There is no exact method of determining the required depth of the voussoirs or of the keystone. The thickness of an arch must be assumed and then the arch investigated in regard to strength.

There are several rules that have been established by recognized authorities for establishing the depth of keystones, but these are admitted to be only empirical. They are a good guide, however, for making a selection for trial.

Trautwine's formula for the depth of the keystone for a first-class cut-stone arch, whether circular or elliptical is

$$\text{Depth of key in feet} = \tfrac{1}{4}\sqrt{\text{radius of intrados} + \frac{\text{span}}{2}} + 0.2$$

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[1] See also Art. 29

For second-class work, this depth may be increased about 1/8 part; and for brick work or fair rubble, about 1/3.

136. Forms of Arches.—Arches are built in a great variety of forms, the most common of which are semicircular, segmental, multi-centered, and elliptical. The name is determined by the curve of the intrados or inner curve of the arch.

The joints of semicircular and segmental arches radiate from a single center. In arches having two or more centers, the joints in each arc radiate from their respective centers. The joints in flat arches radiate from the vertex of an equilateral triangle having the span line at springing as a base.

Semicircular and semi-elliptical arches are full centered—that is, they spring from horizontal beds—while segmental arches spring from inclined beds called skewbacks (see Fig. 251). Multi-centerd arches may have beds either inclined or horizontal. Minor curves joining the arch soffit to pier or abutment are not effective and should not be considered as part of the arch rise. Full centered arches should be used when it is necessary to make the abutments of the arch as small as possible.

A relieving arch is one set immediately above a lintel, to carry the wall above and to relieve the lintel of all except its own weight and the weight of the wall between the lintel and the arch. This form of construction is generally used in brick walls. Some building codes require a relieving arch over the procenium girder in a theatre.

137. Brick Arches.—Arches built of brick are most commonly used over window openings. They are also used to support sidewalks over vaults. In constructing these vaults, brick arches are sometimes sprung between the vertical columns at the curb and make a very effective retaining wall.

When fireproof structures were first used, brick arches, sprung between the flanges of iron beams, were used to support the floors. As this form of construction is very unsightly, it is not used in modern construction, except occasionally in buildings of an unfinished nature, such as in warehouses and mills.

Brick arches can be built either of wedge-shaped bricks made to fit the radius of the soffit, or of common bricks. The former method is, of course, preferable but much more expensive. The common forms of building brick will be found to fill most requirements, and to be the most economical in cost. A brick arch should never be less than 4 in. in depth, and the bricks should be laid on edge supported by a temporary center until they have properly set. In using common size brick the joints at the intrados, will, by necessity, be smaller than at the extrados to accommodate the curvature of the arch. Unless the curvature is very sharp, the mortar will take up the difference in space satisfactorily, in which case small pieces of slate can be driven in the spaces at the extrados of each course of brick.

An arch 4 in. thick will support a considerable load over a span of from 4 to 6 ft. and the span can be made as large as 8 ft. for loads in proportion, with safety. If arches are more than 4 in. thick, the bricks should be alternated by laying one on edge and the next on end to form a bond.

For arches supported on piers which have not the stability to take the arch thrust, cast-iron skewbacks should be provided from which to spring the arch and the thrust is then taken up by tension rods fastened to the skewbacks. The horizontal thrust of the arch is very closely determined by either of the following formulas and equals the tension produced in the rods:

$$\text{Thrust} = \frac{1.5 \times \text{load per square foot} \times (\text{span})^2}{\text{rise of arch in inches}}$$

or

$$\text{Thrust} = \frac{\text{load on arch} \times \text{span}}{8 \times \text{rise of arch in feet}}$$

Good proportions of rise to span occur when the radius is equal to the span, or 1/8 of the span equals the rise.

The required minimum thicknesses of brick arches in proportion to the span is covered by the various building codes.

For all brick arches carrying floors, tie rods should be provided between the supporting beams or walls to take up the thrust.

138. External Forces.—Let P_1 and P_2, Fig. 252, represent the resultants of all the loads on the left and right halves of the arch respectively, the loads being equal in amount and applied symmetrically with respect to the span of the arch. Let R_1 and R_2 represent the vertical reactions. As the loads are equal and symmetrically placed with respect to the span of the arch, then R_1 and R_2 are equal to each other and equal to loads P_1 and P_2. Let R_3 and R_4 represent the horizontal thrust at the supports which will both be equal.

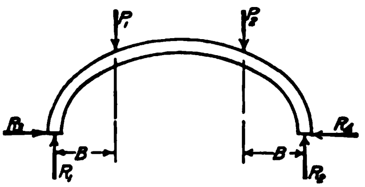

Fig. 252.

Now assume one-half of the arch to be taken away as in Fig. 253. To preserve equilibrium in the half shown, a force must be applied at the crown as R_5, which must be equal to R_3. The algebraic sum of the vertical forces, and likewise the sum of the horizontal forces, must equal zero in order to produce equilibrium.[1] Then R_1 must equal P_1, and R_5 must equal R_3. Also the sum of the moments about any point must equal zero.[1] Therefore, taking moments about the abutment,

$$R_5 = R_3 = \frac{P_1(B)}{C} = \frac{P_2(B)}{C}$$

Any number of loads can be treated in the same manner and if they are equal and symmetrical about the center of the arch, only one-half of the arch need be investigated as both halves will be alike. If, however, the loads are not equal, or are not placed symmetrically, or if the arch is unsymmetrical, the thrust at the crown will not be horizontal. Only symmetrical conditions will be considered in this chapter as is usually the case with arches in building construction.

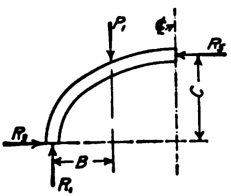

Fig. 253.

139. Determining the Line of Pressure.—To get a fair idea of the nature of the stresses and the line of pressure in an arch, consider the following conditions:

Suppose a cord, fastened at each end supports a number of loads as in Fig. 254. The cord will take a position of equilibrium, depending on the amount and location of the loads. In a case like this, the cord is in tension. For an inverted case, as shown in Fig. 255, the forces are still in equilibrium, but in place of a cord in tension, the broken line between the points of loadings, must be members capable of taking compression. The latter case represents the condition that exists in an arch, and the line intersecting the vertical load lines, forms the *line of pressure* or *line of resistance*.[2] The material of which the arch is constructed must be of such strength and so disposed as to safely resist the compressive forces acting along this line—that is, the maximum intensity of pressure at any point must not exceed the allowable stress.[3]

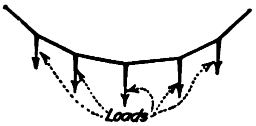

Fig. 254.

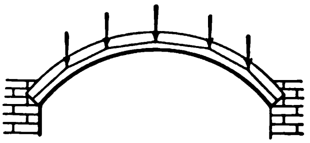

Fig. 255.

The line of pressure for a masonry arch should lie within the middle third of the arch ring. For instance, with an arch 3 ft. deep, the line of pressure should be within a space 6 in. on either side of the center of the depth. If the line of pressure falls outside of the middle third, the joints tend to open, which condition will tend to make the arch, unsightly, and cause cracks in the masonry above the arch; also, the pressure line may make an angle with some of the joints between voussoirs such as to cause the voussoirs to slide on their surfaces of contact—in other words, the tangent of the angle between the line of pressure and the normal to any joint may be greater than the coefficient of friction.

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[1] See Sect. 1, Art. 43b.

[2] Since loads are distributed in an arch, the line of pressure is in reality a continuous curve, but differs very little from an equilibrium polygon for the concentrated loads as usually assumed. For method of drawing equilibrium polygon, see Sect. 1, Art. 43(a).

[3] See Sect. 1, Art. 103, for explanation as to how the maximum unit stress may be obtained at any given section provided the normal component of the resultant thrust on the section is known in position and amount.

To determine the line of pressure or equilibrium polygon for any voussoir or plain concrete arch, a point on this line must be determined at the crown and one at the abutment, otherwise an indefinite number of lines of pressure could be drawn. The *true* line of pressure is usually considered to be the one lying nearest to the center line of the arch. It follows, therefore, that if a line of resistance can be drawn within the middle third of the arch ring, the true line of resistance will lie within the middle third. It is not always possible to determine at first trial as to whether a line of pressure can be drawn which will be wholly within the middle third. By using good judgment, however, in the selection of controlling points through which to pass the equilibrium polygon or line of pressure, two or three trials will usually suffice. If a line of pressure cannot be drawn so as to pass through the middle third, either the thickness of the arch must be increased or the shape of the arch ring changed.

For the first trial the middle points at the crown and skewback may be assumed as points on the line of pressure. For other trials, however, the upper limit of the middle third should be used at one joint and the lower limit of the middle third at the other joint.

The following is quoted from the American Civil Engineers' Pocket Book and shows how one may proceed in determining as to whether a line of pressure may be drawn within the middle third of the arch ring after a first trial is made and the first pressure line found to lie outside of the middle third:

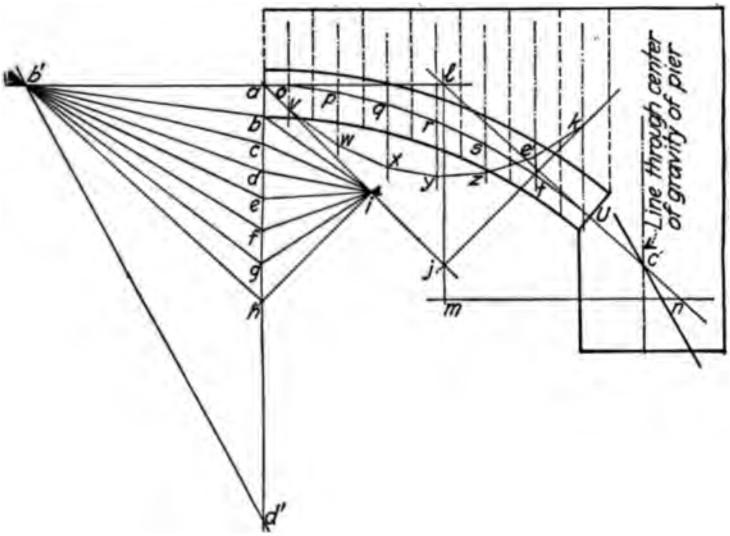

Fig. 256.

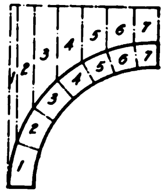

Fig. 257.

After having drawn a resistance line which passes outside of the middle-third at one or more places, an attempt should be made to find another one which lies within it. For this purpose find on the drawing the two joints where the resistance line departs most widely from the neutral axis and select two points A_1 and A_2 on those joints which are nearer that axis, A_1 being on the joint which is the nearer to the crown. Let P_1 and P_2 be the sum of all loads between the crown and A_1 and A_2 respectively, a_1 and a_2 be the horizontal distances from A_1 and A_2 to the lines of action of P_1 and P_2, h = vertical distance from crown to A_2, and h' = vertical distance between A_1 and A_2; then the horizontal thrust H' for the new resistance line and the distance t from the crown to its point of application are (Cain's Voussoir Arches, 1904)

$$H' = \frac{(P_2a_2 - P_1a_1)}{h'} \quad t = h - \frac{P_2a_2}{H'}$$

With this new horizontal thrust a second resistance line may be drawn and this should pass through the points A_1 and A_2.

In taking the loads on arches, all weights must be reduced to the same standard. The loads are made equivalent to masonry weighing in pounds per cubic foot, the same as the masonry of the arch ring. Usually 1-ft. width of the arch is considered. To determine the loads to consider in investigating flat segmental arches, the arch ring and its load may be divided into vertical slices, as shown in Fig. 256. For full-centered arches, however, it is more accurate to divide the arch ring into a certain number of voussoirs, the rest of the load being divided vertically, as shown in Fig. 257. In this case, it is less easy to find the position of each load than in the vertical-slice method but the method of investigation is the same.[1]

139a. Graphical Method.—Begin by drawing, to scale, a diagram of one-half the arch. The load upon one-half the arch must next be determined. Lay off, to scale, a height of masonry whose weight will represent this load. Commencing at the crown, divide the load into, say, 2-ft. sections as far as possible. The weight of each slice will be its contents multiplied by the weight per cubic foot, and is marked on the diagram. Next, fix a point at the crown and one at the spring of the arch, through which the pressure curve or equilibrium polygon is assumed to pass. The points may lie anywhere within the middle third of the width; but the point "*a*" at the crown has been taken at the outer edge, and the point "*u*" at the spring at the inner edge, of the middle third.

Lay off from "*a*" on the vertical *ad'*, the distances *ab*, *bc*, *cd*, etc., which represent the weight of the slices from the crown to the spring. Next draw 45-deg. lines from *a* and *h*, intersecting at *i*; and from *i* draw *ib*, *ic*, *id*, etc. Through the center of gravity of each slice, draw a vertical, as *ov*, *pw*, *qx*, etc. Starting from *a*, draw *av* parallel to *ai*; from *v*, draw *vw* parallel to *bi*, etc. These lines form a broken line, which changes its direction on the vertical line through the center of gravity of each slice. From the last point *k*, draw *kj* parallel to *ih*, and intersecting

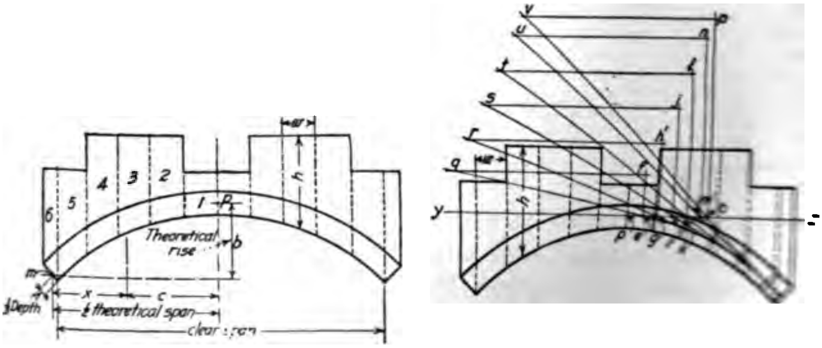

Fig. 258.

ai, extended, at *j*; from *j* draw a vertical line *jl*, which will pass through [illegible] of the half arch and load.[2] From *l*, lay off a distance *lm* equal to *ai*, [illegible] of all the slices. From *l* draw a line through the point *u*; and from [illegible] secting *lu*, extended, at *n*. Then *mn* will be the horizontal thrust [illegible] maintain the half arch in equilibrium when the other half is removed. [illegible] tion and amount of the oblique thrust at the skewback. On [illegible] tance *ab'* equal to *mn*. From *b'*, draw lines to *b*, *c*, *d*, etc., [illegible] center of gravity of each slice. From *a*, draw *ao*, parallel [illegible] *b'b*, etc., then *a*, *o*, *p*, etc., will be points on the line of pressure. [illegible] third, the arch will be stable, provided the pressure is [illegible] is found by measuring *b'h* with the same scale as [illegible]

Having calculated the weight of the pier or wall, [illegible] from *h* to *d'*, and draw *d'b'*. Draw a vertical line through [illegible] cutting *ln* at *c'*; also, a line from *c'*, parallel to [illegible] of the arch, after being influenced by the weight of the [illegible] of the pier, at the ground line, the pier will [illegible] order that a pier [illegible] his line, well with[illegible]

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[1] For [illegible] r to

[2] See [illegible]

139b. Algebraic Method.—In the arch shown in Fig. 258 the pressure curve is considered as passing through the points at the abutments ⅓ the depth of the voussoirs from the intrados, and through the center of depth at the crown. The arch and load are divided by dotted lines into sections, which, for convenience are numbered.

If w be the width of any section and h its average height, then its area "a" is $w \times h$ Also, if "c" is the distance from the crown to the center of gravity of a section, the moment m of any section about the crown is $a \times c$. Call A the sum of all the a's from the crown up to and including the section considered. Call M the total of the m's. Then the distance C from the crown to the center of gravity of the portion between the crown and the section considered is $\frac{M}{A}$ of that section. The above values may be tabulated as follows:

Section	w	h	$a = w \times h$	c	$m = a \times c$	$A = \Sigma a$	$M = \Sigma m$	$C = \frac{M}{A}$
1								
2								
3								
4								
5								
6								

The horizontal thrust at the crown, $Q = \frac{(x)(P)}{b}$, in which x is equal to one-half the theoretical span, minus the value of C for the sixth section. P is equal to A for the last section, and b equals the theoretical rise of the arch. Hence, taking moments about m,

$$Q = \frac{(x)(P)}{b}$$

Multiplying by the weight of the masonry per cubic foot, the horizontal thrust is obtained.

The line of pressure may now be determined as follows: Draw through point p in Fig. 258 the horizontal line yz; lay off to scale from p, in order, the distances C obtained from table. At these points lay off the vertical distance ef, gh', ij, etc., equal respectively to the values of A for each section, from the column headed A. From f, h', j, etc., to the same scale, mark off the constant horizontal thrust Q, as at fq, $h'r$, js, etc. Thus the vertical and horizontal forces at each section being given, the resultant of these two forces in each case is eq, gr, is, etc. Extending each until it intersects the joint beyond e, g, i, etc., the pressure curve may be drawn through these latter points of intersection, as shown by the heavy black line, and the thrust at the joints may be found by measuring eq, gr, is, etc., with the scale to which the diagram was drawn.

Since in this case the pressure curve falls well within the middle third of the arch ring, the arch may be considered satisfactory, provided the safe crushing strength of the masonry is not exceeded.

The influence of the last oblique thrust, which is the resultant thrust of the arch upon the pier, or abutment, is explained in the preceding article on the graphic solution of the pressure curve.

140. Arches of Reinforced Concrete.—Concrete arches reinforced with steel are but rarely used in building construction so it has been thought advisable to omit the treatment of same. Arches of this type are treated at length in Concrete Engineers' Handbook by Hool and Johnson. Plain concrete arches may be designed as described in this chapter.

PIERS AND BUTTRESSES

By Frank C. Thiessen

141. Methods of Failure.—A pier *ACDB* upon which a thrust *P* acts, as shown in Fig. 259, may move from its position by sliding on any section, or by overturning when the moment of the thrust about a point at the edge exceeds the moment of the weight about the same point. A heavy superimposed load on a pier, or an inclined thrust, as from an arch, a rafter, or a truss, may cause an intensity of stress at a point in the outside edge sufficient to crush the masonry. If the pier is stable against sliding along any bed-joint and also along its foundation, a thrust would shift the resultant of the vertical loads, *W*, so that the center of pressure on the foundation would no longer pass through the center of gravity of the pier. The pressure at one side of the base would become greater than at the other side. If the foundation is not firm, excessive pressure may cause the structure to overturn bodily.

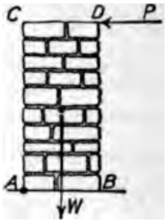

Fig. 259

142. Principles of Stability.—Proper provision can be made in the design and construction of a pier to safeguard against failure as described above. The underlying principles are quite simple.

In Fig. 260, let *W* represent the weight acting through the center of gravity of the rectangular pier, and let *P* represent a force tending to overturn the structure. Drawing a parallelogram of forces (see Sect. 1, Art. 42*a*), the resultant is seen to cut the base *AB* at a point *Q*. If the force *P* is increased sufficiently, the resultant will pass through *A* and the structure will then be at the point of rotating about *A*. A slight crushing of the mortar at the edge would be sufficient to cause rotation. Therefore, in order to insure safe stability against overturning and to secure a satisfactory distribution of pressure, it is customary to limit the position within which the resultant should cut the base. In ordinary masonry piers the action line of the resultant of all forces should intersect the base within the middle section, or *middle-third* as it is called, assuming the base to be divided into three equal sections.

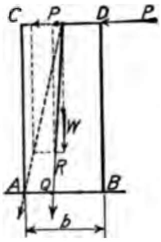

Fig. 260.

If the force *P* (Fig. 260) is not acting, the downward pressure on the foundation due only to the weight *W* is uniform and its intensity is equal to $\frac{W}{b}$ assuming the pier to have a length *b* and a width of unity in the direction perpendicular to the plane of the paper. The horizontal force *P*, acting as shown, tends to increase the pressure at *A* and decrease it at *B*. Considering the pier as a short cantilever, free at the upper end, the bending moment due to the force *P* will cause compression at *A* and tension at *B*. The maximum pressure at *A* will be equal to that due to the weight of the pier plus the compression due to flexure; and the pressure at *B* will be the compression due to the weight of the pier minus the tension due to flexure.

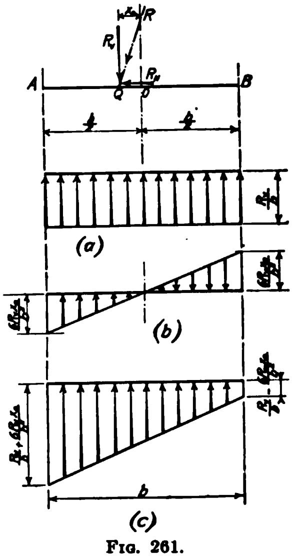

Fig. 261.

In Fig. 261 let *AB* represent the base of a pier with the resultant of all forces (*R*) intersecting the base line at *Q*. Resolve the inclined force *R* into its horizontal and vertical components, R_H and R_V (see Sect. 1, Art. 42*b*). The effect of these two forces will be the same as the single force *R*. The horizontal component, R_H, tends to cause the pier to slide along the base. The vertical component, R_V, is equivalent in effect to an equal R_V acting at *O* and a couple whose moment is R_Vx_0. At any point distant *x* from *O*, according to the common flexure formula (see Sect. 1, Art. 61*b*) the intensity of stress (or pressure) due to this moment is $\frac{R_Vx_0x}{I}$, in which *I* is the moment of inertia of the base plane about a line through *O* perpendicular to the plane of the paper $\left(\frac{bd^2}{12}\text{, see Sect. 1, Art. 61}c\right)$. The maximum values of this expression occur when $x = \frac{b}{2}$. At the edges A and B the intensity is $\frac{6Rvx_0}{b^2}$. The total intensity of pressure at A is

$$p_1 = \frac{Rv}{b} + \frac{6Rvx_0}{b^2} = \frac{Rv}{b}\left(1 + \frac{6x_0}{b}\right)$$

This value should not exceed the safe working strength of the mortar or other materials of which the structure is built.

At the edge B

$$p_2 = \frac{Rv}{b}\left(1 - \frac{6x_0}{b}\right)$$

The diagrams of Fig. 261(*a*), 261(*b*), and 261(*c*) show, respectively, the uniform intensity of pressure, the intensity due to flexure, and the combination of the two. From an inspection of these diagrams it will be seen that the intensity at the edge B will become zero when $\frac{Rv}{b} = \frac{6Rvx_0}{b^2}$. Solving, $x_0 = \frac{b}{6}$, that is, the resultant intersects at one-third the distance AB from A. For this condition the intensity of pressure at the edge A will be $\frac{2Rv}{b}$, or double the average intensity. If the resultant falls outside the middle-third point, some tension might occur at the edge B but, as the tensile strength of masonry with mortar joints is nearly a negligible quantity, the tendency would be to have a greatly increased pressure at the edge A with compression extending over only a part of the joint. When the resultant intersects within the limits of the middle-third, the full width of the joint acts in supporting the structure, the entire joint being in compression.

In many cases architectural considerations may determine the preliminary proportions. With the dimensions given, the pier or buttress is then tested for stability. If upon trial it is found that the resultant passes outside the middle-third section of a joint, the general proportions of the pier, the position of superimposed loads or both, should be changed to bring the resultant within the desirable limits.

The horizontal components of the forces acting tend to slide the structure over a joint or plane of weakness, and are resisted by the friction of the surfaces in contact. For any horizontal joint, motion will occur when $H = fW$, where f is the coefficient of friction, H the sum of the horizontal components of forces acting above the joint, and W the weight of the portion above the joint. In the following table are given a number of frequently required values of the coefficient of friction, with the corresponding values of the angle of inclination at which motion occurs:

	$f = \tan \phi$	ϕ
Masonry upon masonry	0.65	33°
Hard limestone on hard limestone	0.65	33°
Common brick on common brick	0.65	33°
Concrete blocks on concrete blocks	0.65	33°
Common brick on hard limestone	0.65	33°
Masonry upon dry clay	0.50	26°40
Masonry upon moist clay	0.33	18°20′
Masonry upon sand	0.40	21°50′
Masonry upon gravel	0.60	31°

To make sure that the structure is stable against sliding, a safety factor, commonly two, is employed. This is equivalent to providing sufficient resistance so that the structure will remain stable under the action of at least twice the sliding force. Ordinarily, with the dimensions given, the problem is to determine the safety factor, testing the pier or buttress for its stability against sliding at the various bed-joints or planes of weakness. If the value of the safety factor is found to be below two, added resistance should be provided. Stability can be secured by giving the structure sufficient weight, by increasing the frictional resistance, by bringing vertical loads to bear upon the upper portions, and, if necessary, by proper bonding, doweling, or inclining the joints. In building foundations upon a moist clay soil, it is not uncommon to add a projection below the base.

143. Designing for Stability.—The stability of a given pier or buttress is usually determined graphically, or by means of some algebraic work combined with a graphical analysis. The entire problem may also be solved algebraically but the graphical method is rapid and gives sufficient accuracy if the scale is well chosen. In order to illustrate clearly the method of procedure, a structure having the simple form shown in Fig. 262 will be tested for stability about its base under the action of a thrust T applied at the upper corner and acting in the direction shown.

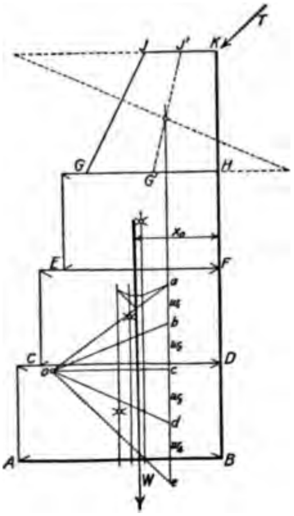

FIG. 262.

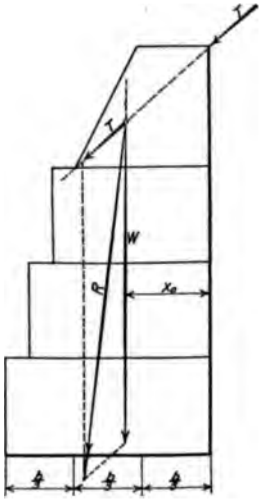

FIG. 263.

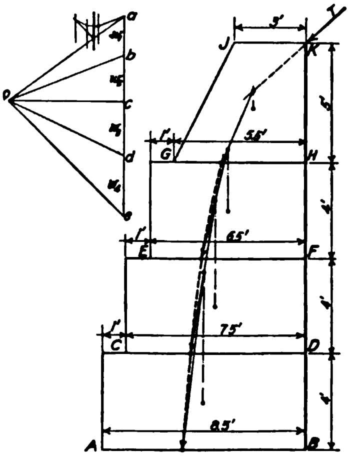

FIG. 264.

The first step is to determine the position of the action line of the weight (W) of the entire structure with respect to some vertical line, such as the face KB. Divide the pier or buttress into triangles or quadrilaterals of which the centers of gravity and areas may be readily determined. For a rectangle, the center of gravity is found at the intersection of the diagonals. For a trapezoid, a simple method is as follows: Bisect JK and GH, Fig. 262, and draw the medial line $J'G'$. On the line KJ extended, lay off from J the distance HG; from H lay off to the right along GH extended, the distance KJ. Connect the extremities as shown. The intersection with the medial line is the center of gravity desired. Through each center of gravity draw a vertical line representing the action line of the weight of the respective portion. Starting (at the extreme right) at a convenient point a on the action line of w_1, lay off to a convenient scale $ab = w_1$, $bc = w_2$, $cd = w_3$, and $de = w_4$ representing the weights of the various portions of the structure. Choose a pole O and draw the rays Oa, Ob, Oc, Od, and Oe of the force polygon. The action line of the weight (W) of the entire structure is found by the aid of an equilibrium polygon (see Sect. 1, Art. 43a).

The distance of the action line of W from the face KB (Fig. 262) may also be obtained by the method of moments. If the sections into which the buttress is divided are simple areas, such as triangles or rectangles, the centers of gravity may be readily found. Let the distances from KB to the vertical lines through the center of gravity of w_1, w_2, w_3, and w_4 be represented by x_1, x_2, x_3, and x_4, respectively. Then the distance x_0 is found by taking a summation of moments about the line KB and dividing by the total weight. Thus, for the buttress of Fig. 262

$$x_0 = \frac{(w_1.x_1) + (w_2.x_2) + (w_3.x_3) + (w_4.x_4)}{w_1 + w_2 + w_3 + w_4}$$

Prolong the action line of the thrust T beyond the intersection with the action line of W (Fig. 263). As a force may be considered as applied at any point along its action line, lay off to a convenient scale the forces T and W, using the same scale for both. Complete the parallelogram of forces. In this case the action line of the resultant of all forces is seen to intersect the base line within the middle-third section. If the point of intersection had been outside the middle-third point, it would have been necessary to have increased the base or otherwise rearranged the vertical loads to bring the intersection within the proper limits.

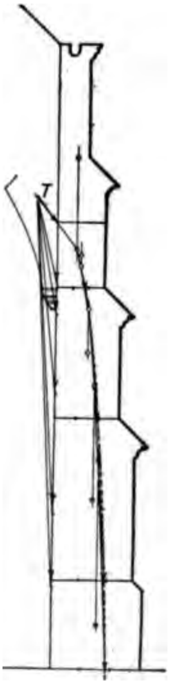

FIG. 265.

If the structure of Fig. 262 had been composed of a number of separate parts such as *JKHG*, *GHFE*, etc., failure might occur by sliding, overturning, or crushing at any joint. Even if no joints existed, the imaginary joints of all points of weakness would be subject to the same principles and hence should be investigated for stability. In Fig. 264 the pressure on the joint *GH* is due to the thrust *T* and the weight of the portion *JKHG*. The point of application of the resultant of these two forces on the portion *GHFE* is indicated by the arrowhead. To find the point of application of the pressure on *EF* this resultant is combined with the weight of the portion *GHFE*. The points of application for the other joints are found in a similar manner. The dotted line connecting the points of intersection of the various joints is called the *line of pressure*, the *line of resistance*, or the *resistance-line*. If the structure is properly designed the resistance line will lie inside the middle-third section of the structure.

In church structures it is common to find parallel walls with vaulted roofs, hammer-beam trusses, or other types having no tie rod or bottom tension member to take the full thrust of the curved or inclined roof. In such cases, the outer walls must be increased in thickness or supplied with buttresses to resist the outward thrust. Ordinarily a trial buttress, satisfying the architectural requirements, is first sketched and tested for stability by drawing a pressure line and determining the factor of safety against sliding at the weakest joint. Fig. 265 shows the construction of a pressure line for such a buttress. It will be noted that the structure is divided into a number of sections and that one of the lines previously drawn serves for the load line of the force polygon. The construction is similar to that required for the buttress of Fig. 262.

TIMBER DETAILING

By Henry D. Dewell

Timber detailing differs from steel detailing in that there are no generally accepted standards of connections for timber structures, as in the case of steel framed buildings. In making this statement, the writer is not forgetting certain trade or stock joist hangers, post caps, etc., the specifications of building ordinances, and the generally accepted types of details of mill construction. In recent years, the lumber manufacturers, notably the Southern Pine Association and the West Coast Lumbermen's Association, are doing much toward securing a better class of construction in timber. "The Southern Pine Manual" of the Southern Pine Association and the "Structural Timber Handbook of Pacific Coast Woods" of the West Coast Lumbermen's Association are excellent aids in design, and should be in the hands of all those designing and constructing in timber.

144. Information to be Given by a Set of Plans.—Every set of plans of a timber framed structure should fulfill the following conditions: (1) It should give such information that the cost of the work may be accurately computed; (2) it should be in sufficient detail that every stick of timber, every rod, bolt, or other piece of iron or steel may be listed and ordered; and (3) every important detail should be shown so that the carpenter may have no excuse for framing it incorrectly. The lack of proper details on a plan or in a set of plans is many times due to the ignorance of the designer with regard to timber joints, and a consequent effort to shift the responsibility to the carpenter.

In a steel framed building an engineer usually prepares the plans and specifications of the structural features of the building; and, in most cases, the engineer's work is confined to the steel frame and foundations. The structural plans thus prepared are known as "contract plans" in distinction to detail plans or shop drawings. Floor framing plans, sections and elevations of wall framing may be shown with details of important connections given. But ordinary connections, as of I-beams framing into I-beams, are not shown, as these connections are standardized by the steel companies. In total, in the case of a steel framed building, a set of contract plans may be but little else than diagrammatic sketches with sizes of members and stresses shown in other members, leaving the details to be worked out in the shop of the contractor securing the job, subject to the engineer's or architect's approval.

Turning to the timber framed building, one sometimes sees plans where the same procedure has been attempted. Such a method cannot be satisfactory, is a certain source of trouble, and may be disastrous. Such a thing as shop or detail plans in timber framed buildings is practically unknown. Consequently, the contract plans in this case should be complete in every detail. The one possible exception to this general statement is in the case of the iron and steel work. If the designer shows the sizes of rods, bolts, etc., with typical details of other steel members, as bases, castings, etc., and calls for detail drawings in accordance with his plans and specifications to be approved by him, the result may be satisfactory. Even in this case, however, the chances for trouble are many. The iron work is of such small amount that a small steel shop with no drafting force will probably furnish the material, and the details are likely to be disappointing.

The writer believes that time and money are eventually saved, and annoyance prevented, if the contract plans show all details carefully worked out. It may be stated that no important detail be left to the discretion of the carpenter. With all due respect for his experience and care, he seldom understands the requirements of any detail but the simplest, and many times in his endeavor to improve on a detail but hazily indicated actually weakens the structure.

For the proper presentation of the work, there should be given a general plan, framing plans of roof and all floors, wall elevations, cross sections and longitudinal sections, elevations and sections of any special features, and details of all connections except the very simplest. These latter may be covered in the specifications. It is obvious that the exact number of drawings must depend wholly on the particular building.

145. Scales.—Ordinarily, the general plan and framing plans should be to the scale of ⅛ or ¼ in. to the foot. In many cases, the larger scale will be necessary in order to bring out the different parts clearly. Often, too, plans of special features may well be made to an even larger scale, say ½ in., in addition to the general plans which may include such special features. However, the general plan to a small scale should always be made, as this may be the one place where all parts are assembled as a whole, and where the entire structure may be seen at a glance. Elevations and sections may be shown to a ⅛ or ¼-in. scale.

146. Plans Required.—Assume the case of a timber framed building of the mill building type, 100 ft. long and 40 ft. wide, roof trusses spanning from wall to wall supported on posts; corrugated iron walls and roof, and floor of timber construction 3 or 4 ft. above the ground, supported by posts resting on concrete footings. The following plans, if properly drawn, will, with specifications, show the work completely·

1. Grading plan to 1/16-in. scale.
2. Foundation plan to ⅛-in. scale, showing size and location of all piers and wall footings, with details of the individual footings and piers to ¼ or ⅛-in. scale. On this sheet any sewers, water or other pipes may be shown, provided that such pipes and connections are so numerous as to merit a special plan.
3. Elevations of four walls, drawn to ⅛-in. scale, showing all window and door openings, the doors and windows being lettered or numbered to correspond with details of same. On these elevations can also be shown any other openings, gutters, downspouts, any ornamental features, etc.
4. Floor framing plan, to ⅛-in. scale, showing sizes of joists, girders, and posts, with all dimensions and spacing of same.
5. Roof framing plan, to ⅛-in. scale, showing main trusses, bracing trusses, with their proper letters or numbers, roof joists, bracing and bridging.
6. General roof plan, to ⅛-in. scale, showing roof covering, downspouts, parapet walls, monitors, roof slopes, etc.
7. Wall elevations, to ⅛-in. scale, showing framing of wall, posts, girts, studding and bracing.
8. Cross section of building, to ⅛-in. scale, completely detailed as to roof joists, trusses, columns, and floor construction.
9. Miscellaneous details to ½-in. scale.
10. Details of all steel to 1-in. scale.

To the above, if completely detailed plans are to be made, should be added:

11. Wall elevations to ⅛-in. scale, showing number and size of corrugated steel.
12. Material lists.

If the material lists are made, the designer may feel sure that his plans have had a thorough checking. There is no better check on the accuracy and completeness of one's work than a detailed bill of materials; conversely, one can never feel certain that all parts are clearly shown until a complete bill of materials has been taken off.

Drawings should never leave the office if badly out of scale. This is a general statement applicable to all construction; it holds particularly in timber construction, as the carpenter is almost certain to scale some lengths of timbers.

A general and comprehensive note should be placed on all structural drawings, even repeating certain important clauses in the specifications. On the job the specifications may be lost; the plans are never lost. One note on the drawings is worth two clauses in the specifications.

STRUCTURAL STEEL DETAILING

By Chas. D. Conklin, Jr.

The material in this chapter will deal exclusively with the work of that part of the drafting room of a structural steel fabricating concern wherein shop detail drawings are prepared. The work of the designing and estimating departments of necessity precedes the work described and illustrated in this chapter. Designing methods for structural steel members have been adequately covered, both from theoretical and practical points of view, in previous chapters and for such, the reader is referred thereto. It is generally understood among structural engineers that structural steel detailing knowledge can best be acquired by actual experience in the drafting room where details are made. In fact, among our best detailers may be classed many of those who have entered the drawing room as apprentices, and with little or no theoretical training, have acquired their ability by practice, observation, and contact with experienced draftsmen, templet makers and shopmen. The following description and illustrations are given with the thought of presenting to the less experienced draftsmen, some practical suggestions and methods that may be of value to them. It is further hoped that the more experienced may find herein some valuable data.[1]

147. Drafting Room Organization and Procedure.—Shop detail drawings are the working drawings by means of which structural steel is fabricated in the shop. They form the medium by which the architect's or engineer's sketches or general drawings are interpreted to the fabricating shop, in order that the latter may intelligently and quickly manufacture the required product. Structural steel, unlike many other materials, is not readily worked in the field or on the job. Hence accurate drawings, showing the sizes and lengths of all materials, size and location of all holes and rivets, all cuts, coping, and in fact every detail of a structure, must be made from which the shop can accurately work. A complete structure must be divided into sections of such dimensions that they can be readily handled, shipped, and erected and these sections must be marked with identifying marks, called erection or shipping marks, which are shown on a sketch of the completed structure for use of the erector. All this drafting work is done under the direction of the chief draftsman, who has entire charge of the drafting room and should be a man of unquestioned and practical ability. The draftsmen under the chief are usually divided into squads of from six to eight men, who are under the direction of a squad chief. Those under the squad chief may be divided into checkers, draftsmen and tracers, although sometimes checkers work independent of squad chiefs. After the drawings are made and checked, final bills of material are made therefrom for purposes of determining accurate weights for payment, shipping, etc. Shop lists and shipping lists are also made. These bills are prepared in a separate department, called the billing department, under the direction of a chief bill clerk.

The procedure of the drafting room is somewhat as follows: Information, including sketches, design sheets, general drawings, surveys, copy of estimate and other miscellaneous data which have been worked up in the designing and estimating department is handed to the chief draftsman, who examines same, assigns a contract number to the job, prepares his files for correspondence, etc. and assigns work to squad best able to get out the details. The squad chief studies the work thoroughly and in detail, so that he has in mind every point that may arise in the preparation of the shop detail drawings. He usually makes a preliminary bill of material required for the job, so that the material can be ordered from the mill or reserved from stock. In preparing this preliminary bill, it may be necessary for the squad chief or an assistant to

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[1] For more elaborate treatment of this subject, the reader is referred to "Structural Steel Drafting and Elementary Design" by Chas. D. Conklin, Jr., published by John Wiley & Sons.

accurately lay out to large scale (say 3 in. to 1 ft.) any details which cannot be determined by inspection. The preliminary bill is passed on to the stock clerk, who reserves from stock any desired material and hands a list of the balance to the purchasing agent to be purchased from mill. This is in the form of a requisition, copies of which together with copies of the material reserved from stock, are handed to the chief draftsman and squad chief. The squad chief then apportions the work among his men, according to their ability to handle it. After drawings are prepared, they are handed to the checker, who goes over them in detail, noting any corrections or desired changes. Drawings are then returned to draftsmen, who back check corrections or changes, make them, and return drawings to checker for approval. Drawings are then sent to billing department for billing, and are then blue printed for the shop.

A list of all drawings and blue prints made should be kept, usually on printed forms, by the squad chief. Extremely complicated drawings may be made in pencil on detail paper and traced in ink by a less experienced man. The more usual and simpler method, however, consists of making a pencil drawing directly on the dull side of tracing cloth and inking it in, all work being done by the same draftsman. It is very common now to have drawings made on either tracing paper or a specially prepared cloth, in pencil only, using a medium pencil and making lines very heavy. These drawings make very good blue prints, and effect a large saving of time. Some drafting rooms require their draftsmen to make a complete bill of material of the work detailed on a sheet, on the extreme right hand side of the same sheet. This greatly simplifies the work of the billing department.

148. Ordering Material.—In the preparation of the preliminary order of material from which structural shapes and plates may be ordered from the rolling mill or reserved from stock, the following rules may be used as they represent average practice:

1. Order main material first.
2. Beams and channels should be so ordered that a variation of ⅜ in. in length either way will not affect the detail. If an exact length is desired, so state in order and an extra charge may be made.
3. Beams and Channels.
   - For wall bearing beams, and foundation beams, order neat length.
   - For beams framing into other beams, order 1½ in. less (to the nearest ½ in.) than the center to center distance.
   - For beams framing into columns, order 1 in. less (to the nearest ½ in.) than the metal to metal distance.
   - For beams framing into riveted members, order 1 in. less than the metal to metal distance.
   - Crane runway beams, order 1 in. less than the distance center to center of columns.
   - Purlins, order 1 in. short (to nearest ½ in.) of distance center to center of trusses.
   - If the end connections on beams are milled after riveting, increase thickness of connecting angles to allow for this.
4. Columns.
   - Order column material milled one end ½ in. longer than figured length.
   - Order column material milled two ends, ¾ to ⅞ in. longer than figured length.
   - Order column details in 30-ft. lengths (base angles, cap angles, shelf angles, etc.).
   - Order lattice bars in 20-ft. lengths.
5. Root Trusses.
   - Order chord angles ¾ in. long.
   - For web angles, lay out to scale, scale the length, add about 1½ in. and multiple to 30-ft.
   - For gusset plates, order in multiple lengths of about 20 ft., arranging for as little waste as possible if corners are sheared.
6. Plate Girders.
   - Use an even inch depth of web plate and make distance back to back of angles ½ in. greater.
   - Order web plate of girder not milled on the ends, ¾ in. shorter than overall length. If milled on the ends, order ½ in. longer than overall length for one milled end, and ¾ in. for two milled ends.
   - Order flange angles ¾ in. longer than overall length.
   - Order full length cover plates ¾ in. longer than overall length
   - For cover plates less than full length, order the neat length.
   - Mark cover plate U.M. (universal mill or rolled edges).
   - Order stiffener angles with fillers ¼ in. longer than neat distance between outstanding legs of flange angles.
   - For crimped stiffener angles, order length equal to distance back to back of flange angles plus 1 in.
   - For heavy fitted stiffeners, allow ½ in. for one fitted end and ¾ in. for two fitted ends.
   - Order fillers under stiffeners ¼ in. clear of flange angles.
   - For diagonal bracing angles, scale length and add 1½ in.

Miscellaneous.

Plates planed top or bottom should be ordered 1/16 in. thicker than finished thickness, for each planing.

Plates having diagonal cuts may be ordered to sketch when over 36 in. wide and say ¾ in. thick, depending somewhat on the equipment of the shop for which material is ordered.

Channels, I-beams, and Z-bars are seldom ordered in multiple lengths.

In arranging multiple lengths make lengths about 30 ft. and not over 32 ft. Allow about 1 in. more than product of length times number required. Make all multiples end with the nearest ¼ in.

Order plates to the nearest whole inch in width. Use stock sizes when possible.

149. Layouts—*Riveted Connections.*—When the preliminary bill of material (for ordering purposes) has been completed, the next logical step in the preparation of shop details consists of designing the riveted connections and making layouts of difficult points, if such have not already been made for ordering purposes. The methods of designing riveted connections have been described in a previous chapter. All connections should be carefully investigated so that there may be no weak links in an otherwise strong structure. Difficult connections should be drawn out in pencil to a large scale, say 3 in. to 1 ft., in order to determine clearances, end distances, and other necessary data for detailing. These layouts are sometimes made and riveted connections designed by squad chiefs although often such are left to the detailer. Layouts consume much time and should not be made unless absolutely necessary. The usual scale to which shop detail drawings are made is ¾ in. to 1 ft.; sometimes 1 in. to 1 ft. is used. In such cases, it is unnecessary to make layouts of simple truss connections or other diagonal connections of similar nature. A careful draftsman can readily determine all necessary data from the shop detail drawing, which for trusses and similar work should be made accurately to scale. All shop details should be drawn to scale in so far as possible, the only exception to this being the length of beam sketches which may be distorted to save space and time.

Theoretically, the working lines or skeleton upon which a truss or similar structure is laid out, should be the gravity lines of the members composing the truss. Practically, however, for light roof trusses, the rivet lines are used, thus much simplifying the work for draftsman and shop. The skeleton diagram for the truss is laid out first to scale and the angles or other truss members are drawn around the skeleton using the latter as the rivet lines of the angles, the proper gages (as found in the steel handbook) being used. For heavy trusses, or similar structures, in order to avoid excessive moments at the connections, the gravity lines should be used as working lines.

150. Shop Detail Drawings.—After all layouts have been made and connections designed, the draftsman proceeds to make the shop detail drawing to scales as indicated below. In preparing shop detail drawings, the draftsman might well keep in mind the following rules, which are typical of modern practice:

Make shop details to scale of ¾ in. to 1 ft. or 1 in. to 1 ft. In exceptional cases, ½ or 1½ in. to 1 ft. may be used.

Use care in placing drawing on sheet to avoid unnecessary crowding of sketches or dimensions.

Size of sheet for large drawings is usually 24 × 36 in. Small sheets may be used for detailing beams, channels, pins, etc. Printed beam and channel sheets, with outline of beams and channels and dimension lines printed in black ink, save considerable time in this type of detailing.

Title of sheet should be placed in lower right-hand corner.

Detail members as nearly as practicable in the position which they occupy in the finished structure. Horizontal members should be detailed lengthwise and vertical members, crosswise on the sheet. Inclined members and vertical members, such as columns, may be detailed lengthwise on the sheet in which case the lower end should be placed to the left.

Show elevations, sections, and other views in their proper positions. Place top view directly above and bottom view below the elevation. The bottom view is always drawn as a horizontal section as seen from above.

For member symmetrical about a center line, draw only the left-hand half and note that it is symmetrical about the center line.

Several members, when similar, but slightly different, may be detailed on one sketch, the difference being shown by notes. Make such notes positive. Do not use the word "omit." If such notes become cumbersome and lead to ambiguity, avoid them and make another sketch.

Eliminate all unnecessary views and lines. Show just enough to express to shop what is intended. A shop detail is just a working drawing and not a masterpiece of art. Do not cross hatch, blacken or otherwise elaborate a shop detail unless it is absolutely necessary to make the drawing clearly understood.

On the other hand, make all work shown clear and distinct and all dimensions in large figures so that all can be easily followed. If a detail is worth making, it is worth making right and in such manner that shop will have no difficulty in interpreting it.

Make the part representing the steel work detailed of heavy black lines. Do not show hidden parts unless necessary for clearness and then show these parts by heavy dotted lines.

In detailing members which connect to others, the latter may be shown in red lines, in order to illustrate their relative position. Avoid the use of colored inks on shop drawings except in this case.

Dimension lines and rivet lines should be made of fine black lines, full and not dotted. Dimensions should be placed above dimension lines, and not in or on them. Make fractions with horizontal dividing lines.

Holes for field connections should be blackened. All holes in a group should be shown, as a rule. Rivet heads of shop driven rivets shall be shown only when necessary, as at the ends of members, when countersunk, flattened, or adjacent to field connections. Make open holes smaller in diameter (on the drawing) than the circles representing shop driven rivets.

When part of one member to be detailed is the same as another already detailed, it is unnecessary to repeat dimensions, etc. It is only necessary to refer to the previous sketch, describing the parts that are the same.

Main dimensions, such as story heights, center to center distances, etc., when given on a detailed drawing, are very helpful to a checker.

The size and length of material should be given close to the part which it represents, in clear, neat figures. If placed to one side, an arrowhead should indicate material referred to.

If a series of dimension lines are given adjacent to a sketch, largest dimensions should be given farthest from sketch, and small dimensions next to the sketch. Dimension lines should be drawn from ¼ to ⅜ in. apart.

Refer to steel handbook or Art. 121*a* for conventional signs for rivets; that is, for method of representing, on detail drawings, the various kinds of rivet heads, such as button head, countersunk one or both sides, etc.

The usual maximum sizes for shipping by railway in one freight car are 8 ft. for width, 10 ft. for height, and 30 to 40 ft. for length. In detailing structures, field connections should be placed such as to keep the member shop rivets within the above sizes. In exceptional cases, members may be made longer than the above and shipped on two or more cars. In export work, structures are usually shipped knocked down (in small pieces) to facilitate shipping by boat.

Each piece that is shipped separately should have an erection or shipping mark which shall consist of capital letters and numerals or numerals only. Do not use small letters for erection marks. Pieces which are absolutely alike may have the same erection mark. Trusses are usually marked *T*1–*T*2, etc.; columns *C*1–*C*2, etc.

For purposes of assembling the various parts of one member in the shop, assembling marks should be used for each plate or shape. These shall consist of small letters and numerals. No capital letters should be used. One system of assembling marks in common use is given below.

Members which are absolutely similar but opposites are called rights and lefts. The member detailed in such cases is called the right-hand piece and the opposite one, the left-hand piece. The erection mark of the former is followed by a large *R* and the erection mark of the latter by a large *L*.

The number of members required should be distinctly stated on a drawing. In a list giving the required number of members, write the word "one" out.

Parts of members which must be shipped bolted so that they can be taken off during the erection should be marked "Bolt for shipment."

The size of rivets, open holes, nature of shop paint, and other notes should be specified near the lower right-hand corner of each sheet.

For title, main dimensions, and shipping or erection marks, letter in heavy type. Use plain lettering, medium type, for other data.

Usual size of rivets for building work is ¾ in. in diameter. Other sizes may be used in exceptional cases.

In writing shop bills, main material should be billed first, followed by smaller pieces. Begin at the left end of a girder or truss and at the bottom of a column. Do not bill all angles and then all plates; group the material together that is assembled together. In case of a column containing brackets, bill each different bracket complete by itself. The shop bill is used as a guide in laying out and assembling the member in the shop as well as list of material required, and should be made accordingly. Members radically different should be billed separately an d not bunched together.

Use standard beam connections for connecting beams to beams, as indicated in steel handbook or Art. 129*a* except in special cases. Watch the limiting values of such connections to see that they are not exceeded.

In beam details, it is usual to make the distance center to center of end connection holes 5½ in. In a beam' detail showing the elevation of the web of a beam, it is usually understood that the horizontal distance center to center of lines of holes, when this distance is not given on drawing, is 5½ in. and the vertical distance between holes, when not given, is 2½ in.

Most structural steel shops have numerous standard details which should be followed when possible.

Avoid unnecessary countersunk rivets, as they are very costly. Use the least possible number of such in the bases of columns.

Steel handbooks give standard gages (distances center to center of lines of holes for flanges of beams and columns or distances from back of angle to lines of holes for angles) for beams, columns, and angles and these gages should be used when possible.

Rivets should be so spaced that they can be readily driven in a shop or field as may be necessary. Proper clearances and spacing can be obtained from the steel handbook.

Holes for anchor bolts are usually ¼ to 5⁄16 in. larger than the size of the bolts, to allow for discrepancies in setting bolt.

The usual minimum shop clearance between diagonal steel members and chords, as in truss work, is ¼ in. Filled clearance, minimum, in such cases, should be ½ in. A beam framing to other steel members by means of connection angles should have an overall length ⅛ in. less than the figured distance between surfaces against which beam frames.

When one beam frames into another with flanges at the same elevation, the flange of the former must be cut out or "coped" to fit against the flange of the latter. It is not customary to dimension a cope on a detailed drawing, but merely to call for the size of beam to which one detailed must be coped (see typical beam details). The shop does the rest in such cases.

An erection diagram, usually a line diagram of the completed structure, should be made with the erection or shipping marks thereon, to enable the erector to easily assemble the work in the field.

Lettering should be simple, straight line Gothic style, preferably inclined although vertical lettering is frequently used. Drawings should be neat and clear so as to inspire confidence in their accuracy.

Dimensions given on a column, when not otherwise shown, are measured from the top of the base plate to the point indicated.

Wherever a note on a drawing will help the erector, by all means use it. It is quite common to place a mark on a member showing the position of one end of the member in the finished structure so that the erector will erect the member as intended.

151. Assembling Marks.—The system of assembling marks which follows is in very common use. It has been used in the typical details at the end of the chapter.

Shop Assembling Marks

Typical letter	Where used
a	For base and cap angles on columns.
b	For bottom seat angles supporting beams and girders, connecting to columns or girders.
c	For base plates, cap plates, and splice plates.
d	For fillers with two or more lines of holes.
f	For fillers with single line of holes.
g	For gusset plates on columns or trusses.
h	For all bent angles and plates.
k	For stiffener angles fitted at one end only, such as angles under beam seats or at column bases.
m	For miscellaneous angles and shapes not covered by the above.
n	For miscellaneous plates not covered by the above; also tie plates.
p	For pin plates.
s	For stiffener angles fitted at both ends.
t	For top connection angles tying beams or girders to columns.
v	For purlin clips.
w	For web members of trusses, laterals in girders or angles in cross frames unless such material is shipped loose without being connected to any other part.
y	For lattice bars.

Material that appears on two or more sheets shall be identified as standard pieces. Standard pieces will be identified by the typical letter given under shop assembling marks and a figure, followed by the letter "*x*." The letter "*x*" indicates that the pieces are standard. For example, a series of standard stiffener angles, fitted at one end only will be given as "*k*1*x*," "*k*2*x*," etc., the letter *k* indicating a stiffener angle fitted at one end only, the numerals 1, 2, etc., being the identifying marks, and the letter *x* making them standard pieces.

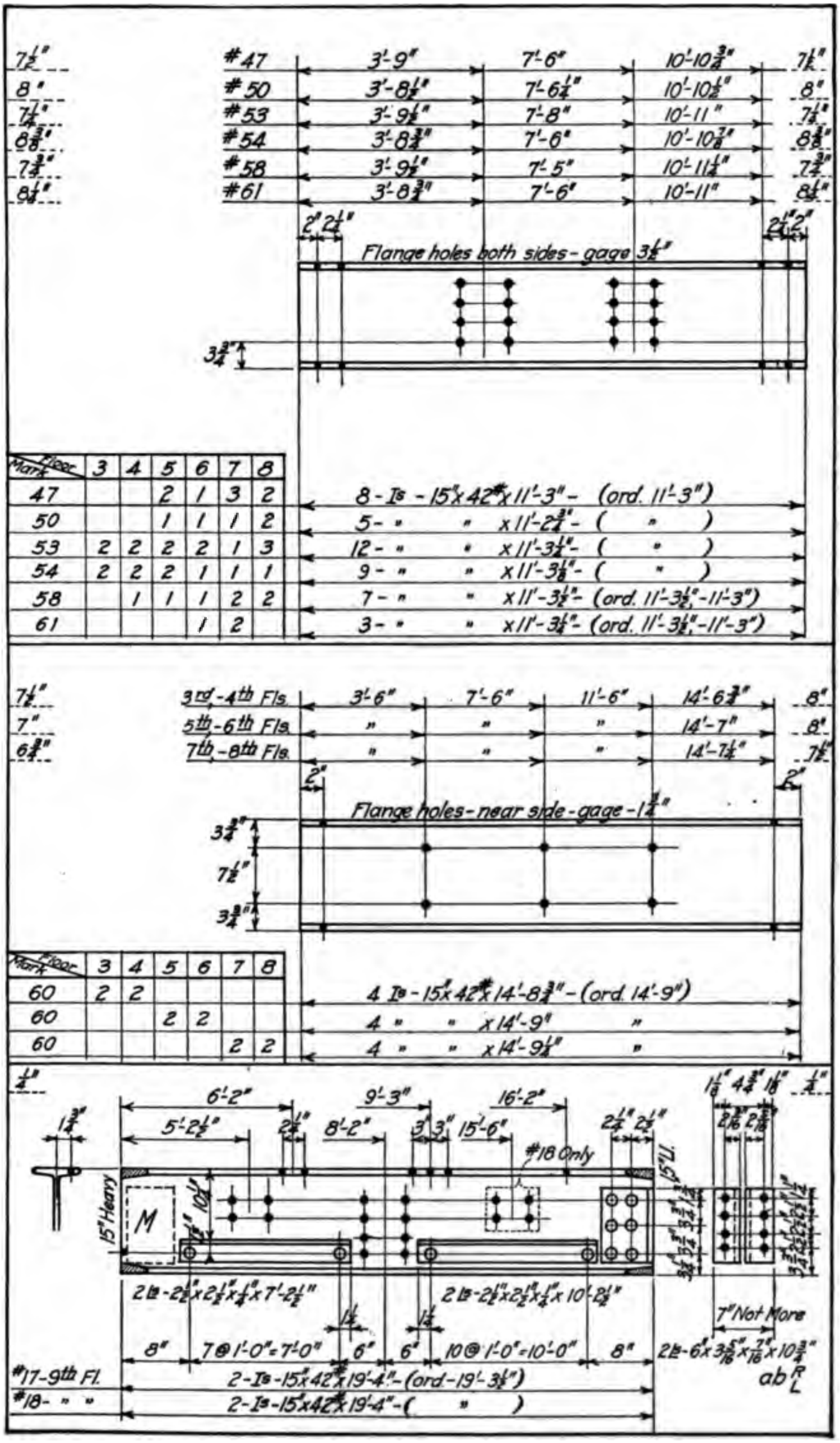
For all standard pieces on an order, a summary shall be prepared. This summary must give the number of pieces, size, length, mark, and the sheet number on which the piece is first detailed. All pieces having the same typical letter shall be grouped together as far as possible in the summary, the numbers to follow each other consecutively. Summary sheets shall be numbered consecutively $X1 - X2$, etc. Summary of standard pieces shall be made for each tier or shipment.

Pieces not standard are pieces that occur only on one sheet. They will be identified by the typical letter given under the shop assembling marks followed by a small letter and the sheet number. For example, an odd seat angle shown on sheet number 1 is marked "*ba*1." The numeral "1," giving the sheet number, should not be given on the drawing; it should only be given in the marking column provided in the shop bill. Hence the angle "*ba*1" would appear on the drawing as "*ba*" and in the shop bill as "*ba*1". Additional seat angles on the same sheet would be marked "*bb*1" "*bc*1," etc. No summary is made for pieces not standard.

All material shipped loose shall have a shipping mark.

The material ordered from the rolling mill must be so noted in the last column of the shop bill.

152. Typical Detail Drawings.—Figs. 266 to 271 inclusive are here presented as being typical shop detail drawings of members most frequently met with in building construction.



Mark \ Floor	3	4	5	6	7	8	
47			2	1	3	2	8 - Is - 15"x42#x11'-3" - (ord. 11'-3")
50			1	1	1	2	5 - " " x11'-2 3/4" - (")
53	2	2	2	2	1	3	12 - " " x11'-3 1/4" - (")
54	2	2	2	1	1	1	9 - " " x11'-3 1/8" - (")
58		1	1	1	2	2	7 - " " x11'-3 1/2" - (ord. 11'-3 1/2"-11'-3")
61				1	2		3 - " " x11'-3 1/4" - (ord. 11'-3 1/2"-11'-3")

7 1/2" 7" 6 3/4"
3rd-4th Fls. 3'-6" 7'-6" 11'-6" 14'-6 3/4" 8"
5th-6th Fls. " " " 14'-7" 8"
7th-8th Fls. " " " 14'-7 1/4" 7 1/2"
2" 2"
Flange holes-near side-gage-1 1/2"
3 3/4" 7 1/2" 3 3/4"

Mark \ Floor	3	4	5	6	7	8	
60	2	2					4 Is - 15"x42#x14'-8 3/4" - (ord. 14'-9")
60			2	2			4 " " x14'-9" "
60					2	2	4 " " x14'-9 1/4" "

1/4" 1 3/4" 15" Heavy M
6'-2" 5'-2 1/2" 2 1/4" 8'-2" 9'-3" 3" 3" 15'-6" 16'-2" #18 Only 2 1/4" 2 1/4" 15" I
10 1/2" 3 1/2"
2 Ls-2 1/2"x2 1/2"x1/4"x7'-2 1/2"
2 Ls-2 1/2"x2 1/2"x1/4"x10'-8 1/2"
1 1/4 1 1/4
8" 7@1'-0"=7'-0" 6" 6" 10@1'-0"=10'-0" 8"
#17-9th Fl. 2-Is-15"x42#x19'-4"-(ord-19'-3 1/2")
#18- " " 2-Is-15"x42#x19'-4"-(")
1 1/8" 4 3/4" 1 1/8" 1/4" 2 3/16 2 3/16
3 3/4 3 3/4 3 3/4 1 1/2
7" Not More
2 Ls-6"x3 5/16"x7/16"x10 3/4"
ab R L

Fig. 266.

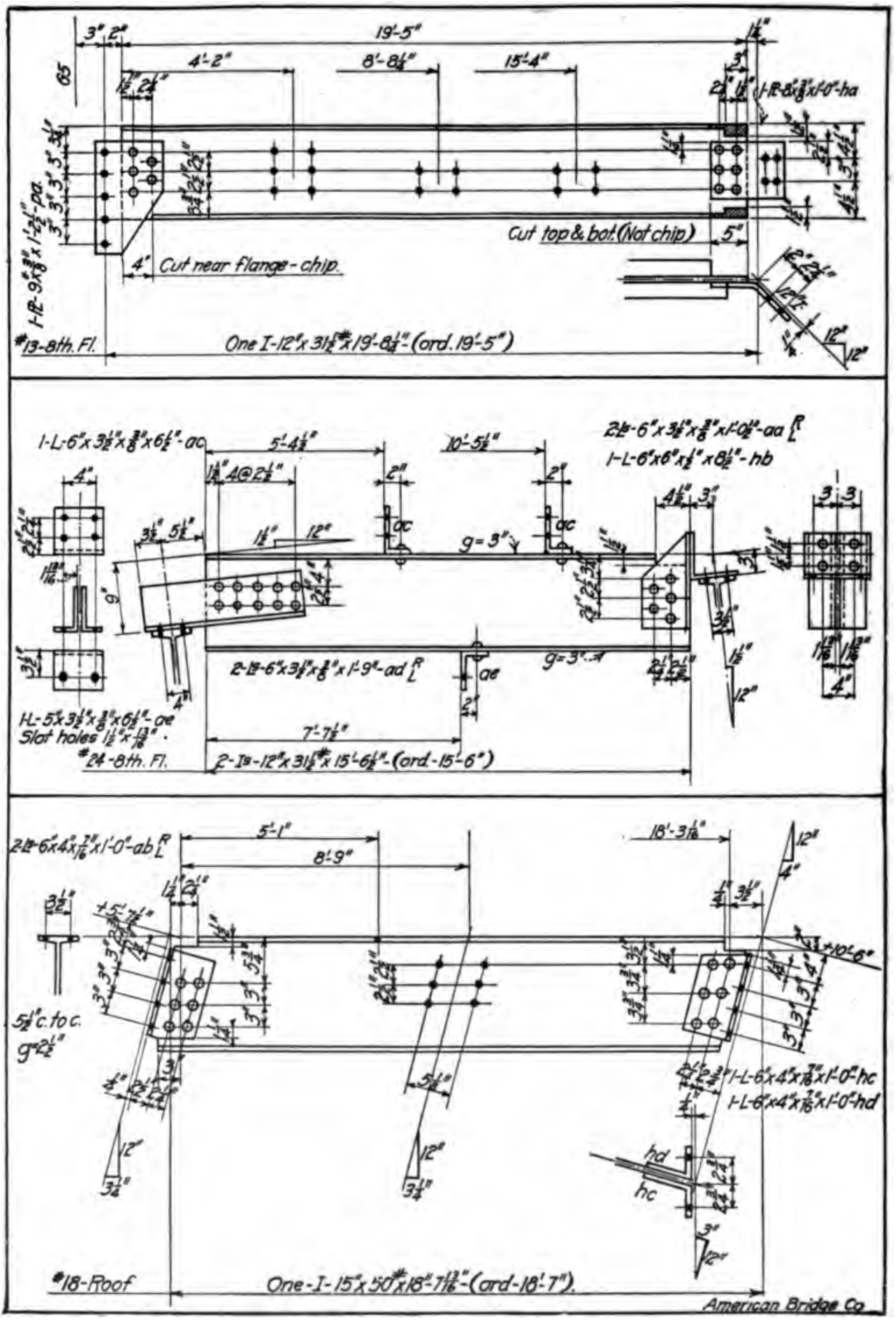

3" 2"
19'-5"
1/2"
4'-2"
8'-8 1/4"
15'-4"
3"
65
1 1/2" 2 1/4"
2 1/4" 1 1/2"
1-Pl-8"x3/8"x1'-0"-ha
1-Pl-9"x3/8"x1'-2 1/2"-pa.
Cut top & bot. (Not chip)
5"
4" Cut near flange - chip.
#13-8th. Fl.
One I-12"x31 1/2#x19'-8 1/4"-(ord. 19'-5")
12"
12"
1-L-6"x3 1/2"x3/8"x6 1/2"-ac
5'-4 1/2"
10'-5 1/2"
2L-6"x3 1/2"x3/8"x1'-0 1/2"-aa R L
1-L-6"x6"x1/2"x8 1/2"-hb
4"
1 1/2" 4@2 1/2"
2"
2"
ac
ac
g=3"
4 1/2" 3"
3 3
3 1/2" 5 1/2"
1 1/2" 12"
9"
2-L-6"x3 1/2"x3/8"x1'-9"-ad R L
ae
g=3"
1-L-5"x3 1/2"x3/8"x6 1/2"-ae
Slot holes 1 1/2"x13/16".
#24-8th. Fl.
7'-7 1/2"
2-I-12"x31 1/2#x15'-6 1/2"-(ord-15'-6")
1 13/16"
4"
12"
2L-6"x4"x7/16"x1'-0"-ab R L
5'-1"
8'-9"
18'-3 1/16"
12"
4"
3 1/2"
1 1/4" 2 1/4"
+10'-6"
5 1/2" c. to c.
g=2 1/2"
1-L-6"x4"x7/16"x1'-0"-hc
1-L-6"x4"x7/16"x1'-0"-hd
hd
hc
5 1/2"
12"
3 1/4"
12"
3 1/4"
3"
12"
#18-Roof
One-I-15"x50#x18'-7 13/16"-(ord-18'-7")
American Bridge Co.

FIG. 267.

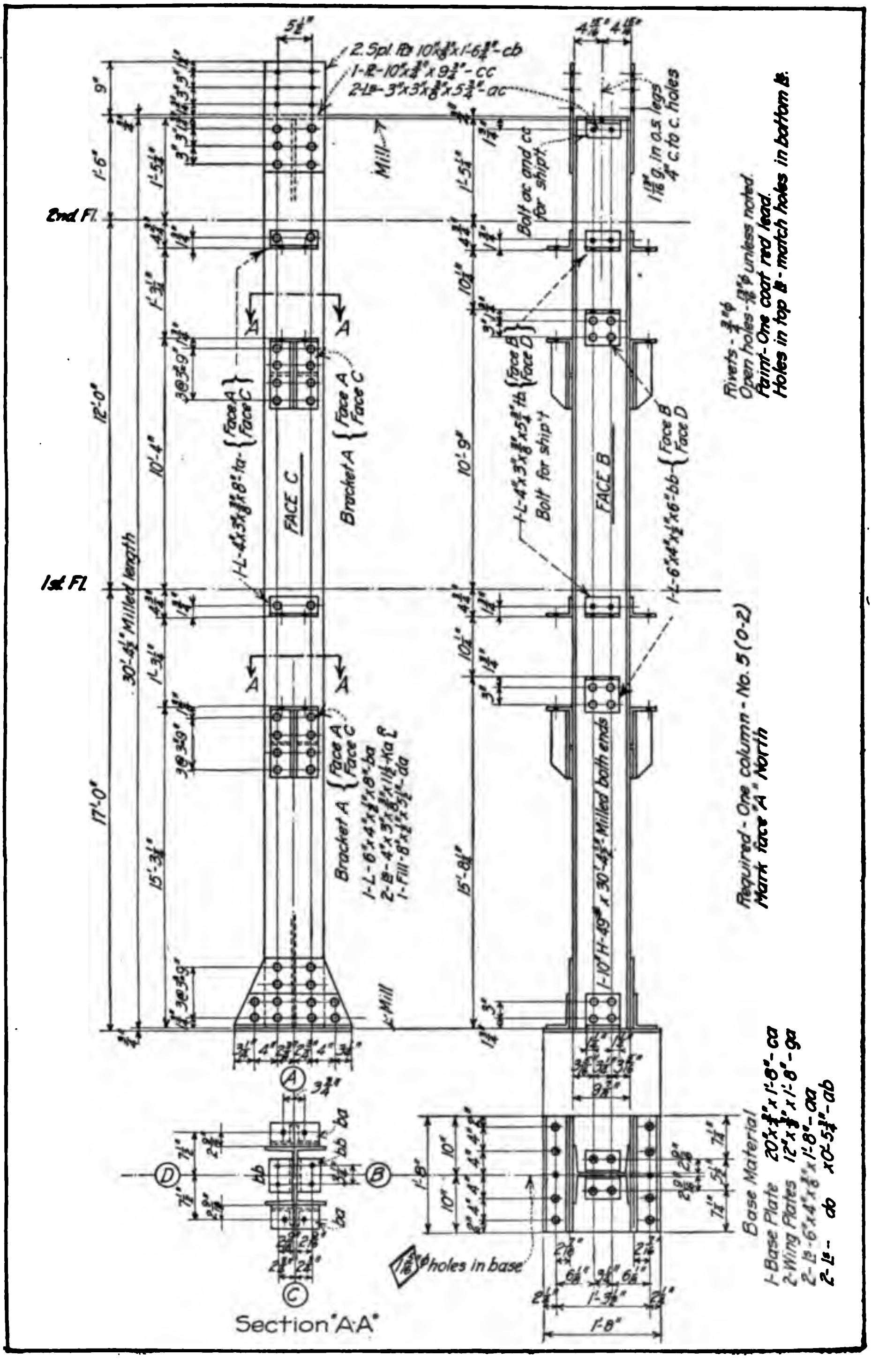
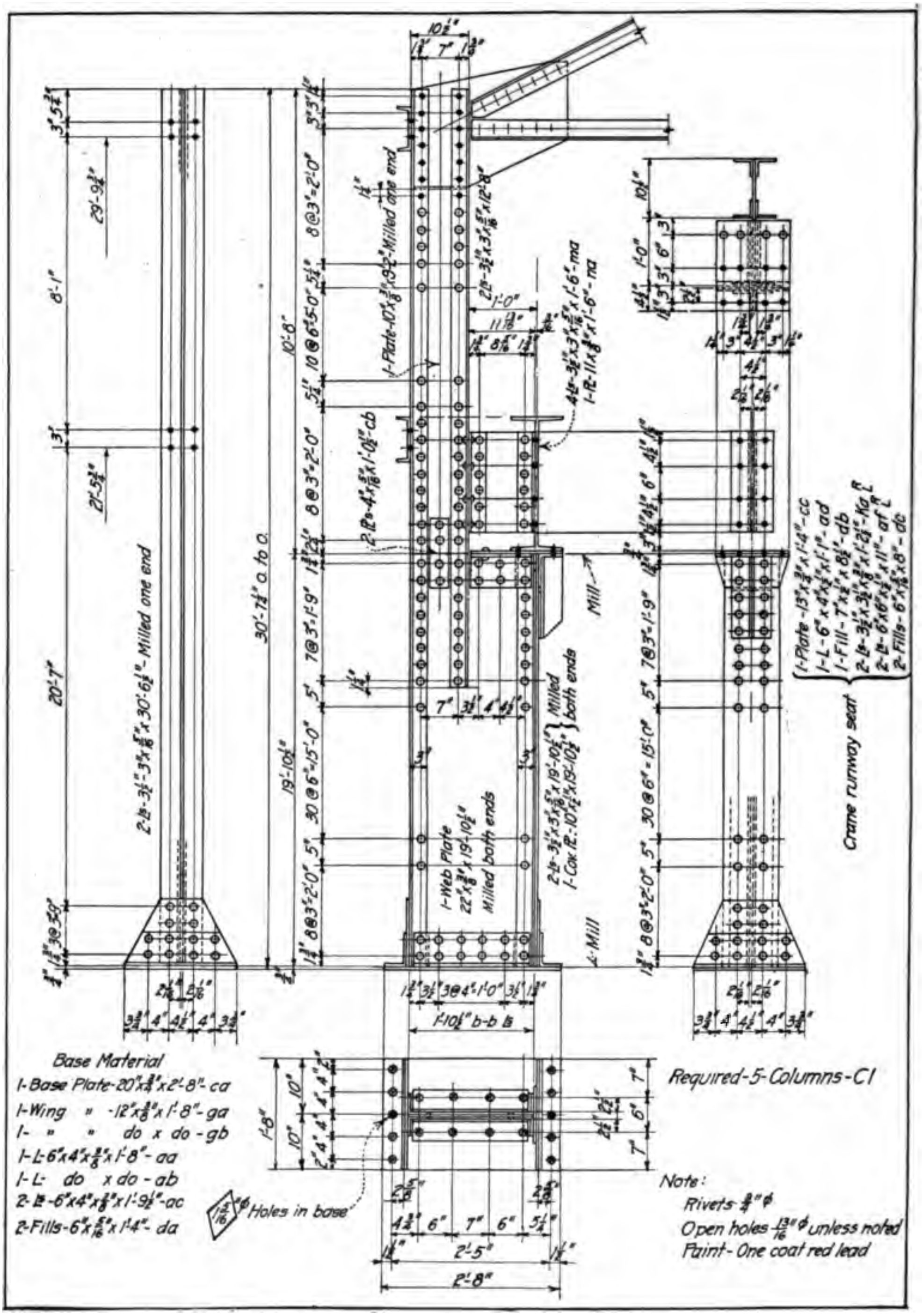

2-Spl. Pls 10"×3/8"×1'-6 1/2"-cb
1-Pl-10"×3/4"×9 1/2"-cc
2-Ls-3"×3"×3/8"×5 3/4"-ac
Mill
2nd Fl.
1st Fl.
30'-4 1/2" Milled length
9"
1'-6"
12'-0"
17'-0"
1'-5 1/4"
1'-3 1/4"
10'-4"
15'-3 1/4"
10'-9"
15'-8 1/4"
3@3 9"
5 1/2"
A
FACE C
FACE B
1-L-4"×3"×3/8"×8"-la- {Face A Face C}
Bracket A {Face A Face C}
Bracket A {Face A Face C}
1-L-6"×4"×1/2"×8"-ba
2-Ls-4"×3"×3/8"×11 1/2"-ka
1-Fill-8"×1/2"×5 1/2"-da
1-L-4"×3"×3/8"×5 3/4"-lb {Face B Face D}
Bolt for shipt
Bolt ac and cc for shipt.
1 1/16" g. in o.s. legs 4" c. to c. holes
1-L-6"×4"×1/2"×6"-bb- {Face B Face D}
1-10" H-49# × 30'-4 1/2" Milled both ends
Rivets - 3/4" φ
Open holes - 13/16" φ unless noted
Paint - One coat red lead.
Holes in top Pl - match holes in bottom Pl.
Required - One column - No. 5 (0-2)
Mark face "A" North
Base Material
1-Base Plate 20"×1 3/4"×1'-8"-aa
2-Wing Plates 12"×3/8"×1'-8"-ga
2-Ls-6"×4"×3/8"×1'-8"-aa
2-Ls- do ×0'-5 3/4"-ab
13/16" φ holes in base
1'-8"
1'-3 1/2"
9 3/4"
6 1/2"
A
B
C
D
ba
bb
Section "A-A"

FIG. 268.



Base Material
1-Base Plate-20"×3/4"×2'-8"-ca
1-Wing " -12"×3/8"×1'-8"-ga
1- " " do x do -gb
1-L-6"×4"×3/8"×1'-8"-aa
1-L- do x do - ab
2-L's-6"×4"×3/8"×1'-9½"-ac
2-Fills-6"×5/16"×1'-4"- da

Required-5-Columns-C1

Note:
Rivets ¾"ϕ
Open holes 13/16"ϕ unless noted
Paint- One coat red lead

FIG. 269.

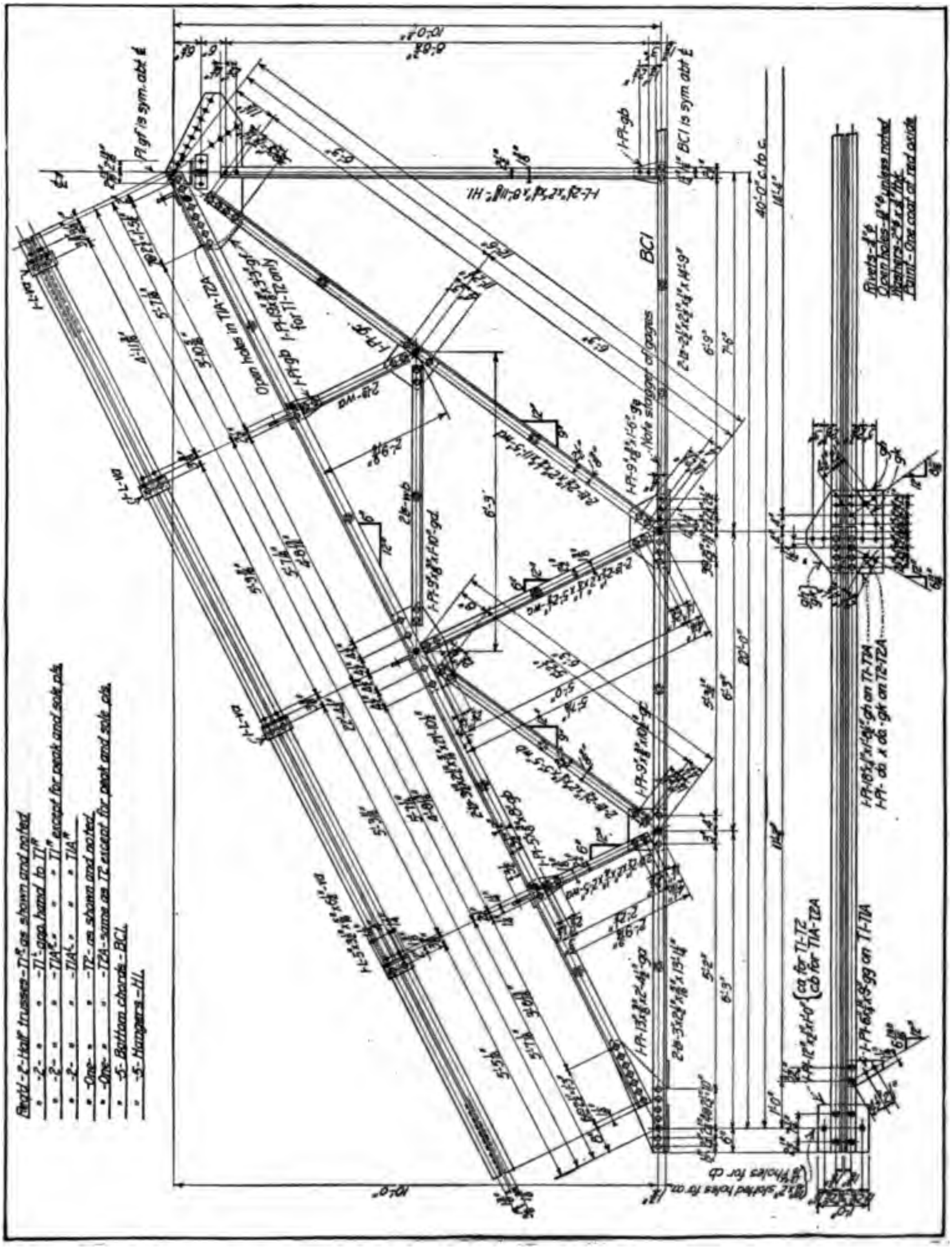

Reqd.-2-Half trusses-T1^R as shown and noted
" -2- " -T1^L opp. hand to T1^R
" -2- " -T1A^R " " T1^R except for peak and sole pls.
" -2- " -T1A^L " " T1A^R
" -One- " -T2- as shown and noted
" -One- " -T2A-same as T2 except for peak and sole pls.
" -5- Bottom chords-BCl
" -5- Hangers-H1
Rivets-3/4"
Open holes-13/16" unless noted
Washers-2¼"x 3/16" x 13/16"
Paint-One coat of red oxide
BCl
BCl is sym. abt. ℄
Pl. gt. is sym. abt. ℄
Open holes in T1A-T2A
Note stagger of gages
40'-0" c. to c.
20'-0"
10'-0"
H1

Fig. 270.

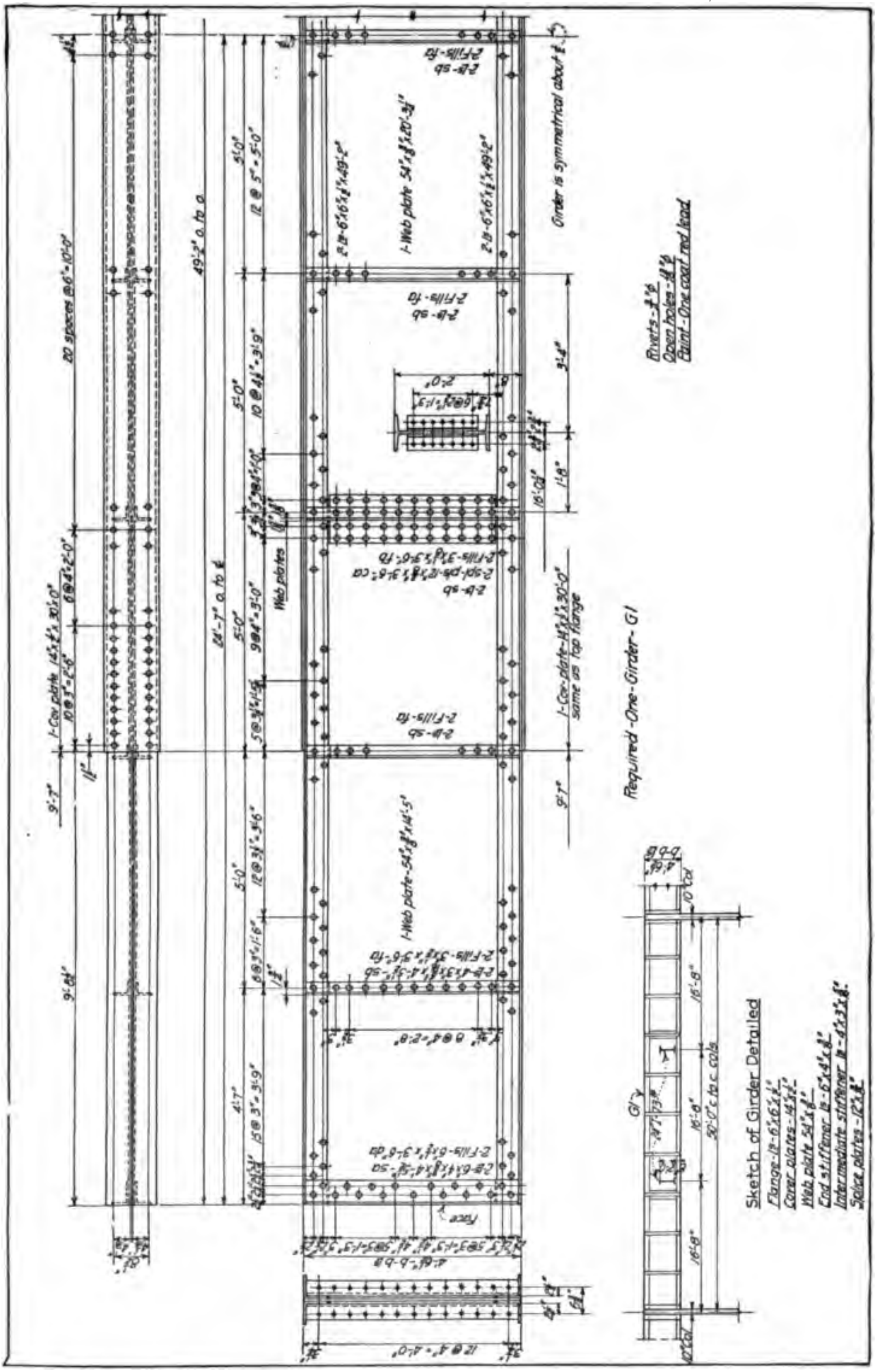

Girder is symmetrical about ℄
Rivets-3/4"φ
Open holes-13/16"φ
Paint-One coat red lead
Required-One-Girder-G1
1-Web plate 54"x3/8"x20'-3"
1-Web plate-54"x3/8"x14'-5"
1-Cov-plate-14"x1/2"x30'-0" same as top flange
2-Fills-fg
2-∟s-sb
Web plates
Face
49'-2" o. to o.
24'-7" o. to ℄
20 spaces @6"=10'-0"
Sketch of Girder Detailed
Flange-4∟s-6"x6"x1/2"
Cover plates-14"x1/2"
Web plate 54"x3/8"
End stiffener 2∟s-6"x4"x3/8"
Intermediate stiffener 2∟s-4"x3"x3/8"
Splice plates-12"x3/8"
16'-8"
50'-0" c to c cols
G1
10" Col.

FIG. 271.

Simple members were selected for these illustrations because of their simplicity but the methods of laying out and arrangement of sketches and dimensions might be studied to advantage and applied to more complicated structures. These methods are typical of modern practice and are easily and quickly applied and readily understood by shop workmen.

Figs. 266 and 267 give typical beam details. Where horizontal distance between holes is omitted, distance center to center is understood to be 5½ in. When vertical distance between holes is omitted, such distance center to center is understood to be 2½ in. These beam sketches are taken from The American Bridge Company's standard and are typical of current practice. In general detailing, which might be used by any shop, it is better to provide the omitted dimensions, size of angles, etc. on the drawing.

Figs. 268 and 269 show shop detail drawings of Bethlehem *H* and built-up mill building columns. Fig. 270 is a shop detail drawing of modern roof trusses, and Fig. 271 of a building plate girder. Figs. 266, 267, and 270 have been taken from Conklin's "Structural Steel Drafting and Elementary Design."

The details shown in Fig. 270 are those for a series of steel roof trusses for a building roof, the complete connections for purlins, struts, and bracing being shown. Trusses of this type and size are usually shipped in halves, the hanger at center and center bottom chord being shipped loose. Note the open holes to provide for this.

CONCRETE DETAILING

By Walter W. Clifford

Concrete detailing, as a branch of structural drafting, is young, and pitifully weak as compared with steel detailing. This is particularly unfortunate, as the grade of labor used on concrete and reinforcement is usually less skilled than that used on steel. Up to the present time, credit for the success of much concrete construction has belonged more to the superintendent or foreman of construction than to the architects or engineers who designed the work.

In concrete detailing, two things must be considered: (1) the outlines of concrete which give necessary information for the forms, and (2) reinforcement details used in the bending shed to get out steel, and on the floor to place it.

153. Outlines.—Outlines, or outside dimensions of concrete, are invariably given by the architect or engineer designing the work. For this part of concrete detailing the common rules of drafting usually suffice. In general, outlines and reinforcement can be taken care of on the same drawing. But where the outlines are very complicated, separate outline and reinforcement drawings avoid confusion and save time in the drafting room as well as in the field. Common cases of this kind are wells and pits, and complicated floors. For wells and pits "outline drawings" are made giving all information for forms, and then in making the reinforcement drawings, the outlines as represented by forms being defined, reinforcement is located from them. In the case of floors, so-called "surface plans" are often made. Upon these plans, together with necessary sections, openings and pedestals are located and dimensioned; surface slope, if any, is shown; and beams are marked, sized, and located. In a few cases floors have been so extremely complicated that it was found advisable to add to surface and reinforcement plans, a machine bolt location plan.

154. Dimensions.—In dimensioning similar members, such as beams or columns, a logical and consistent location of dimensions will simplify both office and field work. On beam details, for example, give the locations of intersecting beams in a line of dimensions above the elevation; the clear span and support width in the first line of dimension below the elevation; and the span center to center of supports below this (see Fig. 279, p. 325). Give stirrup spacing near the center of the elevation; list the cambered or bent steel just below right end; the straight steel below the left end; stirrups and spacers under the center of the beam, etc. Consistency of this kind is essential for good details. The location of the information, so long as it is clearly given is of less importance than the consistency in placing it in a given location.

155. Framing Plans.—Where there is no surface plan, framing plans are usually combined with slab reinforcing plans. Framing plans should show clearly: all column center lines, location of all beams, size of all beams (in case of sloping floor surface, note grade from which beam depth is given), beam marks, column marks, and preferably the sizes of the columns, below the floor. Concrete beams are shown to scale on ¼-in. scale plans and as a single heavy line on ⅛-in. scale plans. Steel beams supporting concrete slabs are well shown by a very heavy dash line.

Beam and column marks are of considerable importance. The common custom of numbering them in sequence is open to objections. In the course of the changes which most plans undergo, No. 92 is likely to land between Nos. 5 and 6 and it is then difficult to locate. The coördinate system while it seems complicated at first, is really simple and easy to learn. In this system column lines vertical on the plan are lettered and horizontal lines are numbered. Beams can then be marked with the mark of the column at the lower left-hand corner of the bay in which they occur together with *H* for horizontal on the plan, or *V* for vertical. Fig. 272 illustrates this system. Intermediate beams may be designated by primes. Typical beams which repeat a number of times may have single numbers—odd for horizontal, and even for vertical beams on the plan—in place of location marks. The floor number may precede the mark. With this system any member added during the making of the drawings has a mark ready for it and cross reference between framing plans and details is greatly facilitated.

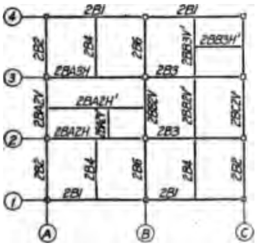


Fig. 272.

Floor grades and references to the sheets on which details will be found are useful additions to framing plans.

156. Reinforcement Details of the Architect.—There are two kinds of reinforcement details, those of the architect and those of the engineer or contractor. The architect is necessarily interested only in giving the information essential for carrying out his design, while the engineer has to give complete information for the bending shop. The information which the architectural office must give, is in general: size and location of all main reinforcement together with the angle and location of all cambers and bends; also the size, shape and location or spacing of auxiliary rods such as stirrups, hoops, and spacers. The architect must remember that if he is to justify himself as a designer of his work he must at least give such information that details can be made in only one way and then he must check bending details to see that they are properly made.

Some of the necessary information can be covered by notes such as:

All main slab steel shall be centered ¾ in. above the forms for bottom steel and ¾ in. below the rough slab grade for top steel.

The lower layer of beam steel shall be centered 2 in. above the forms in all beams and 3 in. in all girders. The top layer of negative reinforcement shall be centered 2 in. below the rough slab grade for all beams and 3 in. for all girders.

Chairs or supports for reinforcement may be covered by note or in specifications in the following manner:

Chairs of an approved type shall be used to support all slab steel. At least one chair shall be used for each 15 sq. ft. of floor.

157. Reinforcement Details of the Engineer or Contractor.—Detailing by the contractor is analogous to steel shop drawing. Assembly drawings should be made on which each piece is given a mark, with the place it is to occupy in the form definitely indicated. Complete schedules should also be given with bending diagrams. A number of engineers, whose business arrangements with clients permit it, detail the concrete fully and schedule the reinforcement. This is the most satisfactory method, for the designer of concrete should be entirely responsible for the details. Details of various parts of concrete construction will now be considered somewhat from the viewpoint of the contractor or the fortunate engineer able to detail his own work.

158. Scale and Conventions.—Scale for concrete details is quite commonly ½ in. = 1 ft., and this is satisfactory for most work. Sections may be indicated by shading on the back of the tracing with a soft pencil. This is quicker than the conventional symbol and at least as effective. Full heavy lines are used for reinforcement in the details given in this chapter, and this is most satisfactory on drawings. The distinction between the rods and the outline of the concrete is in the weight of the line. Dash lines as sometimes used are slower to draw and often lead to confusion where rods cross at angles.

It should be borne in mind that concrete reinforcement details are largely diagrams. Clear indication of the way rods are to go, is vastly more important than true orthographic projection. For example, the rods shown over a beam support in actual projection in Fig. 273 may be in diagram as shown in Fig. 274 or as shown in Fig. 275. They should be diagrammed correctly as shown in one of the later views. The cross section will indicate that they are at the same elevation, and proper scheduling will bring them there.

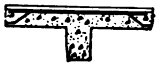


Fig. 273.

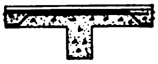


Fig. 274.

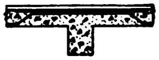


Fig. 275.

159. Slabs and Walls.—Slabs and walls are similar in detail and vary only in position. They have in general main reinforcement perpendicular to a system of beams, and spacers at right angles to the main rods. The main steel may be cambered to give negative reinforcement, or the so-called *loose-rod system* of separate bars to take care of negative moment may be used. In walls, vertical rods are placed outside (nearer the face) wherever possible. This is better for concrete placing.

159a. Listing.—Steel in plan, or elevation if in walls, is best indicated by considering bands consisting of rows of evenly spaced identical bars. The outside bars of the band are shown and the band listed as shown in Fig. 276.

In architectural detailing the bands may be similarly shown and listed simply "⅝ ϕ 6″c. to c."

A diagram of two adjacent rods will be noted in Fig. 276 in the center of the bays. This is often an advantage in working out the detail and may save separate sections to a large extent.

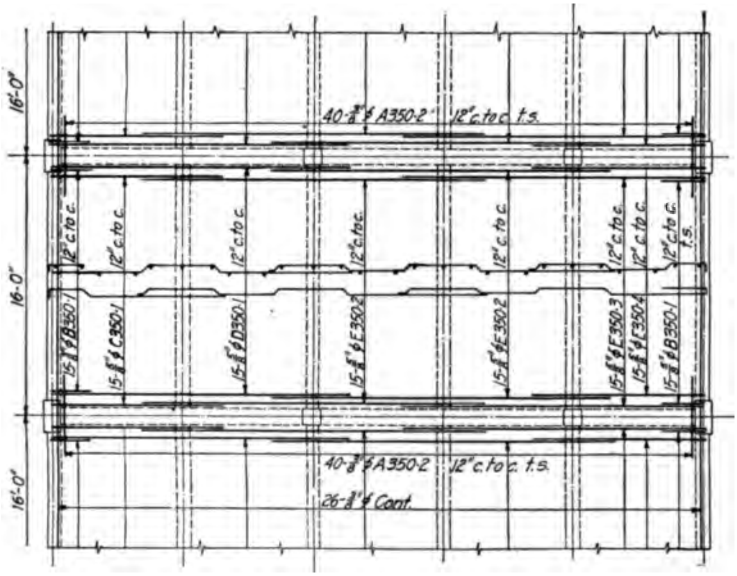
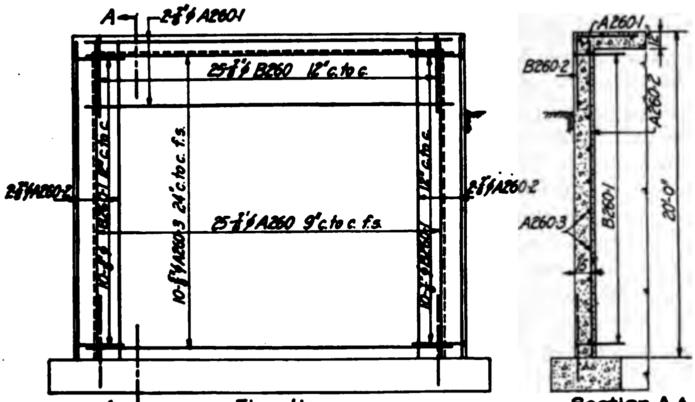


Fig. 276.—Slab detail.

To differentiate clearly between steel in top and bottom or far and near side, a method successfully used is to add to the listing *f.s.* or *t.s.* thus "29–⅝″ ϕ–A42–6″ c. to c.–t.s." Then use as a general note: "All rods marked *t.s.* are in the top of the slab, all other rods are bottom or cambered steel" or "All rods marked *f.s.* are in the far side, all other rods are in the near side."

In listing bands, the number of rods, type, and spacing are obviously needed for setting the steel on the floors. The size should also be given because rods are ordinarily stored by sizes on the job, and this information is, therefore, helpful in finding them. Schedules are ordinarily not used in setting, and if used, cross reference between plan and schedule is a nuisance.

159b. Spacers.—Spacers are very commonly ⅜-in. rounds, 2 ft. on centers, for ordinary slabs. In walls a size smaller than the main reinforcement is commonly used with a maximum of ¾ in. and a minimum of ⅜ in., and with spacing 18 in. to 3 ft. They are ordinarily random length for the smaller rods, scheduled as total length and cut on the floor. They may be covered by a note, or indicated in the diagram (see Fig. 276). The larger spacers (⅝ or ¾ in.) are best listed and typed in bands like main reinforcement.



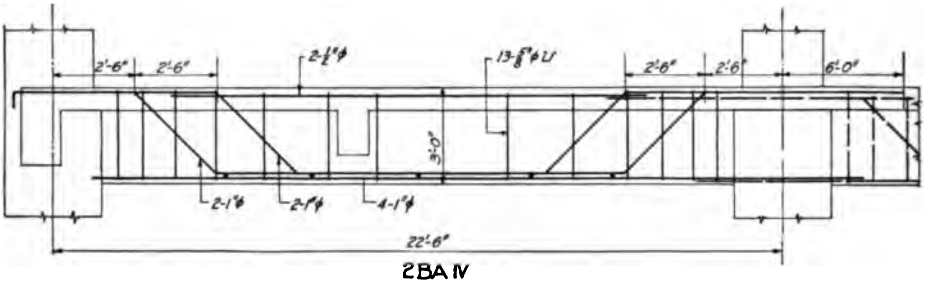
Elevation Section A-A

Fig. 277.—Wall detail.

159c. Rod Spacing.—Rod spacing in slabs is limited in the Joint Committee's report to 2½ times the slab thickness and the minimum should be as in beams. Common practice for ordinary work is 1 to 1½ times the slab thickness.

159d. Sections.—In addition to slab plans and wall elevations, sufficient sections must be given to clearly indicate the location of all steel (see Fig. 277).

159e. Flat Slabs.—Flat slab construction is detailed like other slabs, except that typical bands may well be listed "Band A," etc., the schedule indicating the makeup of the various bands This is sometimes possible with beam-and-slab construction. The S. M. I. flat-slab system makes use of units of spider type over columns and in the center of bays. On reinforcement plans of this system each unit is completely shown once and elsewhere simply a circle is shown (the outside ring) and marked "Unit C," etc. Where separate units are used for positive and negative reinforcement, different weights of lines may be used for top and bottom steel This helps greatly in the clearness of the drawings.



2BA IV
Beam width-15" — Cambers-45°

Fig. 278.

160. Beams.—A typical beam detail from an architectural office is shown in Fig. 278. The same beam is fully detailed in Fig. 279. The best practice is to detail beams and columns as separate units or members, as is done in steel detailing. This is preferable to covering them by various and sundry sections through the floor. Some conventions are used. The dash line is used in the section to indicate cambers in elevation; in the elevation it is used to indicate rods belonging to another detail. A somewhat lighter line is used for stirrups than for main steel. The open circle at the top of the camber is used for a horizontal rod in elevation while the solid circle is used for the rods cut by the section.

160a. Rod Spacing.—Rod spacing in beams is discussed from the theoretical point of view in Sect. 1, Art. 63*h*. In addition to this the detailer should know that the clear distance between rods should be not less than twice the largest aggregate size. Rods are often used in two layers, very seldom more than two. Layers of beam rods are usually separated 1 in. by short spacer bars. The distance between these spacers depends on the size of the main steel. Fifty times the diameter of the main steel is reasonable. There should be at least two spacers under each rod of the top layer.

160b. Connections.—The intersection of beam, girder, and column steel over the column head must be carefully studied. With a beam centered on a column, careless detailing often has a rod in the center of the column and one in the center of the beam. Small rods (½ in. or less) are easily offset, but this is not the case with larger rods. Beam and girder intersections must also be detailed with care to see that interference is not caused by rods at the same grade.

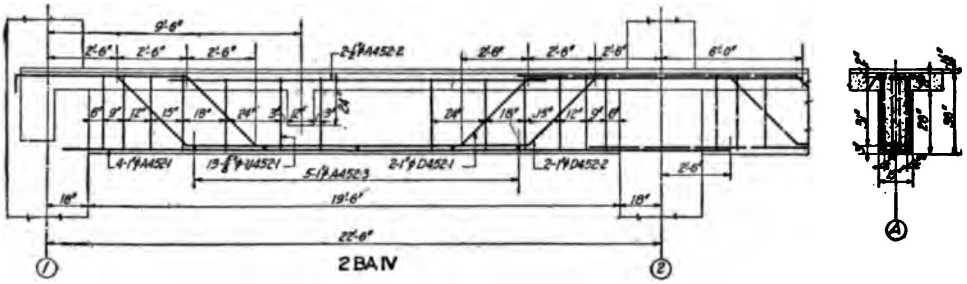


Fig. 279.

160c. Inflection Points.—Certain parts of concrete theory are particularly the province of the detailer. He should be familiar with the use of reinforcement to take tension and know which is the tension side of beams in all cases—as well as in slabs and walls. He should also have a general idea, at least, of the location of inflection points. See "Restrained and Continuous Beams," Sect. 1.

160d. Stirrups.—Shear and stirrups are also very much the province of the detailer. He should know the variation of shear with uniform and concentrated loads (see "Shears and Moments," Sect. 1, and "Restrained and Continuous Beams," Sect. 1). He should be familiar with the method of determining stirrup spacing (see "Reinforced Concrete Beams and Slabs," Sect. 2). In addition to theoretical consideration the following practical points are useful: It is good practice to place stirrups 4 or 6 in. from the face of all intersecting beams. The first stirrup is located by many engineers about ¼ to ⅓ of the depth of the beam from the face of the support, diagonal tension cracks almost never starting at the support. In very wide beams where stirrups of more than four legs would be needed it is better from a practical standpoint to use several U's or W's as shown in Fig. 280. Rods larger than ⅝ in. should not be used as stirrups, unless absolutely necessary, on account of the difficulty of bending.

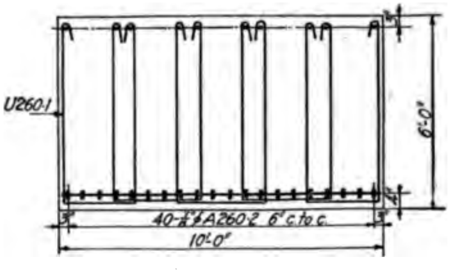
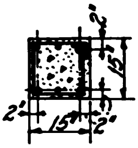
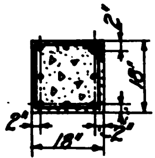
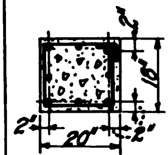
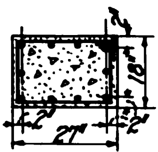
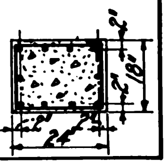


Fig. 280.

160e. Bond.—Bond is seldom an important item in beam and slab design. Most properly designed beam reinforcement is sufficient for bond. In beams continuous over supports, part of the main reinforcement is usually cambered. The balance is continued across the support as compression steel in T-beams, and this use determines the lap rather than bond (see right-hand support, Fig. 279). At end supports, straight steel is often hooked.

It is good practice to hook the ends of tension rods at all end supports. The ends of stirrups usually need hooks for bond and it is good practice to hook all of them.

161. Columns.—Columns can, if simple, be covered by a column schedule of the type shown in Fig. 281. The rod schedule and a few notes will complete the necessary information. In the architectural type of detailing, main steel may be listed as *long rods* and *short rods*, and notes added such as "Short rods shall be 6 in. shorter than the distance floor to floor," "Long rods shall be 50 diameters longer than the distance floor to floor," "All columns are to be concentric, except those on the *A*, *C*, 1, and 10 lines, which are to be flush on the outside face or faces." In the case of columns having complications such as brackets, an elevation should be drawn similar to beam elevations and the necessary sections added.

COLUMN SCHEDULE					
		Col. Nos A1, A2,		A5, A7, A9	
		Steel	Section	Steel	Section
18'-0"	3rd Story	6-7/8"φ A450·5 16-1/2"φ 0450·3 12" c. to c.	Section same as 2nd Story	6-3/4"φ A450·6 16-1/2"φ 0450·3	
20'-0"	2nd Story	6-7/8"φ A450·3 2-7/8"φ A450·4 18-1/2"φ 0450·3 12" c. to c.		Steel same as A1	Section same as A1
20'-0"	1st Story	6-1"φ C450·2 2-1"φ A450·2 18-1/2"φ 0450·2 12" c. to c.		Steel same as A1	Section same as A1.
24'-3"	Basement	8-1"φ C450·1 2-1"φ A450·1 20-1/2"φ 0450·1 12" c. to c.		Steel same as A1	

All splice rods to be lapped 40 diameters.

Fig. 281.

161a. Rod Spacing.—The rod spacing of the main rods usually takes care of itself with standard percentages of steel and commercial rod sizes. The maximum spacing of vertical rods allowed by good practice is about 10 or 12 in. In the case of large columns with high percentages of steel it is difficult to get all that are required in one band. The largest rod easily available in most localities is 1¼ in. In large columns these should be spaced at least 6 in. apart, and where spiral hooping is used at least 8 in. Where too many rods are required for this spacing, two rows of rods should be used or some of the rods should be placed in the form of a cross inside the core. Hoops are limited by the Joint Committee's report to a maximum spacing of 12 in., or 16 times the diameter of the longitudinal bars. Light rods suffice for this hooping, ¼ to ½ in. being the common sizes; ⅜-in. round the most used.

161*b*. Spiral Hooping.—Spiral hooping for columns is expressed in percentage of volume of hooping to volume of core per unit of length. The design of hooping is discussed in Arts. 85 and 96.

Hooping has great possibility of irregularity when the core is of large diameter. In order to ship flat, two vertical ties only are used, and this leads to deformation in handling. One-inch cover may do on 12 to 16-in. columns but on 3-ft. cores or larger at least 3 in. of cover should be allowed and preferably 4 in., irrespective of fire risk.

161*c*. Splices.—Horizontal joints in columns ordinarily occur at the bottom of the deepest girder, at the rough floor grade, and in some cases at the top of upstanding spandrel beams. The top of the rough floor is usually a splice point, and good practice requires rods, to the number of those in the upper section, run up from the lower section, the distance required for bond. These rods should preferably be so located that the rods in the upper section can be wired directly to them. In the case of large rods some engineers require rods to be faced and held in a sleeve. It is very difficult, however, to so place and hold faced rods for the direct transfer of load. Where offsets are required in extended rods on account of change of column sections, they should be at least a foot below the splice, and offsets should not be by slopes of more than 30 deg. with the vertical.

162. Miscellaneous Concrete Members.—The general principles enumerated can be followed to detail most miscellaneous structures. In miscellaneous structures, as in slabs, there is danger of putting so much information on a single view that it becomes confusing to draftsman and builder. Rods usually appear in more than one view. They will, of course, be listed in one view only, and be noted in the others. It is important for good detailing that they be listed in the best place. Ordinarily, this is in the view in which the rods appear in projection as a straight line. Whenever a structure is detailed in parts, however, rods which run into two parts should always be listed with the part which will be poured first. For example, in a tunnel, angle rods from the floor into the walls should be listed in the floor detail. The more common miscellaneous members are footings, pits and tunnels, engine foundations, and retaining walls

162*a*. Footings.—Footings vary so greatly in complexity that it is difficult to lay down general rules. Usually a plan and one or more sections will be needed. Sometimes they are simply large beams and can well be detailed as such. Stirrups should never be used in footings where it is possible to avoid them. They are exceedingly difficult to place.

162*b*. Pits and Tunnels.—Pits and tunnels which are complicated are best separated into members, and each slab and wall detailed independently. Where they are simple, general views and sufficient sections will suffice. Simple structures of considerable length like some power house intake and discharge tunnels, are conveniently detailed by giving all the different cross-sections, and longitudinal sections through the ends, and showing a small scale key plan indicating the extent and location of the parts where each section applies. This method is also applicable to some grade beams, spandrel details, and some retaining walls.

162*c*. Engine Foundations.—Engine foundations where they are only pedestals. can be detailed with the floors. Larger foundations such as those ordinarily required for large turbo-generators should be detailed as separate structures. The larger ones should be broken up, and slabs, beams, and columns detailed separately, like any similar units.

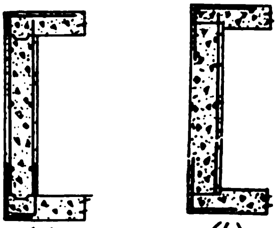
162*d*. Retaining Walls.—Retaining walls, if of uniform section, may be detailed in the method suggested for long tunnels. Where counterfort or buttress walls are used, separate details of vertical slab, footing slab, counterfort or buttress, etc., are needed.

162*e*. Construction Joints.—Construction joints should be included in some details. For example, tunnels are usually poured in three parts—floor, walls, and roof. If the walls are subject to pressure, it is important that they have bearing on floor and roof. Details such as those shown in Fig. 282 should be designed for shear and shown on the drawings.

162*f*. Spacers.—Spacers in miscellaneous members need more attention than is often given them. In addition to their theoretical use for temperature, or to distribute loads, they have the important function of holding the main steel rigidly in place during the pouring of the concrete. Some practical thought of how the steel is to be placed and held, is necessary in locating spacers. For example, bands of L-shaped rods need three spacers at least, one in the angle and one near each end, if the band is to be held rigid.

162g. Rod Splices.—Construction joints must also be considered in reinforcement detailing. It is bad practice to have rods extend through a construction joint with only a small part of their length imbedded in the first pouring. This is especially bad in the case of vertical rods. They are difficult to support and very likely to be bent out of shape. As far as possible, where rods would project 6 ft., or more than half their length beyond a joint, they should extend only the bond distance. They should then be splieed by another rod starting at the joint. Fig. 283 shows a typical illustration of this.

In the case of footings there are no vertical rods to extend up through the joint. Special short rods called *stubs* are used in such cases. They extend a distance required for bond, each side of the joint, and act as dowels (see Fig. 283). Vertical rods should always start at a construction joint when possible, so that they may be set directly on the old concrete when placed (see Fig. 282). Design factors sometimes overrule the foregoing; for example, high walls often require vertical steel from top to bottom while one or more construction joints are necessary. Care must be used in all such cases to conform to design requirements and at the same time make placing as simple as possible.



(a) (b)

Fig. 282.

163. Reinforcement Cover.—The cover over reinforcing rods, as, for example, under slab or beam rods or outside of column rods, serves to protect them from fire and weather and also to develop bond on the entire surface of the rod. Detailers should be familiar with common fireproofing requirements. Too little cover means danger from fire or sometimes moisture, too much in beams and slabs means cracks in the concrete below. A ½-in. clear cover for slabs 4 in. thick, with rods not over ½ in., and a small fire risk, is the minimum. A 1-in. clear cover is about the maximum for slabs. For beams and girders 1½ to 3 in. is used according to the importance of member and the fire risk. In columns, from 1 to 4 in. is used.

164. Shop Bending.—Every concrete detailer should be familiar with reinforcement in place in the forms, and as far as possible with the process of bending and placing. With odd-shaped rods, bending difficulties should receive careful consideration. Radius bends larger than 4 in. are difficult and expensive to obtain. Small bends are made around pipe sleeves or blocks. An exception to this is spirals, and circles such as are used in the S.M.I. flat slab system. Special machines in well equipped yards take care of these economically. It should be remembered that on large rods a precision on offsets closer than 1 in. is difficult to obtain. Details should not, therefore, be made which require such precision. Angles in rods, except parallel offsets, cannot be made with great precision and accurately bent rods will spring in handling unless very heavy compared to their length. Details therefore in which a slight variation in the angle of the rod would cause trouble should not be made. For example, Fig. 284 is bad. The detail should be as shown in Fig. 285. In addition to the practical weakness it is of course poor design to carry a rod around the face of a reëntrant angle as shown in Fig. 284 since the resultant of the tension in the two legs acts against the fireproofing only. Cambers, in slab rods (⅝ in. or under) may be as many as four, within reason. With larger rods, as used in beams, not more than two cambers should be used in a single rod.

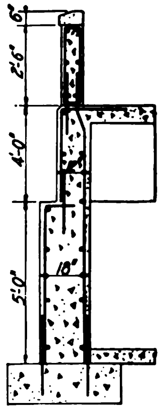


Fig. 283.

165. Reinforcement Assembly.—Bending may be done in the contractor's yard or on the job. In either case the bent rods tagged with type numbers are stored, usually by sizes, in racks or, if space is available, on the ground opposite the place where they are to be used.

Column steel is usually assembled on horses and placed as a unit. Beam steel may be handled in this way but where beams intersect over the columns at least part of them must be assembled in the forms. Beam rods hooked into spiraled columns should therefore be avoided on account of the difficulty of placing. When beam steel is assembled in the form, stirrups are first placed and it is a good idea to provide loop bars (⅜ or ½-in. rods) the full length of the beam to be placed under the hook of the stirrups, by which to support them.

In slabs, assembly by units is generally impracticable except occasionally in some types of flat-slab construction. Spacers are laid down, preferably on suitable chairs, and the main reinforcement is placed on them and wired.

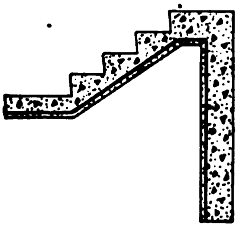


Fig. 284.

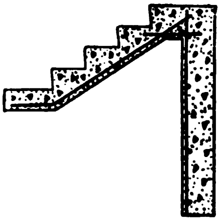
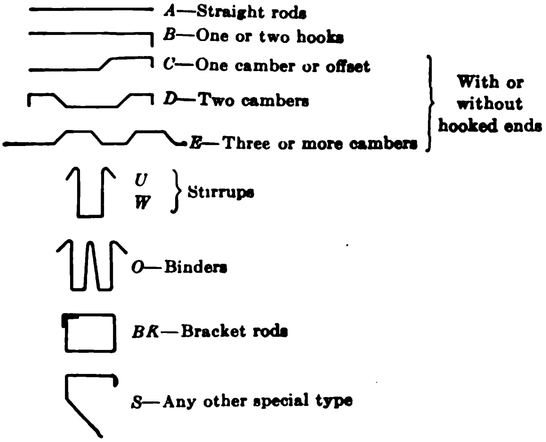


Fig. 285.

In wall reinforcement, vertical rods are usually placed first and then the horizontal rods tied to these. In slab and wall reinforcement, deformed rods are held more rigidly in place by wiring than plain rounds, which have a tendency to slip through the ties.

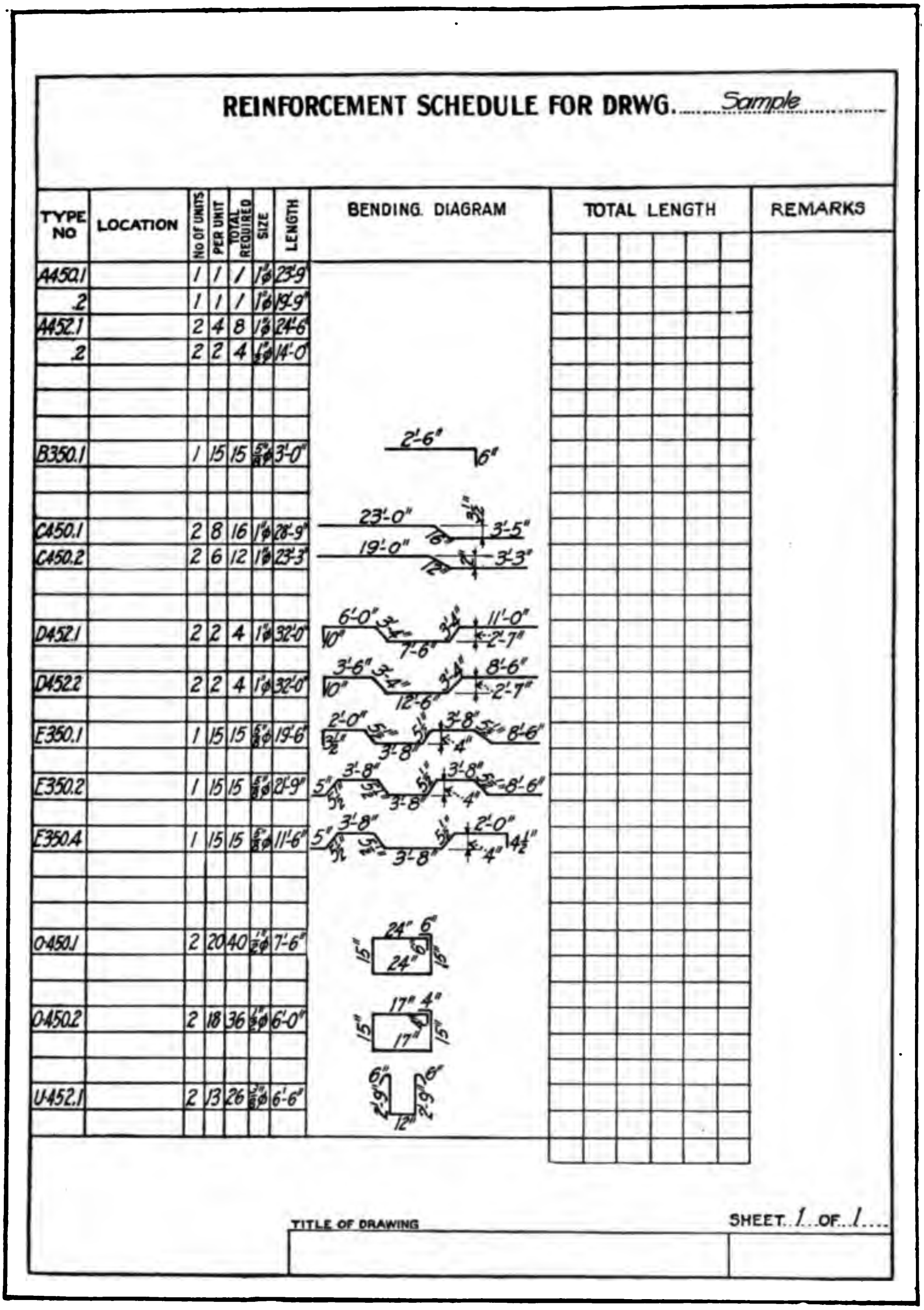
166. Rod Sizes.—In the choice of rods there are a few points to be considered. In the first place, rods of ¾ to 1-in. diameter have base price, *i.e.*, the lowest price per pound, and are therefore, other things being equal, the cheapest. $\frac{1}{16}$-in. sizes with the possible exception of $\frac{5}{16}$-in. square are not commercial sizes. ⅜ to 1¼ in. are the readily available sizes. Good detailing limits the sizes in the various units and as far as possible on the whole job, to avoid confusion. Squares and rounds are best not used together.



167. Schedules.—Rod schedules are sometimes made as a table on the drawing itself, but best practice is a separate sheet which is commonly about 12 × 21 in. This size is easily handled in the yard. A sample of a good schedule form is given in Fig. 286.

Type members must be considered in connection with rod schedules. Letters for various types are convenient. The scheme shown is in successful use. The individual rods are given separate numbers and great care is necessary to avoid duplication of numbers. The use of the number of the sheet on which the detail of the rod occurs, as part of the type number is open to the objection of giving a long number, but it automatically avoids duplication. This is illustrated on the schedule given.

Schedules include, of course, the lengths of bar in each run, *i.e.*, the distance between angles. The curves in Figs. 287 and 288 are convenient for finding camber lengths. At the intersection of the vertical line for the camber height, with the horizontal line for the horizontal projection of the camber, read the slope lengths with the arcs as a scale. For 30 or 45-deg. cambers the slope distance can be read at the intersection of either height or distance with the corresponding slope line.



2'-6"
6"
23'-0"
3½"
3'-5"
16"
19'-0"
3'-3"
12"
6'-0"
3'-4"
11'-0"
10"
7'-6"
2'-7"
3'-6"
8'-6"
12'-6"
2'-0"
3'-8"
5½"
8'-6"
4"
5"
8'-6"
2'-0"
4½"
24"
6"
15"
17"
4"
6"
2'-9"
12"

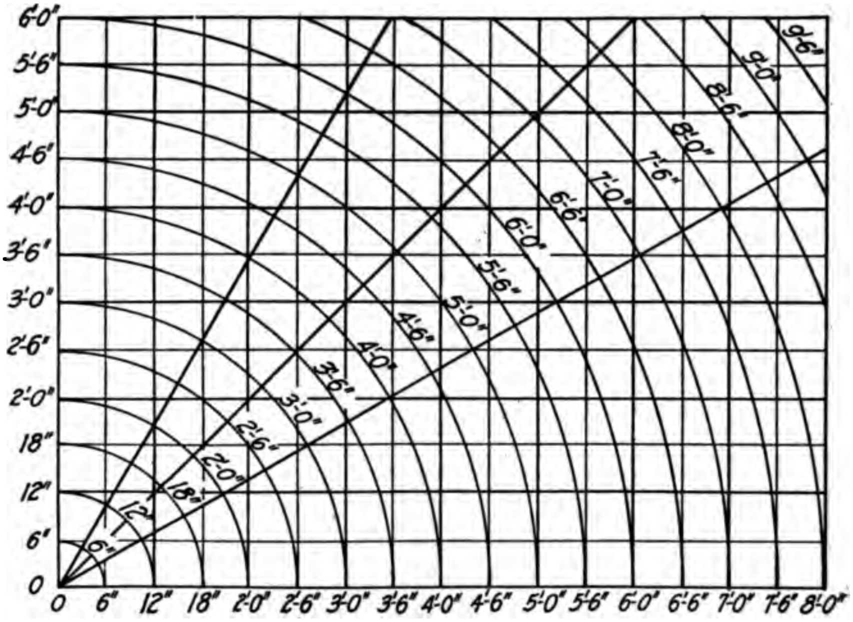
REINFORCEMENT SCHEDULE FOR DRWG. Sample

TYPE NO	LOCATION	No OF UNITS	PER UNIT	TOTAL REQUIRED	SIZE	LENGTH	BENDING DIAGRAM	TOTAL LENGTH	REMARKS
A450.1		1	1	1	1"φ	23'-9"			
.2		1	1	1	1"φ	19'-9"			
A452.1		2	4	8	1"φ	24'-6"			
.2		2	2	4	¾"φ	14'-0"			
B350.1		1	15	15	⅝"φ	3'-0"			
C450.1		2	8	16	1"φ	28'-9"			
C450.2		2	6	12	1"φ	23'-3"			
D452.1		2	2	4	1"φ	32'-0"			
D452.2		2	2	4	1"φ	32'-0"			
E350.1		1	15	15	⅝"φ	19'-6"			
E350.2		1	15	15	⅝"φ	21'-9"			
E350.4		1	15	15	⅝"φ	11'-6"			
O-450.1		2	20	40	½"φ	7'-6"			
O-450.2		2	18	36	½"φ	6'-0"			
U-452.1		2	13	26	⅜"φ	6'-6"			

TITLE OF DRAWING

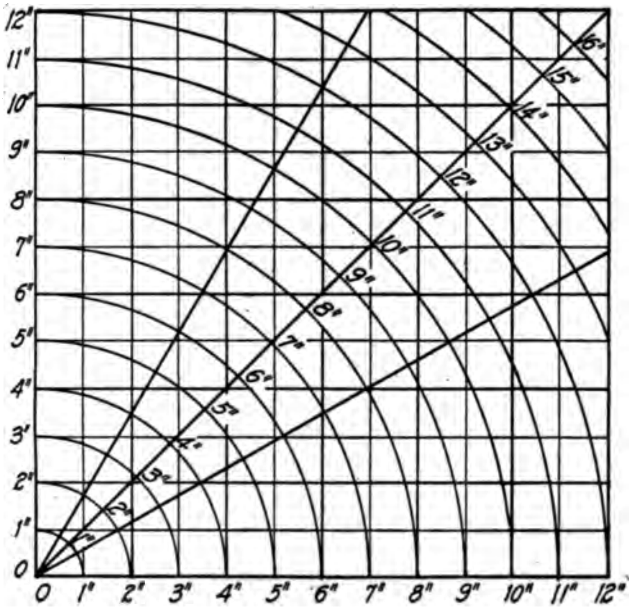
SHEET 1 OF 1

Fig. 286.



Use for Beam Rods

Fig. 287.



Use for Slab Rods

Fig. 288.

SECTION 3

STRUCTURAL DATA

BUILDINGS IN GENERAL

1. Types of Buildings.—Buildings, according to the building law of the City of Boston, are divided into three classes, as follows:

First-class Building.—A first-class building shall consist of fireproof material throughout, with floors constructed of iron, steel, or reinforced concrete beams, filled in between with terra cotta or other masonry arches or with concrete or reinforced concrete slabs; wood may be used only for under and upper floors, window and door frames, sashes, doors, interior finish, hand rails for stairs, necessary sleepers bedded in the cement, and for isolated furrings bedded in mortar. There shall be no air space between the top of any floor arches and the floor boarding.

Second-class Building.—All buildings not of the first class, the external and party walls of which are of brick, stone, iron, steel, concrete, reinforced concrete, concrete blocks, or other equally substantial and fireproof material.

Third-class Building.—A wooden frame building.

Composite Building.—A building partly of second-class and partly of third-class construction. Composite buildings may be built under the same restrictions as, and need comply only with the requirements for, third-class buildings as to fire protection and exterior finish.

Another type of building adapted to mills, factories, warehouses, etc., is the so-called "Slow-Burning Timber Mill Construction," developed by mill owners and the New England FactoryMutual Insurance Companies. This type is described in detail in a separate chapter in this section.

2. Floor Loads.—Floor loads vary with the class of material to be stored. In calculating dead and live loads for buildings, the following, quoted from the Boston Building Law, is good practice. However, the figures given should be checked by the ordinances of the locality in which the building is to be erected.

Dead loads shall consist of the weight of walls, floors, roofs, and permanent partitions. The weights of various materials shall be assumed as follows:

	Pounds per cubic foot
Beech	42
Birch	42
Brickwork	120
Concrete, cinder, structural	108
Concrete, cinder, floor filling	96
Concrete, stone	144
Douglas fir	36
Granite	168
Granolithic surface	144
Limestone	150
Maple	42
Marble	168
Oak	48
Pine, southern yellow	42
Sandstone	144
Spruce	30
Terra cotta, architectural, voids unfilled	72
Terra cotta, architectural, voids filled	120

	Pounds per square foot
Gravel or slag and felt roofing	6
Plastering on metal lath, exclusive of furring	8

Live loads shall include all loads except dead loads. Every permit shall state the purpose for which the building is to be used, and all floors and stairs shall be of sufficient strength to bear safely the weight to be imposed thereon in addition to the dead load, but shall safely support a minimum uniformly distributed live load per square foot, as specified in the following table:

Class of Building	Pounds per square foot
Armories, assembly halls and gymnasiums	100
Fire Houses:	
Apparatus floors	150
Residence and stable floors	50
Garages, private, not more than two cars	75
Garages, public	150
Grandstands	100
Hotels, lodging houses, boarding houses, clubs, convents, hospitals, asylums and detention buildings:	
Public portions	100
Residence portions	50
Manufacturing, heavy	250
Manufacturing, light	125
Office buildings:	
First floor	125
All other floors	75
Public buildings:	
Public portions	100
Office portions	75
Residence buildings, including porches	50
Schools and colleges:	
Assembly halls	100
Class rooms never to be used as assembly halls	50
Sidewalks	250
(Or 8000 lb. concentrated, whichever gives the larger moment or shear)	
Stables, public or mercantile:	
Street entrance floors	150
Feed room	150
Carriage room	50
Stall room	50
Stairs, corridors, and fire escapes from armories, assembly halls, and gymnasiums	100
Stairs, corridors, and fire escapes except from armories, assembly halls, and gymnasiums	75
Storage, heavy	250
Storage, light	125
Stores, retail	125
Stores, wholesale	250

Every plank, slab, and arch, and every floor beam carrying 100 sq. ft. of floor or less, shall be of sufficient strength to bear safely the combined dead and live load supported by it, but the floor live loads may be reduced for other parts of the structure as follows:

In all buildings except armories, garages, gymnasiums, storage buildings, wholesale stores, and assembly halls, for all flat slabs of over 100 sq. ft. area, reinforced in two or more directions and for all floor beams, girders, or trusses carrying over 100 sq. ft. of floor, 10% reduction.

For the same, but carrying over 200 sq. ft. of floor, 15% reduction.

For the same, but carrying over 300 sq. ft. of floor, 25% reduction.

These reductions shall not be made if the member carries more than one floor and therefore has its live load reduced according to the table below.

In public garages, for all flat slabs of over 300 sq. ft. area reinforced in more than one direction, and for all floor beams, girders, and trusses carrying over 300 sq. ft. of floor, and for all columns, walls, piers, and foundations, 25% reduction.

In all buildings except storage buildings, wholesale stores, and public garages, for all columns, girders, trusses, walls, piers, and foundations.

Carrying one floor	No reduction.
Carrying two floors	25% reduction.
Carrying three floors	40% reduction.
Carrying four floors	50% reduction.
Carrying five floors	55% reduction.
Carrying six floors or more	60% reduction.

Roofs shall be designed to support safely minimum live loads as follows:

Roofs with pitch of 4 in. or less per foot, a vertical load of 40 lb. per sq. ft. of horizontal projection applied either to half or to the whole of the roof.

Roofs with pitch of more than 4 in. and not more than 8 in. per ft., a vertical load of 15 lb. per sq. ft. of horizontal projection and a wind load of 10 lb. per sq. ft. of surface acting at right angles to one slope, these two loads being assumed to act either together or separately.

Roofs with pitch of more than 8 in. and not more than 12 in. per ft., a vertical load of 10 lb. per sq. ft. of hori-

zontal projection and a wind load of 15 lb. per sq. ft. of surface acting at right angles to one slope, these two loads being assumed to act either together or separately.

Roofs with pitch of more than 12 in. per ft., a vertical load of 5 lb. per sq. ft. of horizontal projection and a wind load of 20 lb. per sq. ft. of surface acting at right angles to one slope, these two loads being assumed to act either together or separately.

All buildings and structures shall be calculated to resist a pressure per square foot on any vertical surface as follows:

For 40 ft. in height	**10 lb.**
Portions from 40 to 80 ft. above ground	**15 lb.**
Portions more than 80 ft. above ground	**20 lb.**

3. Weights of Merchandise.—The following table taken by permission from data of the Boston Manufacturers Mutual Insurance Company gives approximate weights and dimensions of packages. In designing storehouses it is important to provide for the greatest load which can be placed in the building.

WEIGHTS OF MERCHANDISE

Material	Measurements		Weights		
	Floor space (sq. ft.)	Cu. ft.	Gross	Per sq. ft.	Per cu. ft.
Wool					
In bales, Australia In bales, East India In bales, New Zealand	8.6	19.4	350	40	18
In bales, So. America	12.5	47	1000	80	22
In bales, Oregon	7.5	33	550	73	17
In bales, California — Fleece pulled scoured	7.0	33	480	70	15
In bales, Texas	7.0	33	480	70	15
In bags, Domestic	15.5	18	250	16	14
In bags, scoured or noils	15.5	18	100	6.4	5.5
Woolen goods					
Case, flannels	5.5	12.7	220	40	17
Case, flannels, heavy	7.1	15.2	330	46	22
Case, dress goods	5.5	22.0	460	84	21
Case, cassimeres	10.5	28.0	550	52	20
Case, underwear	7.3	21.0	350	48	16
Case, blankets	10.3	35.0	450	44	13
Case, horse blankets	4.0	14.0	250	63	18
Cotton					
Bale, ginned	9.32	46.6	550	60	12
Bale, compressed	5.25	25.2	550	106	22
Bale, Planters Compress Co.	1.80	5.4	250	139	47
Bale, American Cotton Co.	2.60	7.8	270	104	35
Bale, Egyptian	4.7	20.0	820	170	41
Bale, Indian	4.7	20.0	860	176	43
Cotton goods					
Bale unbleached jeans	4.0	12.5	300	72	24
Piece duck	1.1	2.3	75	68	33
Bale brown sheetings	3.6	10.1	235	65	23
Case bleached sheetings	4.8	11.4	330	69	30
Case quilts	7.2	19.0	295	41	16
Bale print cloth	4.0	9.3	175	44	19
Case prints	4.5	13.4	420	93	31
Bale tickings	3.3	8.8	325	99	37
Skeins cotton yarn	.....	....	.....	.....	11
Carpet					
Roll of carpet	4.1	10.9	129	31.5	11.8
Rug (with pole)	0.44	4	48	.....	12.0

WEIGHTS OF MERCHANDISE—(*Continued*)

Material	Measurements		Weights		
	Floor space (sq. ft.)	Cu. ft.	Gross	Per sq. ft.	Per cu. ft.
Silk					
Bale, silk cocoons	12.5	31.5	260	20.4	8.25
Bale, silk frisons (average)	13.2	34.3	325	24.6	9.50
Bale, dressed silk	12	24	400	33.4	16.6
Bale, raw silk (average)	7.0	8.5	221	31.6	26
Bale, spun silk	5	7.5	235	47.0	31.4
Case broad silk cloth	6.5	10.4	180	27.7	17.3
Case ribbons	8	16	175	21.0	10.9
Jute, etc.					
Bale, jute	2.4	9.9	400	170	40
Bale, jute lashings	2.6	10.5	450	172	43
Bale, Manila	3.2	10.9	280	88	26
Bale, hemp	8.0	30.0	650	81	20
Bale, Sisal	7.5	27.0	400	53	15
Burlaps, various packages	.....	.....	.....	.....	43
Jute bagging	2.3	7.0	100	43	14
Bags in bales					
White linen	8.5	39.5	910	107	23
White cotton	9.2	40.0	715	78	18
Brown cotton	7.6	30.0	440	59	15
Paper shavings	7.5	34	500	68	15
Sacking	16.0	65	450	38	7
Woolen	7.5	30.0	600	80	20
Jute butts	2.8	11.0	400	143	36
Spruce chips, wet, tightly packed	....	.....	.....	.....	18
Spruce chips, wet, loosely packed	.....	.....	.....	.....	14
Spruce chips, dry	.....	.....	.....	.....	10
Paper					
16 × 21, 30 lb. ledger	2.4	5.3	210	130	60
16 × 21, 24 lb. calendered book	2.4	4.4	250	105	57
16 × 21, 29 lb. super-cal. book	2.4	4.3	300	125	70
18½ × 29, 26 lb. news	3.7	5.9	270	73	46
32 × 42, No. 38 straw board	9.3	3.9	130	14	33
24 × 31, 52 lb. Manila wrapping	5.2	10.8	530	102	49
Sheets in bundles, with wood frames	5.4	4.0	120	22	30
Sheets in bundles, without wood frames	6.3	4.2	140	22	33
Roll newspaper	4.8	28.8	1200	250	41
Sulphite pulp	.....	.....	.....	.....	17
Average pile of paper, in bundles	.....	.....	.....	.....	40
Tobacco					
Bale Sumatra wrapper	6.1	6.0	150	24.5	24.7
Hogshead of tobacco	8.0–13.4	36.0–80.4	1000–2200	.....	28
Grain					
Wheat in bags	4.2	4.2	165	39	39
Wheat in bulk	.....	.....	.....	.....	44
Wheat in bulk	.....	.....	.....	.....	39
Wheat in bulk mean	.....	.....	.....	.....	41
Barrels flour on side	4.1	5.4	218	53	40
Barrels flour on end	3.1	7.1	218	70	31
Corn in bags	3.6	3.6	112	31	31
Cornmeal in barrels	3.7	5.9	218	59	37
Oats in bags	3.3	3.6	96	29	27
Bale of hay	5.0	20.0	284	57	14
Hay, dederick compressed	1.75	5.25	125	72	24
Straw, dederick compressed	1.75	5.25	100	57	19
Tow, dederick compressed	1.75	5.25	150	86	29
Excelsior, dederick compressed	1.75	5.25	100	57	19

WEIGHTS OF MERCHANDISE—(*Continued*)

Material	Measurements		Weights		
	Floor space, (sq. ft.)	Cu. ft.	Gross	Per sq. ft.	Per cu. ft.
Dye stuffs, etc.					
Hogsheads bleaching powder	11.8	39.2	1200	102	31
Hogsheads soda ash powder	10.8	29.2	1800	167	62
Box indigo	3.0	9.0	385	128	43
Box cutch	4.0	3.3	150	38	45
Box sumac	1.6	4.1	160	100	39
Caustic soda in iron drum	4.3	6.8	600	140	88
Barrel pearl alum	3.0	10.5	350	117	33
Box extract logwood	1.06	0.8	55	52	70
Barrel lard oil	4.3	12.3	422	98	34
Miscellaneous					
Rope	.....	.....	.....	.....	42
Box tin	2.7	0.5	139	99	278
Box glass	.....	.....	.....	.....	60
Crate crockery	9.9	39.6	1600	162	40
Cask crockery	13.4	42.5	600	52	14
Bale leather	7.3	12.2	190	26	16
Bale goatskins	11.2	16.7	300	27	18
Bale raw hides	6.0	30.0	400	67	13
Bale raw hides compressed	6.0	30.0	700	117	23
Bale sole leather	12.6	8.9	200	22	16
Pile sole leather	.....	.....	.....	.....	17
Barrel granulated sugar	3.0	7.5	317	106	42
Barrel brown sugar	3.0	7.5	340	113	45
Cheese	.....	.....	.....	.....	30
Pitch	.....	.....	.....	.....	72

4. Fire Prevention and Fire Protection.—In the design of important structures, especially industrial and commercial, the architect or engineer should consult the local insurance boards, as they maintain laboratories and a large engineering force which is at the disposal of interested parties without charge. In many cases, insurance costs may be materially reduced by their assistance.

Mills, factories, warehouses, stores, or any structures having extensive areas containing quantities of inflammable materials, should first of all be protected with a complete automatic sprinkler system. All large buildings should have standpipes with hose reels or racks conveniently located in stairways, etc., where they are easily accessible, in case of fire and so placed that the hose stream or streams will reach every part of the floor or section to be protected. Chemical fire extinguishers or pails of water, or both, should also be placed where easily accessible.

A sprinkler system should have its own water supply, usually a tank of proper capacity either on the roof or on an independent tower. In locating a tank on the roof, care must be taken that it is amply supported, preferably on the walls of the building. Where a city fire department is available, an outside connection for fire engines is also installed. In one fire protection system designed for a large steamship pier, there was a connection at the land end for fire engines, and another at the water end for fire-boats.

Fire pumps should be of the Underwriters pattern of approved make. Approved rotary and centrifugal pumps may be used instead of steam pumps, but should be driven by independent motors. The pump room and boiler room should be cut off from the rest of the plant by fire walls and fire doors, and so loacted that in case of fire, men may stand by the boilers and pumps to the end.

Industrial plants covering extensive ground area should have a system of water piping and hydrants with fire hose in suitable hose houses.

The following is quoted from a report of the Associated Factory Mutual Insurance Companies, detailing the necessary equipment for proper fire protection. Requirements of other insurance boards do not differ materially from these.

The extent and capacity of the fire apparatus depends largely upon construction, height, area, occupancy, and arrangement of a plant, and also upon its surroundings. The more important requirements for an ideal plant are as follows:

Water Supply: (*a*) Public water supplied by gravity at good pressure and ample quantity is best. A pressure of about 60 lb. maintained in the mill yard while 1000 to 1500 gal. or more are flowing is ordinarily considered excellent. Such a public water supply is always preferred to an elevated tank.

(*b*) Pump supply from one or two Underwriter pumps according to the size of the plant. Pumps to draw from supply capable of furnishing water during a fire of long duration and independent of the public water works.

(*c*) Steam boilers should have two absolutely independent sources of water supply. A direct connection from fire pump to the boilers is often desirable and may be considered as one of these. The steam supply to pump should be taken off behind a valve or valves controlling supply to engines or other factory service, and all controlling valves should be in the boiler house. The pipe should be so located that it can not be broken by falling walls or other accident at a fire.

Hydrants: Placed at sufficiently frequent intervals so that the full capacity of the water supply available may be concentrated at any point of the plant without the use of long lines of hose.

Generally hydrants at intervals of about 200 ft. are required, two-way hydrants to have at least 5-in. gate opening and barrel, and hydrants with more than two outlets to have a 6-in. gate opening and barrel, and independent gates for each outlet.

Roof hydrants are of value in fighting outside fires either in adjoining properties or where buildings adjoin one another in a crowded mill yard.

Hose standpipes properly located are of great value in buildings of over two or three stories especially when fire is beyond control of sprinklers.

Sprinklers: (*a*) Automatic sprinklers throughout all rooms including storehouses, elevators, and stairs, all closets, enclosures, etc., also to be covered. There should be no part of the floor area, ceilings, or roofs without ample protection, and heads must be so spaced as to satisfactorily cover all places. It is required that detail sprinkler plans showing protection proposed be submitted to the Insurance Companies before the installation begins. Dry pipe valves should be used only when it is impracticable to heat the building, as their installation considerably increases the time before discharge of water on the fire, and therefore correspondingly weakens the protection.

(*b*) Each sprinkler connection into buildings to be provided with outside post indicator gate, safely located, and sufficient connections are required for large areas so that there may not be over 200 sprinklers in one room on a single 6-in. supply. Pipe connections into buildings should not be less than 6 in., even when supplying risers of smaller size, except in especial cases where only 30 or 40 heads are supplied per floor in low buildings.

Yard Pipes: Of ample size to carry the water available to sprinklers and hydrants without serious loss of pressure. For the mill shown, and 8-in. loop pipe is sufficient. Should the loop not be practicable, the pipe in a part of the yard system may need to be 10 in. For large mills with extended yard area, 10-in. pipe or even larger may be necessary. Class *E* pipe N.E.W.W. Association is required. Pipes to be in such location that hydrants and post indicator valves may be at a good distance from the walls of very high buildings or those of large area. Pump check valves should be safely located below floor level. The brick well is merely to make it more readily accessible.

Circuit controlling valves are advisable at intervals in extensive yards so as not to necessitate shutting off the entire yard system at one time in case of repairs or alterations.

Hose: (*a*) Outside equipment to consist of 2⅝-in. Underwriter cotton rubber-lined hose of one of the approved brands which, together with spanners, 1⅛ in. Underwriter nozzles, axes, bars, lantern, etc., must be kept in the hose houses.

(*b*) Inside equipment to be provided in all rooms, fed preferably from a system of small standpipes independent of sprinkler system, that it may be available if the sprinklers are shut off on account of accident or after they are shut off at fire to save water damage. In some cases, it may be attached to 1-in. nipples from sprinkler pipes not less than 2½ in. in diameter, but is then not available at a time when it may be most needed. Hose and couplings to be for 1¼-in. Underwriter linen hose and nozzles ⅝-in. smooth bore.

(*c*) For tower standpipes 2⅝-in. best Underwriter linen hose of approved brands to be provided.

PROTECTION OF STRUCTURAL STEEL FROM FIRE

By Frank C. Thiessen

5. Effects of Heat on Steel.—Structural steel, used for the framework of modern buildings, loses its rigidity at a relatively low temperature. At 600 deg. F. the material begins to lose its strength; as the temperature is increased above this point, tests show that the strength decreases rapidly and at or about 1000 deg. F. the steel has little or no value in supporting loads. At approximately 1500 deg. F. the material softens and fails of its own weight.

6. Intensity of Heat in a Fire.—Fused and distorted metals, indicating temperatures of 1700 deg. F., and in many cases in excess of 2000 deg. F., are found in buildings after fires. In the Edison fire on Dec. 9, 1914, at West Orange, N. J., evidences of temperatures ranging from 2000 to 2500 deg. F. were found. In the sub-basement and on the third floor of the Wax House where inflammable materials were stored, the heat was sufficient to fuse portions of the trap concrete. Even a moderate fire or a small hot fire confined to a portion of a building may cause failure of improperly protected columns or floor beams with resulting partial or total loss.

7. Protection of Steel From Failure.—Protection of steel from failure consists in encasing it in a non-heat conducting material so that the temperature of the structural steel framework does not reach a point endangering its strength.

The ideal material for protective coverings should conduct heat very slowly and should be of a quality and thickness such that in the course of burning of the contents of the building no serious damage will result, either to the members encased or to the material itself. The protective covering must be adapted to resist not only the destructive action of the fire but also the action of the fire streams used in extinguishing the fire. No material can resist the continued alternate action of heat and the sudden cooling by water. Brick, concrete, terra cotta tile, plaster, and gypsum products, when properly made and properly used, have withstood laboratory tests and ordinary fires to a satisfactory degree.

8. Fire-resistance of Materials.

8a. **Brick.**—The fire-resisting qualities of brick have been demonstrated in many fires. When used in large units, particularly in thin walls, damage may result in severe fires from expansion. Thick walls suffer less damage from expansion although the bricks may crack, spall, or fuse under the action of fire or water. In small units, as for example in floor arches or protection for columns, properly made brickwork is an excellent fire-resistant material. To be first class in this respect the chemical properties of the clay should be such that a temperature of at least 2200 deg. F. is required to vitrify it. The burning of the brick should proceed to a point just short of vitrifaction.

8b. **Concrete.**—The wonderful development of concrete construction and the behavior of plain and reinforced concrete in fires and conflagrations offers sufficient evidence of its value as a constructive and fire-resistive material. The low heat conductivity of concrete is due partly to its porosity and partly to the process of dehydration which begins at a temperature of 500 to 600 deg. F. The process is slow because the surface material, having become a poorer conductor of heat, remains in place and retards the progressive action of the dehydration of material in the interior. At corners or edges exposed to intense heat, the calcined material may spall to a maximum depth of ¾ to 1 in., but in ordinary fires this action is rarely of importance. The character of the aggregate is an important factor. Stone or gravel containing quartz grains tends to disintegrate and should not be used.

Cinders are light in weight, porous, and when mixed wet and very well mixed by machine, forms a concrete having excellent fire-resisting qualities. The cinders should be hard, free from fine, powdery ash or other soft material, and for maximum strength and quality should preferably be a porous, vitreous clinker. Anthracite coal cinders are obtainable in some cities. In general, carefully selected bituminous coal cinders from buildings or plants in which no waste or refuse is burned will be satisfactory if the particles are well coated with cement in mixing. The presence of unburned coal may cause slight pitting of a surface in a fire but the porosity of the aggregate and the dehydration of the cement in well mixed concrete will ordinarily protect the covering from serious damage. Blast furnace slag is a very good aggregate.

8c. **Terra Cotta Tile.**—The tile for fire protective coverings and structural purposes is made in three grades, "porous," "semi-porous," and "dense." The porosity is obtained by mixing sawdust with clay, the sawdust being removed in the process of burning. Semi-porous tiling is also made of fire clay to which a percentage of coarsely ground bituminous coal is added before burning. Of the three grades, porous hollow tile is the best nonconductor of heat and the lowest in compressive strength. The chief weakness of hollow tile with thin walls and webs lies in the liability of the breaking away of the exposed face due to sudden and unequal expansion. Its many advantages, however, have led to its wide use in building construction.

8d. Plaster.—Ordinary lime plaster is a good non-conductor of heat but in severe fires does not remain in place. A single layer may be considered as a fire-retardant or coating for other fire -resisting materials. Very little reliance should be placed upon a single layer; a double covering, of plaster on metal lath, separated by air spaces, is much more effective as a covering for steel.

8e. Gypsum.—The calcination of gypsum ($CaSO_4.2H_2O$) forms Plaster of Paris ($CaSO_4.\frac{1}{2}H_2O$). This material is used in various forms of protective coverings. Its coefficient of expansion is low and as a non-conductor of heat is one of the best materials. Plain blocks have a tendency to become calcined in intense heat and the softened surface does not withstand the action of hose streams. The prepared or hard wall plasters, being similar in composition to gypsum blocks, form a better bond for the joints than cement mortar and are more satisfactory.

9. Selection of Protective Covering.—The fire risk will vary, depending upon the contents, the use of the building, and the external hazards. A machine shop, foundry, or structural shop, containing no combustible material and having no external hazard, may require no protection of its framework from fire. The lower floors of office or store buildings are more often subject to fire because of the location of the heating system or accumulation of waste or inflammable material in basements. Partial protection is of some value. Plaster on metal lath will protect structural steel for a while in a fire but the destruction of the covering and the exposure of the steel to the fire becomes merely a question of the intensity and duration of the exposure. Many considerations besides the character of the materials affect the selection of the fireproofing. Too often the first cost governs the selection and the result is a low-grade covering. As a rule, if it is decided that reinforced concrete is the cheapest and best for the floor construction, the same material will be used for the protection of the columns—likewise for hollow tile. Combinations, however, are frequently used. Portland cement concrete and hollow tile besides having excellent fire resisting qualities serve for the structural parts and are the materials most commonly used.

10. Thickness of Protective Covering.—The thickness of the covering required varies with the exposure and the importance of the member. Floors on which quantities of combustible materials are stored should have protection in proportion to the severity and duration of the fire. Columns are the most vital members of a building and should receive the most protection. Steel near exterior window or door openings is subject to severe exposure and should be covered with a thickness greater than for the floor joists. The sections of the Chicago Building Ordinance[1] relating to columns and floors are as follows:

Fireproof Material.—The material which shall be considered as filling the conditions of fireproof covering are: (1) burnt brick; (2) tiles of burnt clay; (3) approved cement concrete; (4) terra cotta.

In all cases, the brick or hollow tile, solid tile or terra cotta shall be bedded in cement mortar close up to the iron or steel member and all joints shall be made full and solid.

Exterior Columns.—(*a*) All iron or steel used as vertical supporting members of the external construction of any building exceeding 50 ft. in height shall be protected against the effects of external change of temperature, and of fire by a covering of fireproof material consisting of at least 4 in. of brick, hollow terra cotta, concrete, burnt clay tiles, or of a combination of any two of these materials, provided that their combined thickness is not less than 4 in. The distance of the extreme projection of the metal, where such metal projects beyond the face of the column, shall be not less than 2 in. from the face of the fireproofing; provided, that the inner side of external columns shall be fireproofed as hereafter required for interior columns.

(*b*) Where stone or other incombustible material not of the type defined in this ordinance as fireproof material is used for the external facing of a building, the distance between the back of the facing and the extreme projection of the metal of the column proper shall be at least 2 in., and the intervening space shall be filled with one of the fireproof materials.

(*c*) In all cases, the brick, burnt clay, tile, or terra-cotta, if used as a fireproof covering, shall be bedded in cement mortar close up to the iron or steel members, and all joints shall be made full and solid.

Interior Columns.—(*a*) Covering of interior columns shall consist of one or more of the fireproof materials herein described.

(*b*) If such covering is of brick it shall be not less than 4 in. thick; if of concrete, not less than 3 in. thick; if of burnt clay tile, such covering shall be in two consecutive layers, each not less than 2 in. thick, each having one air space of not less than ½ in., and in no such burnt clay tile shall the burnt clay be less than ⅝ in. thick; or if of porous clay solid tiles, it shall consist of at least two consecutive layers, each not less than 2 in. thick; or if con-

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[1] Revised Building Ordinances of the City of Chicago, as amended Feb. 20, 1911.

stituted of a combination of any two of these materials, one-half of the total thickness required for each of the materials shall be applied, provided that if concrete is used for such layer it shall not be less than 2 in. thick.

(c) In the case of columns having an "H" shaped cross section or of columns having any other cross section with channels or chases open from base plates to cap plates on one or more sides of the columns, then the thickness of the fireproof covering may be reduced to 2½ in., measuring in the direction in which the flange or flanges project, and provided that the thin edge in the projecting flange or arms of the cross sections does not exceed ¾ in. in thickness. The thickness of the fireproof covering on all surfaces measuring more than ¾ in. wide and measuring in a direction perpendicular to such surfaces shall not be less than that specified for interior columns in the beginning of this section, and all spaces, including channels or chases between the fireproof covering and the metal of the columns, shall be filled solid with fireproof material. Lattice or other open columns shall be completely filled with approved cement concrete.

Wiring Clay Tiling on Columns.—(a) Burnt clay tile column covering shall be secured by winding wire around the columns after the tile has been set around such columns. The wire shall be securely wound around tile in such manner that every tile is crossed at least once by a wire. If iron or steel wire is used it shall be galvanised and no wire used shall be less than number twelve gage.

(b) In places where there is trucking or wheeling, or handling of packages of any kind, the lower 5 ft. of every column encased with hollow tile shall be encased in a protective covering of No. 16 U. S. gage steel embedded in concrete.

Pipes Enclosed by Covering.—(a) Pipes shall not be enclosed in the fireproofing of columns or of other structural members of any fireproof building; provided, however, gas or electric light conduits not exceeding ¾-in. diameter may be inserted in the outer ¾ in. of the fireproofing of such structural member, where such fireproofing is entirely composed of concrete.

(b) Pipes of conduits may rest on the tops of the steel floor beams or girders, provided, they are embedded in cinder concrete to which slaked lime equal to 5% of the volume of the concrete has been added before mixing or their being embedded in stone concrete.

Coverings of Beams, Girders, and Trusses.—(a) The metal beams, girders, and trusses of the interior structural parts of a building shall be covered by one of the fireproof materials hereinbefore specified so applied as to be supported entirely by the beam or girder protected, and shall be held in place by the support of the flanges of such beams or girders and by the cement mortar used in setting.

(b) If the covering is of brick, it shall be not less than 4 in. thick; if of hollow tiles or if of solid porous tiles, or if of terra cotta, such tiles shall be not less than 2 in. thick applied to the metal in a bed of cement mortar; hollow tiles shall be constructed in such manner that there shall be one air space of at least ¾ in. by the width of the metal surface to be covered within such clay coverings; the minimum thickness of concrete on the bottom and sides of the metal shall be 2 in.

(c) The tops of all beams, girders, and trusses, shall be protected with not less than 2 in. of concrete or 1 in. of burnt clay bedded solid on the metal in cement mortar.

(d) In all cases of beams, girders, or trusses, in roofs and floors, the protection of the bottom flanges of the beams and girders and so much of the web of the same as is not covered by the arches shall be made as hereinbefore specified for the covering of beams and girders. In every case the thickness of the covering shall be measured from the extreme projection of the metal, and the entire space or spaces between the covering and the metal shall be filled solid with one of the fireproof materials, excepting the air spaces in hollow tile.

(e) Provided, however, that all girders or trusses when supporting loads from more than one story shall be fireproofed with two thicknesses of fireproof materials or a combination of two fireproof materials as required for exterior columns, and such covering of fireproof material shall be bedded solid in cement mortar.

FIRE-RESISTIVE COLUMN CONSTRUCTION

By Frank C. Thiessen

11. Reinforced Concrete Columns.—Reinforced concrete columns are treated in Sect. 2. The Joint Committee on Concrete and Reinforced Concrete recommends that concrete reinforcement be protected by a minimum of 2 in. of concrete.

12. Covering for Cylindrical Columns.—Cross-sectional forms of tile for encasing cylindrical columns are shown in Figs. 1 to 3 inclusive. These blocks are made in segments of a circle and of varying sizes, allowing a space between the block and the surface of the column. The tile should be arranged to break joints. The designs shown in Figs. 3 and 4 have ribs on the inner face to aid in the setting of the tile and to maintain a space of uniform width around the column. If the columns are of cast iron, the space may be left unfilled to act as a "dead air space." To be effective in this respect, however, the space should be sealed tight. For steel columns, the space should be filled solid as a protection against corrosion. To make the anchorage of the tile covering to the column more secure against the action of fire streams or falling débris during a fire, galvanized iron wire should be tightly wound around the column so as to cross each tile at least once. Fig. 5 shows an effective method of protection if plaster is to be used. It consists of a double covering of cement plaster on metal lath separated by and attached to metal furring strips, forming two air spaces. A single layer is not considered fireproof. The double layer with the air spaces not only makes the construction more fire-resistant but also forms a better arrangement to resist the action of fire streams. It will be noted that this column is not thoroughly protected from corrosion.

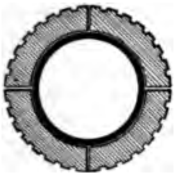

Fig. 1.

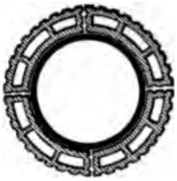

Fig. 2.

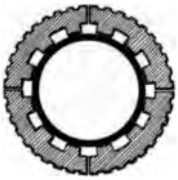

Fig. 3.

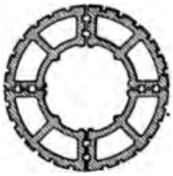

Fig. 4.

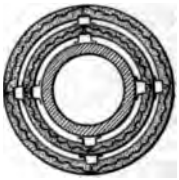

Fig. 5.

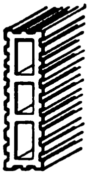

Fig. 6.

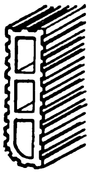

Fig. 7.

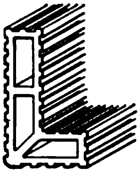

Fig. 8.

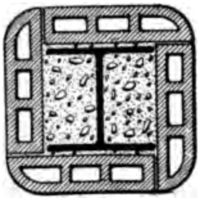

Fig. 9.

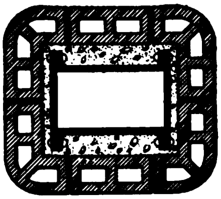

Fig. 10.

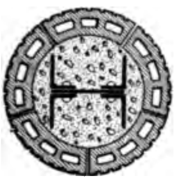

Fig. 11.

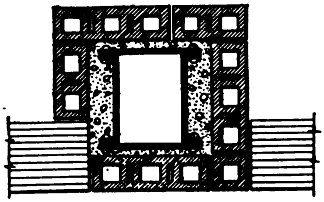

Fig. 12.

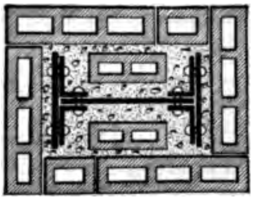

Fig. 13.

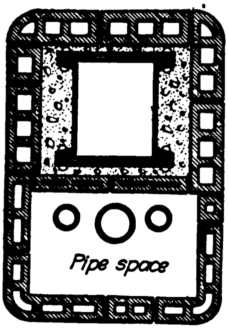

Fig. 14.

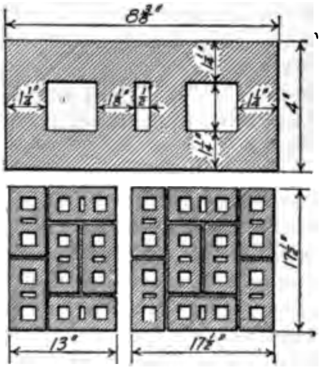

Fig. 15.—"Monarch" tile block.

13. Coverings for Various Steel Columns.—Three sections of hollow tile used for column covering are shown in Figs. 6, 7, and 8 Two of these shapes have a rounded corner. The application of tile to various common shapes of columns is shown in Figs. 9, 10, 11, 12, and 13. If pipes or wiring are to be protected or concealed in a space alongside a column, the column, nevertheless, should be encased on all sides as shown in Fig. 14. Failure to provide the inner layer adjacent to the steel column has been demonstrated to be bad practice. With the arrangement shown, temporary removal of the casing around the pipe space for the purpose of inspection for repairs will not leave the column exposed. The protection of the pipe is ordinarily not as important as that of the main strength members and accordingly the thickness of covering required may be somewhat less provided the pipes are set 3 or 4 in. inside the casing.

14. Hollow Tile Columns.—Fig. 15 shows a form of hollow tile having webs and walls about twice as thick as ordinary hollow tile. These blocks are made in one size, 8⅜ × 4 × 8 in., or about the size of 4 ordinary building bricks. Columns of these blocks may be built up in square or rectangular cross section, varying from 8½ to 31 in. square. The height of the column should not exceed 12 times the least dimension.

TABLE FOR "MONARCH" TILE BLOCK COLUMNS

Size of column (inches)	Safe load (pounds)	No. of tile in cross section	No. of tile per lin. ft.	Weight of column per lin. ft.
31 × 31	612,500	24½	36¾	612
31 × 26½	525,000	21	31½	525
26½ × 26½	450,000	18	27	450
26½ × 22	375,000	15	22½	375
22 × 22	312,500	12½	18¾	312½
22 × 17½	250,000	10	15	250
17½ × 17½	200,000	8	12	200
17½ × 13	150,000	6	9	150
13 × 13	112,500	4½	6¾	112½
13 × 8½	75,000	3	4½	75
8½ × 8½	50,000	2	3	50

FIRE-RESISTIVE FLOOR CONSTRUCTION

BY FRANK C. THIESSEN

15. Requirements of a Fire-resistive Floor.—A fire-resistive floor should withstand a fire destroying the combustible contents of a building with no damage to the structural parts and with no more than slight damage to the material used for the protective covering. It goes almost without saying that the floor should support its full safe load at all times without excessive deflection. The floor should be water-tight to prevent damage by water to the contents of floors below. As ordinarily constructed, floors of hollow tile or brick are very permeable; water will make its way through cinder fill; cracks in concrete or tiled floors may allow water to reach the floor below. Ordinary plaster is usually removed either by the fire or by hose streams. Most forms of plaster or gypsum blocks, although serving to protect the steel framework from heat, may require reconstruction after the combined action of fire and water. Some repairs are to be expected even with the best of materials for no material can resist the prolonged action of intense heat and water applied when the parts are hot.

16. Fire Tests.—The proper manner of using the various fire-resisting materials in the construction of fire-resistive floors has been developed by observation and study of many buildings after fires or conflagrations and by fire tests of small units. By far the greatest number of tests of types of floor panels has been made under the auspices of the New York City authorities according to specifications of the New York Building Ordinance. A brief description of the essential features of tests and the requirements for acceptance will indicate what is expected of a fire-resisting floor. A platform or floor is constructed within enclosure walls with the same quality of materials and workmanship employed in actual practice. This floor, designed for and carrying a distributed load of 150 lb. per sq. ft., is subjected to a continuous wood fire below the floor maintained at an average temperature of 1700 deg. F. for 4 hr. At the end of that time the underside of the hot floor is subjected to a 1⅛-in. stream of water at 60-lb. nozzle pressure for 5 min.; after which the upper side of the floor is flooded with water at low pressure;

and then the stream of water under pressure is again applied to the underside of the floor for 5 min. After cooling, the distributed load is increased to 600 lb. per sq. ft. and the deflections noted. The Standard Test[1] for fireproof floor construction of the American Society for Testing Materials, which is essentially the same as the test of the New York City Bureau of Buildings and the British Fire Prevention Committee, prescribes that "the tests shall not be regarded as successful unless the following conditions are met: No fire or smoke shall pass through the floor during the test; the floor shall safely sustain the loads prescribed; the permanent deflection shall not exceed ⅛ in. for each foot of span in either slab or beam."

17. Scuppers.—The floors of storage warehouses, mills, or factories, containing merchandise or stock subject to damage by water, should be impervious and should be provided with interior drains or scuppers placed in the exterior walls for the ready and quick escape of water from sprinkler heads, bursted pipes, or hose. The scuppers should be of cast iron with an opening at the floor level of about 4 × 12 in., sloping downward, at a pitch of 2½ in. to the foot to the opening beyond the edge of the wall. Brackets or guards may be used to prevent the opening from being covered or clogged by material being placed against it. Flap covers allowing the water to escape readily without permitting a circulation of air along the surface of the floor are used at the openings. Two designs of scuppers are shown in Figs. 16 and 17.

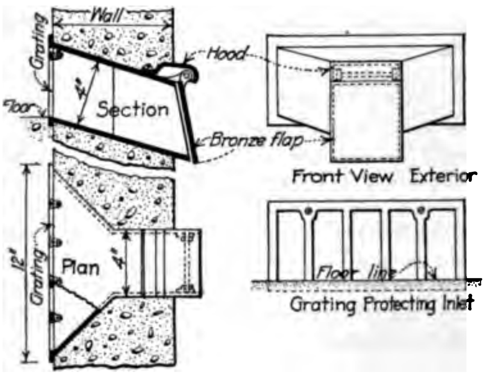

FIG. 16.

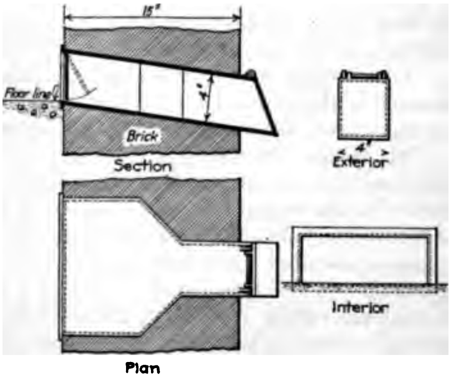

FIG. 17.

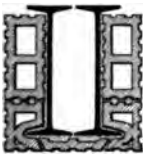

FIG. 18.

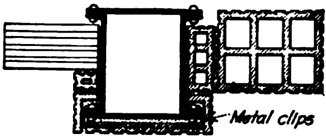

FIG. 19.

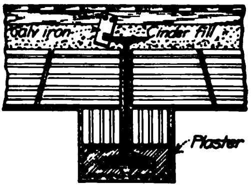

FIG. 20.

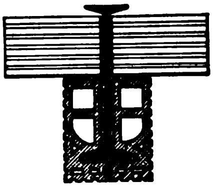

FIG. 21.

18. Reinforced Concrete Floors.—Reinforced concrete floors are treated in other chapters in this section and in Sect. 2. The Joint Committee on Concrete and Reinforced Concrete recommends that concrete reinforcement be protected by a minimum of 2 in. of concrete on girders, 1½ in. on beams, and 1 in. on floor slabs.

19. Protection of Steel Girders.—Steel girders having a greater depth than the floor joists and projecting below the floors may be subject to extremely severe exposure during a fire. The lower flange should be covered with at least 2½ in. of solid tile construction to 4 in. of hollow tile, depending on the exposure and the importance of the member. If the member is deep enough so that the web is exposed below the floor, the space above the flange or flanges should be filled flush with the fire-resisting material. Sharp corners are subject to unequal heating and usually spall more than flat surfaces or rounded corners. Figs. 18 to 21 inclusive show typical

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[1] Year Book, Am. Soc. for Testing Materials.

coverings for various requirements of girders used in floor construction. If concrete is used for the fire-protective covering the steel girders should be wrapped with a wire mesh to reinforce and bond the covering to the member. See Art. 68 (*c*) for various types of steel frame floors fire-proofed with concrete.

20. Brick Arch Floor Construction.—A brick arch may be built between steel floor beams to support heavy loads. Tie-rods, connecting the beams, are used to take the thrust and should be covered with a thickness of at least 2½ in. of fire-resistive material. The brick are laid in cement mortar and set so as to break joints. The space between the arch and the floor is filled to a level with one of the fire-resistive materials, usually concrete. Although this type of construction is excellent in its resistance to fire, it is heavy and expensive. It has been used in the warehouse type of building where appearance of the underside of the floor is not objectionable.

21. Terra Cotta or Tile for Floor Arches.—Hollow terra cotta or tile blocks are made in a great variety of shapes and sizes for the various requirements of floor construction. Having parallel sides or edges, the blocks are adapted to use between the floor members of square or rectangular floor panels. Irregular shaped panels or irregular spaces created by openings in the floor are somewhat difficult to fill with the regular units of tile. If the space is so irregular that much patchwork is required, the covering of the steelwork may be imperfectly done and there is also the possibility of tile not being placed in position to develop its maximum strength. If the floor beams are parallel, or nearly so, the tile are easily and rapidly laid, and without great interference or delay to other work in the building.

Porous tile is the best from the standpoint of resistance to fire but does not possess as great strength as the harder grades. Semi-porous tile is extensively used for floor arches because it combines adequate strength with satisfactory fire-resistive qualities.

22. Hollow Tile Flat Arch.—In Fig. 22 is shown a perspective view of a hollow tile flat-arch floor with the tile laid side to side and breaking joints. The openings or cells of the tile run parallel to the beams. In this type, called side-construction, the breaking of a single block or its removal will not greatly impair the strength of the arch beyond the block. Fig. 23 is an illustration of so-called end construction of a flat arch, using a key block placed as in the side construction. In this type the tile is placed in the proper position to transmit the thrust directly through the webs and walls to the steel beam. It is evident that the blocks should be set in line and that the joints should be well bedded with cement mortar.

TABLE OF WEIGHTS AND SPANS FOR END-CONSTRUCTION ARCH[1]

Depth of arch (inches)	Weight (pounds per square foot)	Maximum safe spans	
		(feet)	(inches)
6	26	4	0
7	30	4	6
8	32	5	0
9	36	6	0
10	38	6	6
12	44	8	0
14	50	9	0
15	54	9	6
16	55	10	0

The strength of any arch depends as largely on workmanship as on materials, therefore the maximum spans given can be used only where experienced workman are employed and the work is guaranteed by a responsible contractor.

The end block, shown enlarged in Fig. 24, is objectionable because it may not offer as great protection from fire to the lower flange of the beam, and may not be smoothly and firmly bedded at the floor member. Using the skew shown in the side construction and combining with a key block and lengtheners set endwise, we have the type of floor arch most commonly used (Fig.

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[1] National Fireproofing Co.

25). The bottom flange is covered with a soffit block having an air space and which is attached to the flange by clips and thoroughly bedded in cement mortar. The tile are scored to provide a bond for the plaster which is applied directly to the tile. The screeds or sleepers, to which the flooring is nailed may be of 2 × 2 in., 2 × 3 in., or 2 × 4 in. beveled or dovetailed to remain in place in the concrete filling over the tile. These nailing strips may rest directly on the steel joists or may be held in position above the upper flanges by sheet metal clips notched to fit the upper flange and nailed to the sides of the nailing strips. Cinder concrete is commonly used for the filling.

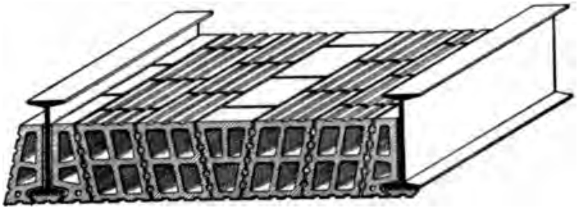

Fig. 22.—Hollow tile flat arch—side construction.

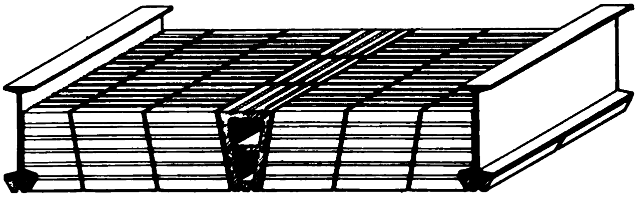

Fig. 23.—Hollow tile flat arch—end construction.

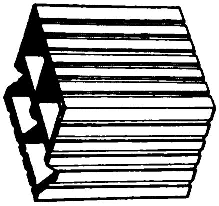

Fig. 24.

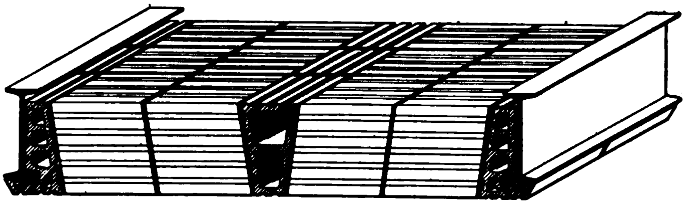

Fig. 25.—Common type of hollow tile flat arch.

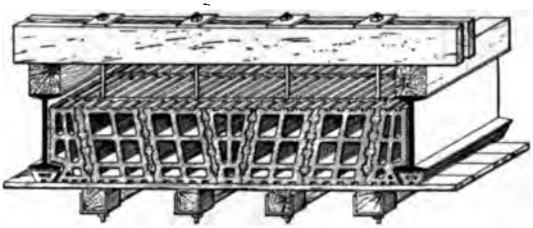

Fig. 26.—Simplex floor arch.

23. Simplex Floor Arch.—This flat arch is of the side-construction type having tile with lugs at the bottom edge to form a space or recess into which cement mortar may be grouted with a trowel. Fig. 26 shows a cross section of the arch with a form of support or centering used in setting tile in flat-arch floors.

24. New York Reinforced Tile Floor.—A type of construction known as the "New York" Reinforced End-construction Arch is shown in Fig. 27. It is intended to be used in light floors, especially for residences, apartment houses, and hotels. It is adapted to wide spans, in which some tension may exist at the center of the span. A woven wire reinforcement (Fig. 28) is embedded in the cement mortar between rows and near the lower surface of the tile. This steel is shipped in reels and is cut to the proper length on the job as required. Tests by the Bureau of Buildings of New York City have indicated that a live load of 150 lb. may be used for 6-in. tile of 6-ft. span, and for 8-in. tile of 7 ft. 6-in. span.

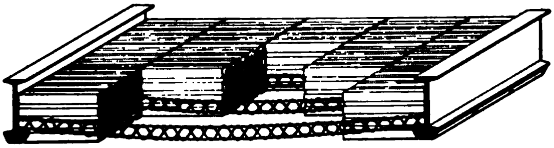

Fig. 27.—New York reinforced tile floor.

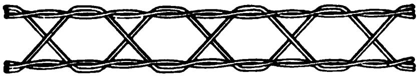

Fig. 28.—Reinforcement for New York reinfoiced tile floor.

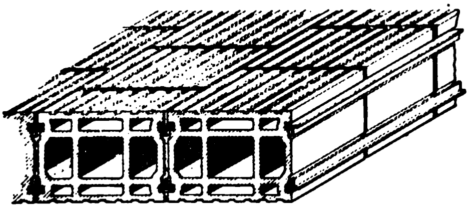

Fig. 29.—Herculean flat arch tile floor.

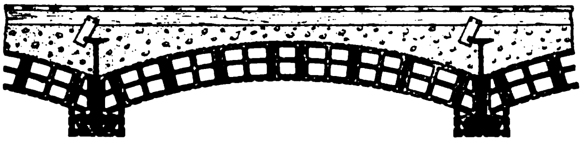

Fig. 30.—Segmental arch floor.

25. Herculean Flat Arch.—This system consists of 12 × 12-in. blocks of semi-porous terra cotta, of 6, 8, 10, or 12-in. depth according to span, combined with steel reinforcement. It is adapted to wide spans in which beam action requires the use of steel at the top or bottom. The reinforcement consists of a T-shaped steel bar, $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{3}{16}$ in., embedded in cement more tar in a groove in the side of the block. For arches of greater depth than 8 in., two T-bars are used as shown in Fig. 29.

26. Segmental Arches.—Fig. 30 shows a hollow tile arch. This type of floor construction may be used where loads are heavy, as in warehouses, factories or lofts. Tie-rods are required to take the thrust. The setting of the tile and the placing and covering of the tie-rods make the segmental arch type much more difficult to construct than the flat arches. A plastered ceiling may be suspended from the arch.

FOUNDATIONS

By T. Kennard Thomson

The foundation, as applied to buildings, bridges, etc., is considered as that portion of the structure resting on the rock or soil. The foundation work generally includes the excavation to, and preparation of, the rock or subsoil and the placing of concrete, brick, or other footings thereon.

27. Preliminary Investigations.

27*a*. Personal Survey of Site.—Before making any plans, a *personal inspection* of the site is necessary. No rules or regulations can take the place of this, for every site has its own peculiar environments which greatly affect its adaptability for foundations. A site in a vacant block, for instance, requires very different treatment to one with high buildings around it; likewise, a site near a stream of water, or even in the bed of an old stream long since diverted, requires more than ordinary consideration.

If the plot has high hills surrounding or nearby, an enormous unexpected pressure may be exerted on the foundations. For example, a well built culvert having walls 10 ft. thick and supported by 1600 piles, under an embankment on the Erie Railroad, was badly wrecked after completion by the piles being forced sideways by the movement of a soft strata, which caused one end of the tunnel to move 10 ft. horizontally and then back 2 ft., while the other end moved 2½ ft. in the opposite direction. The cause of this distortion was the action of the water from the surrounding hills on a soft bed of clay some distance below the surface. The tops of these hills were 200 ft. or so above the culvert. In this case the probabilities are that if the piles had been omitted the culvert would not have been destroyed, as the movement was in a strata below the surface and carried the piles with it. It is interesting to note that evidence of glacial deposits of hardpan were found on the adjacent hills over 1200 ft. above the sea level.

The above case is cited simply to show that a careful inspectioh by a trained observer should always precede the mechanical investigations, or much better still, before the site is even purchased. Such precautions would save in the aggregate many millions of dollars, as good locations can often be as easily and cheaply secured as bad or unsafe ones.

27*b*. Rod Test.—If the site for the building has already been selected where the ground is more or less soft, it would be advisable to ascertain the approximate depth of the soft strata, for if it were only a few feet, with a good gravel, rock, or other stable material near the surface, it would be worth while to continue the excavation to the more reliable material. A simple way to ascertain this is to drive a steel rod or crowbar into the ground. If the rod only penetrates a few feet, more definite means should be taken to ascertain the nature of the material under the surface, whereas if it penetrates many feet, the nature of the building might be such that it would not pay to carry the foundations to a hard bottom at that site, and the character of the building might also be such that there would be no object in going deeper than the frost or other requirements necessitate. In some cases, the rod may be driven 30 ft. or more, but at the best, this method simply indicates that a hard foundation cannot be obtained at a reasonable depth.

27*c*. Auger Borings.—The driving of a steel rod or crowbar stops on the first obstruction and would not indicate that below this obstruction, be it clay, gravel, boulder, or stump, there is not another soft strata. An ordinary wood auger is often used where more definite information is required. The auger will often penetrate 100 ft. or more and brings up fairly reliable samples. The auger, however, is chiefly of use in fine sand or clay and stops on the first obstruction encountered.

27*d*. Wash Borings.—When the material is too hard or compact to get good results from the rod or auger, wash borings are frequently made. The simplest method is to use a gas pipe into which water is forced and allowed to escape at the bottom as the pipe is worked up and down by one or two men holding it. A more effective method is to have a larger pipe—say, 2 to 4 in. in diameter—which is driven down by a sort of miniature pile driver (generally in the shape of a tripod) with a smaller water jet pipe working inside of the larger or casing pipe. The continual flow of water brings the material to the surface where it is carefully collected and tabulated so that a plan can be prepared showing the various stratas passed through.

In washing up the materials, clay is apt to disappear and the coarse material to be separated from the finer so it is rather difficult to be sure that the samples really show the nature of the ground. . Wash borings, however, are in many cases sufficiently reliable for the purpose; cost very much less than core borings; and may be carried down 100 ft. or more.

As a general rule, men who make wash borings claim that they stopped on rock or a boulder—but it is nearly always a boulder. An experienced man who knows the nature of the rock at that site can often tell if he has really reached bed rock, especially if it is a soft rock, like micaceous gneiss which easily chips off and is washed out. One of the few cases where wash borings always reached bed rock was for the Pennsylvania Tunnel in New York City, under Thirty-third Street. In this case wherever a boulder was encountered a small stick of dynamite was dropped down the hole to shatter and remove the boulder. In lower New York the operator nearly always claims that he has reached bed rock when, as a matter of fact, he is at or near the top of the hardpan. After being badly deceived once or twice, an experienced contractor will never agree to carry his foundations to bed rock on the evidence of wash borings—but will only contract to go to the depth indicated by the borings, if for a lump sum, with so much per cubic yard for extra work below these depths.

27*e*. Diamond Drill Borings.—Diamond drill or core borings are used where it is necessary to be absolutely sure as to the depth of the bed rock and the nature of it. These borings are obtained by having a cutter which is hard enough to cut out a core of even the hardest rock and bring it to the surface. The cutting tool is made of diamond, shot, or fragments of chilled cast iron. These cores are sometimes about 1 in. in diameter and from a fraction of an inch to 5 or 10 ft. long.

An experienced operator should never have any difficulty in telling whether his sample is from a boulder or bed rock—for, in the first place, he should know, or soon find out, the nature of the bed rock at the site he is working, and, in the second place, boulders are usually of a much harder material than the rock and are naturally limited in size. The reason for this is that what we call boulders are big gravel, having been brought down and deposited in the glacial period—all the rough corners and soft pieces being ground off in the process. New York gneiss, for instance, would have been pulverised long before it could have been formed into a boulder.

Diamond drill borings are naturally much more expensive than the other methods described, but on the other hand they are conclusive evidence, as far as they go, although they do not show the variation of the rock level between the borings. For instance, in the Ohio River, at Mingo Junction, the rock is almost as level as the water, while in New York the rock is tilted as if it has been thrown into place and is, therefore, exceedingly uneven in elevation. In lower New York, the top of the hardpan is usually nearly level for considerable distances—but the top of the rock is very irregular, varying as much as 14 ft. vertical in the same number of feet of horizontal distance.

As it is much cheaper to get a contractor to rig up a plant who makes a specialty of making borings, than it is to get one to do it who is not familiar with the operation, it is hardly worth while to give details of these devices of which there are an unlimited number of designs.

27*f*. Test Pits.—Digging a small test pit will often take the place of boring or supplement the information obtained thereby. But test pits are not usually made under the ground water level nor to more than a few feet in depth.

27*g*. Test of Soil for Bearing Capacity.—Where the local conditions are not well understood, it is well to make special tests of the soil by putting a platform on the ground and loading it. The larger the area covered by the testing platform the more reliable the results, but even the most careful experiments of this nature require a great deal of personal judgment, not only that the conditions may be thoroughly understood, but also that the present conditions will really represent future conditions. For instance, a test on dry hard clay would be valueless if the clay subsequently became wet; or, on the other hand, if the test were made on wet clay—that could not squeeze out and the clay afterwards became dry—the shrinkage resulting might be serious.

It is often good judgment to dig a hole and put the loading platform on the bottom of this hole, provided the excavation for the test hole fairly represents the conditions of the proposed foundations. The reason for this is that the weight of surrounding material holds foundation soil in place, so where only 2 to 4 tons would be allowed on sand when the foundations were to be near the surface, if the excavation, say by pneumatic caisson or cofferdam, were carried 30 or 40 ft. down, 6 to 10 tons per sq. ft. might be safe.

28. Characteristics of Soil, Rock, Etc.—If the sand, clay, or other material had been prepared by man so that he knew the exact constituents, how it had been placed, how rammed, rolled, or tamped, it would be comparatively easy to say how much load could safely be applied, but as these materials have been placed by Nature, sometimes by gentle sedimentation and sometimes under enormous hydraulic pressure, and as they are often placed in layers of more or less thickness, with or without water present, all we can do is to give general rules as above and then make tests and use one's best judgment. In fact, no part of a structure is so dependent on good judgment and so little bound by cast-iron rules as the foundations.

Sand.—Clean sand has been packed in such a manner by hand that it safely carried 100 tons per sq. ft., or more, and yet as it is found in nature, it cannot be loaded with more than from 2 to 4 tons except in deep excavations.

Sand varies from pure silica in very fine particles, to gravel, or it may be mixed in various proportions with many different materials, as clay, loam, decayed vegetable matter, minerals, etc., and most important of all, water. Sometimes Nature makes a thorough mixture; while there are many places where successive layers are found. These may be thick or thin, of sand, clay, gravel, etc., and may be repeated over and over again. A shaft has been sunk through about 40 ft. of distinct layers many of which were less than 1⁄16 in. thick. The clay acts as a lubricant to help the sand to slide into any accessible opening.

If the sand is confined so that it cannot escape, it will safely sustain great loads whether it be dry or wet, and sand of coarse grain may be alternately wet and dry provided no sand is lost or carried away in the process of wetting or drying, the coarser grains being much less liable to be carried off.

The disintegration of rocks (especially igneous rock, containing silica and calcium) by the action of the weather, wave, or wind, forms pure sand. After being separated from the rock the grains are carried by the rivers, waves of the oceans, or wind, to a new bed and often many other substances, such as clay, mud, minerals, etc., are deposited at the same time or in between the different layers of sand. Calcareous sands are formed generally by the waves of the seashore, which act on limestone beds, shells, corals, etc. Much sand comes from pulverized quartz as the softer rocks will not stand the grinding action necessary to form clean white sand.

On the desert, the sand particles have their rough edges ground off by being blown over and over each other by the wind, which like the waves and floods, tend to separate the larger or heavier from the smaller and lighter fragments—often to be mixed up again with other grades of sand and with other material. Even such hard substances as diamonds, are rounded when carried along with sand. The banks of a river may contain many kinds of rock and the same kind of rock in many places, some making sand, others gravel, mud, clay, etc., all of which may be mixed together in transit. Even a coarse sand is carried on a current of less than one-half a mile per hour, the heavier grains sinking first and the finer grades being carried much farther.

In North America and other places, much sand was brought down with the ice during the glacial period. The particles of this sand are often more angular than the particles of sand washed down with gravel in the rivers or blown about by the wind. The treatment which makes sand, would make clay or mud of the softer rocks.

All kinds of metals, diamonds, earthy matter, etc., are found mixed with the sand at different places, gold and other heavy metals working their way to the bottom.

Heat accelerates the chemical action in the disintegration of rocks.

Clay.—Nearly all rocks if pulverized fine enough, would be found to have some of the qualities of clay. Hard rocks, like quartz, as a rule are not so easily decomposed by the weather and are more apt to form sand than clay. In New York, however, rock containing quartz has been found under 30 ft. of hardpan so rotten that it could be shovelled; whether this deterioration occurred before the hardpan was deposited or was due to subsequent chemical action, would be hard to ascertain.

Clay is a combination of silica and alumina with all sorts of impurities mixed with it. When mixed wet and dried out it becomes very hard, and shrinks in volume. Being so much finer in particles than sand, it is held in suspension and carried much farther out to sea than the coarser grained sand or gravel, which are deposited first. The finest particles of all are carried, often, far out into the ocean as mud. This fine material may become shale by pressure or some other means. The shale may be uplifted and exposed to weather where it will disintegrate and again become mud or clay.

Clay is deposited, layer after layer, with sand, gravel, or other material (such as decayed vegetable matter, animal matter, minerals, etc.) mixed in between and often acts as a lubricant for the more compact or heavier material to slide upon, and is undoubtedly the cause of nearly all great land slides. It is at the best a very treacherous material to deal with. When dry it will carry 4 tons per sq. ft., or much more, but when wet its carrying capacity is extremely uncertain to say the least, and often it would not be safe to place ½ ton per sq. ft. on it, unless a considerable settlement would not be injurious to the buildings.

Clay, unlike sand, is softened by water and liable to move under pressure. In a case at Hudson, N. Y., a 225-ft. chimney, power houses, and other buildings were wrecked, all of which were located on rising ground near a creek, and 12 acres dropped 20 ft. in 2 min. Fifty auger borings failed to indicate the cause of the disaster, but a shaft, about 4 ft. square, sunk to a depth of 35 ft. disclosed a very soft layer of clay at about the same level as the bed of the creek. The probabilities are that the excessive rains of that season had reached this bed of clay from the surrounding hills, causing the sudden collapse which moved the creek bodily, about 100 ft., in addition to the sinking of the 12 acres. This layer of clay as disclosed in the shaft, was entirely inadequate when softened by the excessive rains, to carry the weight of the soil above it even without considering the buildings at all; and as a proof of this it might be stated that a similar slide occurred nearby in the Virgin Forest.

Loam.—Loam is a mixture of decomposed organic matter with sand, clay, etc., and is treacherous enough material even when not full of worm holes. As a rule, it is not compacted by Nature as most sands and clays are by the glacial or other floods, and does not extend to any great depths. No building of any importance should be founded on it.

Marl.—Marl is composed of clay and carbonate of lime in different proportions, the carbonate of lime often making it valuable as a fertilizer. Like clay and sand, it contains many impurities, fossils, etc. Soft marl is called earthy; hard marl, indurated.

Hardpan.—Hardpan is usually a mixture of sand, clay, and gravel. In New York, for instance, it was evidently formed in the glacial period and seems to be free from vegetable or animal deposits, for if any such were originally in the mass, all traces thereof seem to have disappeared. Generally this hardpan lies directly on the rock (in New York) with from 30 to 80 ft. of quicksand on top of it, but occasionally a layer of from 2 to 20 ft. of clean sand, gravel, and boulders is found between the hardpan and the rock. The proportions and consistency of this hardpan vary from mud to a natural concrete which is so hard that it has been mistaken for good Portland cement concrete. As a rule, however, it can be removed by pick and shovel. In one case only, when sinking caissons in New York City, a vacant space of about 8 cu. ft. was found in the middle of the hardpan removed. This may have been formed by some matter which was afterwards decomposed allowing the space to be filled with water. Most hardpan is much harder when dried out than when in its original bed, under water, but any good hardpan will support in its natural bed more than 15 tons per sq. ft. provided it is not underdrained. Some hardpans are water-tight, others water-bearing.

Peat, Bog, Etc.—It is sometimes necessary to put floating foundations for railroads or other structures on these materials, but as the risk is great, it should only be taken when unavoidable and then with great care. Peat is vegetable matter not fully carbonized. It has been used for embankments on canals where the question as to the safety of having an inflammable material for the banks of a canal was gravely debated.

Silt.—The Hudson River silt is so fine that a 23-ft. diameter shield of a tunnel could be driven across the Hudson River without excavating any material whatever (see James Forgie, *Eng. News*, Feb. 28, 1917, p. 228). In this material 90-ft. piles have been driven in 6 min., without reaching any harder materials; and then a test was made by capping 4 of these piles a week after being driven, when they held a test load of 160 tons without any further penetration whatever. The Hudson River silt is very much finer and more treacherous than the New York quicksand.

Gravel.—Gravel is generally obtained by screening from mixed deposits the sand, mud and boulders; occasionally the run of the quarry can be used as found either for gravel or for concrete, without removing the sand.

Rock.—A good rock when lying in its original bed will support any load which is liable to be placed upon it. The chief danger is where the stratification is inclined and in such a position that it can break on its natural cleavage plane, allowing the structure to slide into a valley or adjoining excavation; a condition to be guarded against in a city like New York, where the stratification is very tilted and very irregular, and where subways and excavations for other purposes remove the rock by blasting many feet below the foundations of the adjoining buildings.

29. Loads on Foundations.—New York Building Code, as of March 14, 1916, gives a good summary for loads per square foot on different soils, excluding mud, as follows:

Wet clay	1 ton
Wet sand	2 tons
Firm clay	2 tons
Sand and clay mixed or in layers	2 tons
Fine and dry sand	3 tons
Hard dry clay	4 tons
Coarse sand	4 tons
Gravel	6 tons
Soft rock	8 tons
Hardpan	10 tons
Medium rock	15 tons
Hard rock	40 tons

When the Superintendent of Buildings is in doubt as to the quality of the soil, he demands at proper tests shall be made to determine the safe bearing capacity. He will also consider y tests the owner may wish to make under the supervision of the Department.

In New Orleans, where the subsoil is all alluvial, the building laws specify that only 1400 . per sq. ft. will be allowed on any foundation. Buffalo allows 3½ tons per sq. ft. Cambria eel Handbook, 1919, pp. 327–349, quotes Building Laws, Foundations, etc., for 31 cities.

The writer is satisfied that almost any material that deserves to be called rock will bear, in its original posi- n, practically any load that can be placed upon it, provided that the rock is not inclined and lying in such a sition that it can slip off its base and take the building with it. When the rock is so rotten that it can be shovel- l out, it is hardly fair to call it rock. Usually concrete is placed on top of the rock, and 15 tons per sq. ft. is a fe allowance for good concrete. This load is the same as 208 lb. per sq. in., or 104 lb. on ½ sq. in. Now imagine girl weighing 104 lb. standing on a French heel of ½ sq. in. She could not make any impression on a wood floor, uch less on bed rock; or, in other words, the foundations for the Singer Tower in New York City, 612 ft. high, ly cover half the area of the lot, and so if the weight of the Singer Building were doubled, the weight on the hole area would be only 104 lb. per ½ sq. in. First class concrete would carry safely much more than 15 tons r sq. ft., but owing to liability of poor workmanship, etc., it is safer not to allow more than this amount. The d allowed on mortar or concrete will generally govern the load on the rock since, apart from the expense of reling off the rock to get a direct bearing for the steel columns, it is usually advisable to have some waterproof aterial, such as sheet copper or lead under the columns and to have several inches of mortar between this mate- l and the column base. Copper should never be in contact with steel as the steel may be destroyed by ectrolysis, and tar and felt are too compressible to be put under heavy columns.

30. Dead, Live, and Wind Loads.—There are many empirical rules for estimating the loads foundations, especially as regards live and wind loads. The dead load is, of course, a fixed em being the weight of the structure itself.

Most building laws do not anticipate that all of the floors will be loaded to their maximum one time, but while the floors of an office building, for instance, must be sufficiently strong to rry heavy safes and a crowd of people and there is little probability of all the floors of such ilding being so loaded at the same time, a warehouse or factory on the contrary might have s capacity taxed to the utmost, so the only safe way is to take each case by itself and design ch foundation for the total load which it will probably be subjected to, including wind and ow. Many cities specify that the foundations shall be designed to carry 60 % of the assumed re load in addition to the dead load, snow load, and wind pressure.

In designing steel buildings there seems to be a greater variation in provision for wind stresses than for any her item, for some buildings seem to have been built without making any provision at all—while others, like e Singer Building Tower, not only have ample knee braces and other connections, but have in addition, anchor e-bars extending many feet into the concrete caissons in such a manner that the whole caisson would have to lifted or the column broken before the building could blow over (see *Trans.* Am. Soc. C. E., vol. LXIII, pp. 30). Very few buildings are so anchored and very few would need any provision against uplift. On the other nd, however, it is often advisable to add the wind loads to the dead and live loads on the leeward side of the ilding. For tall chimneys or isolated buildings, the entire wind pressure might reach the foundations while in a ilt up section of a city only a fraction of the maximum wind pressure would probably do so.

31. Building On Old Foundations.—When it is desired to add 3 or 4 stories to an old ilding, it will often be found that a building which has been in existence for many years, sting on sand, clay, etc., has so compressed its foundation that the additional weight will not use any settlement or cracks in the building at all. This, however, can be determined only by a careful investigation of the site, making borings and other observations. The National City Bank on New York quicksand, and the Methodist Book Concern, Fifth Ave., on sand, clay, etc., are examples of this. Both had been built many years and neither settled the slightest when new stories were added to the old.

32. Effect of Climate.—Foundations are not usually exposed to the weather and are not therefore as much affected by the climate as the rest of the building, but the results of expansion and contraction must always be considered. Some reinforced concrete buildings have been built from 100 to 300 ft. long without any expansion joints, but if the foundations had been continuous for that length, the upper part of the structure would have expanded more than the base with disastrous results. Cast-iron cylinder piers, 6 to 8 ft. in diameter, have been filled with masonry which did not contract as quickly as the cast-iron shells, with the result that the shells split open. This has occurred in several places.

A large hospital was founded on shale, and had a 4-in. concrete slab for a floor, without any expansion joints, although the building was over 100 ft. square. Under the floor were numerous tunnels, or subways, 4 ft. deep by 5 ft. wide, for steam pipes. The floor was constructed in January; hospital opened in July; thermometer stood at 102 deg. in shade outside and 128 deg. in the subways on account of the steam pipes being required for sterilizing purposes. As the heavy building was on a solid foundation, the floor was held on its four sides by the heavy building, so it just naturally buckled up—smashing various light partition walls, etc., and causing thereby considerable discussion as to whether (1) the building had settled, (2) the building had risen in places, or (3) an explosion of coal gas had occurred. This discussion lasted for months before the real cause of the trouble—expansion—was discovered. The object of having such large floors without expansion joints was to avoid the danger of germs finding their way into the joints where they could not be scrubbed out. Needless to say, the above object could have been obtained and proper provision made for expansion and contraction at the same time.

Heat.—Concrete while setting should be protected from excessive heat of the sun and in some places it would be advisable to keep the foundation so protected until the building is constructed over it.

Concrete like rock or soils, is much more liable to disintegration from chemical action when at the same time subjected to heat. This has been found to be so at Panama, Long Island Sound, New York City, and many other places.

In the writer's opinion, pure salt water does not injure dense Portland cement concrete, but chemicals from sewage or other sources, especially when heated by the sun or other means, do destroy it. For an example, the discharge tunnel from a power house was built of concrete. The impure water, so discharged, was very hot and it was found that no concrete could last in this position. A wood lined tunnel was tried and up to date seems to give satisfaction.

Cold.—A porous concrete which allows the water to enter and freeze or to carry chemicals in or out is in much more danger from climatic changes than an impervious concrete. Where necessary, steel reinforcing should be used to prevent danger from expansion and contraction. Foundations should always be carried deep enough, unless on bed rock, to prevent the material under the foundation from freezing and thus expanding so as to lift and destroy the work.

It is a very safe rule not to place concrete when the temperature is much below freezing. Good concrete, however, has been laid in from 10 to 15 deg. or more below freezing by heating the ingredients before mixing and covering the concrete while setting. It is always advisable to prevent the concrete from freezing before or while it is setting, as the distortion is liable to be injurious.

33. Waterproofing.—The nearer concrete is to being waterproof, the better, as it will be less liable to be damaged by frost, etc., and one of the surest ways of accomplishing this is to have enough cement to fill all the voids in the sand. This generally means a mixture of one part of cement to two, or less, parts of sand. A concrete of good Portland cement, sand, and stone, or gravel, with no voids will come very near to being waterproof, but at the same time this is a very hard condition to obtain.

There are numerous substances which it is claimed, when mixed with the cement, will keep the water out. Other methods, such as, tar and felt, sheet copper, sheet lead, etc., are well known and reliable if properly applied, but as a rule contractors for waterproofing do not try to waterproof against a head of water, preferring to put drains under the floors or behind the walls which are to be protected. These drains lead to sumps and the pumping therefrom as a rule is not serious. Where there is a persistent leak in a wall, it is a common practice to cut a groove in the wall and then cover it over, thus forming a blind drain to carry the water from the leak down to the sump.

Foundations, retaining walls, etc., should have the concrete poured continuously from the base to the top of the wall, for if the work is suspended until the concrete has begun to set, water will always be able to find its way through horizontal cracks formed where the stops are made in pouring. As there is generally a certain amount of milk of lime or laitance on the top of wet concrete, a small seepage of water will eventually greatly enlarge these horizontal cracks, by washing out the soft mortar or milk of lime. An examination of almost any retaining wall along a railroad will prove this statement. The writer never allows his work to stop over night, in cases where such leakage would be objectionable.

34. Allowances for Uneven Settlements.—Buildings founded on sand, clay, or other material liable to compress under the weight of the building, should be designed so as to have fairly uniform loads per square foot on the foundations, otherwise one part of the building will settle more than the other parts. A low or light building attached to a high or heavy or old building, should have an open joint, not necessarily exposed to view, so that if the heavier building settles it would not make an unsightly crack between it and its addition. Lack of this precaution resulted in a fine church breaking away from a one story extension although the load was not over $\frac{3}{4}$ ton per sq. ft. on the foundation of either.

In Chicago many high buildings were built on spread footings on the clay, which were sometimes carried a considerable distance from the surface by means of vertical shafts or open cofferdams. Great care was exercised to design these foundations so that each footing under the building would have the same load per square foot on the clay. But in spite of all precautions the settlements have not been uniform, varying from 2 to 4 ft. On account of the trouble which resulted, the more recent buildings have been or are being carried to bed rock.

The sinking of the buildings in Chicago started long before the day of subways, so the trouble is liable to get worse instead of better. The tunnel construction will undoubtedly continue in Chicago and all other large cities and every deep cellar or excavation must more or less affect the ground water conditions with disastrous results.

After having tried so unsuccessfully the founding of buildings of 18 and more stories in height on clay in Chicago, the plan of driving pile foundations or better still, carrying the foundations to hardpan or bed rock was adopted for the higher buildings, and of limiting the height of the buildings on the clay foundations to 6 or 8 stories, the foundations of which only covered about half of the area of the lot instead of the whole of it. When only a portion of the lot is covered by foundations in this material, the load can naturally be larger per square foot of surface covered.

35. Foundations as Regards Character of Structure.

35*a*. Residences.—In determining what load can safely be placed on the foundations one must know to what use the building will be put. For instance, a country dwelling would require very little spreading of the foundations assuming an ordinary cellar or where the foundations are deep enough to be below the frost line. If, however, the ground has previously been levelled up with a rock fill, on top of which more or less dirt has been placed, the rocks may settle to a certain extent due to soft ground underneath or to breakage of the stones which were loosely packed, and, what more frequently occurs, the rain may wash the superimposed earth into the crevices of the rock allowing the residence to settle, badly cracking the plaster and wall paper and jamming the doors and windows. This sometimes continues for many years.

Even with light buildings, it is advisable to see that the rains or streams are not liable to draw sand, loam, or clay from underneath or to soften the clay by wetting it, or causing it to shrink by drying it out.

35*b*. Factories.—When near other buildings, in addition to the above requirements, factory buildings need to be ensured against shock or vibrations from destroying otherwise perfectly safe foundations. For instance, a building containing a gas engine, built on silty ground and having a large number of compresol piles under it, vibrated so badly that other buildings 700 ft. away moved as much as $\frac{1}{16}$ in., vertically and horizontally with each motion of the engine. These compresol piles had been formed by dropping a pear shaped weight from a pile driver until a hole 3 or 4 ft. in diameter had been made some 12 ft. deep. Occasionally sand, ashes, or clay were dropped into the hole and rammed aside to keep the water from troubling. Finally the holes were filled with concrete and it was thought that a shock-proof foundation had been obtained, but the very roughness of the piles seemed to assist in transferring the shock to the soft ground. Subsequent borings indicated that an ordinary cofferdam could have been carried about 4 ft. farther, where it would have reached a much harder and more satisfactory material. The company had on its own ground in just as convenient a location, a site where this engine could have been built on hard ground and at a less cost. In fact, the most feasible way of remedying the error would be to build an entirely new engine house on the higher site and use the old building for other purposes, that is, for stationary loads which would cause no shock to be transmitted through the ground.

35c. Churches.—Special pains have to be taken with churches which are often very heavy with high unsupported walls and long span roof trusses or arches. The beautiful and historic St. Paul's Church, London, England, has long been a source of worry on account of the settling of the foundations, aggravated by the construction of subways which lowered the water level, thereby injuriously affecting the stability of the clay sub-strata.

35d. City Buildings.—The efforts to economize on the foundations for buildings in Chicago with the very unsatisfactory results due to the continual settlement, both even and uneven, have already been noted in Art. 34. Buildings up to 8 or 10 stories, as a rule, would hardly seem to justify foundations of 40 to 80 ft. or more in depth, although there are a few buildings in New York of from 4 to 6 stories in height, above the curb, which have pneumatic caisson foundations carried to bed rock under them. In these cases, however, the work was so designed that many more floors could be added to the building later on without tearing it down or adding to the foundations.

A very fine cathedral, recently built, had a foundation on coarse sand, within a foot or so of the street level. The ground level between the street and the building was then raised some 3 ft. The towers had a load of 4 tons per sq. ft., while adjacent walls had only 1 ton per sq. ft. The uneven settlements caused serious cracks between the towers and the walls.

In large cities, like New York, one must not only consider the existing structures in the neighborhood, but also those of the future. In this respect many 12 to 16-story buildings in New York were founded on piles or on floating foundations, the excavation being carried almost to the surface of ground water, with the result that excavations for other buildings and for subways have seriously imperiled them by lowering the water level.

Wooden piles or steel shells filled with concrete will last indefinitely if kept always under water, but will soon rot or rust out if the water is withdrawn. On 33d St., New York City, the construction of the Pennsylvania R. R. diverted an old stream and left wooden piles high and dry, which were originally 30 ft. under water, thus destroying their value and making expensive underpinning necessary. Similar results, but not to such a great extent, have been noticed in many parts of the city. Recently in lower Broadway where the material above the hardpan is the so-called New York quicksand, the water level suddenly rose 9 ft. and then dropped back 10 ft. almost as suddenly. This high water caused the flooding of several buildings over a block away. As this was the site of a 12-story building which rested on the very fine sand, the danger can readily be seen. The most plausible explanation is that the ground water level, which used to be from 6 to 9 ft. above the high tide level, had been lowered by some nearby construction, either the subways or deep cellars, and that a broken water main temporarily raised the water to its old level only to be quickly drained off again. Needless to say, such periodic occurrences must be very unsafe to the buildings. A designer of foundations should have a danger signal running through his mind—Water! Water! Look out for water!

Every here and there skyscrapers are erected with so-called "earth scrapers" under them, which have from 3 to 4 floors below the water level, and it is very hard indeed to prevent some seepage into the cellar drains. Again, the subways are in many cases below the water level and it will be only a question of time before the railroads will want to tunnel under the subways to cross Manhattan from Jersey to Long Island, so any new building which does not take into account the future changes of the ground water level will probably pay for the lack of foresight. It has been proposed to cofferdam around the lower end of the city and to pump the water out, which would surely have very interesting results, to the onlooker, if ever attempted.

Similar results may be expected in all large cities founded on fine sand with a high water level, or on clay as in Chicago, or on alluvial deposits as in New Orleans.

As before stated, sand of various degrees of fineness or coarseness, wet or dry, will carry very considerable loads—say, from 2 to 10 tons per sq. ft.—the greater loads being permissible where the excavation is carried to a considerable depth below the surface, but this advantage would of course partly disappear if adjoining buildings were subsequently built to the same depth.

The Municipal Building in New York City has its tower and south section founded on pneumatic caissons which were carried 112 ft. below the water level or 143 ft. below the street level to bed rock. After the contract was let, borings disclosed the fact that rock under the north end of the site was at very much greater depths and therefore unattainable by pneumatic caissons; so it was decided to sink caissons through from 40 to 50 ft. of sand, where they would safely carry 10 tons. The tower and south wing of the building were founded on bed rock at the depths stated above. Danger of slight settlement of the north end of the buildings, which would cause slight cracks, was easily taken care of by concealed joints in the masonry between the two sections.

Sand makes an excellent foundation provided the water level remains the same, and as long as the sand cannot escape into adjoining excavations. This contingency is a very vital one, for many sands which have various amounts of clay mixed with them, will flow almost as freely as water. The sand under the Municipal Building is very coarse and water flows through it very freely, and it was found impossible to lower the water level by pumping.

A 14-story building founded on quicksand was nearing completion when the pneumatic caisson foundations on the adjoining lot caused the north end of the 14-story building to settle 4 in., while the south end remained where it was. The floors were all leveled up and the subsequent tenants never knew the difference.

36. Electrolysis and Rust.—Electrolysis is one of the most serious dangers to foundations of modern steel buildings to be guarded against. The trouble occurs where the electric current enters or leaves the building or where dissimilar metals in the presence of water form an electric couple. An example of this was shown on the removal of some old brick piers with long anchor bolts. Electrolysis had corroded these bolts and in doing so had cracked the brick piers as if by an explosion.

It might be stated that in many large cities there is considerable electric current in the ground, having escaped from trolleys, subways, and elevated railroads, especially the latter in old days before the return current was taken care of. The result is that there is always a chance of the current escaping from or entering the buildings, especially when the foundations are under water.

The simplest manner of taking care of this is to have wires attached to each column and "grounded" where no harm can be done, and making sure that the ground water can not reach the columns or their bases. This precaution against electrolysis has unfortunately seldom been taken.

The writer has seen steel girders under buildings from 12 to 25 stories high, in very bad condition from rusting. The most inexcusable case was where 24-in. I-beams and 4-ft. plate girders carrying a high building were buried in the earth without any concrete around them. Needless to say, there was no paint left on the steel, and the rusting was making rapid progress when discovered, which was just in time to save the building by embedding the beams and girders in concrete.

When wrecking the 17-story Gillender Building, on the corner of Wall and Nassau Streets, 14 yr. after its erection, it was noticed that wherever the concrete was in direct contact with the steel no rusting had commenced, but that wherever there was the slightest space between the steel and concrete, rusting had started and in some places made rapid progress. This applied to the steel columns, girders, and foundations. Base plates and shim plates showed much rust. The columns rested on heavy plate girders which had been painted, covered with tar and embedded in concrete. These girders showed not the slightest sign of rust. Underneath the girders were 12-in. I-beams which had been painted and buried in concrete and were also in perfect state of preservation.

Under the adjoining buildings were some 14-in. diameter underpinning cylinders or pipes which had been driven to hardpan and filled with concrete. These steel pipes had of course nothing on the outside of them—not even paint—but were entirely under the water line, in the sand, and were found to be in a perfect state of preservation. This would seem to indicate that New York quicksand will preserve steel from rusting if it is not disturbed, mixed up with chemical impurities, or subject to electric currents. It might be remarked here that the concrete only extended to within about 2 ft. of the bottom of the 14-in. underpinning pipes or cylinders which had been jacked down under the buildings, and that the writer has never seen a case yet where it was possible to get all the sand out of the pipes. In some cases, more or less gravel remained in the pipes. This means that the foundation of the pipes has all the bearing on the steel shell, and that if the friction on the shell is reduced, the pipe will cut into the hardpan or sand and cause some settlement. This has happened a number of times.

37. Foundations Partly on Rock.—Sometimes it is necessary but never desirable to have part of the foundations on bed rock and part on sand, clay, or mud. Whenever this is the case, the building should be so designed that a settlement in the softer material will not crack walls, plaster, paper, etc. In many cases the bulk of the settlement will occur during construction, and the balance can be taken up by the blind joints in the walls, etc.

If the building is to be subject to vibration from machinery, etc., serious trouble will result, unless separate foundations either entirely on or entirely off the rock can be secured for the machinery. Some years ago a building was erected facing an elevated railroad, with the front of the building on sand and the rear on ledge. The owner sued the elevated for damage to his building. It is doubtful if he could have recovered damage even if his house had been built first instead of after the railroad, as was the case.

38. Teredo.—Any structure with a foundation resting on wood in salt water must be protected from the teredo and limnoria. Both of these borers have cut off piles 45 ft. under water, in Fall River, Mass., although the piles were only 150 ft. from a small sewer. Two years after erection, these piles had been completely eaten through allowing the bridge pier to drop 2 ft. over night. It will be noted that these animals started work 45 ft. below the water although they are only supposed to start between high and low tide. At present, the harbors of such cities as New York and Philadelphia are too polluted with sewage to permit teredo or limnoria to live, but some day the sewage will be diverted and used as fertilizer, and then the damage will begin. The teredo and limnoria are found in many places on Long Island Sound as well as on the

39. Eccentric Loading.—When heavy walls have been built on the property lines it has been the custom to spread the base on the inside of the building only, thus having a much greater load on the outside of the base than on the inside. The only defense for such design is that it has been much used. It would be very much better to carry the foundation deeper and use higher unit loads, or to use piles or caissons.

One disadvantage of eccentric loading of this kind developed when it was necessary to underpin with 3-ft. diameter cylinders, old walls having a base of 10 to 12 ft. in width. The cylinders were, of course, placed directly under the wall or the outside of the base, leaving 7 to 9 ft. of the base overhanging the underpinning cylinders. Another disadvantage is that these eccentric bases take up an enormous amount of cellar room. It would often be cheaper to get deeper and better foundations even without allowing anything for the rental value of the space saved or lost.

40. Cantilever Construction.—Eccentric or wide footings with the walls carried on one side making the pressure so much greater on the outside of the footing than on the inside, are obviously incorrect in principle and unsafe on soft grounds. A much better arrangement is a system of cantilevers. This simply means placing a cantilever from the outer column base to one of the interior bases so that the cantilever girders or beams will have a bearing on the center of both bases, be they spread footings, cofferdams filled with concrete, caissons, or piles.

The cantilever will thus support the outer column with a short leverage arm, usually not over a few feet, and as the inner arm of the cantilever will be held down by the interior column, the anchor arm leverage is generally from 5 to 10 times the overhanging leverage, so the plan is simple and safe as long as the girders or beams are protected from rust and electrolysis.

On soft ground, exactness is required in this design, but in some cases where the concrete caissons form a continuous wall around the lot, and are carried to bed rock or good hardpan, the cantilever girders might be considerably cut down on the assumption (1) that the concrete caisson would distribute much of the weight over the base many feet below the column; and (2) that the strength of the concrete caisson is really so much greater than assumed, that it would safely carry the load without overturning or crushing.

When the foundation rests on clay or sand, it is often customary to use combined footings (see Art. 50).

41. Bearing Pressure, Gross and Net.—When the foundations are comparatively near the surface of the ground, the total or gross pressure only need be considered; but in some cases of very expensive foundations, it is customary to allow for the surrounding earth, or water, or earth and water pressure combined, to deduct this from the gross pressure, and call the result the net pressure. For instance, if the excavation has been carried to a considerable depth, the probabilities are that the material founded on would not be compressed and could not be squeezed out without lifting the surrounding material. If the depth were 100 ft. and the material water, the amount to be deducted would be 6200 lb., or say, 3 tons per sq. ft. If in earth and water, the amounts to be deducted might be 50% more than this.

Some consider deducting for the friction of the earth on the side of the pier but this is too uncertain an item to be relied upon, and excavation on adjoining property might reduce this friction to almost nothing. Friction on the sides of caissons has been accurately calculated and varied on one job from 30 to 650 lb. per sq. ft. of surface.

42. Wooden Pile Foundations.—Wooden piles have, up to this date, been used much more than other kinds of piles, and vary all the way from a 3-ft. block to a 90-ft. pole. In some cases, a hole is dug 2 or 3 ft. deep and a pile is placed in the hole with its big end down. But it seems foolish, in such a case, not to enlarge the hole so that a mud sill can be put under the pile, which is, in this case, really a post. Failure to use such mud sills has resulted in a bad collapse in many places.

Probably the shortest driven piles, for an important building, were those under the Campanile in Italy. These were only about 3 ft. long and were used to compress the soil. As subsequently proved, longer piles there would have broken through into the water-bearing soil and caused much damage.

42*a*. Frictional Resistance.—Wooden piles generally depend on the frictional resistance of the ground since a pile would not have very much strength as a long column, even if resting on rock. Piles are simply long straight trees driven, of course, with the small end down and the small end is often not more than 4 or 5 in. in diameter.

The frictional resistance of a pile varies very greatly according to the material driven through and the quality of the timber itself. The only safe proceeding in a strange locality is to drive a few piles and put a test load on them.

If water is withdrawn from piles, the frictional resistance is apt to be destroyed.

42b. Safe Load.—The Building Laws of most cities specify that the maximum load allowed on a wooden pile shall be 20 tons (New York City and others) while a few allow 25 tons or even a little more.

All books on pile driving give a drop hammer test for ascertaining the safe load to allow on piles, and the favorite formulas are those of Wellington, former Editor-in-Chief of the *Eng. News.* They are:

For a pile driven with a drop hammer, $P = \frac{2Wh}{s + 1}$

For a pile driven with a steam hammer, $P = \frac{2Wh}{s + 0.1}$

in which P is the safe load in pounds, W the weight of hammer in pounds, h the fall of hammer in feet, and s the penetration or sinking in inches under the last blow, on sound wood.

Judgment must be exercised in using this method of determination, for more piles have been destroyed by over driving than by any other cause. Over and over again, when a contractor knows perfectly well that the penetration of a pile has been sufficient, he has been told by some young well meaning inspector who is making use of one of the above formulas, that he must keep on driving, when all of a sudden, a few blows of a hammer sends the pile down anywhere from 3 to 8 ft. Then the inspector joyfully exclaims, "There, you were on a thin shell which you have broken through; now keep on driving until you reach a hard bottom." But what has really happened is that the pile has been broken, split, or bushed, often in such a way as to make it absolutely useless as a pile (see Figs. 31 and 32). Some piles which were so butchered in Back Bay, Boston, and afterwards removed and photographed looked more like a lot of hemp than pieces of timber.

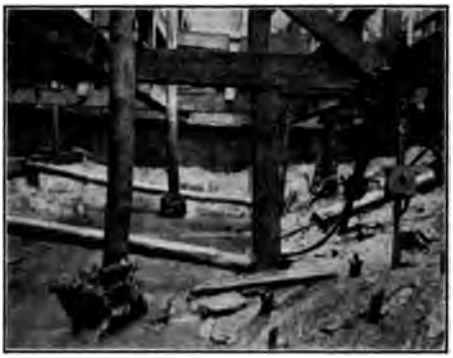

FIG. 31.—Piles, showing result of too much driving.

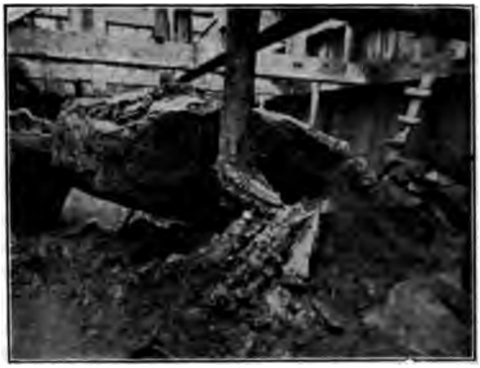

FIG. 32.—Piles, showing result of too much driving.

The *Eng. News*, Jan. 14, 1909, has an illustrated article of some piles in Columbus, Ohio, which were afterwards removed showing that 38% of the piles (oak) had been destroyed by the driving—some telescoped, some split, some broken, and some bushed, while many had no bearing value left.

The proper place for piles is in soft ground, sand or clay, for in hard ground or gravel, etc., a spread footing of concrete would probably be better. When used in soft ground, the pile should be driven until the frictional resistance is sufficient to hold, say 20 tons, or until a harder strata has been reached. The depth of the harder strata should be determined by borings and tests.

If the borings indicate a great depth of silt or other soft material, then a cluster of four or more piles should be driven, capped, allowed to stand for a week or so, and then tested. For instance, at Perth Amboy, 90-ft. piles were driven (the steam hammer followed the pile 30 ft. under water) in 6 min., without reaching any harder material. Then a test load of 160 tons was placed on four of these piles (40 tons on each) which had been properly capped, but no settlement occurred.

In cases where hardpan or other impenetrable strata exists within driving distance, a water-jet pipe should be put down for each pile, so that the length to be driven will be known before starting.

42c. Spacing of Piles.—The best spacing for wooden piles under buildings is 3 ft., center to center. This does not apply to bents for railroad trestles where the spacing is usually greater. To put piles much closer than this is to destroy the frictional resistance and sometimes to disturb the ground to such an extent that piles, previously driven, are forced up.

Close spacing was adopted under the Park Row Building, New York, with the result that it was found impossible, with the hammer, etc., used, to drive the piles as far as expected and 10 or 15 ft. or more were cut off the top of many of the piles, which were none too long to start with. And, in addition, some groups of piles were noticeably out of plumb.

In another case, the owner and contractor were so sure that the piles under their building were driven to hardpan that they were quite confident of the safety of their building, but the first caissons on the adjoining lot disclosed the fact that the piles were not only not plumb, but were also not within 15 ft. of the hardpan. The owner of the old building paid many thousands of dollars to have his structure underpinned safely.

42*d*. Cutting Off Piles.—Wood, when wholly under water, has remained perfectly sound for centuries, but if wet and dry alternately, will soon be destroyed. Consequently piles should be cut off so that they will always be under water. If wood caps are used, the caps also should be permanently under water.

The difficulty is to ascertain the lowest probable elevation of water. For instance, in New York City in many places the ground water stands from 6 to 9 ft. above high tide. New excavations are apt to lower and have lowered this level, at least temporarily, even below the high tide level. (Since the above was written, the ground water level has been found to be 2 ft. below the high tide level.) In one case, the piles were driven in the bed of an old creek, still running under ground, and a tunnel permanently lowered the water level 34 ft. A great many similar cases could be cited.

42*e*. Capping Piles.—In early days, the ordinary cap for a pile was of wood or stone. Now, however, wherever concrete can be readily made, it is by far the best material for capping wood or concrete piles. It is stronger, does not rust out, and if necessary, can be strengthened by reinforcing with steel. It is also a protection against the teredo and limnoria.

42*f*. Kind of Wood for Piles.—The kind of wood used for piles will generally be determined by what is most easily obtained and by the cost. Pine, hemlock, spruce, and many soft woods make admirable piles. Cedar, hickory, oak, etc., are, of course, much tougher and more durable, and therefore desirable when they can be obtained of proper lengths and at reasonable cost.

42*g*. Size of Piles.—The size of piles depends entirely on the character of the structure, material at hand, etc. The most common requirement for building purposes is given by the New York Building Laws, which specify that the diameter at the point shall be not less than 6 in. and at the butt 10 in. for piles not over 25 ft. long, and 12-in. diameter at the butt for piles over this length.

42*h*. Water Jet.—In some soils, like New York quicksand, it is a great advantage to water jet the site of each pile and even to work a jet pipe (ordinary gas pipe through which water is forced under pressure) up and down as the pile is being driven. In such a soil, the driving is greatly facilitated, and the disturbance to the adjoining soil much reduced. While the pile is thus easily forced down, the material flows back and binds or sticks to the wood, increasing the frictional resistance enormously. In solids where, on the contrary, the water jet would merely make a hole which would not fill itself up again, the jetting would not be desirable.

42*i*. Advantages of Wood Piles.—Wood for permanent piles should be used only where it will always be under water, in which condition it will practically last forever, and if properly designed and driven, will afford an absolutely safe foundation. But as wooden piles should and do depend mainly on the frictional resistance of the ground, any withdrawal of the ground water will not only cause the wood to rot, but would also remove the greater part of its sustaining capacity.

One very important advantage wood has over steel or concrete for piles is "safety in numbers"—that is, as a wooden pile is supposed to carry only about 20 tons, which is the proper working limit, a number of piles are used for each support, so if one pile of the group is out of plumb, or broken, or bushed, the foundation will still be safe; whereas, if only two or three piles of the stronger materials are used, a defect in one or two of them would jeopardize the safety of the structure.

Wooden piles, at present at least, in most places, are cheaper than concrete or steel piles, although concrete is usually cheaper than the same volume of wood.

43. Concrete-pile Foundations.—Concrete piles may be divided roughly into two classes—"pre-cast" and "made in place"—and they may be reinforced or not, though pre-cast piles always should be and probably always are.

The advantages of concrete piles are their great strength and durability. They are practically free from danger of deterioration if wholly in the ground and cannot be attacked by the teredo or other borers.

If used in harbors and extended above the low water lines, the chief trouble is weathering from frost, chemical action, etc. The trouble from chemical reaction increases as the climate becomes warmer—that is, in tropical climates. Freezing is much more apt to destroy piles which have less cement than one part cement to two parts of sand, which proportion is required to ensure the voids of the sand being filled with cement.

One disadvantage of these piles is the practice of allowing very much greater loads on concrete than on wood, thereby reducing the number of piles used. For instance, a good structural steel designer knows that two rivets do not make an ideal joint for there always ought to be at least two bolts to hold the shapes together, while a rivet is being driven in the third hole. Similarly, the writer does not consider that two piles will ever be a good design for column footing, for in this case, if one pile is out of plumb (and it is hard matter indeed to drive piles plumb or to detect a deflection), then a very unsafe condition may exist without being even suspected; whereas, with a large number of piles in the unit, if a few were out of plumb and in different directions, they would simply act as batter piles and strengthen the foundation unless, as unfortunately sometimes occurs, they all assume the same batter in the same direction.

Another disadvantage of concrete and steel piles is that the smooth surfaces do not afford the same frictional resistances as wood, and more reliance is placed on their value as long or short columns, so they would have to be fairly long to obtain enough frictional resistance to develop the full strength of the reinforced concrete.

To act as columns, piles should have a fair bearing on the bottom, and as they are usually made flat instead of pointed, this means that if a pile is driven to hardpan or gravel and boulders, etc., it would very likely strike a boulder on one side. This might result in breaking off one or more corners of the pile, or in deflecting the pile itself, in which case, it might even break the pile, as has frequently happened with wooden piles. With only two or three piles under a column and one or two of them battered or resting partly on a boulder, the frictional resistance might be sufficient to hold the building until some adjoining excavation withdrew the water, thereby removing the adhesion of the soil to the pile with a resulting settlement of the building. These are not imaginary conditions but those that have been known to occur over and over again with wooden piles.

It might be noted here that boulders in New York hardpan are sometimes as much as 7 ft. thick so they could not be displaced by the driving of the pile or pipe.

43a. Pre-cast Piles.—Pre-cast piles are reinforced with steel rods and are of rich concrete and are then driven like wooden piles. The New York Building Laws stipulate that "the pile shall be not less than 8 in. at the bottom and not average less than 12 in. in thickness; shall not contain more than 4 % of steel reinforcement; that the length shall not exceed 20 times the average thickness, if driven to rock, nor 40 times if not driven to rock."

"When driven to rock, the allowable load shall not exceed 500 lb. per sq. in. of concrete per average cross section, and 6000 lb. per sq. in. on the steel longitudinal reinforcement. When not driven to rock, the carrying capacity is to be determined by test."

The New York Building Laws also require that if a pile is to be driven to rock, it shall have an iron shoe. If the iron shoe has a flat bottom 8 in. wide, then the probabilities are that only one point would bear on the rock, as bed rock cannot be assumed to be level. If, on the other hand, it has a pointed shoe, there would be danger of the shoe hitting a rock or boulder and deflecting the pile.

One of the advantages of a pre-cast concrete pile is that it can be made of uniformly varied cross section as required, while a wooden pile cannot often be found so.

In the navy supply warehouse in Brooklyn, which consists of vast reinforced concrete buildings resting on fine concrete piles, no borings were made to ascertain the nature of the subsoil before driving the piles, with the result that the buildings settled some 15 in., requiring the underpinning of the new reinforced concrete building.

43b. Piles Built in Place—*Raymond Pile.*—The Raymond pile is formed by driving a steel shell into the ground on a mandrel that can be collapsed and withdrawn. Then the hole is filled with concrete—reinforced, or not, as desired. The permanent steel shell used outside of the mandrel has the great advantage of preventing any sand from flowing in as the mandrel is withdrawn.

The Simplex Pile.—The Simplex pile is made by driving down a closed steel pipe and withdrawing it while concrete is forced out at the bottom.

Pedestal Piles.—Pedestal piles are supposed to have a spread footing obtained by driving the concrete out at the bottom of the shaft, at the same time compressing the surrounding soil.

Chenoweth Pile.—A Chenoweth pile is made by spreading mortar over a wire mesh and then rolling the wet mass into the shape of a pile which, after setting, is placed in an ordinary pile driver.

Breuchaud Pile.—The Breuchaud pile consists of driving an open steel pipe into the ground, washing out the sand or blowing it out by air pressure, and then filling the pipe with concrete. If the steel is always under water, it will never rust out and the pipe can be filled with good concrete almost to the bottom.

Compresol Pile.—A compresol pile is formed by making a hole in the ground with a pear shaped weight operated by a pile driver, and tamping concrete in the hole.

44. Sand-pile Foundations.—Sand piles are hardly to be recommended, as a more reliable foundation can nearly always be obtained. They simply consist of making holes in the ground by means of a wooden pile or some other method, and then ramming sand into the hole. The French probably originated this method and found it desirable before the days of good cheap Portland cement concrete.

45. Excavating.—When making excavations for foundations above the water line, the amount of bracing required will depend entirely on the judgment of the man in charge. The older or more experienced men are apt to use the heavier bracing.

In a rush job in Brooklyn, once the writer saw a contractor dig holes 5 or 6 ft. square, some 12 to 15 ft. deep, almost plumb sides, without any timbering or shoring of any kind; but while it was in good stiff ground (clay, sand, and boulders) it was taking a big risk for the slightest slide would have killed the men in the bottom of the shaft.

In a few cases, it might pay to excavate to depths of, say 5 or 6 ft., by sloping the sides and then back filling instead of timbering. As a rule, however, if the ground is at all soft, it will pay to timber the sides.

45*a*. Wooden Sheet-piling.—The old method was to set 1 or 2-in. planks, and as the men excavated, to drive these planks into the ground, holding them in place with rectangular bracing. These planks were usually 6 to 8 ft. long, and when they had been driven, a fresh set was started inside (about 6 or 8 in., according to the size of the bracing timbers) and so on down, the hole not only getting smaller and smaller as each tier of plank was driven, but also very often being forced out of line. This was generally a haphazard method and often it was not known how far the excavation was to be carried when it started.

Nowadays, the best practice is to ascertain, by borings, etc., just how far the sheeting is to be driven and then driving it in one length, properly braced. The thickness of this sheeting will depend entirely on the nature of the ground and the depth required. For holes up to 10 ft., from 2 to 3-in. plank will usually be sufficient; with from 6 to 8-in. plank, up to about 20 ft.

In the Harlem River tunnel, three 12 × 12-in. timbers were bolted together with a tongue on one of the outside timbers made of a 3 × 4-in. timber and a corresponding groove on the other outside 12 × 12 made of two 3 × 4-in. timbers; each pile being 12 × 36 in. by about 40 ft. long. On account of the bolting, the pile driver was able to force 3 ft. of horizontal sheet piling down at a time. These were driven about 40 ft. under the water and, after the roof of the tunnel had been sunk on two lines of this sheeting, compressed air was used to enable the excavation to be completed. This piling is known as the Wakefield sheet-piling and is nothing more than a built-up tongue and groove sheeting. The original Wakefield sheeting consisted of bolting three planks together in such a way that the center plank formed a tongue at one side and the other two a groove.

In some cases, 12 × 12-in. sheeting driven for a 30-ft. excavation, and heavily braced every 8 ft. horizontally and from 3 ft. (at the bottom) to 5 ft. (at the top) vertically, have been badly distorted, sometimes being shoved in 2 or 3 ft., the bracing timbers cutting into each other.

Generally, where the worst damage occurs, the excavated material is more or less plastic and is dumped right outside of the cofferdam. Every bucket of soft material dumped seems to act like a hydraulic ram with accumulative action, until no amount of bracing will stand the strain. It always pays to have a reasonable excess strength in the sheeting and bracing, and to avoid dumping too much of the excavated material outside of the cofferdam.

45*b*. Steel Sheet-piling.—In recent years, many different kinds of interlocking steel sheet-piling have been used successfully. This kind of sheeting was first tried out in

Chicago by Friestedt, Jackson, and others. It works to its best advantage in soft material, clay, sand, etc., where it can be assisted by the water jet, if necessary.

Steel sheeting is not adapted to hard ground containing boulders, etc., unless the excavation can precede the driving. In Brooklyn, some very heavy steel sheeting was driven for a dry dock and after a failure, was abandoned and the work completed by pneumatic caissons. The steel sheet-piling, when removed by the caisson work, was found to have been twisted and rolled up until it would have been hard to guess as to the original shape.

Sometimes, sheet-piling is driven in double lines as much as 25 ft. apart, and the space between filled with sand, clay, etc., to make a water-tight cofferdam. In this case, the piling is driven in a series of half circles tied together, giving a strength that could never be obtained by parallel lines. This plan was adopted by General Black for raising the Maine; then used by his son for the dam in the Hudson River near Troy. It was also used for the big docks in New York City at 46th St. and Harlem River. These cases have been illustrated in the *Eng. News*.

45*c*. Poling Board Method.—In Chicago many shafts have been sunk by the vertical poling board method—that is, inserting the lining, timber or steel, as the shaft is excavated. This is like constructing a tunnel vertically, and has been carried as deep as 100 ft.

45*d*. Cofferdams.—Cofferdams are generally constructed by driving steel or wooden sheeting in advance of the excavation, or simultaneously with it, and inserting sufficient bracing to keep the sheeting in place. The amount of this bracing is often seriously under-estimated, with the result that the sides are bulged in from 2 to 5 ft., and much trouble follows. Open cofferdams are rarely used where the water is over 30 ft. deep, as pneumatic caissons would generally be more economical.

A common construction is to have double walls and pack mixtures of clay, gravel, etc., between the walls. But when a leak starts under these walls it is very hard to stop. Where the current is not too strong, much earth has been dumped outside of the cofferdams in an endeavor to stop the flow of water.

Open cofferdams were tried in 19 ft. of water where there was practically no earth or mud on top of the rock, but were abandoned for pneumatic caissons which proved to be cheaper and quicker. In other places where the cofferdams could not be made water-tight, 5 ft. of concrete was dumped under water, and after the concrete had set for a couple of weeks, the cofferdams were pumped out, and the rest of the work was done in the dry. Unfortunately, in many cases such concrete seems to set hard except around the edges, where it is really needed, and the cofferdams still leak.

45*e*. Pneumatic Caissons.—Caisson comes from the French word "caisse," a box, and in foundation work a pneumatic caisson has four sides (or it may be circular) and a roof, but no bottom. The roof has one or more holes for shafts, usually about 3 ft. in diameter, for the passage of men or material from the outer air into the working chamber. An air lock prevents the air pressure in the working chamber from being seriously reduced while men or material are passing in or out.

The air pressure in the working chamber is kept just high enough to balance the water pressure. If the air pressure is too high, it blows out and allows the water, sand, etc., to rush in, while if the air pressure is too low, the water rushes in, drowns the men, and probably fills the working chamber with mud, etc. A cubic foot of water weighs about 62.5 lb., giving a pressure on its base of 0.434 lb. per sq.in. If the water is 10 ft. deep, the air pressure required will be 4.34 lb. per sq. in. If 100 ft. deep, it will be 43.3 lb. per sq. in., which is nearly the limit of human endurance.

For the Municipal Building of New York City, the maximum pressure actually worked in was 49 to 50 lb., at a depth of 112 ft. French experiments have raised the pressure in a specially constructed glass cage to 75 lb. per sq. in., keeping the men who did no work, under close personal observation.

The first very large caissons built in this country were of massive wooden construction having wooden decks 10 ft. thick. Subsequent designers even used oak decks (roofs of caisson) 10 or 12 ft. or more in thickness. Later wooden caissons have been built with decks 3 ft. thick and finally only 1 ft. Complete designs for the wooden caisson used for the extension of the Manhattan Life Building were given in the Trans. Can. Soc. C. E. vol. XXIII, 1909, pp. 320–341.

The first high building to be founded on pneumatic caissons was the Manhattan Life Building, New York City, 1893. The caissons were built of steel plates and shapes of a massive construction about 9 ft. high (published in the *Eng. Rec.*). The deck was 7 ft. high and carried the brick piers which were built around the working shafts as the caisson sunk. It was found, however, that the friction of the earth on the sides of this brick masonry was so great that the joints were forced open, so the next advance was to build cofferdams of steel from the caissons up, and to fill the space with concrete.

Steel caissons, round and rectangular, have been much used, one of the principal buildings being the Mutual Life, described in the *Eng. News*, pp. 221–227, March 28, 1901. The great cost of the steel work has nearly eliminated steel caissons, sending designers first back to the wood, then to reinforced concrete, and sometimes back to wood again.

As concrete cost less than wood, many caissons have been built without any wood in the permanent construction, using steel rods for reinforcing. At first it was thought that the concrete would not hold air, but on the contrary it has been found that the concrete does hold air much better than the wooden caissons and does not require the expensive caulking of joints nor is a concrete caisson subject to fire. A fire in a wooden caisson, many feet under water, was always one of the hardest things to extinguish, the compressed air simply feeding it. Even flooding the working chamber with water sometimes failed to extinguish the fire.

When reinforced concrete caissons can be built from the cutting edge to the top (up to 35 ft. in height so far) before sinking commences, they are the most economical; but if the work has to be done by successive "build ups" where the first section is built, pig iron or other weights added for sinking, then the sinking stopped while the pig iron is removed, a second section of concrete added to the first, requiring more pig iron for sinking, and this operation repeated several times, it will be found that the omission of all wood would be very expensive. A very much cheaper and quicker job could be obtained by having a light cofferdam of say 2-in. planks from the caisson up, so that the penetration of the caisson would not have to stop after once starting until a firm bottom is reached.

The cofferdam method, therefore, saves rehandling much material; saves pumping compressed air while building up the different sections; and requires much less weight in pig iron or cast-iron blocks to overcome the friction caused by the material settling around and binding the caisson during the long waits, which waits have amounted to from 2 to 60 days.

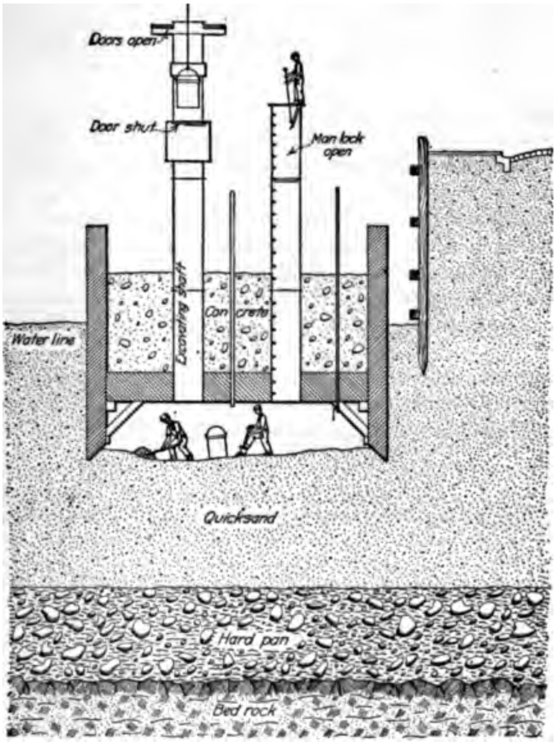

FIG. 33.—Sinking pneumatic caisson.

Designs.—The design of a pneumatic caisson is almost entirely a matter of experience and good judgment, for while theoretically, when a caisson is being sunk, the air pressure in the working chamber is high enough to balance the water pressure on the outside—which leads some to think that there is practically no pressure on the chamber walls—it is known that the air pressure is frequently lowered to normal, purposely or accidentally, in which event the water pressure from the full head would tend to collapse the caisson before the water flows into the working chamber.

This is a condition that is sure to occur, and if the caisson is truly vertical, which it almost never is, and in uniform material, such as sand, the maximum stress might be obtained; but it is known from experience that it is very far from being the maximum. It is a common occurrence for boulders, hard masses of clay, etc., to be encountered on one side of the caisson or the other with the result that the caisson is thrown out of plumb, the effect being like the "hogging of a ship." In one case at least this was sufficient to break the walls of the working chamber away from the deck when the cutting edge was still 20 ft. above hardpan. It was then found necessary to continue the excavation like a vertical shaft, putting in timber lining all the way down and leaving the cutting edge where it was. The steel caissons of the Commercial Cable Building, 1896–7, had ½-in. steel side plates with heavy angle-iron supports every 3½ ft. in the walls of the working chambers. These plates buckled inward about 2 to 3 in.

In caissons of from 20 to 30 ft. horizontal lengths, it is good practice to put in two cross struts about a foot or so above the cutting edge. For caissons up to 10 ft. in width, these struts should be the equivalent of a 12 × 12 timber with a 1-in. square or round steel tie rod. In wide caissons, these struts have been made to act as trusses with the roof or deck. While it is of the utmost importance to prevent a possible collapse of the side walls, it must also be remembered that every strut put in the working chamber greatly adds to the cost of the excavation, interfering with the handling of the bucket, making digging more difficult, and frequently making it necessary to shovel the material twice or more to put it into the bucket.

Cutting Edges.—More money has been wasted on elaborate cutting edges than on any other part of the caisson. Theoretically, the cutting edge should be a knife edge, penetrating the material easily and permitting the pick and shovel to get directly up to the outside of the cutting edge. This effort has resulted in many cutting edges being designed of steel plates (vertical) stiffened by angles, etc. The only place that such a cutting edge will work is in soft ground where it is hardly needed, and when it is really needed, that is, in hard ground where the pick or crowbar is used, it will not answer because the weight of the caisson above is sure to buckle it so badly that it will have to be removed.

These plate and angle cutting edges are not only useless but also very expensive, and it is better to use a 6 or 8-in. channel iron laid flat with the flanges turned up. This works well for either wood or concrete caissons.

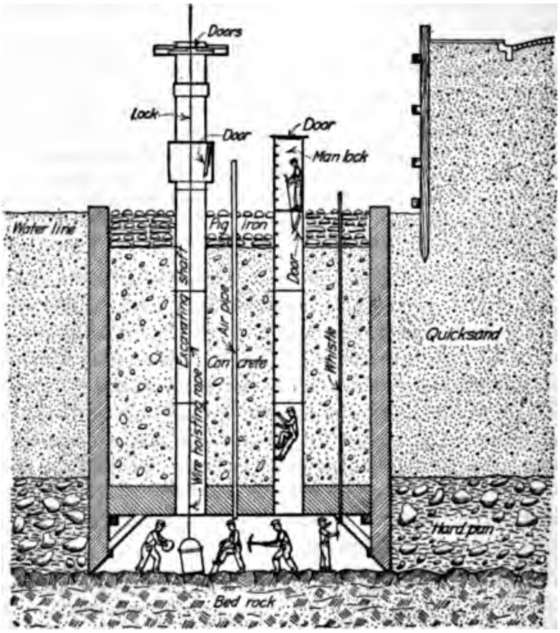

FIG. 34.—Pneumatic caisson sunk to bed rock.

A 6-in. angle iron with one leg horizontal and the other leg vertical and above the horizontal leg, the horizontal leg being firmly attached to the wood or concrete above by 3/4-in. round bolts every 3 ft., also makes a good cutting edge.

In most places a 6 or 8-in. oak or pine timber will be perfectly satisfactory though the steel angle or channel works out a little better with concrete caissons.

The four corners of the cutting edges should be strongly braced to avoid danger of the caisson being twisted out of its rectangular shape.

Many caissons—especially when of wood or steel—have their surfaces badly warped, which makes the sinking much more difficult, increasing enormously the frictional resistance to be overcome.

Steel Caissons.—For rectangular steel caissons, ⅜-in. side plates should be used with stiffener brackets made up of 4 angles 3 × 3½ × ⅜ in., the vertical pair being riveted to the side plates and the other inclined pair resting on a 6 × 6 × ¾-in. shelf angle which is riveted to the side plates all around, the horizontal flange of the 6 × 6 × ¾-in. angle being 12 in. above the cutting edge, the vertical leg of this angle being below the horizontal leg. The top of the inclined angles of the brackets are riveted to the deck about 2 ft. or more from the side walls. These brackets should be spaced about 4 to 5 ft. centers depending on the depth to be sunk, material, etc.

For the circular steel caisson, the shell should be from ¼ to ⅜ in. thick, unless the depth is very great and in bad soil. These caissons should also have a bottom shelf angle from 3½ ×

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As concrete cost less than wood, many caissons have been bu
tion, using steel rods for reinforcing. At first it was thought that

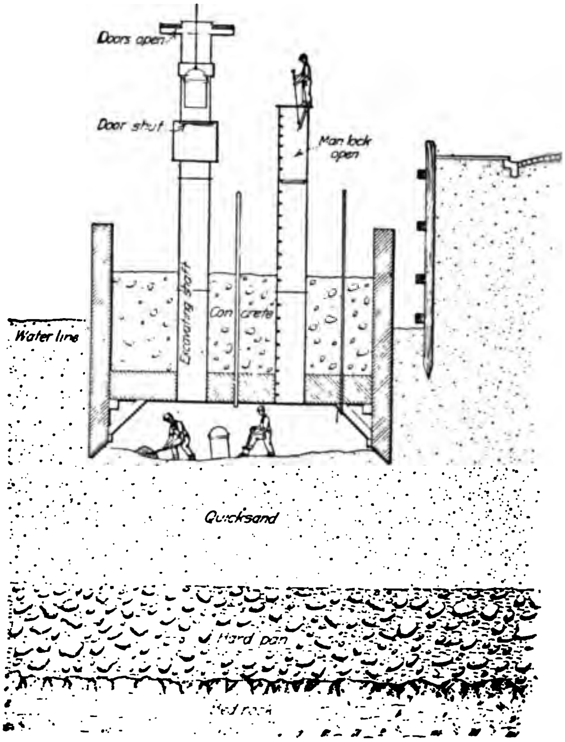

FIG. 33.—Sinking pneumatic caisson.

Designs.—The design of a pneumatic caisson is
good judgment, for while theoretically, when a cai
working chamber is high enough to balance the wate
to think that there is practically no pressure on th
pressure is frequently lowered to normal, purposely
pressure from the full head would tend to collapse
working chamber.

This is a condition that is sure to occur, and if
never is, and in uniform material, such as sand, the
known from experience that it is very far from being
for boulders, hard masses of clay, etc., to be encoun
with the result that the caisson is thrown out of pl
ship." In one case at least this was sufficient to b
from the deck when the cutting edge was still 20 ft
sary to continue the excavation like a vertical shaf

meter of the caisson. No brackets are
er. but a 3½ × 3½ × ⅜-in. ring angle
the bottom shelf angle and the deck.
he bottom of the side plate all around.

plates. All rivet heads on the outside of the
from the inside against air pressure, and from
get properly done.
I-beams to support the weight of the concrete
the working chamber as in the case of a con-

of the caisson and especially whether or not the
the caisson. For caissons in cities, the concrete
in the shape of iron blocks or pig iron are re-
hand, are often so heavy in comparison with
the deck in the cofferdam is often 20 or 30 ft.
must have very ample bracing.
working chamber of small caissons and makes
often be cheaper to use larger caissons of either

ft. above the cutting edge.

made of vertical tongued-and-grooved
nsion for the Manhattan Life Building
caissons—that is, of over 15 ft. in width
a solid wall of 12 × 12-in. timbers, laid
de of the horizontal 12 × 12-in. timbers,
placed vertically to reduce the frictional
y extend some 14 ft. above the cutting
2-in. posts decreases, until near the top,
pport the waling pieces for the cofferdam
uld also be placed vertically, with the

timber 10 to 12 ft. thick, thoroughly bolted, and
idth with a deck of 3 ft. thick, the top and bot-
g longitudinally. Under the deck a 2-in. plank
al trusses have been used about 20 ft. apart.
lace 12×12-in. knee braces every 5 ft. from the

om the inside against air pressure and from the
mon material for this purpose.

est material for caisson construction.
wood or steel as the occasion requires.
material is used. Beginners generally
ed so that the bottom horizontal area
n the side walls will be reduced. Ex-
material rolling in against the caisson,
tons extra pig iron to break the friction.
dged through hard clay, the opposite
ion, and the caisson wabbled so much
ween the cylinder and the clay was back
process of sinking was resumed.

nd more than ½ or ¾ in. beyond the sides of
ing edges, as already noted.
the inside from the cutting edge to the roof of
izontal distance can be considerably increased.
to the working shaft, for facility in filling the

concrete to prevent (1) the side walls of the from buckling in and (2) the friction on the sides of the caissons from opening cracks in the con- rods should be about ⅝ or ¾ in. square and 4 in. center to center for a distance of 10 or 12 ft. above and 12 in. center to center above that height. Similar rods should run from the cutting edge, on the working chamber wall, up to the deck and extend several feet above the deck into the concrete. also be horizontal reinforcing rods about the same size and distance apart as the vertical rods. of rods required in the deck would of course depend on the span, etc., but in most cases ¾-in. square ter to center in each direction would be more than ample.

—Small caissons have only one shaft which is used for both men and material. The ons have at least two, one for material and one for men, and sometimes as many as . The cost of the shaft is balanced against the extra cost of handling the material king chamber if fewer shafts are used.

y the shafts were made of steel but now steel shafts are used only at the top and timber or metal forms are used to make the shafts in the concrete. The concrete shafts should have a recess about 6 in. ft. wide in which round rods are inserted to form a ladder. ll caissons it is very necessary to have vertical and horizontal reinforcing rods around these shafts to concrete between the shaft and the outside of the caisson from opening up serious cracks.

ng the Caisson.—When the caisson has reached its final resting place either on rock, or in a few places on sand or clay, it is necessary to fill the working chamber with . The old method was to deposit the concrete by hand until it was about 4 ft. below and then by means of timber forms to bench the concrete all around until only a space under the shaft was left and also a space of 3 or 4 in. under the deck. This space ked with very dry mortar and rammed into place using a hammer on a small plank. thod was expensive and never satisfactory, one trouble being that the benching required ncrete which is exceptionally undesirable in compressed air work and another trouble difficulty of getting the tedious work of ramming properly done.

writer some 10 yr. ago abandoned the old method for the following which he has used ever since: The work- bers are filled with wet concrete to within 1 ft. or better, 2 ft., from the deck, under air pressure of course, the compressed air is kept on for 48 hr., after which the air is taken off and the rest of the space under the the shafts themselves is rapidly filled with wet concrete dumped from the top of the shaft. It is very im- have the concrete under the deck mixed very wet. always necessary to have vent pipes as far from the shaft as possible so that no air can be trapped under to cause voids in the concrete. When the work is properly done the grout will be found to have been forced vent pipes from the working chamber to from 15 to 25 ft. above the deck. As the working chamber is d it is very necessary to reduce the air pressure gradually. Neglect to do this has resulted in much con- g blown out under the cutting edge.

ater-tight Cellars.—A number of buildings have been constructed in New York with from floors below the water level. These are made water-tight by sinking pneumatic caissons the lot, the caissons having a width of from 5 to 8 ft. and lengths up to 30 or 40 ft. and sealing the joints between the caissons.

ne method is to use a compressed air shaft some 3 ft. in diameter which is a more or less lt matter. A better method as far as economy, safety, and good results are concerned, ink the caissons about 6 in. apart, holding the distance by having two 6 × 8-in. timber tors, preferably of oak, attached from the cutting edge to top of the first caisson sunk. pace between these separators, about 2 ft., is stock-rammed. This is accomplished by a heavy 4-in. pipe down to the level of the cutting edge; then pellets of clay are d into the pipe, and the clay is forced out at the bottom by an iron piston rod, g enough to work easily inside of the pipe, the piston being operated by a pile driver. driving becomes harder, the pipe is raised a foot or so, and the operation is continued e entire pipe has been removed, section by section, and the space well packed with clay. y has been thus rammed so hard that it resembles shoe leather. Care is required to t the ramming is not overdone as the accumulative effect is very great—enough to he caisson bodily out of place. This has successfully held the water back for depths . and permitted the placing of concrete or brick work in the joints after the cellar has g. For further details, see the writer's article in Railroad Age Gazette, Aug. 7–14, 1908.

45f. Open Caissons.—Open caissons are constructed on the surface like pneuma- sons and sunk into position where they are held down by weights.

45g. Dredged Wells.—Where the depths are too great for pneumatic work, dredged wells are often used. These sometimes consist of double steel cylinders with concrete filling the space between the inner and outer cylinder. Ordinary clam shell or orange peel buckets are used for dredging the material through the inner cylinders. Reinforced concrete is often used, having steel forms for temporary purposes only.

The Phoenix Construction Company used a number of these for the Erie R. R. at Penhorn Creek and elsewhere. These were 6 ft. outside diameter, and 3 ft. 6 in. inside diameter, and were sunk through 90 ft. of sand, gravel, etc.

FOOTINGS

By W. Stuart Tait

46. Temporary Wood Footings.—Where temporary wood footings are installed, the construction usually consists of a sill or longitudinal timber *A* (Fig. 35), transverse timbers *B*, and sometimes a layer of longitudinal boards under the transverse timbers. The transverse timbers are usually laid close together. The boards *C* are desirable in soft ground to prevent unequal settlement of the timbers *B*. In temporary work the following stresses may be used: extreme fiber stress, 1600 lb. per sq. in.; bearing across the grain, 500 lb. per sq. in.

Illustrative Problem.—Load on *A* (Fig. 35) is 10,000 lb. per lin. ft. and the soil pressure is 2000 lb. per sq. ft. Design a temporary wood footing.

$$a = \frac{10{,}000}{2000} = 5 \text{ ft.}$$

Assume *A* to be a 12 × 12-in. timber. Then

M in B = (2000) (2) (12) = 48,000 in.-lb. per lin. ft.

$M = fS = 1600S$

$$S = \frac{48{,}000}{1600} = 30 = \frac{bd^2}{6} = \frac{12d^2}{6} = 2d^2$$

$d^2 = 15$, or $d = 3.88$ in. Use 4-in. timber for B

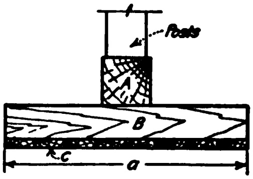

Fig. 35.

47. Plain Concrete Footings.

47a. Light Wall Footings.—Under walls carrying no great load the minimum projection of the footing beyond the face of the wall should be equal to one-half the wall thickness. The depth of the footing should be twice the projection (Fig. 36). The weight per square foot occurring at the bottom of the footing should be checked to make sure that it does not exceed the allowable bearing pressure on the soil (see Art. 29).

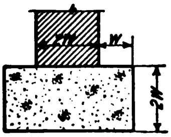

Fig. 36.

47b. Heavy Wall Footings.—Under walls carrying a considerable load, such as a party bearing wall in a six-story warehouse, a footing similar to that for light walls may be used. Assume the basement wall to be of concrete 20 in. thick. Soil pressure = 4000 lb. per sq. ft.

Load per lin. ft. of wall at top of footing	= 24,000 lb.
Weight of footing	= 4,000 lb.
Total load	= 28,000 lb.

$$\text{Width of footing} = \frac{28{,}000}{4000} = 7 \text{ ft.}$$

The footing is stepped down as shown in Fig. 37, the depth of any step being twice its projection.

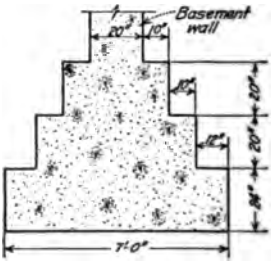

Fig. 37.

47c. Piers.—This type of footing usually occurs in mill buildings. Refer to Fig. 38.

Lowest story column load	= 350,000 lb.
Weight of footing	= 90,000 lb.
Total load	= 440,000 lb.

Soil load = 3500 lb. per sq. ft.

$$\text{Area of cast base} = \frac{350{,}000}{700} = 500 \text{ sq. in.} = 22.4 \text{ in. square. Use 24-in. square.}$$

$$\text{Area of footing} = \frac{440{,}000}{3500} = 126 \text{ sq. ft. Use 11 ft. 4 in.} \times \text{11 ft. 4 in.}$$

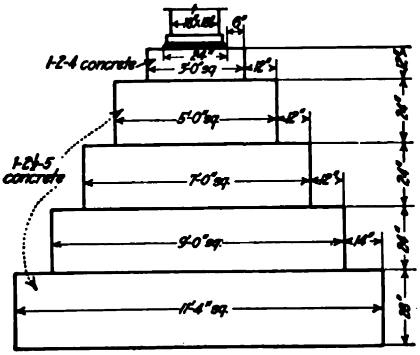

FIG. 38.

48. Stone and Brick Footings.—Masonry, whether of stone or brick, is not suitable for footings except for very light loads. Even for such loads, it is recommended that concrete be used.

49. Reinforced Concrete Pier Footings.

49a. Theory of Design.—The method of design given in bul. No. 67 of the University of Illinois, is generally accepted and used in the design of pier footings of reinforced concrete. The concrete is usually a 1-2-4 mix. Assuming gravel concrete, the following stresses are recommended by the Joint Committee (see *Appendix A* and *J*).

f_s = 16,000 lb. per sq. in.
f_c = 650 lb. per sq. in.
Bond = 80 lb. per sq. in. for plain bars.
Bond = 100 lb. per sq. in. for deformed bars.
Shear = 40 lb. per sq. in. as a measure of diagonal tension.
Punching shear = 120 lb. per sq. in.
$n = 15$

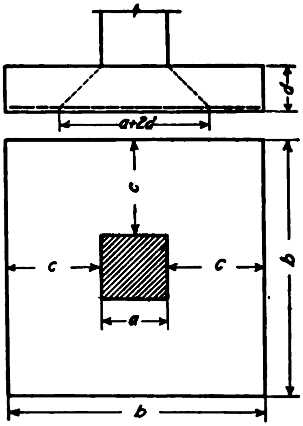

FIG. 39.

Bending moment in one direction (see Fig. 39) is

$$M = (\tfrac{1}{2}ac^2 + 0.6c^3)w$$

where w is the upward unbalanced pressure in pounds per square foot.

Steel effective is that within width $= a + 2d + \tfrac{1}{2}(b - a - 2d)$

Load producing punching shear $= (b^2 - a^2)w$

$$\text{Unit punching shear} = \frac{(b^2 - a^2)w}{4ad}$$

Loading producing diagonal tension in one direction

$$V = \tfrac{1}{4}\left[b^2 - (a + 2d)^2\right] w \text{ and } v = \frac{V}{4(a + 2d)jd}$$

Bond stress

$$u = \frac{V}{mojd} = \frac{(ac + c^2)w}{mojd}$$

where m = number of bars and o the periphery of one bar.

The chapter on the design of "Reinforced Concrete Beams and Slabs" in Sect. 2 should be referred to for explanation of the above formulas.

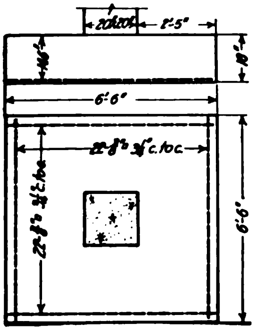

FIG. 40.

49b. Single Slab Footings.—This type of footing is not used for large loads as it is not economical in concrete.

Illustrative Problem.—Soil pressure, 4000 lb. per sq. ft. Column size, 20 × 20 in. (see Fig. 40).

Column load	= 150,000 lb.
Footing weight	= 10,000 lb.
Total load	= 160,000 lb.

$$\text{Area} = \frac{160{,}000}{4000} = 40 \text{ sq. ft., say 6 ft. 6 in. square.}$$

Now the weight of the footing balances a certain amount of upward soil pressure. In this case the upward unbalanced pressure

$$w = \frac{150{,}000}{(6.5)(6.5)} = 3560 \text{ lb. per sq. ft.}$$

The depth required for punching shear

$$d = \frac{(b^2 - a^2)w}{(4a)(120)} = \frac{(39.5)(3560)}{(4)(20)(120)} = 14.6 \text{ in.}$$

Now shear as a measure of diagonal tension produced in this case, using $d = 14.6$ in.,

$$v = \frac{[b^2 - (a + 2d)^2]w}{4(a + 2d)jd} = \frac{(25.45)(3560)}{(4)(49.2)(0.87)(14.6)} = 36.2 \text{ lb.}$$

Thus the depth required for punching shear satisfies the requirement for diagonal tension. Had this not been the case, d would have to be increased, as it is not good practice to use stirrups in footings on account of the probability that they would be improperly placed or omitted.

$$M = (\tfrac{1}{2}ac^2 + 0.6c^2)w$$
$$= [(\tfrac{1}{2})(1.67)(2.42)^2 + (0.6)(2.42)^2](3560)(12) = 572{,}000 \text{ in.-lb.}$$
$$A_s = 2.83 \text{ sq. in.} = 20 - \tfrac{3}{8}\text{-in. square bars.}$$
$$p = \frac{2.83}{(78)(14.6)} = 0.0025, \text{ which is satisfactory.}$$
$$u = \frac{(ac + c^2)w}{mojd} = \frac{(9.88)(3560)}{(20)(1.5)(0.87)(14.6)} = 92 \text{ lb.}$$

Thus deformed bars must be used. The 20 – ⅜-in. square bars must be placed in a width of

$$a + 2d + \tfrac{1}{2}(b - a - 2d) = 63.6 \text{ in.}$$

and have a spacing of

$$\frac{63.6}{19} = 3.35 \text{ in., say } 3\tfrac{1}{4} \text{ in. on centers.}$$

The 20 bars will be placed equally on each side of the center at 3¼ in. on centers. The outer bar will be 8 in. from the edge of the footing so one bar will be added on each side making a total of 22 bars each way.

Total depth of footing = 14.6 + 0.375 + 3.0 = 17.975 in., say 18 in.

49c. Multiple Slab Footings.—This is a type of footing very generally used for large column loads. In structures where the column loads are fairly large, some provision should be made in the design to allow for a greater percentage of dead load on an exterior than on an interior column. If the ground at the bottom of the footing is hardpan, hard shale, or solid gravel, this provision is not essential, but where a certain amount of settlement is probable, it is a necessity.

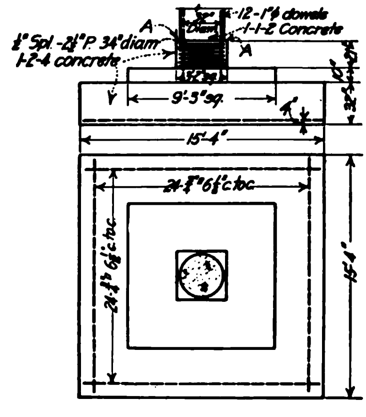

FIG. 41.

It is good practice to design the columns for the full dead load and a proportion of the live load depending upon the number of stories in the structure. In Chicago, the basement story columns in a 6-story and basement building would be designed for the full dead load, the roof load and 72½% of the live load for which the floors were designed. The footings are designed for the basement story column load. Some designers proportion the footings on the basis of the dead load only. The writer always recommends using the full dead load and one-half of the live load used in the design of the basement story columns. (For loads coming on columns, see Sect. 1, Art. 86.) The following example is worked out on this basis:

Interior column: Size, 32 in. diameter; 1-1-2 concrete; 1% spiral. 11—1-in. round bars.
  Dead load = 297,000 lb.
  Live load = 423,000 lb.

Exterior column: Size, 30 × 30 in.; 1-2-4 concrete; 1% spiral. 10—⅞-in. round bars.
  Dead load = 280,000 lb.
  Live load = 196,000 lb.

Maximum soil pressure = 3500 lb. per sq. ft.

Allowing 15% of column load for weight of footing, area of interior footing $= \frac{828{,}000}{3500} = 236$ sq. ft. = 15 ft. 4 in. square (see Fig. 41). Now using half of the live load and all the dead load, we have a pressure of $\frac{617{,}000}{236} = 2620$ lb. per sq. ft. The area required for the exterior column would then be $\frac{(547{,}000 - 98{,}000)}{2620} =$ 172 sq. ft., or say 13 ft. 0 in. square (see Fig. 42).

Following through the above, it will be noted that the area of the interior column footing, which is the one having the highest percentage of live load, was first obtained using the soil pressure allowed. A new soil pressure is then obtained by using all the dead load and one-half of the live load. All other footings are then proportioned by using this reduced soil pressure and applying it upon the full dead load and one-half of the live load.

Having determined the footing area, the design will be carried out in the usual way using the total column load occurring at the top of the foundation. In case the live load for which the floors are designed exceeds 400 lb. per sq. ft., it would be well to take ¼ of the live load instead of one-half. The reason for this is that the settlement, if any, will probably occur during construction and not after the building is fully loaded.

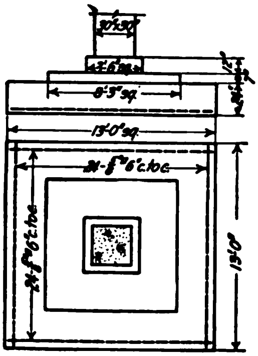

Fig. 42.

Interior Footing:

Total column load = 720,000 lb.

Area of footing = 235 sq. ft. $w = \frac{720,000}{235} = 3060$ lb.

The column diameter is 32 in., which is equivalent to 28 in. square. Consequently in the formulas used, a (see Fig. 39) will be taken as 28 in. except for punching shear.

The depth for punching at edge of column

$$d_1 = \frac{(b^2 - a^2)w}{(\pi)(32)(120} = \frac{(229.5)(3060)}{(\pi)(32)(120)} = 58 \text{ in.}$$

The cap or top layer is of 1–2–4 concrete, while the column is 1–1–2. The cap design must be such that the bearing on A–A is within the limit allowed on 1–2–4 concrete, *i.e.*, 700 lb. per sq. in. This may be taken on the full section of the column and the resistance of the dowels added. Then

Concrete...804 × 700 = 562,800 lb.
Steel.......................................11 × 0.785 × 700 × (15 − 1) = 84,500 lb.

657,300 lb.

We must therefore either add a spiral in the cap, increase the steel, or use 1–1½–3 concrete. The best thing to do is to add a spiral. Using 1% spiral and 1% vertical, we have an average stress of 793 lb. per sq. in. The spiral will be 34-in. diameter, giving a core of 908 sq. in. with 12—1-in. round bars. The spiral will be ½-in. wire at 2¼-in. pitch. The cap will be 34 + 4 = 38 × 38 in. square. The lowest member of the footing is 15 ft. 4 in. square. The middle member will be made the mean of the cap and base, or 9 ft. 3 in.

For punching shear at edge of cap

$$d_2 = \frac{(235 - 10)(3060)}{(4)(38)(120)} = 37\tfrac{1}{2} \text{ in.}$$

For punching shear at edge of middle member

$$d_3 = \frac{(235 - 85)(3060)}{(4)(111)(120)} = 8.7 \text{ in.}$$

Diagonal tension below cap

$a = 3$ ft. 2 in. $c = 6$ ft. 1 in. $d = 3$ ft. 1½ in.

$a + 2d = 9$ ft. 5 in.

The middle member is 9 ft. 3 in. in width, which is less than $a + 2d$, so we will take $a + 2d = 111$ in.

$$v = \frac{[b^2 - (a + 2d)^2]w}{4(a + 2d)jd} = \frac{(148)(3060)}{(4)(113)(0.87)(d_3)} = 40 \text{ lb.} \qquad d_3 = 28 \text{ in.}$$

Now we have $d_1 = 58$ in. and $d_2 = 37.5$ in., and we will make $d_3 = 28$ in.

Moment about A–A = $(\frac{1}{2}ac^2 + 0.6c^3)w = (193)(3060)(12) = 7,080,000$ in.-lb. $d = 37.5$ in., so $A_s =$ 13.6 sq. in. 24—¾-in. square bars.

$$p = \frac{13.6}{(111)(37.5)} = 0.0033, \text{ which is satisfactory.}$$

These bars are placed in a width $= a + 2d + \frac{1}{2}(b - a - 2d) = 12$ ft. 4½ in. Use 24—¾-in. square bars, 6½ in. on centers each way.

$$u = \frac{(ac + c^2)w}{mojd} = \frac{(56)(3060)}{(24)(3)(0.87)(37.5)} = 72.5 \text{ lb.}$$

The depth of cap = 58 − 37.5 = 20½ in., say 21 in. The depth of middle member = 37.5 − 28 = 9½ in., say 10 in. The depth of bottom member = 28 + 4 = 32 in.

Exterior Footing:

Column load = 476,000 lb.

Area of footing = 169 sq. ft. $w = 2820$ lb.

The design will be carried out the same as above and we obtain the design shown in Fig. 42.

49*d*. Sloped Footings.—This type of footing is favored by some designers. It requires less concrete than the slab type, but the sloping sides are more difficult to shape.

Illustrative Problem.—Column load, 600,000 lb. Column size, 30 × 30 in. Soil pressure = 6000 lb. Allow 70,000 lb. for the dead weight of footing, making a total load of 670,000 lb. on the soil.

Footing area $= \frac{670,000}{6000} = 112$ sq. ft. = 10 ft. 6 in. square (see Fig. 43).

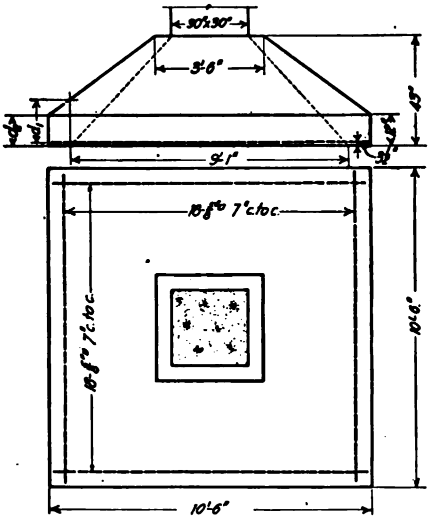

FIG. 43.

$$w = \frac{600,000}{110} = 5460 \text{ lb.}$$

$$\frac{600,000}{(30)(30)} = 666 \text{ lb.}$$

Make area of top = (2)(30)(30) = 1800 sq. in. = 3 ft. 6 in. square

$$\text{Depth for punching} = \frac{(110 - 6)(5460)}{(4)(30)(120)} = 39.5 \text{ in.}$$

Diagonal tension: With $d = 39.5$ in. at column

$$v = \frac{[b^2 - (a + 2d)^2]w}{4(a + 2d)jd}$$

$$d_1 = \frac{(110 - 82)(5460)}{(4)(109)(0.87)(40)} = 10 \text{ in. required.}$$

$d_2 = 6.5$ in. Use 12 in.

$$M = (\tfrac{1}{2}ac^2 + 0.6c^3)w = 3,830,000 \text{ in.-lb.}$$

$d = 39.5$ in. $A_s = 7.00$ sq. in. = 18—⅝-in. square bars

$$p = \frac{7.00}{(42)(39.5)} = 0.0042, \text{ which is satisfactory.}$$

$$u = \frac{(10 + 16)(5460)}{(18)(2.5)(0.87)(39.5)} = 92 \text{ lb. per sq. in.}$$

The 18—⅝-in. square bars must be placed in a width $= a + 2d + \tfrac{1}{2}(b - a - 2d) = 42 + 79 + \tfrac{1}{2}[126 - (42 + 79)] = 123$ in.

Spacing of bars $= \frac{123.5}{17} = 7\tfrac{1}{4}$ in. on centers. Use 7 in. on centers and then outside rods will be 3½ in. from outside edge of footing.

Total depth of footing = 39.5 + 0.625 + 3.0 = 43 in.

49e. Rectangular Footings.—Since a footing is a very stiff rigid member, no appreciable deflection will take place at the edges, and uniform pressure will prevail throughout the foundation. In footings in which the length does not exceed the breadth by more than 50%, the design will be carried out in the same manner as in a square footing. Thus, referring to Figs. 44 and 45,

$$M_{(1-1)} = (\tfrac{1}{2}ac^2 + 0.6c^2b)w$$
$$\text{and } M_{(2-2)} = (\tfrac{1}{2}ab^2 + 0.6cb^2)w$$

The shearing values, etc. can be followed through similarly.

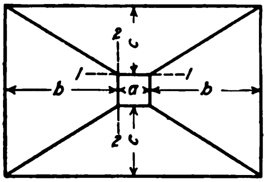

FIG. 44.

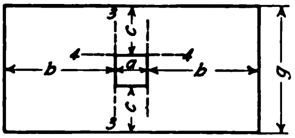

FIG. 45.

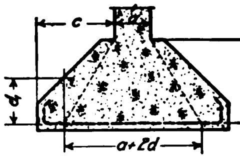

FIG. 46.

* In cases where the length is more than 50% greater than the width, the footing should be designed as follows:

$$M_{(3-3)} = \frac{wb^2}{2}g$$

$$M_{(4-4)} = \left(\frac{ac^2}{2} + \frac{bc^2}{2}\right)w$$

The reinforcement across 4 - 4 should be placed within a width equal to $(a + b)$.

49f. Wall Footings.—Continuous footings of this type may either have a sloped top as shown (Fig. 46) or be constructed with a level top. If w is the unbalanced upward earth pressure, then punching shear $= \frac{wc}{bjd}$. Also, $d_1 = \frac{(c - d)\,w}{bj\,(40)}$. Max. $M = \frac{wc^2}{2}$ per lin. ft.

Bond $= \frac{wc}{mojd}$, where m is the number of bars per lin. ft.

It will be usually found that in this type of footing the reinforcing bars must be hooked as indicated.

50. Reinforced Concrete Combined Footings.—In case a column occurs very close to the property line, it is probable that a symmetrical footing cannot be constructed without encroaching upon the adjoining property. In this case a cantilever footing as described in Art. 51, or a combined footing for the exterior and next adjacent interior footing may be constructed.

If the exterior column load is less than the interior column load, it is most economical to use a rectangular footing, though, if conditions do not permit of this, a trapezoidal shape may be used. In case the exterior column load is greater than the interior, a trapezoidal footing must be used.

50*a*. Rectangular Combined Footings.—In this design (Fig. 47) the foundation will be proportioned upon the basis of one-half the live load used in the basement column and all the dead load, as was done in Art. 49*c*. The footing area will be determined and located with respect to the column upon the above basis and the design of the reinforced concrete then prepared on the basis of the full column load.

Interior Column (1)..........34 × 34 in., 1-2-4 concrete.
Dead load = 297,000 lb. Dead load plus one-half the live load = 509,000 lb.
Live load = 423,000 lb.

Total = 720,000 lb.
Exterior Column (2)..........30 × 30 in., 1-2-4 concrete.
Dead load = 280,000 lb. Dead load plus one-half the live load = 378,000 lb.
Live load = 196,000 lb.

Total = 476,000 lb.

Maximum soil pressure = 4000 lb. per sq. ft. Allow 12½ % for weight of footing.

Now column1 has the greater percentage of live load. Area required at 4000 lb. soil pressure on total load =

$$\frac{(720{,}000)(1.125)}{4000} = 203 \text{ sq. ft.}$$

Now with one-half the live load, we have a pressure =

$$\frac{599{,}000}{203} = 2950 \text{ lb. per sq. ft.}$$

Area required for exterior column $= \frac{438{,}000}{2950} = 149$ sq. ft.

Total area of footing = 352 sq. ft.

Now the center of gravity of this area must coincide with that of the loads 509,000 and 378,000 lb. Column centers are 18 ft. 0 in.

c. g. is $\frac{(18.0)(509{,}000)}{887{,}000} = 10.35$ ft. from center of column 2

Now side of column 2 is on the lot line so c. g. is 10.35 + 1.25 = 11.60 ft. from end of footing.

Footing is to be rectangular, so length will be 2 × 11.6 = 23.2 ft.

Width of footing $= \frac{352}{23.2} = 15.2$ ft.

The size of the footing and its location with respect to the two columns has now been determined. In the design of the footing, the full column loads will be used. The sum of the column loads = 1,196,000 lb.

$$w = \frac{1{,}196{,}000}{352} = 3400 \text{ lb.}$$

With the full loads the pressure at *ab* would be somewhat greater than this, and at *cd* somewhat less. It will be satisfactory, however, to design the footing for the average pressure.

$$M_{(3-3)} = (15.2)(3.95)\left(\frac{3.95}{2}\right)(12)(3400) = 4{,}800{,}000 \text{ in.-lb.}$$

Max. M between columns 1 and 2 $= \frac{WL}{8} - \frac{1}{2}M_{(3-3)} = (3400)\,(15.2)\,(18)^2(1.5) - 2{,}400{,}000 = 22{,}600{,}000$ in.-lb.

$$d^2 = \frac{22{,}600{,}000}{(108)(15.2)(12)} = 1150 \qquad d = 34 \text{ in.}$$

A_s = 48 sq. in. = 31—1¼-in. square bars.

These bars must be investigated for bond. Shear at the side of columns 1 and 2 = 396,000 lb., so

$$u = \frac{396{,}000}{(31)(5)(0.87)(34)} = 87 \text{ lb. per sq. in.}$$ Use deformed bars.

At column 1: $A_s = 10.2$ sq. in. = 17 – ⅞-in. round bars.

$$\text{Bond stress} = \frac{(3400)(3.95 - 1.42)(15.2)}{(17)(2.75)(0.87)(34)} = 94 \text{ lb.}$$

Use deformed bars.

In this case it will be assumed that there is no basement wall at column 2. Transverse reinforcement must be provided at both columns 1 and 2.

At column 1:

$$M = \frac{720{,}000}{2} \times \frac{(15.2-2.83)}{15.2} \times \frac{6.2}{2} \times 12 = 10{,}900{,}000 \text{ in.-lb.}$$

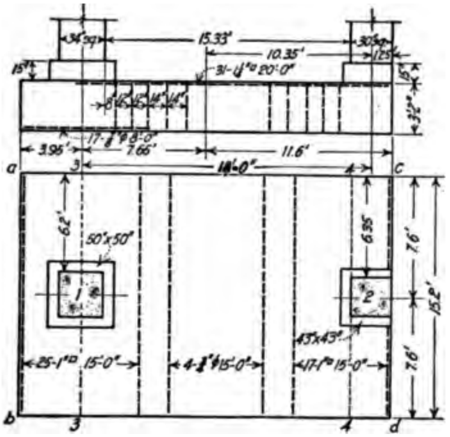

FIG. 47.

$A_s = 24$ sq. in. Use 24—1-in. square rods, 4 in. on centers, giving a width of 7 ft. 8 in. = column width plus 4.5 ft., or less than $2d$ (see formula for width within which steel is effective in Art. 49a), which is satisfactory.

$$\text{Bond} = \frac{291{,}000}{(24)(4)(0.87)(34)} = 103 \text{ lb.}$$ Use 25 – 1-in. square deformed bars.

At column 2:

$$M = \frac{476{,}000}{2} \times \frac{(15.2-2.5)}{15.2} \times \frac{6.35}{2} \times 12 = 7{,}600{,}000$$ in.-lb.

$A_s = 16.7$ sq. in. Use 17—1-in. square bars, or width required = 64 in., which is satisfactory.

$$\text{Shear at 4–4} = \frac{15.33}{2}(15.2)(3400) = 396{,}000 \text{ lb.}$$

$$v = \frac{396{,}000}{(15.2)(12)(0.87)(34)} = 74 \text{ lb.}$$

$$A_s \text{ for stirrups} = \frac{396{,}000}{16{,}000} \times \frac{2}{3} = 16.3 \text{ sq. in.}$$ in 34-in. length of footing. Use 5 – ½-in. square stirrups, each having 22 verticals in each end of footing. The stirrup spacing will be as shown in Fig. 47 (see chapter in Sect. 2 on Reinforced Concrete Beams and Slabs).

Punching shear at column 1 $= \dfrac{720{,}000 - (8)(3400)}{(4)(34)(0.87)(34)} = 175$ lb. Size of cap required $= (34)\dfrac{175}{120} = 50$ in. square. Depth required at column 1 $= \dfrac{693{,}000}{(4)(34)((0.87)120)} = 49$ in. Cap at column 1 is 50 × 50 × 15 in.

Depth required at column 2 $= \dfrac{476{,}000 - (6.25)(3400)}{(3)(30)(0.87)(120)} =$ 48.5 in. Size of cap required $= \dfrac{455{,}000}{(3)(34)(0.87)(120)} = 43$ in. square.

This footing might have been designed with a heavy beam running on the column centers and a thinner slab of the area shown used, reinforced in a transverse direction. On account of shear this beam would have to have a cross section of 3800 sq. in. or a beam about 60 × 66 in. In practically all cases the design illustrated will be found to be the most economical.

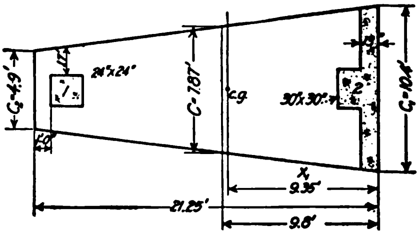

FIG. 48.

50b. Trapezoidal Combined Footings.—In this case the foundation will simply be proportioned directly to the basement story column loads.

Interior column 1 .... load = 390,000 lb., column size 24 × 24 in.
Exterior column 2 .... load = 476,000 lb., column size 30 × 30 in.

Total load = 866,000 lb.

Soil pressure = 6000 lb. per sq. ft.

Allowing 12½ % for weight of footing, the area required = 163 sq. ft. Column spacing is 18 ft. 0 in. In this case we have a concrete basement wall at the edge of the footing. The weight of this is included in the column load.

The center of gravity of column loads is $\dfrac{(18)(390{,}000)}{866{,}000} = 8.1$ in. from column 2, or 9.35 ft. from end of footing. The footing will be continued 1 ft. 0 in. past the edge of column 1. The length of the footing is therefore 21 ft. 3 in. (see Fig. 48).

The widths C_1 and C_2 must be such that the area of the footing is 163 sq. ft. and the center of gravity of this trapezoid is at 9.35 ft. from the end as shown. Then

$$\frac{(C_1 + C_2)(21.25)}{2} = 163 \quad \text{or} \quad C_1 + C_2 = 15.3 \text{ ft.} \qquad (1)$$

Using the common equation for the center of gravity in a trapezoid

$$x_1 = \frac{21.25}{3} \times \frac{C_1 + 2C_2}{C_1 + C_2} = 9.35 \text{ ft.} \tag{2}$$

Solving equations (1) and (2)

$$C_1 = 10.4 \text{ ft. and } C_2 = 4.9 \text{ ft.}$$

$$w = \frac{866{,}000}{163} = 5300 \text{ lb.}$$

The line of maximum moment will be at the line of zero shear across the footing. This line is at 9.8 ft. from right end of footing.

Neglecting the small negative moment at column 1, we have

$$M(\text{max.}) = \left[476{,}000 - (10.2)(1.25)(5300)\right](9.8 - 1.25) - 5300\left[(7.87)\left(\frac{8.55^2}{2}\right) + \left(\frac{2.2}{2}\right)(8.55)^2\left(\frac{2}{3}\right)\right]$$

$$= 1{,}700{,}000 \text{ ft.-lb.} = 20{,}400{,}000 \text{ in.-lb.}$$

Since $C = 7.87$ in.

$d = 45$ in. $A_s = 32.7$ in. $= 21$—1¼-in. square rods.

Checking for bond, we have

$$u = \frac{400{,}000}{(21)(5)(0.87)(45)} = 100 \text{ lb. per sq. in.}$$ Use deformed bars.

The ends of one-half the bars will be bent down as shown (see Fig. 49)

Shear at column 2 = 409,000 lb.

$$v = \frac{409{,}000}{(45)(0.87)(10)(12)} = 87 \text{ lb.}$$

Stirrups will be used as shown.

Punching shear at column 2 may be considered along the entire face of the wall and is the same as v. No cap is required.

At column 1 the punching shear is 104 lb. per sq. in., so no cap is needed.

Cross bending need not be considered at column 2 as the rigid concrete wall will distribute the load at that end. At column 1, we have

$$M = \left(\frac{390{,}000}{2}\right)\left(\frac{3.4}{5.4}\right)\left(\frac{1.7}{2}\right)(12) = 1{,}260{,}000 \text{ in.-lb.}$$

$A_s = 2.0$ sq. in. Use 4—¾-in. square rods.

For bond, we have

$$\frac{1}{2} \cdot \frac{(390{,}000)(3.4)}{(100)(3)(0.87)(45)(5.4)} = 11.$$ Use 11—¾-in. square deformed bars.

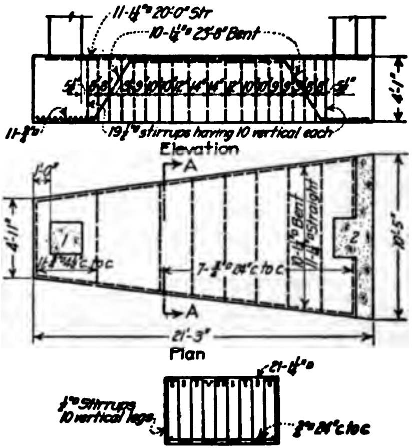

FIG. 49.

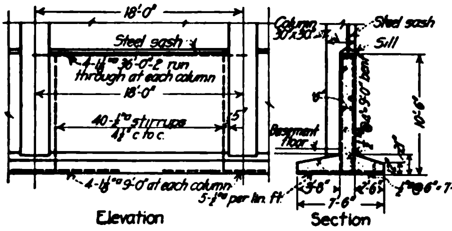

FIG. 50.

50c. Continuous Exterior Column Footings.

—In many cases where we have a continuous concrete basement wall it will prove economical to use the basement wall as a beam to distribute the column load to a continuous footing of relatively small width. A footing of the type is shown in Fig. 50. The following is an example of its design. Note that the footing is concentric with the column and that consequently the projections from the wall vary.

Basement story column load = 480,000 lb. Column size, 30 × 30 in.
Soil pressure = 4000 lb. Columns are spaced 18 ft. c. to c.

Basement wall	9 ft. × 16 in.	= 1800 lb.
Footing	about 8 ft. × 16 in.	= 1600 lb.
		3400 lb. per lin. ft. = 60,000 lb.

Total load = 540,000 lb.

$$\text{Area} = \frac{540{,}000}{4000} = 135 \text{ sq. ft.}$$

$$w = 3560 \text{ lb.}$$

End shear in wall

$$v = \frac{V}{bjd'} \text{ or } b = \frac{(7.5)(15.5)(3560)}{(2)(120)(0.87)(122)} = 16 \text{ in.}$$

The maximum moment in the back part of the footing

$$M(\text{max.}) = (3560)\left(\frac{3.67^2}{2}\right)(12) = 288{,}000 \text{ in.-lb.}$$

d required = 15 in. Use 18 in. depth = 21 in.

A_s = 1.14 sq. in. Use 5—½-in. square rods per linear foot of wall.

$$\text{Bond} = \frac{(3.67)(3560)}{(5)(2)(0.87)(18)} = 84 \text{ lb. Use deformed bars.}$$

Embedment of 3 ft. 6 in. = 84 diameters. Embedment of 2 ft. 4 in. = 56 diameters

The maximum moment in the front part of the footing

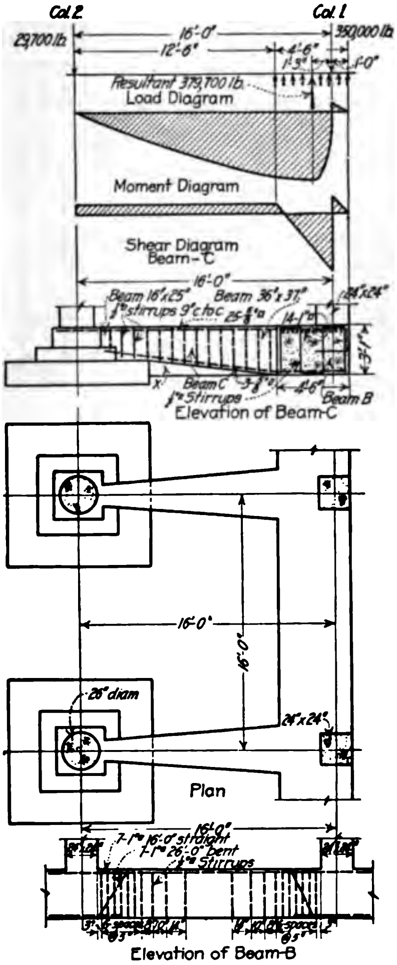

Fig. 51.

$$M(\text{max.}) = (3560)\left(\frac{2.5^2}{2}\right)(12) = 134{,}000 \text{ in.-lb.}$$

Unbalanced M = 288,000 − 134,000 = 154,000 in.-lb.

A_s in wall = 0.75 sq. in.

A_s in outer side = 0.53 sq. in.

End shear = (80)(120)(16.5) = 158,500 lb. on stirrups.

A_s = 10.0 sq. in. in 10 ft. = 20—½-in. square stirrups = 40 in the beam.

M in beam = $(7.5)(3560)(18)^2$ = 8,650,000 in.-lb.

A_s = 5.15 sq. in. = 4—1⅛-in. square rods.

51. Reinforced Concrete Cantilever Footings.—This type of footing may be used with economy in some cases when the exterior columns are adjacent to the property line. In this type of design it is not good practice to use the basement wall as a beam to distribute the column load.

Take an exterior column load of 350,000 lb., column size 24 × 24 in. Soil pressure = 6000 lb. Column centers, 16 ft. each way.

The load, moment, and shear diagrams (Fig. 51) illustrate the action of the cantilever beam C. In addition to this beam, we have a continuous footing beam B. This footing is not concentric with column 1 and beam C resists the bending moment caused by this eccentricity. By moments we find that, in order to balance the moment in beam C, a load of 29,700 lb. must be applied at column 2. The total pressure exclusive of the weight of the footing to be provided for will be 379,700 lb. Allowing 12½ % for the footing weight, a footing 16 ft. 0 in. × 4 ft. 6 in. is required.

Beam B:

$$M = \frac{WL}{12} = (379{,}700)(16) = 6{,}080{,}000 \text{ in.-lb.}$$

b = 54 in., so d = 32.5 in. A_s = 13.5 sq. in. = 14 − 1-in. square bars.

End shear

$$V = \left(\frac{379{,}700}{2}\right)\left(\frac{14}{16}\right), \quad \text{or} \quad v = 113 \text{ lb.}$$

The stirrups will be as shown.

Beam C:

M(max.) = (350,000)(1.25)(12) = 5,250,000 in.-lb.

d = 32.5 in., so A_s = 11.6 sq. in. = 21 − ¾-in. square bars.

M at edge of beam B = (29,700) (12.5)(12) = 4,450,000 in.-lb.

d = 32.5 in. A_s = 9.85 sq. in. Use 25 − ⅝-in. square bars. b = 36 in.

The section of C at column 2 will be governed by shear. Take shear at 100 lb. and we have

$$bd = \frac{29{,}700}{(100)(0.87)} = 342 \text{ sq. in.}$$

Beam, 16 × 25 in. is satisfactory.

Note that the 25 − ⅝-in. square bars used in beam C all have large hooks at the end near column 1 and are deformed bars. Note also particularly that beam C is not to bear on the ground. The space marked X in the elevation must be left open under the beam. This can be arranged in the formwork.

52. Concrete Raft Foundations.—Where the safe soil load is very low it is sometimes pos- ble and desirable to use concrete raft or mat covering the entire building site. This type of undation is usually more economical than piles. The raft may be designed either as a flat ab or as beam and slab construction. The beam and slab usually more expensive but as the advantage over flat slab hat the piping below the basement floor may be installed fter the foundation work is one instead of before. There nothing unusual about the reparation of these designs. n Figs. 52 and 53 two such esigns are illustrated. The ead weight of the foundation ill simply balance a certain mount of upward soil pressure nd so the weight of the foundaion will not enter into the slab esign. If the column bases or nverted caps shown in Fig. 52 re objectionable, they may be liminated and the slab in- reased in thickness for the ncreased moments and shear resulting.

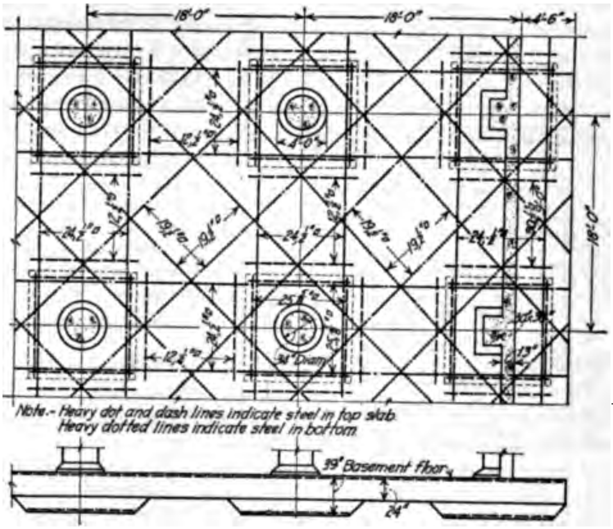
18'-0"
18'-0"
4'-6"
18'-0"
4'-0"
24'-0"
12½"
24½"
19½"
25½"
38" Diam.
13"
Note.- Heavy dot and dash lines indicate steel in top slab
Heavy dotted lines indicate steel in bottom
39' Basement floor
24"

Fig. 52.

53. Piers Sunk to Rock or Hardpan.—The most desirable foundation for high buildings is he concrete pier sunk to rock or to a very hard formation. It has also been found that for -story buildings where the site consists of a soft clay or other material overlying a hard formation at a depth of 30 or 40 ft., that these piers are more economical than piles or spread footings. In Chicago these piers are called caissons and are widely used.

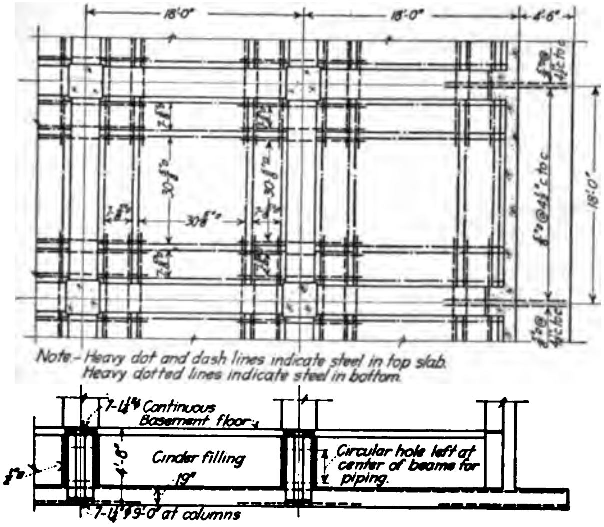
18'-0"
18'-0"
4'-6"
18'-0"
30½"
⅝"@4½" c. to c.
Note.- Heavy dot and dash lines indicate steel in top slab
Heavy dotted lines indicate steel in bottom
7-1¼" Continuous
Basement floor
Cinder filling
19"
4'-6"
Circular hole left at center of beams for piping.
7-1¼" 9-0" at columns

Fig. 53.

If 1–2–4 concrete is used, the Chicago Ordinance allows the cross section of the pier to be determined by using a concrete stress on the entire section of 400 lb. per sq. in. for the basement-story column load. Where the piers are sunk to hardpan and not to rock, the bottom of the pier is belled out so that a bearing on the hardpan of 13,000 lb. per sq. ft. is obtained. The weight of the pier must be added for this calculation. The height of the conical portion is equal to the difference in diameter between the bottom and the pier shaft.

54. Reinforced Concrete Footings on Piles.—These footings may be of the same types as those designed under Arts. 49, 50, and 51. They may also be constructed as mattresses or raft foundations as illustrated in Art. 52. The only difference in the design from that of the spread footings is that the pile loads are treated as concentrated loads. Most building ordinances require that the top 6 in. of the piles be enclosed in concrete which is not considered as contributing to the footing strength. The footing must be deep enough at all points so there is sufficient punching shear resistance. In most ordinances wood piles are figured at a maximum bearing value of 20 tons. Concrete piles are figured at the top section the same as a concrete column. The following is an example of a reinforced footing design over wood piles (see Fig. 54).

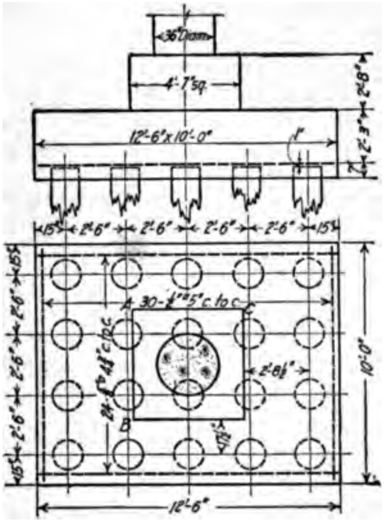

Fig. 54.

Column load = 700,000 lb. Column size, 36-in. diameter. 1-2-4 concrete.

Pressure on base of column = 690 lb. per sq. in. Area of cap = 2034 sq. in. = 45 × 45 in. Use 55 in. square.

We must use the weight of the footing in addition to the column load in designing for punching shear, but for bending and diagonal tension, we will use the column load only.

$$d \text{ at column for punching shear} = \frac{800{,}000}{(\pi)(36)(120)} = 59 \text{ in.}$$

$$d \text{ at edge of cap} = \frac{800{,}000}{(4)(55)(120)} \cdot \frac{18}{20} = 27 \text{ in.}$$

Minimum d over a pile for punching:

Load on 1 pile = 40,000 lb. Take pile 14-in. diam. at cut off.

$$d = \frac{40{,}000}{(\pi)(14)(120)} = 7.6 \text{ in.}$$

Diagonal Tension: Considering long side we have the reaction from three piles and two half piles = 4 piles. Using 40 lb. as the allowable shearing stress per sq. in.

$$d = \frac{4}{20} \cdot \frac{700{,}000}{(12.5)(12)(0.87)(40)} = 27 \text{ in.}$$

Considering short side, we have the reaction from three piles, or

$$d = \frac{3}{20} \cdot \frac{700{,}000}{(120)(0.87)(40)} = 25 \text{ in.}$$

$$\text{Mom. at } AB = \left(\frac{3}{20}\right)(700{,}000)(32\tfrac{1}{2}) = 3{,}400{,}000 \text{ in.-lb.}$$

$$\text{Mom. at } AC = \left(\frac{4}{20}\right)(700{,}000)(17\tfrac{1}{2}) = 2{,}450{,}000 \text{ in.-lb.}$$

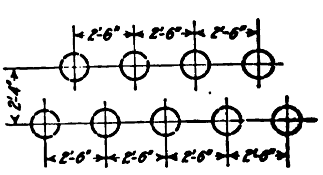

Fig. 55.

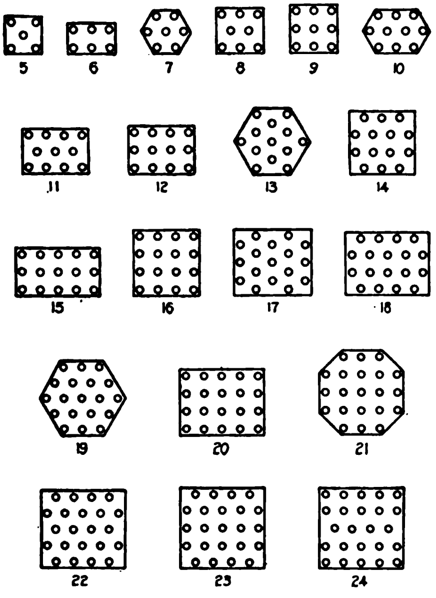

Fig. 56.—Arrangement of piles and shapes of footings.

For bending

$$d = \sqrt{\frac{3{,}400{,}000}{(108)(120)}} = 17 \text{ in. We have 27 in.}$$

A_s in the long direction = 9.1 sq. in. = 24 — ⅝-in. square bars.

$$\text{Bond} = \frac{3}{20} \cdot \frac{700{,}000}{(24)(2.5)(0.87)(27)} = 75 \text{ lb. per}$$

sq. in., which is satisfactory.

A_s in short direction = 6.55 sq. in. = 26—½-in. square bars.

$$\text{Bond} = \frac{4}{20} \cdot \frac{700{,}000}{(26)(2)(0.87)(27)} = 115 \text{ lb.}$$

We must use $26 \times \frac{115}{100}$ — ½-in. square deformed bars = 30 — ½-in. square bars.

The various building departments have their specifications covering the distance of piles center to center. In the example worked out 2 ft. 6 in. was used and this should be the minimum for wooden piles except where piles are spaced as in Fig. 55 where the centers may be 2 ft. 6 in. × 2 ft. 4 in.

Concrete piles are usually spaced about 3 ft. 0 in. on centers. The design of foundations over them is similar in every way to the footing design for wooden piles except that the load per pile will be much greater in most cases.

For concrete piles placed as in Fig. 55 a spacing of 3 ft. 0 in. in one direction and 2 ft. 7 in. to 2 ft. 9 in. normal to it is used.

The diagrams of pile arrangements given in Fig. 56 will be found convenient. The spacing of the piles is not given as the designer must comply with the local specifications in this matter.

55. Steel Beam and Girder Footings.—Steel beam footings are not now used to any great extent. The footing consists of tiers of steel beams placed side by side and embedded in concrete, as shown in Fig. 4, p. 118. The method of design for steel beam pier footings is described in the illustrative problems on pp. 121 and 122. Steel girders are used in combined and cantilever footings of this type to distribute the loads. The method of designing a steel girder for a combined footing is given in the illustrative problem on p. 187.

FLOOR AND ROOF FRAMING—TIMBER

By Henry D. Dewell

56. Floor Construction.

56*a*. Thickness of Sheathing and Spacing of Joists.—The type and intended use of the building will in a great measure determine the general arrangement of floor system, the thickness of sheathing, and the approximate spacing of joists. For timber floors carrying light loads, as dwelling houses, apartment houses, schoolhouses, and office buildings, the sheathing is usually of double thickness, consisting of an under floor of rough 1 × 6-in. boards, laid diagonally with the joists, and an upper floor of ⅞-in. tongue and grooved flooring. The joists for this class of buildings are usually 2 to 3 in. nominal thickness, spaced 16 in. on centers, and of such depth as is necessary for strength and stiffness. The spacing of 16 in. for the joists must be maintained when a ceiling of wood lath and plaster is supported from the under side of joists. Usually, the span of the joists will not exceed 20 ft. Floor joists 2 × 8 in. are the smallest size that should ordinarily be used, while the maximum depth for a 2-in. thickness should not exceed 16 in. If a stronger joist than a 2 × 16-in. is required, the thickness should be increased to 3 in. with a maximum depth of 18 in., or the spacing decreased to 12 in. With a ceiling supported from the floor joists, the size of joists must be sufficient to keep the deflection of the joists when fully loaded to $\frac{1}{360}$ of the span of the joists. In making such a computation for deflection the load of ceiling, joists and bridging, flooring and any partitions is considered as the constant or "dead" load, and the modulus of elasticity used should not exceed ¾ that given in Sect. 7, Art. 10 for the particular kind of timber used. The deflection for live load is computed, using the full value of the modulus of elasticity. The total deflection to be expected is the sum of the two partial deflections.

In buildings where floors carry much heavier loads, as warehouses, lofts, etc., the flooring is usually 1½ in. thick as a minimum. If such a building has no ceiling, the spacing of joists may profitably be increased over 16 in. In general, the most economical floor will occur with short spans for joists and girders, and consequently small size joists. On the other hand, many other factors enter which may warrant longer spans for both joists and girders, and the most important of these factors is the advantage of having as few posts inside of a building as possible. In the framing of the first floors of buildings where such floors are but a few feet off the ground, it will usually be found, for example, that for a live load approximating 100 lb. per sq. ft., the most economical system of framing will be 6 × 6-in. posts, 6 × 8, or 6 × 10-in. girders, 2 × 8-in. joists, the floor bays being approximately 10 × 10 ft. In the above statement, it is assumed that the footings rest on the soil; for pile foundations the situation would be entirely different. In the latter case economy will dictate the use of long spans to utilize the full capacity of pile.

Comparing 2-in. joists with 3-in. joists of equivalent strength, it may be pointed out that, since the actual finished thickness of a 3-in. joist when surfaced one side is 2⅝ in., and the finished thickness of a 2-in. joist is 1⅝-in., the loss of strength by surfacing is 18.75% in a 2-in. joist and 12.5% in the 3-in. joist, or an economy of 6.25% for the 3-in. joist, although the price of the 3-in. timber will be slightly higher than the 2-in. stock. Only a comparison of several schemes for an actual case will indicate the cheapest construction.

For proper spiking the thickness of joist should be somewhat greater than the thickness of single floor spiking to it. Using floor boards of 2-in. thickness, the joists should be 3 in. thick.

56*b*. Bridging.—Bridging consists of timbers placed between joists to support them laterally. Bridging is either solid or of the cross or herring-bone type. The latter method, shown in Fig. 57, is the more effective of the two types, since it not only supports the joists laterally; but, in the event that a concentrated load comes on one joist, the bridging will effectively assist the flooring in distributing a portion of the load to the joists at either side.

For joists 2 × 10 in. and under, cross bridging 1 × 4 in. or 1 × 3 in. will be sufficient. For joists 2 × 12 in. and larger, the cross bridging should be at least 2 × 3 in., and for the larger sizes of joists, 2 × 4 in.

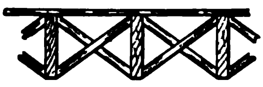

Fig. 57.—Detail of herring-bone bridging.

Solid bridging consists of pieces of planks of the same depth as the joists, cut and fitted between the joists. Solid bridging should never be less than 2 in. thick.

All bridging should be neatly and snugly fitted between the joists and well nailed thereto. It should be continuous throughout a line of joists having a common span. Cross bridging should be placed at intervals not to exceed 8 ft. All joists should be solid bridged over supports.

56c. Arrangement of Girders.—With a rectangular floor bay, the economical arrangement of girders and joists is to make the girders span the short side of the rectangle, the joists taking the longer span.

For general stiffness of the building, the girders, where possible, should run parallel to the transverse axis of the building. It may be advisable, if clearances will permit, to use knee braces from girders to columns, but in any case the span of girder should always be taken as the distance between center lines of end bearing on columns or walls. Knee braces should preferably be fitted or attached to girders and columns after the full dead load of floor is in place; otherwise even the slight deflection of girder may put heavy bending stresses in the columns.

Openings for stairs, etc., make the case of non-uniform loading more likely to be encountered in the case of floor girders than in the case of joists.

If double girders are necessary, an air space should be left between them, and the two girders connected at short intervals, say 2 ft., by pairs of bolts, using cast-iron separators between the girders. This air space is necessary to prevent dry rot taking place, although for fire protection, such air space is undesirable.[1]

56d. Connections to Columns.—To prevent the girders in falling from pulling the columns with them, in case of fire, standard practice recommends that the attachment of girders to columns be made self-releasing. The writer believes, however, that in the event of a fire serious enough to burn through the girders, the interior posts of the building are almost certain to fall. For this reason, where it is necessary to secure lateral stiffness in a building, he believes it well to design the connections of girders to columns, and joists to columns, relatively strong, providing continuity across the columns. Details of such connections are discussed in Sect. 2, Art. 123.

56e. Connections to Walls.—All girders and joists entering masonry walls should rest upon steel or iron bearing plates, well painted. An air space should be left around the ends of joists and girders. In order to allow the girders or joists to fall without pulling the walls over in case of fire, the ends of the timbers are usually cut back, as in Fig. 58. For tying the girders and joists into the walls, iron or steel anchors are used, as illustrated in Fig. 58. These anchors should be approximately ¼ × 1½-in. straps, one end forged into a lug to fit into a notch in the upper side of girder. The portion within the wall may be bonded into the masonry. Sometimes an anchor consisting of a round rod is passed through the wall, and is fitted with an exterior ornamental cast-iron washer on the outside. The other end of the rod may be forged into a flat strap with a lug as before.

Every girder should be anchored into the wall. In the case of joists, at least every sixth joist should be so anchored. Building ordinances usually prescribe in detail the size and arrangement of wall anchors.

Joists, closely spaced, entering a masonry wall weaken the walls. Further, unless very careful inspection is maintained, one can never be certain that proper air spaces will be left around the timbers entering the wall. For this reason, there have been developed wall boxes, made of malleable iron, steel, and cast iron, which insure an air space around the joist or girder, and at the same time allow the timber to be self-releasing in case of fire. The tie between timber and wall is secured by a lug on the base of the anchor which engages a notch on the under side of joist or girder. Typical box anchors are shown in Figs. 59 to 62 inclusive. Fig. 63 shows a Duplex wall plate.

A third method for support of joists and girders is the wall hanger shown in Figs. 64 and 65. With the wall hanger, no hole is left in the wall. Since the joists and girders with this device extend only to the inner surface of

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[1] In mill construction, this air space is considered objectionable by many since it forms a concealed space, which, in the event of fire, cannot be reached by water from the sprinklers.

the wall, a saving in timber is made. Since lumber comes in lengths of multiples of 2 ft. only, the use of the wall hanger as compared to the box anchor may mean a saving, in many cases, of 2 ft. in the length of timber—a very considerable item.

56f. Typical Floor Bay Design.—The following example will illustrate the necessary computations for designing the joists and girders of a typical floor bay. The framing plan of the bay is shown in Fig. 66.

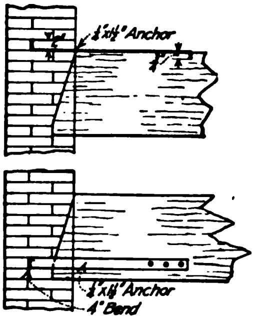

Fig. 58.—Details of connections—timber joists to brick walls.

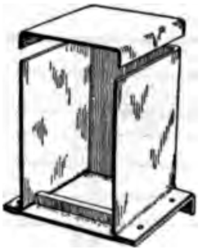

Fig. 59.—Van Dorn box anchor.

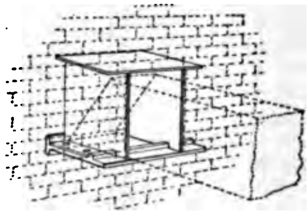

Fig. 60.—"Ideal" wall box.

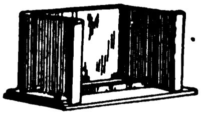

Fig. 61.—Lane wrought steel wall box.

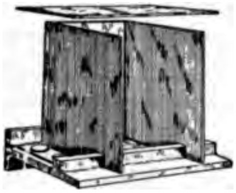

Fig. 62.—Duplex wall box.

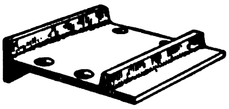

Fig. 63.—Duplex wall plate.

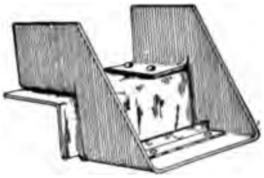

Fig. 64. Duplex wall hanger.

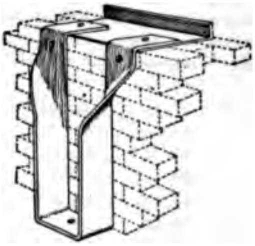

Fig. 65.—"Falls" joist hanger.

Data: Office floor; partitions 2 × 4 in., plastered both sides, 12 ft. high; flooring double, under floor rough 1 × 6 in., upper floor 1 × 4 in., T & G; ceiling plastered; joists 16 in. on centers; live load for joists, 60 lb. per sq. ft.; live load for girders, 48 lb. per sq. ft.; live load for stairs, 75 lb. per sq. ft.

For approximate dead load, call flooring 2 in. thick at 3 lb. per board foot; assume joists 2 × 16 in.—16 in. on centers; allow 1 lb. per sq. ft. for bridging; assume plaster ceiling weight 5 lb. per sq. ft.; assume girder weight as 2 lb. per sq. ft.

Timber: Douglas fir, dense structural grade, all timbers to be taken as S1S1E,[1] working stress 1800 lb. per sq. in. in flexure and 175 lb. in horizontal shear.

[1] Surfaced one side and one edge.

Loadings:

	Joists	Girders
Flooring	6	6
Joists	6	6
Bridging	1	1
Ceiling	5	5
Girder	0	2
Total dead load	18	20
Live load	60	48
Total dead and live load	78 lb. per sq. ft.	68 lb. per sq. ft.

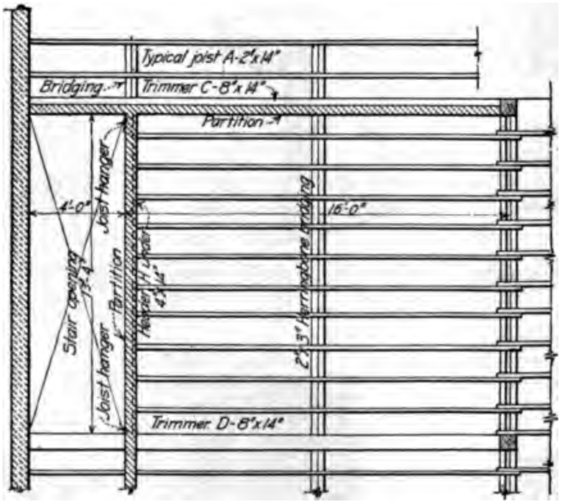

Fig. 66.—Framing plan of typical floor.

Typical Joist A.—Span 20 ft.; load = (20)($1\frac{1}{3}$)(78) = 2080 lb. From Table 7, p. 110, it is found that a 2 × 12-in. joist on a 20-ft. span will carry 2149 lb., limited by bending. The load producing a deflection of $\frac{1}{30}$ in. per foot of span is 1236 lb., so that a deeper joist must be chosen. Since for dead load a modulus of elasticity may be used of only $\frac{3}{4}$ of that used for live load, the dead load of 18 lb. per sq. ft. will be multiplied by the factor $\frac{4}{3}$ giving 24 lb. per sq. ft., making a total loading of 84 lb. per sq. ft., and a total load of 2240 lb. to be considered as producing deflection. Again, entering the tables it is found that the safe load for a 2 × 14-in., as limited by deflection, is 2153 lb. This load, while slightly under the required loading, will be taken as satisfactory, and 2 × 14-in. joists used.

TABLE 1.—STUD PARTITIONS[1]

Weight and strength based on actual size
Board measure based on nominal size
Add weight of plaster or ceiling
Single plate top and bottom included, same size as studs

SAFE LOAD BASED ON STUDS BEING BRIDGED AT CENTER

Nominal size	Actual size	Distance on centers (inches)	Height (feet)	Per linear foot of partition			
				Safe load* (pounds)		Weight (pounds)	Board feet
2 × 4	1⅝ × 3⅝	12	8	2060	3,723	16.30	6.66
		12	10		3,180	19.56	8.00
		12	12		2,631	22.82	9.33
		16	8	1540	2,793	13.04	5.33
		16	10		2,385	15.50	6.33
		16	12		1,974	18.00	7.33
2 × 6	1⅝ × 5⅝	12	8	3200	5,767	25.30	10.00
		12	10		4,926	30.56	12.00
		12	12		4,076	35.42	14.00
		16	8	2400	4,326	20.24	8.00
		16	10		3,699	24.03	9.50
		16	12		3,057	27.83	11.00
2½ × 6	2¼ × 5½	12	8	4330	9,079	34.30	12.50
		12	10		8,250	41.16	15.00
		12	12		7,422	48.02	17.50
		16	8	3250	6,808	27.44	10.00
		16	10		6,187	32.59	12.00
		16	12		5,566	37.73	13.75
3 × 6	2¾ × 5½	12	8	5300	11,823	42.00	15.00
		12	10		10,992	50.40	18.00
		12	12		10,175	58.80	21.00
		16	8	3970	8,868	33.60	12.00
		16	10		8,244	39.90	14.25
		16	12		7,630	46.20	16.50
2 × 8	1⅝ × 7½	12	8	4260	7,692	33.80	13.33
		12	10		6,570	40.56	16.00
		12	12		5,436	47.32	18.66
		12	14		4,315	54.08	21.33
		16	8	3200	5,769	27.04	10.66
		16	10		4,927	32.11	12.66
		16	12		4,077	37.18	14.66
		16	14		3,236	42.25	16.66
2½ × 8	2¼ × 7½	12	8	5900	12,382	46.80	16.66
		12	10		11,252	56.16	20.00
		12	12		10,122	65.52	23.33
		12	14		9,008	74.88	26.66
		16	8	4420	9,286	37.44	13.33
		16	10		8,439	44.46	15.83
		16	12		7,591	51.48	18.33
		16	14		6,756	58.50	20.83
3 × 8	2¾ × 7½	12	8	7220	16,124	57.20	20.00
		12	10		14,990	68.64	24.00
		12	12		13,877	80.08	28.00
		12	14		12,743	91.52	32.00
		16	8	5420	12,093	45.76	16.00
		16	10		11,242	54.34	19.00
		16	12		10,408	62.92	22.00
		16	14		9,557	71.50	25.00

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[1] From the Southern Pine Manual (modified).

* Safe loads in first column as limited by bearing on top and bottom plates at 350 lb. per sq. in. Safe loads in second column as limited by column action (Winslow's formula with $p = 1000$ lb. per sq. in.).

Typical Joist B.—Since the ceiling must be continuous, the same size of joists will be continued for the shorter span.

Header H.—The load coming on this beam from the floor is a girder load. Consequently, the uniformly distributed floor load = (14)(8)(68) = 7616 lb. The partition lumber will weigh 18 lb. per lin. ft. (see Table 1). Adding plaster for two sides at 5 lb. per sq. ft. per side, gives a total load per linear foot of 18 + (12)(10) = 138 lb. The partition load on the header therefore = (14)(138) = 1930 lb. Total load on header = 9546 lb. From Table 9, p. 113, it is found that a 4 × 14-in. timber on a 14-ft. span will carry 9764 lb. in bending, and 9415 lb. as limited for deflection. Again reducing the dead load to equivalent live load, we have,

$$\begin{aligned} (14)(8)(20)(1\tfrac{1}{3}) &= 2{,}987 \\ (1930)(1\tfrac{1}{3}) &= 2{,}570 \\ \text{Live load} = (14)(8)(48) &= 5{,}370 \\ & \overline{10{,}927 \text{ lb.}} \end{aligned}$$

This load is 16% in excess of the limiting load for deflection for a 4 × 14 in. On the other hand, the safe load as limited by deflection for a 6 × 14 in. is 13,808 lb., which is 47% too heavy, and the actual span is 13 ft. 8 in. instead of 14 ft. 0 in. A 4 × 14 in. will therefore be used.

Trimmer C.—Uniform partition load = (138)(20) = 2760

$$\text{Uniform floor load} = \frac{(20)(1\tfrac{1}{3})(78)}{2} = 1040$$

$$\text{Total uniform load} = \overline{3800} \text{ lb.}$$

Since there is a concentrated load on this header, also a portion of a uniform load, in addition to the uniform floor load figured above, we will compute the maximum bending moment. Fig. 67 represents the actual loadings diagrammatically.

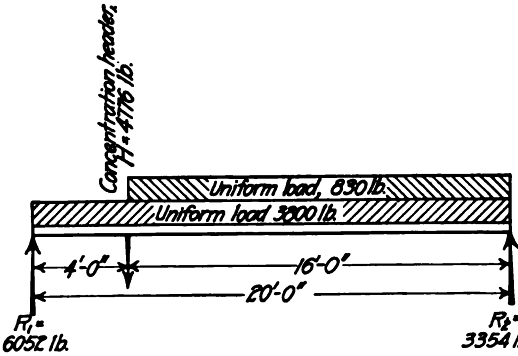


Fig. 67.—Diagram of loads on Trimmer C.

The live load acting as a concentration (the reaction of Header *H*) is a girder load for which a 20% reduction may be taken from the live load for joists.

The concentrated load at *P* is, therefore,

$$\begin{aligned} \text{Floor} = (7)(8)(68) &= 3810 \\ \text{Partition} = (138)(7) &= 966 \\ & \overline{4776 \text{ lb.}} \end{aligned}$$

The portion of uniform load on the trimmer not yet considered = (78)(16)($\frac{2}{3}$) = 830 lb.

Bending moments and reactions:

Uniform load of 3800 lb.

$M = (\frac{1}{8})(3800)(20) = 9500$ ft.-lb.

$R_1 = R_2 = 1900$ lb.

Concentrated load:

$$\begin{aligned} R_1 &= \frac{(4776)(16)}{20} = 3820 \\ R_2 &= \frac{(4776)(4)}{20} = \underline{956} \\ & \qquad 4776 \text{ lb.} \\ M &= (3820)(4) = 15{,}280 \text{ ft.-lb.} \end{aligned}$$

Small uniform load:

$$\begin{aligned} R_1 &= \frac{(830)(8)}{20} = 332 \text{ lb.} \\ R_2 &= \frac{(830)(12)}{20} = 498 \text{ lb.} \end{aligned}$$

$M = (332)(4) = 1328$ ft.-lb. (approximately)

Fig. 68 shows the bending moment curves plotted graphically.

The construction of the parabola of uniform moments is simple, a rectangle being erected on the span with a height of 9500 ft.-lb. to scale. The ends and half spans are divided into the same number of equal parts (in this case 4), ordinates erected on the span length at these division points, and radiating lines drawn from the center of upper side of rectangle to the division points on the sides. The intersection of corresponding radiating lines and ordinates fix points on the parabola. The triangle of moment for the concentrated load is indicated by the dotted line. This triangle is increased for the moment of the small uniform load (increase in moment = 1328 ft.-lb. at a point 4 ft. from left support). The moment of the small load is also computed at a point 8 ft. from the right end of trimmer. $M = (12)(332) - (4)(415) = 2324$ ft.-lb. The ordinate to the triangle of the moment of P is therefore increased by 1328 ft.-lb., and the full line drawn to represent the increased bending moment, passing through the point 8 ft. from left support that represents the increased ordinate of 1328 ft.-lb.

From the diagram, the maximum bending moment is 22,680 ft.-lb. Since the depth of floor construction is limited to 14 in., it is evident from the computations for the joists that a fiber stress of 1800 lb. per sq. in. cannot be used without exceeding the allowed deflection. In the case of Joist "A" a 2 × 14-in. joist was used when for strength a 2 × 12 in. was found to be satisfactory. The ratio of the strengths of these two joists is 3190/2149. In other words, the fiber stress in the 2 × 14-in. joist approximately = (2149/3190)(1800) = 1215 lb. per sq. in. A fiber stress of 1200 lb. per sq. in. will therefore be used for an approximate solution. Entering Table 6, p. 108, we find that an 8 × 14-in. beam, sized to 7½ × 13½, has at 1200 lb. per sq. in. a safe resisting moment of 22,781 ft.-lb., which is satisfactory.

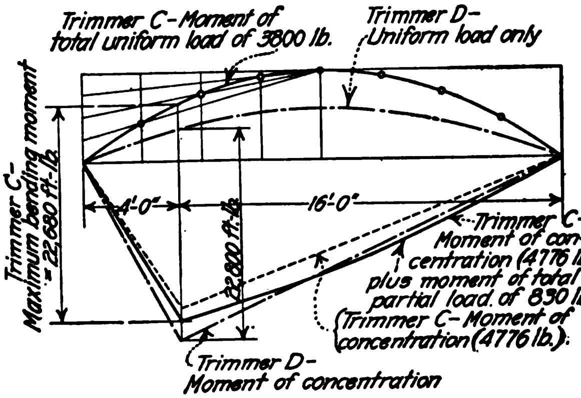

Trimmer C-Moment of total uniform load of 3800 lb.
Trimmer D-Uniform load only
Trimmer C-Maximum bending moment = 22,680 ft.-lb.
4'-0"
16'-0"
22,800 ft.-lb.
Trimmer C-Moment of concentration (4776 lb.) plus moment of total partial load. of 830 lb.
Trimmer C-Moment of concentration (4776 lb.)
Trimmer D-Moment of concentration

Fig. 68.—Diagram of bending moments for Trimmers C and D.

Trimmer D.—The calculations for Trimmer D are similar to those for Trimmer C. No uniform partition load occurs on the trimmer. However, there exists a stair load at the left-hand end. The dead and live load for the stairs will be assumed at 75 lb. per sq. ft. [(L. L. 75)(80%) + (D. L. 15)] = 75 lb. per sq. ft. The reaction of the stairs will therefore = (7)(4)(75) = 2100 lb., carried by two stringers. Only the reaction of one stringer applied 4 ft. out from the left end, need be considered. This concentration, added to the concentration from Header H, gives a total concentration of 4776 + 1050 = 5826 lb.

For simplicity it will be assumed that Trimmer C takes a load equal to that of Joist "A," or 2080 lb.

$$M = (\tfrac{1}{8})(2080)(20) = 5200 \text{ ft.-lb.} \quad R_1 = R_2 = 1040 \text{ lb.}$$

Concentrated load:

$$R_1 = \frac{(5826)(16)}{20} = 4660 \text{ lb.}$$

$$R_2 = \frac{(5826)(4)}{20} = 1165 \text{ lb.}$$

$$M = (4660)(4) = 18{,}640 \text{ ft.-lb.}$$

The diagram for bending moments is shown by the dot and dash lines in Fig. 68. The maximum bending moment is approximately 22,800 ft.-lb., so an 8 × 14-in. timber will be used.

The maximum vertical shear is 5700 lb. The maximum intensity of horizontal shear is therefore $\frac{(5700)(1\frac{1}{2})}{(7\frac{1}{2})(13\frac{1}{2})}$ = 86 lb. per sq. in., which is well within the permissible unit stress.

57. Roof Construction.

57*a*. Thickness of Sheathing.—Except in mill construction, the thickness of roof sheathing is seldom over 1 in. nominal, or $\frac{13}{16}$ in. finished. For roofs with a finish of tar or asphalt and gravel, or prepared roofing, either built up on the job or ready roofing, the sheathing should be dressed and matched and of good quality, not less than No. 2 Common. The span of sheathing of this size is usually limited by deflection, rather than strength, although the strength should always be investigated. Roofs are always walked upon at some time or another, and appreciable deflection of the sheathing will tend to break off the tongues of tongue-and-grooved lumber. Shiplap, instead of tongue-and-grooved lumber, may be used. The two sections are shown in Figs. 69 and 70.

57*b*. Spacing of Roof Joists.—If the roof joists support the ceiling also, their spacing should not exceed 16 in., as this is the limiting span for wooden laths with plaster ceiling.

On the Pacific Coast, where no snow, or at most very light snow occurs, the spacing of roof joists, when no ceiling must be provided for, is commonly taken at 24 in., and in cheap construction the spacing is made 32 in.

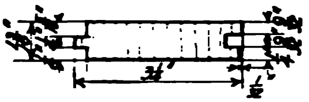


Fig. 69.—Section of 1 × 4-in. tongue and grooved flooring.

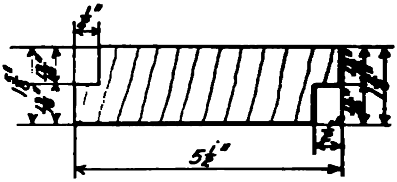


Fig. 70.—Section of 2 × 6-in. shiplap.

57c. Arrangement of Girders or Trusses.—The arrangement of girders and trusses is a matter worthy of study in any building. Usually there are requirements of interior arrangement which dictate the spacing of columns.

Trusses are most economically spaced at approximately 16 to 20 ft. Three methods of framing the roof joists or rafters may be adopted: (1) Supporting the joists directly on the upper chords; or (2) placing roof girders or purlins at the panel points of the trusses and spanning the bays between purlins by light rafters; or (3) providing purlin trusses at certain panel points and spanning between the purlin trusses by means of rather heavy rafters, or roof joists. There are, naturally, advantages and disadvantages to each system. Considering vertical loads above, the particular building involved may carry with it some special reason for adopting one method in preference to the others. From the standpoint of cost alone, it will usually be found upon investigation, that, if the different systems are designed correctly and consistently, there will be little difference in cost. In some localities, the relatively high price of steel compared to lumber may warrant a minimum of truss work and the employment of larger sizes of lumber. In other localities the cost of securing the larger sizes of joists may make small spans advisable. No hard and fast rule can be laid down.

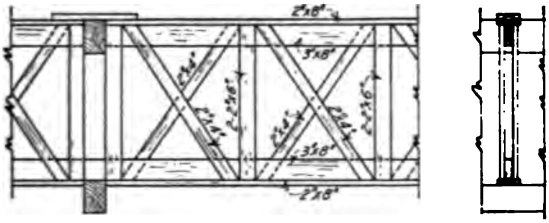


Fig. 71.—Detail of typical roof bracing truss.

57d. Bracing Trusses.—Bracing trusses are a necessity in long truss spans; in fact, the writer recommends that all roof trusses over 20-ft. span be provided with at least one bracing truss, and that, in general, bracing trusses be placed at a spacing not greater than 15 or 16 ft. The bracing trusses may be utilized as purlin trusses if properly proportioned. They should be of the full depth of the main truss, and well connected thereto. The compression chord of a main roof truss needs to be supported laterally for column action; the lower chord should also be stayed laterally for general stiffness of the building, if for no other reason. Such bracing trusses may be made up of dimension lumber and spiked or bolted together, and thus give a comparatively cheap, and at the same time, effective construction. A typical example of such a bracing truss is shown in Fig. 71. Attention is called to the "*T*" section of chords, also to the details for connection to the main trusses.

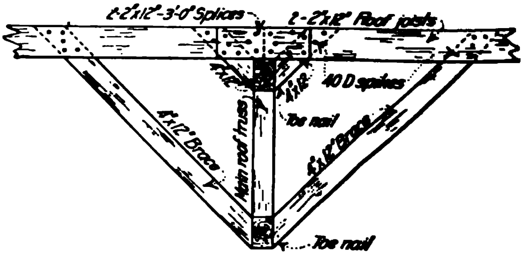


Fig. 72.—Knee brace system of truss bracing.

Another method for providing general stiffness in the roof framing is shown in Fig. 72. In this detail the roof joists are doubled at certain intervals; braces or struts are framed be tween the double joists, and the bottom of these struts fitted against and attached to the lower chords of the truss.

The actual stresses coming upon a bracing truss are usually indeterminate. With study of the roof framing plan, however, a definite scheme of wind bracing may be provided, in which the bracing trusses will play a vital part. The whole roof, or one side of the roof, may be regarded as a horizontal beam, or truss, transferring the wind reactions delivered thereto from the side walls to the end walls, or to columns and walls. Following out this scheme, diagonal rods may be placed in the plane of the upper chords of the roof trusses.

Fig. 73 shows an arrangement of roof trusses, bracing trusses and diagonal rods for an assumed small building of the mill-building type. When the length of a building is three or more times its breadth, and such building is only moderately high, the diagonal rods may very frequently be omitted in some of the outer side bays. It may also be possible, without endangering the rigidity of the building, to make some of the lines of bracing trusses non-continuous throughout the length of the building. For example, in Fig. 73, were the building twice as long as shown, it might be entirely consistent with safety to omit alternate bracing trusses in the first and third lines, keeping the center line of bracing continuous. It must be obvious that the exact arrangement of bracing in a roof is almost entirely a matter of judgment, but judgment based on an understanding of the fundamental principles of structural mechanics and experience in design and construction. While it is granted that the actual stresses in a roof due to wind are impossible to find, an assumption of a reasonable wind pressure and a definite and logical system of bracing consistently followed out in all details will insure a much safer structure than a "hit-or-miss" or "rule-of-thumb" procedure, and will also result in a more economical building than one composed of heavier sections, poorly braced.

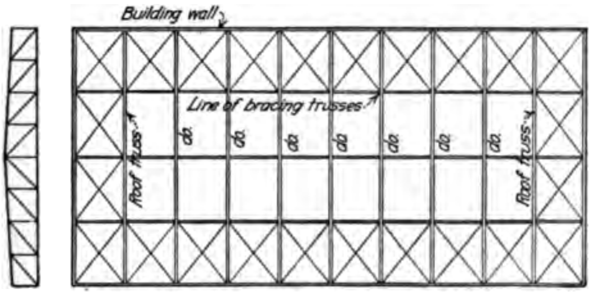


Fig. 73.—Diagrammatic plan of typical roof bracing.

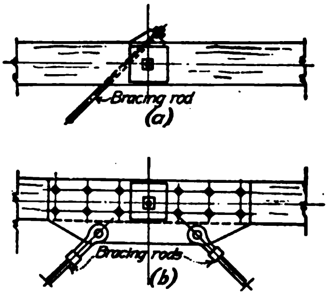


Fig. 74.—Typical details of connection of bracing rods to upper chord of roof truss.

Two typical details of connections of such diagonal rods to the roof trusses are shown in Fig. 74. In Fig. 74(a) the rods are passed through holes bored diagonally through the chord, and fitted with special beveled cast-iron washers. In Fig. 74(b) a steel plate is lag-screwed to the chord, and connection between plate and rods is secured by means of clevises and pins. If the roof joists are supported directly upon the upper chord, these plates will probably have to be attached to the lower side of chord. In such a case, the plates should be fastened to the chord while the truss is on the ground. It may be taken for granted that such connection, if made after the truss is erected, will be poor. It is difficult, at best, to make a carpenter screw lag-screws into place, and it is almost certain, if placed by a man on a scaffold, that the work will be poorly done.

Obviously, the system of diagonal bracing rods just described may be placed in the plane of the lower chords of the trusses, provided that bracing trusses exist to form the chords of the wind resisting truss. Provision must be made for supporting the rods to prevent them from sagging.

Diagonal rods in the plane of the roof framing, placed in the outer bays, are an excellent thing; they enable the building to be "squared up" and will do much to prevent racking of the roof due to wind, with possible consequent breaking of skylights. Re-tightening of these bracing rods will be necessary from time to time as shrinkage of the timber takes place.

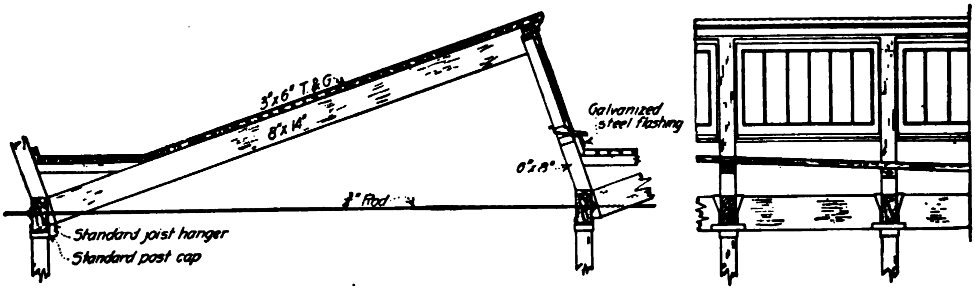
57e. Saw-tooth Roof Framing.—Saw-tooth roofs are constructed with inclined or vertical faces, the former being perhaps more generally used than the latter on account of better diffusion of light. From the standpoint of maximum efficiency in diffused lighting, the saw-teeth should face north with the faces inclined at an angle of 25 to 30 deg. with the vertical.

The saw-tooth with vertical face is somewhat easier to construct and is less likely to give trouble through leakage and condensation than the inclined face construction. In the latter type, there should be no horizontal mullions in the windows, since water would stand on these and eventually leak through. Further, condensation will tend to take place on the inner side of the inclined glass and drop vertically on the contents of the building.

In both types of construction, careful attention must be given to the design of the windows, whether fixed or movable sash. The flashing should run under the window sill and form an inside condensation gutter discharging into conductors. Double glazing is sometimes employed in the more northerly latitudes on account of its nonconducting qualities.

Some typical details of saw-tooth roofs are shown in Figs. 75, 76, 77, 78 and 79.

The roof planking should be at least 3 in. in thickness, tongued-and-grooved or splined, spanning 8 to 10 ft. between the inclined roof beams. The valleys between the saw-teeth should have an inclination of not less than ½ in. to the foot, and the conductors should be spaced not more than 50 ft. apart. The construction of the sloping valleys is easily accomplished by blocking between the structural members of the frame.



Cross section through typical saw tooth.

Partial elevation of saw tooth.

FIG. 75.

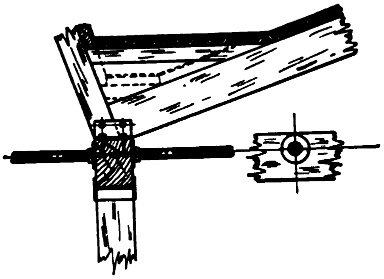


FIG. 76.—Detail of saw-tooth frame—inclined face with pipe ties.

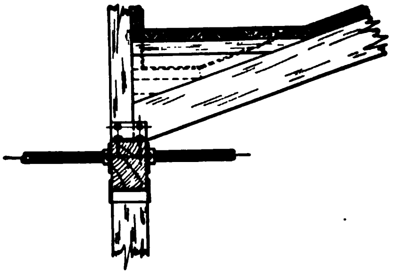


FIG. 77.—Detail of saw-tooth frame—vertical face with pipe ties.

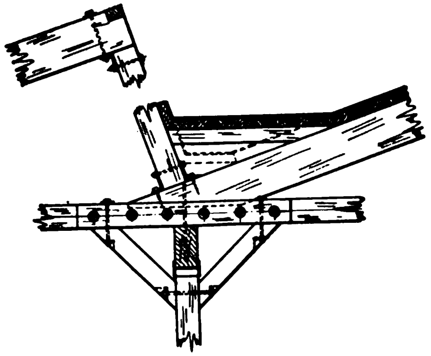


FIG. 78.—Detail of saw-tooth frame—inclined face with timber ties.

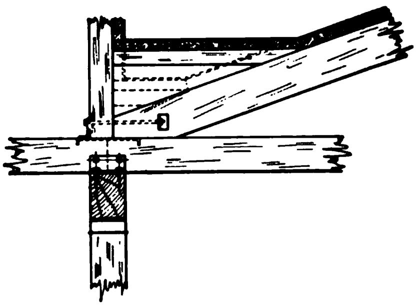


FIG. 79.—Detail of saw-tooth frame—vertical face with timber ties.

Fig. 75 illustrates a typical construction with inclined faces. The roof joists are supported at their upper ends on inclined posts, and at their lower ends by joist-hangers on the roof girders. Tie rods are shown at the foot of each inclined roof beam to prevent the roof from spreading. While the construction shown in this figure may be termed standard, objection can be raised (1) to the use of joist-hangers, (2) to the use of small tie rods exposed to fire and tending to sag, (3) to the absence of any horizontal members at the top of posts to take thrust, and (4) to the absence of general stiffness of frame to horizontal forces.

In Figs. 76 and 77 the above objections are largely met by bringing the inclined roof beams to rest on the top of the girders and the substitution of pipe ties between the roof girders. These pipe ties, fitted with standard flanges and bolted through the girders, have the advantage over rods of being able to take both tension and compression and also of not requiring hangers to prevent them from sagging. These pipes, however, must be of fairly large size in order that they may be of value as compression members. The ratio of length of member to radius of gyration should not exceed 175. This construction, however, still gives metal exposed to fire.

Figs. 78 and 79 illustrate an all-timber type of construction. These details, drawn for both the inclined and vertical face types of saw-tooth, furnish a simple and effective construction. A somewhat higher building is required by this construction than with that of Fig. 75 but the general stiffness gained, and the absence of exposed metal, will more than offset the cost of increased height of walls.

58. Mill Construction.[1]—The preceding discussion in this chapter has related to timber framed floors and roofs in general. This article treats very briefly and in a general way of the special type of construction known as "Mill Construction," or "Slow-burning Mill Construction," so-called because it was developed for use in factory or mill buildings in the New England states. In this construction all timbers, as posts, girders, and joists, are made of large section; joists are eliminated as far as possible, by substituting a heavy thick floor sufficient in strength to span some feet. The result gives a building having large areas of flat ceilings, and heavy, solid masses of timber in girders and posts. Such a structure in case of fire will tend to char rather than burn, and all parts are easily reached by the water from the automatic sprinklers. This type of building, properly sprinkled, takes a comparatively low insurance rate.

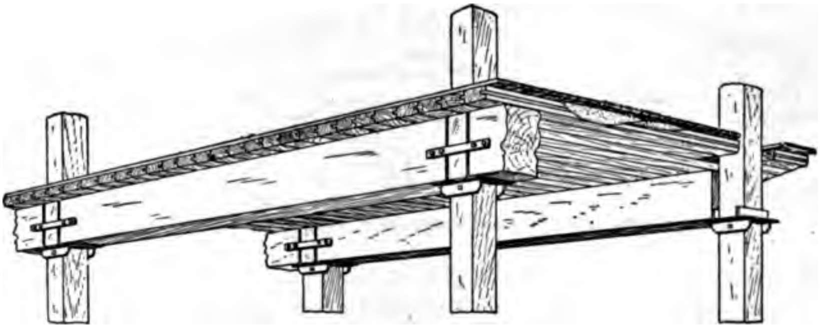


Fig. 80.—Standard mill construction.

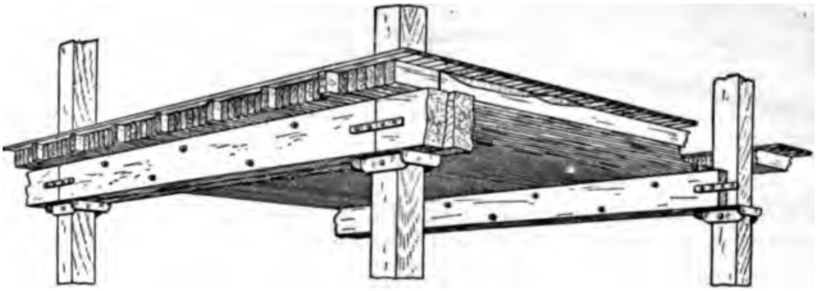


Fig. 81.—Mill construction with laminated floor.

In the bulletin, "Heavy Timber Mill Construction Buildings," published by the Engineering Bureau of the National Lumber Manufacturers Association, mill construction is divided into three classes as follows (see Figs. 80 to 84 inclusive):

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[1] See also the following chapter by F. W. Dean.

1. Floors of heavy plank laid flat upon large girders which are spaced from 8 to 11 ft. on centers. These girders are supported by wood posts or columns spaced from 16 to 25 ft. apart. This type is often referred to as "Standard Mill Construction."

2. Floors of heavy plank laid on edge and supported by girders which are spaced from 12 to 18 ft. on centers. These girders are supported by wood posts or columns spaced 16 ft. or over apart, depending upon the design of the structure. This type is called "Mill Construction with Laminated Floors."

3. Floors of heavy plank laid flat upon large beams which are spaced from 4 to 10 ft. on centers and supported by girders spaced as far apart as the loading will allow. These girders are carried by wood posts or columns located as far apart as consistent with the general design of the building. A spacing of from 20 to 25 ft. is not uncommon for columns in this class of framing where the loading is not excessive. This type is more generally known as "Semi-Mill Construction."

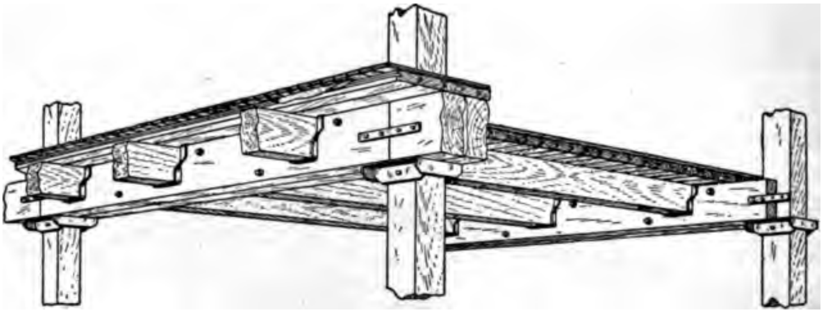

Fig. 82.—Semi-mill construction, beams in hangers.

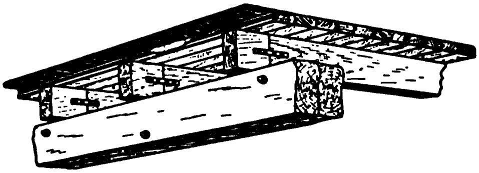

Fig. 83.—Semi-mill construction, beams on top of girders.

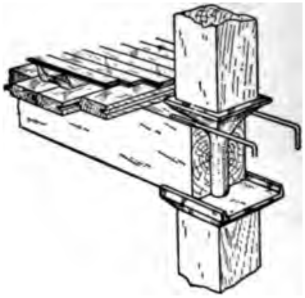

Fig. 84.—Detail of column and girder construction with cast-iron pintle.

The following clauses from the Building Code recommended by the National Board of Fire Underwriters, also define in detail the timber construction classed as mill constructions

Definition: "Mill" Construction (also called "Slow-burning Construction") is a term applied to buildings having masonry walls and heavy timber interior construction with no concealed spaces. Such buildings are usually occupied for factory purposes, and should always be protected by a system of automatic sprinklers.

Columns and Girders or Floor Timbers:

1. Columns, if of timber, shall be not less than 8 in. in smallest cross-sectional dimensions and all corners shall be rounded or chamfered.

2. Wooden columns shall be superimposed throughout all stories on iron or steel post caps with brackets.

Note: Columns should never rest on timbers, as shrinkage may cause them to sag.

3. Iron or steel columns or girders may be used if protected, as follows: Steel girders and steel or iron columns which support masonry walls, other than those facing upon a street, shall be protected by at least 2 in. of fireproofing. . . . or by 2 in. of metal lath and cement plaster; the latter being applied in two layers with an air space between them. All other iron or steel columns shall be protected by at least 1 in. of metal lath and cement plaster or its equivalent.

4. Wooden girders or floor timbers shall be suitable for the load carried, but in no case less than 6 in. either dimension, and shall rest on iron plates on wall ledges and where entering walls shall be self-releasing. Walls may be corbeled out to support floor timbers where necessary. The corbeling shall not exceed 2 in.

5. So far as possible, girders or floor timbers shall be single stick.

6. Where wooden beams enter walls on opposite sides, there shall be at least 12 in. of masonry between ends of beams, and in no case shall they enter more than one-quarter the thickness of the wall.

7. Width of floor bays shall be between 6 and 11 ft.

Note: The practice in "mill" construction of supporting the ends of beams on girders by means of metal stirrups or bracket hangers is objectionable. Experience has shown that such metal supports are likely to lose their strength when attacked by fire and so cause collapse.

Floors:

1. Floors shall be not less than 3-in. (2⅝-in. dressed) splined or tongued and grooved plank covered with 1-in. (⅞-in dressed) flooring laid crossways or diagonally. Top flooring shall not extend closer than ½ in. to walls so as to allow for swelling in case floor becomes wet. This place shall be covered by a moulding so arranged that it will not obstruct movement of the flooring.

2. Waterproofing shall be laid between the planking and the flooring in such manner as to make a thoroughly waterproof floor to a height of at least 3 in. above floor level. When there are no scuppers, the elevator or stairwells may be used as drains for the floors, in which case the waterproofing material need not be flashed up at these points.

3. All exposed woodwork in interior construction shall be planed smooth.

Roofs, Skylights, and Cornices:

1. Roofs shall be of plank and timber construction and flat, except for pitch necessary for proper drainage. Plank shall be not less than 2½ in. (2⅛ in. dressed) splined, or tongued and grooved. Timbers shall be not less than 6 in. either dimension and shall be single stick.

Both roof timbers and planks shall be self-releasing as regards walls.

Note: The saw-tooth form of roof is considered satisfactory, although not quite the equivalent of a flat mill constructed roof.

Partitions:

Partitions shall be constructed of incombustible material or of 2-in. matched plank or double matched boards with joints broken, preferably coated with fire retarding paint.

Note: Ordinary paint is not objectionable, but varnish or shellac is very undesirable.

The following description of laminated floors is taken from the bulletin of the National Lumber Manufacturers Association referred to above:

If heavy loads are to be carried on long spans, planks 6 or 8 in. wide are set on edge close together, firmly nailed at each end and at about 18-in. intervals with 60-D nails, alternating top and bottom, thus forming a "laminated floor." Each of these floors is covered with two or more thicknesses of waterproof paper or similar material and then by a top, or wearing, floor. laid at right angles to the direction of the underfloor. Material is surfaced on all sides and edges of plank beveled to serve as a finish on the ceiling below.

Where plank floors are laid flat, the boards should be two bays in length if possible and laid to break joints every 4 ft. With laminated floors, it may be difficult to obtain plank two bays in length. In such a case, the planks may be laid with the ends extending between centers of girders with one plank laid across the girder at frequent intervals (every sixth or eighth piece) to act as a tie in the floor. Or, by another method, the ends of planks should join at or near the quarter point of the span between girders, taking care to break joints in such a way that no continuous line across the floor will occur.

In laying laminated floors, it is advisable to omit the last two planks at walls until after glazing and roofing have been completed. Then these spaces should be filled in close against the walls. It is often recommended that laminated floors be laid without nailing to the girders which support the floor, so that expansion in the floors due to dampness will not cause movement in the girders at the walls.

The top-floor may be of softwood or hardwood as use demands. Tongued and grooved flooring is used almost entirely. Square-edged flooring is easier to replace when repairs are needed, but wears less around nails, thus making an uneven floor. Some of the best buildings have a double top-floor, the lower part of softwood laid diagonally upon the plank under-floor, and the hardwood upper part laid lengthwise. This latter method allows boards in alleys or passages to be easily replaced when worn, and the diagonal boards brace the floors, reduce vibration, and distribute the floor load evenly. The top-floor should always be laid so that the length of the pieces is parallel to the direction of the traffic or trucking. Usually this is lengthwise of the building.

When a laminated floor is constructed of material surfaced four sides, or of material surfaced two sides, there is great danger of dry rot, unless the lumber is thoroughly air seasoned or kiln dried. On account of this feature, many engineers prefer to use only rough lumber for laminated floors, the slight unevenness of the boards or planking providing enough air spaces to prevent dry rot. It is the writer's opinion that the rough flooring, besides being cheaper, will give additional security against the decay of the timber.

Tables 2 and 3 give the maximum spans for timber mill laminated floors for thicknesses varying from 3 in. nominal to 12 in. nominal, fiber stresses from 1200 to 1800 lb. per sq. in., and loads from 50 to 400 lb. per sq. ft.

In both these tables, the limiting span is given for a deflection of 1/30 in. per foot of span, based on a modulus of elasticity of 1,620,000. Since mill floors in general have no ceiling, the deflections taken from this table may be used directly, although, if the permanent deflection is desired, a reduced modulus of elasticity for the constant loads should be used.

TABLE 2.*—MAXIMUM SPANS FOR TIMBER MILL FLOORS

Fiber stress 1200, 1300, 1500, 1600 and 1800 lb. per sq. in.; modulus of elasticity, 1,620,000 lb. per sq. in.
The sum of the live load and the weight of the floor was used in calculating the spans.
In the line marked deflection is given the span which has a deflection of 1/30 in. per foot of span.
Made of planks on edge, laid close.

Fiber stress (lb. per sq. in.)	Span in feet Live load in pounds per square foot											
	50	100	125	150	175	200	225	250	275	300	350	400
	(3 in. Nominal thickness—2⅝ in. actual thickness											
1200	13' 8"	10' 1"	9' 1"	8' 4"	7' 9"	7' 3"	6'10"	6' 6"	6' 3"	6' 0"	5' 7"	5' 2"
1300	14' 3"	10' 6"	9' 6"	8' 8"	8' 1"	7' 7"	7' 2"	6'10"	6' 6"	6' 3"	5' 9"	5' 5"
1500	15' 4"	11' 3"	10' 2"	9' 4"	8' 8"	8' 2"	7' 8"	7' 4"	7' 0"	6' 8"	6' 2"	5'10"
1600	15'10"	11' 8"	10' 6"	9' 7"	8'11"	8' 4"	7'11"	7' 7"	7' 2"	6'11"	6' 5"	6' 0"
1800	16' 9"	12' 4"	11' 2"	10' 3"	9' 6"	8'11"	8' 5"	8' 0"	7' 8"	7' 4"	6' 9"	6' 4"
Defl.	9' 0"	7' 4"	6'11"	6' 6"	6' 2"	5'11"	5' 8"	5' 6"	5' 4"	5' 2"	4'11"	4' 9"
	(4 in. Nominal thickness—3⅝ in. actual thickness)											
1200	18' 5"	13' 8"	12' 4"	11' 5"	10' 7"	10' 0"	9' 5"	9' 0"	8' 7"	8' 3"	7' 7"	7' 2"
1300	19' 2"	14' 3"	12'11"	11'10"	11' 0"	10' 4"	9'10"	9' 4"	8'11"	8' 7"	7'11"	7' 5"
1500	20' 7"	15' 4"	13'10"	12' 9"	11'10"	11' 2"	10' 6"	10' 0"	9' 7"	9' 2"	8' 6"	8' 0"
1600	21' 3"	15'10"	14' 4"	13' 2"	12' 3"	11' 6"	10'11"	10' 4"	9'11"	9' 6"	8'10"	8' 3"
1800	22' 7"	16' 9"	15' 2"	13'11"	13' 0"	12' 2"	11' 7"	11' 0"	10' 6"	10' 1"	9' 4"	8' 9"
Defl.	12' 3"	10' 1"	9' 5"	8'11"	8' 6"	8' 2"	7'10"	7' 7"	7' 4"	7' 2"	6'10"	6' 6"
	(5 in. Nominal thickness—4⅝ in. actual thickness)											
1200	22'10"	17' 8"	15' 7"	14' 5"	13' 5"	12' 7"	11'11"	11' 4"	10'10"	10' 5"	9' 8"	9' 1"
1300	23'10"	17'11"	16' 3"	14'11"	13'11"	13' 1"	12' 5"	11'10"	11' 4"	10'10"	10' 1"	9' 5"
1500	25' 7"	19' 3"	17' 5"	16' 1"	15' 0"	14' 1"	13' 4"	12' 8"	12' 2"	11' 8"	10'10"	10' 2"
1600	26' 5"	19'11"	18' 0"	16' 7"	15' 6"	14' 7"	13' 9"	13' 1"	12' 6"	12' 0"	11' 2"	10' 6"
1800	28' 0"	21' 1"	19' 1"	17' 7"	16' 5"	15' 5"	14' 7"	13'11"	13' 4"	12' 9"	11'10"	11' 1"
Defl.	15' 4"	12' 9"	11'11"	11' 3"	10' 9"	10' 4"	10' 0"	9' 8"	9' 4"	9' 1"	8' 8"	8' 4"
	(6 in. Nominal thickness[1]—5⅝ in. actual thickness[1])											
1200	....	20' 8"	18' 9"	17' 4"	16' 2"	15' 3"	14' 5"	13' 9"	13' 2"	12' 8"	11' 9"	11' 0"
1300	....	21' 6"	19' 6"	18' 0"	16'10"	15'10"	15' 0"	14' 3"	13' 8"	13' 1"	12' 2"	11' 5"
1500	....	23' 1"	21' 0"	19' 4"	18' 1"	17' 0"	16' 1"	15' 4"	14' 8"	14' 1"	13' 1"	12' 3"
1600	....	23'10"	21' 8"	20' 0"	18' 8"	17' 7"	16' 7"	15'10"	15' 2"	14' 7"	13' 6"	12' 8"
1800	....	25' 3"	23' 0"	21' 2"	19'10"	18' 8"	17' 8"	16'10"	16' 1"	15' 5"	14' 4"	13' 6"
Defl.	....	15' 4"	14' 5"	13' 8"	13' 0"	12' 6"	12' 1"	11' 8"	11' 4"	11' 0"	10' 6"	10' 1"

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* From Southern Pine Manual.

[1] Use for laminated floors when made of 2 × 6 and 4 × 6 pieces.

TABLE 3.*—MAXIMUM SPANS FOR TIMBER LAMINATED FLOORS

**Fibers stress 1200, 1300, 1500, 1600 and 1800 lb. per sq. in.; modulus of elasticity, 1,620,000 lb. per sq. in.
The sum of the live load and the weiget of the floor was used in calculating the spans.
In the line marked deflection is given the span which has a deflection of $\frac{1}{30}$ in. per foot of span.
Made of planks on edge, laid close.**

Fiber stress (lb. per sq. in.)	Span in feet Live load in pounds per square foot										
	100	125	150	175	200	225	250	275	300	350	400
(6 in. Nominal thickness[1]—5½ in. actual thickness)											
1200	20′ 3″	18′ 4″	16′11″	15′10″	15′ 1″	14′ 1″	13′ 5″	12′10″	12′ 4″	11′ 6″	10′ 9″
1300	21′ 1″	19′ 1″	17′ 8″	16′ 5″	15′ 8″	14′ 8″	14′ 0″	13′ 4″	12′10″	11′11″	11′ 2″
1500	22′ 7″	20′ 9″	18′11″	17′ 8″	16′10″	15′ 9″	15′ 0″	14′ 4″	13′ 9″	12′10″	12′ 0″
1600	23′ 4″	21′ 3″	19′ 7″	18′ 3″	17′ 5″	16′ 4″	15′ 6″	14′10″	14′ 4″	13′ 3″	12′ 5″
1800	24′ 9″	22′ 6″	20′ 9″	19′ 4″	18′ 5″	17′ 3″	16′ 5″	15′ 9″	15′ 1″	14′ 0″	13′ 2″
Defl.	15′ 0″	14′ 1″	13′ 4″	12′ 9″	12′ 3″	11′ 9″	11′ 5″	11′ 1″	10′ 9″	10′ 3″	9′10″
(8 in. Nominal thickness—7½ in. actual thickness)											
1200	26′10″	24′ 6″	22′ 8″	21′ 2″	20′ 0″	19′ 0″	18′ 1″	17′ 4″	16′ 7″	15′ 6″	14′ 7″
1300	27′11″	25′ 6″	23′ 7″	22′ 1″	20′10″	19′ 9″	18′10″	18′ 0″	17′ 4″	16′ 1″	15′ 2″
1500	30′ 0″	27′ 5″	25′ 4″	23′ 9″	22′ 4″	21′ 2″	20′ 3″	19′ 4″	18′ 7″	17′ 4″	16′ 3″
1600	31′ 0″	28′ 3″	26′ 2″	24′ 6″	23′ 1″	21′11″	20′10″	20′ 0″	19′ 2″	17′10″	16′ 9″
1800	32′10″	30′ 0″	27′ 9″	26′ 0″	24′ 6″	23′ 3″	22′ 2″	21′ 2″	20′ 4″	19′ 0″	17′10″
Defl.	20′ 1″	19′ 4″	17′11″	17′ 2″	16′ 6″	15′11″	15′ 5″	15′ 0″	14′ 7″	13′11″	13′ 4″
(10 in. Nominal thickness—9½ in. actual thickness)											
1200	.....	.....	.....	.....	.....	.....	.....	.....	20′10″	19′ 5″	18′ 3″
1300	.....	.....	.....	.....	.....	.....	.....	.....	21′ 9″	20′ 3″	19′ 1″
1500	.....	.....	.....	.....	.....	.....	.....	.....	23′ 4″	21′ 9″	20′ 5″
1600	.....	.....	.....	.....	.....	.....	.....	.....	24′ 1″	22′ 5″	21′ 2″
1800	.....	.....	.....	.....	.....	.....	.....	.....	25′ 7″	23′10″	22′ 5″
Defl.	.....	.....	.....	.....	.....	.....	.....	.....	18′ 4″	17′ 6″	16′10″
(12 in. Nominal thickness—11½ in. actual thickness)											
1200	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	22′ 0″
1300	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	22′11″
1500	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	24′ 7″
1600	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	25′ 4″
1800	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	26′11″
Defl.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	20′ 3″

SLOW-BURNING TIMBER MILL CONSTRUCTION[2]

BY F. W. DEAN

Slow-burning mill construction[3] is the name applied to a long-used type of fire-resisting timber building common in New England, especially in textile mills. As now designed by the best mill engineers, it consists of brick walls, with heavy transverse wood beams, on top of which, for floors, are spiked thick planks at right angles to the beams, and these planks are covered with a top floor at right angles to the planks. The planks are grooved on both edges and so-called

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*From Southern Pine Manual.

[1] Use for 2½ × 6, 3 × 6, and 6 × 6 pieces, for 2 × 6 and 4 × 6 use table for mill floors (Table 2).

[2] Appeared in *Engineering News-Record*, Vol. 79, No. 26, Dec. 27, 1917.

[3] See also the preceding chapter by Henry D. Dewell.

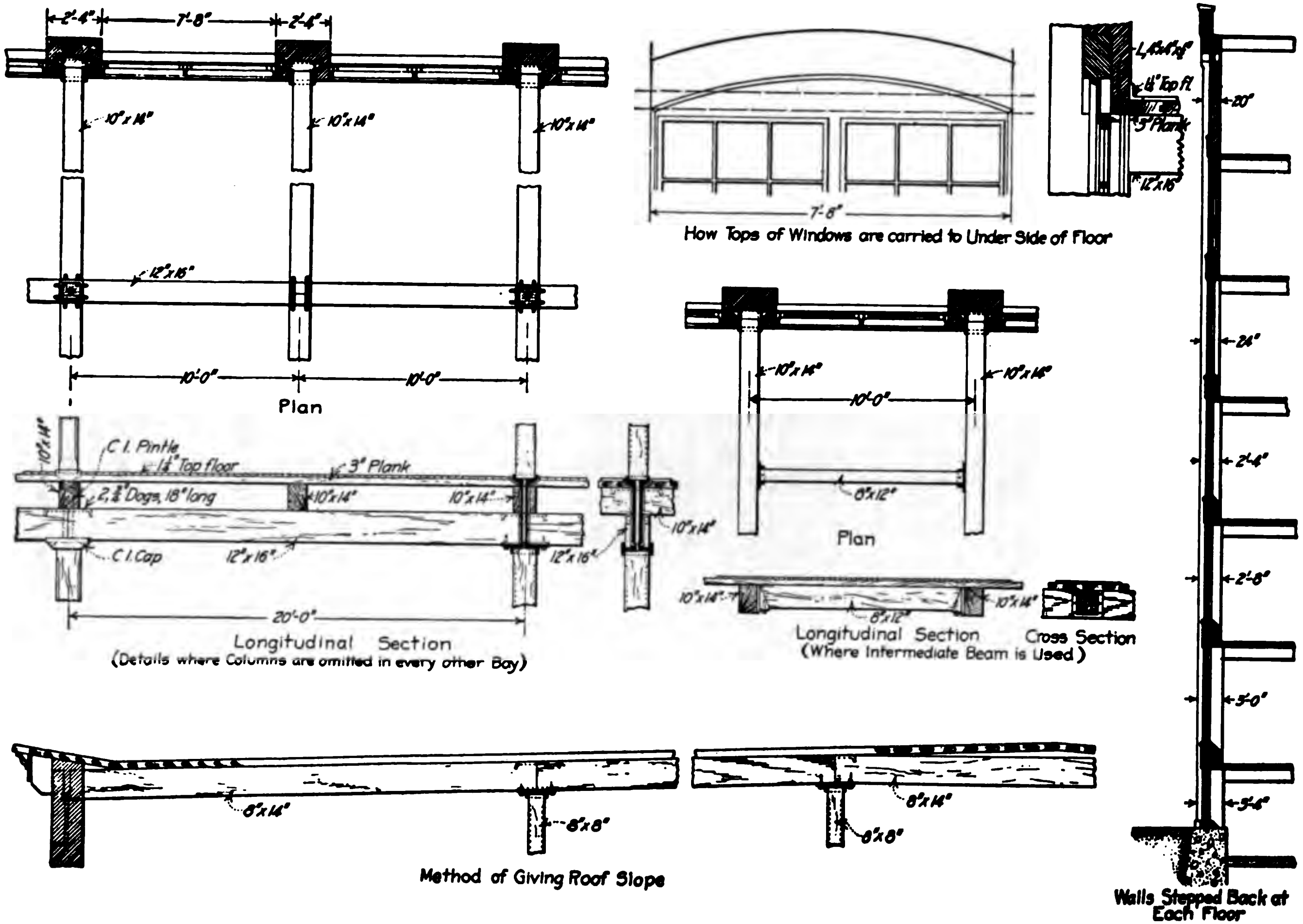

FIG. 85.—Wall, column and floor details of typical mill construction.

wood splines are tightly driven into the grooves of adjoining planks so that one plank will assist in the support of the next, thus stiffening the floor for isolated loads and preventing one plank from moving vertically relatively to the next (Fig. 86). The spaces between the beams or the "bays" should not be so wide as to require beams at right angles to the main beams, or any subdivision of the bays. A maximum bay width of 10 ft., except to accomplish a special object, is advisable. Wherever any metal is used it should always be deeply buried within the wood so that fire cannot reach it at first.

From the above it will be seen that real mill construction contemplates the smallest practicable number of heavy smooth beams covered with heavy smooth plank in turn covered with a top floor. The mass of such construction, the small amount of surface and the smoothness of the surfaces make this type of construction fire resisting, and merit the name often applied to it, of being "slow-burning." Compared with this, the floor and roof construction consisting of planks on edge for beams and a foot or two apart are kindling wood. Mill construction also contemplates the entire absence of concealed spaces and the use of such spaces as can readily be reached by the spray from the smallest number of automatic sprinklers. It will readily be seen that the spaces between the beams of mill construction can be reached by a few sprinklers, while with the older construction, many times as many sprinkler pipes and heads are required to give protection, as every part must be reached by the spray.

The beams of mill construction afford opportunity for supporting shaft hangers, and the shaft hangers and the spaces between them give room for pulleys and belts. If short countershafts are to be put up, the wide flat surfaces between the beams afford an opportunity for attaching them.

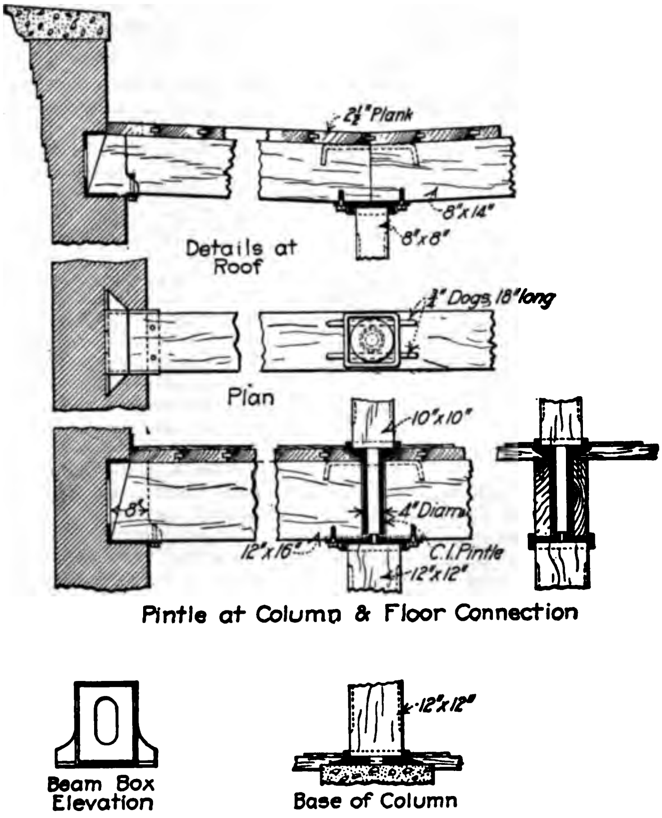

Fig. 86.—Some special details used in framing mill construction.

59. Pintles Over Columns are Fundamental to Type.—The method of fastening the beams to each other where they butt together, and to the walls, is of great importance in securing rigidity. This must be considered in connection with the columns, and it is with respect to these and connecting the beams together that architects unversed in this type invariably fail. It is well understood that columns should rest end to end upon each other from top to bottom of buildings, but the columns themselves should not pass through the floors and between the ends of the beams, as is often done. Proceeding upward they should stop at the bottoms of the beams, and begin again at the tops. Between the top of one column and the bottom of the one above it there should be a short separate cast-iron column known as a "pintle" (Fig. 86). Being of cast iron, which is a material of great compressive resistance, it may be very small in diameter, and requires only a small hole through the beam to accommodate it, half of the hole being in the end of one beam and half in the other. The lower end of the pintle rests on the cap of the lower column and the top of the pintle receives the lower end of the column above.

There are several advantages in this construction. In consequence of it, the beams actually butt against each other, and having only a small hole through them (not much over 4 in. in diameter), the ends of the beams are actually over the body of the column and are not supported by the overhanging ends of the column cap. If a cap end is burnt off or breaks off the beam is held as securely as ever. It is a common thing for architects to carry the lower end of a column to the top of the one below, and sometimes both columns are of the same size. The result is that the beams are supported by the overhanging ends of the column caps. This is dangerous construction, in respect to both strength and fire resistance. The end may break if of cast iron, bend if of steel, become soft in a fire and cause the floor to fall. In this construction most of the cap, and the whole of the part which supports the beam are exposed to the fire.

The pintle construction, as before stated, permits the beams to butt against each other and thus become perfect struts to transmit pressure from one side of the building to the other, and it also gives room to put two iron dogs, or ties in the tops of the beams, one on each side of the pintle, to tie the beams together. Thus the beams become not only struts but rigid and continuous ties to keep the sides of the building in their proper relative positions. At the same time the pintles and dogs fulfill the necessary conditions before mentioned of being surrounded by heavy wood, for the pintles are within the beams and the dogs are embedded in grooves in the tops of the beams and covered by floor planks. Moreover, the dogs cannot work out because they are beneath the planks (Fig. 86).

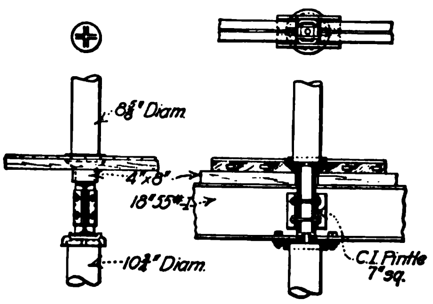

FIG. 87.—Pintle design for steel beams.

Where the pintles enlarge at the top to take the upper column only, the top edges should be exposed to fire and can scarcely be injured. It should not be overlooked that when the beams and planks shrink the pintle tops become more exposed than at first, and allowance should be made for this. It should be observed also that the enlarged hole for the top of the pintle is in the plank and not in the beam (Fig. 86).

Still another advantage of pintle construction is that if a floor falls and a column below is knocked over by the falling floor or a heavy piece of machinery, it simply tips over on top of the pintle. A column which goes down between the beams if knocked over would pry the beams apart, punch a hole through the wall, possibly push it over, and cause the beam to drop off the column and fall. Thus the building might be wrecked on account of the absence of pintle construction.

60. Rigidity of Connection is Necessary.—The beams must be connected to the wall in such a way that the walls will not be pushed or pulled until after this connection is made, such effort only coming from wind pressure or manufacturing strain. The beam ends should rest in cast-iron boxes with side wings firmly built into the walls (Fig. 86). The beams should then be made to butt firmly over the columns and be drawn together by driving in the dogs which for this purpose have their ends inclined where entering the wood. When this is done one or two lag screws should be screwed into the beams through holes in projecting lips of the beam boxes, which completes the connection across the building. After this is done the lips of the column caps are lag-screwed to the beams thus making the columns stable and preventing the beams from pressing against the pintles. Thus the column caps as well as the dogs hold the beams firmly together. No attempt should be made to have the pintle fit the hole, as it should be free to maintain its position as the beams are moved slightly when the dogs are driven. In fact, the hole for the pintle should be at least ½ in. larger in diameter than the pintle (Fig. 86).

Beams usually end over columns, but if they do not, a hole is bored through for the pintle, and dogs are not required.

Floor beams when double should have no space between them as was formerly provided in order to permit air to circulate between, as these spaces hold fire tenaciously.

There is a very pernicious practice of supporting intermediate cross beams so that their upper surfaces are level with those of the beams which they join. This is effected by the use of forged stirrups or commercial beam hangers. When a fire occurs they are easily softened, and give way if they support any material weight, which they often do. Beams should never be supported in this way if it is possible to avoid such construction, and if they are, heavy cast-iron beam-sockets should be used lag-screwed to the beams (Fig. 85). The commercial beam-hanger is a great menace to the safety of buildings.

Roofs are framed, supported and planked after the manner of floors, using dogs, and the latter should be driven before the brickwork is built around the anchors in the walls (Fig. 85). When there is not a row of columns in the center of the room, the roof beams should not be carried on the slant to the center of the building and there fastened together, with the expectation that a stable roof will result. Horizontal beams should run between the two rows of columns next to the center and the roof slant given by wedge-shaped pieces spiked to the beams (Fig. 85). Roof beams are not usually secured to the walls by means of beam-boxes, but they might be advantageously employed (Fig. 86), especially if parapet walls are used. Wrought-iron anchors spiked or screwed to the beams are generally used (Fig. 85).

61. Special Beam Arrangements Possible.—Sometimes it is desired to have columns omitted in every other bay, and the beams that do not rest on columns must be supported by longitudinal stringers resting on top of the columns that are used. Many architects in this case yield to the temptation to use the beam-hangers disapproved of above, but instead of this the stringers should be lower than the transverse beams by the depth of the latter, and the intermediate transverse beams should rest on top of them, and be fastened thereto (Fig. 85). Thus slow-burning construction is fully realized in this detail. The stringers are separated from the floor by the depth of the transverse beams, and the space thus provided is very convenient for the upper strands of belting. In this construction vertical shrinkage of the beams is considerable, and the pintles, which are long enough to go through both longitudinal and transverse beams, should be rather short, so that after the shrinkage the top will not appear materially above the floor.

Floor planks are usually 2½ to 5 in. thick and in widths not exceeding 10 in. They should be at least two bays long, but there must be enough one-bay lengths to cause breaking of joints. It is not necessary to have every other plank break joints; four or five planks of the same length can be laid side by side and the next set can break joints with these. Where floor planks are interrupted by pintles they should be fitted under the pintle to some extent, so that their ends will rest on the beams. This with the splines and top floors will support them. Otherwise they should rest on a stick secured to the adjoining planks by lag screws.

62. Location of Beams.—It is inadvisable to have beams at right angles to the main transverse beams in a factory—that is, parallel to the sides of the mill. One objection to this is, that if they are not at or near the center of the building they cut off the top light. Some architects wrongly place such beams against the sides of the building above the windows, and thereby prevent the tops of the windows from being as high as they might be, and close to the underside of the floor. These beams are hung to the transverse beams by means of the objectionable hangers already commented upon, and even intermediate transverse beams are sometimes hung to them. If the bays are not too wide intermediate beams are unnecessary, but architects often make the bays so wide that they are compelled to use intermediate beams, and this causes them to run the planks the wrong way.

The tops of the windows can be brought to the underside of the floor by building the arch in the next story above. The opening which would thus be made above the upper floor is closed by not carrying the arch clear through the wall. One course of brick carried down to the springing of the arch is sufficient to close the opening, and this is supported by an angle iron spanning the window opening (Fig. 85). If a straight arch is used this is supported by angles or other forms of structural material.

The beams are usually made of long-leaf Southern pine, which formerly came chiefly from Georgia, but the name "Georgia pine" is now chiefly a name, as such pine comes from any state south of North Carolina, and even from Cuba. Beams should be chamfered on the lower edges between bearings for the sake of appearance, and, some persons have stated, to do away with corners which readily ignite.

63. Floor Details.—Floor planks are made of spruce or pine, planed on three sides, grooved for splines, and for appearance slightly bevelled or beaded on the bottom edges. The splines are made of clear yellow pine and are always ¾ × 1½ in. and, as already stated, should fit tight enough to require driving in. The planks should end on the centers of the beams, and be nailed to each beam with two nails of such lengths that two or three inches should penetrate the beams.

On each side of a room a plank should be left out until the building is well dried, as the planks sometimes swell enough to push out the walls.

On the planks there should be one or two layers of tarred paper, or, better, a paper covered with an elastic material which will fit around the nails securing the top floor, in order to make the floor waterproof. Such a lining is intended to continue to be tight around nails after the floors shrink.

In Canada, and to some extent in this country, it is the practice to use for floors, planks on edge nailed together horisontally. It is not customary to end these planks over the beams, but anywhere. This weakens the floor seriously and should not be permitted. Sometimes, if such floors are very thick, they are not fastened to the beams.

Top floors are usually of square-edged maple of ⅞-in. nominal thickness, but sometimes thicker. The boards are commonly 5 in. wide and should not be less than 6 ft. long. They should be kiln dried, wedged together when laid, nailed with two nails 16 in. apart on diagonal lines, with two nails at the end of each board independent of the diagonal nailing. Sometimes top floors are laid diagonally, but little or nothing is gained by this and the cost is a little more. All nails should be set and the floor planed if it is not smooth enough without it.

Steel beams are used somewhat in mill construction buildings, but are not liked by the insurance companies as well as wood. They tolerate them, however, trusting to sprinklers to keep them cool in case of fire, and consider that a fire will be confined to one story. Their advantages are that piers are not cut away by their use as in the case of wood and can therefore be narrower, thus increasing the window width, and columns can be farther apart. The beams should be laid on the walls as the work proceeds, with one brick course fitted around them in the face of the wall, and the pocket thus formed filled with cement grout. The brickwork can then proceed and the wall is not reduced in cross section where the beam enters. If steel beams are used, pintles should not be omitted.

64. Anchoring of Steel Beams.—The anchoring of steel beams in walls is not quite so desirably accomplished as with wood. The common way is to have a couple of short pieces of steel angle riveted vertically to the web near the end of the beam to anchor it, and build the beam in as described above. The beam can be drawn up to the pintle before this is done. If the beam falls in a fire it will pry out some of the brickwork. A beam-box could be used, as in the case of wood beams, and bolted to the lower flange of the beam before the box is built into the brickwork. The beam and box could then be slid as the beam is drawn up to the pintle.

Square or rectangular pintles look better with steel beams than round ones, as the beam ends fit against them better (Fig. 87). Sometimes the lower flange is bolted to a bracket cast on the bottom of the pintle and sometimes the web is bolted to an appropriate projection on the pintle. The best way is to rivet angles to the web, and bolt the beams together by means of bolts passing through oblong holes cast in the pintle, as in Fig. 87. Care must be taken to have the beam rest over the top of the column and avoid transverse stress in the pintle brackets.

65. Roofs.—The remarks on floors will apply to roofs, except that spruce sometimes warps and turns up its edges so that it may injure the roof paper. The standard slope of mill roofs is ½ in. per foot.

Concerning roof coverings, there are many different makes, any of which will be furnished with a guaranty of five or ten years. Tar and gravel are very satisfactory and should be five or six plies thick. Thick roof coverings of this kind are important in some places on account of their insulating qualities which assist in preventing condensation of humid atmosphere on the underside in cold weather.

The undersides of roofs and floors are sometimes painted white, but the cracks between the planks make them look bad, although the lighting effect is good. Likewise, brick walls can be painted, but the natural brick looks better. Brick looks best when washed down with acid and oiled.

66. Columns and Walls.—Columns are usually made of long-leaf Southern pine and should be carefully selected for straightness of grain and freedom from defects. It is very important that the ends should be square with the axis, and when the columns are round this is easily accomplished in the lathe. Wood columns are often made as small as 6 in. square, but they are very apt to spring and in hot factories this is true of columns of any size. Practically, it is better to have 8 in. the minimum size. Pipe columns filled with concrete are used, but the mutual insurance companies consider wood columns a better fire risk, and where the pipe concrete columns are used they prefer to have a reinforcement placed inside, this being strong enough to support the load (Fig. 87). The stock companies do not require this. This type of column without interior reinforcement went through the Salem fire successfully. Even with these columns, or those of cast iron, pintles should be used. Both ends of pintles should be faced off square and likewise the surfaces with which they come in contact, and pintles for square columns should have a circular face on which the column rests so that it can be faced in a lathe or boring mill (Fig. 86).

Wood columns were formerly bored and ventilating holes made at top and bottom. The benefit of this cannot be identified and the practice has been generally abandoned.

The upper and lower ends of wood columns should be treated with a preservative, especially the lower ends, as they are frequently wet from washing the floors.

In building such a factory some designers have slanted the piers between windows inward on the outside of the building. This is expensive and useless, and it should be kept in mind that the center of pressure coming from the weights should be as near the center of the foundation as possible. By stepping the walls back 4 in. or more at each floor on the inside of the building, or at every other floor, this is partly accomplished and the outside of the pier can be kept vertical (Fig. 85). If the pier is inclined or made like a stepped buttress on the outside, the result is that the foundation will be eccentrically loaded. These inclined or buttressed pieces accomplish nothing desirable.

67. Basement Floors.—If a wood floor is desired in the basement the best way is to make a tar concrete and wood floor, as follows: The earth should be filled-in layers 6 in. thick and rammed level. On top of this there is to be a layer 3 in. thick of hot tar concrete laid and rolled firmly and level, the upper ½ in. being of fine material laid hot and well rolled to prevent moisture from coming through. On this there is to be a layer of unplaned square-edged plank 2½ to 4 in. thick, laid close. The plank should be kyanized or treated with other preservative to prevent decay. A top floor is then laid at right angles to the plank and well nailed. The plank need not be splined, because there is no chance for vertical movement.

It is not well to use sleepers, as it is difficult to surround them properly with tar concrete and difficult to get them level, and they accomplish nothing. A floor as above described is a heavy solid mass and is bound together by the top floor. Experience shows that it is satisfactory without being fastened to anything and is suitable for holding any machinery that does not require foundations. It is good where wet processes are carried on.

FLOOR AND ROOF FRAMING—STEEL

By H. J. Burt

68. Floor Construction and Fireproofing.—Steel floor framing may be used with almost any form of floor construction. The design of the steel work is governed thereby. Hence, the details of the floor construction including fireproofing, if any, must be determined as a preliminary to the design of the steel.

68a. Wood Floors.—It is not usually desirable to use steel with wood floor construction, but occasionally conditions warrant it. The following combinations may occur:

(*a*) Ordinary wood joist construction with steel girders, the joists being closely spaced for supporting a plastered ceiling; and for supporting a sub-floor and finished floor of ⅞-in. boards. There may be a layer of concrete or cinders between the sub-floor and the finish floor.

(*b*) Mill construction having wood joists and steel girders, the joists being spaced 4 to 6 ft. apart.

(*c*) Mill construction having steel joists and steel girders, the joists being spaced 4 ft. or more apart.

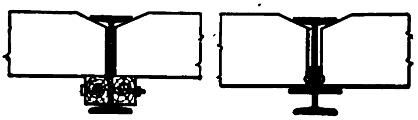

Fig. 88.—Detail of framing of wood joists to steel girders.

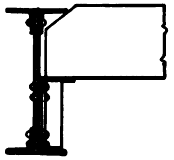

Fig. 89.—Bracket on web of steel girder to support wood joist.

Although in the above cases fireproofing is seldom used, it is, nevertheless, very desirable. Tile is most economical for this purpose, but concrete may be used. To provide complete protection, it must be put on before the wood is placed. In case (*a*), some protection for the lower flange can be provided by covering it with metal lath and cement plaster.

In case (*a*), the simplest detail is to rest the joists on top of the girders. If headroom under the girders is a consideration, the joists may be framed to the sides of the girders, resting on wood strips, shelf angles, or the bottom flange of wide flange beams (Fig. 88). If the girder is fireproofed, stirrups must be used.

In case (*b*), the wood joists may rest on top of the girders, or, if headroom governs, be carried in stirrups. If the depth of girder permits, brackets may be riveted to the girder web (Fig. 89).

In case (c), the wood floor may rest directly on the steel joists and be fastened thereto by small railroad spikes driven from below so as to engage the flange of the beam (these can be readily driven with a compressed air hammer); or a nailing strip may be bolted to the top flange. In this construction, it is not practicable to fireproof the top flange of the girder, but fairly good protection can be had by encasing the bottom flange and the web with tile after the floor is laid. The wood will furnish considerable protection to the top flange.

68b. Tile Arch Floors.—Tile arch floors serve to furnish the sub-floor construction and the fireproofing of the steel joists and girders (Fig. 90). The finish floor may be concrete or a wood flooring.

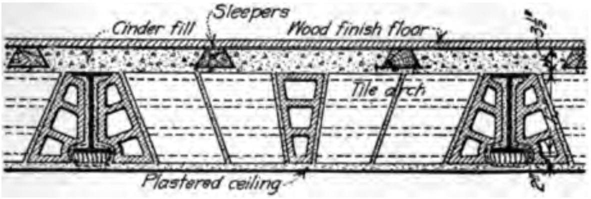

Fig. 90.—Section of flat tile arch floor.

A practical rule for the relation of depth to span is that the span in feet shall not be more than ¾ the depth of tile in inches, or a ratio of 9 to 1. The depth of tile, depth of joists, and spacing of joists (or span of tile arch) are so related that they must be considered together, taking into account the following:

For a given spacing of girders, there is greater economy of steel if deep joists are used spaced as far apart as their strength will permit. It is desirable to space joists so that they will divide the panel equally, having joists on column lines. The depth of joist controls the total thickness of floor construction, and the greater this thickness, the greater is the dead load and its cost. The arrangement is indicated in Fig. 90 which shows the total depth to be 6 or 7 in. more than the depth of joist.

Tile arch sets, in place, weigh approximately as given below, but these weights will vary and must be checked locally.

Thickness (inches)	Weight per square foot (pounds)	Max. span
8	28	6 ft. 0 in.
10	32	7 ft. 6 in.
12	36	9 ft. 0 in.
14	40	10 ft. 6 in.
16	46	12 ft. 0 in.

As an illustration, assume a panel 20 × 20 ft. It may be divided into 2, 3, 4, or 5 sub-panels, having widths respectively of 10 ft. 0 in., 6 ft. 8 in., 5 ft. 0 in., and 4 ft. 0 in. Assume a live load of 100 lb.

For trial, assume a 12-in. joist with a total floor thickness of 19 in. Then the loads may be computed as follows:

Wood flooring	⅞ in.	4
Cinder fill	3½ in.	28
Arch set	14 in.	40
Steel joists		7
Plaster	⅝ in.	6
	19 in.	85
Partitions (average)		25
Total dead load		110
Live load		100
Total load		210 lb.

For a 10-in. beam, the tile arch will be 12 in., decreasing the load 4 lb. per sq. ft. and making the total 206 lb. For a 15-in. beam, the tile arch will be 16 in. and the cinder fill 4½ in., increasing the load 14 lb. per sq. ft. and making the total 224 lb. For an 18-in. beam, the tile arch will be 14 in. with a 6-in. filler tile and 3½ in. of cinders, increasing the load 22 lb. per sq. ft. and making the total 232 lb.

For these loads the beam sections required and their comparative weights for the respective sub-panel widths, are:

Spacing	Beam sections	Comparative weights (lb. per sq. ft.)
10 ft. 0 in.	18-in. 46-lb. I	4.6
6 ft. 8 in.	15-in. 36-lb. I	5.4
5 ft. 0 in.	12-in. 35-lb. I (scant)	7.0
4 ft. 0 in.	12-in. 27½-lb. I	6.88
4 ft. 0 in.	10-in. 40-lb. I	10.00

A comparative estimate of costs can now be compiled to determine which floor is cheapest. The figures here used are for illustration only; and are given in cents per square foot of floor:

Spacing	10 ft. 0 in.		6 ft. 8 in.		5 ft. 0 in.		4 ft. 0 in. 12-in. I		4 ft. 0 in. 10-in. I	
Steel in place at 3¢........	4.6 lb.	13.8¢	5.4 lb.	16.2¢	7.0 lb.	21.0¢	6.88 lb.	20.6¢	10.0 lb.	30.0¢
Tile in place at 0.6¢.......	62.0 lb.	37.2¢	46.0 lb.	27.6¢	40.0 lb.	24.0¢	40.0 lb.	24.0¢	36.0 lb.	21.6¢
Cinder concrete at 2¢......	3½ in.	07.0¢	4½ in.	09.0¢	3½ in.	07.0¢	3½ in.	07.0¢	3½ in.	07.0¢
Excess cost of columns, girders and foundations to carry extra weight at 0.2¢...................	14 lb.	02.8¢	22.0 lb.	04.4¢						
Totals............	......	60.8¢	....	57.2¢	....	52.0¢	....	51.6¢	....	58.6¢

This tabulation indicates the 4-ft. spacing with 12-in. joists to be cheapest, but a closer analysis would probably show in favor of the 5-ft. spacing because of the smaller number of pieces of steel and tile to be handled.

If there happens to be close competition between two depths of beams, the effect of the increased height of walls and columns may be a determining factor.

Where the height of buildings is limited by law, the floor thickness may become very important, possibly affecting the number of stories for the building. This may justify the increase in cost of the floor resulting from the use of shallower but heavier beams.

As a conclusion of the foregoing analysis, it is determined that 12-in. joists will be adopted as typical.

To prevent joists from spreading from the thrust of the arches during construction and in outside panels, tie rods are used spaced 5 to 7 ft. apart. The details are shown in Fig. 91.

If one end of an arch is supported by a girder deeper than the typical joist, a shelf angle may be used, or the skew-back may be blocked up from the bottom flange of the girder (Fig. 92).

The typical joist having been settled for a given case, the ceiling line is thus established and a deeper joist cannot be used in any special situation without projecting below the ceiling line.

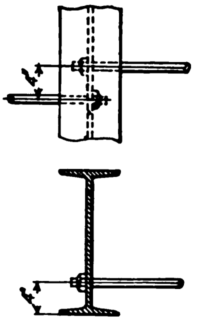

Fig. 91.—Detail of tie rods in tile arch floor.

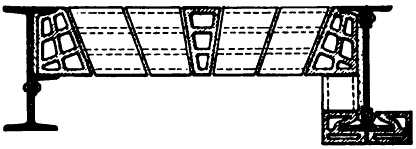

Fig. 92.—Support of tile arch at girder.

If a shallower joist is used, it is placed flush on the bottom with the typical joist. This is illustrated by Fig. 93.

68c. Concrete Floors.—When a concrete floor is used on steel framing, the concrete is also used for fireproofing the steel. Whether or not the concrete provides the floor finish is not pertinent to the subject under discussion, only as the weight may be affected. Wood or other floor finish may be placed on top of the slab. If flat ceiling finish is required, some form of suspended ceiling will be attached to or suspended from the bottom flanges of the joists.

The combinations of concrete floor and steel framing most frequently occurring are:

(*a*) Concrete slab resting on steel joists.
(*b*) Concrete slab, or slab with concrete joists spanning from girder to girder.
(*c*) Concrete slab supported by girders on four sides.

The fireproofing of the steel beams is accomplished by encasing them in the concrete, using a minimum cover of 2 in. or such other amount as specified by proper authority. No special details of the steel beams are required for supporting the casing. On deep plate girders, however, it may be desired to save weight of concrete by paneling the sides, in which case it may be desirable to punch the web plate for anchors. Some form of steel fabric on the bottom flanges and vertical wires on the sides are used to secure the fireproofing in place and are provided in detailing the concrete.

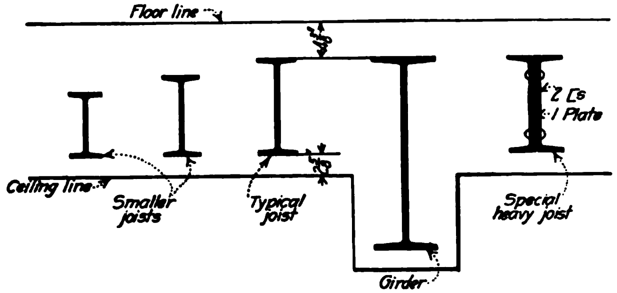

FIG. 93.—Diagram showing the relative position of joists and girder in tile arch floor.

The thickness of concrete on top of the beams should be not less than 3 in. and more may be required if many pipes are to be embedded. If the slab used is greater than the amount determined as necessary over the tops of the beams, the bottom of the slab may be below the top of the beams. The tops of all the joists and girders are placed at one level unless some special condition requires otherwise (compare with tile arch construction, Fig. 93).

In case (*a*), if the thickness of slab is settled as previously specified, the greatest economy of steel will be effected by spacing the joists as far apart as the slab will span, being limited of course, by equal divisions of the panel, so that joists will occur on column lines. If not so established, an analysis must be made of all the possible spacings to determine the cheapest combination.

As an illustration, assume a panel 20 × 20 ft. and a live load of 100 lb. per sq. ft. The panel may be divided into 2, 3, 4, or 5 sub-panels, having widths respectively of 10 ft. 0 in., 6 ft. 8 in., 5 ft. 0 in., and 4 ft. 0 in. The thickness of slabs and weight of reinforcing required, are:

Span of slab	Thickness (inches)	Weight of reinforcement (lb. per sq. ft.)
10 ft. 0 in.	5½	1.65
6 ft. 8 in.	4	1.10
5 ft. 0 in.	3	0.85
4 ft. 0 in.	3	0.85

The approximate loads per square foot of floor can now be computed from which to determine the beam sizes.

Spacing	10 ft. 0 in.	6 ft. 8 in.	5 ft. 0 in.	4 ft. 0 in.
Slab	68	50	38	38
Beam casing	20	24	25	32
Steel	5	6	6	6
Ceiling	8	8	8	8
Partitions	25	25	25	25
	126	113	102	109
Live load	100	100	100	100
Totals	**226 lb.**	**213 lb.**	**202 lb.**	**209 lb.**

From these loads, the beam sections required and their comparative weights for the respective sub-panel widths are:

Spacing		Beam section	Comparative weights (lb. per sq. ft.)
10 ft. 0 in.		18-in. 46-lb. I	4.6
	or	15-in. 60-lb. I	6.0
6 ft. 8 in.		15-in. 36-lb. I	5.4
5 ft. 0 in.		12-in. 31½-lb. I	6.3
4 ft. 0 in.		12-in. 37½-lb. I	6.88
	or	10-in. 35-lb. I	8.75

A comparative estimate of costs can now be compiled. The figures here used are for illustration only, and are given in cents per square foot of floor:

Spacing	10 ft. 0 in.(18-in.I)		10 ft. 0 in.(15-in.I)		6 ft. 8 in.	
Steel in place at 3.0¢	4.6 lb.	13.8¢	6.0 lb.	18.0¢	5.4 lb.	16.2¢
Concrete in slab and beam casing at 30¢	0.60 cu. ft.	18.0¢	0.58 cu. ft.	17.4¢	0.50 cu. ft.	15.0¢
Reinforcing in place at 3.0¢	1.65 lb.	5.0¢	1.65 lb.	5.0¢	1.10 lb.	3.3¢
Forms for slab at 9.0¢	1.0 sq. ft.	9.0¢	1.0 sq. ft.	9.0¢	1.0 sq. ft.	9.0¢
Forms for beams at 9.0¢	0.40 sq. ft.	3.6¢	0.34 sq. ft.	3.1¢	0.51 sq. ft.	4.6¢
Excess cost of columns, girders, and foundations to carry excess weight at 0.2¢	24 lb.	4.8¢	20 lb.	4.0¢	1 lb.	0.2¢
Totals		54.2¢		56.5¢		48.3¢

Spacing	5 ft. 0 in.		4 ft. 0 in.(12-in.I)		4 ft.0 in.(10-in.I)	
Steel in place at 3.0¢	6.3 lb.	18.9¢	6.9 lb.	20.7¢	8.8 lb.	26.4¢
Concrete in slab and beam casing at 30¢	0.42 cu. ft.	12.6¢	0.47 cu. ft.	14.1¢	0.42 cu. ft.	12.6¢
Reinforcing in place at 3.0¢	0.85 lb.	2.5¢	0.85 lb.	2.5¢	0.85 lb.	2.5¢
Forms for slab at 9.0¢	1.0 sq. ft.	9.0¢	1.0 sq. ft.	9.0¢	1.0 sq. ft.	9.0¢
Forms for beams at 9.0¢	0.61 sq. ft.	5.5¢	0.77 sq. ft.	7.0¢	0.60 sq. ft.	7.2¢
Excess cost of columns, girders, and foundations to carry excess weight at 0.2¢	0 lb.	0.0¢	7 lb.	1.4¢	......	0.0
Totals		48.5¢		54.7¢		57.7¢

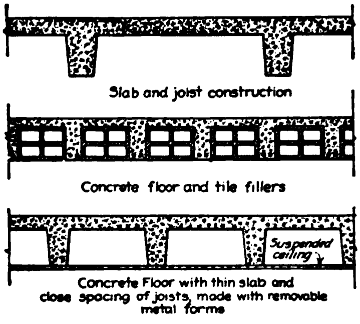
Slab and joist construction
Concrete floor and tile fillers
Suspended ceiling
Concrete Floor with thin slab and close spacing of joists, made with removable metal forms

Fig. 94.—Three types of concrete floor.

The foregoing computations show little choice between the 5 ft. 0-in. spacing with a 12-in. joist, and the 6 ft. 8-in. spacing, with a 15-in. joist. If the clear height of story is fixed, the shallower beam would probably be selected as there would be relative saving in columns, walls, and partitions. Comments in the preceding article, regarding limits of building heights, also apply here.

As a conclusion of the foregoing analysis, it is determined that 12-in. joists will be adopted as typical.

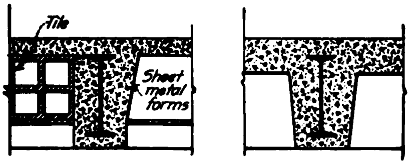

Fig. 95.—Sections showing relation of girders to concrete floors.

For case (b), there may be a flat slab heavy enough to span from girder to girder, no joists being required; or there may be a thin concrete slab with concrete joists. Fig. 94 shows three types of floors with concrete joists. Fig. 95 shows sections through the girder.

The top of the girder must be at least 3 in. below the top of the slab. No special details of the girder are required.

In buildings several stories high where the girders are steel and the joists concrete, it may be necessary to provide steel members on column lines to act as struts for bracing the columns. If not used, temporary bracing must be provided to hold the columns accurately plumb until the concrete is in place.

Case (*c*) occurs when a flat slab is used, reinforced in two directions. It requires no special details of the girder. For load effect on girder, see Sect. 2, Art. 39*c*.

69. Design of Joists.—The method of determining the proper spacing of joists for various kinds of floor construction, has been described on the preceding pages. The unit loads can now be accurately computed. The area supported is, of course, the spacing multiplied by the length. The loads used are the full dead and live loads.

The joist is designed as a simple beam, no account being taken of the restraint furnished by the end connections. The joist section is usually designed for bending resistance, the standard tables being used for this purpose.

For long spans with light loads, the deflection needs to be considered. The practical limit of length is 24 times the depth, if the beam is loaded to its capacity. For short spans with heavy loads, the strength of the standard end connection may govern the depth of beam or a special connection may have to be designed.

Concentrated loads may occur on joists from partitions, around stair and elevator shafts, etc. The resulting bending moments and shears must be computed for such cases and combined with the bending moments and shears from the uniformly distributed loads. As this occurs more generally with girders, it is discussed further in the next article.

The I-beam is the proper section to use for joists, except in special cases. The minimum weight section of a given depth is most economical, and should, if possible, be selected as the typical joist.

Having adopted a typical joist, there will be found cases where a shallower joist can be used and ordinarily there will be no objection to its use (see Fig. 93). There will be found other cases where the typical joist is not strong enough. Then, if it is not permissible to have it project below the ceiling level, a heavier joist of the same depth will be used. If the heaviest weight I-beam will not suffice, a special section can be built up of two-channels, or two channels and a web-plate (see Fig. 93).

70. Design of Girders.—In addition to the loads brought to it by the joists, the girder carries its own weight and its fireproofing. It may also have special loads from partitions, stairs, etc. The joist loads are concentrated, the weight of the girder is uniformly distributed, and the special loads may be either concentrated or distributed.

The total load on the girder is not the whole panel load, as some joists connect directly to the columns, but the effect on the girder resulting from the joist concentration is nearly the same as if the whole panel load were applied as uniformly distributed. This latter method of applying the load (*a*) is exact, if the length of girder is from center to center of columns and the number of sub-panels is even; (*b*) is excessive, if the length of girder is substantially less than the center to center distance of columns, or, if the number of sub-panels is odd.

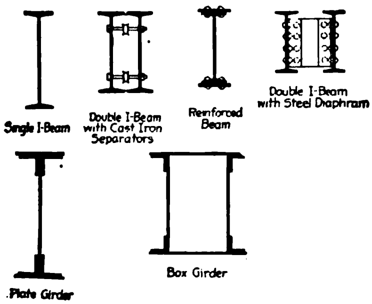
Single I-Beam
Double I-Beam with Cast Iron Separators
Reinforced Beam
Double I-Beam with Steel Diaphram
Plate Girder
Box Girder

FIG. 96.—Girder sections.

In making the final design of a girder, it is usually worth while to make accurate calculations, taking advantage of the actual length of the girder, and the concentration of the loads.

A concrete floor spanning from girder to girder, gives a uniformly distributed load on the girder, unless concrete joists are used with wide spacing, in which case the comments relating to steel joists will apply.

If a slab reinforced in two directions is supported on four sides, the panel load is equally divided between the girders, but is not uniformly distributed along the girders (see Sect. 2, Art. 39*c*).

The preferred section for a girder is a single I-beam or a plate girder. A double I-beam, a double plate girder, or a box girder, is used when the allowable depth is not sufficient for a single beam or plate girder (see Fig. 96).

71. Arrangement of Girders and Joists.—It is assumed that column locations and consequently the sizes of floor panels are governed by other considerations than the floor construction. With the panel arrangement fixed, it must be decided in which direction to place the girders. There are a number of considerations: (1) The girders can best be enclosed in cornices if over partitions, as along corridors; (2) they intercept less light if placed at right angles to the outside walls; (3) they will be shallower if used on the shorter span; and (4) economy may be the important factor. All of these considerations must be weighed.

The following is taken from Burt's "Steel Construction" by permission of the American Technical Society.

Fig. 97 illustrates a typical floor panel in a building. It is desired to investigate the various possible arrangements of framing for this panel. Assume that the dead load on the joists is 80 lb. per sq. ft. including the weight of joists (but not the weight of the girders and their fireproofing); and that the live load is 100 lb. per sq. ft. on joists, and 85 lb. per sq. ft. on girders.

Scheme (*a*).—Scheme (*a*) places the girders on the longer span and divides the panel into four parts. The joists are spaced 5 ft. 4½ in. center to center.

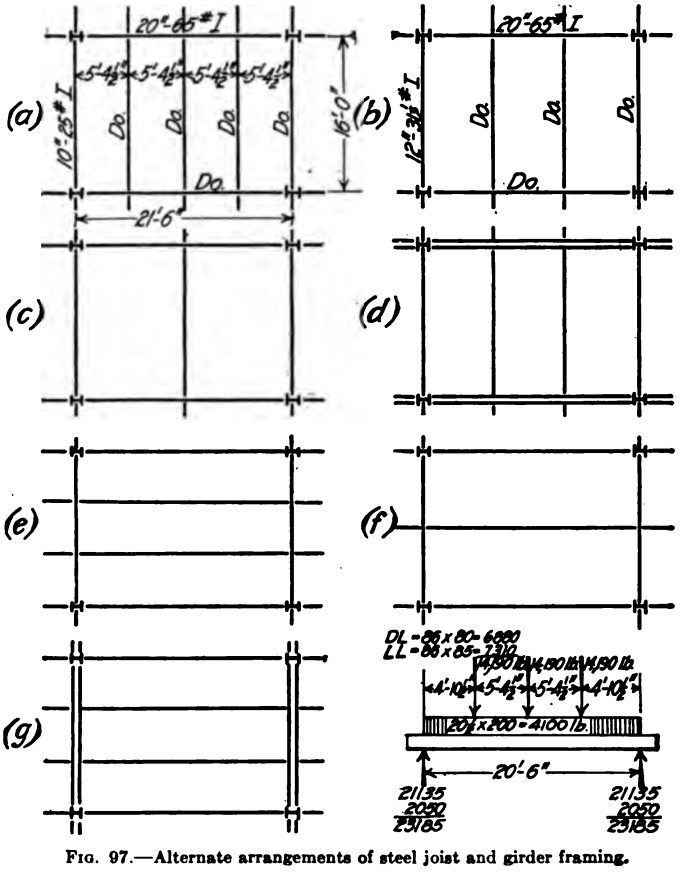

FIG. 97.—Alternate arrangements of steel joist and girder framing.

Area supported by one joist = 16 × $5\frac{3}{8}$ = 86 sq. ft.

Dead load on one joist = 86 × 80 = 6880 lb.
Live load on one joist = 86 × 100 = 8600 lb.

Total load = 15,480 lb.

This total load, 15,480 lb., is uniformly distributed on a span of 16 ft. The table of safe loads in the steel handbook indicates a 10-in. 25-lb. I.

The girder carries the reactions of the joists on each side and the weight of itself and of its fireproofing (assumed at 200 lb. per lin. ft.). On the theory that the whole floor will not be loaded at one time, the live load on the girder is taken at 85 lb. per sq. ft. The length of span is taken at 20 ft. 6 in. (allowance being made for the width of the column). Then the loads on the girder are as indicated in the figure and the bending moments are:

$$\text{For uniformly distributed load, } M = \frac{(4100)(20\tfrac{1}{2})}{8} = 10{,}5000 \text{ ft.-lb.}$$

$$\text{For concentrated loads}\begin{cases} +\ 21{,}135 \times 10\tfrac{1}{4} = 216{,}634 \\ -\ 14{,}190 \times 5\tfrac{3}{8} = \underline{76{,}271} = \underline{140{,}363} \end{cases}$$

$$\text{Total bending moment} = 150{,}863 \text{ ft.-lb.}$$

From the table of resisting moments in the steel handbook, a 20-in. 65-lb. I is indicated.

Scheme (*b*).—Scheme (*b*) places the girders on the longer span and divides the panel into three parts. This requires a 12-in. 31½-lb. I for the joist and a 20-in. 65-lb. I for the girder.

Similarly the other schemes can be designed and comparative costs estimated as in the previous articles.

Choice of Scheme.—A number of considerations will affect the final decision as to the scheme to be adopted. The character of the floor construction will limit the spacing of the joists. It might eliminate schemes (*b*), (*c*), (*d*), and (*f*). The thickness of floor construction may be important, in which case scheme (*a*) would be preferred as to joists and scheme (*g*) as to girders. The thickness of floor may affect its cost and also the dead load to be carried by joists, girders, and columns, making the thinner floor preferable on this account. A flat ceiling may be required over the entire area, in which case, scheme (*g*) is applicable.

72. Details of Connections.[1]

72*a*. Connection of Beams to Beams.—When one beam bears on top of another, the only connection required is rivets or bolts through the flange, as shown in Fig. 98. No stress is transmitted by these rivets or bolts. They serve simply to hold the beams in position. Steel clips are sometimes used for this purpose (Fig. 99), but as they are not positive in holding the beams in position, they are not as good, especially when lateral support is required. When this is not important, the clips can be used and may effect a saving in cost. These clips are most useful for attaching tees and angles to beams in ceiling and roof construction.

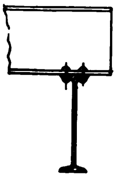

FIG. 98.

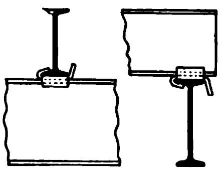

FIG. 99.

Angle Connections.—The most common method of connecting one beam to another is by means of angles riveted to the web. There are several sets of standard connections, various concerns having their own standards. The standard connections given in the latest edition of the Carnegie Pocket Companion, are recommended. The two- angle connection is generally used, but when beams are used in pairs, or when for any reason the two-angle connection cannot be used, the one-angle connection is employed. The rivets used in the standard connections are ¾ in. in diameter.

The strength of the two-angle connection may be limited by

(1) Shop rivets in double shear.
(2) Field rivets in single shear.
(3) Shop rivets in bearing in web of joist.
(4) Field rivets in bearing in web of girder.

For example, take the connection for a 15-in. 42-lb. I:

(1) 6 shop rivets in double shear
$6 \times 10{,}300 = 61{,}800$ lb.
(2) 8 field rivets in single shear
$8 \times 4420 = 35{,}360$ lb.
(3) 6 shop rivets in bearing in web of joist
$6 \times 0.41 \times 0.75 \times 25{,}000 = 46{,}125$ lb.
(4) 8 field rivets in web of girder.

The thickness of the web is not given. It must be at least 0.30 in. for a connection on one side only, or of twice this thickness if an equal connection is on the opposite side, in order to have the same strength as the field rivets in shear. The shearing strength of this connection, 35,360 lb., corresponds to the maximum safe uniformly distributed load on a span of about 9 ft. It is less than the shearing strength of the web of the beam. It rarely happens that the strength of the connection is less than required, and occurs only when the beam is short and heavily loaded, or when a heavy load is applied near the end. Lack of bearing in the web of the girder is more likely to occur, but this is not frequent. If it does happen, however, angles with 6-in. legs may be used to provide space for more rivets, or a reinforcing plate may be riveted to the web of the girder (Fig. 100).

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[1] From Burt's "Steel Construction" by permission of the American Technical Society.

Special Connections.—When beams on the two sides of a girder do not come opposite or are of different sizes so that the standard connections do not match, it is necessary to devise a special connection. If a beam is flush on the top or on the bottom with the one to which it connects, the flange must be coped (Fig. 101). A number of special connections are shown in Fig. 102 and need no explanations.

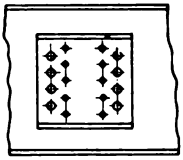


FIG. 100.

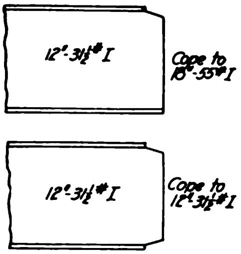


FIG. 101.

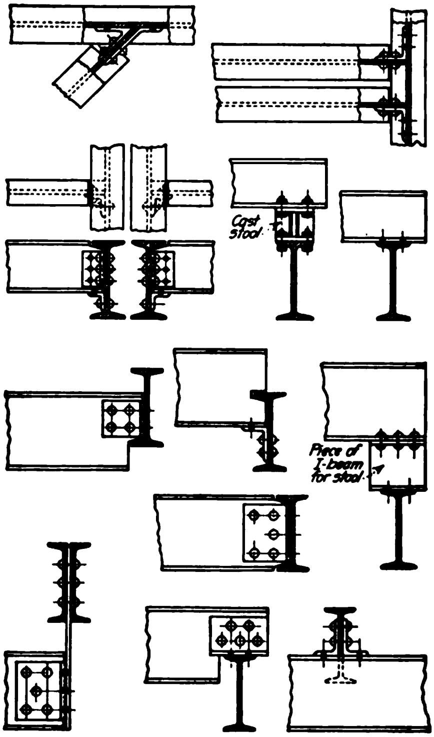


FIG. 102.—Details of beam connections.

72b. Connections of Beams to Columns.—A beam may connect to a column be means of a seat or by means of angles on the web. The great variety of conditions that may by encountered make it impracticable to have standards for these connections, though the work of each shop is standardized to some extent.

Seat Connections.—The seat connection is shown in Fig. 103. This seat or bracket is made up of a shelf angle, one or two stiffener angles, and a filler plate. The load is transmitted by the rivets, acting in single shear, which connect the bracket to the column. The number of rivets used is proportioned to the actual load instead of being standardized for the size of the beam. The stiffener angles support the horizontal leg of the shelf angle and carry the load to the lower rivets of the connection.

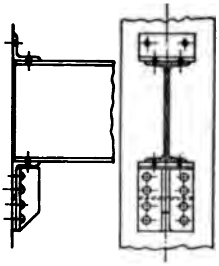


FIG. 103.—Seated connection of beam to column.

Shelf angles are 6, 7, or 8 in. vertical, and 4 or 6 in. horizontal, having a thickness of 7/16 to 3/4 in., depending on the size of beam and the load. The leg of the stiffener angle parallel to the web of the beam is usually 1/2 or 1 in. less in width than the horizontal leg of the shelf. The leg against the column is governed by the gage line of the rivets in the column. The filler is the same thickness as the shelf angle. An angle connecting the top flange of the beam to the column is generally used. It is not counted as carrying any of the load, but serves to hold the top of the beam in position and stiffens the connection. The rivets connecting the bottom flange of the beam to the shelf serve only to hold the members together and make a stiff connection. Usually there are only two rivets in each flange but sometimes larger angles and more rivets are used to develop resistance to wind stresses. Fig. 104 gives a number of examples of seat connections.

The advantages of the seat connections are:

(1) All shop riveting is on the column which is a riveted member. No shop riveting is required on the beam which thus needs only to be punched.

(2) The seat is a convenience in erecting.

(3) The rivets which carry shear are shop driven.

4) The number of field rivets is small.

Web Connections.—The web connection is made by means of two angles (Fig. 105). The legs parallel to the beam, rivet to the web, and the outstanding legs to the columns. The connection to the web of the beam is governed by the same conditions as the standard beam connection. The length of the outstanding leg is governed by the gage lines of the rivets in the column or the space available for them. Usually the angles are shop riveted to the beam and field riveted to the column. If the angles were shop riveted to the column, it would be difficult or impossible to erect the beam. However, one angle may be shop riveted to the column and the other furnished loose. In this case, the number of field rivets generally will be the same as if the angles were shop riveted to the beam, but the shop riveting on the beam will be eliminated, which is an advantage. When this connection is used, a small seat angle is provided for convenience in erecting.

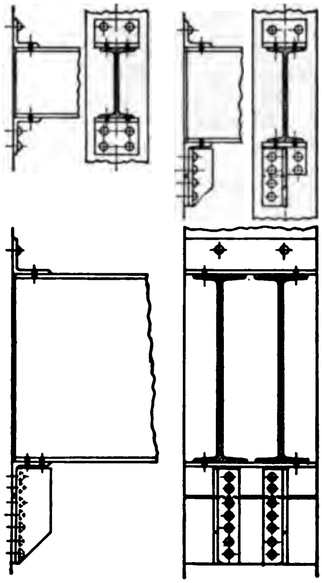


Fig. 104.—Types of seat connections.

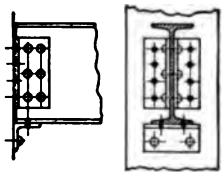


Fig. 105.—Web connection of beam to column.

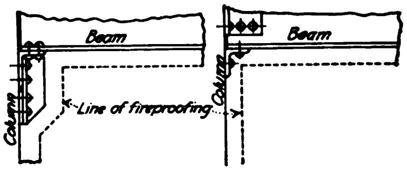


Fig. 106.

The advantage of the web connection is the compactness of the parts, keeping within the limits of the fire-proofing and plaster, whereas the seat connection may necessitate special architectural treatment to fireproof it or conceal it (Fig. 106).

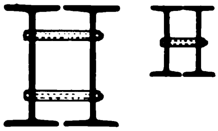


Fig. 107.

72c. Separators.—When beams are used in pairs or groups, some connection is usually made between them at short intervals. The connecting piece is called a *separator*. If the purpose to be served is merely to tie the beams together and keep them properly spaced, the gas-pipe separator is used (Fig. 107). This consists of a piece of gas pipe with a bolt running through it. This form is used in lintels and in grillage beams. For beams 6 in. or less in depth, one separator and bolt may be used; for greater depth, two should be used.

The separator most commonly used is made of cast iron (Fig. 108). It not only serves as a spacer but it stiffens the webs of the beams and, to a limited extent, transmits the load from one beam to the other in case one is loaded more heavily. It seldom fits exactly to the beam, so it cannot be relied upon to transmit much load. One bolt is used for beams less than 12 in. deep and two bolts for 12-in. and deeper beams. The dimensions and weights of separators and the bolts for them are given in the steel handbooks. They can be made for any spacing of beams and special shapes can be made for beams of different sizes (Fig. 109).

The individual beams of a pair or group should be designed for the actual loads which they carry, if it is practicable to do so. If it is necessary to transfer some load from one to the other, a steel separator or diaphragm should be used. This may be made of a plate and four angles, or of a short piece of I-beam or channel (Fig. 110). If the beams are set close together, the holes must be reamed and turned bolts must be used in order to get an efficient connection. If the beams are set with 4-in. or more clearance between the flanges, the separator can be riveted to the beams.

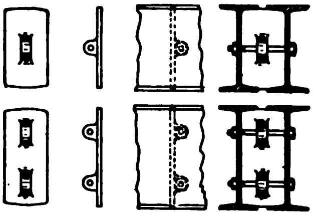


Fig. 108.

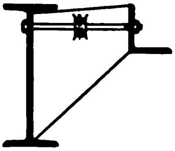


Fig. 109.

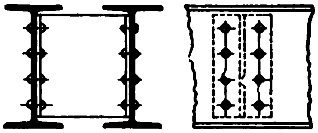


Fig. 110.

Specifications usually require that separators be spaced not further than 5 ft. apart. They should be placed at points of concentrated loads and over bearings.

73. Special Framing.—The typical arrangement of joists and girders must be modified to meet special requirements.

73*a*. Stair Wells.—The exact dimensions and location of the stair-well opening must be determined from the architectural plan. Fig. 111 illustrates a case. It shows a well for a double-run stairway. It is placed against an outside wall as indicated.

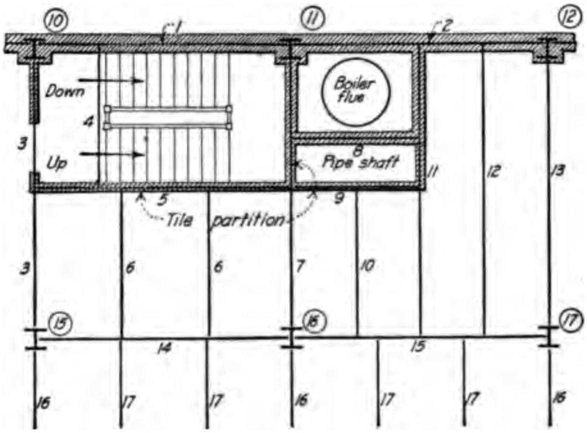


Fig. 111.—Framing around stairwell, chimney, and pipe shaft.

Beam (1) is placed off center of the column on this account. In addition to the wall load it gets a load from beam (4) and from the intermediate stair landing (not shown).

Beam (4) carries a small area of floor and also the weight of the stairs, both up and down. It must be so designed and so placed as to provide convenient connections for the stair stringers if steel stringers are used.

Beam (5) carries the reactions from beams (4) and (6). It may also carry an enclosing partition and a part of the intermediate stair landing.

Beam (3) carries the reactions from beam (5) in addition to floor load, and it may also carry an enclosing partition.

73b. Elevator Wells.—Fig. 112 shows a bank of three elevators with provision for a fourth. In this instance they are placed against the outside wall. The width of elevator has been adjusted to suit the column spacing. The locations of nearby partitions and proposed ceiling treatment will influence the arrangement of the framing.

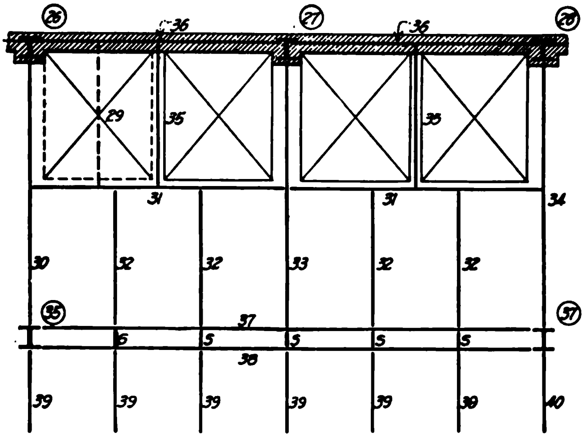


Fig. 112.—Framing around elevator openings.

No loads come from the elevators at the floor levels, the entire weight being carried by the overhead framing. There will be loads from the elevator enclosure.

Beams (35) provide lateral support for the elevator guides and carry dividing partitions, if any.

In this case, column 36 is omitted to give a clear lobby. This requires a heavy girder between columns 35 and 37. To save headroom below, a double girder is used consisting of beams (37) and (38). Two steel beams will be used. As they are not equally loaded, they must be designed separately; however, both beams may be the same size if provided with steel separators as indicated. In any event, such separators should be used so as to avoid unequal deflection in the beams.

All other beams are easily designed to meet the conditions indicated.

73c. Pipe Shafts, Etc.—Fig. 111 shows a pipe shaft and chimney space. Both are enclosed in fireproof walls which must be supported by the floor framing. Pipes or cables in the shaft may impose loads on certain floors. Such loads must be provided for where they occur. The chimney does not impose any load on the typical floor framing. As the chimney changes length with variations of temperature, it must be supported at one level only. Special framing may be provided for this support at the first or basement floor.

Innumerable variations of the foregoing special situations will occur in floor framing. Each must be treated as a separate problem. The important thing is to ascertain all the limiting conditions. When this it done, the designing is generally a simple operation.

74. Framing for Flat Roofs.—The problems involved in designing the steel framing for flat roofs are essentially the same as for floors. But there are some additional conditions. Special loads on roof framing come from elevator machinery, tanks, pent-house walls, signs, flag poles and kindred items. These having been determined from the architectural requirements, the roof framing is designed in the same manner as the floor framing.

If the top story is to have a finished ceiling, it becomes a problem to determine whether the framing shall be set level at the ceiling elevations or set on a slope at the roof elevation. If future stories are contemplated, the framing will be set level at the ceiling elevation, and so arranged as to serve as the future floor framing.

Unless there are special considerations indicating to the contrary, it is usually better to place the framing at roof elevation and place the beams parallel to the roof surface as nearly as practicable. This involves beveled connections for many of the joists and girders, but these are not difficult to make. The ceiling can be suspended from the roof framing or from the roof slab or arches by wire or rods.

In case an attic space is provided, the ceiling may still be suspended if no attic floor is to be used, or an independent set of framing may be provided. The latter will be necessary if loads are to be placed on the attic floor.

75. Framing for Pitched Roofs.—The shape of the roof surface and the kind of covering are usually determined as a part of the architectural design. The problem is, therefore, to provide framing to support a roof whose shape and covering have been determined.

Certain roof coverings are attached directly to the purlins and require no sheathing—such are corrugated steel concrete tiles, and some earthen tiles. Most other roof coverings require a sheathing, interposed between the roofing and the framing (see chapter on "Roof and Roof Coverings").

Having selected the kind of sheathing, the next step is to determine the most economical purlin spacing. An analysis of costs similar to that used in the study of floor joists (Art. 68b) will aid in determining the spacing. A spacing of approximately 5 ft. is a convenient one and suits most of the roof materials. However, a wider spacing may be cheaper for reinforced concrete cast in place and for some types of precast concrete.

75a. Design of Hip and Valley Rafters.—Where two roof planes intersect, they form a ridge, valley, or hip. In Fig. 113, $a-a'$ and $b-b'$, are ridges, $b-c$ is a valley, and $a-d$ is a hip. This figure shows the arrangement of trusses, rafters, and purlins. The trusses are designated by *T*, the ordinary rafters by *R*, the hip rafters by *HR* and the valley rafters by *VR*.

The hip and valley rafters are designed in the same manner as ordinary rafters, taking nto account the shape of the oaded area.

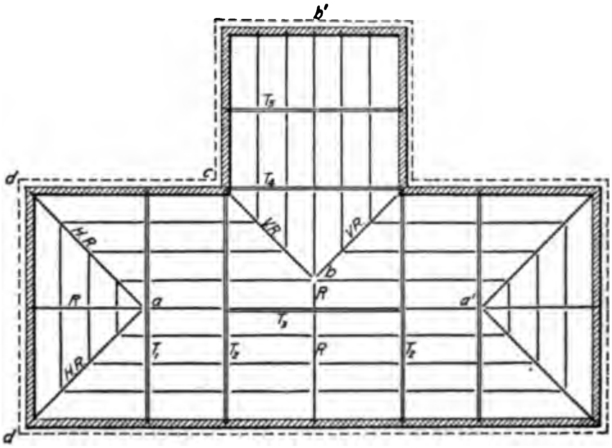


Fig. 113.—Showing roof framing with hip and valley rafters.

In the case illustrated in Fig. 113, russ T_1 supports the purlins, as indicated, and also the three rafters which converge at its apex. Truss T_3 spans between trusses T_2, its top chord servng as the ridge purlin, and supporting the intermediate rafters. A ridge purlin extends from truss T_4 to truss T_5, supporting the valley rafters at *b*, and also the lower end of a short rafter at the same point.

76. Saw-tooth Skylights.—Saw-tooth roofs are used to admit light through the roof without allowing direct sunlight to come through. To accomplish this, the glass must be to the north (in the Northern Hemisphere). The glass surface may be either vertical or inclined slightly to the south of the vertical. The maxmum inclination which can be used and still keep out direct sunlight at noon, is 23 deg. less than the latitude of the place. The inclined surface admits more and stronger light, but is more subject to leakage. The vertical surface is generally preferred. The area of glass surface to be provided will be determined by the lighting requirements.

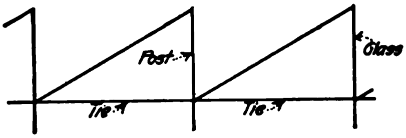


Fig. 114.—Saw-tooth skylight framework with I-beam rafter.

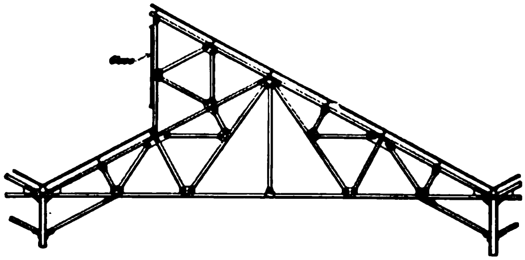


Fig. 115.—Saw-tooth skylight truss as designed by M. S. Ketchum.

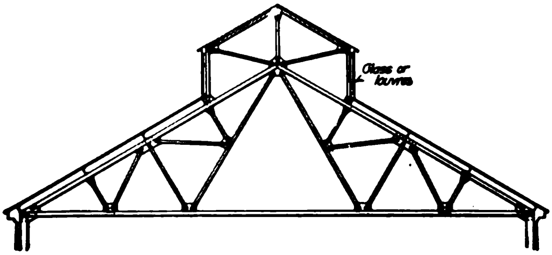


Fig. 116.—Roof truss with monitor frame.

If the spacing of the supports is such as to permit the use of beam framing, the arrangement of members shown in Fig. 114, may be used. The tie shown should be a rigid member for bracing purposes.

For wider spacing of supports, trusses are used. The most satisfactory form is that devised by M. S. Ketchum and shown in Fig. 115. Its important advantage is that it allows ample gutter space, being in this regard very much better than the design shown in Fig. 114.

The design of the trusses and purlins does not involve any principles that have not been explained.

77. Monitors.—Lighting and ventilation of mill buildings are often provided through a monitor on the roof. The monitor frame is mounted on the rafters or the trusses as shown in Fig. 116. It is made up of light angles as the loads to be carried are small. In the case shown, the gravity loads are carried direct to the main truss by the vertical members. The diagonal members take wind stresses only. If the monitor is wide, the top chord member of its frame may need to be trussed.

FLOOR AND ROOF FRAMING—CONCRETE

By W. J. Knight

78. Practical Considerations.—Competition in the economical design of reinforced concrete structures has reached such proportions, that few engineers can afford to neglect the practical and economic features of design. On every hand the engineer is confronted with the problems of economy, when serving his clients to the best advantage. Every prospective owner, with few exceptions, demands the best structure at the cheapest price. Therefore, the economy of arrangement, or the selection of a floor system that will result in giving the least comparative cost consistent with strength for any proposed structure, cannot be over-estimated in importance. A thorough knowledge of the costs of materials and labor that will be applicable to the various types of construction which can be used, may be termed vital considerations in the design of any structure. To design a building of sufficient strength, without considering cost, is not a difficult accomplishment, but to produce an arrangement that will afford both strength and economy in combination, is decidedly another problem. Theory by itself is a deceiving form of enlightenment and cannot well be applied intelligently until the many practical conditions governing design and application are learned through experience and made an integral part of theoretical knowledge.

It will often be found expedient to make comparative estimates of a typical panel for two or more different arrangements to ascertain the relative cost of concrete, steel bars and centering per square foot of superficial surface, and then the most economical system may be selected from these calculations.

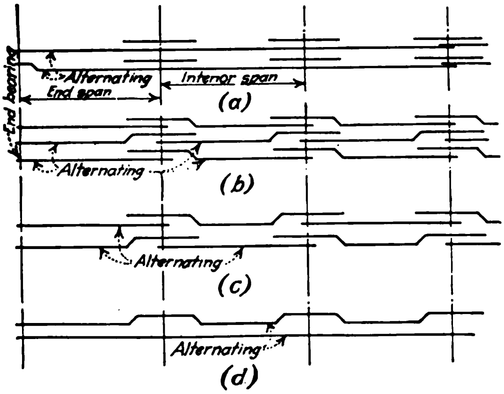


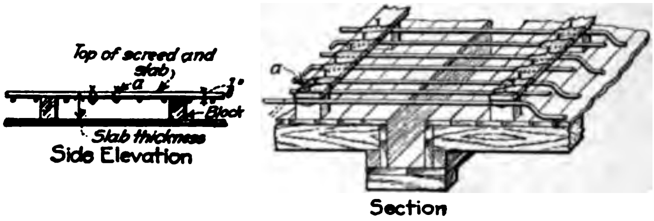
Fig. 117.

79. Slab Steel Arrangement—Ordinary Type.—The arrangement for slab steel can be accomplished in several ways. Fig. 117(*a*) shows an arrangement consisting of straight rods in the bottom and loose rods in the top over supporting members. This arrangement, though eliminating to a great extent the cost of bending, is objectionable on account of the difficulty of placing properly the loose rods in the top. Loose rods of this nature should be avoided when possible. This method has been employed in a great many buildings, but the actual position occupied by the top rods after the concrete has been placed is a question. Loose rods of this type are often placed after the slab has been poured to its full thickness, and the rods relied upon to remain near the top surface of the wet concrete. Laborers walking about engaged in screeding the concrete surface, can hardly be expected to avoid forcing them into the bottom of the slab.

Fig. 117(*b*) shows an arrangement used very often in short and long span slabs. The amount of steel over the supports is the same as at the center of the span. This arrangement requires the bending of all rods with the exception of alternate rods of end spans. Fig. 117(*c*) shows another arrangement that gives equal steel area over supports and in the center of span. The tonnage to be bent in this case is less than is required in Fig. 117(*b*) and is just as satisfactory.

In very short spans where arch action is considered to exist, alternate rods only may be bent up into the top of slab over the supports, which afford only one-half the steel area over the supports (see Fig. 117*d*).

To properly place slab rods has always been a very annoying question for engineers and superintendents to solve. Slab rods when only tied to temperature rods cannot be held permanently in position. The most severe stages when disarrangement takes place, occur before and during concreting operations. Electricians placing conduits and outlet boxes, plumbing and heating contractors installing sleeves, etc., all contribute to the disarrangement of slab rods, unless some device is employed to permanently space the rods and to hold them up the required distance from the false-work when concrete is being placed. To prevent the bent-up portion of slab rods from turning sidelong is another source of difficulty. The bent-up portion may be held in position by wiring the rods to the under surface of screeds placed on both sides of the beam. These screeds also serve the purpose of forming a gage by which the specified thickness of the slab is maintained (see Fig. 118). Wires (*a*) of small gage are employed to hold the rods in position. While the surface of slab is being leveled off, the wires are detached as the finishing off of slab proceeds, and this should be done in such a manner as to avoid treading on the rods which have been disengaged



Side Elevation

Section

Fig. 118.

The specified thickness of a floor slab cannot be maintained unless contrivances similar to the commonly known "screeds" are constructed and installed with depth equal to the thickness of slab desired. Screeds should be installed prior to the pouring of any concrete and at such intervals as will permit the "blockmen" to level the surface with a straight edge from one screed to the other.

It is regrettable that more stress has not been laid in the past upon practical engineering matters of this nature. Seemingly a great many engineers of the past have contended themselves in the knowledge of theoretical design to the exclusion of things practical, laboriously applying formulas, and in many instances, intricate theories, to establish to a hair's thickness the depth of a slab. It should be realized that lax methods applied during construction, discompose completely the intended accuracy of correct design. There is little engineering logic displayed in applying formulas to derive the exact thickness of a slab, the size of a beam, or the exact location and spacing for the reinforcement, when there is wanting a specific means of fulfilling the intent and purpose of the drawings.

The University of Illinois published a very interesting bulletin in May, 1918, describing conditions found in a reinforced concrete building which was removed to clear the site for the New Passenger Station in Chicago, Ill. The original thickness of flat slab shown was 8½ in. The variation in thickness over the test area ranged from 7.5 to 9.8 in. A considerable variation in thickness was found in nearly all parts of the test area. At the columns the distance of the centers of the bars of the top layer from the upper surface varied from 0.90 to 2.00 in., and that of the lower layer from 3.60 to 4.00 in. At points between columns, the centers of the bars of the rectangular bands were from 2.30 in. in one case to 4.20 in. above the lower surface of the slab. In the region of the center of the panel the centers of the lower layer of diagonal bars were from 1.20 to 2.20 in. above the lower surface of the slab. This incident is cited to show that some form of screed to gage the specified thickness of a slab is indispensable to correct application, and some device for supporting and spacing the bars is as necessary to guarantee good execution, as the fundamental principles of theory are indispensable to good design.

80. Marking of Bent Rods.—A great many serious errors have been made in the past by installing the wrong bent rods in beams and slabs, principally due to the absence of some indestructible form of tag that should serve as a means of ready identification for each bent rod used in a structure. When rods are bent at the rolling mill or at the building site it is most difficult to identify them and avoid errors, unless painstaking care is exercised in giving each bent rod or bundles of identical bent rods a clear, indestructible mark stamped on tags made of non-corrosive metal. Cloth tags have been experimented with and found decidedly unsatisfactory. Marks on such tags with the use of ordinary or even indelible ink cannot survive the wear and tear of shipping and handling, without becoming disfigured, detached from the rods, or illegible from the effects of water and rust. It is common practice for high-priced iron workers to spend part of each day searching for and measuring bent rods, endeavoring to locate the material desired. At this time of writing, iron workers have increased their wages to one dollar per hour in the central west, which from a monetary standpoint alone should even further emphasize the importance of providing suitable tags where necessary.

The enforcement of this essential requirement by engineers executing designs or superintending the erection of structures, is simply another step forward in making more practical the application of theory and giving added assurance that the design will be carried out with reasonable accuracy.

The following simple method has been used with success where employed, and consists of stamping metal tags with numbers that designate each different bent rod, besides indicating by the first figure of the mark number the size of the rod. To illustrate: Reduce all merchantable bar sizes to fractions of eighths, the dividend of the fraction for each bar size always representing the first figure of the mark number as follows:

$\frac{1}{4}$ in. = $\frac{2}{8}$ = Mark 200
$\frac{3}{8}$ in. = $\frac{3}{8}$ = Mark 300
$\frac{1}{2}$ in. = $\frac{4}{8}$ = Mark 400
$\frac{5}{8}$ in. = $\frac{5}{8}$ = Mark 500
$\frac{3}{4}$ in. = $\frac{6}{8}$ = Mark 600
$\frac{7}{8}$ in. = $\frac{7}{8}$ = Mark 700
1 in. = $\frac{8}{8}$ = Mark 800
$1\frac{1}{8}$ in. = $\frac{9}{8}$ = Mark 900
$1\frac{1}{4}$ in. = $\frac{10}{8}$ = Mark 1000

Any bent rods found marked 200, 201, 202, etc., will indicate at once a $\frac{1}{4}$-in. rod, or marks 700, 701, 702, etc., a $\frac{7}{8}$-in. rod, and so on. This system used in connection with metal tags is very simple and effective, and when applied by workmen will reduce to a minimum the chance of placing bars in the wrong location.

81. Special T-Beam Design.—A minimum specified clearance or head room will often control the depth of T-beams of long spans. An example of long span T-beam constructon is given below, which illustrates the special provision made to obtain the requisite flange area for compression. The design of this beam was one of a large number required to span a theater auditorium in connection with a large structure built in 1916. The floor supported by these beams was designed for a dancing pavilion.

Illustrative Problem.—The beams are 8 ft. on centers and span 48 ft. center to center of column supports. The maximum depth allowed was 33 in. The live load from the floor to be supported by the beam was assumed at 75 lb. per sq. ft., consideration having been given to the additional safety factor afforded by the heavy dead load of the beam, which is about 16 tons. Assumptions used in the design, $f_s = 20{,}000$, $f_c = 800$ and $n = 15$.

The slab spanning 8 ft. was designed to support a live load of 100 lb. per sq. ft. for the reason that in a building of this character the slab in all probability will receive its full live load at intervals, whereas the supporting beams will not.

Slab Design when $M = \frac{wl^2}{12}$.—A minimum slab of 4 in. and reinforcement of $\frac{3}{8}$-in. rounds 6 in. c. to c. was selected. In the design of this slab the supporting beams were also considered, to obtain cross reinforcement that would assist the T-action of the members.

Using a 4-in. slab the theoretical requirements would be:

Live load = 100
4-in. slab dead load = 50
$\frac{1}{2}$-in. finish = 6

Total = 156 lb. per sq. ft.

$$M = \frac{(156)(8)^2(12)}{12} = 9980 \text{ in.-lb.}$$

Selecting a minimum slab of 4 in. and $d = 3$ in.,

$$9980 = K(12)(3)^2$$
$$K = 93$$

Diagram 2, p. 153 shows that the value of $K = 93$, with $f_s = 20{,}000$ requires a percentage, $p = 0.0053$. Then the actual area of steel required per foot width is

$$A_s = (0.0053)(12)(3) = 0.191 \text{ sq. in.}$$

To find the unit stresses in the steel and concrete assumed in the design: $\frac{3}{8}$-in. rounds 6 in. c. to c. = 0.221 sq. in. per 12-in. width.

$$p = \frac{0.221}{(12)(3)} = 0.0061$$

Using Table 2, p. 150, $p = 0.0061$, $k = 0.346$ and $j = 0.885$.

$$f_s = \frac{M}{A_s jd} = \frac{9980}{(0.221)(0.885)(3)} = 17{,}000 \text{ lb. per sq. in.}$$

$$f_c = \frac{(2)(9980)}{(0.346)(0.885)(12)(3)^2} = 603 \text{ lb. per sq. in.}$$

Or referring to Diagram 2 when $K = 93$ and $p = 0.0061$, the stress in the concrete and steel will be found to agree with values determined above for f_s and f_c.

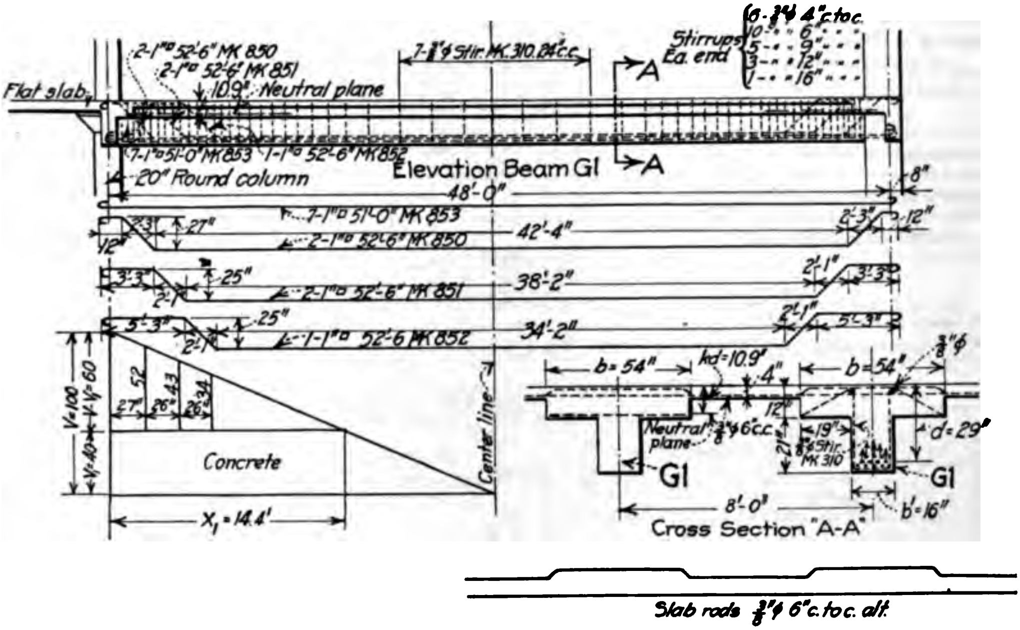


FIG. 119.

Considering the shortness of the slab span and the increased effective depth near the supports (Fig. 119), on account of the depression for flange section, the bent rods were arranged as shown.

T-Beam Design:

Live load per linear foot = (8)(75)	=	600
Dead load of slab per linear foot = (8)(50)	=	400
Dead load of finish per linear foot = (8)(6)	=	48
Dead load of beam including depression below slab, per linear foot = (4.34)(150)	=	650
Total	=	1698 lb. per lin. ft.

$$M = \frac{(1698)(48)^2(12)}{8} = 5{,}868{,}300 \text{ in.-lb.}$$

$$b' = 16 \text{ in.} \qquad d = 29 \text{ in. (Fig. 119)}$$

$$v = \frac{(1698)(24)}{(16)(\frac{7}{8})(29)} = 100 \text{ lb. per sq. in.}$$

Twelve 1-in. squares used in the design give a section of 12 sq. in.

$$p = \frac{12}{(54)(29)} = 0.0076$$

In the design of this member where the depth is small in proportion to span length, it was considered of prime importance to obtain a rigid construction and not rely on the 4-in. slab flange to resist any part of the compressive stress. Therefore, a flange thickness of 12 in. was chosen and a flange width of $b = 54$ in. $\frac{t}{d} = \frac{12}{29} = 0.414$.

Diagram 6, p. 166, shows the neutral axis is in the flange when $p = 0.0076$ and $\frac{t}{d} = 0.414$. Hence, Case I applies.

Table 2, p. 150, gives the following values of k and j for $p = 0.0076$:

$$k = 0.376 \qquad j = 0.875$$

Referring to Table 3, it is found that these values give about equal strength for the steel and concrete, or solving for f_s and f_c:

$$f_s = \frac{M}{A_s jd} = \frac{5{,}868{,}300}{(12)(0.875)(29)} = 19{,}300 \text{ lb. per sq. in.}$$

$$f_c = \frac{2f_s p}{k} = \frac{(2)(19{,}300)(0.0076)}{0.376} = 780 \text{ lb. per sq. in.}$$

or

$$K = \frac{M}{bd^2} = \frac{5{,}868{,}300}{(54)(29)^2} = 129.2$$

When K = 129.2 and $p = 0.0076$, the above values for f_s and f_c may be checked by Diagram 2, p. 153. Points at which bends in rods may be made can be readily obtained from Diagram 8. At the worst section, 5 out of the 12 rods are bent, or 42 % of the total at a point 2 ft. 10 in. from the center of each support. Diagram 8 shows when $M = \frac{wl^2}{8}$, 42 % may be bent up at $0.17l$ or 8 ft. 2 in. from center of support. Further investigation in this respect is unnecessary.

Bond stress in straight rods with hooked ends: The perimeters of seven 1-in. squares = (7)(4) = 28 in.

$$u = \frac{V}{\Sigma o \frac{7}{8} d} = \frac{40{,}750}{(28)(\frac{7}{8})(29)} = 58 \text{ lb. per sq. in.}$$

Theoretically, hooks were unnecessary, but the idea of securing the greatest rigidity for the structure dictated the use of hooks for the ends of all bent and straight rods. The locality in which this structure was erected is subject to periodical storms and wind of great velocity, hence judgment was exercised in anchoring the structural parts wherever it was deemed advisable.

Provision for shearing stresses: The unit shearing stress has been determined above. $v = 100$. Shearing value assumed for concrete $v_1 = 40$.

$$x_1 = \frac{(100 - 40)48}{(2)(100)} = 14.4 \text{ ft.}$$

In Fig. 119 it will be noted that the bent rods were so arranged that the diagonal tension at the ends could be taken principally by these rods, but regardless of this fact 3/8-in. round stirrups were introduced extending from end to end of the beam as shown. Referring to diagram Fig. 119, the total stress in the two bent 1-in. square rods, mark 850, is

$$\frac{\left(\frac{60 + 52}{2}\right)(27)(16)}{\sqrt{2}} = \frac{24{,}200}{1.414} = 17{,}100 \text{ lb.}$$

The unit stress in bent rods, Mark 850, is

$$\frac{17{,}100}{2.00} = 8550 \text{ lb. per sq. in.}$$

The total stress in the two 1-in. squares, Mark 851, is

$$\frac{\left(\frac{52 + 43}{2}\right)(26)(16)}{\sqrt{2}} = \frac{19{,}760}{1.414} = 13{,}970 \text{ lb}$$

The unit stress in bent rods, Mark 851, is

$$\frac{13{,}970}{2.00} = 6980 \text{ lb. per sq. in.}$$

The stress in the one 1-in. square rod, Mark 852, found in a similar manner, is 11,400 lb. per sq. in.

All beams $G1$ were cambered 1½ in. at the center, to avoid the delusive appearance of a straight beam soffit of this span.

The effective depth of these beams is about one-twentieth of the span length, and although this proportion of depth to span is somewhat unusual, little or no deflection was noted after the removal of supports. Swiss deflectometers were employed to detect any deflection, with the result that no movement was recorded. All steel bars used in the design were hard grade with a minimum elastic limit of 50,000 and a minimum ultimate strength of 75,000 lb. per sq. in. Minimum elongation in 8 in., 10 %.

82. Long Span Rectangular Beams.—The example of long span rectangular beam design given below was used in connection with the same structure as the long span T-beams described in the preceding article. The purpose of these beams (Fig. 120) is to support a passage for pedestrians over a thoroughfare below.

Illustrative Problem.—The depth of these beams was restricted to a total depth equal to one-tenth of the span length, and the width b proportioned accordingly.

$f_s = 20{,}000$, $f_c = 800$ and $n = 15$. Width b assumed = 18 in.

$$\begin{array}{lll} \text{Dead load passage} = (75)(6)(68) & = & 30{,}600 \\ \text{Live load passage} = (50)(6)(68) & = & 20{,}400 \\ \text{Dead load beam} = (1.5)(7)(150)(68) & = & 107{,}100 \\ \hline \text{Total}\ldots\ldots\ldots\ldots\ldots\ldots & = & 158{,}100 \end{array}$$

$$M = \frac{(158{,}100)(70)(12)}{8} = 16{,}600{,}000 \text{ in.-lb.}$$

The design was first tried out assuming balancing values for f_s and f_c. From Table 3, p. 150, when $f_s = 20{,}000$, $f_c = 800$, and $n = 15$.

$$k = 0.375,\ j = 0.875,\ p = 0.0075, \text{ and } K = 131.25$$
$$M = Kbd^2 = (131.25)(18)(80)^2 = 15{,}120{,}000 \text{ in.-lb.}$$

This is less than the moment required. It is desired to retain the width $b = 18$ in. The steel bars used, or ten 1-in. and two 1⅛-in. squares have a sectional area equal to 12.53 sq. in. Then

$$p = \frac{12.53}{(18)(80)} = 0{,}0087$$

The steel in compression must take (Sect. 2, Art. 37*a*)

$$M_2 = 16{,}600{,}000 - 15{,}120{,}000 = 1{,}480{,}000$$
$$p_2 = \frac{1{,}480{,}000}{20{,}000(1 - 2/80)(18)(80)^2} = 0.00066$$
$$p = 0.0075 + 0.00066 = 0.00816 \text{ or}$$
$$A_s = (0.00816)(18)(80) = 11.75 \text{ sq. in.}$$
$$p' = 0.00066\frac{1 - 0.375}{0.375 - 2/80} = 0.00118 \text{ or}$$
$$A' = (0.00118)(18)(80) = 1.70 \text{ sq. in.}$$

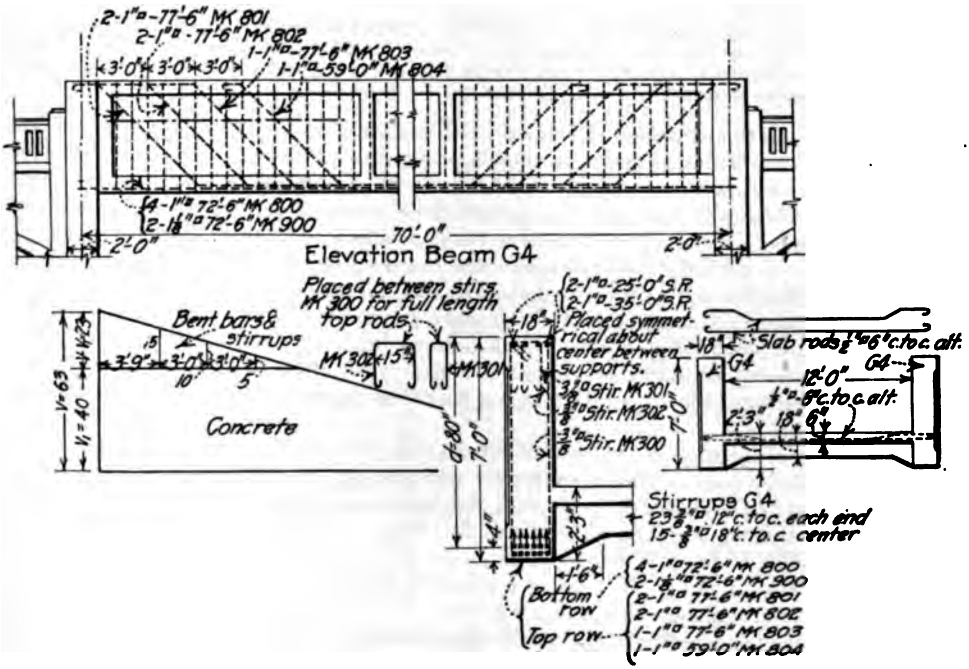


FIG. 120.

Comparing the values found for A_s and A' with the values used in the design it will be noted that the sectional area of tensile steel is slightly more than the theoretical requirements, and the compession steel, four 1-in. squares or 4 sq. in. exceeds the computed area. Compression steel was added to give a stiffer member. The section of member Fig. 120 shows the arrangement of stirrups employed to anchor the compression rods into the body of the beam.

The shearing stress is equal to

$$v = \frac{79{,}000}{(18)(7/8)(80)} = 63 \text{ lb. per sq. in.}$$

After observing the arrangement of bent rods and stirrups in elevation, Fig. 120, it is evident that resistance to diagonal tension is amply provided for.

The total stress taken by two bent 1-in. square rods, Mark 801, is

$$\frac{\left(\frac{23 + 15}{2}\right)(45)(18)}{\sqrt{2}} = \frac{15{,}390}{1.414} = 10{,}900 \text{ lb.}$$

The unit stress in bent rods, Mark 801, is

$$\frac{10{,}900}{2.00} = 5450 \text{ lb. per sq. in.}$$

To investigate the resistance of the other bent rods is unnecessary. 3/8-in. square stirrups were used as shown, to mechanically tie together all parts of the member. Theoretically the stirrups used were not required, but from a practical view point the member may be considered a stronger unit.

The shearing stress v being only 63 lb., the bond stress in the bottom rods at the supports should be comparatively small. The sum of the perimeters of four 1-in. and two 1 1/8-in. square rods is equal to 25 in.

$$u = \frac{79{,}000}{(25)(7/8)(80)} = 45 \text{ lb. per sq. in.}$$

The slab connecting the two beams was designed for a live load of 100 lb. per sq. ft.

$$\begin{aligned} \text{Live load} &= 100 \\ \text{Dead load, 6-in. slab} &= 75 \\ \text{Dead load, } 1/2\text{-in. finish} &= 6 \\ \hline & \ 181 \text{ lb. per sq.ft.} \end{aligned}$$

Dead and live load per linear foot of slab is

$$(12)(181) = 2172 \text{ lb.}$$

$$M = \frac{(2172)(12.5)(12)}{8} = 40{,}725 \text{ in.-lb.}$$

Referring to Table 9, p. 161, when $f_s = 20{,}000$, $f_c = 800$ and $n = 15$, a 6-in. slab with 1/2-in. square bars 6 in. c. to c. is required, when $M = 40{,}725$ in.-lb. The bars have hooked ends extending into the beams. To insure further rigidity, three intermediate cross beams 12 × 18 in. dividing the span into four equal parts were employed as shown in Fig. 120. The soffits of beams *G4* were cambered 1 1/2 in.

83. Hollow-tile Construction.—Hollow-tile construction is extensively used in light buildings such as hotels, office buildings and appartments, and has to a great extent superseded the one way solid slab construction for spans over 12 or 14 ft. Comparative estimates with other forms of solid slab construction will demonstrate the economy of this arrangement for floors. The economy is not only found in the cost of the floor alone, but also in the reduction in the structural sizes of all the supporting members including beams, columns, and footings, by reason of the dead weight, which is much less than for solid slabs designed for equivalent strength. Tile may also be obtained which make possible a two-way reinforced panel with supporting beams along the four sides. Although the function of the tile is only to create a void in the concrete, considerable strength is added to the ultimate capacity of such panels. Tests of combination hollow tile and concrete floors have given surprising results in stiffness and strength.

Tile produced by the different manufacturers will give a large variation in results when subjected to intense heat in kilns prepared for test purposes. Tests show that some tile will not melt at 3000 deg. F., whereas the product of other manufacturers will disintegrate almost to a cinder under this temperature. The resistance to heat that tile will offer in a floor panel is not so satisfactory as when heated uniformly over all surfaces. The lower soffit of the tile exposed to the heat, in many cases has been known to fall out, and no doubt this is due to the expansion of heated surface, while the other portion of the tile protected from the heat remains nearly at normal temperature. The result of this condition will cause the exposed face to shear away from the vertical ribs.

The tile should be thoroughly wetted just before concreting operations are begun. Dry tile readily absorbs moisture from the concrete and for this reason are most objectionable. A thorough sprinkling of the tile should be insisted upon, especially in dry, hot weather. When the tile are placed in position on the falsework, intervals between the ends of tile should be avoided, to prevent loss of the concrete and the added dead weight. The ends of the tile at beam flanges should be closed with cardboard, plaster of Paris or by other satisfactory means.

The accompanying table gives the sizes and weights of commercial tile together with the cubic feet of concrete and the combined weight of tile and concrete per square foot of floor surface when the rib widths and thicknesses of top are as indicated. Particular care should be exercised when pouring the concrete ribs between 4, 5 and 6-in. tile. On account of the light weight of these sizes the concrete should be placed simultaneously in each rib, otherwise the tile will be forced toward the side where the least pressure is exerted. Poor alignment of tile, and the consequent reduction of rib width specified often occurs during construction by neglecting to heed this precaution. The loss of tile on account of breakage due to shipping, hauling and handling ranges from 2 to 5%.

Hollow Tile and Concrete Floors

Cu. Ft. of Concrete Per Sq. Ft. and Weight In Lb. Per Sq. Ft. of Combination Hollow Tile and Concrete Floors When Width of Ribs and Thickness of Top Are As Follows:

Tile		1½" top 4" ribs		2" top 4" ribs		2" top 4½" ribs		2" top 5" ribs		2" top 5½" ribs		2" top 6" ribs		2½" top 4" ribs		2½" top 4½" ribs		2½" top 5" ribs		2½" top 5½" ribs		2½" top 6" ribs	
Size (inches)	Weight	c.f.co.[1]	Wgt.	c.f.co.	Wgt.	c.f.co.	Wgt.	c.f.co.	Wgt.	c.f.co.	Wgt.	c.f.co.	Wgt.	c.f.co.	Wgt.	c.f.co.	Wgt.	c.f.co.	W t.	c.f.co.	Wgt.	c.f.co.	Wgt.
4×12×12	16	0.21	43	0.25	50	0.26	51	0.265	51	0.27	52	0.28	54	0.29	56	0.29	56	0.31	57	0.315	58	0.32	60
5×12×12	20	0.23	49	0.27	56	0.28	57	0.29	57	0.30	59	0.31	59	0.31	62	0.32	63	0.33	60	0.34	65	0.35	65
6×12×12	22	0.25	55	0.29	60	0.30	61	0.31	63	0.32	64	0.33	65	0.33	67	0.34	68	0.36	69	0.37	70	0.38	71
7×12×12	27	0.27	61	0.31	67	0.33	69	0.34	70	0.35	72	0.36	72	0.35	73	0.37	75	0.38	76	0.39	78	0.40	78
8×12×12	30	0.29	67	0.33	70	0.35	74	0.36	75	0.38	78	0.39	79	0.38	79	0.39	81	0.40	82	0.42	84	0.43	85
9×12×12	33	0.31	72	0.35	78	0.37	80	0.39	81	0.40	83	0.42	85	0.40	84	0.41	86	0.43	87	0.44	90	0.46	91
10×12×12	35	0.33	76	0.38	81	0.39	84	0.41	87	0.43	88	0.45	90	0.42	89	0.44	90	0.45	93	0.47	95	0.49	96
12×12×12	40	0.38	86	0.42	91	0.44	95	0.46	97	0.48	99	0.50	102	0.46	99	0.48	101	0.50	103	0.52	105	0.54	108

[1] c.f.co. = cu. ft. of concrete per sq. ft

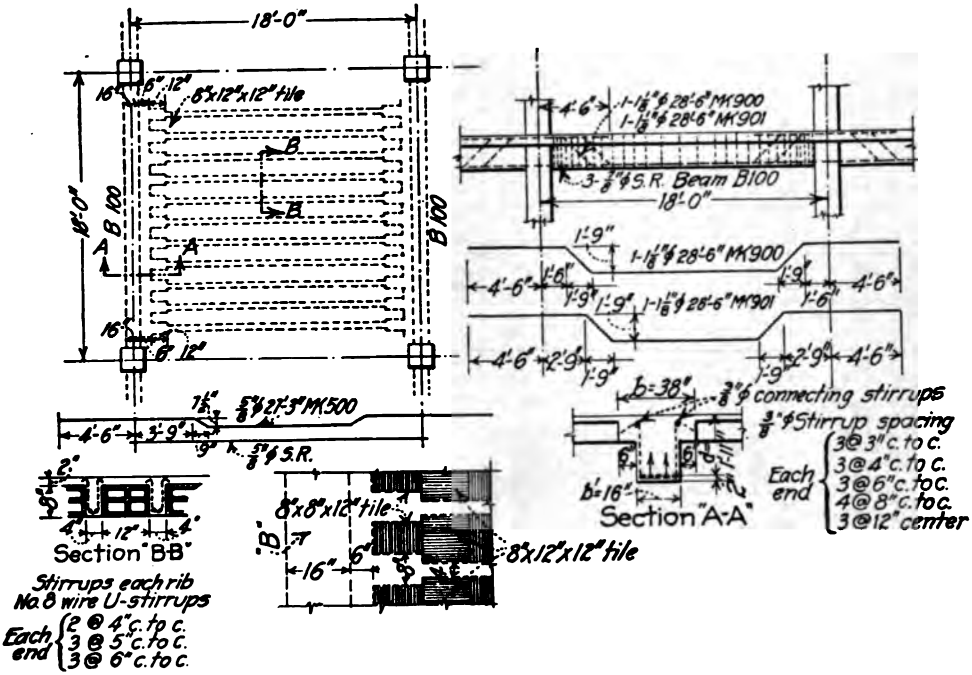


Fig. 121.

Illustrative Problem.—Fig. 121 represents a typical panel in a building, to be designed for combination hollow tile and concrete joists, with supporting beams extending continuous in one direction between columns. Live load assumed = 100 lb. f_s = 16,000, f_c = 650 and n = 15. Maximum v = 110 lb. v_1 = 40.

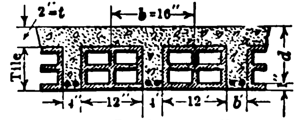
The combination slab will be designed for the following loads in pounds per square foot:

Live load	= 100
Wood floor and fill	= 18
Total superimposed load	= 118

One-way System

Safe Superimposed Loads in Pounds per Sq. Ft. for Unit Steel Stress = 16,000 lb.

$M = \frac{WL}{12}$ $n = 15$	4″ × 12″ × 12″ Tile, 4″ Ribs, 16″c., 2″ Top					6″ × 12″ × 12″ Tile, 4″ Ribs, 16″c., 2″ Top				
	Weight Fl. per sq. ft. = 50#					Weight Fl. per sq. ft. = 60#				
	Concrete per sq. ft. 0.25 cu. ft.			Tile per sq. ft. 0.75–4″ Tile		Concrete per sq. ft. 0.292 cu. ft.			Tile per sq. ft. 0.75 –6″ Tile	
Values p k j	.00276 .249 .918	.00351 .276 .908	.00491 .3172 .8943	.00625 .349 .884	.00767 .378 .874	.0025 .235 .921	.00351 .274 .909	.0045 .305 .90	.00548 .334 .893	.00697 .372 .887
Reinforcement each rib	2-⅜″ϕ	2-⅜″ϕ	2-½″ϕ	2-½″ϕ	2-⅝″ϕ	2-⅜″ϕ	2-½″ϕ	2-½″ϕ	2-⅝″ϕ	2-⅝″ϕ
Span in feet 10	•44 71	•56 103	•78 161	•100 215	•122 272	•56 157	•79 240	100 318	128 400	
11	•40 51	•51 76	•71 124	•91 169	•111 216	•51 119	•71 189	•91 252	111 320	
12	•37 34	•47 56	•65 96	•84 134	•102 174	•47 91	•66 148	•88 202	•102 260	
13	34 22	•48 41	•60 74	•77 107	•94 140	•48 68	•60 118	•77 163	•94 212	•120 284
14		•41 28	•56 57	•71 85	•88 114	•40 51	•56 93	•71 133	•88 175	•111 237
15			•52 44	•67 68	•82 93	•38 37	•52 73	•66 108	•82 145	•104 199
16			•49 32	•62 53	•77 76		•49 57	62 88	76 119	98 167
17				•58 42	•72 61		•46 43	58 70	72 99	92 142
18				•55 32	•68 49		•48 32	•56 57	68 82	87 119
19					•65 39			52 44	64 67	82 101
20									61 55	78 85
21									58 44	74 72
22										71 60
23										68 50
24										65 40
25										
26										
27										
28										
29										
30										
Resisting moment, in.-lb. (M_s)	*16,230*	20,410	28,120	35,360	42,930	28,980	40,010	50,400	61,370	77,890



Typical Detail

When value of "k" is less than $\frac{t}{d}$, Case I applies.

When value of "k" is greater than 0.3786, M_c controls.

When value of "k" is less than 0.3786, M_s controls.

*Indicates neutral axis in the flange.

Note: This table is based on $M = \frac{WL}{12}$. Top steel over support for negative "M," same area A_s as for positive at center of span, top steel over supports extending ¼ or ⅓ of span length. For end spans, when $M = \frac{WL}{10}$, use ⅚ of the combined superimposed load and dead weight of floor given.

For simple spans, when $M = \frac{WL}{8}$, use ⅔ of the combined table values as for end spans.

The unit shear $v = \frac{V}{b'jd}$ is given for each load value in small type.

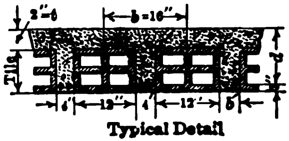
Combination Tile and Concrete Floors.
Unit Concrete Stress = 650 lb. **Continuous Spans**

8″×12″×12″ Tile, 4″ Ribs, 16″c., 2″ Top					10″ × 12″ × 12″ Tile, 4″ Ribs, 16″c., 2″ Top					12″×12″×12″ Tile, 4″ Ribs, 16 c., 2″ Top			
Weight Fl. per. sq. ft. = 70					Weight Fl. per sq ft. = 81#					Weight Fl. per. sq. ft. 91#			
Concrete per sq. ft. 0.334 cu. ft. Tile per sq. ft. 0.75–8″ Tile					Concrete per sq. ft. 0.375 cu. ft. Tile per sq. ft. 0.75–40″					Concrete per sq. ft. 0.417 cu. ft. Tile per sq. ft. 0.75–12″			
.00273 .250 .920	.00347 .280 .914	.00426 .310 .910	.00542 .350 .906	.00614 .372 .905	.00284 .264 .925	.00348 .294 .922	.00443 .334 .920	.00502 .358 .920	.0057 .381 .919	.00295 .283 .933	.00375 .324 .931	.00425 .348 .930	.00483 .372 .930
2-½″φ	2-½″ϕ	2-⅝″φ	2-⅝″ϕ	2-¾″φ	2-½″ϕ	2-⅝″φ	2-⅝″ϕ	2-¾″φ	1-¾″φ 1-¾″ϕ	2-⅝″φ	2-⅝″ϕ	2-¾″φ	1-¾″φ 1-¾″ϕ
78 320	100 424	123 523			100 529	122 665				122 801			
71 255	91 338	111 428			91 424	111 536				112 647			
65 201	83 273	102 349			83 343	102 437				102 529			
60 161	77 222	94 286			77 280	94 361	120 480			94 438	120 580		
56 129	71 182	88 237	111 320		71 231	88 300	111 403			88 365	112 488		
52 103	67 149	81 198	104 269		67 190	81 250	104 340	118 396		82 305	104 413		
49 82	62 123	77 165	98 228	110 267	62 158	77 211	97 289	110 338		77 258	98 352	110 410	
46 65	59 101	72 139	91 194	104 230	59 130	72 178	92 247	104 290		72 218	92 301	104 353	118 413
43 50	55 82	68 116	87 165	98 197	55 107	68 149	87 211	98 250	110 290	68 184	87 259	98 305	112 359
41 38	52 66	65 97	82 141	93 169	52 88	64 126	82 181	93 216	104 251	64 156	82 223	93 264	106 312
	50 53	61 81	78 120	89 146	50 71	61 106	78 156	88 188	99 219	61 132	78 192	88 229	100 273
	47 42	58 66	74 103	84 126	47 57	58 88	74 134	84 162	94 191	58 111	74 166	84 200	95 239
		56 55	71 87	80 108	45 45	56 73	71 115	80 141	90 167	56 93	71 143	80 174	91 210
		53 44	68 74	77 93	43 34	53 60	68 98	77 121	86 146	53 78	68 123	77 151	87 184
		51 34	65 63	73 80		51 48	65 83	74 106	82 127	51 64	65 106	73 132	83 162
			62 52	70 68		49 38	62 70	70 91	79 111	49 51	62 90	70 114	80 142
			59 42	67 58			60 59	68 78	76 97	47 41	59 75	68 99	77 124
				65 48			58 49	65 66	73 83	45 31	58 65	65 84	74 109
				63 40			56 40	63 55	70 72		56 54	63 72	72 95
								61 47	68 61		54 44	61 61	69 82
								59 38	66 52		52 35	59 51	67 71
960	65,810	80,400	101,890	115,200	81,400	99,570	126,460	143,140	160,100	119,080	151,240	171,000	194,270

One-way System

Safe Superimposed Loads in Pounds per Sq. Ft. for Unit Steel Stress = 18,000 lb.

$M = \frac{WL}{12}$ $n = 15$	4" × 12" × 12" Tile, 4" Ribs, 16" c., 2" Top					6" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top				
	Weight Fl. per sq. ft. = 50#					Weight Fl. per sq. ft. = 60#				
	Concrete per sq. ft. 0.25 cu. ft.			Tile per sq. ft. 0.75 cu. ft.		Concrete per sq. ft. 0.292 cu. ft.			Tile per sq. ft. 0.75–6"	
Values p k j	.00276 .249 .918	.00351 .276 .908	.00491 .3172 .8943	.00625 .349 .884	.00767 .378 .874	.0025 .235 .921	.00351 .274 .909	.0045 .305 .900	.00548 .334 .893	.00697 .372 .887
Reinforcement each rib	2-3/8"φ	2-3/8"φ	2-1/2"φ	2-1/2"φ	2-5/8"φ	2-3/8"φ	2-1/2"φ	2-1/2"φ	2-5/8"φ	2-5/8"φ
Span in feet 10	*49 87	*68 122	*88 187			*68 184	*88 288	112 365		
11	45 62	*58 92	*80 146	*102 197		*58 142	*80 219	102 291		
12	*41 45	*52 70	*74 115	*95 157	*115 202	*52 110	*74 174	94 235	115 300	
13	*38 31	*49 52	*69 90	*86 126	*106 164	*49 73	*68 140	86 191	106 246	
14		*45 38	*63 72	*81 102	*99 134	*45 64	*63 112	80 157	98 204	
15		*41 27	*60 55	*75 82	*92 111	*41 48	*59 89	75 129	92 170	117 231
16			*55 43	*70 67	*86 92	*39 35	*55 71	70 106	86 142	110 196
17			*52 32	*66 54	*81 75		*51 56	66 87	81 119	103 166
18				*68 43	*76 61		*49 44	62 71	77 99	98 143
19				*59 33	*73 50		*47 34	59 57	72 83	92 121
20								56 46	68 69	87 103
21								53 36	65 57	84 89
22									62 46	79 75
23									59 38	76 63
24										73 53
25										70 44
26										67 37
27										
28										
29										
30										
Resisting moment, in.-lb (M_s)	*18,180*	*22,940*	*31,620*	39,780	48,300	32,600	45,030	56,650	69,010	87,280



Typical Detail

When value of "k" is less than $\frac{t}{d}$, Case I applies.
When value of "k" is greater than 0.3846, M_c controls.

When value of "k" is less than 0.3846, M_s controls.
*Indicates neutral axis in the flange.

Note: This table is based on $M = \frac{WL}{12}$. Top steel over support for negative "M" same area A_s as for positive at center of span, top steel over supports extending 1/4 or 1/5 of span length.
For end spans, when $M = \frac{WL}{10}$, use 5/6 of the combined superimposed load and dead wt. of floor given.
For simple spans, when $M = \frac{WL}{8}$, use 2/3 of the combined table values as for end spans.

The unit shear $v = \frac{V}{b'jd}$ is given for each load value in small type.

COMBINATION TILE AND CONCRETE FLOORS
Unit Concrete Stress = 750 lb.

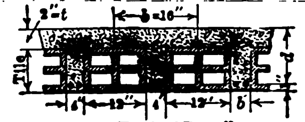
Continuous Spans

8″ × 12″ × 12″ Tile, 4″ Ribs, 16″ c. 2″ Top					10″ × 12″ × 12″ Tile, 4″ Ribs, 16″ c., 2″ Top					12″ × 12″ × 12″ Tile, 4″ Ribs, 16″ c., 2″ Top			
Weight Fl. per sq. ft. = 70#					Weight Fl. per sq. ft. = 81#					Weight Fl. per. sq. ft. = 91#			
Concrete per sq. ft. 0.334 cu. ft. Tile per sq. ft. 0.75–8″					Concrete per sq. ft. 0.375 cu. ft. Tile per sq. ft. 0.75–10″					Concrete per sq. ft. 0.417 cu. ft. Tile per sq. ft. 0.75–12″			
.00273 .250 .920	.00347 .280 .914	.00426 .310 .910	.00542 .350 .906	.00614 .372 .906	.00284 .264 .925	.00348 .264 .922	.00443 .334 .920	.00502 .358 .920	.0057 .381 .919	.00295 .283 .933	.00375 .324 .931	.00425 .348 .930	.00483 .372 .930
2-½″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ
88 370	112 485				112 606								
80 293	103 388				102 487								
74 235	94 315	115 401			94 395	115 502							
68 190	86 258	106 331			86 325	106 416				106 504			
64 154	80 214	98 277			80 269	98 347				98 422			
59 125	75 177	92 232	117 312		75 224	92 293	117 393			92 355			
55 101	70 147	86 195	110 267		70 188	86 247	110 336			86 301	110 408		
52 82	66 123	81 165	104 228	117 266	66 157	81 209	104 288	117 337		81 256	103 350		
49 66	62 101	76 139	97 196	111 230	62 131	77 178	97 248	110 292		76 220	97 303	110 354	
47 51	59 84	72 117	93 169	105 200	59 109	73 152	92 215	104 254	119 298	72 187	93 263	105 309	
44 39	56 69	69 99	88 145	99 173	56 90	69 129	87 185	99 221	118 262	69 160	87 228	99 269	118 318
	53 55	66 84	84 125	95 150	53 74	66 109	84 161	94 193	107 229	66 137	83 199	94 237	107 280
	51 45	63 70	80 107	90 131	51 61	63 93	79 140	90 168	105 202	62 116	80 172	90 207	102 247
	49 35	60 58	77 93	86 114	49 49	59 77	76 122	86 148	98 178	59 99	76 150	86 182	98 219
		58 47	73 79	83 98	47 39	57 65	73 104	83 128	94 157	57 83	73 130	83 159	94 193
		55 39	70 68	79 85		55 54	70 90	79 112	90 138	55 69	70 112	79 139	90 172
			67 57	77 73		53 43	67 77	76 97	87 121	53 58	67 98	76 122	86 151
			65 48	75 64		51 34	65 66	73 85	83 107	51 48	65 84	74 107	84 134
			63 40	71 54			62 55	71 73	80 94	49 37	63 72	71 93	80 118
			60 32	68 45			60 46	68 63	78 82		60 61	68 80	77 103
				66 38			56 38	66 53	75 71		58 51	66 69	75 91
58,570	74,030	90,450	114,620	129,600	91,570	112,010	142,260	161,030	182,740	133,960	170,140	192,370	218,560

One-way System

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQ. FT., FOR Unit Steel Stress = 20,000 lb.

$M = \frac{WL}{12}$ $n = 15$	4" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top					6" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top				
	Weight Fl. per sq. ft. = 50#					Weight Fl. per sq. ft. = 60#				
	Concrete per sq. ft. 0.25 ft. Tile per sq. ft. 0.75–4" Tile					Concrete per sq. ft. 0.292 cu. ft. Tile per sq. ft. 0.75–6"				
Values p	.00276	.00351	.00491	.00625	.00767	.0025	.00351	.0045	.00548	.00697
k	.249	.276	.3172	.349	.378	.235	.274	.305	.334	.372
j	.918	.908	.8943	.884	.874	.921	.909	.900	.893	.887
Reinforcement each rib	2-⅜"φ	2-⅜"φ	2-½"φ	2-½"φ	2-⅝"φ	2-⅜"φ	2-½"φ	2-½"φ	2-⅝"φ	2-⅝"φ
Span in feet 10	*55 102	*70 141	*98 213			*70 211	*98 315			
11	*50 75	*64 108	*89 168	*118 224		*64 164	*89 250	*118 330		
12	*45 55	*58 83	*82 133	*105 180		*58 129	*82 200	*104 268		
13	*42 40	*54 63	*76 106	*96 146	*117 184	*54 99	*70 162	*96 219	*118 280	
14		*50 48	*70 85	*90 119	*108 152	*50 78	*70 131	*89 181	*109 233	
15		*46 35	*66 67	*68 97	*101 126	*46 60	*65 106	*88 150	*102 195	
16			*61 53	*78 80	*94 104	*48 46	*61 86	*78 124	*95 164	*122 224
17			*58 41	*78 65	*89 87	*41 34	*57 69	78 103	90 139	114 191
18				*70 53	*88 72		*55 56	69 86	85 117	109 165
19				*66 42	*79 59		*52 44	65 *70	80 99	102 141
20					*75 49			62 58	76 83	97 121
21					*71 39			59 47	72 70	92 105
22									69 58	88 90
23									66 49	85 77
24										81 66
25										78 56
26										75 48
27										
28										
29										
30										
Resisting moment, in.-lb. (M_c)	20,200	25,490	35,130	44,200	52,860	36,100	50,030	62,940	76,680	96,980



Typical Detail

When value of "k" is less than $\frac{t}{d}$, Case I applies.
When value of "k" is greater than 0.375 M_c controls.
When value of "k" is less than 0.375 M_s controls.
* Indicates neutral axis in the flange.

NOTE: This table is based on $M = \frac{WL}{12}$. Top steel over supports for negative M same area A_s as for positive at center of span. top steel over supports extending to ¼ or ⅓ of span.

For end spans, when $M = \frac{WL}{10}$, use ⅚ of the combined superimposed load and dead wt. of floor given.
For simple spans, when $M = \frac{WL}{8}$, use ⅔ of the combined table values, as for end spans.

The unit shear $v = \frac{V}{b'jd}$ is given for each load value in small type.

COMBINATION HOLLOW TILE AND CONCRETE FLOORS
Unit Concrete Stress 800 lb. Continuous Spans

8″×12″×12″ Tile, 4″ Ribs, 16″ c., 2″ Top					10″× 12″× 12″ Tile, 4″ Ribs, 2″ Top					12″×12″×12″ Tile, 4 Ribs 16c.,″ 2″ Top			
Weight Fl. per sq. ft. = 70#					Weight Fl. per sq. ft. = 81#					Weight Fl. per sq. ft. = 91#			
Concrete per sq. ft. 0.334 cu. ft.		Tile per sq. ft. 0.75–8″			Concrete per sq. ft. 0.375 cu. ft.		Tile per sq. ft. 0.75–10″			Concrete per sq. ft. 0.417 cu. ft.		Tile per sq. ft. 0.75–12″	
.00273 .250 .920	.00347 .280 .914	.00426 .310 .910	.00542 .350 .906	.00614 .372 .905	.00284 .264 .925	.00348 .294 .922	.00443 .334 .920	.00502 .358 .920	.0057 .381 .919	.00295 .283 .933	.00375 .324 .931	.00425 .348 .930	.00483 .372 .930
2-½″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ
98 419							To find reinforcement and moment for any other width						
89 333	114 439				118 550		of rib than 4″, multiply moment and steel area "A." each by distance center to center of ribs and divide by						
82 269	104 358				104 448		16, total lb. per sq. ft. remaining same. The unit shear for any other width of rib = 4″ divided by width of rib × shear sq. in. in table × distance c.c.						
76 219	96 295	118 376			96 370	118 471				118 570	ribs divided by 16.		
71 179	89 245	109 315			89 308	109 395				109 479			
65 147	88 204	102 265			88 258	102 334				102 405			
61 120	78 171	96 224	122 304		78 218	96 283	122 382			96 345			
58 99	78 144	90 191	115 261		73 183	90 241	115 329			90 295	115 399		
55 81	69 120	85 162	108 225	128 263	69 155	85 207	108 285	122 333		85 254	108 347		
52 65	66 101	80 138	108 195	117 230	66 130	81 178	102 248	116 291		80 218	108 302	117 353	
49 51	62 84	77 118	98 169	110 200	62 109	77 152	97 215	110 254	122 289	77 188	97 263	110 309	
47 41	59 69	73 101	98 147	105 174	59 91	78 130	98 188	105 223	116 254	78 162	98 231	105 273	119 321
	57 58	70 86	89 127	100 153	57 77	69 112	88 164	100 196	111 224	69 139	89 201	100 240	114 285
	55 47	67 72	85 111	96 134	54 63	66 95	85 144	96 173	106 199	66 120	85 177	96 212	108 253
		64 60	81 96	92 117	52 52	64 81	81 125	92 151	101 175	64 102	81 155	92 187	104 225
		61 51	78 83	88 102		61 69	78 109	88 133	98 156	61 87	78 135	88 165	100 201
		59 42	75 71	85 89		59 57	75 94	85 117	94 138	59 74	75 119	85 146	96 178
			72 61	83 79		57 47	72 82	81 103	90 121	57 63	72 103	82 129	93 159
			70 52	79 68			69 70	79 90	87 107	54 51	70 90	79 113	89 141
			67 43	76 58			67 60	76 79	84 95	52 42	67 78	76 99	86 125
				73 50			65 51	73 68	81 83		65 67	73 87	84 111
65,080	82,260	100,500	*127,360*	*144,000*	*101,750*	124,460	158070	178,920	197,220	148,850	189,050	213,750	242,840

table on p. 418 shows for an 18-ft. span that 6 × 12 × 12 tile, 4-in. ribs and 2-in. top, with two ⅝-in. square rods to each rib, will give a safe superimposed load of 119 lb. per sq. ft. when the shear is 87 lb. Or 8 × 12 × 12 tile, 4-in. ribs, 2-in. top and two ⅝-in. rounds will give a superimposed load value of 116 lb. per sq. ft. and 68 lb. shear. The latter combination will be accepted in this case for illustration. The value v = 68 will require web reinforcement for each end of each rib. d = 9 in. Referring to Sect. 2, Art. 34c,

$$x_1 = \frac{(68 - 40)(18)}{(2)(68)} = 3.70 \text{ ft.}$$

$$V_1 = \frac{(68 - 40)(4)(3.70)}{2}(12) = 2490 \text{ lb.}$$

A ¼-in. round stirrup at 10,000 will have a value of 980 lb., which would require only say three stirrups at each end. The resultant spacing may be considered unsatisfactory, spaced over the distance 3.70 ft. To give greater economy in the weight of stirrups and in order to preserve the proper spacing, it will be necessary in this case to use wire of smaller gage than ¼ in. A No. 8 gage wire has a cross-sectional area equal to 0.023 sq. in. Assuming the use of No. 8 wire, each stirrup will have a value

$$(2)(0.023)(10{,}000) = 460 \text{ lb.}$$

$$\frac{2490}{460} = \text{say 6 stirrups at each end.}$$

Now the closest spacing at the end of rib is

$$s = \frac{(0.046)(10{,}000)}{(68 - 40)(4)} = 4 \text{ in.}$$

No. 8 wire U-stirrups spaced two at 4 in., three at 5 in. and three at 6 in. will be satisfactory, which will be two more at each end than obtained above.

The above values for shear and moment at the center line of supports do not consider the additional strength produced by the flange of the T-shaped beams. In determining the negative compression in ribs at supports, allowance for this may be made. The moment for each rib at the edge of flange may be assumed to be about $\frac{6}{7}$th of maximum positive moment found at the center of ribs. Table *A* gives the moment 80,400 in.-lb. The moment at the support for the rectangular section of rib will then be

$$M = (80{,}400)\left(\tfrac{6}{7}\right) = 68{,}900 \text{ in.-lb.}$$

One ⅝-in. round of each rib will extend straight in the bottom and one ⅝-in. round will be bent up at both ends at the quarter points, and will extend along the top over beams to the quarter points of adjoining spans. This arrangement will give an equal steel area for positive and negative moments. When stirrups are used at the ends of each rib the straight rods in the bottom may be considered to act in compression, but when stirrups are not used (which is more in accord with general practice for this type of floor construction, the shear for each rib being reduced to about 40 lb. by widening the ribs) the straight rods in the bottom cannot be expected to act effectively in compression. Stirrups in small ribs of this kind are very awkward to install and almost impossible to hold in position during construction, therefore a simple method of widening the ribs at the flange of beams will be illustrated ignoring the value of rods in compression. Referring to Fig. 121, 8 × 8 tile 12 in. long will be used at the ends which will increase the width of concrete ribs to 8 in. instead of 4 in. 8×8×12 tile may be readily obtained from manufacturers. The top steel at supports for each rib has an area equal to 0.60 sq. in. The percentage p for the section where ribs are 8 in. wide will be

$$p = \frac{0.60}{(9)(8)} = 0.0083$$

From Table 2, p. 150,

$$p = 0.0083,\ k = 0.388 \text{ and } j = 0.871$$

Now the stress in the top steel is

$$f_s = \frac{M}{A_s jd} = \frac{68{,}900}{(0.60)(0.871)(9)} = 14{,}700 \text{ lb. per sq. in.}$$

Referring to Diagram 2, p. 153, when p = 0.0083 and f_s = 14,700, the concrete stress is found to be slightly less than 650 lb. per sq. in. This method gives a more definite assurance that the proper resistance to negative compressive stresses will be carried out in actual construction, whereas the use of stirrups invites carelessness in execution.

T-Beam Design.

Weight of tile and concrete floor = 70 lb. per sq. in.
Superimposed load = 118 lb. per sq. ft.

Total floor load = 188 lb. per sq. ft.
Load per linear foot on beam = (188)(18) = 3380
Load of beam per linear foot assumed = 450

Totol load = 3830 lb. per lin. ft.

$$M = \frac{(3830)(18)^2(12)}{12} = 1{,}240{,}900 \text{ in.-lb.}$$

As a general rule, beams in connection with hollow tile and concrete floors come under Case I (see Sect. 2, Art. 40c). The flange is made the same thickness as the floor, which in this case is 10 in.

Buildings are usually planned to obtain the least story height. Beams that extend too far beneath the lower surface of slab will lessen the clearance required between the underside of beam and floor level and therefore are objectionable.

After making rough trials it will be found that a section 16 in. wide by 23 in. effective depth will fulfill the requirements for shear, or,

$$v = \frac{(3830)(9)}{(16)(7/8)(23)} = 107 \text{ lb. per sq. in.}$$

A beam 16 in. wide and with $d = 23$ in. will be considered satisfactory.

$$\frac{t}{d} = \frac{10}{23} = 0.435$$

Now the approximate steel area A_s required will be

$$A_s = \frac{1{,}240{,}900}{(0.87)(23)(16{,}000)} = 3.88 \text{ sq. in.}$$

The flange will be assumed to extend 6 in. beyond each face of web, then $b = 28$ in

$$p = \frac{3.88}{(28)(23)} = 0.0060$$

Referring to Diagram 6, p. 166, when $\frac{t}{d} = 0.435$ and $p = 0.006$, it is at once determined that the neutral plane is in the flange. Case I applies. Since rectangular beam formulas apply, Table 3, p. 150, shows that the controlling value for p is 0.00769 when $f_s = 16{,}000$, $f_c = 650$ and $n = 15$. The value $p = 0.006$ indicates that the concrete stress will be less than the assigned value for f_c and that the steel will control. To confirm this understanding, the formulas governing this case will be used to check the above results.

Using Table 2

$$p = 0.006,\ k = 0.344, \text{ and } j = 0.885$$

The unit stress in the steel and concrete will be

$$f_s = \frac{1{,}240{,}900}{(3.88)(0.885)(23)} = 15{,}710 \text{ lb. per sq. in.}$$

$$f_c = \frac{(2)(15{,}710)(0.0060)}{0.344} = 548 \text{ lb. per sq. in.}$$

The flange width $b = 28$ in. will be used as it is better to have more flange area than is required in this kind of construction on account of working conditions at the building, which make it a difficult matter to maintain an accurate specified space between the ends of tile and the beam sides.

The steel bars will now be selected to conform to the steel section. $A_s = 3.88$. Three ⅞-in. rounds straight in the bottom and two 1⅛-in. rounds bent will give a combined area equal to 3.80 sq. in. The bent rods will be arranged as shown in Fig. 121 and extending to the one-fourth point of adjoining beams. Diagram 8 shows that the two 1⅛-in. rounds or 52% of the total area may be bent up at point 0.21 or 3 ft. 9 in. from the center line of support.

The shear v has been found to be 107 lb. per sq. in. After applying the formulas the following results are obtained: $x_1 = 5.63$ ft., $V_1 = 36{,}210$ lb., and assuming ⅜-in. square U-stirrups at 10,000 lb. per sq. in., the total number of stirrups for each end will be 13, and $s = 2.6$ in. The stirrups at each end may be spaced 3 at 3, 3 at 4, 3 at 6 and 4 at 8 in. center to center. As bent rods will not be used at the supports to resist diagonal tension, the stirrups are proportioned to take the entire shear represented by triangle with height $v - v_1 = 67$ and base $x_1 = 5.63$.

Additional bent rod units may be used to take the entire shear, but a practical arrangement for them is more difficult to obtain than in the case of stirrups at continuous ends of beams.

A simple trial will first be made to ascertain if the rectangular section for negative moment is sufficient without considering the compression rods. The four 1⅛-in. rounds in the top over supports have an area $A_s = 3.97$ sq. in.

$$p = \frac{3.97}{(16)(23)} = 0.0108$$

$$K = \frac{1{,}240{,}900}{(16)(23)^2} = 146$$

Diagram 2 shows, with $p = 1.08\%$ and $K = 146$, that the concrete is stressed to slightly less than 800 lb. and the steel to less than 16,000 lb. With the presence of compression rods, it will be noted from the values obtained that the section at the support will give adequate strength, without resorting to further investigation. It has been noted in Sect. 2, Art. 40*f*, that the negative moment decreases rather abruptly from the point of greatest intensity over the supports and hence only a small portion of a continuous member will be subjected to the greatest stress. For this reason higher working stresses may be assumed at this point, without endangering the strength of the member.

The more accurate formulas for double-reinforced rectangular beams could be applied to obtain the accurate stresses, but it is hardly worth the while, if the section is known to afford safe resistance for negative stress.

The bond stress along the four 1⅛-in. rounds at the top of beam near support is

$$u = \frac{34{,}470}{(14.14)(7/8)(23)} = 121 \text{ lb. per sq. in.}$$

The tension rods in continuous beams over the supports, in important cases, require inverted stirrups to anchor them into the body of the beam. These inverted stirrups should be separate from the stirrups which are designed primarily to resist diagonal tension at the ends. It is essential that the main stirrups engage the straight rods in the bottom at supports, otherwise the value of straight rods as compressive reinforcement, may be compared with the value of longitudinal rods of a column without bands.

When designing a structure composed of many different ordinary members of simple construction, the experienced engineer as a general rule, has not the time at his disposal or the inclination to engage in long theoretical calculations to determine what is required to safely and economically support the dead and superimposed loads. The engineer who has been engaged in the design of practical structures for a number of years develops judgment, intuition, perception and a quick comprehension of the proper proportion required for members when ordinary problems of design arise for solution. In the absence of tables, simple cases of design may be solved by the use of approximate formulas, making it unnecessary to resort to the more complex and longer methods of calculation.

In many forms of construction it is possible to prepare tables that will give directly the requirements desired for given conditions, such as Tables 11, 12 and 13 for combination hollow tile and concrete joists.

84. Metal Floor-tile Construction.—Metal floor tile, although made by a comparatively few manufacturers, are used to no little extent as a substitute for hollow tile. Fig. 122 shows a typical cross section of combination metal tile and concrete floor construction. This type of floor gives a smaller dead weight than hollow tile construction per unit of area and the economy of one over the other should be determined by making comparative estimates.

The upper surface of the metal tile is corrugated or depressed at intervals to prevent sagging when exposed to working conditions after being placed in position on the formwork. If the gage of the metal is too light or the corrugations are not of sufficient depth and spacing, sagging will inevitably occur, resulting in a material loss of concrete, by increasing the specified thickness of the top.

As in the case of tile construction, the metal domes create voids in the concrete and form a system of small T-beams. The design of this type of floor is identieal to that of tile and concrete rib floors. In the case of Hy-Rib ceilings the bottom edges of the metal tile are serrated to straddle the ribs. This type of flat metal ceiling is laid in place on the formwork before the metal tile are placed.

Metal tile are also manufactured in the shape of domes for two-way reinforced panels.

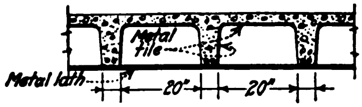


FIG. 122.

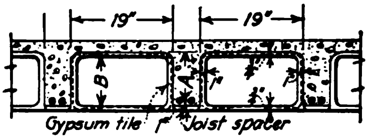


FIG. 123.

85. Gypsum Floor-tile Construction.—Gypsum is one of the best known non-conductors of heat and cold. Besides being used for partitions in buildings, it is now extensively employed in the form of floor tile in combination with concrete for long-span floor construction. Gypsum floor tile are cast from molds, and are made from dense, hard gypsum, with sides, bottom, top and ends cast integral. The end feature of these tile insures against waste of concrete in the event tile is displaced during construction. Fig. 123 illustrates this type of floor. The joist spacer in the bottom of each concrete rib which preserves intact the specified width of rib, is one of the cardinal advantages of this system. Metal lath ceilings are eliminated by the use of this construction and the plaster is applied directly on the gypsum surface. Each tile is reinforced throughout with metal fabric to prevent breakage beyond reasonable expectations, during shipment and handling.

Size of gypsum floor tile (see Fig. 123)

A = Depth of joist	6 in.	8 in.	10 in.	12 in.
B = Height of tile	7 in.	9 in.	11 in.	13 in.
Weight per lin. ft.	24 lb.	27 lb.	30 lb.	33 lb.

Flat gypsum tile are manufactured principally for the roofs of factory buildings. The tile are reinforced in the bottom and are designed for a safe uniform load of 100 lb. per sq. ft. Each unit is 30 × 12 × 3 in. thick and weighs 13 lb. per sq. ft.

86. Beam Schedules.—Fig. 124 shows two typical arrangements for beam schedules, which concentrate in detail the information desired for the preparation of steel order lists, and to simplify the work of the superintendent during the erection of a structure. With such schedules available the superintendent may select in advance the material desired for any one member or collection of members. Knowing the number of beams required and the dimensions for the sections, false-work for the beam sides and bottoms may be readily constructed in advance for the entire building. Schedules are especially adapted for beams of simple design, or those that have a uniform section throughout, with reinforcement bent symmetrical about the center line of the member. The location of "rod bends" from the center line of bearings should be indicated for special reinforcement as shown for B. 30 and *B*. 31, Fig. 124. There is little excuse for wrong installation if the drawings are made clear, concise, and entirely convenient for ready reference.

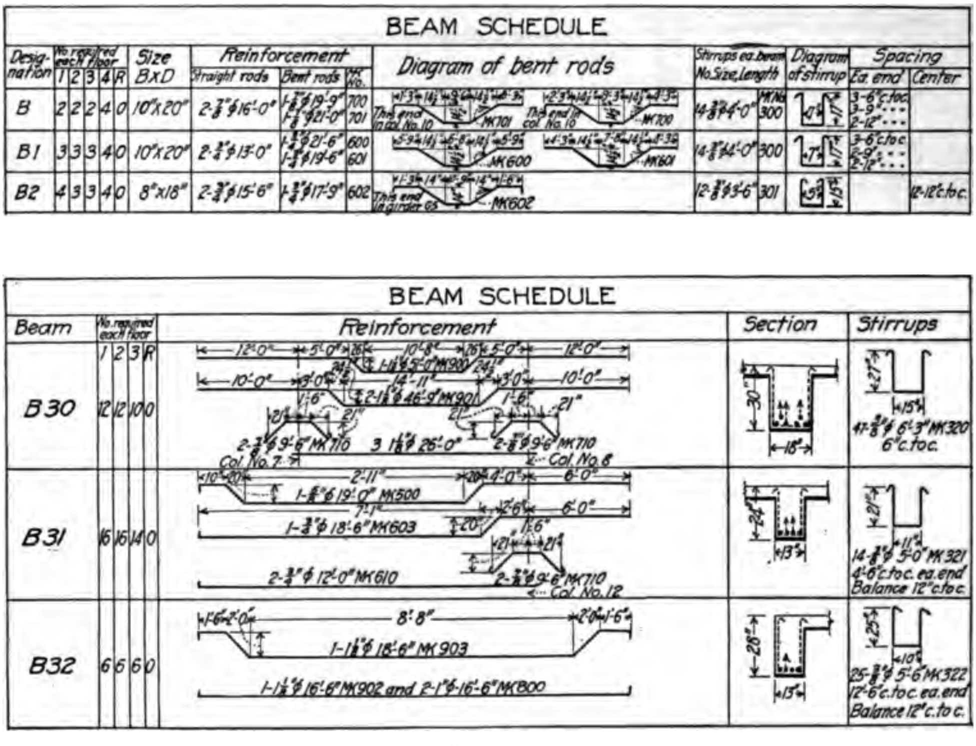

BEAM SCHEDULE
Designation
No. required each floor
Size BxD
Reinforcement
Straight rods
Bent rods
Diagram of bent rods
Stirrups ea. beam No. Size, Length
Diagram of stirrup
Spacing
Ea. end
Center
B
B1
B2
10"x20"
10"x20"
8"x18"
MK700
MK701
MK600
MK601
MK602
BEAM SCHEDULE
Beam
Reinforcement
Section
Stirrups
B30
B31
B32
Col. No. 7
Col. No. 8
Col. No. 12
MK710
MK500
MK603
MK610
MK903
MK320
MK321
MK322

FIG. 124.

It is often necessary to prepare complete details for complicated beams or girders and project the location of straight and bent rods from the elevation. Details with projected reinforcement, such as indicated in Fig. 126, clearly show the relative position and bends for each rod. Some drawings prepared without due regard for accuracy, require the most expert interpretation to fathom the probable intentions of the designer. Superintendents have often been observed making their own interpretations by guessing at the requirements. After the concrete is poured no one else will be any the wiser unless failure occurs. In the event of failure, the designer is deserving of blame and not the superintendent.

87. Unit Construction.

87 *a*. Recognized Systems.—Two systems of Unit Construction are generally recognized as embodying features of designs and installation that represent the correct applica-

PLATE 1

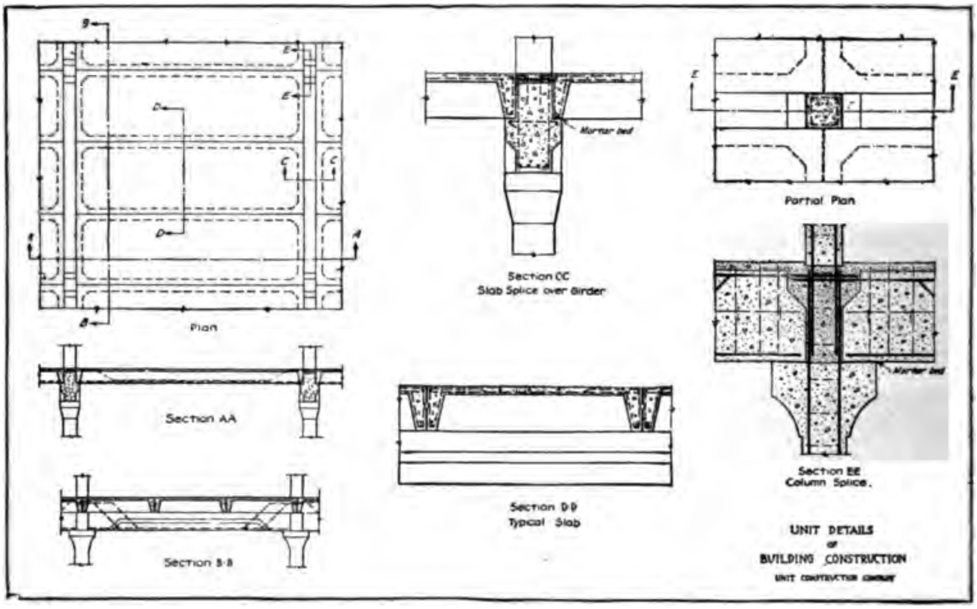
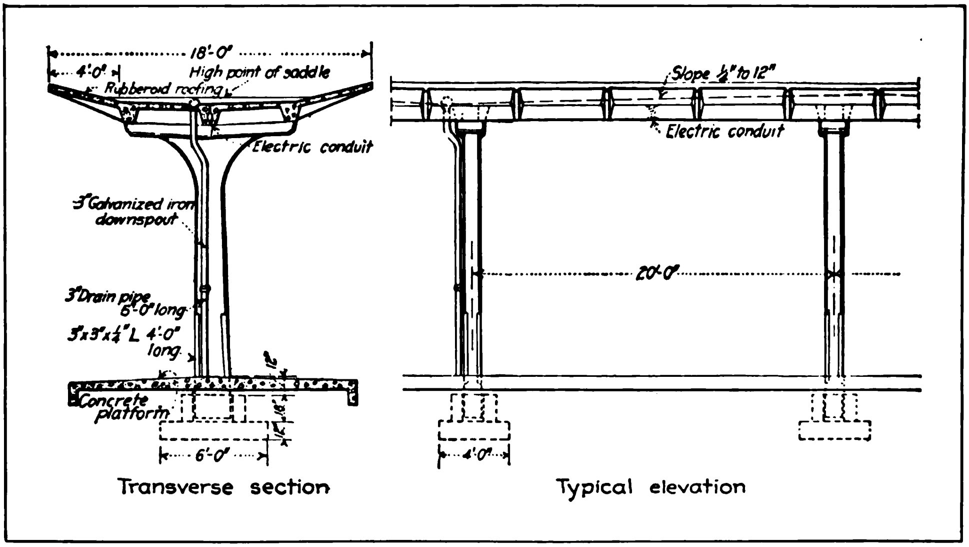
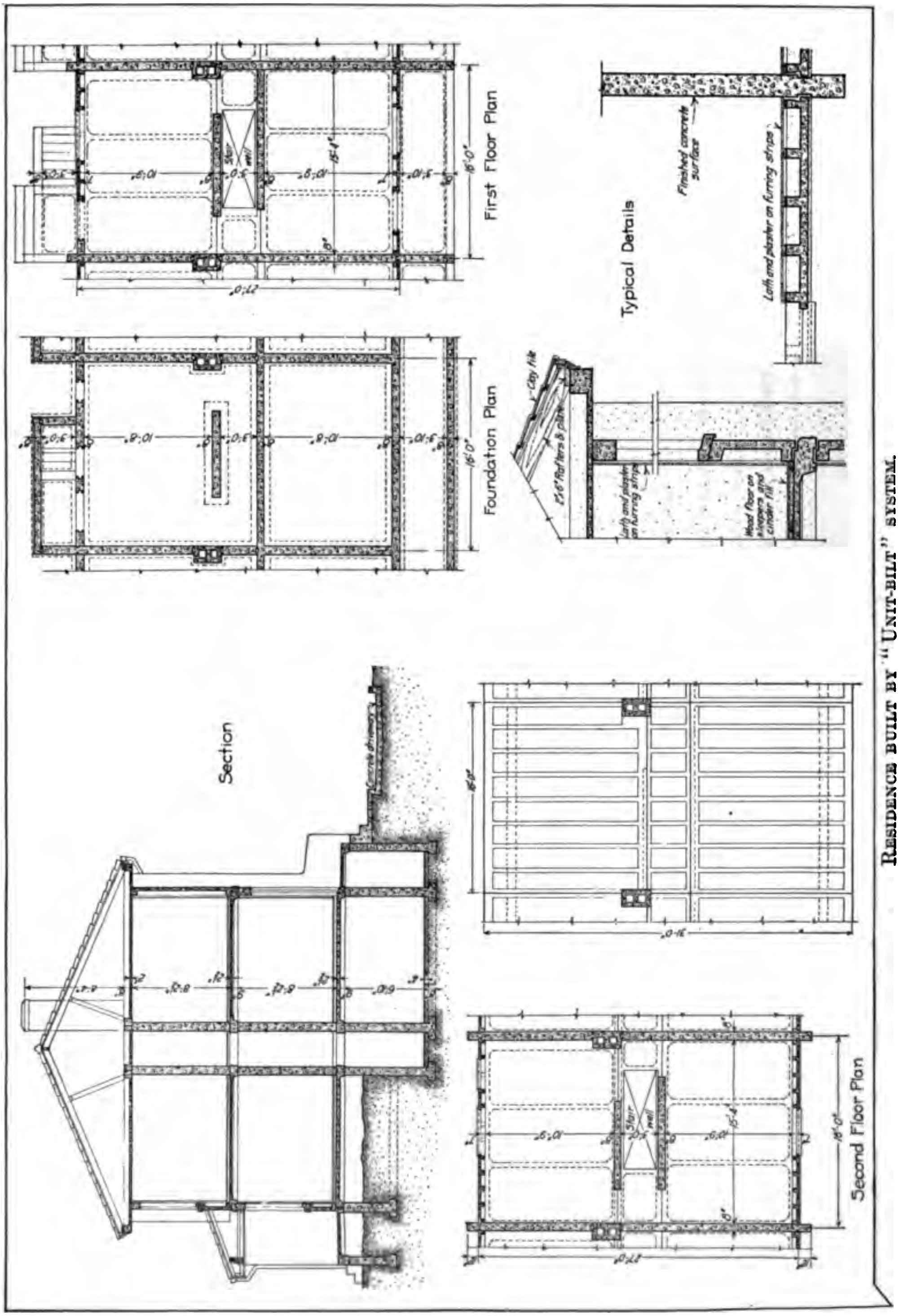


PLATE 2



TRAINSHED BUILT BY THE "UNIT-BILT" SYSTEM.

PLATE 3



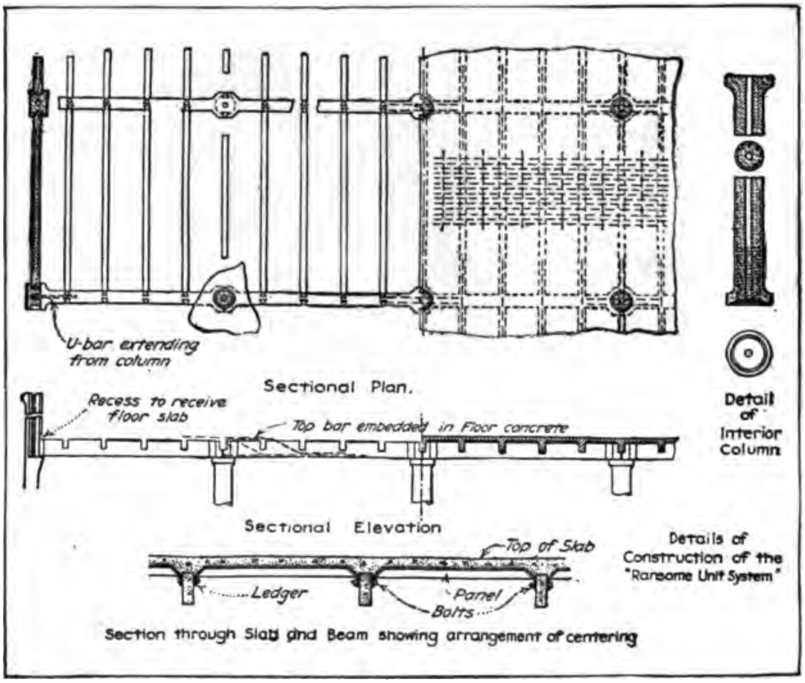
RESIDENCE BUILT BY "UNIT-BILT" SYSTEM.

tion of this type of construction. These systems are the "Unit-bilt" system and the Ransome Unit system. In the "Unit-bilt" system, the slabs, beams, girders, columns, etc. are built apart from the structure, and bonded together in the structure by means of projecting rod anchors, whereas in the Ransome system the slabs are poured in place after the other supporting members have been erected.

87b. Unit-bilt System.—Plates 1, 2 and 3 show the principal details of this type of construction. The unit method of building is especially adapted to train sheds, round houses, one and two story factories and warehouses, where there is a large duplication of members.

The Unit-bilt method as perfected by the Uuit Construction Co. of St. Louis, consists of casting the walls, floors, and roofs in units and erecting these units by means of derricks, in much the same manner as structural steel is erected.

Plate 4


U-bar extending from column
Sectional Plan.
Recess to receive floor slab
Top bar embedded in Floor concrete
Sectional Elevation
Top of Slab
Ledger
Panel Bolts
Section through Slab and Beam showing arrangement of centering
Detail of Interior Column
Details of Construction of the "Ransome Unit System"

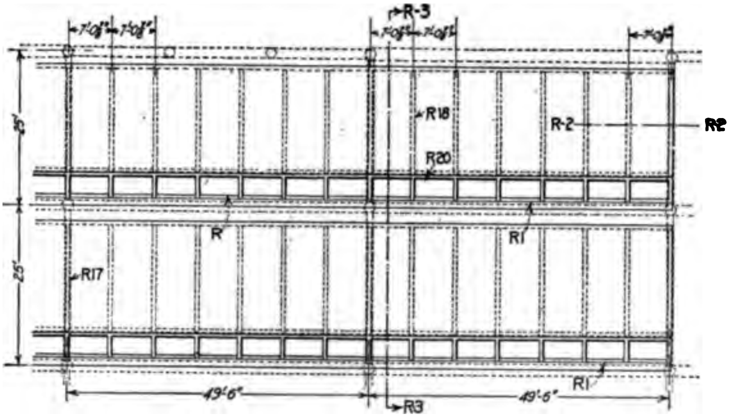
The forms for the units are laid in a casting yard, adjacent to the building site and there all the concrete for the various members is poured, with the exception of the foundations. The advantages of this construction are: (1) the great number of uses possible from one set of forms, especially on large operations; (2) the small number of men required due to the extensive use of locomotive cranes, motor trucks, derricks, etc.; (3) the remarkable speed of erecting attained; (4) the use of local materials for aggregates, whether the aggregate be stone, gravel, or slag; (5) the ease with which the units may be inspected while being poured and before entering the building; and (6) the fact that all shrinkage takes place before the units enter the structure, thus eliminating all shrinkage cracks in the building.

During the year 1918 the construction of fireproof houses for workingmen by this system proved an innovation highly suitable for this purpose. Plate 3 shows the details of construction.

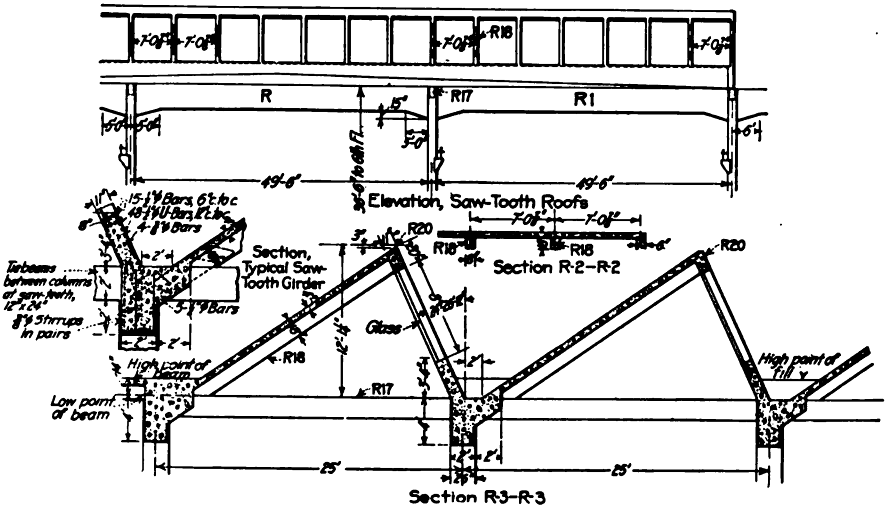
One derrick with its crew of five men can erect an average of thirty units per day, and this is the equivalent of 200 lin. ft. of trainshed, one story of a round house, 2000 sq. ft. of factory or warehouse, or one complete 4-room house. Maximum economy cannot be obtained on a building operation of less than 80,000 sq. ft., as economy is obtained by the greater use of forms and the familiarity of the erecting crews with the particular type of building. However, under favorable conditions, a very great economy can be shown on an operation of as little as 50,000 sq. ft.

Plate 1 gives a general idea of the details of this unit construction. Plate 2 shows the application of this method to trainshed construction.

87c. Ransome Unit System.—Plate 4 shows the main details of this system. The girders are notched along the top at intervals of about 4 ft. to receive the beams. The stirrups and bent rods of these girders are so arranged as to insure a mechanical bond between the girder and slab. The ends of girders are widened, so as to practically cover the cap of the column. The ends of beams which fit into the pockets of the girders are dove-tailed to increase the anchorage at these points.



Typical Plan, Saw-Tooth Roofs



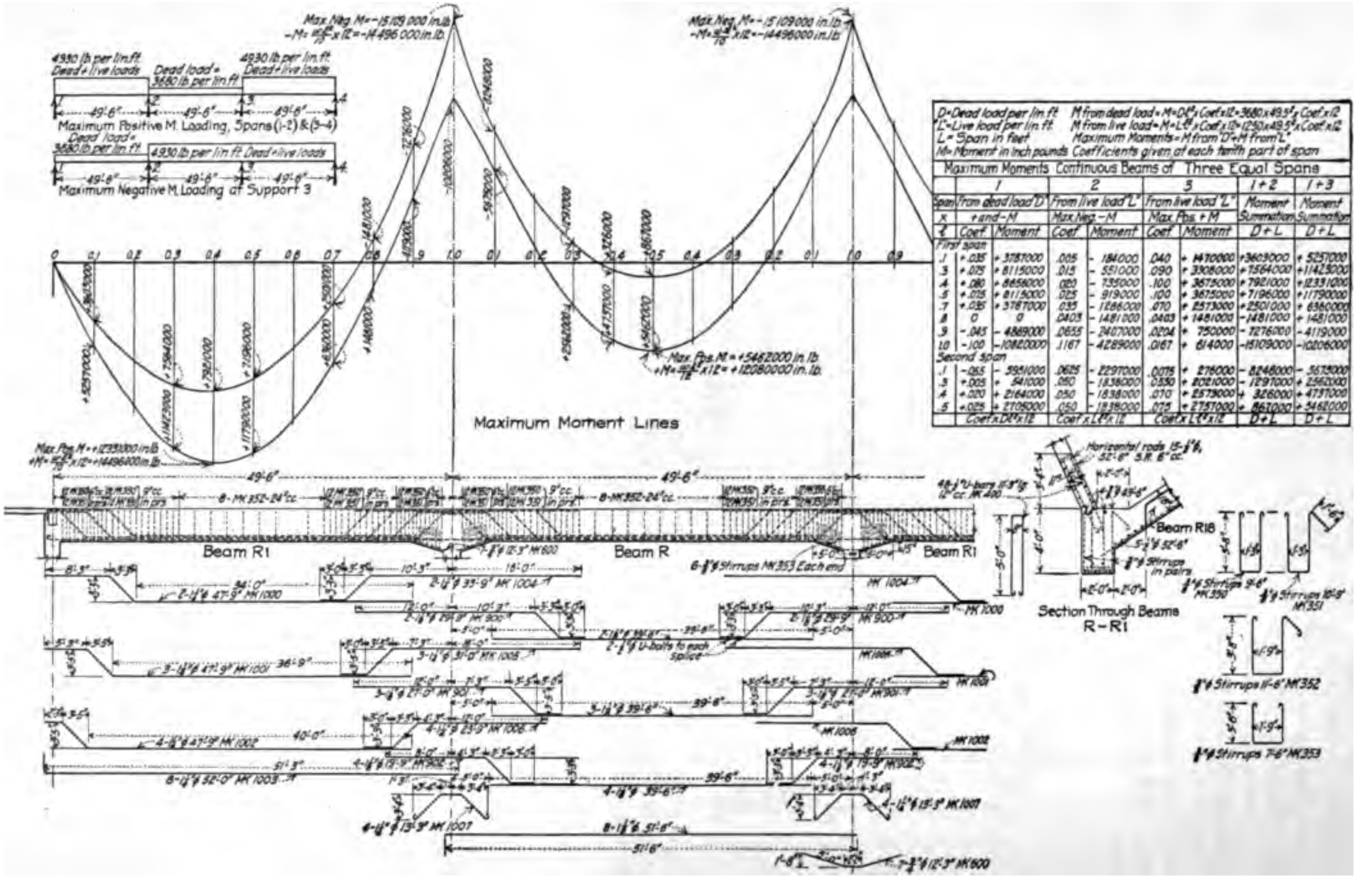
Section R-3-R-3

Fig. 125.

The slabs which span an average of 4 ft. are poured on forms previously erected between the beams. Ledgers are bolted to the sides of the beams upon which the slab forms rest, thus eliminating vertical shores from the floor below. As a consequence of this procedure in the construction of this type of floor, the beams and girders are designed to carry their own dead weight, the weight of the floor slab, and the construction loads incident to building operations.

The shortness of span and the nature of the construction permit of removing all forms in the shortest time.

The columns are reinforced with longitudinal rods and bands or hoops in addition to a longitudinal rod inserted in a cored hole extending through the center of the column. The holes are grouted from the top after the beams and girders are set in place. The cored hole is made larger and flared out at the base of column to give an even bed for bearing. The loads from columns in one story to that of the other beneath are transferred entirely by means of flared caps and bases and are not assisted by the lapping of any longitudinal rods, as is ordinarily done in monolithic construction.


Max. Neg. M = -15109000 in. lb.
-M = wL²/10 x 12 = -14496000 in. lb.
4930 lb. per lin. ft. Dead + live loads
Dead load = 3680 lb. per lin. ft.
4930 lb. per lin. ft. Dead + live loads
49'-6"
Maximum Positive M. Loading, Spans (1-2) & (3-4)
Dead load = 3680 lb. per lin. ft.
4930 lb. per lin. ft. Dead + live loads
Maximum Negative M. Loading at Support 3
Max. Pos. M = +12331000 in. lb.
+M = wL²/10 x 12 = +14496000 in. lb.
Max. Pos. M = +5462000 in. lb.
+M = wL²/12 x 12 = +12080000 in. lb.
Maximum Moment Lines
Beam R1
Beam R
Beam R1
Section Through Beams R-R1
Beam R18
Horizontal rods 15-⅜" φ, 52'-6" S.R. 6" cc.
48-½" U-bars 11'-8" lg. 12" cc. MK 400
6-⅜" φ Stirrups MK 353 Each end
2-⅞" φ U-bolts to each splice
⅜" φ Stirrups 9'-6" MK 350
⅜" φ Stirrups 10'-9" MK 351
⅜" φ Stirrups 11'-6" MK 352
⅜" φ Stirrups 7'-6" MK 353
8-1⅛" φ 52'-0" MK 1003
8-1⅛" φ 51'-6"
51'-6"

D = Dead load per lin. ft. M from dead load = M = DL² x Coef. x 12 = 3680 x 49.5² x Coef. x 12
"L" = Live load per lin. ft. M from live load = M = L'L² x Coef. x 12 = 1250 x 49.5² x Coef. x 12
L = Span in feet Maximum Moments = M from "D" + M from "L"
M = Moment in inch pounds Coefficients given at each tenth part of span

Maximum Moments Continuous Beams of Three Equal Spans

	1		2		3		1 + 2	1 + 3
Span	From dead load "D"		From live load "L"		From live load "L"		Moment Summation	Moment Summation
x	+ and - M		Max. Neg. - M		Max. Pos. + M			
ℓ	Coef.	Moment	Coef.	Moment	Coef.	Moment	D + L	D + L
First span								
.1	+.035	+ 3787000	.005	- 184000	.040	+ 1470000	+3603000	+ 5257000
.3	+.075	+ 8115000	.015	- 551000	.090	+ 3308000	+7564000	+11423000
.4	+.080	+ 8656000	.020	- 735000	.100	+ 3675000	+7921000	+12331000
.5	+.075	+ 8115000	.025	- 919000	.100	+ 3675000	+7196000	+11790000
.7	+.035	+ 3787000	.035	- 1286000	.070	+ 2573000	+2501000	+ 6360000
.8	0	0	.0403	- 1481000	.0403	+ 1481000	-1481000	+ 1481000
.9	-.045	- 4869000	.0655	- 2407000	.0204	+ 750000	- 7276000	- 4119000
10	-.100	-10820000	.1167	- 4289000	.0167	+ 614000	-15109000	-10206000
Second span								
.1	-.055	- 5951000	.0625	- 2297000	.0075	+ 276000	- 8248000	- 5675000
.3	+.005	+ 541000	.050	- 1838000	.0550	+ 2021000	- 1297000	+ 2562000
.4	+.020	+ 2164000	.050	- 1838000	.070	+ 2573000	+ 326000	+ 4737000
.5	+.025	+ 2705000	.050	- 1838000	.075	+ 2757000	+ 867000	+ 5462000
	Coef. x DL² x 12		Coef. x L'L² x 12		Coef. x L'L² x 12		D + L	D + L

FIG. 126.

88. Saw-tooth Roof Construction.—Saw-tooth roofs arranged to provide a diffusion of north light and ventilation have been found especially adapted for factories and machine shops. The cost of this type of roof is somewhat in excess of the ordinary flat arrangement of reinforced concrete construction, or saw-tooth roofs built of other materials, but the advantages gained in efficiency, fireproofness, and maintenance in the case of concrete more than offset the additional cost entailed.

Fig. 125 shows a typical arrangement for reinforced concrete saw-tooth roof construction. The effectiveness of light afforded will depend to a considerable extent on the angle at which the sash and glass are placed. In the example given in Fig. 125, the glass surface has an angle of 24 deg. 26 min. 12 sec. with the vertical, which has proven entirely satisfactory. Then again the lower edge of sash should be a sufficient distance above the surface of trough formed by the saw-tooths over the main supporting girders, to prevent leaks from occurring when snow is banked over the area. All troughs should be arranged for proper drainage.

The saw-tooth roofs shown in Fig. 125 are supported by beams *R* and *R*1, each having a span of 49 ft. 6 in. center to center of supports. The design of these members is shown in Fig. 126. On account of the loads from the 8x8-in. posts being distributed through the 11-in. walls to the beams, the entire dead and live loads were considered uniformly distributed when deriving the maximum positive and negative moments for three spans.

It will be interesting to note that since the dead load of the construction is considerably greater than the live load (in this case approximately three times the live load), the maximum positive moment at the center of interior span is much less than the moment obtained by formula $M = \frac{wl^2}{12}$. The load assumptions used in the design, Fig. 126, could hardly be realized under normal conditions for a roof subjected only to strains occasioned by dead load, snow, wind, and water, but were used and moment lines plotted accordingly to provide a more accurate distribution for the steel reinforcement than could be obtained by the approximate moment assumptions usually employed in the design of important members.

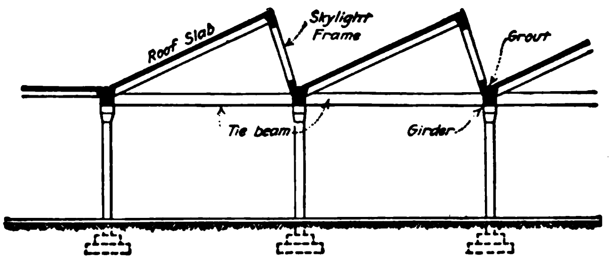


Fig. 127.—Cross-section showing typical arrangement of units in sawtooth construction, *Unit-bilt* system.

The design of long-span continuous members frequently requires the splicing of the reinforcing bars, due to the difficulty of securing the bar length desired in single units. In the design of beams *R* and *R*1, Fig. 126, the bars were spliced as shown at points where the moments would permit. Each rod splice was secured together by two ½-in. U-bolts, which proved more practical and effective in this instance than wire of small gage.

As in the case of Beams *B* and *B*1, Fig. 46, p. 144, the reinforcing bars for maximum positive and negative moeents in beams *R* and *R*1, Fig. 126, were proportioned for moments $M = \frac{wl^2}{10}$ and $\frac{wl^2}{12}$, on account of the building ordinance requirements which had to be complied with.

To insure fireproofness and permanency, saw-tooth skylights are preferably glazed with ¼-in. wired glass securely fastened with glazing clips in metallic frames. Movable sash are mechanically controlled by operating devices.

The Unit Construction Co. of St. Louis has developed a saw-tooth roof construction using separately molded members. Fig. 127 is a cross section of this "Unit-bilt" construction showing the typical arrangement of the pre-cast units. The roof portion of saw-tooth at its lower end rests on a ledge cast in the main supporting girders, and the upper end on a ledge in the skylight frame. ·The lower end of frame also rests on a ledge in the girder. Tie beams between the girders are provided to make a rigid construction and may also be used for the purpose of supporting shafting and other installations if desired.

FLAT SLAB CONSTRUCTION

By W. Stuart Tait

89. In General.—Flat slab construction consists of a concrete slab of practically uniform thickness so designed that the slab carries and transfers the load coming upon it directly to the columns. This form of construction has become very widely used during the past ten years until today it is used to a far greater extent for warehouse and manufacturing buildings than all other types of concrete construction. It is also used in railroad track elevation, in bridges, hotels, apartment buildings, and offices.

The correct method of design for this type of concrete construction has been a contentious point among engineers for a number of years. In spite of a lot of research work, flat slab construction must still be classed as a statically indeterminate structure. The methods of design now in general use must be considered as empirical but we have now had a sufficiently wide experience with their application to be certain of the results to be obtained.

The Joint Committee recently adopted a ruling for the design of flat slab construction but this ruling will not be treated here owing to the fact that it is rather too flexible to be considered as a design method. In addition to this, it has had very little practical application and results obtained in years of experience with other rulings do not justify the higher moments given under the Joint Committee report. The proposed American Concrete Institute ruling agrees closely with many building codes which have been in effect for a number of years, and which have given highly satisfactory results. The A.C.I. ruling, however, is more complete than any city code so far as the writer is aware and covers more completely many of the secondary features of the design. There will be given later a number of examples of the different forms of flat slab construction, fully worked out, so that by following through and understanding the various steps, an engineer will have no difficulty in applying any of the various city codes now in effect. It is almost an impossibility to cover all the points of flat slab design in a handbook such as this, which may determine the difference between a highly satisfactory structure and one which is simply passable. Furthermore, long experience is necessary before a designer may be able to produce the most economical design for a given purpose. It is, therefore, desirable to have designs of this class prepared by an engineer who has had wide experience in flat slab construction and who has proved by the satisfactory structures to his credit that he is an authority on the subject.

A number of systems of flat slab construction have patented features which may or may not contribute to the efficiency, economy, and strength of a design, but it is not the writer's intention to elaborate on these various systems but rather to explain and show examples of flat slab design which can be taken as guides by practicing engineers.

In multiple-story warehouse and factory construction the flat-slab type of design shows marked economy over other types of concrete construction. In most cases too, it offers many physical advantages. Its execution is thoroughly understood by the greater proportion of concrete contractors and, owing to its simplicity, good construction and accurate adherence to the designs are easily obtained. Designing engineers would do well to give this method of construction very careful consideration before deciding upon the type of design to be used for any building, particularly where the structure has large floor areas with fairly regularly spaced columns. It has also been found that in many hotels, offices, and apartment buildings where regular column spacing can be obtained and in which spans of about 18 ft. or less can be used, that the type of flat slab construction, in which large columns without any projecting capitals are used, offers economy and some advantages.

90. American Concrete Institute Ruling.—The diagram, Fig. 128, together with the following notes, is a summary of this proposed ruling.[1] It is inserted so that designers may easily follow the examples worked out later. The general notation is given in *Appendix A.*

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[1] While the following matter was in the hands of the printers, some slight modifications to this proposed ruling were made at the 1919 convention. These proposed modifications are not shown in this chapter.

Slab Thickness.—t shall not be less than $0.02L\sqrt{w} + 1$ in., nor less than $L/32$ for floors and $L/40$ for roofs.

Design Moments.—Numerical sum of positive and negative moments shall not be less than $0.09\ wl_1\ (l_2 - qc)^2$. The report allows a slight variation in the distribution of this total moment. A reasonable division of this moment in percentage is shown in Fig. 128. Note that a slightly different distribution applies in the case of drop construction from that in cap construction. Corresponding moments shall be figured at right angles to those shown in Fig. 128. The moments shown in Fig. 128 are calculated for a value of the cap diameter $c = 0.225L$, and are for interior panels.

For exterior panels the negative moment at the first row of interior columns and the positive moments at the center of the exterior panels on sections parallel to the wall shall be increased 20% over those specified for interior panels. The negative moment at the exterior column parallel to the wall shall not be less than 50% of that for the interior panel.

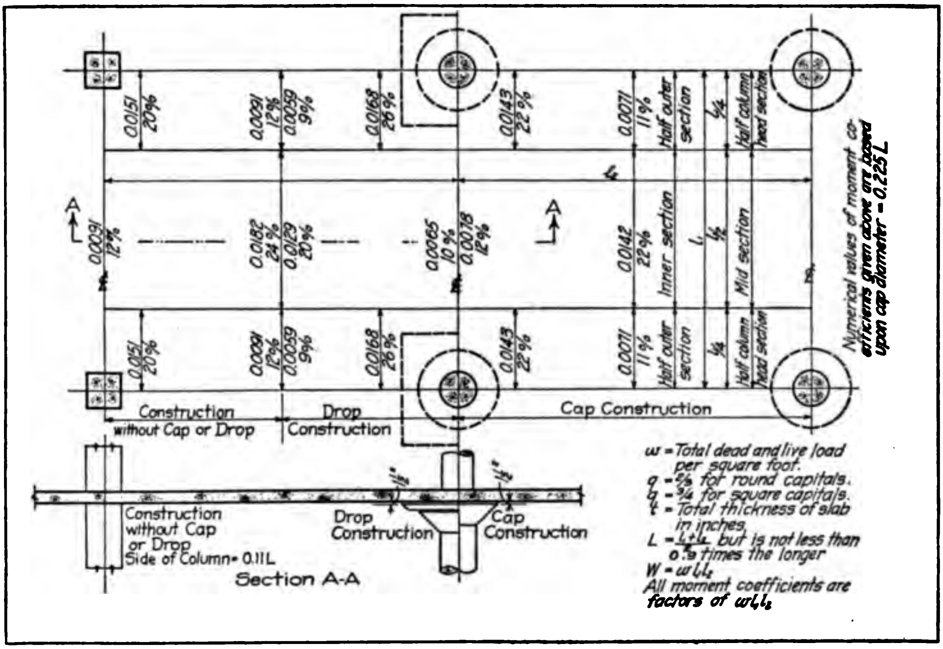

0.0151
20%
0.0091
12%
0.0059
9%
0.0168
26%
0.0143
22%
0.0071
11%
Half outer section
¼
Half column head section
l_1
A
A
0.0091
12%
0.0182
24%
0.0129
20%
0.0065
10%
0.0078
12%
0.0142
22%
Inner section
l_2
Mid section
0.0151
20%
0.0091
12%
0.0059
9%
0.0168
26%
0.0143
22%
0.0071
11%
Half outer section
¼
Half column head section
Numerical values of moment coefficients given above are based upon cap diameter = 0.225L
Construction without Cap or Drop
Drop Construction
Cap Construction
Construction without Cap or Drop
Side of Column = 0.11L
Drop Construction
Cap Construction
Section A-A
w = Total dead and live load per square foot.
q = ⅔ for round capitals.
q = ¾ for square capitals.
t = Total thickness of slab in inches.
L = $\frac{l_1 + l_2}{2}$ but is not less than 0.9 times the longer
W = wl_1l_2
All moment coefficients are factors of wl_1l_2

Fig. 128.

Shear.—The shearing stress which is used as a measure of diagonal tension stress is calculated on a width equal to $L/2$, and the formula used in this calculation is $v = \frac{0.25W}{bjd}$ for cap construction, and $v = \frac{0.30W}{bjd}$ for drop construction. Punching shear at the edge of the drop and at the column cap is calculated by multiplying the total panel load occurring outside the area under consideration by 1.25 and dividing this load by the perimeter of the cap (or drop as the case may be) and by d.

Columns.—Both interior and exterior columns shall be designed for bending. The moment in a column shall not be less than $0.022w_1l_1\ (l_2 - qc)^2$ where w_1 is the designed live load. In the case of exterior columns, the total dead and live load (w) whould be used in the above formula instead of w_1. For top story columns, this amount is all applied at one section of the column. For columns continuous through the story above, the moment is to be divided between the upper and lower column in proportion to their stiffness. Stress used in calculations for direct load and bending may exceed the direct load stresses allowed by 50%.

Stresses.—In the examples worked out, the stresses recommended by the Joint Committee based on 1–2–4 gravel concrete (see *Appendix J*) are used as follows: f_c for positive moment = 650 lb. per sq. in., f_c for negative moment = 750 lb. per sq. in., f_s = 16,000 lb. per sq. in., shear as a measure of diagonal tension = 40 lb. per sq. in. on plain concrete. Punching shear = 120 lb. per sq. in.

91. Example of Design—Drop Construction, Four-way Arrangement.—Take a panel 20 ft. square for a live load of 300 lb. per sq. ft., with cement finish laid with the slab.

Live load = 300 lb.
Dead load = 115 lb.
w = 415 lb.
Use 9¼-in. slab. Fireproofing 1 in.

$t = 0.02L\sqrt{w} + 1 = (0.02)(20)(\sqrt{415}) + 1 = 9.15$ in.
t not less than $L/32 = 7.5$ in.

d for outer section = 9.25 − 1.25 = 8.00 in. (one layer of steel)
d for inner section = 9.25 − 1.50 = 7.75 in. (two layers of steel)

Column capital = $0.225L$ = 4 ft. 6 in.
M − column head section = $0.0336wl_1 \times l_2^2 = (0.0336)(415)(20)(20)^2(12)$ (in.-lb.)
$M = \text{K}bd^2$. $b = 0.3L$ = 6 ft. 0 in. K = 134 (see Sect. 2, Art. 31*a*).

$$d^2 = \frac{(0.0336)(415)(20)(20)^2(12)}{(134)(6)(12)} = 139 \text{ in., or } d = 11.8 \text{ in.}$$

= 11.8 + 1.00 + 1.00 (4 layers steel) = 13.8 in. Use 14 in. Slab = 9¼ in. Drop = 14 − 9¼ = 4¾ × 6 ft. 0 in. × 6 ft. 0 in. Note with 14-in. thickness, d at column becomes 12 in. (see later increase).

$$\text{Shear at column} = \frac{0.3W}{bjd} = \frac{(0.3)(415)(20)(20)}{(120)(0.86)(12)} = 40 \text{ lb.}$$

$$\text{Punching shear at edge of drop} = \frac{(415)(400 - 36)(1.25)}{(4)(72)(7.75)} = 90 \text{ lb.}$$

$$\text{Punching shear at edge of capital} = \frac{(415)(400 - 16)(1.25)}{(\pi)(54)(12)} = 99 \text{ lb.}$$

From this, it is noted that both punching shear and diagonal tention stress are within the limits prescribed.

M − column head section (see above) = 1,340,000 in.-lb.

$$A_s = \frac{1,340,000}{(0.86)(16,000)(12)} = 8.12 \text{ sq. in.}$$

$$M - \text{mid. section} = 0.0065\, wl_1 \times l_2^2$$
$$= (0.0065)(415)(20)(20^2)(12) = 258,000 \text{ in.-lb.}$$

$$A_s = \frac{258,000}{(0.86)(16,000)(8.00)} = 2.34 \text{ sq. in.} = 12\text{—}\tfrac{1}{2}\text{-in. round bars.}$$

$$M - \text{at outer section} = 0.0118wl_1 \times l_2^2$$
$$= (0.0118)(415)(20)(20)^2(12) = 470,000 \text{ in.-lb.}$$

$$A_s = \frac{470,000}{(0.86)(16,000)(8.00)} = 4.27 \text{ sq. in.} = 22\text{—}\tfrac{1}{2}\text{-in. round bars.}$$

$$d^2 \text{ required} = M/Kb = \frac{470,000}{(108)(120)} = 36.2 \text{ in.} \qquad d = 6 \text{ in., where we have 8.00 in.}$$

$$M - \text{at inner section} = 0.0129\, wl_1 \times l_2^2$$

In this design we are using the four-way arrangement of steel, and consequently each bar in each diagonal band cuts the inner section line at 45 deg. The A. C. I. ruling specifies that the sectional area of bars, crossing any section at an angle multiplied by the sine of the angle between these bars and the section may be considered as effective. Now we have two diagonal bands of rods, so the effective area of steel to resist the moment at the inner section = 0.7 × 2 bands of rods = 1.4 bands. Therefore

$$A_s \text{ each diagonal band} = \frac{514,000}{(0.86)(16,000)(8.0)(1.4)} = 3.45 = 18 - \tfrac{1}{2}\text{-in. round bars.}$$

We, therefore, have the following reinforcing for the interior panels:

Direct bands........22—½-in. rounds = 4.30 sq. in.
Diagonal bands.....18—½-in. rounds = 3.53 sq. in.
Across direct bands..12—½-in. rounds = 2.35 sq. in.

If general practice is followed, and we bend up all bars at the column, we have 4.3 + (1.4)(3.53) = 9.24 sq. in. effective, and we found above that 8.12 sq. in. were required at the column head section.

Exterior Panel.—In case the exterior panel is the same size as the interior, for which the design above is shown the moment at the first interior column would be increased by 20% and becomes 1,340,000 × 1.2 = 1,610,000 in.-lb. To resist this increased moment the depth of the drop or the width must be increased. For the sake of uniformity, it is good practice to make all drops the same size and to let all other interior drops be governed by the size of the first interior. If b is kept 6 ft. 0 in.,

$$d^2 = \frac{1,610,000}{(134)(72)} = 167 \qquad d = 12.92 \text{ in., say 13 in.}$$

The drop then becomes $13 + 2 - 9\frac{1}{4} = 5\frac{3}{4}$ in. This increase in the interior column drop thickness woud permit less steel to be used at the column section, but it is better practice to allow the number of rods given above to remain, as short bars should be avoided.

$$\text{Now } A_s \text{ at 1st int. column} = \frac{1,610,000}{(0.86)(16,000)(13)} = 9.0 \text{ sq. in.}$$

A_s direct band normal to wall becomes $= (1.2)(4.3) = 5.1$ sq. in. $= 26 - \frac{1}{2}$-in. round bars.
A_s diagonal band in the exterior panels $= (1.2)(3.45) = 4.14 = 21 - \frac{1}{2}$-in. round rods.

If we therefore band up all bars to top of slab at the 1st interior column from the exterior span, we have $5.1 + (1.4)(4.1) = 10.8$ sq. in. which is satisfactory. Since the moment is the same on each side of the column the extra bars in the exterior panel must continue past the first interior column to the quarter point of the next span. This is shown in diagram, Fig. 129.

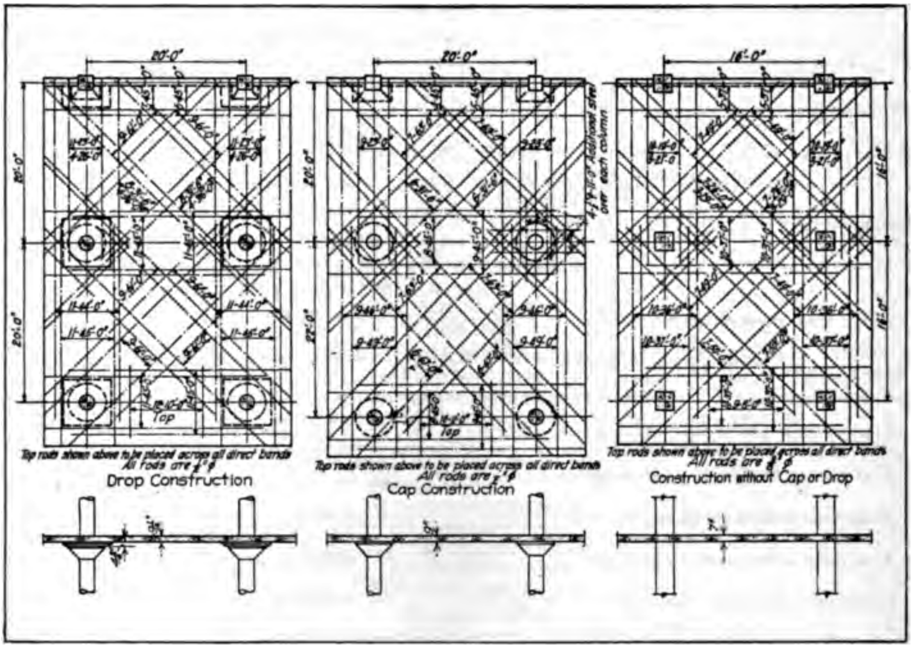

20'-0"
20'-0"
16'-0"
20'-0"
20'-0"
16'-0"
16'-0"
Top
Top
Top rods shown above to be placed across all direct bands
All rods are ½" φ
Drop Construction
Top rods shown above to be placed across all direct bands
All rods are ½" φ
Cap Construction
Top rods shown above to be placed across all direct bands
All rods are ½" φ
Construction without Cap or Drop

FIG. 129.

The A. C. I. ruling specifies a bending moment at the column head section parallel to the wall at the exterior column of 50 % of the interior column head section moment, *i.e.*, $0.0168wl_1l_2^2 = 670,000$ in.-lb. The area of the steel which must be provided to resist the moment across this section $= \frac{670,000}{(0.86)(16,000)(12)} = 4.06$ sq. in. We have available, $5.1 + (1.4)(4.1) = 10.8$ sq. in. which is more than is required. It is good practice to allow about one-half the bars in the exterior direct band to pass through in the bottom of the slab, and the other half to be bent up to top of slab. The moment which this steel is resisting occurs at the edge of the column capital and the distance from this point to the end of the bars is usually ample to develop 16,000 lb. in the steel in bond. It is, however, good practice to bend the ends of some of the bars down into the column or beam.

Now the moment in a direction normal to this is one-half the moment of an interior column head section, since there exists but one-half a section along the wall. Therefore, the cap and drop construction will be similar to that used at an interior column. The steel required at this section = 4.06 sq. in. We have available $2.15 + (0.7)(4.1) = 5.02$ sq. in. So as to provide sufficient imbedment to develop the bars in the exterior diagonal bands, it is generally advisable to continue the ends of the bars along the wall a short distance. The steel arrangement for this design is shown in Fig. 129. Note that one-half of the bars in each band only are broken at each column. This is a recommendation of the A. C. I.

Column Moments.—The ruling specifies a bending moment of $0.022w_1l_1(l_2 - qc)^2$ for interior columns. In this case $M = 0.0158w_1l^3 = (0.0158)(300)(20)^3(12) = 455,000$ in.-lb. Top-story interior columns should be designed for this moment combined with the direct load. The lower-story columns, if of equal size above and below the floor considered, should be designed for half of this moment. If of different sizes, the moment should be divided directly as the stiffness of the columns, *i.e.*, in proportion to the value $\frac{I}{h}$ for each column, where I is the moment of inertia and h the height of the column. Similarly the exterior column moment

$$M = 0.0158wl^3 = (0.0158)(415)(20)^3(12) = 620,000 \text{ in.-lb.}$$

must be provided for. Note particularly that the A. C. I. ruling allows an extreme fiber stress combining direct load and bending 50% greater than the direct stress allowed for columns. While no ruling or ordinance is distinct on this point, it is the writer's opinion that in designing columns for direct load and bending, the entire concrete section may be considered. His reason for this is the fact that we are not required to deduct any portion of the concrete in a beam in designing for negative bending and the lower side of a beam at the supports is just as liable to damage from fire as is the column it rests upon.

92. Example of Design—Cap Construction, Four-way Arrangement.—In the previous example the design was accompanied by many explanations but in this case these will be eliminated, as they would simply be repetition. Take a panel 20 × 22 ft. for a live load of 100 lb. per sq.ft. with a maple floor finish laid on sleepers with a cinder concrete fill between.

$$\begin{aligned} \text{Live load} &= 100 \\ \text{Floor finish} &= 20 \\ \text{Dead load} &= 112 \\ \hline w &= 232 \text{ lb.} \end{aligned}$$

$$t = 0.02L\sqrt{w} + 1 = (0.02)(21)(\sqrt{232}) + 1 = 7.4 \text{ in.}$$

t not less than $L/32 = 7.85$ in.

Column cap $= (0.225)(21) = 4$ ft. 9 in.

$$M - \text{column head section} = 0.0286wl_1 \times l^2_2 = (0.0286)(232)(20)(22)^2(12) \qquad \text{(in.-lb.)}$$

$$d^2 = \frac{(0.0286)(232)(20)(22)^2(12)}{(134)(0.5)(20)(12)} = 47.5. \qquad d = 6.90 \text{ in.}$$

t required $= 6.9 + 1 + 1$ (4 layers of steel). Use 9-in. slab.

$$\begin{aligned} d \text{ at column head section} &= 9 - 2 &&= 7 \text{ in.} \\ d \text{ at mid-section and outer section} &= 9 - 1.25 &&= 7.75 \text{ in.} \\ d \text{ at inner section} &= 9 - 1.5 &&= 7.5 \text{ in.} \end{aligned}$$

$$\text{Max. shear at column} = \frac{0.25w}{bjd} = \frac{(0.25)(232)(20)(22)}{(120)(0.86)(7)} = 35 \text{ lb. per sq. in.}$$

$$\text{Punching shear at column} = \frac{(232)(440 - 17.7)(1.25)}{(\pi)(57)(7)} = 98 \text{ lb. per sq. in.}$$

From this we find that 9-in. slab satisfies the shear requirements.

$$A_s \text{ at column head section across span } l_2 = \frac{(0.0286)(232)(20)(22)^2(12)}{(0.86)(16{,}000)(7)} = 7.95 \text{ sq. in.}$$

$$A_s \text{ at column head section across span } l_1 = \frac{(0.0286)(232)(22)(20)^2(12)}{(0.86)(16{,}000)(7)} = 7.22 \text{ sq. in.}$$

$$A_s \text{ at outer section across span } l_2 = \frac{(0.0142)(232)(20)(22)^2(12)}{(0.86)(16{,}000)(7.75)} = 3.6 \text{ sq. in.} = 18 - \tfrac{1}{2}\text{-in. round rods.}$$

$$A_s \text{ at outer section across span } l_1 = \frac{(0.0142)(232)(22)(20)^2(12)}{(0.86)(16{,}000)(7.75)} = 3.27 \text{ sq. in.} = 17 - \tfrac{1}{2}\text{-in. round rods.}$$

$$A_s \text{ at inner section both directions} = \frac{(0.0142((232)(21)(21)^2(12)}{(0.86)(16{,}000)(7.5)} = 3.53 \text{ sq. in.}$$

$$A_s \text{ each diagonial band} = \frac{3.53}{1.4} = 2.52 = 13 - \tfrac{1}{2}\text{-in. round rods.}$$

If all bars are bent up to top of slab at column, the steel we have available across span $l_2 = 3.6 + 3.53 = 7.13$ sq. in. The steel required = 7.95 sq. in. We must therefore provide 7.95 − 7.13 = 0.82 sq. in. or 4 − ½-in. round bars extra. The steel available across $l_1 = 3.27 + 3.53 = 6.8$ sq. in., and we require 7.22 sq. in. We must therefore provide in this direction 0.42 sq. in. For the sake of uniformity we will add 4 − ½-in. round bars in each direction and these bars will be made 11 ft. 0 in. in length.

The exterior panels and the bending moments in the column will be found and treated in a manner similar to the case where drop construction was used. It must be borne in mind, however, that the bending moment at the first row of interior columns will have to be increased across the section parallel to the wall. Since we must maintain the same thickness of slab, namely, 9 in., it will be necessary to introduce compression steel in this direction provided the exterior span is the same as the interior It is convenient where the layout permits, to slightly reduce the exterior span so that the moment at the first interior column head section is the same as the others. This has been done in Fig. 129 which is a plan of this design.

93. Example of Design Where Neither Drop nor Cap Are Used.—It will have been noted that a smaller percentage of the total bending moment was used at the column head section in the case of cap construction than in the case of drop construction. This is on account of the fact that drop construction is slightly stiffer than cap construction at the supports. Now if in addition we eliminate the capital, we have still a smaller amount of stiffness at the column section. Accordingly, a slightly smaller percentage of the total moment may be used at the column head section. A satisfactory distribution of moments for the four-way arrangement is shown in Fig. 128. Square columns are generally used in this design, as partitions fit up to them better than other shapes. The writer has found that a square column having a size of 0.11L usually proves economical and satisfactory. The bending moment coefficients shown in

Fig. 128 for this class of design are based on this value. Designers will find it economical to maintain the same size of columns for a number of stories in this design. In most cases of designs of this class, the writer has maintained one size of column throughout the structure, simply varying the mix and steel for the increased loads. Take a panel 16 ft. square for a live load of 50 lb. per sq.ft., a partition load of 25 lb., plaster ceiling, and cement finish 1½ in. thick.

Live load = 50
Partitions = 25
Plaster ceiling = 8
Cement finish = 18
Dead load (7-in. slab) = 88
189 lb.

$L/32 = 6$ in.
$0.02L\sqrt{w} + 1 = 5.35$ in.

Minimum column size = (0.11)(16) = 20 in. square. We will use 22 in. square as it will be found later that this size is necessary on account of punching shear.

Fireproofing below steel = 1 in.
Fireproofing above steel = ½ in. (Note cement finish above the structural slab).
d at column head section = 7–½–¾ = 5.75 in.
d at outer section = 7–1–$\frac{3}{16}$ = 5.80 in.
d at mid-section = 7–½–$\frac{3}{16}$ = 6.30 in.
d at inner section = 7–1–⅜ = 5.62 in.
M – column head section = $0.0302wl_1 \times l_2^2$

$$= (0.0302)'189)(16)(16)^2(12) = 278{,}000 \text{ in.-lb.}$$

Had this design been for cap construction, the diameter of the cap would have been about $0.225L$, or 3.6 ft., and b used in the resistingmoment at the column head section would have been $L/2$, or 8 ft. In this case we have a column 1.8 ft. in width and we should therefore use a width of beam = 8.00 – (3.6 – 1.8) = 6.2 ft., or 74 in. Another good rule is to limit b to the width of the column plus $8t$. In this case we would have 22 + 56 = 78 in. In this case we will use the smaller value, namely, 74 in. At the column head section, then $d^2 = \frac{278{,}000}{(134)(74)} = 28.0$ in. and $d = 5.28$ in., so the 7-in. slab assumed above is satisfactory.

$$\text{Shear at column} = \frac{0.25w}{bjd} = \frac{(0.25)(189)(16)(16)}{(74)(0.86)(5.75)} = 33 \text{ lb. per sq. in.}$$

$$\text{Punching shear} = \frac{(189)(256 - 3)(1.25)}{(4)(22)(5.75)} = 118 \text{ lb. per sq. in.}$$

The 7-in. slab is satisfactory for shear.

$$A_s \text{ at column head section} = \frac{278{,}000}{(0.86)(16{,}000)(5.75)} = 3.52 \text{ sq. in.}$$

$$A_s \text{ at mid-section} = \frac{(0.091)(189)(16)^3(12)}{(0.86)(16{,}000)(6.3)} = 0.98 \text{ sq. in.} = 9 - \tfrac{3}{8}\text{-in. round rods.}$$

$$A_s \text{ at outer section} = \frac{(0.182)(189)(16)^3(12)}{(0.86)(16{,}000)(5.8)} = 2.12 \text{ sq. in.} = 20 - \tfrac{3}{8}\text{-in. round rods.}$$

$$A_s \text{ at inner section} = \frac{(0.182)(189)(16)^1(12)}{(0.86)(16{,}000)(5.62)} = 2.18 \text{ sq. in.}$$

$$A_s \text{ each diagonal band} = \frac{2.18}{1.4} = 1.56 \text{ sq. in.} = 14 - \tfrac{3}{8}\text{-in. round rods.}$$

We have available at the column head section, 2.12 + 2.18 = 4.30 sq. in., where we require 3.52 sq. in. provided all the bars are raised to the top of the slab at the column. It would be good policy, however, to run the excess steel, *i.e.*, 4.30 – 3.52 = 0.78 sq. in. or 7 – ⅜-in. round bars in the direct band, through in the bottom of the slab and raise the remaining 13 bars at the column.

The exterior panel may be designed in a manner similar to that given under drop construction. In this case, as in the case of cap construction, it is desirable where the layout permits to decrease the size of the exterior panel. Unless this can be done it will usually be necessary to determine the slab thickness by using the bending moment for the column head section which applies to the first interior columns. Where relatively thin slabs are used, as in this case, compressive reinforcement to provide the increased resisting moment necessary at the first row of interior columns is very inefficient. In the above case, the moment at the first tier of interior columns is such that a depth of 5.8 in. is required. We have practically this depth available due to the fact that punching shear governed the slab thickness. In cases where the increased moment warrants the addition of compression steel or increased slab thickness at the first interior tier, increasing the slab will usually be found to be the most economical. Fig. 129 is a plan of this design.

Other arrangements of steel, such as the two-way or combinations of two and four-way, should be treated in the same manner as the above .Some of the systems of reinforcing prefer slightly different distributions of the total bending moments from those shown in Fig. 128. The distribution shown will, however, give satisfactory results.

94. Construction in Which Brick Bearing Walls are Used Instead of Exterior Columns.—This class of support for flat slab construction should be avoided by engineers wherever possible. In the case of relatively short spans it can usually be relied upon to give satisfactory results. Engineers who have not had extensive experience in the design of flat slab construction would, however, be wise to avoid its design. Neither the Joint Committee nor the American Concrete Institute make any recommendations covering the design of these exterior panels. Flat slab construction relies to a marked extent upon the stiffness of the exterior columns to prevent undue deflection in the exterior panels. If brick walls are used for the exterior support, this restraining action is practically eliminated and the slab itself must therefore be stiffened up. The Chicago Code specifies that the positive moments in these wall panels shall be increased to 50% in excess of those used for interior panels—it does not, however, specify that the slab thickness must be increased. The Chicago Code formula for minimum slab thickness is $t = 0.023L\sqrt{w}$. The minimum slab thickness used for wall bearing construction should be $0.025L\sqrt{w} + 1$, where L is in feet and the result in inches. It would also be well to increase the minimum thickness to about $L/28$ for both floors and roofs.

Pilasters with substantial corbels on line with the interior columns should be used in the wall. The total pilaster and wall thickness should be at least equal to the minimum size of column permitted ($L/12$) plus 4 in. The width of pilaster should be at least equal to the thickness of the wall and pilaster. The corbel should have a vertical depth of at least two courses before the offsets begin. The corbel projection should be determined in the same way as that of a column cap for the same length of span. It will be found that the brick wall will be subjected to some bending in a similar manner to a concrete column. The amount of this bending will probably be less than that occurring in a concrete column. It is well, however, to make an investigation of the stresses occurring in the pilaster by combining the direct load with the bending moment given previously for exterior columns. The pilaster size used should be such that little or no tension is found upon combining the direct load and bending, and also that the maximum compression is within that allowable upon the kind of brickwork used.

95. Rectangular Panels.—Flat slab construction proves most economical in panels which are approximately square and engineers should endeavor to make their layouts accordingly. Most codes and rulings provide that the methods of analysis given are limited to panels in which the long side is not greater than 1.33 times the short side. It has been the writer's practice in cases where this proportion was exceeded to a slight extent, to increase the length of the short side for design purposes only so that this proportion of spans was maintained. Thus, a panel 20 × 28 ft. would be treated in the design as if it were a panel 21 × 28 ft. It will be noted by referring to the total bending moment formula given previously that the moment in any band is a function of the total panel load times the first power of its span. This form of moment equation is recommended by both the Joint Committee and the American Concrete Institute. In some of the older building codes the bending moment in a band is a function of the load per square foot times the cube of the span of the band. It will be found upon examining this method that the moment in a band running in the long direction of the panel is exactly the same as that in a square panel of the same span. In the 21 × 28-ft. panel referred to above, under this method of analysis, we would have the same moment in the band running in the 28-ft. direction that we would have in a panel 28 × 28 ft. This is obviously incorrect for the width of the panel would only be 21 ft. and not 28 ft. In order to insure that sufficient reinforcement is introduced in the short span direct band, the code usually further requires that the steel in that direction shall not be less than 75% of the steel in the long span direction.

Most codes and rulings allow panels, in which the long side is not more than 1.05 times the short side, to be treated as square panels having a span equal to the mean of the length of the two panel sides.

The drop panel in rectangular panels should be made rectangular since with this arrangement we tend to stiffen up the slab on the long span. Thus, in a panel 21 × 28 ft., the drop panel would be about 6 ft. 6 in. × 8 ft. 8 in., the width and length both being directly proportionate to the width and length of the panel.

96. Unequal Adjoining Spans.—In flat slab construction, as in any form of design where we have continuity over a number of unequal spans, the correct bending moments must be obtained by applying the Theorem of Three Moments. In flat slab construction since the moments used are empirical we cannot apply the theorem directly but must increase or decrease the bending moment coefficients used for equal spans by applying certain factors to these moments. The following is a method of applying the Theorem of Three Moments and obtaining the factors referred to for the case shown in Fig. 130.

In this layout we have a series of panels 20 ft. in length and varying from 16 to 25 ft. in width. While the arrangement is somewhat irregular, it will be noted that the length of the panels is in no case greater than 1.33 times the breadth. We will assume that drop construction is to be used and that the column caps are 0.225L in diameter. The bending moment coefficient for uniform spans, then, will be as shown in Fig. 128. As explained previously

$M_1 = \frac{1}{2}M$ for interior column head section
$= 0.0168wl_1 \times l_2^2 = (0.0168)(w)(X)(T_1)^2$
or per foot of width $= 0.016wT_1^2$ .... (1)

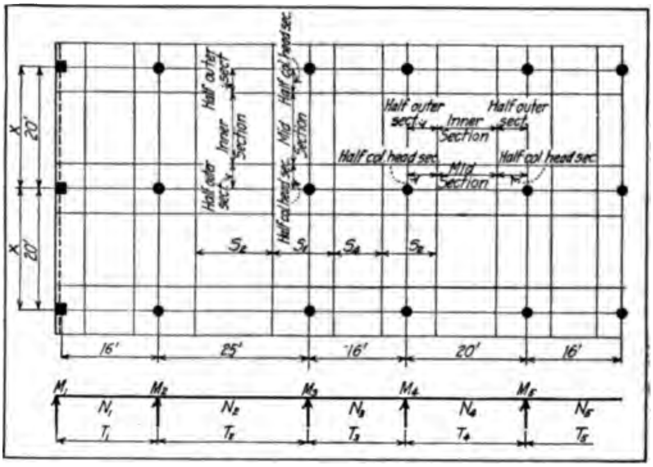


FIG. 130.

Now applying Clapeyron's theorem, we have

$$M_1T_1 + 2M_2(T_1 + T_2) + M_3T_2 = \frac{-w(T_1^3 + T_2^3)}{4} \quad \text{........} (2)$$

$$M_2T_2 + 2M_3(T_2 + T_3) + M_4T_3 = \frac{-w(T_2^3 + T_3^3)}{4} \quad \text{........} (3)$$

$$M_3T_3 + 2M_4(T_3 + T_4) + M_5T_4 = \frac{-w(T_3^3 + T_4^3)}{4} \quad \text{........} (4)$$

$$M_4T_4 + 2M_5(T_4 + T_5) + M_6T_5 = \frac{-w(T_4^3 + T_5^3)}{4} \quad \text{........} (5)$$

Now substituting in these equations for T_1, T_2, etc., we have values per foot width in the direction of span X as follows:

Equation (1) becomes $M_1 = -4.3w$
(2) becomes $82M_2 + 25M_3 = -4861.9w$
(3) becomes $25M_2 + 82M_3 + 16M_4 = -4930.2w$
(4) becomes $16M_3 + 72M_4 + 20M_5 = -3024\ w$
(5) becomes $20M_4 + 88M_5 = -3024\ w$

Solving these simultaneous equations:

$M = -4.3w$
$M_2 = -46.8w$
$M_3 = -41w$
$M_4 = -24.9w$
$M_5 = -28.6w = M_6$

Now find the positive moments N_1, N_2, etc., as follows:

$N_1 = \frac{wl^2}{8} - \frac{1}{2}(M_1 + M_2)$
$N_1 = 6.47w$
$N_2 = 34.23w$
$N_3 = -0.95w$
$N_4 = 23.25w$
$N_5 = 3.40w$

The quantities M_1, M_2, etc., and N_1, N_2, etc., are the bending moments per foot of width at their respective points shown in the diagram and are for one-way construction. Now, if we obtain a value for N_1 similarly to the above but for a series of spans equal to T_1, and also a value for N_2 for a series of spans T_2, etc., and designate these values by Q_1, Q_2, etc., we can by dividing N_1 by Q_1, N_2 by Q_2, etc., obtain factors C_3, C_4, etc., which are the coefficients giving the influence of the adjoining unequal spans upon the bending moments for equal spans. By solving equations (1) to (5) for equal spans and writing Q_1, Q_2, etc., for N_1, N_2, etc., we find

$$Q_1 = 0.066wT_1^2 = 0.066w \times 16^2 = 16.9w$$
$$Q_2 = 0.035wT_2^2 = 0.035w \times 25^2 = 21.8w$$
$$Q_3 = 0.043wT_3^2 = 0.043w \times 16^2 = 11.0w$$
$$Q_4 = 0.041wT_4^2 = 0.041w \times 20^2 = 16.4w$$
$$Q_5 = 0.042wT_5^2 = 0.042w \times 16^2 = 10.8w$$

$$\text{Now } C_2 = \frac{N_1}{Q_1} = \frac{6.47w}{16.9w} = 0.384$$
$$C_4 = \frac{N_2}{Q_2} = \frac{34.23w}{21.8w} = 1.57$$
$$C_6 = \frac{N_3}{Q_3} = \frac{-0.95w}{11.0w} = -0.09$$
$$C_8 = \frac{N_4}{Q_4} = \frac{23.25}{16.4} = 1.42$$
$$C_{10} = \frac{N_5}{Q_5} = \frac{3.4w}{10.8w} = 0.315$$

By means of these coefficients we may determine the correct moments across the spans T_1, T_2, etc., for the inner and outer sections. Take the outer section in span T_1. By referring to Fig. 128 we find that on this section

$M = 0.0118wl_1l_2^2$, which in this case
$= 0.0118wXT_1^2 \times 1.2$ for an exterior panel with equal spans adjoining.
$= 0.0118wXT_1^2 \times C_2 \times 1.2$ for unequal panels.
$= (0.0118)(w)(20)(16)^2(0.384)(1.2)$ in this case. (ft.-lb.)

Similarly across T_2 we find in this case
$M = (0.0118)(w)(20)(25)^2(1.57)$ (ft.-lb.)

and across T_3 we find
$M = (0.0118)(w)(20)(16)^2(-0.09)$ (ft.-lb.)

Note particularly that the coefficient C_6 is negative and that in consequence we have a negative moment at the inner and outer sections across T_3.

The Theorem of Three Moments assumes knife edge supports at the columns. While this is not strictly correct the assumption will give slightly higher moments in the slab on the side of the column adjoining the short span than actually occur. The bending moment occurring in the column will be taken up later.

We previously found numerical values for M_1, M_2, etc. for the negative moments, considering one-way construction for the arrangement of spans shown. By solving equations (1) to (5) we may also obtain numerical value for these moments for a series of equal spans. For these negative moments we will write P_1, P_2, etc. Then by dividing M_1 by P_1, M_2 by P_2, etc., we will obtain coefficients C_1, C_3, etc., which are measures of the influence of the unequal spans upon the negative moments.

In obtaining the numerical values of M_2 and P_2, it is immaterial whether we use the span length of T_1 or T_2, provided in our calculations for the moments occurring in the construction, we use the same value for l_2 in the equation $M = 0.0336wl_1l_2^2$.

The best method is to use in all calculations a span equal to the mean of the spans adjoining the column at which the negative moment is being calculated.

The moment M_1 is not effected by the unequal span arrangement and in consequence C_1 is unity.

Solving equations (1) to (5) for a series of spans of $\frac{T_1 + T_2}{2}$ for P_1, $\frac{T_2 + T_3}{2}$ for P_2, etc., we have

$$P_1 = -0.0168wT_1^2 = (-0.0168)(w)(16)^2 = -4.3w$$
$$P_2 = -0.101w\left(\frac{T_1 + T_2}{2}\right)^2 = (-0.101)(w)(20.5)^2 = -42.5w$$
$$P_3 = -0.079w\left(\frac{T_2 + T_3}{2}\right)^2 = (-0.079)(w)(20.5)^2 = -33.3w$$
$$P_4 = -0.085w\left(\frac{T_3 + T_4}{2}\right)^2 = (-0.085)(w)(18)^2 = -27.5w$$
$$P_5 = -0.083w\left(\frac{T_4 + T_5}{2}\right)^2 = -(0.083)(w)(18)^2 = -26.9w$$

$$C_1 = \frac{M_1}{P_1} = \frac{-4.3w}{-4.3w} = 1 \qquad C_5 = 1.23$$
$$C_3 = \frac{M_2}{P_2} = \frac{-46.8w}{-42.5w} = 1.1 \qquad C_7 = 0.9$$
$$C_9 = 1.06$$

By means of these coefficients we may determine the correct moments at the column head and mid-sections across the series of spans T_1, T_2, etc. Take the column head section between spans T_1 and T_2. By referring to Fig. 128 we find that on this section

$M = 0.0336wl_1l_2^2$ which in this case
$= (0.0336)(w)(X)\left(\frac{T_1 + T_2}{2}\right)^2 \times 1.2$ for an exterior panel with spans $\left(\frac{T_1 + T_2}{2}\right)$ adjoining.
$= (0.0336)(w)(X)\left(\frac{T_2 + T_2}{2}\right)^2 \times 1.2 \times C_3$ for unequal spans
$= (0.0336)(w)(20)(20.5)^2(1.2)(1.1)$ in this case. (ft.-lb.)

Similarly at column head section between span T_2 and T_3, we have in this case

$$M = (0.0336)(w)(20)(20.5)^2(1.23) \qquad \text{(ft.-lb.)}$$

Proceeding as above, the moments occurring in the slab at all sections across the span T_1, T_2, T_3, etc., may be determined. The moments at right angles to these will be entirely unaffected by the inequality of the spans T_1, T_2, etc. and may be obtained in the usual manner. The design may then be treated in the usual way. For the sake of uniformity in the construction, the maximum slab and drop thickness should be determined for the worst cases of bending moment and panel size, and these thicknesses allowed to govern in all cases. The dimensions of the drops will be laid out from the column center lines in each direction and the projection from these center lines made the same proportion of the span in which each part of the drop occurs.

In this analysis it will probably be found that the moment at the inner section across the spans T_1, T_2, etc., is not the same as that found across the span X. In two-way construction, then, the steel in these two sections will vary. In four-way construction the steel used in each diagonal band in a rectangular panel should be the same. The designer will, therefore, take the mean of the two bending moments obtained across the inner sections in calculating the steel required in each diagonal band.

By following the methods of examples given above, all of the moments across the spans T_1, T_2, etc., can be found without doubt arising in the designer's mind. For the sake of entire clearness, a few examples of the method of obtaining the moments across the spans X will be given. Take the moment S_1 as indicated in Fig. 130. This moment is made up of the moments in two half outer sections, in one case the panel width being 25 ft. and in the other 16 ft.—the span in both cases being 20 ft. Assuming the same column capital proportion shown in Fig. 128, we have for drop construction M for half outer section $= 0.059wl_1l_2^2$. In this case

$$\begin{aligned} S_1 &= (0.059)(w)(T_2)(X)^2 + (0.059)(w)(T_3)(X)^2 \\ &= 0.059wX^2(T_2 + T_3) \\ &= 0.118wX^2\left(\frac{T_2 + T_3}{2}\right) \end{aligned}$$

Similarly $S_2 = 0.118wX^2\left(\frac{T_3 + T_4}{2}\right)$, $S_3 = 0.129wT_2X^2$, and $S_4 = 0.129wT_3X^2$. The negative moments at the column head and mid-sections may be found in the same manner.

The analysis given above assumes knife edge supports as stated previously. This means that if we have uniform loading throughout the structure, there will be no bending in the columns. This is not strictly true, but the departure of the moments obtained under this method of analysis from the precise moments is not sufficient to warrant the application of the extremely laborious calculation necessary if the method or slopes and deflections were to be applied. The bending occurring in any interior column, then, will be that due to any entire panel being unloaded while the adjacent panel is loaded fully. For equal adjoining spans the A. C. I. recommends the use of the formula

$$M = 0.022w_1l_1(l_2 - qc)^2$$

where w_1 is the live load per square foot. Where unequal adjoining panels occur, the dead load moments at the column do not cancel each other. In this case, therefore, the moment in the column between spans T_1 and T_2 would become

$$M = 0.022wX(T_2 - qc)^2 - 0.022DX(T_1 - qc)^2t$$

where D is the dead load per square foot of the structure.

The moment in the exterior column will be found in the usual way.

97. Openings.—Providing for openings is one of the places where the designer's judgment and experience come into play. In general, small openings may be placed in the slab in the regions of the outer and inner sections without varying the design in any way. The same opening, however, could not in most cases be introduced within the area occupied by the drop panel without making special provisions in the design. Circular openings of reasonable diameter may be carried through the column capital at the location of sprinkler risers, downspouts, etc., without damage. No definite rules about relatively small openings can be laid down; it is all a matter of judgment and experience.

98. Use of Beams.—Where large openings occur in a flat-slab floor, beams must be used. In some cases also where heavy concentrated loads occur it is advisable to introduce beams. Beams around openings must be designed to carry the loads coming upon them and in addition a portion of floor adjoining. The width of the floor strip to be used cannot be governed by definite rules. The engineer's judgment and experience must be relied upon in this. Wherever possible, it is desirous to use broad flat beams of depth equal to the depth of the slab and drop panel. Some city codes require that spandrel beams be designed to carry a portion of the floor load in addition to the weight of the brick spandrel. Since the spandrel beam is usually much stiffer than the adjoining slab, it is true that a portion of the floor load will be carried by the beam. The only live load falling upon the spandrel beam is that coming from the narrow strip of floor which is carried by the beam. This is generally a small proportion of the total load on the beam. Since the load is practically all dead and is uniformly distributed, the positive moment for an interior beam approaches $\frac{WL}{24}$. We are bound by code, however, to figure for $\frac{WL}{12}$, which is almost twice as much. Under these conditions, then, it is satisfactory to design spandrel beams to carry the wall load only.

99. Capitals at Exterior Columns.—Exterior columns are usually square or rectangular and, in place of the usual half capital, a bracket, the width of the column, is used. This is good and safe practice where reasonably large spandrel beams are used. Where no spandrel beams are used, or where they are relatively thin or shallow, the full half column capital should be used.

100. Drop at Exterior Column.—A half drop panel should be used at the exterior column in practically all cases where drop construction is used at the interior columns.

101. Omission of Spandrel Beams.—In particular cases as in cold storage buildings, where the enclosing walls are built as independent structures, no spanderl beams need be used. In some other cases the beam may be upturned above the ceiling either forming a concrete spandrel or carrying a brick spandrel. In factory construction it will usually be found that a beam having a depth equal to the depth of the slab plus the drop will be sufficient, and in other cases where cap construction is used the slab alone may be of sufficient depth.

102. Narrow Buildings.—The bending moments given in the A.C.I. proposed ruling are for structures over three spans in width. For structures of less width the moments should be increased by factors obtained by comparing the actual negative and positive moments applying in one-way construction with those occurring in an interior span.

103. Minimum Column Size.—Neither the Joint Committee nor the A.C.I. rulings provide a minimum column size as a function of the span. It will be usually found that if the bending moments specified for columns are provided for that a column having a diameter or least size of $L/12$ is required. It is good practice in any case, however, to limit the minimum column size to $L/12$.

104. Width of Bands.—In four-way construction the widths of the bands of steel are usually made $0.4L$. In rectangular panels where the width is much less than the length of the panel, the band widths should be made proportionate to the width of the panel and not a proportion of the span of the band. Thus in a panel 20 × 24 ft. the bands spanning the 24 direction should be 8 ft. wide. In two-way construction, the bands are made $0.5L$ in width with the same provision as above for rectangular panels.

105. Kinds of Bars to Use.—Either deformed or plain bars may be used but the use of square twisted bars should be entirely avoided. A round bar is better than a square for the reason that it packs better at the column and also that the concrete will flow round the intersecting bars more completely.

106. Construction Notes.

106*a*. Pouring Columns and Slabs.—If it is convenient, it is well to pour the columns including the capital up to the underside of the drop or slab before placing the slab steel. If the columns are to be poured after the slab steel is in place, they should be filled up to the top of the capital and allowed to set for about two hours before the slab above is placed.

106*b*. Construction Joints.—Construction joints should be made at the center of the span in all cases. Bulkheads should be set up to form vertical joints in these locations and any concrete which has passed under the bulkhead running out to a feather edge should be carefully removed before pouring the next section.

106*c*. Supporting and Securing Steel.—At the center of the span the steel should be held securely in place at the correct distance above the forms by means of one of the many devices of this nature now on the market. The device used should be in one piece for each band so that the bars may be securely held to an accurate spacing. Two of these spacing bars should be used on each band of steel in the region of the mid span. At the column head, spacing bars are not necessary but substantial supporting bars should be used. The bars must be supported at the correct distance above the formwork and while many metal devices for this purpose have been placed on the market, a concrete block about 3 in. square serves this purpose in the most satisfactory manner. The supporting bars are placed just outside the drop panel and are carried on three of these blocks. The blocks should have the wires imbedded in the top to wire down the supporting bars. For supporting bars, ⅝-in. rounds may be used for spans up to 20 ft., ¾-in. bars for spans between 20 and 25 ft., and ⅞-in. bars for spans between 25 and 30 ft. One more block about ¾-in. higher than those under the supporting bars should be used at the middle of the other sides of the drop panel allowing the slab steel to rest directly upon it. The steel in the mid-section should be securely supported in a manner similar to the steel at the column head section.

106*d*. Placing Steel.—The A.C.I. gives a formula for the distance from the column center line to the point of inflection as $\frac{1}{5}(l_2 - qc) + \frac{1}{2}qc$ for cap construction and a distance equal to $\frac{1}{4}(l_2 - qc) + \frac{1}{2}qc$ for drop construction. In a square panel in which the diameter of column cap is 0.225L, these distances become $0.25l_2$ and $0.3l_2$ respectively. The steel should bend down to the bottom of the slab in approximately these locations. It is essential for good construction that the negative reinforcement be securely supported with the minimum cover allowed in the upper part of the slab and be carried out parallel to the top of the slab to approximately the line of inflexion. Arrangements of steel in which the reinforcement droops away from the top of the slab and is some distance below the top in the region of the line of inflexion will lead to unsatisfactory results.

106*e*. Floor Finish.—Satisfactory results from a structural point of view can be obtained by either applying the floor finish with the slab or applying it after the main slab has set. In general, however, the best and most economical results can be obtained by finishing the structural slab with a mixture of the same mortar proportions as used in the slab, before the slab has set.

106*f*. Future Extensions.—Future extensions can be provided for by introducing a spandrel beam along the side to be extended and leaving in the upper part of the beam a seat about 6 in. in width to receive the new slab. Sufficient steel should be left projecting in the top of the slab to satisfy the moments at the column head and mid-sections. This steel should be structural grade material. It should project beyond the edge of the slab about 80 diameters for bond. After the concrete in the first portion of the building has set, the steel may be bent up and enclosed in the spandrel wall. The usual column capital should be built on the columns projecting out for the future extension and these capitals should be reinforced with bracket bars. This is not an entirely satisfactory method of providing for future extensions. A far better method is to build the foundation only to allow for future extension and construct new independent columns to support the extension later allowing the existing columns to remain supporting the original structure.

FLAT-SLAB FLOORS—AMERICAN CONCRETE INSTITUTE RULING

f_s = 16,000; f_c for positive moment = 650; f_c for negative moment = 750. (See Fig. 128 for distribution of moments)

Interior panel—Superimposed load 100 lb. per sq. ft.

Panel size (feet)	Capital diameter	Size of drop panel	Depth of slab (inches)	Depth of drop (inches)	Concrete in cubic feet per sq. ft.	Steel in each band			Steel in. lb. per sq. ft.
						Direct (inches)	Diagonal (inches)	Across direct (inches)	
16×16	3′6″	4′10″×4′10″	6	2¼	0.52	14-⅜φ	11-⅜φ	8-⅜φ	1.88
17×17	3′9″	5′ 2″×5′ 2″	6½	2¼	0.56	16-⅜φ	13-⅜φ	9-⅜φ	2.00
18×18	4′0″	5′ 6″×5′ 6″	6¾	2½	0.58	18-⅜φ	15-⅜φ	10-⅜φ	2.12
19×19	4′3″	5′ 8″×5′ 8″	7¼	2½	0.62	21-⅜φ	16-⅜φ	11-⅜φ	2.22
20×20	4′6″	6′ 0″×6′ 0″	7½	2¾	0.65	13-½φ	11-½φ	7-½φ	2.42
21×21	4′9″	6′ 4″×6′ 4″	8	2¾	0.69	15-½φ	12-½φ	8-½φ	2.58
22×22	5′0″	6′ 8″×6′ 8″	8¼	3¼	0.71	16-½φ	13-½φ	9-½φ	2.67
23×23	5′3″	7′ 0″×7′ 0″	8¾	3½	0.76	18-½φ	14-½φ	10-½φ	2.77
24×24	5′6″	7′ 4″×7′ 4″	9	3½	0.78	20-½φ	16-½φ	11-½φ	2.95
25×25	5′9″	7′ 6″×7′ 6″	9½	3½	0.82	22-½φ	18-½φ	12-½φ	3.14
26×26	6′0″	7′10″×7′10″	9¾	3¾	0.84	24-½φ	19-½φ	13-½φ	3.27

Interior panel—Superimposed load 150 lb. per sq. ft.

Panel size (feet)	Capital diameter	Size of drop panel	Depth of slab (inches)	Depth of drop (inches)	Concrete in cubic feet per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct (inches)	Diagonal (inches)	Across direct (inches)	
16×16	3′6″	4′10″×4′10″	6	2¾	0.52	18-⅜ϕ	15-⅜ϕ	10-⅜ϕ	2.42
17×17	3′9″	5′ 2″×5′ 2″	6½	2¾	0.56	20-⅜ϕ	16-⅜ϕ	11-⅜ϕ	2.46
18×18	4′0″	5′ 6″×5′ 6″	6¾	3¼	0.59	17-7/16ϕ	14-7/16ϕ	10-7/16ϕ	2.70
19×19	4′3″	5′ 8″×5′ 8″	7¼	3¼	0.63	19-7/16ϕ	15-7/16ϕ	10-7/16ϕ	2.76
20×20	4′6″	6′ 0″×6′ 0″	7½	3½	0.65	17-½ϕ	13-½ϕ	9-½ϕ	2.98
21×21	4′9″	6′ 4″×6′ 4″	8	3½	0.69	18-½ϕ	15-½ϕ	10-½ϕ	3.16
22×22	5′0″	6′ 8″×6′ 8″	8¼	4	0.72	20-½ϕ	17-½ϕ	11-½ϕ	3.36
23×23	5′3″	7′ 0″×7′ 0″	8¾	4	0.76	22-½ϕ	18-½ϕ	12-½ϕ	3.43
24×24	5′6″	7′ 4″×7′ 4″	9	4¼	0.78	24-½ϕ	20-½ϕ	14-½ϕ	3.63
25×25	5′9″	7′ 6″×7′ 6″	9½	4½	0.83	27-½ϕ	22-½ϕ	15-½ϕ	3.84
26×26	6′0″	7′10″×7′10″	9¾	4¾	0.85	30-½ϕ	24-½ϕ	16-½ϕ	4.05

Interior panel—Superimposed load 200 lb. per sq. ft.

Panel size (feet)	Capital diameter	Size of drop panel	Depth of slab (inches)	Depth of drop (inches)	Concrete in cubic feet per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct (inches)	Diagonal (inches)	Across direct (inches)	
16×16	3′6″	4′10″×4′10″	6½	3	0.57	20-⅜ϕ	17-⅜ϕ	11-⅜ϕ	2.70
17×17	3′9″	5′ 2″×5′ 2″	6¾	3¼	0.59	24-⅜ϕ	19-⅜ϕ	13-⅜ϕ	2.90
18×18	4′0″	5′ 6″×5′ 6″	7¼	3¾	0.63	19-7/16ϕ	16-7/16	11.7/16ϕ	3.03
19×19	4′3″	5′ 8″×5′ 8″	7½	4	0.66	22-7/16ϕ	18-7/16ϕ	12-7/16ϕ	3.22
20×20	4′6″	6′ 0″×6′ 0″	8	4	0.70	19-½ϕ	15-½ϕ	10-½ϕ	3.36
21×21	4′9″	6′ 4″×6′ 4″	8¼	4¼	0.72	21-½ϕ	17-½ϕ	12-½ϕ	3.60
22×22	5′0″	6′ 8″×6′ 8″	8¾	4½	0.76	23-½ϕ	19-½ϕ	13-½ϕ	3.79
23×23	5′3″	7′ 0″×7′ 0″	9¼	4½	0.81	25-½ϕ	20-½ϕ	14-½ϕ	3.85
24×24	5′6″	7′ 4″×7′ 4″	9½	5	0.83	28-½ϕ	23-½ϕ	15-½ϕ	4.15
25×25	5′9″	7′ 6″×7′ 6″	10	5	0.87	30-½ϕ	24-½ϕ	17-½ϕ	4.22
26×26	6′0″	7′10″×7′10″	10½	5¼	0.92	33-½ϕ	27-½ϕ	18-½ϕ	4.50

Interior panel—Superimposed load 250 lb. per sq. ft.

Panel size (feet)	Capital diameter	Size of drop panel	Depth of slab (inches)	Depth of drop (inches)	Concrete in cubic feet per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct (inches)	Diagonal (inches)	Across direct (inches)	
16×16	3′6″	4′10″×4′10″	7	3¼	0.61	22-⅜ϕ	18-⅜ϕ	12-⅜ϕ	2.85
17×17	3′9″	5′ 2″×5′ 2″	7½	3½	0.65	18-7/16ϕ	15-7/16ϕ	10-7/16ϕ	3.04
18×18	4′0″	5′ 6″×5′ 6″	7¾	4	0.68	21-7/16ϕ	17-7/16ϕ	12-7/16ϕ	3.28
19×19	4′3″	5′ 8″×5′ 8″	8¼	4¼	0.72	18-½ϕ	15-½ϕ	10-½ϕ	3.48
20×20	4′6″	6′ 0″×6′ 0″	8½	4½	0.74	21-½ϕ	17-½	11-½ϕ	3.75
21×21	4′9″	6′ 4″×6′ 4″	9	4½	0.79	23-½ϕ	18-½ϕ	12-½ϕ	3.84
22×22	5′0″	6′ 8″×6′ 8″	9½	4¾	0.83	25-½ϕ	20-½ϕ	14-½ϕ	4.04
23×23	5′3″	7′ 0″×7′ 0″	10	5	0.87	27-½ϕ	22-½ϕ	15-½ϕ	4.20
24×24	5′6″	7′ 4″×7′ 4″	10½	5	0.91	30-½ϕ	24-½ϕ	17-½ϕ	4.40
25×25	5′9″	7′ 6″×7′ 6″	11	5¼	0.96	33-½ϕ	28-½ϕ	18-½ϕ	4.80
26×26	6′0″	7′10″×7′10″	11¼	5¾	0.98	36-½ϕ	29-½ϕ	20-½ϕ	4.90

Interior panel—Superimposed load 300 lb. per sq. ft.

Panel size (feet)	Capital diameter	Size of drop panel	Depth of slab (inches)	Depth of drop (inches)	Concrete in cubic feet per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct (inches)	Diagonal (inches)	Across direct (inches)	
16×16	3′6″	4′10″×4′10″	7½	3¾	0.66	18-7/16 φ	14-7/16 φ	10-7/16 φ	3.15
17×17	3′9″	5′ 2″×5′ 2″	7¾	4¼	0.68	16-½ φ	13-½ φ	9-½ φ	3.48
18×18	4′0″	5′ 6″×5′ 6″	8¼	4¼	0.72	18-½ φ	14-½ φ	10-½ φ	3.58
19×19	4′3″	5′ 8″×5′ 8″	8¾	4¾	0.77	20-½ φ	16-½ φ	11-½ φ	3.76
20×20	4′6″	6′ 0″×6′ 0″	9¼	4¾	0.81	22-½ φ	18-½ φ	12-½ φ	3.97
21×21	4′9″	6′ 4″×6′ 4″	9¾	5	0.85	24-½ φ	20-½ φ	13-½ φ	4.12
22×22	5′0″	6′ 8″×6′ 8″	10	5½	0.88	27-½ φ	22-½ φ	15-½ φ	4.40
23×23	5′3″	7′ 0″×7′ 0″	10½	5¾	0.92	30-½ φ	23-½ φ	16-½ φ	4.50
24×24	5′6″	7′ 4″×7′ 4″	11	6¼	0.96	33-½ φ	26-½ φ	18-½ φ	4.76
25×25	5′9″	7′ 6″×7′ 6″	11½	6½	1.01	35-½ φ	29-½ φ	20-½ φ	5.00
26×26	6′0″	7′10″×7′10″	12	7	1.05	39-½ φ	31-½ φ	22-½ φ	5.24

Interior panel—Superimposed load 350 lb. per sq. ft.

Panel size (feet)	Capital diameter	Size of drop panel	Depth of slab (inches)	Depth of drop (inches)	Concrete in cubic feet per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct (inches)	Diagonal (inches)	Across direct (inches)	
16×16	3′6″	4′10″×4′10″	7¾	4¾	0.68	15-½ φ	13-½ φ	8-½ φ	3.56
17×17	3′9″	5′ 2″×5′ 2″	8¼	5	0.73	17-½ φ	14-½ φ	9-½ φ	3.68
18×18	4′0″	5′6″ ×5′ 6″	8¾	5¼	0.77	19-½ φ	15-½ φ	10-½ φ	3.80
19×19	4′3″	5′8″ ×5′ 8″	9¼	5¾	0.82	21-½ φ	17-½ φ	11-½ φ	3.97
20×20	4′6″	6′ 0″×6′ 0″	9¾	6	0.86	23-½ φ	19-½ φ	13-½ φ	4.13
21×21	4′9″	6′ 4″×6′ 4″	10¼	6¼	0.90	26-½ φ	21-½ φ	14-½ φ	4.40
22×22	5′0″	6′ 8″×6′ 8″	10¾	6¾	0.95	28-½ φ	23-½ φ	16-½ φ	4.58
23×23	5′3″	7′ 0″×7′ 0″	11¼	7¼	0.99	31-½ φ	25-½ φ	18-½ φ	4.80
24×24	5′6″	7′ 4″×7′ 4″	11¾	7¾	1.04	34-½ φ	28-½ φ	19-½ φ	5.05
25×25	5′9″	7′ 6″×7′ 6″	12¼	8¼	1.09	37-½ φ	30-½ φ	21-½ φ	5.22
26×26	6′0″	7′10″×7′10″	12¾	8½	1.13	41-½ φ	33-½ φ	22-½ φ	5.53

In these panels the steel is lapped to develop the strength of the bar by bond. The steel is considered to be in approximately 2 panel lengths. The necessary supporting bars are included in the steel weights. The concrete in the slab and in the drop panel are included in the concrete quantities.

FLOOR SURFACES

By Allan F. Owen

107. Wood Floor Surfaces.

107*a*. Softwood Flooring.—Soft pine is not used for flooring except some northern pine for very cheap work. It is called 1 × 6-in. matched and dressed, but comes 13/16 × 5¼ in. It is apt to have sap in it and be subject to warping and twisting.

Hard pine, or yellow pine, comes flat sawed and quarter sawed (see Figs. 131 and 132). The flat-sawed flooring should never be used as it splinters badly with use. The quarter-sawed or edge-grain flooring is good flooring and can be used for residences, factories, and warehouses, although it will not wear as well as hard wood. The best yellow pine flooring is cut from logs having the largest number of circular rings per inch of diameter and with the largest proportion of hard summer wood in the rings and the smallest proportion of soft spring growth. Longleaf yellow pine generally has more than 8 rings per inch, and short leaf and loblolly pine generally have less than 8—sometimes only 2 or 3 rings per inch. Yellow pine flooring comes in the following sizes:

Nominal	Actual Thickness	Face
1 × 3	13/16	2¼
1 × 4	13/16	3¼
1 × 6	13/16	5¼
1½ × 3	1 11/32	2¼
1½ × 4	1 11/32	3¼
1½ × 6	1 11/32	5¼
2 × 6	1⅝	5⅛
2½ × 6	2⅛	5⅛
3 × 6	2⅝	5⅛

Yellow pine also comes 4 × 8, 5 × 8, and 6 × 8, grooved for splines (see Fig. 133). This flooring is seldom used for a wearing surface, being used as a structural floor spanning from girder to girder, spacings 6 to 16 ft. When so used a wearing surface of maple is usually added.

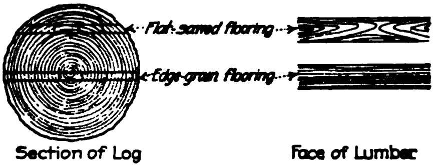


Fig. 131.—Flat sawed and edge grain flooring.

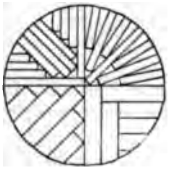


Fig. 132.—Four methods of cutting a quarter sawed log.

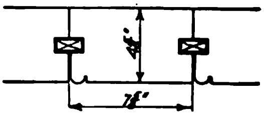


Fig. 133.—Splined flooring.

107*b*. Hardwood Flooring.—Hard maple flooring is most suitable for kitchens, stores, offices, factories, warehouses, and assembly halls. It is smooth and hard, wears well, and can be waxed and polished for dancing, or oiled to keep down dust, or left bare and scrubbed to make it white and clean. Standard grades in maple flooring are:

"Clear"—for the finest work.
"No. 1"—good for all commercial work.
"Factory"—for cheap work.

Maple flooring can be had selected for color by specifying "White Clear." The standard sizes are 13/16 in. thick with 1½, 2, 2¼, and 3¼-in. face; 1 1/16 in. thick with 2, 2¼, and 3¼-in. face; ⅝ in. thick with 1½, 2, and 2¼-in. face.

Beech and birch flooring are manufactured in the same sizes as maple. They do not wear as well as maple, but are better than pine.

Oak flooring is usually considered the most desirable for fine residence work. The standard grades are:

Quarter sawed	"Clear"—(finest grade)
Quarter sawed	"Sap clear"
Quarter sawed	"Select"
Plain sawed	"Clear"
Plain sawed	"Select"
Plain sawed	"No. 1 common"
Plain sawed	"No. 2 common"—(poorest grade)

Standard sizes are 13/16 in. thick with 1½, 2, and 2¼-in. face; ⅜ in. thick with 1½ and 2-in. face. Quarter-sawed oak is sawed so that the face is on a radial line of the log and, as this is parallel to the "silver ray" in the wood, a very beautiful and varied marking is the result (see Fig. 132). The principal advantage of quarter sawing is in securing this mottled grain effect. Oak floors can be filled with a white or colored paste filler to produce natural wood or color effects, and varnished or waxed. Varnish lasts rather longer on oak than on any other floor.

Other hard woods are used only for special ornamental patterns in room borders, show window floors, etc.

107*c*. Parquetry.—The best parquetry is made up of 13/16 in. thick hardwood, cut in short lengths to suit the pattern, dressed, matched and end matched. This class of work must be laid on a very good underfloor and must be scraped and sandpapered after being laid to get a good surface.

107*d*. Refinishing Wood Floors.—In refinishing old floors, thin hardwood strips are used. Flooring 3/8 in. thick comes with tongue and groove, and may be blind nailed. Strips 5/16 in. thick may be had in beech, birch, maple, or oak and are face nailed to the under floor. In connection with this thin flooring "wood carpet" can be had. This consists of ornamental borders, using small pieces of wood glued on a cloth back, each piece to be nailed to the underfloor where the "carpet" is laid. These patterns can be had in a single wood or in a combination of two or more woods, and may include walnut, cherry, white holly, and mahogany.

107*e*. Wood Blocks.—Wood block floors are used in factories where the floor is subject to very rough usage. Standard paving blocks 4 in. thick can be used, and these are usually set in asphalt.

A thinner wood-block flooring has lately come into use which consists of blocks dovetailed and glued to a yellow pine flooring strip. The most used size is 2 1/8 in. thick with 3 1/8-in. face, in lengths up to 8 ft. The sides of the strips are grooved for splines and the strips are blind nailed to joists, nailing strips, or underflooring. This flooring is used where creosoting or asphalt is not wanted and it stays in place through wet and dry weather better than paving blocks. It is a strictly utilitarian floor as the end grain wood tends to hold enough dirt never to look very clean.

107*f*. Supports for Wood Floors.—Softwood and hardwood floors may be nailed direct to joists in ordinary construction buildings or to sleepers bedded in concrete in fireproof buildings. Better floors are built with an underfloor nailed to joists or sleepers and with the finished floor laid diagonally or at right angles to the underfloor. Parquetry and wood blocks must have an underfloor. On a concrete floor construction the finished wood floor may be laid in asphalt direct on the concrete without any nailing strips.

107*g*. Floors for Trucking Aisles.—Special precautions are necessary in building floors where heavy trucking is to be done. Wood block flooring can be used if otherwise satisfactory. Maple flooring has been used more than any other and is probably the most satisfactory in the long run if properly built. It should be laid on a very substantial wood underfloor so that every part of the maple floor is supported, and there is no chance of the truck wheels breaking the floor where they run over a strip near its end. 1 1/8-in. flooring is much stronger than the 13/16-in., and is well worth the difference in cost.

In some warehouses it has been found necessary to lay steel plates on top of the wood floor in the trucking aisles and fasten them down with long countersunk wood screws. This makes a floor that will wear a very long time but it is always noisy. The screws pull out and must be replaced from time to time and the plates buckle up in the center. They wear slippery and the truckers sprinkle the plates to get a film of rust which is easier to work over.

107*h*. Loading Platforms.—Floors exposed to the weather must have provision for drainage and expansion and contraction. 3 × 6-in. oak plank, laid with 1/4-in. open joints, meet these requirements. Cypress and yellow pine are also used.

108. Brick Floors.—Brick is used for floors of packing houses, storage battery rooms, factories, and warehouses where the floor must resist acid, hot and cold water, grease, etc. They are laid edge up for strength where heavy trucking occurs, and the joints must be filled with acidproof or waterproof cement. For this purpose the bricks must be smooth and very dense, preferably vitrified shale brick. Special brick are made from 1 to 4 in. thick and in sizes from 3 × 3 in. to 12 × 12 in., square and rectangular. The foundations for brick floors are the same as for tile floors (see Art. 109*i*).

109. Tile Floors.

109*a*. Cork Tile.—Cork tile are made from cork shavings compressed under very heavy pressure and baked. The blocks thus made are cut in two to make tiles 1/2 in. thick. The tile are cemented to concrete floors, or glued and nailed to wood floors. On account of its durability and non-slip quality, cork tile is especially recommended for the working space in banks, for elevator cars, the space in front of elevators on each floor, for kitchens and bath rooms, and for stair treads and landings.

Cork brick 2 or 2½ in. thick are used for stable floors where the best is wanted regardless of cost.

109*b*. Rubber Tiling.—Interlocking rubber tiling is used for stair halls, elevator floors, and spaces in front of elevators on account of its non-slip property. It is usually ¼ in. thick and is to be cemented to a wood or concrete base.

109*c*. Quarry Tile.—Thin square brick are known as quarry tile. The most used sizes are 6 × 6 in., 8 × 8 in., and 12 × 12 in.; all sizes about 1 in. thick. They are used for fireplace hearths, conservatory floors, engine room floors, hotel grill rooms and for many ornamental purposes. The best red quarry tile were formerly imported from Wales.

109*d*. Ornamental Tiles.—Vestibule and corridors of public buildings are sometimes paved with ornamental tiles which have an embossed pattern (see Sect. 7, Art. 174). The embossment is of value in making a non-slip floor.

109*e*. Ceramic Mosaic.—Probably the most widely used fireproof flooring is ceramic mosaic (see Sect. 7, Art. 174). The standard tile is ¾ in. square and ⅜ in. thick. It comes in white and black, and many colors. The mosaic is usually furnished glued to sheets of paper which are soaked with water and removed after the tile are in place. The combinations of design and color, ornamental borders, and plain fields are unlimited. This tile also comes in large pieces, 2-in. squares and hexagons being largely used.

109*f*. Marble Mosaic.—Marble mosaic is superior in texture and color to ceramic mosaic, but is comparatively little used at the present time.

109*g*. Marble Tile.—The corridor floors of our best public buildings and office buildings are paved with marble tile. This tile is also used for floors in monumental buildings, museums, art galleries, public rooms in fine hotels, club houses, etc., and for toilet room floors. The standard thickness is ⅞ in. and, as the tile are cut for each particular job, there is no standard size. Light colors are preferred for floor tile though verde antique is sometimes used for borders, in spite of the fact that the washing compounds used in cleaning the floors eat away the softer parts. The best wearing floor marble in this country is Tennessee grey or pink.

109*h*. Terrazo Tile.—Marble chips mixed with colored cement and sand are manufactured into tile, then ground and polished. This tile makes good substitute for marble tile or mosaic. It is made in plain colors and also "tutti colouri," the latter being a mixture of different colored marbles.

109*i*. Foundation for Tile Floors.—Any brick, mosaic, or tile floor may be laid over concrete, hollow tile, or wood floor construction, but ample strength and stiffness must be provided to support the finished floor properly and keep it from cracking. When used over wood construction, 2½ in. of concrete foundation should be provided, the top being leveled and left rough at the exact depth below the finished floor line necessary for the kind of finish to be employed. For tile or mosaic ½ in. thick this depth should be 1 in. to allow for the ½-in. setting bed of mortar. For the heavier tile and brick, an allowance of 1 in. should be made for the setting bed. For cork tile, the foundation may be wood or concrete and must be placed the exact thickness of the cork below the level of the finished floor.

110. Cement Floors.—For many purposes a cement floor is the most economical and satisfactory finish, especially for a reinforced concrete building. A great deal of trouble in the past has been caused by the cement finish "dusting." In other words, the top surface wears off rapidly in use and produces a large amount of dust in so doing. To remedy this defect many concrete "hardeners" have been put on the market and some of them have been of value. But their greatest value has been in the extra care taken to procure the necessary grade of workmanship to produce a good cement finish. Where cement sidewalks are laid on cinder foundation, the excess water in the concrete dries out from below as well as above and the rich top dressing of cement and sand can be mixed with just the right amount of water to be troweled to a hard smooth surface. But in reinforced concrete work where the concrete is poured in a semi-fluid state into tight wood forms, the excess of water comes to the top and brings with it laitance (excess hydrated lime) which produces the objectionable dusty floor.

The following method of producing a hard, dense, dustless cement floor is now being used with perfect success:

The forms are poured full of concrete and screeded with a straight edge to bring the surface of the slab up to the grade of the finished floor. Cement finishers then float this down thoroughly while it is still liquid or in a plastic state, bringing in this manner the surplus water present in all concrete to the surface, which carries with it the hydrated lime or laitance in the cement. This is then darbied or floated off to one side. A dry mixture of Portland cement and clean sharp sand (1 to 1½) is then added to the slab and worked into the top of it, filling up all depressions and replacing settlement caused by the removal of the excess water, and enriching the topping, thereby making a more dense wearing surface. After this mixture is thoroughly floated and incorporated into the slab it is given a hard fanning or burnishing, using a steel trowel, polishing and eliminating all trowel marks, producing a hard, unabrasive wearing surface. If the work is properly done, it will be hard, back-breaking work to trowel and polish so dry a surface, but on this depends the success of the cement finish. The floor must be covered within 24 hr. with a heavy layer of sawdust thoroughly wet down and left in place until the building is completed. This sawdust protects the floor from premature use and abuse and, what is of more importance, retards the setting of the cement and improves the quality of the concrete.

The top ¾ in. of a concrete slab may be made of 1 to 1½ Portland cement and ¼-in. granite screenings. This (called *granitoid*) makes an excellent floor for hard usage, but the same precaution must be taken to avoid dusting as described above.

111. Terrazo Finish.—Where terrazo finish is to be used, the foundation is left 2½ in. below the finished floor. 2 in. of concrete is poured on the foundation and then about 1 in. of terrazo finish (Portland cement, sand, and marble chips, mixed almost dry) is spread, rolled, and worked into the top until the proper finished grade is obtained. The surface is polished after the cement has hardened. Color effects are produced by the use of the desired color of marble and by use of colored cement.

Ornamental effects can be had by the use of colored cement. Care must be taken to get colors that are not chemically affected by the cement. The colors should be obtained from a reliable manufacturer of cement colors and used strictly in accordance with his instructions.

112. Composition Floors.—Composition floors, or sanitary floors, are much used for toilet rooms, kitchens, restaurants, etc. There are many varieties on the market, known by various trade names, and they can be had in almost any color, the red and brown probably being the most satisfactory. Magnesia is the basic material in each floor mixture.

When used over a wood floor, wire mesh is laid and tacked down, and about ⅝ in. of Portland cement and sand laid first and ¼-in. composition floor on top of that. When used over a concrete foundation, ¼ in. of cement and sand and a ¼ in. composition floor are sufficient.

When composition floors are finished, they are given a finish of paraffin or wax. This can be washed or mopped over for two or three months before the floor begins to show signs of wear. At that time the floor should be thoroughly washed with warm water and soap or "gold dust" and allowed to dry and then given a coating of oil. Two parts of boiled linseed oil thinned with one part of kerosene should be used. The oil should be applied with a brush or cloth and allowed to dry for about ½ hour and then any surplus oil wiped off. The linseed oil tends to toughen the surface of the composition floor and prevents its becoming rough from wear. The kerosene makes the oil thin enough to soak into the pores of the flooring.

113. Asphalt Floors.—Asphalt is used for waterproof floors in packing houses, canning factories, and wherever it is frequently necessary to flush the floor with water to clean it. When used over a wood foundation, heavy paper is laid and on top of this is placed 2 in. or more of a mixture of hot asphalt and sand which is rolled to a hard, even finish. Not less than 2 in. of the mixture should be used over a concrete foundation.

114. Glass Inserts in Sidewalks.—Glass is used in sidewalks to light the basement space underneath. The pieces of glass are small, generally 3¼ in., round or square, flat top and bottom, or with prisms on the bottom to deflect the light toward the back of the basement. The lights are set in cement on steel, iron, or reinforced concrete frames. When metal frames are used, the lights are generally assembled at the building. Reinforced concrete sidewalk light slabs are made at the factory and shipped to the building ready to be set in place. Care must be taken to have all joints caulked with oakum and waterproofed with asphalt cement.

FLOOR OPENINGS AND ATTACHMENTS

By Allan F. Owen

115. Floor Openings.—Special framing must be used around openings through floors for elevator shafts, stairways, dumb waiters, wire shafts, and plumbing spaces. Figs. 134 to 139 inclusive, show typical framing.

In concrete floors, wrought-iron and galvanized-iron sleeves are built into the construction work for all steam, return, sprinkler, sewer, gas, and similar pipes. All floor sleeves should be flush with the ceiling line and should extend about 2 in. above the floor line. Pipe-risers should not be allowed to come up through columns as repairs and alterations are difficult, if not impossible, under such an arrangement; small size electric conduits, however, form an exception to this rule. Special shafts with fireproof walls are sometimes used for plumbing and vent pipes, and this practice has much to commend it since a floor to be a perfect fire cutoff should be solid from wall to wall, with stairways, elevators, and all openings enclosed in vertical fireproof walls.

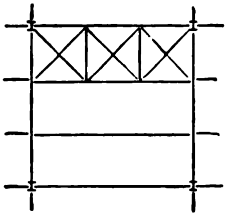


Fig. 134.—Elevator openings in steel frame construction.

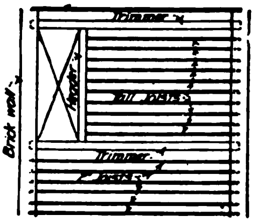


Fig. 135.—Stair opening in ordinary construction.

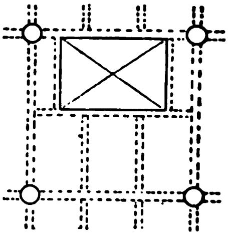


Fig. 136.—Stair opening in concrete beam and girder construction.

Special pits are required for platform scales and it is best to get the details of the scales to be used and include the framing for the scales in the general plans of the building.

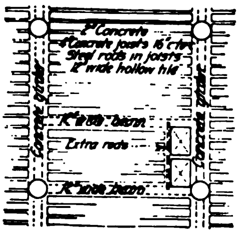


Fig. 137.—Shaft openings in tile and concrete construction.

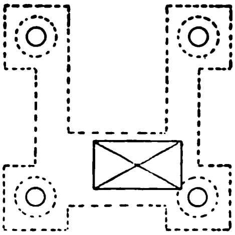


Fig. 138.—Opening in flat slab concrete construction.

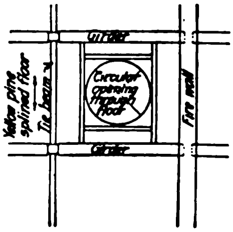
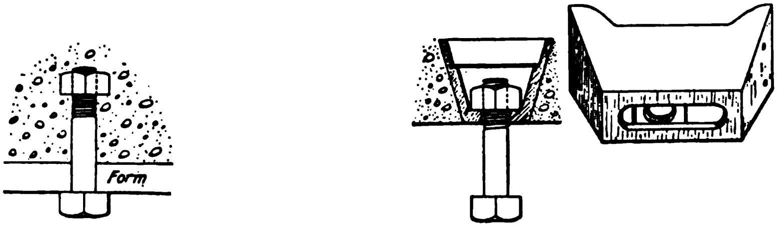


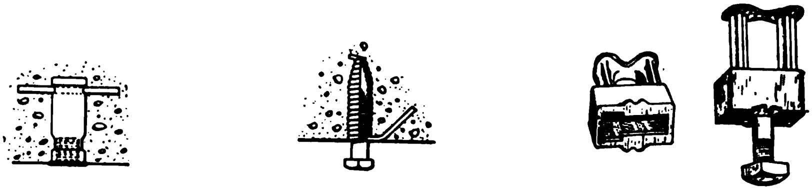
Fig. 139.—Opening for spiral conveyor in mill construction.

116. Floor Attachments.—Machinery, shafting, sprinkler pipes, steam pipes, etc., are often hung from the ceiling. In wood construction, blocks are usually attached to the ceiling joists by lag screws and machinery hangers bolted to these blocks. In steel construction, clamps are used around the lower flanges of the floor beams. In concrete construction, some form of *insert* is used to support these utilities. Where permanent pipes, machinery, etc., are to be placed, it is possible to lay out the inserts to care for these. But in a building in which there is much machinery, provision should be made for changing conditions, the shifting of departments, and the installation of improved machines. For this purpose, it is well to spot inserts at regular intervals over the entire ceiling. In a recent machine manufacturing plant, inserts were provided 4 ft. on centers each way over the entire ceiling, and this has proved a satisfactory arrangement. In Fig. 140 are illustracted the common types of inserts.



Bolt insert. Bolt is removed before forms are taken down, leaving the nut in the concrete.

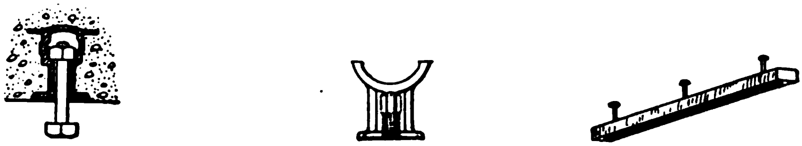
Wrialco insert.



Kohler pressed steel insert.

Barton steel spiral socket for lag screws.

Dayton insert.



Security insert.

Havemeyer socket insert.

Truscon slotted insert.

Fig. 140.

GROUND FLOORS

By Allan F. Owen

117. Drainage.—Ground floors at sidewalk shipping platform level, or in basements, must be protected against dampness. The most important item in the prevention of dampness is drainage. Where the floor is above the sewer, a system of tile drains is installed under the floor and connected to the sewer. Lines of drain tile should be laid near the outside walls and about 20 ft. apart under large floors. Where the sewer is above or very close to the floor, it is necessary to connect the drain tile to an ejecter pit and provide an automatic sewage ejector connected to the sewerage system. Where the floor is below water level, in water bearing soil, no drainage can be used.

118. Underfloor.—Under the finished floor a porous layer of cinders, stone, or gravel should be laid to allow water to run to the drains and to insulate the floor from the damp earth beneath. Where the floor is below water level, the underfloor must be waterproofed and reinforced against water pressure. A damp proofing course should then be laid on the top of the underfloor and under the finished floor. The water pressure to be reinforced against is equal to 62½ lb. per sq. ft. of floor times the depth from the top of the highest known water level to the waterproofing course. The weight of reinforced concrete above the waterproofing course may be deducted from the total pressure to be reinforced against. The waterproofing course must extend up the outside walls above water level.

119. Waterproofing.—Ground floors should be waterproofed as explained in Sect. 5, Art. 29.

120. Floor Finish.—Finished concrete floors are most widely used for ground floors, but any of the wood, tile, marble, composition, or asphalt floors described in the chapter on "Floor Surfaces" may be used.

ROOF TRUSSES—GENERAL DESIGN

BY W. S. KINNE

121. Roof Trusses in General.—A roof truss is a frame work designed to support the roof covering or ceiling over large rooms, thereby avoiding the use of interior columns. Fig. 141 shows the relative position of the roof trusses, the walls of the building, and the roof covering.

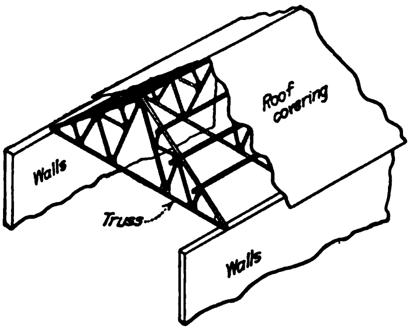
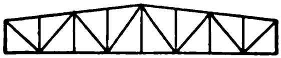


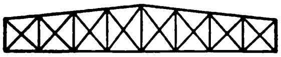
FIG. 141.

When the nature of the supporting forces is such that the reactions are vertical under vertical loading, or the reactions due to inclined loading can be determined by the methods of simple statics, the frame work is known as a "simple truss." Where the reactions are inclined, even under vertical loading, and where they can not be determined by simple statics, the frame work is known as an "arch." The discussion of this chapter will be confined to simple trusses; arches will be considered in the chapter on "Arched Roofs."

Simple roof trusses can be further divided into two classes based on the methods of supporting the trusses. In one class can be placed the trusses which are supported on rigid walls of masonry, or other material forming a wall which is able to resist lateral forces without additional bracing. In a second class can be placed the trusses which are supported on steel columns carrying a light curtain wall in addition to the trusses. The construction of these columns is such that, unaided, they do not offer any considerable resistance to lateral forces. To secure a rigid structure, it is necessary to join the trusses and the columns by a member known as a "knee-brace," thus forming a rigid framework which is known as a "knee-braced bent." Further discussion of this type of structure is given in the chapter entitled: "Detailed Design of Truss With Knee-braces."



(a) Single Web System



(b) Double Web System

FIG. 142.

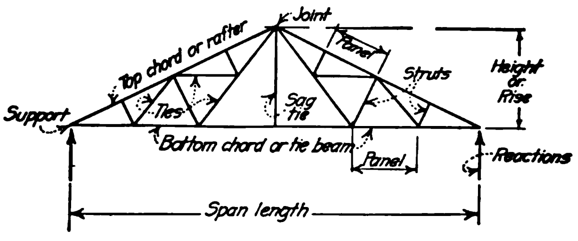


FIG. 143.

In general, a roof truss should consist of a simple framework composed preferably of a system of triangles. The members of the frame work are usually so arranged that they are in direct tension or compression. Trusses composed of a single web-system, as shown in Fig. 142(a), are preferable to those with a double web-system, as shown in Fig. 142(b). The stresses in the truss of Fig. 142(a) are readily determined by the principles of simple statics, as given in Sect. 1. In the truss of Fig. 142(b), the stresses are statically indeterminate. An exact determination of the stresses can be made, but the work of stress calculation is long and tedious. Approximate methods of stress calculation are generally used, but as the distribution of the load to the various members is uncertain, such methods are unsatisfactory.

Fig. 143 shows the component parts of a truss. The names of the several parts are indicated in position. As shown on Fig. 143, the upper members are known as the top chords, or rafters, and the lower members are known as the bottom chords, or tie beams. The interior compression members are known as struts, and the interior tension members are known as ties. Points of intersection of chord members are known as joints, and the distance between adjacent joints is known as a panel, or panel length. A sag tie is a member provided to form a support for a long horizontal member which would deflect excessively under its own weight if not so supported.

122. Form of Trusses.—A great variety of trusses are used in building construction, the form depending upon the character of the roof covering and the architectural features of the structure. Fig. 144 shows some of the forms of simple trusses in common use for trusses supported on rigid walls. Types of knee-braced bents and arches are shown in later chapters.

In Fig. 144 the forms shown in Figs. (*a*) to (*m*) are well adapted to construction in steel, while those of Figs. (*n*) to (*q*) are suited for construction in wood. The trusses of Figs. (*a*) to (*m*) are so arranged that the compression members, shown by the heavy lines, are the shortest members in the truss, while the tension members, shown by the light lines, are the longest members. This results in a considerable saving of material, for a compression member requires a greater sectional area for a given stress than a tension member. Also, the greater the length of a compression member, the greater the required area.

In the trusses of Figs. (*n*) to (*q*), the top and bottom chord members and the interior diagonals are usually made of wood, while the vertical tension members are made of steel rods. Since compression joints between wooden members are easier to frame than tension joints, or splices, it follows that these types are well adapted for construction in wood.

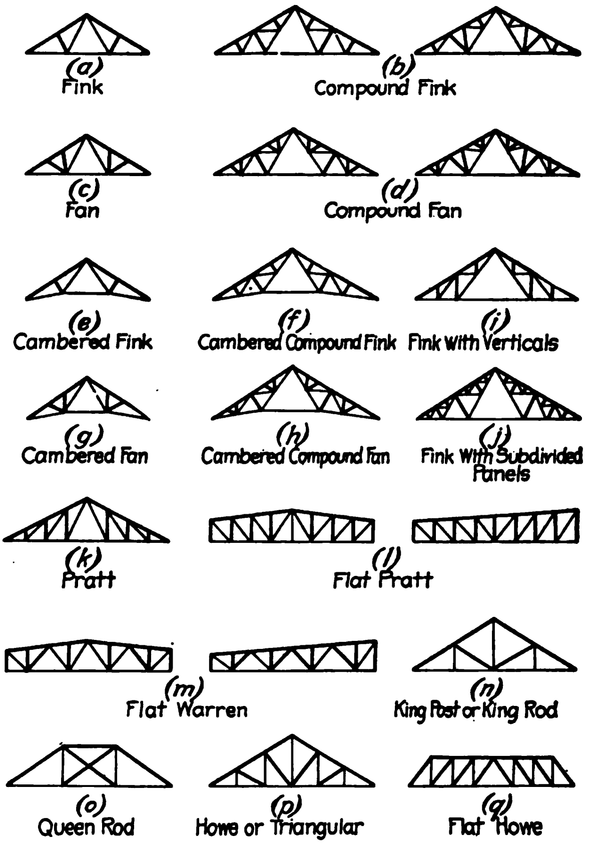

(a) Fink
(b) Compound Fink
(c) Fan
(d) Compound Fan
(e) Cambered Fink
(f) Cambered Compound Fink
(i) Fink With Verticals
(g) Cambered Fan
(h) Cambered Compound Fan
(j) Fink With Subdivided Panels
(k) Pratt
(l) Flat Pratt
(m) Flat Warren
(n) King Post or King Rod
(o) Queen Rod
(p) Howe or Triangular
(q) Flat Howe

Fig. 144.

The form of truss is dependent to some extent upon the span length, for in order to avoid bending stresses in the top chord, it is desirable to have a panel point of the truss directly under each purlin. To avoid the use of excessive areas in the top chord sections, it will probably be best to limit the length of these members to about 8 ft. as a maximum. With this limitation, the advisable maximum spans for the several types shown in Fig. 144 are about as follows: Figs. (*a*) and (*e*), 30 ft.; (*c*) and (*g*), 40 ft.; (*b*) and (*f*), 50 to 60 ft.; (*d*) and (*h*), 70 to 80 ft.; and (*j*), 80 to 90 ft. The forms shown in Figs. (*k*), (*l*), and (*m*) can be used for spans of from 20 to 80 ft. by varying the number of panels. Wooden trusses of the type shown in Figs. (*n*) and (*o*) can be used for spans up to about 25 or 30 ft., while those of Figs. (*p*) and (*q*) can be used for spans of from 20 to 80 ft. by varying the number of panels.

The type of truss to be used with a given roof covering is determined by the allowable slope of roof for the roof covering in question. Table 1 gives the minimum allowable slope of roof for some of the common types of roof coverings.

TABLE 1

Asphalt or asbestos	Rise 1/12 of span.
Corrugated steel	Rise 1/4 of span.
Slate	Rise 1/6 to 1/4 of span.
Tar and gravel	Flat, or sufficient slope for drainage.
Tile	Rise 1/3 of span.
Tin	All slopes.
Wood shingles on sheathing	Rise 1/4 of span.

The trusses shown in Figs. (*l*), (*m*), and (*q*) are suitable for tar and gravel, or for tin roofs. For these types of covering it is necessary to give the roof only enough slope to provide proper drainage. A slope of more than 1 in. to the foot is not desirable for a gravel and tar roofing, due to the fact that the material will flow when laid, and that intense summer heat will also cause it to flow if the slope is greater than that mentioned. All of the other forms shown in Fig. 144 are adaptable to roofs with a rise equal to from 1/6 to 1/2 of the span.

Trusses with a cambered lower chord, as shown in Figs. (*e*) to (*h*) incl., are used for the sake of appearance. A long line of trusses with exposed horizontal chords appear to sag. This effect can be overcome by cambering the lower chord. In other cases the architectural treatment of the ceiling calls for a cambered truss. Where a moderate camber is required, one of the forms shown in Fig. 144 can be used. In churches and similar structures, the architectural treatment often calls for an ornamental truss, which is considered in the chapter on "Ornamental Roof Trusses."

In general it can be said that the selection of the type of truss is just as important as any other feature of the design. Having fixed upon the span length and the height of truss, that type of framing should be adopted in which the members are well placed with respect to the loads which are to be carried.

123. Pitch of Roof Truss.—The pitch of a roof truss is usually defined as the ratio of the height, or rise, of the truss to the span length, and is usually designated by a fraction. Thus in the truss of Fig. 143, suppose the height to be 15 ft. and the span to be 60 ft. As defined above

$$\text{pitch} = \frac{\text{height}}{\text{span}} = \frac{15}{60} = \frac{1}{4}$$

In the preceding article the effect of character of roof covering on the ratio of rise to span length has been considered. As the pitch of roof, as defined above, is the same as the rise divided by the span, the values given in Table 1 will indicate the minimum desirable pitch of a roof truss for a given roof covering.

The pitch of the truss should also be determined with reference to the loads to be carried. As shown by the tables of wind and snow load given in Arts. 135 and 136, a roof with a 1/3 pitch has a smaller snow load but a greater wind load per sq. ft. of roof than one with a 1/4 or 1/6 pitch. Also from the stress tables of the following chapter, the stresses in the trusses of 1/3 pitch are less than those of 1/4 or 1/6 pitch. However, in trusses of 1/4 pitch, the interior compression members are somewhat shorter than those in trusses of 1/3 pitch, which results in a considerable saving in material, in spite of the greater stress. Trusses of 1/6 pitch have greatly increased stresses, which call for added material in spite of the reduced length of the compression members. Considering all factors, it seems that the truss of 1/4 pitch is the most economical.

124. Spacing of Trusses.—The theoretical spacing of trusses for least total cost of trusses, purlins, and roof covering depends upon the relative cost of the component parts. As the spacing increases, the cost of the trusses per unit of covered area will decrease, as small changes in spacing have little effect on the weight of a truss; the cost therefore varies inversely as the spacing. The size of purlin is determined by the moment to be carried; this varies as the square of the span. Therefore the cost of the purlins can be considered to vary as the square of the spacing. The roof covering cost varies directly as the spacing. To determine the theoretically most economical spacing, all of these factors must be given proper consideration.

The relation between the quantities given above for minimum cost can be expressed approximately in the following manner:

As stated above, the cost of the trusses can be assumed to vary inversely as the spacing of the trusses, which relation can be written, $t = k/s$, where t = cost of trusses per sq. ft. of roof, k = a constant, and s = spacing of trusses. Again, the cost of the purlins varies directly as the square of the spacing of trusses, or $p = ns^2$, where p = cost of purlins per sq. ft. of roof, n = a constant, and s = spacing of trusses. Also, the cost of roof covering varies directly as the spacing of trusses, or $c = ms$, where c = cost of roofing per sq. ft. of roof, m = a constant, and s = spacing of trusses. If X be the total cost of the roof, per sq. ft., we have

$$X = t + p + c = k/s + ns^2 + ms$$

By the methods of the Differential Calculus it can be shown that the relation existing between the terms of the above expression at the time the cost of the roof is a minimum is

$$t = 2p + c$$

That is, for least cost, the spacing of trusses must be such that the cost of the trusses per sq. ft. of roof is equal to twice the cost of the purlins per sq. ft. of roof plus the cost of roof covering per sq. ft. of roof.

The relation given above can not be used directly for the determination of the truss spacing for the spacing does not appear in the equation. However, by means of the above expression, a given design can be tested out to see if it answers the theoretical conditions. A study of the formula will aid in forming conclusions regarding the proper truss spacing.

The cost of materials and labor is such that the cost of the trusses per sq. ft. of roof is usually several times greater than that of the purlins. Roof covering costs vary with the nature of the covering, but will probably not exceed that of the purlins. These facts point toward a rather wide spacing of trusses, in order to secure maximum economy. If it were possible to obtain rolled sections which would provide exactly the required areas for all truss members, it would be possible to use rather a small truss spacing. But as can be seen from the design given in the chapters on the design of steel and wooden roof trusses, the sizes of many members are determined by the specifications, or by the requirements of standard practice. These requirements add considerably to the weight of the structure. From this discussion it can be seen that the cost of the trusses controls the economy of the design, and the spacing of the trusses should be determined accordingly.

Comparative estimates of cost, made by comparing the total cost of roof trusses of the same span length but with varying spacing indicate that for spans up to 50 ft. the most economical spacing is about 15 ft. for light loads (about 30 lb. per sq. ft.), or about $\frac{1}{4}$ of the span. For spans of from 50 to 100 ft., the spacing should be about $\frac{1}{4}$ of the span for the shorter spans and about $\frac{1}{5}$ of the span for the longer spans, or from 15 to 20 ft. In many cases local conditions govern and determine the spacing of the trusses regardless of the economical conditions.

125. Spacing of Purlins.—The spacing of the purlins is governed to a large extent by the roof covering, and to some extent by the type of roof truss. In the first place, the strength of the roof covering, considered as a beam spanning the distance between purlins, determines the allowable span of the roofing, and in the second place, the position of the joints of the truss determines the possible points of support for purlins, and in this way determines the possible span of the roof covering. This assumes that the top chord of the truss acts only as a compression member. In some cases where the type of the truss is such that the distance between top chord joints is greater than the allowable span of the roof covering, purlins are placed at points between the chord joints. This arrangement has the disadvantage of subjecting the chord section to bending as well as direct stress, for the chord section must act as a beam as well as a chord member. But this is probably offset by the saving in weight of purlins made possible by the use of smaller closely-spaced sections.

Roof coverings are often laid on sheathing, which is in turn supported by rafters laid parallel to the top chord of the truss and resting on purlins. By using suitable rafters, the purlin spacing can be made as desired. This construction is apt to result in a heavy roof. To avoid this, the sheathing is sometimes laid directly on the purlins, thus limiting the spacing of purlins to the safe span of the sheathing. This safe span is to be determined with reference to the bending stress in the sheathing, and also with respect to the allowable deflection of the sheathing, for in some cases the roof covering, as tile or slate, is likely to crack if the sheathing is subjected to excessive deflection. The allowable deflection is about $\frac{1}{400}$ part of the clear span.

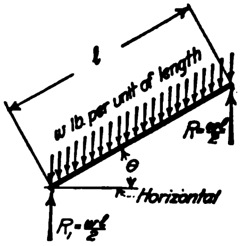


Fig. 145.

Fig. 145 shows an inclined beam subjected to a vertical uniform load of w lb. per ft. of beam. Assuming that the sheathing is continuous over several purlins, the maximum moment is $M = \frac{1}{10} wl^2 \cos \theta$, and the fiber stress is given by the formula $f = Mc/I$. Placing the value of M in the formula for fiber stress and solving for l, the limiting span length, we have, for a rectangular section of width b and depth d,

$$l = \left(\frac{5}{3} \frac{bd^2f}{w} \sec \theta \right)^{\frac{1}{2}}$$

In terms of the fiber stress, the deflection of a rectangular beam under a uniform load is given by the formula $\Delta = 5/24 \frac{fl^2}{Ed}$ where E is the modulus of elasticity of the material, and the other terms have the same values as before. Substituting in this expression the value of f, and solving for l, the limiting span, we find for an allowable deflection of $\frac{1}{360}$ of the span, that

$$l = \left(\frac{1}{45}\frac{bd^3E}{w}\sec\theta\right)^{1/3}$$

The smaller of the values given by the above equations is the allowable span for the sheathing under consideration. Table 2 gives the limiting spans for sheathing in common use for several load capacities and varying slope of roof, as determined by the above equations.

TABLE 2.—LIMITING SPANS FOR ONE INCH SHEATHING FOR VARIOUS LOAD CAPACITIES AND SLOPES

f = 1000 lb. per sq. in.; E = 1,000,000 lb. per sq. in.; d = 1 in.

(Limiting spans given in feet)

Capacity in pounds per sq. ft.	Slope of roof in inches per foot						
	0	2	4	6	8	10	12
20	9.13 4.53	9.20 4.56	9.35 4.60	9.66 4.71	10.02 4.81	10.43 4.95	10.85 5.08
25	8.17 4.19	8.22 4.22	8.37 4.25	8.65 4.35	8.97 4.45	9.35 4.58	9.72 4.70
30	7.45 3.95	7.51 3.97	7.64 4.00	7.89 4.11	8.17 4.20	8.52 4.32	8.86 4.43
40	6.46 3.59	6.51 3.61	6.62 3.64	6.84 3.73	7.20 3.82	7.39 3.92	7.69 4.03
50	5.77 3.34	5.82 3.36	5.92 3.40	6.00 3.47	6.34 3.55	6.60 3.65	6.86 3.75
60	5.27 3.13	5.32 3.15	5.41 3.17	5.58 3.25	5.78 3.33	6.03 3.42	6.27 3.52

NOTE.—Upper values = limiting span in feet due to bending. Lower values = limiting span in feet due to deflection.

For limiting spans due to fiber stresses other than 1000 lb. per sq. in., multiply upper values in table by the ratio $\sqrt{\frac{f}{1000}}$.

For limiting spans due to deflection for values of E other than 1,000,000 lb. per sq. in., multiply lower values in table by the ratio $\sqrt[3]{\frac{E}{1{,}000{,}000}}$.

For limiting spans for sheathing of other than 1 in. thickness, multiply values given in the table directly by the thickness of the sheathing in inches.

The limiting span for corrugated steel roofing, considered as a horizontal beam, is given by the Rankine formula as

$$l = \left(0.178\frac{shbt}{w}\right)^{1/2}$$

where S = working stress in lb. per sq. in., h = depth of corrugations in inches, b = width of sheet in inches, t = thickness of sheet in inches, w = safe load in lb. per ft., uniform load, and l = allowable span in feet. Table 3 gives the allowable spans of corrugated steel for several load capacities per sq. ft. of roof. The values are computed from the above formula.

TABLE 3.—LIMITING SPANS FOR CORRUGATED STEEL

From formula $l = \left(0.178 \dfrac{Shbt}{w}\right)^{1/2}$

$S = 12{,}000$ lb. per sq. in.; $b = 12$ in.; $h = 5/8$ in.

Gage	t (in.)	Values of l in feet					
		$w = 20$	$w = 25$	$w = 30$	$w = 40$	$w = 50$	$w = 60$
16	$1/16$	7.08	6.32	5.77	5.00	4.47	4.08
18	$1/20$	6.32	5.65	5.16	4.47	4.00	3.65
20	$3/80$	5.50	4.91	4.48	3.88	3.47	3.17
22	$1/32$	5.00	4.47	4.08	3.54	3.16	2.88
24	$1/40$	4.49	4.01	3.66	3.17	2.84	2.59

126. Spacing of Girts.—Girts are members, similar to purlins, which are used to support the siding in a building in which the walls are formed by siding or corrugated steel carried on the columns which support the roof trusses. The design of girts is carried out by the same methods as given in Sect. 2 for purlins.

The spacing of girts is governed by the same considerations as given in the preceding article for purlins. Allowable spacing of girts can be determined by the tables of the preceding article. Design methods are given in Art. 167.

127. Purlin and Girt Details and Connections.—Wooden purlins can be made up of a single piece, or can be built up by placing several narrow pieces side by side. When properly fastened together, either by nailing or bolting, built-up beams are equally as strong as a single piece, and are cheaper and easier to obtain. Such purlins are used either with wooden or steel roof trusses.

The connection of wooden purlins to the roof truss depends upon the type of roof construction and the kind of truss. For wooden trusses, purlin connections of the type shown in Fig. 146 are in common use. In Fig. (*a*) the purlin is placed on the top of the chord section. This is often done when a deep roof covering is not undesirable. The purlin is held in position and prevented from overturning by means of a block or short piece of angle nailed or bolted to the top chord, as shown in Fig. (*a*). Where the depth of the roof construction is limited, the connection shown in Fig. (*b*) is used. The purlin is suspended by means of a strap hanger, or by means of one of the patent hangers shown in Sect. 2, Art. 122*b*. Figs. (*c*) and (*d*) show details of connections at the apex of the truss and at the wall. For the design of such connections see Art. 146. Fig. (*e*) shows a type of connection used for wooden purlins on steel roof trusses. A short clip angle is riveted to the top chord and the purlin is fastened to this clip angle by means of lag screws.

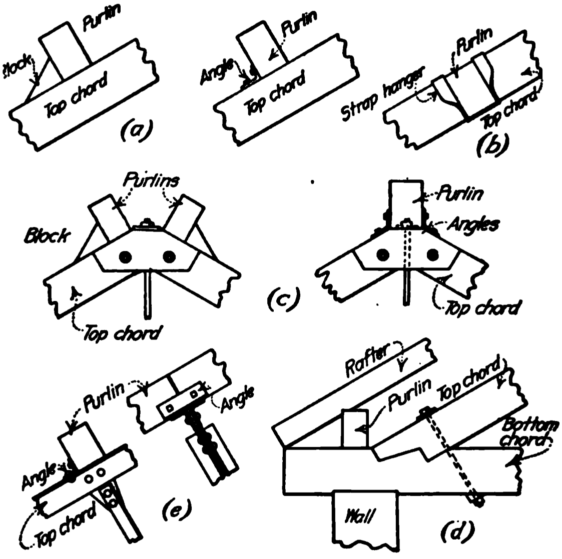


FIG. 146.

Purlins for steel roof trusses are generally made of rolled sections, although in some cases wooden purlins are used, as shown by the detail of Fig. 146 (*e*). The rolled sections most used as purlins are the I-beam, the channel, and the angle. T-bars and Z-bars are sometimes used, but their use is limited, as Z-bars are hard to obtain, except in large orders, and as pointed out in Sect. 1, Art. 112, the T-bar is not an ideal beam section. In selecting rolled sections from the steel handbooks, it is best to use the section of minimum weight for any given depth, as these sections are stock sizes and are easily obtained. A list of standard sections is given in Art. 130.

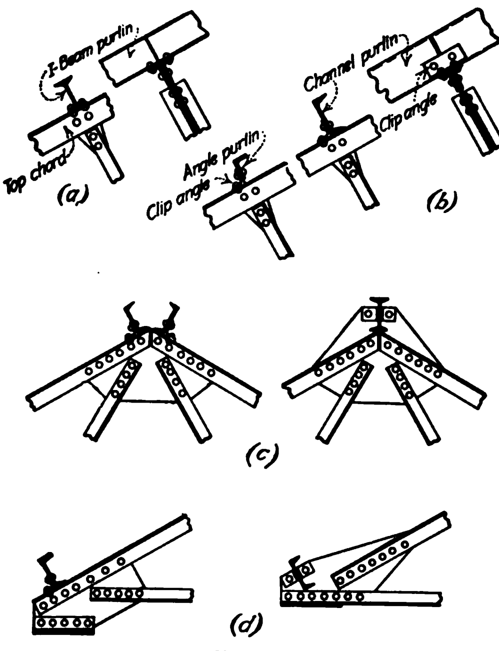


FIG. 147.

Fig. 147 gives details of I-beam, channel, and angle purlin connections. Fig. (a) shows an I-beam connection. The connection is made by rivets or field bolts. Fig. (b) shows the usual type of connections for angles and channels. A clip angle is shop riveted to the truss, as shown. The length of this clip is such that at least one rivet can be placed in the end of each purlin. Fig. (c) shows details of purlin connections at the apex of the truss. Fig. (d) shows the arrangement at the wall for a truss on masonry walls. This arrangement is not always followed, for in many cases a purlin is not used at this point. These sketches show two general classes of details. In one case the purlin is fastened directly to the top chord. In the other, adequate direct connection to the top chord can not be secured. To provide proper connection, the gusset plates are enlarged and the purlin is fastened to the plate by means of a standard I-beam or channel connection. As a great variety of special connections are in use for details at these points, only a few of the more common types are shown.

Purlins for truss spacing greater than about 20 ft. can not be provided economically by single rolled shapes. It is necessary to use a form of plate or trussed girder, or if the span is not too great, a trussed purlin, such as shown in Fig. 148, can be used. Where the girder purlin is used, it is usually placed in a vertical position. A form of roof truss must be selected which contains vertical members so located as to provide proper end connections for the purlin. Trusses of the type of Fig. 144 (i), (k), (l), or (m) provide the necessary vertical members, where a moderate span length is used. Trussed purlins are generally used where a very wide truss spacing is necessary to obtain maximum economy.

Girts are usually made of angle or channel sections. Fig. 149 shows the method of connecting the section to the supporting column. For spans of 15 ft. or more, it is necessary to provide a line of tie rods which extend vertically to the eaves. This relieves the bending stresses in the girts and permits the use of smaller sections.

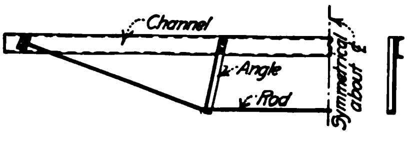


FIG. 148.

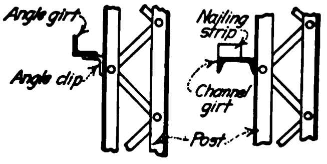


FIG. 149.

128. Connections between Purlins and Roof Covering.—Fig. 150 shows a few of the methods used in fastening the roof covering to the purlins. Fig. (a) shows the details of connections between rolled steel sections and plank sheathing. As shown, a nailing strip is fastened to the section. The sheathing is then nailed to this strip. Where wooden siding is used, it is fastened to the girts in a similar manner.

Corrugated steel roofing and siding are fastened to the purlins or girts by the methods shown in Fig. (b). Clinch nails are used with angle purlins, and sometimes with the smaller channels. The nails are made of soft wire, and are clinched around the purlins. Strap fastenings are used with all sections. The straps are made of No. 18 gage steel about ¾ in. wide, and are fastened to the covering by a stove bolt in each end of the strap. Clip fastenings are made of No. 16 gage steel. The usual dimensions are 1½ × 2½ in. They are fastened to the covering by two stove bolts at one end of the clip to prevent turning. A nailing strip is preferably used with an anti-condensation lining, and also for fastening siding to girts. In all cases the fastenings are spaced about a foot apart.

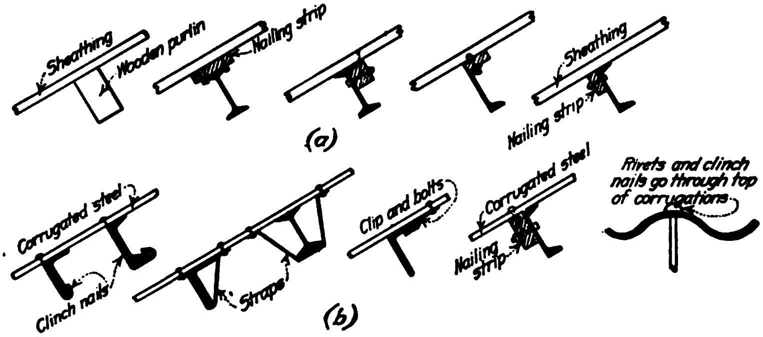
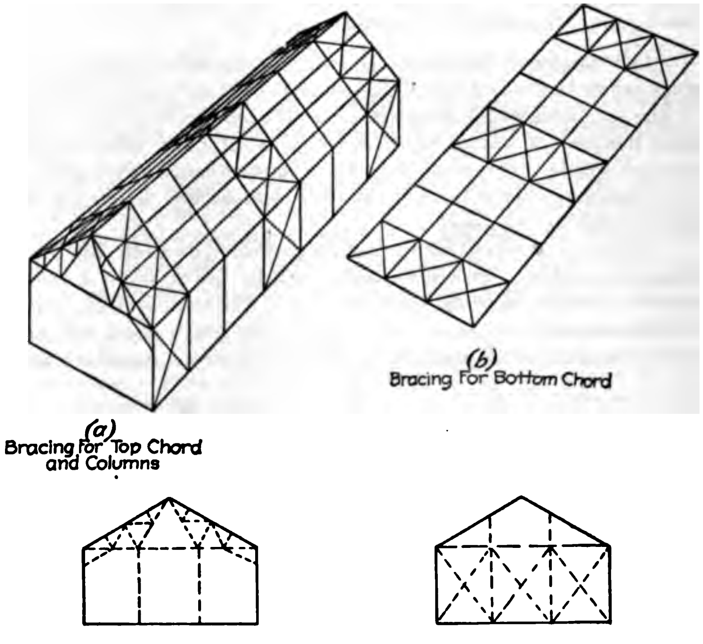


Fig. 150.

129. Bracing of Roofs and Buildings.—The bracing to be provided for a roof depends upon the character and use of the building. For a roof supported on masonry walls, the object of the bracing is to provide a stiff rigid structure which will not be subjected to vibration due to machinery or moving loads, such as cranes, etc. In the case of a roof supported on steel columns, the entire structure is dependent on bracing for stability against lateral forces. The trusses must be thoroughly braced and the columns must be connected by longitudinal and transverse systems of bracing. Without such bracing the structure would collapse in a high wind storm or due to stresses and vibration from moving loads, such as cranes. In general it can be said that bracing should be so located that the lateral forces will be transmitted as directly as possible to the walls and foundations of the building.


(a) Bracing for Top Chord and Columns

(b) Bracing for Bottom Chord

(c) End Bracing (d)

Fig. 151.

Bracing for a roof supported on rigid walls is not subject to analysis for stresses, as the forces acting on the bracing are indefinite in nature. The designer must use his judgment, based on past experience, in the determination of the form of bracing and the make-up of the sections. In the case of roofs supported on columns it is possible to determine approximately the stresses in the bracing. This problem is considered in detail in the chapter on the "Detailed Design of a Truss with Knee-Braces."

Roof trusses supported on columns should be provided with bracing for the trusses and also bracing for the columns. Fig. 151 shows the relative position of the required bracing. Every third or fourth pair of trusses should be rigidly braced with diagonals placed in the planes of the upper and lower chords of the trusses. The unbraced trusses between the pairs of braced trusses should be connected to the others by unbroken lines of struts running the full length of the building and located at the eaves, the apex of the truss, and at several points in the plane of the lower chord of the truss, at distances apart depending upon the width of the building. These distances should be such that the diagonals of the bracing will form angles of about 45 deg. with the loads to be carried.

Column bracing should be provided for the bays in which the trusses are braced, as shown in Fig. (a). This bracing consists of rods or rolled shapes. The bracing should be so arranged that the members make angles of about 45 deg. with the horizontal.

A system of bracing is also to be provided in the plane of the ends of the building. This bracing must assist in carrying the transverse forces. Two forms of such bracing are shown in Fig. 151. Fig. (c) shows a knee-braced bent similar to the others. This truss provides the required bracing for transverse forces, and also supports a set of vertical members which carry the girts and siding. The horizontal forces brought to the lower chord of this truss by the siding are resisted by the horizontal trusses in the plane of the lower chord of the main trusses.

Fig. (d) shows an arrangement of vertical beams which carry the girts and the siding. These beams transfer part of their load to the bracing in the plane of the lower chord of the main trusses. Vertical diagonal bracing is provided in the plane of the end of the building, as shown in Fig. (d).

Buildings with rigid side and end walls of masonry require bracing only in the planes of the upper and lower chords of the trusses. This bracing can be of the same general form as described above for the roof on steel columns, except that a strut is not required at the eaves. A detail design of bracing for a roof of this kind is given in the chapter on the "Detailed Design of Steel Roof Truss."

130. Choice of Sections.—In selecting the rolled shapes with which the members of the truss are to be formed, the designer must be governed not only by the required area but also by the ease with which the section can be obtained from the rolling mills. If any section is in great demand, it will be rolled at frequent intervals, while a section for which there is little demand will be rolled only when the orders on hand will warrant a rolling of the section. It often happens, therefore, that the time element will determine the section to be used instead of the stress to be carried.

The sections which are the easiest to obtain, as a rule, are those of minimum weight for the shape in question. It will be found best to use as small a number of sections and sizes as possible, thereby insuring quick delivery. The various mills and large bridge companies have certain standard and permissible sections for which quick delivery is fairly certain. A short list of standard and permissible sections used by the American Bridge Co. is given in Table 4.

TABLE 4*

Standard angles		Permissible angles	
6" × 6"	6" × 4"	8" × 8"	6" × 3½"
4" × 4"	5" × 3½"	5" × 5"	4" × 3½"
3½" × 3½"	4" × 3"	2¼" × 2¼"	3½" × 2½"
3" × 3"	3½" × 3"	2" × 2"	3" × 2"
2½" × 2½"	3" × 2½"		
	2½" × 2"		
Standard channels			Permissible channels
15"	8"		9"
12"	6"		7"
10"			5"
Standard I-beams			Permissible I-beams
20"	10"		24"
18"	8"		9"
15"	6"		7"
12"			5"

* Steel Mill Buildings, and Structural Engineers' Handbook, by M. S. Ketchum.

131. Form of Members for Roof Trusses.—Members for wooden roof trusses are made preferably of single pieces of timber, square or rectangular in shape. Where single pieces can not be obtained, members are built up of planks securely fastened together so that the parts of the member will act as a unit. The design of members of a wooden roof truss is considered in another chapter.

Fig. 152 shows the form of members in general use for simple roof trusses of the type shown in Fig. 144. Compression chord and web members are made up as shown in Fig. (*a*). For members subjected to moderate stresses, too angles placed back to back, as shown in Fig. (*a*), will provide sufficient area. Angles with unequal legs are preferable, the longer legs to be placed together. In this way the ratio of length to radius of gyration of the combined section for axes *OX* and *OY* of Fig. (*a*) can be made equal, or nearly so. The resulting column is then of equal rigidity in all directions. To make certain that the two angles act as a unit, they must be riveted together at intervals such that the ratio of unsupported length to radius of gyration for a single angle is equal to or less than that for the combined section. This detail will be considered further in Art. 156.

Connections between chord and web members are made by separating the two angles by a small space which will allow a connecting plate to be inserted, as shown in Fig. (*b*). This space between the angles is maintained over their entire length by means of ring fills or washers located at the connecting rivets. The size and shape of the connecting plates, which are known as gusset plates, depend upon the number of rivets to be provided in the connection.

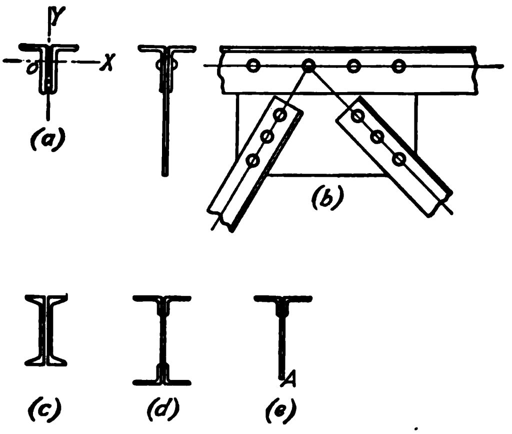


FIG. 152.

Where very large stresses are to be carried, the forms of members shown in Figs. (*c*), (*d*), and (*e*) are used. The form of Fig. (*c*) shows two rolled channels in place of angles, and Fig. (*d*) shows a built-up member consisting of 4 angles and 1 plate. In some cases the form of Fig. (*e*) is used. This form consists of 2 angles and 1 plate. The plate acts as a part of the chord member, and at the joints, it acts as a gusset plate, similar to the arrangement shown in Fig. (*b*).

In some forms of trusses the purlin spacing is such that purlins must be placed at points between the top chord joints. The top chord section is then subjected to bending in addition to direct stress, and the section must be designed as a combined beam and column. Design methods are given in Sect. 1, and in the design of Art. 158. For members subjected to moderate stress and bending, the form of member shown in Fig. (*a*) can be used. Figs. (*c*) and (*d*) show forms adapted for large moments and direct stresses. The form of Fig. (*e*), although often used for members subjected to bending, is not a desirable form of beam section, as pointed out in Sect. 1, Art. 112. This is due to the fact that the top chord member of a roof truss is continuous from end to end, thus forming a continuous girder. As shown in Sect. 1, the moments at points of support are negative. Therefore the narrow edge of the plate at *A*, Fig. (*e*), is in compression. As this plate is not well supported at the joints, it is likely to buckle sidewise. The forms of Figs. (*c*) and (*d*) are not subject to this objection.

Tension members are also made of two angles placed as shown in Fig. (*a*). Equal legged angles can be used for tension members, as it is not necessary to secure equal rigidity in all directions. Where tension members are subjected to bending as well as direct stress, the forms of Figs. (*c*) and (*d*) can be used.

132. Joint Details for Roof Trusses.—The design of joint details of a roof truss is a matter of the greatest importance. An investigation of the causes of roof truss failures will show that in most cases, the failure can be traced to faulty joint details. The same care and study should be devoted to the design of joints as to the design of the main members.

In designing joints, a point of great importance is that the center lines of all members entering a joint should meet at a common point, which should be located at the intersection of the center lines of the truss members, as shown in Fig. 153 (*a*). If this point is overlooked, as shown in Fig. (*b*), where the intersection point of the diagonals is at a distance *a* from the line of action of the remaining members, there is set up a bending moment *Pa*, which tends to twist the joint out of position. This moment must be resisted by the members entering the joint. Proper provision should be made for the increased stresses, or the detail should be changed so as to eliminate the moment.

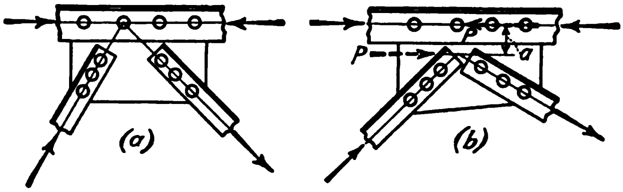


FIG. 153.

The designer, in addition to satisfying the above requirement, should carefully trace the stresses from the several members into the joint, making certain that proper connections have been made, and that all parts are proportioned to care for the stress which they may be called upon to carry.

Most specifications state that symmetrical sections shall be used for principal members. Others allow the use of single angles for members with small stress. Fig. 154 shows a connection made for a member composed of a symmetrical section and another made of a single angle. In Fig. (*b*) is shown a symmetrical member composed of two equal angles, one on each side of the gusset plate. The stress in the member can then be considered as brought directly to the gusset plate. In Fig. (*a*), where a single angle is used, the center line of the member and the plane of the truss do not coincide. The member is then subjected to a direct stress *P* and a bending moment $M = Pa$, where *a* is the distance from the center of gravity of the angle to the plane of the truss. For the conditions shown in Fig. (*a*), the design must be carried out by the methods given in Sect. 1 for bending and direct stress. The usual methods often neglect entirely the effect of the eccentric connection, which leads to a faulty design.

In addition to the large bending stresses in the member in question, as shown in the detail of Fig. 154(*a*), there is also present the effect of the eccentric load on the other truss members. A load applied to the side of a plate, as shown in Fig. (*a*), tends to twist the top chord out of line, thereby setting up additional stresses in the chord section. It therefore seems best to specify that all members carrying calculated stress shall be composed of symmetrical sections, or sections which will allow a symmetrical connection of the form shown in Fig. (*b*) to be made.

The methods of design for joint and member connections, and the general methods of detailing have been given in Sect. 2. Application of the principles of this article and of the chapters quoted will be found in the design of roof trusses given in later chapters.

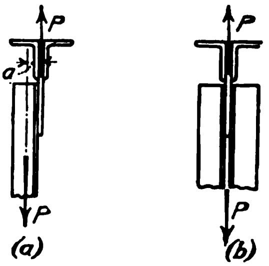


FIG. 154.

133. Loadings for Roof Trusses.—The load to be carried by a roof truss consists of the weight of the truss, the roof covering, purlins, bracing, and any other loads, such as ceilings, suspended floors, and machinery, etc., in factory buildings. In addition to these loads, the roof must be designed to carry the maximum wind and snow loads which experience shows are likely to occur in the particular locality. These loads will be considered in the following articles.

TABLE 5.—WEIGHTS OF BUILDING MATERIALS

(Pounds per square foot)

Copper roofing, sheets	1½
Corrugated iron, painted or galvanized	
No. 26, 1 lb.; No. 24, 1.3 lb.; No. 22, 1.6 lb.; No. 20, 1.9 lb.; No. 18, 2.6 lb.; No. 16, 3.3 lb.	
Felt and asphalt roofing	2
Felt and gravel roofing	8 to 10
Plastered ceiling	10
Sheathing, 1 in. thick	
White pine, hemlock, spruce	3
Yellow southern pine	4
Shingles, common	2½ to 3
Skylights, including frames	
¼-in. glass, 4½ lb.; ⁵⁄₁₆-in., 5 lb.; ⅜-in., 6 lb.	
Tile, corrugated, 8–10; flat, 15–20.	
Tin, sheets or shingles	1 to 1½

When a roof truss is to be designed to carry additional loads of the nature mentioned above, the amount of these loads must be determined, together with their points of application on the truss. The maximum stresses in the members of the truss are then to be determined by the methods of Sect. 1, or in certain cases, the stress coefficients of the following chapter can be used. To assist in the calculation of these loads there is given in Table 5 the weights of building materials in common use for roofing.

134. Weight of Roof Trusses.—The weight of a roof truss must be known before the true maximum stresses can be determined. Since the size of the members, and therefore their true weight, is dependent upon the stresses, it follows that the true weight of the truss must be known before a correct design can be made. The true weight of a truss can be determined by cut and try methods. A preliminary design can be made using an assumed weight. The weight of the structure as designed can then be determined and the assumed and calculated weights compared. If these weghts do not agree within a reasonable limit, another design must be made, using an estimated weight based on the calculated weight of the preliminary design. This process, if repeated, will finally lead to the desired true weight.

In general it will be found that for trusses of moderate size, spans of 80 feet or less, the weight of the truss is a small part of the total load to be carried. The greater part of the load, as the weight of the roofing, purlins, bracing, and the wind and snow loads, can be determined as soon as the local conditions are known. For trusses of the size mentioned, it will be found that the weight of the truss represents about 10 or 15 % of the total load to be carried. Therefore the preliminary estimate of truss weight need not be very accurate, as a relatively large error in the estimated weight will result in a small error in the total load. Thus, if the dead load be 15 % of the total load, and an error of 30 % be made in estimating the dead load, the resulting error is $0.3 \times 15 = 4.5\%$ of the total load. It is therefore probable that the true weight, as determined by the process outlined above, can be found from the second trial design.

Bridge companies and designing engineers have collected the actual shipping weights of roof trusses of moderate span designed for a great variety of loading conditions. From this information empirical formulas have been derived from which it is possible to estimate the approximate weight of a given truss. Instead of using the long process indicated above, the weight of a truss is calculated from a selected formula. If the proper formula has been used, the actual and assumed weights will usually be found to agree within reasonable limits, and a revision will not be necessary.

The factors which influence the weight of a roof truss are the type of truss, pitch of roof, character of roof covering, distance between trusses, amount and distribution of loading, assumed combinations of loading, working stresses, general requirements of the specifications as to details and minimum thickness of material, and the personal equation of the designer. It can be seen, then, that a formula for roof truss weight, in order to yield reliable results, must be used for the conditions for which it was derived. In most cases this information is not given with the formula. As there are so many factors which effect the weight of a truss, it is to be expected that the formulas collected from different sources will not agree. An interesting comparison of this nature made by R. Fleming is given in the *Eng. News-Record*, Vol. 82, No. 12, March 20, 1919, p. 576, to which the reader is referred.

From an examination of the weight data for a large number of simple roof trusses of ¼ pitch supported on masonry walls, the weight per sq. ft. of horizontal covered area was found to range from about 2 to 2.5 lb. for spans of 30 ft. to about 5 or 6 lb. for spans of 100 ft. Within these limits the weight of bracing was found to vary from about 0.3 to 0.8 lb. Trusses of greater or less slope were found to have weights differing from 5 to 25 % of the values given above. The variation in weight due to different loadings was found to be equal to from 25 to 75 % of the change in loading. Trusses with cambered lower chords were found to weigh from 15 to 40 % more than corresponding trusses with flat chords.

The formulas on p. 466 are a few of those proposed for the determination of the weight of roof trusses.

TABLE 6.—FORMULAS FOR WEIGHT OF ROOF TRUSSES

Formulas for Wooden Roof Trusses	
$w = 0.04L + 0.000167L^2$	N. C. Ricker
$w = 0.5 \quad + 0.075L$	H. S. Jacoby
$w = 0.75\ (1 + 0.10L)$	M. A. Howe
Formulas for Steel Roof Trusses	
$w = 0.06L + 0.6$ for heavy loads	C. E. Fowler
$w = 0.04L + 0.4$ for light loads	
$w = 0.20\ (\sqrt{L} + 0.125L)$	Carnegie Handbook

For 40 lb. per sq. ft. capacity. For other loads multiply formula by ratio: Load per sq. ft. ÷ 40.

Formula for steel mill building trusses

$$w = \frac{P}{45}\left(1 + \frac{L}{5\sqrt{A}}\right) \qquad \text{M. S. Ketchum}$$

In the above formulas, w = weight of truss in lb. per sq. ft. of horizontal covered area, L = span in feet, A = distance between centers of trusses in feet, and P = capacity of truss in pounds per sq. ft. of horizontal covered area.

In roof trusses for large structures, such as long span trusses for train sheds or auditoriums, the dead weight of the trusses form a large part of the total load to be carried. The weight of the trusses must then be known within much narrower limits than in the case of short spans. As long span roof trusses are not as common as those of shorter spans, there is available very little weight data from which to derive weight formulas. Also, the conditions to be met differ so widely that a general formula available for all cases is entirely out of the question. The designer must then resort to the cut and try method outlined above for the determination of the weight of the trusses.

135. Wind Loads.—The maximum wind load to be carried by a roof has been determined by experiment and by observation of the results of severe wind storms. Experiments show that the pressure on a plane surface normal to the direction of the wind varies approximately with the square of the wind velocity. From experiments made at Mt. Washington in 1890, Prof. Marvin derived the formula[1]

$$P = 0.004V^2$$

where V = velocity of wind in miles per hour, and P = pressure in pounds per sq. ft. Later experiments made at the Eiffel Tower and at the National Physical Laboratory of England gave results in close agreement, but with somewhat smaller values than obtained by Prof. Marvin. The observed values are expressed by the formula

$$P = 0.0032V^2$$

It was found by observation that the pressure varied greatly over a large area, due to the variable character of the wind. During the erection of the Forth Bridge, Sir Benjamin Baker found that the ratio of unit pressure upon an area of 1½ sq. ft. to that on an area of 300 sq. ft. varied from 1.3 to 2.5, averaging 1.5. During a seven year period the maximum observed pressure on the smaller area was 41 lb. per sq. ft.; while that on the larger area was 27 lb.[2]

No measurements have been made of wind pressures during tornadoes. Damage resulting to structures during the St. Louis tornado of 1896 indicated that there must have been a pressure of 60 lb. per sq. ft. on a length of 180 ft.[3] A study of the effects of tornadoes made by C. Shaler Smith and others leads to the conclusion that the maximum wind pressures are exerted over a comparatively small width, and that pressures exceeding 30 lb. per sq. ft. are not likely to extend over a width exceeding 60 ft.[4]

A study of the above data indicates that a maximum pressure of 30 lb. per sq. ft. is ample for structures in an exposed position. For structures in a protected position, 20 to 25 lb. per sq. ft. is ample.

The results quoted above are for surfaces perpendicular to the direction of the wind, which is assumed as horizontal. In the case of roof trusses, the roof surface is usually inclined to the horizontal, and therefore to the direction of the wind. It is usually assumed that the resultant pressure of the wind is entirely normal to the roof surface. This assumption is reasonable, since the friction of the air on comparatively smooth surfaces is very small. The component of wind pressure parallel to the roof can then be neglected without sensible error.

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[1] *Eng. News*, Dec. 13, 1890.

[2] *Engineering*, Feb. 28, 1890.

[3] *Trans. Am. Soc. C. E.*, Vol. XXXVII, p. 221.

[4] *Trans. Am. Soc. C. E.*, Vol. LIV, p. 37.

The pressure on surfaces inclined to the direction of the wind has been determined by experiment. Experiments made in 1829 by Col. Duchemin, a French army officer, are the basis of the Duchemin formula, which is considered to give the most reliable results and to represent the best knowledge on the subject. The Duchemin formula is

$$P_n = P\frac{2 \sin \alpha}{1 + \sin^2 \alpha}$$

where P = unit pressure in lb. per sq. ft. on a surface perpendicular to the direction of the wind, P_n = component of pressure normal to the roof, and α = angle which the inclined surface makes with the direction of the wind. The vertical and horizontal components of P_n, shown in Fig. 155, are given by the formulas

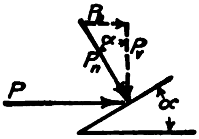

Fig. 155.

$$P_h = P\frac{2 \sin^2 \alpha}{1 + \sin^2 \alpha} \quad \text{and} \quad P_v = P\frac{2 \sin \alpha \cos \alpha}{1 + \sin^2 \alpha}$$

where P_h and P_v are respectively the horizontal and vertical components of the unit pressure. Table 7 gives values of P_n for various angles.

Table 7.—Wind Load in Pounds per Square Foot of Roof Surface

Inclination	Normal pressure, P_n	
	P = 30 lb.	P = 20 lb.
5°	5.1	3.4
10°	10.1	6.7
15°	14.6	9.7
21° 48′ 5″ (⅕ pitch)	19.8	13.1
26° 33′ 54″ (¼ pitch)	22.4	14.9
30°	24.0	16.0
33° 41′ 24″ (⅓ pitch)	25.5	17.0
45° (½ pitch)	28.3	18.9
60°	29.7	19.8
90°	30.0	20.0

Experiments made on small scale models of buildings indicate that the action of the wind causes a suction on the leeward side of the building in addition to the pressure on the windward side. An account of these experiments will be found in the Proc. Inst. Civ. Engrs., Vol. CLVI, p. 78, Vol. CLXXI, p. 175; and in the Journ. Western Soc. Engrs., Feb., 1911, Apr. and Dec., 1912. While this suction undoubtedly exists, as shown by the bursting effect of tornadoes, it is difficult to formulate a set of practical conditions to be used as a basis for designing. The experiments quoted above were made on small models, closed on the leeward side. Open windows on the leeward side of a shop building, or monitors at the ridge, will relieve all or a part of the pressure due to suction. This action should be recognized and provided for to the extent of making all members capable of resisting a reversal of stress, and by providing proper anchorage of trusses.

136. Snow Loads.—The snow load to be carried by a roof truss is a variable quantity, depending upon the slope of the roof, the latitude, and the humidity. Dry freshly fallen snow weighs about 8 lb. per cu. ft., and may attain a depth of 3 ft. on flat roofs. Packed or wet snow weighs about 12 lb. per cu. ft., but seldom will be found at greater depths than 18 in. Table 8 gives snow loads for various latitudes and roof pitches.

Table 8.—Snow Loads for Roof Trusses
(Pounds per sq. ft. of roof surface)

Location	Pitch of roof				
	½	⅓	¼	⅕	Flat
	* †	* †	* †		
Southern States and Pacific Slope.......	0–0	0–5	0–5	5	5
Central States.......................	0–5	7–10	15–20	22	30
Rocky Mountain States..............	0–10	10–15	20–25	27	35
New England States..................	0–10	10–15	20–25	35	40
Northwest States.....................	0–12	12–18	25–30	37	45

* For slate, tile, or metal roofs. † For shingle roofs.

137. Combinations of Loads.—The proper combination of wind and snow load to be used with the dead load for the determination of the maximum stresses in the members of a truss is largely a matter of judgment on the part of the designer. It is generally assumed that the wind pressure acts normal to the windward surface of the roof, there being no pressure on the leeward surface. The unit pressure on a vertical surface is generally taken at 30 lb. per sq. ft. for exposed structures and at 20 lb. per sq. ft. for sheltered structures. Pressures on inclined surfaces are usually determined by the Duchemin formula for which values are given in Table 7 of Art. 135. The snow loads are given by Table 8 of Art. 136.

Some designers assume that the maximum stresses in a roof truss are due to the dead load and a combination of the full wind and snow loads acting at the same time. This does not seem to be a reasonable assumption, for it implies that the snow remains undisturbed under a wind velocity of 100 miles per hour. A wind storm of this intensity would blow all of the snow off a roof as fast as it falls.

Wet snow or sleet is likely to adhere to the roof surface even in a high wind, but the depth of such a deposit will seldom be greater than one-half of the probable maximum for that region. It would then seem best to provide for the maximum wind load and a snow load equal to one-half the value given in Table 8. In some cases the minimum snow is assumed to be 10 lb. per sq. ft. of roof for all slopes. To provide for the condition that a heavy snow storm may be accompanied by a light wind, it is sometimes specified that the maximum snow load shall be combined with a wind pressure of such intensity that the snow load will not be disturbed. This wind pressure is estimated at from ⅓ to ½ of the maximum wind pressure.

Other designers assume that the snow load exists only on the leeward surface of the truss in combining wind and snow loads. This assumption does not seem reasonable, as eddy currents are set up on the leeward surface of the truss due to the reduction of pressure caused by the wind blowing over the top of the roof. These currents of air tend to clear the leeward surface of all snow.

The combinations of loading which seem to be most reasonable, and to approximate actual conditions are:

(*a*) Dead load and maximum snow load.

(*b*) Dead load, maximum wind load, and one-half the snow load or a minimum snow load of 10 lb. per sq. ft. of roof.

(*c*) Dead load, one-half or one-third wind load, and maximum snow load.

The stress to be used in the design of the member is the greatest obtained from these combinations. In a region of moderate snow fall it will be found that the stresses obtained for (*b*) and (*c*) are practically equal for trusses of the type of Fig. 144. For very large roofs of varying slopes both combinations must be tried out to determine the maximum stress. Where a heavy snow fall occurs, as in the far North, it is very likely that cases (*a*) or (*c*) will give the maximum stress.

It has been found that for simple roof trusses of the type shown in Fig. 144 resting on masonry walls, the maximum stresses due to wind and snow loading for cases (*b*) and (*c*) do not differ materially from those determined for a uniform vertical load over the entire roof surface. The great advantage of such a method, for the cases to which it will apply, is the ease with which the stresses can be determined. By means of the tables of stress coefficients given in the chapter which follows, the time spent in stress calculation can be reduced greatly.

Before this short cut method of stress calculation is applied to the determination of the stresses in a given truss, it is necessary to know the limitations of the method. Comparative stress calculations made by the uniform vertical load method and by the normal wind load method for trusses of the Fink, Pratt, and Howe type, as shown in Figs. 144(*a*) to (*k*) incl., and (*p*) show that for wind effect only, the first method of calculation gives chord stresses which are greater than those obtained by the second method, while the second method gives stresses in some of the interior members which are greater than those obtained by the first method. In no case was a reversal of stress found to occur. Since the stresses due to wind form from ⅓ to ½ of the total stress in the members, it was found that when the combined effect of the dead, snow, and wind loads was considered, the total stresses obtained by the two methods were close enough for all practical purposes.

One of the important points in a short cut method of this nature is the selection of the proper equivalent uniform load to be used. This is a matter on which the designer must use his judgment. Before deciding on the load to be used, the designer should make a study of the case in hand. By trial an equivalent load can be determined which will answer the conditions. This load will differ for trusses of different types, a point which must be checked up by the designer. Table 9 gives values of combined wind and snow loads.

TABLE 9.—COMBINED WIND AND SNOW LOADS FOR ROOF TRUSSES

(Pounds per sq. ft. of roof surface)

Location	Pitch of roof					
	60°	45°	⅓	¼	⅙	Flat
Northwest States	30	30	25	30	37	45
New England States	30	30	25	25	35	40
Rocky Mountain States	30	30	25	25	27	35
Central States	30	30	25	25	22	30
Southern and Pacific States	30	30	25	25	22	20

A point which comes up in the determination of the areas of the sections for the members of a roof truss is the working stresses to be used for the different kinds of loadings. Most designers determine the maximum stresses by either of the methods mentioned above and apply the same working stresses for all loadings.

In deciding this point, it should be noted that the loads carried by a roof truss differ in nature. Thus the dead load is always present, and must be included in all combinations of loading. The snow load is not always present, but when present, it can be expected to exist for a considerable time. For loads of the character of the dead and snow loads, which may be considered as permanent loads, the allowable working stresses as specified, should be used. The wind load, on the other hand, is quite variable in nature. From the values given in Art. 135, the specified wind load of 30 lb. per sq. ft. is due to a wind velocity of about 100 miles per hour. Such a wind pressure is then an extreme condition which is encountered but few times in the life of a structure, and then only for very short intervals of time. Maximum wind pressure can then be classed as an occasional loading, and the working stresses modified accordingly. This point has been discussed by R. Fleming in an excellent series of articles on "Wind Stresses."[1] He recommends that the working stresses for wind loads, when combined with dead and snow loads, be increased 50%. This is done by decreasing the intensity of the unit wind pressure by ⅓, and applying the same working stresses as for the dead and snow loads. Further discussion of this question will be found in the chapters on steel roof truss design.

ROOF TRUSSES—STRESS DATA

By W. S. KINNE

138. Stress Coefficients.—Where the stresses are to be calculated for a great many structures, in which the type of truss and the character of loading are exactly the same, the time spent in stress calculation can be reduced greatly by the use of stress coefficients. A type of structure to which the calculation of stresses by coefficients is readily adapted is the roof truss, for which in general the applied loads consist of equal panel loads placed at the panel points of the truss. Since in general it is possible to arrange the calculations so that the only variable is the amount of the equal applied loads, which for convenience are taken as unit loads, the stresses in all members of the truss can be expressed as a function of the form of the truss and the position of the loads. This factor is known as a stress coefficient. If then, the panel loads are determined, subject to conditions depending upon the size of the truss and the intensity of the applied loads, the stress in any member is obtained by multiplying the actual panel load by the stress coefficient for the member in question.

In the present chapter, tables of stress coefficients have been worked out for some of the standard forms of roof trusses. A general formula is given by which the stress coefficient for any member is expressed in terms of the form of the truss. Special numerical values of these coefficients have been calculated and are tabulated for a few of the pitch ratios generally used in practice. A more complete discussion of the conditions to which the tables apply will be given in the following articles.

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[1] *Eng. News*, Vol. 73, No. 5, Feb. 4, 1915, p. 219.

The numerical values of the stress coefficients given in the tables at the end of this chapter have been expressed to three significant figures. Therefore, all stresses calculated from these tables are accurate only to three significant figures. For example: Suppose that the panel load for a given truss, calculated by the methods given in the chapters on the "Detailed Design of Roof Trusses" is 3,520 lb., and suppose that the stress coefficient for the member whose stress is desired is 4.52. Assuming three figure accuracy, the stress in the member is 3,520 × 4.52 = 15,900 lb. It is of course possible to multiply out these quantities, obtaining the result, 3,520 × 4.52 = 15,910.40 lb. But since in calculating the coefficients we retain only three significant figures, the coefficient 4.52 may mean anything from 4.515 to 4.525, and the corresponding products will be 3,520 × 4.515 = 15,892.80, and 3,520 × 4.525 = 15,928.00. However, as the original data is accurate only to three places, it is quite evident that the result of any manipulation of these data can be accurate only to the same number of places. If we decide to retain only three significant figures in the above multiplications, we proceed to discard any figures in the fourth place below a five, and retain any figure in the fourth place above the five by changing the third significant figure to the next higher number. Thus in each case the result is found to be 15,900 lb. It will be noted that in each case the change made is less than 1% of the result. From an examination of the design tables given in the chapters on the "Detailed Design of Roof Trusses" it can be seen that stresses obtained with this degree of accuracy are close enough for all designing conditions.

If the designer desires more accurate results, he can make the proper substitutions in the general formulas for the stress coefficients, retaining the desired number of significant figures.

139. Arrangement of Tables of Stress Coefficients—Notation Adopted.—The tables of stress coefficients given at the end of this chapter have been made up for some of the standard forms of roof trusses of the type shown in Fig. 144, p. 455. In each of these tables, a truss diagram shows the form of the truss and the position of the applied loads. Each member of the truss is represented by a number, which is placed on the truss diagram. By locating the member whose stress is desired, its reference number can be determined, and by looking up this reference number in the table, the stress in the member can be determined. Where several members have equal stresses, the same reference number has been used.

Two methods have been used to indicate the kind of stress in the members. One method indicates the character of the stress by the weight of the lines used in the loading diagram at the head of each table. Heavy lines denote compression, light lines denote tension, and dotted lines denote zero stress. The other method indicates the character of the stress by means of the sign used with the numerical value of the stress coefficient. A plus sign is used to indicate tension, and a minus sign is used to indicate compression. There are a few members in the trusses of Tables 27 and 28 for which a reversal of stress occurs. In such cases the sign given with the stress coefficient must be used to obtain the character of the stress.

In deriving the stress coefficients, it was found convenient to express them in terms of the ratio of span length to height of truss at the span center. The resulting ratio, which is denoted by n, is given by the expression $n = l/h$, where l = span length and h = height of truss. It will be noted that this ratio is the reciprocal of the pitch of the truss, as defined in the chapter on "Roof Trusses—General Design." In calculating the numerical values of the stress coefficients, substitutions were made in the general formulas for the pitch ratios in general use. If values for other pitch ratios are desired, they can be obtained by interpolation from the values given in the tables, or they can be calculated directly from the general formulas.

140. Stress Coefficients for Vertical Loading.—Tables 1 to 26 give stress coefficients due to vertical loading for several of the types of trusses commonly used for roofs. Two general cases will be considered: (a) equal loads applied at all top chord panel points, known also as *roof loads;* and (b) equal loads applied at all lower chord points, known also as *ceiling loads.* These cases will be discussed separately.

140*a*. Roof Loads.—Tables 1 to 17 give stress coefficients for Fink, Fan, Pratt, and Howe trusses of various numbers of panels due to equal vertical loads applied at the top chord points. Tables 15, 16, and 17 are for Fink trusses for which the lower chord has been cambered for the sake of appearance This introduces another variable, k, by means of which the rise of the lower chord member is expressed as a fractional part of the height of the truss. Numerical values of the stress coefficients have been calculated for the usual values of n and for three values of k.

140*b*. Ceiling Loads.—Where the top and bottom chord panel points lie on the same vertical line, as in the Pratt trusses of Tables 7 to 10 and the Howe trusses of Tables 11 to 14, stress coefficients for panel loads applied at the lower chord points can be obtained from those given for *roof loads* by the application of a simple rule. This rule is as follows: Stress coefficients due to ceiling loads for all members in Pratt and Howe trusses, *except verticals*, are the same as given in Tables 7 to 14 for roof loads. Stress coefficients for stresses in *vertical members* due to ceiling loads can be obtained from the values given in Tables 7 to 14 by adding +1 (algebraic addition) to the stress coefficients for roof loads. By adding +1 algebraically, the sign of the result will indicate the character of stress in the vertical (+ = tension, − = compression) and the numerical value will give the amount of the stress.

As an example of the application of this rule, suppose that the stress coefficients are desired for the vertical members of the Howe truss of Table 12. Note that the stresses in vertical members are independent of the value of n. Applying the above rule to member 6, the stress coefficient for a ceiling load is $0 + 1 = + 1$, or a tension of 1, as indicated by the plus sign. Likewise for member 7 we have $+ 1 + 0.5 = + 1.5$, or a tension of 1.5.

Applying the same rule to the Pratt truss of Table 8, the stress coefficient for member 3 due to ceiling loads is $+ 1 - 1 = 0$, or zero stress. For member 4 we have $- 1.50 + 1.00 = - 0.50$, or a compression of 0.50. For member 10, we have $0 + 1.0 = 1.0$, or a tension of 1.

The rule given above does not apply to the trusses of Tables 1 to 6 and 15 to 17. Special tables of stress coefficients for ceiling loads are given for these trusses in Tables 18 to 26. Tables 18 to 21 are for unsymmetrical loads such as lines of shafting, heavy pipe lines, or machinery loads. Tables 22 and 23 are for symmetrical loads, such as ceiling or floor loads, and can be made to include the weight of purlins, floor or ceiling joist, floor and ceiling loads, and live loads applied to an attic floor.

If stresses are desired for all lower chord points loaded, the stresses calculated for the partial loads, as given by Tables 22 and 23 can be added to obtain the total stresses. It will usually be found that stress calculations can be made by this process in less time than is required by the graphical methods given in Sect. 1.

Tables 24 to 26 for a cambered Fink truss are similar to Tables 21 to 23 for the straight chord Fink truss.

141. Stress Coefficients for Wind Loads.—In the discussion in Art. 135, it was pointed out that for trusses of the Fink, Fan, Pratt, and Howe type, wind stresses calculated for a vertical loading represent fairly well the effect of wind loads. The stress coefficients of Tables 1 to 17 can be used for this assumed wind loading.

In case a more exact determination of wind stresses is desired, stress coefficients have been worked out for Fink and Howe trusses for wind loads applied normal to the windward roof surface. Since wind loads acting normal to the roof surface cause reactions which have horizontal components, the stress will depend upn the conditions at the points of support. Fig. 156 shows the conditions assumed at the supports. Cases I, II, and III are intended to represent conditions in steel trusses, where provision for expansion due to temperature changes must be made at the walls. Three common assumptions are shown in Fig. 156. It will be noted that these assumptions affect the stresses in the lower chord member only, and the tabulation of stress coefficients is arranged accordingly. Case IV represents conditions in small steel trusses, and in all spans of wooden trusses, for in these spans expansion due to temperature need not be considered.

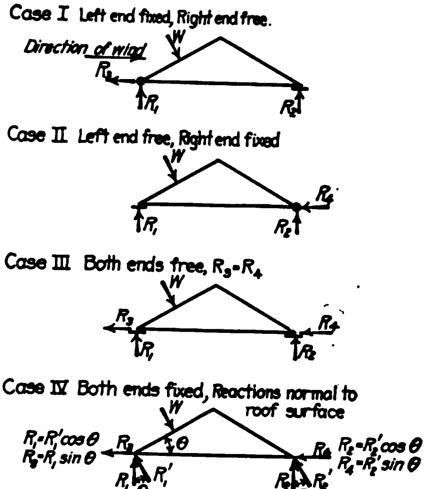
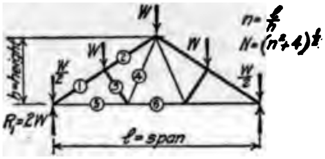


FIG. 156.—Assumed reaction conditions for wind load stresses.

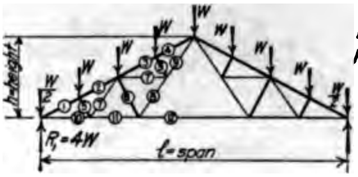
TABLE 1.—STRESS COEFFICIENTS—FINK TRUSS



Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{3}{4}WN$	−2.70	−3.00	−3.35	−4.04	−4.74
2	$-\frac{W}{4N}(3n^2+4)$	−2.15	−2.50	−2.91	−3.67	−4.43
3	$-\frac{Wn}{N}$	−0.832	−0.866	−0.894	−0.929	−0.949
4	$+\frac{1}{4}Wn$	+0.750	+0.868	+1.00	+1.25	+1.50
5	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50
6	$+\frac{1}{2}Wn$	+1.50	+1.73	+2.00	+2.50	+3.00

+ = tension — = compression

TABLE 2.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS

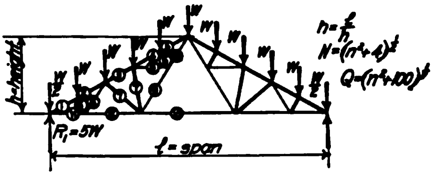


$n = \frac{l}{h}$
$N = (n^2+4)^{\frac{1}{2}}$

Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{7}{4}WN$	−6.31	−7.00	−7.83	−9.42	−11.07
2	$-\frac{1}{4}WN(7N^2 - 8)$	−5.76	−6.50	−7.38	−9.05	−10.75
3	$-\frac{1}{4}WN(7N^2-16)$	−5.20	−6.00	−6.93	−8.68	−10.43
4	$-\frac{1}{4}WN(7N^2-24)$	−4.65	−5.50	−6.48	−8.31	−10.12
5	$-W\frac{n}{N}$	−0.832	−0.866	−0.894	−0.929	−0.949
6	$-2W\frac{n}{N}$	−1.66	−1.73	−1.79	−1.86	−1.90
7	$+\frac{1}{4}Wn$	+0.750	+0.868	+1.00	+1.25	+1.50
8	$+\frac{1}{2}Wn$	+1.50	+1.73	+2.00	+2.50	+3.00
9	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50
10	$+\frac{7}{4}Wn$	+5.25	+6.07	+7.00	+8.75	+10.50
11	$+\frac{3}{2}Wn$	+4.50	+5.20	+6.00	+7.50	+9.00
12	$+Wn$	+3.00	+3.46	+4.00	+5.00	+6.00

+ = tension **− = compression**

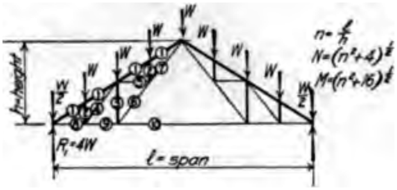
TABLE 3.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS



Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{9}{4}WN$	−8.11	−9.00	−10.06	−12.12	−14.21
2	$-\frac{1}{4}\frac{W}{N}(9n^2 + 28)$	−7.55	−8.50	−9.62	−11.75	−13.91
3	$-\frac{1}{20}\frac{W}{N}(37n^2 + 100)$	−6.00	−6.80	−7.74	−9.52	−11.31
4	$-\frac{3}{4}\frac{W}{N}(3n^2 + 4)$	−6.44	−7.50	−8.72	−11.00	−13.28
5	$-\frac{1}{4}\frac{W}{N}(9n^2+4)$	−5.88	−7.00	−8.28	−10.63	−12.98
6	$-W\frac{n}{N}$	−0.832	−0.866	−0.894	−0.929	−0.949
7	$-\frac{3}{20}W\frac{nQ}{N}$	−1.31	−1.38	−1.45	−1.56	−1.66
8	$+\frac{1}{4}Wn$	+0.750	+0.868	+1.00	+1.25	+1.50
9	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50
10	$+Wn$	+3.00	+3.46	+4.00	+5.00	+6.00
11	$+\frac{9}{4}Wn$	+6.75	+7.79	+9.00	+11.25	+13.50
12	$+2Wn$	+6.00	+6.92	+8.00	+10.00	+12.00
13	$+\frac{5}{4}Wn$	+3.75	+4.34	+5.00	+6.25	+7.50

+ = tension — = compression

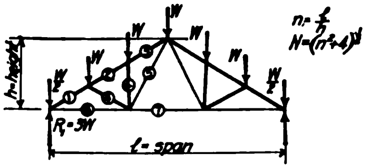
TABLE 4.—STRESS COEFFICIENTS—FINK TRUSS WITH VERTICALS



Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{7}{4}WN$	−6.31	−7.00	−7.83	−9.42	−11.07
2	$-W$	−1.00	−1.00	−1.00	−1.00	−1.00
3	$-2W$	−2.00	−2.00	−2.00	−2.00	−2.00
4	$+\frac{1}{4}WM$	+1.25	+1.32	+1.41	+1.60	+1.80
5	$+\frac{1}{4}Wn$	+0.750	+0.868	+1.00	+1.25	+1.50
6	$+\frac{1}{2}WM$	+2.50	+2.64	+2.82	+3.20	+3.60
7	$+\frac{3}{4}WM$	+3.75	+3.96	+4.23	+4.80	+5.40
8	$+\frac{7}{4}Wn$	+5.25	+6.07	+7.00	+8.75	+10.50
9	$+\frac{3}{2}Wn$	+4.50	+5.20	+6.00	+7.50	+9.00
10	$+Wn$	+3.00	+3.46	+4.00	+5.00	+6.00

+ = tension — = compression

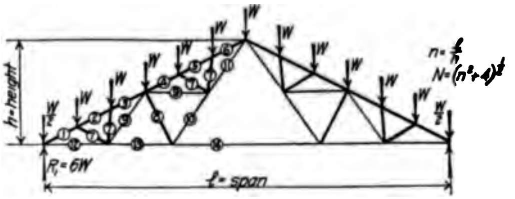
TABLE 5.—STRESS COEFFICIENTS—FAN TRUSS



Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{5}{4}WN$	−4.51	−5.00	−5.59	−6.73	−7.91
2	$-\frac{1}{12}\frac{W}{N}(13N^2 - 16)$	−3.54	−4.00	−4.55	−5.59	−6.64
3	$-\frac{1}{12}\frac{W}{N}(15N^2 - 48)$	−3.40	−4.00	−4.70	−5.99	−7.27
4	$-\frac{1}{6}W\frac{n}{N}(n^2 + 36)^{\frac{1}{2}}$	−0.930	−1.00	−1.08	−1.21	−1.34
5	$+\frac{1}{2}Wn$	+1.50	+1.73	+2.00	+2.50	+3.00
6	$+\frac{5}{4}Wn$	+3.75	+4.33	+5.00	+6.25	+7.50
7	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50

+ = tension — = compression

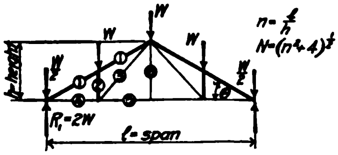
TABLE 6.—STRESS COEFFICIENTS—COMPOUND FAN TRUSS



Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{11}{4}WN$	−9.92	−11.00	−12.30	−14.81	−17.39
2	$-\frac{1}{12}\frac{W}{N}(31N^2 - 16)$	−8.95	−10.00	−11.25	−13.66	−16.13
3	$-\frac{1}{12}\frac{W}{N}(33N^2 - 48)$	−8.81	−10.00	−11.40	−14.07	−16.76
4	$-\frac{1}{12}\frac{W}{N}(33N^2 - 72)$	−8.25	−9.50	−10.96	−13.70	−16.44
5	$-\frac{1}{12}\frac{W}{N}(31N^2 - 88)$	−7.28	−8.50	−9.91	−12.55	−15.18
6	$-\frac{1}{12}\frac{W}{N}(33N^2 - 120)$	−7.14	−8.50	−10.06	−12.95	−15.93
7	$-\frac{1}{6}W\frac{n}{N}(n^2 + 36)^{\frac{1}{2}}$	−0.930	−1.00	−1.08	−1.21	−1.34
8	$-3\frac{Wn}{N}$	−2.50	−2.60	−2.68	−2.79	−2.85
9	$+\frac{1}{2}Wn$	+1.50	+1.73	+2.00	+2.50	+3.00
10	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50
11	$+\frac{5}{4}Wn$	+3.75	+4.33	+5.00	+6.25	+7.50
12	$+\frac{11}{4}Wn$	+8.25	+9.53	+11.00	+13.75	+16.50
13	$+\frac{9}{4}Wn$	+6.75	+7.79	+9.00	+11.25	+13.50
14	$+\frac{3}{2}Wn$	+4.50	+5.20	+6.00	+7.50	+9.00

+ = tension — = compression

TABLE 7.—STRESS COEFFICIENTS—PRATT TRUSS—4 PANELS

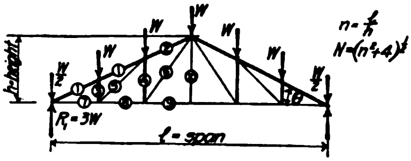


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{3}{4}WN$	−2.70	−3.00	−3.35	−4.04	−4.74
2	$-W$	−1.00	−1.00	−1.00	−1.00	−1.00
3	$+\frac{W}{4}(n^2+16)^{\frac{1}{2}}$	+1.25	+1.32	+1.41	+1.60	+1.80
4	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50
5	$+\frac{1}{2}Wn$	+1.50	+1.73	+2.00	+2.50	+3.00
6	0	0	0	0	0	0

+ = tension — = compression

For loads on lower chord, see Art. 140*b*

TABLE 8.—STRESS COEFFICIENTS—PRATT TRUSS—6 PANELS

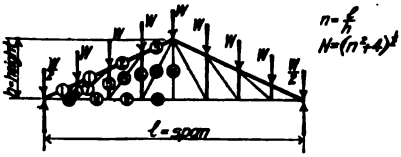


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{5}{4}WN$	−4.51	−5.00	−5.59	−6.73	−7.91
2	$-WN$	−3.61	−4.00	−4.47	−5.39	−6.32
3	$-W$	−1.00	−1.00	−1.00	−1.00	−1.00
4	$-\frac{3}{2}W$	−1.50	−1.50	−1.50	−1.50	−1.50
5	$+\frac{W}{4}(n^2+16)^{\frac{1}{2}}$	+1.25	+1.32	+1.41	+1.60	+1.80
6	$+\frac{W}{4}(n^2+36)^{\frac{1}{2}}$	+1.68	+1.73	+1.80	+1.95	+2.12
7	$+\frac{5}{4}Wn$	+3.75	+4.33	+5.00	+6.25	+7.50
8	$+Wn$	+3.00	+3.46	+4.00	+5.00	+6.00
9	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50
10	0	0	0	0	0	0

+ = tension — = compression

For loads on lower chord see Art. 140*b*

TABLE 9.—STRESS COEFFICIENTS—PRATT TRUSS—8 PANELS

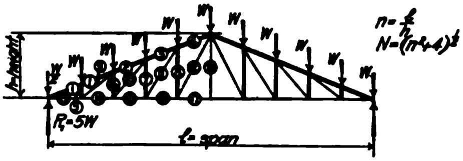


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{7}{4} WN$	−6.31	−7.00	−7.83	−9.42	−11.07
2	$-\frac{3}{2} WN$	−5.41	−6.00	−6.71	−8.08	−9.49
3	$-\frac{5}{4} WN$	−4.51	−5.00	−5.59	−6.73	−7.91
4	$-W$	−1.00	−1.00	−1.00	−1.00	−1.00
5	$-\frac{3}{2} W$	−1.50	−1.50	−1.50	−1.50	−1.50
6	$-2W$	−2.00	−2.00	−2.00	−2.00	−2.00
7	$+\frac{1}{4} W(n^2 + 16)^{\frac{1}{2}}$	+1.25	+1.32	+1.41	+1.60	+1.80
8	$+\frac{1}{4} W(n^2 + 36)^{\frac{1}{2}}$	+1.68	+1.73	+1.80	+1.95	+2.12
9	$+\frac{1}{4} W(n^2 + 64)^{\frac{1}{2}}$	+2.14	+2.18	+2.24	+2.36	+2.50
10	$+\frac{7}{4} Wn$	+5.25	+6.06	+7.00	+8.75	+10.50
11	$+\frac{3}{2} Wn$	+4.50	+5.20	+6.00	+7.50	+9.00
12	$+\frac{5}{4} Wn$	+3.75	+4.33	+5.00	+6.25	+7.50
13	$+Wn$	+3.00	+3.46	+4.00	+5.00	+6.00
14	0	0	0	0	0	0

+ = tension − = compression

For loads on lower chord see Art. 140b

TABLE 10.—STRESS COEFFICIENTS—PRATT TRUSS—10 PANELS

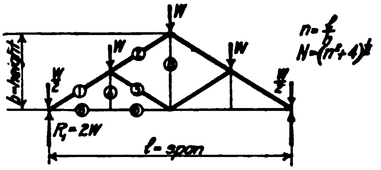


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{9}{4}WN$	−8.11	−9.00	−10.06	−12.12	−14.23
2	$-2WN$	−7.21	−8.00	− 8.94	−10.77	−12.65
3	$-\frac{7}{4}WN$	−6.31	−7.00	− 7.83	− 9.42	−11.07
4	$-\frac{3}{2}WN$	−5.41	−6.00	− 6.71	− 8.08	− 9.49
5	$-W$	−1.00	−1.00	− 1.00	− 1.00	− 1.00
6	$-\frac{3}{2}W$	−1.50	−1.50	− 1.50	− 1.50	− 1.50
7	$-2W$	−2.00	−2.00	− 2.00	− 2.00	− 2.00
8	$-\frac{5}{2}W$	−2.50	−2.50	− 2.50	− 2.50	− 2.50
9	$+\frac{W}{4}(n^2 + 16)^{\frac{1}{2}}$	+1.25	+1.32	+ 1.41	+ 1.60	+ 1.80
10	$+\frac{W}{4}(n^2 + 36)^{\frac{1}{2}}$	+1.68	+1.73	+ 1.80	+ 1.95	+ 2.12
11	$+\frac{W}{4}(n^2 + 64)^{\frac{1}{2}}$	+2.14	+2.18	+ 2.24	+ 2.36	+ 2.50
12	$+\frac{W}{4}(n^2 + 100)^{\frac{1}{2}}$	+2.61	+2.65	+ 2.69	+2.80	+ 2.92
13	$+\frac{9}{4}Wn$	+6.75	+7.79	+ 9.00	+11.25	+13.50
14	$+2Wn$	+6.00	+6.93	+ 8.00	+10.00	+12.00
15	$+\frac{7}{4}Wn$	+5.25	+6.06	+ 7.00	+ 8.75	+10.50
16	$+\frac{3}{2}Wn$	+4.50	+5.20	+ 6.00	+ 7.50	+ 9.00
17	$+\frac{5}{4}Wn$	+3.75	+4.33	+ 5.00	+ 6.25	+ 7.50
18	0	0	0	0	0	0

+ = tension — = compression

For loads on lower chord see Art. 140*b*

TABLE 11.—STRESS COEFFICIENTS—HOWE TRUSS—4 PANELS

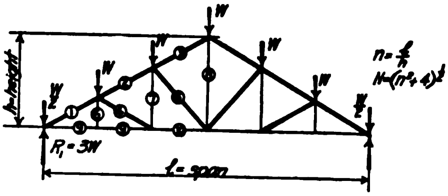


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{3}{4}WN$	−2.70	−3.00	−3.35	−4.04	−4.74
2	$-\frac{1}{2}WN$	−1.80	−2.00	−2.24	−2.69	−3.16
3	$-\frac{1}{4}WN$	−0.900	−1.00	−1.12	−1.35	−1.58
4	0	0	0	0	0	0
5	$+3W$	+3.0	+3.0	+3.0	+3.0	+3.0
6	$+\frac{3}{4}Wn$	+2.25	+2.60	+3.00	+3.75	+4.50

+ = tension — = compression

For loads on lower chord, see Art. 140*b*

TABLE 12.—STRESS COEFFICIENTS—HOWE TRUSS—6 PANELS

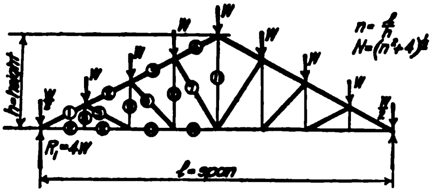


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{5}{4}WN$	−4.51	−5.00	−5.59	−6.73	−7.91
2	$-WN$	−3.61	−4.00	−4.47	−5.39	−6.32
3	$-\frac{3}{4}WN$	−2.70	−3.00	−3.35	−4.04	−4.74
4	$-\frac{1}{4}WN$	−0.900	−1.00	−1.12	−1.35	−1.58
5	$-\frac{1}{4}W(n^2 + 16)^{\frac{1}{2}}$	−1.25	−1.32	−1.41	−1.60	−1.80
6	0	0	0	0	0	0
7	$+\frac{W}{2}$	+0.500	+0.500	+0.500	+0.500	+0.500
8	$+2W$	+2.00	+2.00	+2.00	+2.00	+2.00
9	$+\frac{5}{4}Wn$	+3.75	+4.33	+5.00	+6.25	+7.50
10	$+Wn$	+3.00	+3.46	+4.00	+5.00	+6.00

+ = tension — = compression

For loads on lower chord see Art. 140*b*

TABLE 13.—STRESS COEFFICIENTS—HOWE TRUSS—8 PANELS

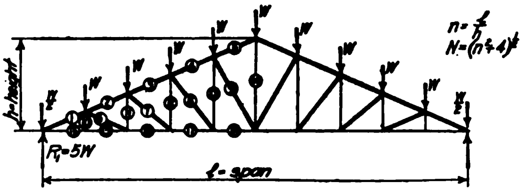


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta - 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{7}{4}WN$	−6.31	−7.00	−7.83	−9.42	−11.07
2	$-\frac{3}{2}WN$	−5.41	−6.00	−6.71	−8.08	−9.49
3	$-\frac{5}{4}WN$	−4.51	−5.00	−5.59	−6.73	−7.91
4	$-WN$	−3.61	−4.00	−4.47	−5.39	−6.32
5	$-\frac{1}{4}WN$	−0.900	−1.00	−1.12	−1.35	−1.58
6	$-\frac{1}{4}W(n^2 + 16)^{\frac{1}{2}}$	−1.25	−1.32	−1.41	−1.60	−1.80
7	$-\frac{1}{4}W(n^2 + 36)^{\frac{1}{2}}$	−1.68	−1.73	−1.80	−1.95	−2.12
8	0	0	0	0	0	0
9	$+\frac{1}{2}W$	+0.500	+0.500	+0.500	+0.500	+0.500
10	$+W$	+1.00	+1.00	+1.00	+1.00	+1.00
11	$+3W$	+3.00	+3.00	+3.00	+3.00	+3.00
12	$+\frac{7}{4}Wn$	+5.25	+6.06	+7.00	+8.75	+10.50
13	$+\frac{3}{2}Wn$	+4.50	+5.20	+6.00	+7.50	+9.00
14	$+\frac{5}{4}Wn$	+3.75	+4.33	+5.00	+6.25	+7.50

+ = tension − = compression

For loads on lower chord see Art. 140*b*

TABLE 14.—STRESS COEFFICIENTS—HOWE TRUSS—10 PANELS

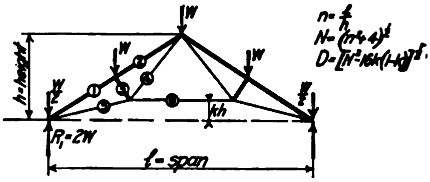


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{9}{4}WN$	−8.11	−9.00	−10.06	−12.12	−14.23
2	$-2WN$	−7.21	−8.00	−8.94	−10.77	−12.65
3	$-\frac{7}{4}WN$	−6.31	−7.00	−7.83	−9.42	−11.07
4	$-\frac{3}{2}WN$	−5.41	−6.00	−6.71	−8.08	−9.49
5	$-\frac{5}{4}WN$	−4.51	−5.00	−5.59	−6.73	−7.91
6	$-\frac{1}{4}WN$	−0.900	−1.00	−1.12	−1.35	−1.58
7	$-\frac{1}{4}W(n^2+16)^{\frac{1}{2}}$	−1.25	−1.32	−1.41	−1.60	−1.80
8	$-\frac{1}{4}W(n^2+36)^{\frac{1}{2}}$	−1.68	−1.73	−1.80	−1.95	−2.12
9	$-\frac{1}{4}(n^2+64)^{\frac{1}{2}}$	−2.14	−2.18	−2.24	−2.36	−2.50
10	0	0	0	0	0	0
11	$+\frac{1}{2}W$	+0.500	+0.500	+0.500	+0.500	+0.500
12	$+W$	+1.00	+1.00	+1.00	+1.00	+1.00
13	$+\frac{3}{2}W$	+1.50	+1.50	+1.50	+1.50	+1.50
14	$+4W$	+4.00	+4.00	+4.00	+4.00	+4.00
15	$+\frac{9}{4}Wn$	+6.75	+7.79	+9.00	+11.25	+13.50
16	$+2Wn$	+6.00	+6.93	+8.00	+10.00	+12.00
17	$+\frac{7}{4}Wn$	+5.25	+6.06	+7.00	+8.75	+10.50
18	$+\frac{3}{2}Wn$	+4.50	+5.20	+6.00	+7.50	+9.00

+ = tension — = compression

For loads on lower chord see Art. 140*b*

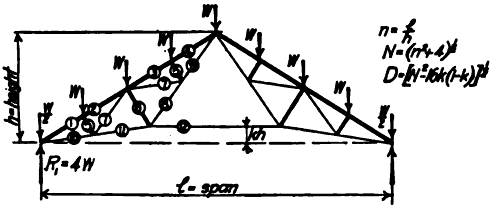
TABLE 15.—STRESS COEFFICIENTS—CAMBERED FINK TRUSS



Member	General formula	k	Value of n 3 $\theta = 33°-41'$	 $2\sqrt{3}$ $\theta = 30°$	 4 $\theta = 26°-34'$	 5 $\theta = 21°-48'$	 6 $\theta = 18°-26'$
1	$-\frac{3}{4}W\frac{(N^2-8k)}{N(1-2k)}$	$\frac{1}{10}$	−3.17	−3.56	−4.03	−4.90	−5.81
		$\frac{1}{8}$	−3.32	−3.75	−4.25	−5.20	−6.17
		$\frac{1}{6}$	−3.64	−4.13	−4.70	−5.78	−6.89
2	$-\frac{1}{4}W\frac{[3n^2-8(1+k)]}{N(1-2k)}$	$\frac{1}{10}$	−2.62	−3.06	−3.49	−4.54	−5.51
		$\frac{1}{8}$	−2.77	−3.25	−3.80	−4.83	−5.85
		$\frac{1}{6}$	−3.09	−3.63	−4.25	−5.41	−6.57
3	$-\frac{Wn}{N}$		−0.832	−0.866	−0.894	−0.929	−0.949
4	$+\frac{1}{4}Wn\frac{D(1+k)}{N(1-2k)(1-k)}$	$\frac{1}{10}$	+1.08	+1.26	+1.48	+1.87	+2.26
		$\frac{1}{8}$	+1.20	+1.40	+1.64	+2.08	+2.52
		$\frac{1}{6}$	+1.43	+1.69	+1.98	+2.52	+3.06
5	$+\frac{3}{4}Wn\frac{D}{N(1-2k)}$	$\frac{1}{10}$	+2.65	+3.09	+3.62	+4.57	+5.52
		$\frac{1}{8}$	+2.79	+3.27	+3.83	+4.85	+5.85
		$\frac{1}{6}$	+3.07	+3.62	+4.24	+5.40	+6.56
6	$+\frac{1}{2}\frac{Wn}{(1-k)}$	$\frac{1}{10}$	+1.67	+1.93	+2.22	+2.78	+3.33
		$\frac{1}{8}$	+1.72	+1.98	+2.29	+2.86	+3.34
		$\frac{1}{6}$	+1.80	+2.08	+2.40	+3.00	+3.60

+ = tension — = compression

TABLE 16.—STRESS COEFFICIENTS—CAMBERED COMPOUND FINK TRUSS

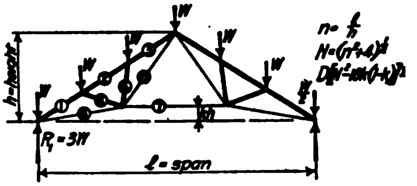


Member	General formula	k	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{4}W\frac{[n^2+4(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−7.39	−8.31	−9.40	−11.46	−13.56
		$\frac{1}{8}$	−7.79	−8.75	−9.93	−12.14	−14.42
		$\frac{1}{6}$	−8.49	−9.63	−10.96	−13.49	−16.04
2	$-\frac{1}{4}W\frac{[7n^2+20(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−6.84	−7.81	−8.95	−11.08	−13.25
		$\frac{1}{8}$	−7.23	−8.25	−9.48	−11.76	−14.10
		$\frac{1}{6}$	−7.94	−9.13	−10.51	−13.11	−15.72
3	$-\frac{1}{4}W\frac{[7n^2+12(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−6.29	−7.31	−8.50	−10.70	−12.94
		$\frac{1}{8}$	−6.67	−7.75	−9.03	−11.38	−13.78
		$\frac{1}{6}$	−7.39	−8.63	−10.06	−12.74	−15.40
4	$-\frac{1}{4}W\frac{[7n^2+4(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−5.74	−6.81	−8.05	−10.32	−12.63
		$\frac{1}{8}$	−6.11	−7.25	−8.58	−11.00	−13.46
		$\frac{1}{6}$	−6.83	−8.13	−9.61	−12.37	−15.08
5	$-W\frac{n}{N}$		−0.832	−0.866	−0.894	−0.929	−0.949

TABLE 16.—(*Continued*)

6	$-2W\frac{n}{N}$		−1.66	−1.73	−1.79	−1.86	−1.90
7	$+\frac{1}{4}W\frac{nD}{N(1-2k)}$	$\frac{1}{10}$	+0.884	+1.030	+1.20	+1.52	+1.85
		$\frac{1}{8}$	+0.933	+1.09	+1.29	+1.62	+1.96
		$\frac{1}{6}$	+1.02	+1.21	+1.41	+1.80	+2.19
8	$+\frac{1}{2}Wn\frac{D(1+k)}{N(1-2k)(1-k)}$	$\frac{1}{10}$	+2.16	+2.52	+2.95	+3.73	+4.51
		$\frac{1}{8}$	+2.40	+2.80	+3.29	+4.15	+5.04
		$\frac{1}{6}$	+2.87	+3.37	+3.96	+5.04	+6.12
9	$+\frac{1}{4}Wn\frac{D(3+k)}{N(1-2k)(1-k)}$	$\frac{1}{10}$	+3.04	+3.57	+4.15	+5.24	+6.34
		$\frac{1}{8}$	+3.32	+3.90	+4.56	+5.76	+7.00
		$\frac{1}{6}$	+3.88	+4.58	+5.37	+6.85	+8.30
10	$+\frac{7}{4}W\frac{nD}{N(1-2k)}$	$\frac{1}{10}$	+6.18	+7.22	+8.45	+10.68	+12.91
		$\frac{1}{8}$	+6.54	+7.64	+8.95	+11.31	+13.71
		$\frac{1}{6}$	+7.17	+8.44	+9.90	+12.61	+15.71
11	$+\frac{3}{2}W\frac{nD}{N(1-2k)}$	$\frac{1}{10}$	+5.30	+6.20	+7.25	+9.15	+11.09
		$\frac{1}{8}$	+5.61	+6.55	+7.68	+9.70	+11.76
		$\frac{1}{6}$	+6.15	+7.23	+8.48	+10.81	+13.49
12	$+W\frac{n}{(1-k)}$	$\frac{1}{10}$	+3.34	+3.85	+4.44	+5.55	+6.66
		$\frac{1}{8}$	+3.43	+3.96	+4.57	+5.72	+6.86
		$\frac{1}{6}$	+3.60	+4.16	+4.80	+6.00	+7.20

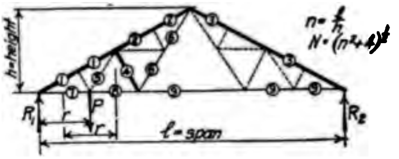
TABLE 17.—STRESS COEFFICIENTS—CAMBERED FAN TRUSS



Member	General formula	k	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{5}{4}W\frac{[n^2 + 4(1 - 2k)]}{N(1-2k)}$	$\frac{1}{10}$	−5.28	−5.94	−6.71	−8.18	−9.70
		$\frac{1}{8}$	−5.55	−6.25	−7.09	−8.67	−10.28
		$\frac{1}{6}$	−6.06	−6.88	−7.83	−9.64	−11.46
2	$-\frac{1}{4}W\frac{[\frac{13}{3}n^2 + 12(1 - 2k)]}{N(1 - 2k)}$	$\frac{1}{10}$	−4.20	−4.81	−5.52	−6.83	−8.19
		$\frac{1}{8}$	−4.44	−5.07	−5.84	−7.26	−8.70
		$\frac{1}{6}$	−4.89	−5.63	−6.48	−8.10	−9.70
3	$-\frac{1}{4}W\frac{[(5n^2 + 4(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−4.17	−4.94	−5.81	−7.45	−9.07
		$\frac{1}{8}$	−4.45	−5.25	−6.20	−7.92	−9.65
		$\frac{1}{6}$	−4.96	−5.88	−6.93	−8.89	−10.81
4	$-\frac{1}{6}\frac{Wn}{N}\frac{[(n^2+36(1-2k)^2]^{\frac{1}{2}}}{(1-2k)}$	$\frac{1}{10}$	−0.981	−1.07	−1.17	−1.34	−1.52
		$\frac{1}{8}$	−1.00	−1.09	−1.20	−1.39	−1.58
		$\frac{1}{6}$	−1.04	−1.15	−1.26	−1.49	−1.71
5	$+\frac{1}{4}W\frac{nD}{N}\frac{(2 + k)}{(1-2k)(1-k)}$	$\frac{1}{10}$	+2.06	+2.41	+2.80	+3.56	+4.31
		$\frac{1}{8}$	+2.26	+2.66	+3.10	+3.93	+4.76
		$\frac{1}{6}$	+2.66	+3.13	+3.67	+4.69	+5.70
6	$+\frac{5}{4}W\frac{nD}{N(1-2k)}$	$\frac{1}{10}$	+4.42	+5.16	+6.03	+7.62	+9.22
		$\frac{1}{8}$	+4.67	+5.45	+6.39	+8.08	+9.80
		$\frac{1}{6}$	+5.12	+6.03	+7.07	+9.01	+10.92
7	$+\frac{3}{4}W\frac{n}{(1-k)}$	$\frac{1}{10}$	+2.50	+2.89	+3.34	+4.17	+5.00
		$\frac{1}{8}$	+2.57	+2.97	+3.43	+4.28	+5.15
		$\frac{1}{6}$	+2.70	+3.12	+3.60	+4.50	+5.40

+ = tension — = compression

TABLE 18.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS

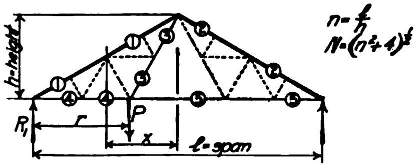


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{16}P\frac{N}{n^2}(7n^2 - 4)$	-1.479	-1.665	-1.889	-2.305	-2.720
2	$-\frac{1}{16}P\frac{N}{n^2}(3n^2 - 4)$	-0.576	-0.667	-0.769	-0.957	-1.140
3	$-\frac{1}{16}P\frac{N^3}{n^2}$	-0.326	-0.333	-0.349	-0.391	-0.438
4	$-\frac{1}{2}P\frac{N}{n}$	-0.602	-0.576	-0.559	-0.539	-0.527
5	$+\frac{1}{4}P\frac{N^2}{n}$	$+1.083$	$+1.160$	$+1.250$	$+1.450$	$+1.667$
6	$+\frac{1}{8}P\frac{N^2}{n}$	$+0.542$	$+0.580$	$+0.625$	$+0.725$	$+0.833$
7	$+\frac{1}{16}P\frac{(7n^2 - 4)}{n}$	$+1.229$	$+1.442$	$+1.688$	$+2.139$	$+2.585$
8	$+\frac{3}{16}P\frac{N^2}{n}$	$+0.813$	$+0.865$	$+0.936$	$+1.088$	$+1.25$
9	$+\frac{1}{16}P\frac{N^2}{n}$	$+0.271$	$+0.288$	$+0.312$	$+0.362$	$+0.417$
R_1	$\frac{1}{8}P\frac{(7n^2 - 4)}{n^2}$	0.819	0.833	0.844	0.855	0.861
R_2	$\frac{1}{8}P\frac{N^2}{n^2}$	0.181	0.167	0.156	0.145	0.139
r	$\frac{1}{8}l\frac{N^2}{n^2} = \frac{1}{8}h\frac{N^2}{n}$	$0.181l$ $0.543h$	$0.167l$ $0.578h$	$0.156l$ $0.625h$	$0.145l$ $0.725h$	$0.139l$ $0.833h$

+ = tension **— = compression**

Stress is zero for dotted members.

TABLE 19.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS

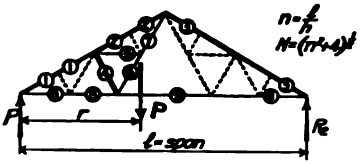


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{8}P\frac{N}{n^2}(3n^2 - 4)$	−1.152	−1.335	−1.538	−1.915	−2.228
2	$-\frac{1}{8}P\frac{N^2}{n^2}$	−0.652	−0.667	−0.699	−0.783	−0.877
3	$+\frac{1}{4}P\frac{N^2}{n}$	+1.083	+1.160	+1.250	+1.450	+1.667
4	$+\frac{1}{8}P\frac{(3n^2 - 4)}{n}$	+0.958	+1.152	+1.372	+1.775	+2.167
5	$+\frac{1}{8}P\frac{N^2}{n}$	+0.542	+0.580	+0.625	+0.725	+0.833
R_1	$\frac{1}{4}P\frac{(3n^2 - 4)}{n^2}$	0.639	0.667	0.688	0.710	0.723
R_2	$\frac{1}{4}P\frac{N^2}{n^2}$	0.361	0.333	0.312	0.290	0.277
r	$\frac{1}{4}l\frac{N^2}{n^2} = \frac{1}{4}h\frac{N^2}{n}$	0.361l 1.086h	0.333l 1.156h	0.312l 1.25h	0.290l 1.45h	0.277l 1.667h
x	$\frac{1}{16}\frac{l}{n^2}(5n^2 - 12)$	0.229l	0.250l	0.266l	0.282l	0.292l

+ = tension. — = compression.

Stress is zero for dotted members.

TABLE 20.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS

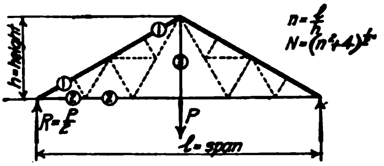


Member	General formula	Value of *n*				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{16}P\frac{N}{n^2}(5n^2-4)$	−1.027	−1.165	−1.329	−1.630	− 1.930
2	$-\frac{1}{16}P\frac{(9n^4-16)}{n^2N}$	− 1.37	−1.667	−2.00	−2.60	− 3.20
3	$-\frac{1}{16}P\frac{N}{n_2}(3n^2+4)$	−0.776	−0.832	−0.910	−1.065	−1.228
4	$-\frac{1}{2}P\frac{(n^2-4)}{Nn}$	−0.231	−0.289	−0.336	−0.390	−0.422
5	$+\frac{1}{4}P\frac{(n^2-4)}{n}$	+0.417	+0.576	+0.750	+1.050	+1.33
6	$+\frac{1}{8}P\frac{(n^2-4)}{n}$	+0.208	+0.288	+0.375	+0.525	+0.667
7	$+\frac{1}{8}P\frac{(3n^2+4)}{n}$	+1.291	+1.44	+1.625	+1.975	+2.333
8	$+\frac{1}{16}P\frac{(5n^2-4)}{n}$	+0.855	+1.014	+1.188	+1.513	+1.833
9	$+\frac{1}{16}P\frac{(3n^2+4)}{n}$	+0.645	+0.720	+0.813	+0.988	+1.167
R_1	$\frac{1}{8}P\frac{(5n^2-4)}{n^2}$	0.570	0.583	0.593	0.605	0.611
R_2	$\frac{1}{8}P\frac{(3n^2+4)}{n^2}$	0.430	0.417	0.407	0.395	0.389
r	$\frac{1}{8}l\frac{(3n^2+4)}{n^2}$	0.430*l*	0.417*l*	0.407*l*	0.395*l*	0.389*l*

+ = tension — = compression

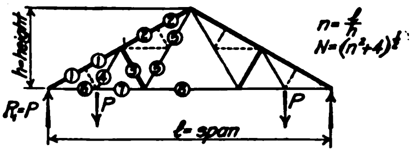
Stress is zero for dotted members.

TABLE 21.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS



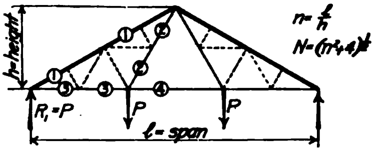
Member	General formula	Value of n				
		3 $\theta = 33^\circ - 41'$	$2\sqrt{3}$ $\theta = 30^\circ$	4 $\theta = 26^\circ - 34'$	5 $\theta = 21^\circ - 48'$	6 $\theta = 18^\circ - 26'$
1	$-\frac{1}{4}PN$	−0.9025	−1.00	−1.117	−1.347	−1.582
2	$+\frac{1}{4}Pn$	+0.75	+0.866	+1.00	+1.25	+1.50
3	$+P$	+1.0	+1.0	+1.0	+1.0	+1.0

TABLE 22.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS



Member	General formula	Value of n				
		3 $\theta = 33^\circ - 41'$	$2\sqrt{3}$ $\theta = 30^\circ$	4 $\theta = 26^\circ - 34'$	5 $\theta = 21^\circ - 48'$	6 $\theta = 18^\circ - 26'$
1	$-\frac{1}{2}PN$	−1.805	−2.00	−2.235	−2.695	−3.163
2	$-\frac{1}{4}PN$	−0.903	−1.00	−1.118	−1.347	−1.582
3	$-\frac{1}{2}P\frac{N}{n}$	−0.602	−0.578	−0.558	−0.538	−0.527
4	$+\frac{1}{4}P\frac{N^2}{n}$	+1.083	+1.152	+1.25	+1.45	+1.667
5	$+\frac{1}{8}P\frac{N^2}{n}$	+0.542	+0.576	+0.625	+0.725	+0.833
6	$+\frac{1}{2}Pn$	+1.50	+1.732	+2.00	+2.50	+3.00
7	$+\frac{1}{4}P\frac{N^2}{n}$	+1.083	+1.152	+1.25	+1.45	+1.667
8	$+\frac{1}{8}P\frac{N^2}{n}$	+0.542	+0.576	+0.625	+0.725	+0.833

TABLE 23.—STRESS COEFFICIENTS—COMPOUND FINK TRUSS

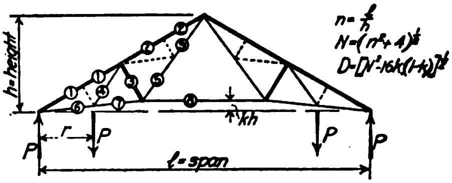


Member	General formula	Value of n				
		3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{2}PN$	−1.805	−2.00	−2.235	−2.695	−3.163
2	$+\frac{1}{4}P\frac{N^2}{n}$	+1.083	+1.152	+1.25	+1.45	+1.667
3	$+\frac{1}{2}Pn$	+1.50	+1.732	+2.00	+2.50	+3.00
4	$+\frac{1}{4}P\frac{N^2}{n}$	+1.083	+1.152	+1.25	+1.45	+1.667

+ = tension — = compression

Stress is zero for dotted members.

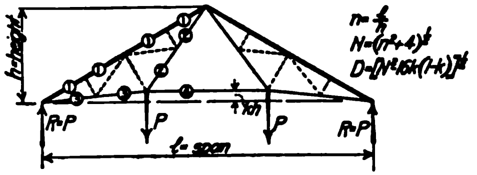
TABLE 24.—STRESS COEFFICIENTS—CAMBERED COMPOUND FINK STRESS


$n = \frac{l}{h}$
$N = (n^2 + 4)^{\frac{1}{2}}$
$D = [N^2 - 16k(1-k)]^{\frac{1}{2}}$
h = height
① ① ② ② ③ ③ ④ ⑤ ⑤ ⑥ ⑦ ⑧
kh
r
P
P
P
P
l = span

Member	General formula	k	Value of n				
			3 θ = 33° −41′	$2\sqrt{3}$ θ = 30°	4 θ = 26° − 34′	5 θ = 21° − 48′	6 θ = 18° − 26′
1	$-\frac{1}{2}P\frac{[n^2+4(1-2k)]}{N(1-2k)}$	1/10	−2.12	−2.38	−2.68	−3.28	−3.88
		1/8	−2.22	−2.50	−2.84	−3.46	−4.12
		1/6	−2.42	−2.76	−3.14	−3.86	−4.58
2	$-\frac{1}{4}P\frac{[n^2+4(1-2k)]}{N(1-2k)}$	1/10	−1.06	−1.19	−1.34	−1.64	−1.94
		1/8	−1.11	−1.25	−1.42	−1.73	−2.06
		1/6	−1.21	−1.38	−1.57	−1.93	−2.29
3	$-\frac{1}{2}P\frac{[n^2+4(1-2k)]}{nN}$	1/10	−0.562	−0.549	−0.537	−0.523	−0.517
		1/8	−0.554	−0.542	−0.532	−0.519	−0.515
		1/6	−0.538	−0.530	−0.522	−0.514	−0.510
4	$+\frac{1}{4}P\frac{D[n^2+4(1-2k)]}{nN(1-2k)}$	1/10	+1.20	+1.31	+1.45	+1.72	+2.01
		1/8	+1.24	+1.37	+1.52	+1.81	+2.13
		1/6	+1.32	+1.48	+1.65	+1.99	+2.35
5	$+\frac{1}{8}P\frac{D[n^2+4(1-2k)]}{nN(1-2k)(1-k)}$	1/10	+0.667	+0.729	+0.806	+0.956	+1.11
		1/8	+0.709	+0.782	+0.867	+1.04	+1.22
		1/6	+0.793	+0.889	+0.990	+1.19	+1.41
6	$+\frac{1}{2}P\frac{nD}{N(1-2k)}$	1/10	+1.77	+2.06	+2.42	+3.04	+3.70
		1/8	+1.86	+2.18	+2.56	+3.24	+3.92
		1/6	+2.04	+2.40	+2.84	+3.62	+4.38
7	$+\frac{1}{4}P\frac{D[n^2+4(1-2k)]}{nN(1-2k)}$	1/10	+1.20	+1.31	+1.45	+1.72	+2.01
		1/8	+1.24	+1.37	+1.52	+1.81	+2.13
		1/6	+1.32	+1.48	+1.65	+1.99	+2.35
8	$+\frac{1}{8}P\frac{[n^2+4(1-2k)]}{n(1-k)}$	1/10	+0.565	+0.610	+0.670	+0.785	+0.910
		1/8	+0.570	+0.620	+0.680	+0.800	+0.930
		1/6	+0.585	+0.635	+0.700	+0.830	+0.965

+ = tension. − = compression.

TABLE 25.—STRESS COEFFICIENTS—CAMBERED COMPOUND FINK TRUSS

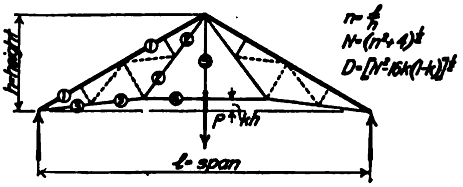


Member	General formula	k	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{2}P\,\frac{[n^2 + 4(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−2.12	−2.38	−2.68	−3.28	−3.88
		$\frac{1}{8}$	−2.22	−2.50	−2.84	−3.46	−4.12
		$\frac{1}{6}$	−2.42	−2.75	−3.13	−3.85	−4.58
2	$+\frac{1}{4}P\,\frac{D}{nN}\,\frac{[n^2 + 4(1-2k)]}{(1-2k)}$	$\frac{1}{10}$	+1.20	+1.31	+1.45	+1.72	+2.01
		$\frac{1}{8}$	+1.24	+1.37	+1.52	+1.81	+2.13
		$\frac{1}{6}$	+1.32	+1.48	+1.65	+1.99	+2.35
3	$+\frac{1}{2}P\,\frac{nD}{N(1-2k)}$	$\frac{1}{10}$	+1.77	+2.06	+2.42	+3.04	+3.70
		$\frac{1}{8}$	+1.86	+2.18	+2.56	+3.24	+3.92
		$\frac{1}{6}$	+2.04	+2.40	+2.84	+3.62	+4.38
4	$+\frac{1}{4}P\,\frac{[n^2 + 4(1-2k)]}{n(1-k)}$	$\frac{1}{10}$	+1.13	+1.22	+1.34	+1.57	+1.82
		$\frac{1}{8}$	+1.14	+1.24	+1.36	+1.60	+1.86
		$\frac{1}{6}$	+1.17	+1.27	+1.40	+1.66	+1.93

+ = tension — = compression

Stress is zero for dotted members.

TABLE 26.—STRESS COEFFICIENTS—CAMBERED COMPOUND FINK TRUSS

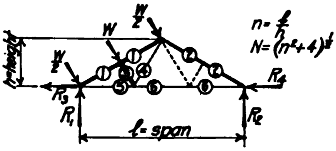


Member	General formula	k	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
1	$-\frac{1}{4}P\,\frac{[n^2 + 4(1-2k)]}{N(1-2k)}$	$\frac{1}{10}$	−1.06	−1.19	−1.34	−1.64	−1.94
		$\frac{1}{8}$	−1.11	−1.25	−1.42	−1.73	−2.06
		$\frac{1}{6}$	−1.21	−1.38	−1.57	−1.93	−2.29
2	$+\frac{1}{4}P\,\frac{nDk}{N(1-2k)(1-k)}$	$\frac{1}{10}$	+0.0980	+0.114	+0.134	+0.169	+0.206
		$\frac{1}{8}$	+0.133	+0.156	+0.183	+0.232	+0.280
		$\frac{1}{6}$	+0.204	+0.240	+0.284	+0.362	+0.438
3	$+\frac{1}{4}P\,\frac{nD}{N(1-2k)}$	$\frac{1}{10}$	+0.884	+1.03	+1.21	+1.52	+1.85
		$\frac{1}{8}$	+0.932	+1.09	+1.28	+1.62	+1.96
		$\frac{1}{6}$	+1.02	+1.20	+1.42	+1.81	+2.19
4	$+\frac{1}{4}P\,\frac{n}{(1-k)}$	$\frac{1}{10}$	+0.675	+0.780	+0.900	+1.13	+1.35
		$\frac{1}{8}$	+0.656	+0.758	+0.875	+1.09	+1.31
		$\frac{1}{6}$	+0.625	+0.722	+0.833	+1.04	+1.25
5	$+P$		+1.0	+1.0	+1.0	+1.0	+1.0

+ = tension — = compression

Stress is zero for dotted members.

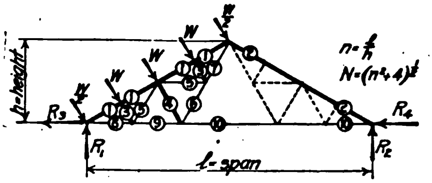
TABLE 27.—WIND STRESS COEFFICIENTS—FINK TRUSS



Case	Member	General formula	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
I, II, III and IV	1	$-\frac{1}{2}W\frac{(n^2-2)}{n}$	−1.17	−1.45	−1.75	−2.30	−2.83
	2	$-\frac{1}{4}W\frac{N^2}{n}$	−0.100	−0.0833	−0.0700	−0.054	−0.0438
	3	$-W$	−1.00	−1.00	−1.00	−1.00	−1.00
	4	$+\frac{1}{4}WN$	+0.900	+1.00	+1.12	+1.35	+1.58
	R_1	$\frac{1}{2}W\frac{(3n^2-4)}{nN}$	1.06	1.15	1.23	1.32	1.37
	R_2	$\frac{1}{2}W\frac{N}{n}$	0.600	0.578	0.559	0.539	0.526
I	5	$+\frac{1}{2}WN$	+1.80	+2.00	+2.24	+2.69	+3.16
	6	$+\frac{1}{4}WN$	+0.900	+1.00	+1.12	+1.35	+1.58
	R_3	$4\frac{W}{N}$	1.11	1.00	0.895	0.742	0.633
	R_4	0	0	0	0	0	0
II	5	$+\frac{1}{2}W\frac{(n^2-4)}{N}$	+0.694	+1.00	+1.34	+1.95	+2.53
	6	$+\frac{1}{4}W\frac{(n^2-12)}{N}$	−0.208	0	+0.224	+0.604	+0.950
	R_3	0	0	0	0	0	0
	R_4	$4\frac{W}{N}$	1.10	1.00	0.895	0.742	0.633
III	5	$+\frac{1}{2}W\frac{n^2}{N}$	+1.25	+1.50	+1.78	+2.32	+2.85
	6	$+\frac{1}{4}W\frac{(n^2-4)}{N}$	+0.347	+0.500	+0.670	+0.975	+1.265
	R_3	$2\frac{W}{N}$	0.555	0.500	0.447	0.371	0.316
	R_4	$2\frac{W}{N}$	0.555	0.500	0.447	0.371	0.316
IV	5	$+\frac{1}{2}WN\frac{(n^2-2)}{n^2}$	+1.41	+1.67	+1.96	+2.46	+2.98
	6	$+\frac{1}{4}WN\frac{(n^2-4)}{n^2}$	+0.502	+0.667	+0.837	+1.13	+1.41
	R_3	$W\frac{(3n^2-4)}{nN}$	0.708	0.667	0.616	0.526	0.458
	R_4	$W\frac{N}{n}$	0.401	0.333	0.280	0.216	0.175

+ = tension — = compression

TABLE 28.—WIND STRESS COEFFICIENTS—COMPOUND FINK TRUSS



Case	Member	General formula	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
I, II, III and IV	1	$-\frac{1}{4}W\frac{(5n^2-8)}{n}$	−3.08	−3.75	−4.50	−5.83	−7.17
	2	$-\frac{1}{2}W\frac{N^2}{n}$	−2.17	−2.31	−2.50	−2.90	−3.33
	3	$-W$	−1.00	−1.00	−1.00	−1.00	−1.00
	4	$-2W$	−2.00	−2.00	−2.00	−2.00	−2.00
	5	$+\frac{1}{4}WN$	+0.902	+1.0	+1.12	+1.35	+1.58
	6	$+\frac{1}{2}WN$	+1.80	+2.0	+2.24	+2.70	+3.16
	7	$+\frac{3}{4}WN$	+2.71	+3.0	+3.35	+4.05	+4.24
	R_1	$W\frac{(3n^2-4)}{Nn}$	2.12	2.31	2.46	2.64	2.74
	R_2	$W\frac{N}{n}$	1.20	1.15	1.12	1.08	1.05

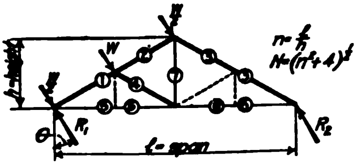
+ = tension — = compression

Stress is zero for dotted members.

Table 28 (*Continued*)

Case	Member	General formula	3	$2\sqrt{3}$	4	5	6
I	8	$+\frac{5}{4}WN$	+4.51	+5.00	+5.59	+6.73	+7.90
	9	$+WN$	+3.61	+4.00	+4.47	+5.39	+6.32
	10	$+\frac{1}{2}WN$	+1.80	+2.00	+2.24	+2.69	+3.16
	R_3	$8\frac{W}{N}$	2.22	2.00	1.79	1.49	1.27
	R_4	0	0	0	0	0	0
II	8	$+\frac{1}{4}W\frac{(5n^2 - 12)}{N}$	+2.28	+3.00	+3.80	+5.26	+6.65
	9	$+W\frac{(n^2 - 4)}{N}$	−1.39	+2.00	+2.68	+3.92	+5.06
	10	$+\frac{1}{2}W\frac{(n^2 - 12)}{N}$	−0.415	0	+0.447	+1.21	+1.90
	R_3	0	0	0	0	0	0
	R_4	$8\frac{W}{N}$	2.22	2.00	1.79	1.49	1.27
III	8	$+\frac{1}{4}W\frac{(5n^2 + 4)}{N}$	+3.40	+4.00	+4.70	+6.02	+7.28
	9	$+W\frac{n^2}{N}$	+2.49	+3.00	+3.58	+4.66	+5.70
	10	$+\frac{1}{2}W\frac{(n^2 - 4)}{N}$	+0.693	+1.00	+1.34	+1.96	+2.53
	R_3	$4\frac{W}{N}$	1.11	1.00	0.894	0.746	0.633
	R_4	$4\frac{W}{N}$	1.11	1.00	0.894	0.746	0.633
IV	8	$+\frac{1}{4}W\frac{N}{n^2}(5n^2 - 8$	+3.71	+4.33	+5.03	+6.31	7.56
	9	$+W\frac{N}{n^2}(n^2 - 2)$	+2.81	+3.34	+3.92	+4.96	+5.97
	10	$+\frac{1}{2}W\frac{N}{n^2}(n^2 - 4)$	+1.01	+1.33	+1.68	+2.25	+2.81
	R_3	$2W\frac{(3n^2 - 4)}{Nn^2}$	1.42	1.33	1.23	1.05	0.915
	R_4	$2W\frac{N}{n^2}$	0.803	0.667	0.558	0.431	0.351

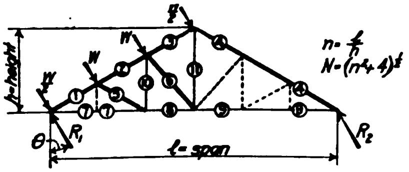
TABLE 29.—WIND STRESS COEFFICIENTS—HOWE TRUSS—4 PANELS



Case	Member	General formula	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
Case IV	1	$-\frac{1}{2}W\frac{(n^2-2)}{n}$	−1.17	−1.45	−1.75	−2.30	−2.83
	2	$-\frac{1}{4}Wn$	−0.750	−0.867	−1.00	−1.25	−1.50
	3	$-\frac{1}{4}W\frac{N^2}{n}$	−1.08	−1.16	−1.25	−1.45	−1.67
	4	$-\frac{1}{4}W\frac{N^2}{n}$	−1.08	−1.16	−1.25	−1.45	−1.67
	5	$+\frac{1}{2}W\frac{N}{n^2}(n^2-2)$	+1.41	+1.67	+1.96	+2.46	+2.98
	6	$+\frac{1}{4}W\frac{N}{n^2}(n^2-4)$	+0.502	+0.667	+0.837	+1.13	+1.41
	7	$+\frac{1}{2}W\frac{N}{n}$	+0.600	+0.575	+0.559	+0.539	+0.526
	R_1	$\frac{1}{2}W\frac{(3n^2-4)}{n^2}$	1.28	1.33	1.375	1.42	1.445
	R_2	$\frac{1}{2}W\frac{N^2}{n^2}$	0.720	0.665	0.625	0.580	0.555

+ = tension. − = compression.

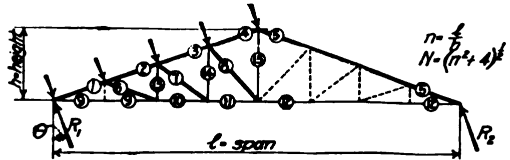
TABLE 30.—WIND STRESS COEFFICIENTS—HOWE TRUSS—6 PANELS



Case	Member	General formula	Value of n				
			3 θ = 33° − 41′	$2\sqrt{3}$ θ = 30°	4 θ = 26° − 34′	5 θ = 21° − 48′	6 θ = 18° −
Case IV	1	$-\frac{1}{8}W\frac{(7n^2-12)}{n}$	−2.12	−2.60	−3.12	−4.07	−5.00
	2	$-\frac{1}{8}W\frac{(5n^2-4)}{n}$	−1.71	−2.02	−2.38	−3.03	−3.67
	3	$-\frac{1}{8}W\frac{(3n^2+4)}{n}$	−1.29	−1.44	−1.63	−1.98	−2.34
	4	$-\frac{3}{8}W\frac{N^2}{n}$	−1.61	−1.74	−1.88	−2.18	−2.50
	5	$-\frac{1}{4}W\frac{N^2}{n}$	−1.08	−1.16	−1.25	−1.45	−1.67
	6	$-\frac{1}{4}W\frac{N}{n}(n^2+16)^{\frac{1}{2}}$	−1.50	−1.53	−1.58	−1.73	−1.90
	7	$+\frac{1}{8}W\frac{N}{n^2}(7n^2-12)$	+2.56	+3.00	+3.49	+4.38	+5.28
	8	$+\frac{1}{8}W\frac{N}{n^2}(5n^2-12)$	+1.66	+2.00	+2.37	+3.04	+3.70
	9	$+\frac{3}{8}W\frac{N}{n^2}(n^2-4)$	+0.752	+1.00	+1.26	+1.69	+2.11
	10	$+\frac{1}{2}W\frac{N}{n}$	+0.600	+0.575	+0.559	+0.539	+0.526
	11	$+W\frac{N}{n}$	+1.20	+1.15	+1.12	+1.08	+1.05
	R_1	$\frac{3}{4}W\frac{(3n^2-4)}{n^2}$	1.92	2.00	2.06	2.13	2.17
	R_2	$\frac{3}{4}W\frac{N^2}{n^2}$	1.08	1.00	0.940	0.867	0.833

+ = tension — = compression

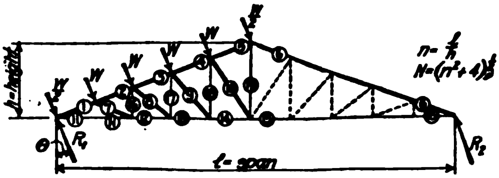
TABLE 31.—WIND STRESS COEFFICIENTS—HOWE TRUSS—8 PANELS



se	Member	General formula	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26'$
	1	$-\frac{1}{4}W\frac{(5n^2-8)}{n}$	−3.08	−3.75	−4.50	−5.83	−7.17
	2	$-W\frac{(n^2-1)}{n}$	−2.67	−2.89	−3.75	−4.80	−5.83
	3	$-\frac{3}{4}Wn$	−2.25	−2.60	−3.00	−3.75	−4.50
	4	$-\frac{1}{2}W\frac{(n^2+2)}{n}$	−1.83	−2.02	−2.25	−2.70	−3.17
	5	$-\frac{1}{2}W\frac{N^2}{n}$	−2.11	−2.32	−2.50	−2.90	−3.33
	6	$-\frac{1}{4}W\frac{N^2}{n}$	−1.08	−1.16	−1.25	−1.45	−1.67
	7	$-\frac{1}{4}W\frac{N}{n}(n^2+16)^{\frac{1}{2}}$	−1.50	−1.53	−1.58	−1.73	−1.90
	8	$-\frac{1}{4}W\frac{N}{n}(n^2+36)^{\frac{1}{2}}$	−2.02	−1.97	−2.01	−2.11	−2.24
	9	$+\frac{1}{4}W\frac{N}{n^2}(5n^2-8)$	+3.71	+4.33	+5.03.	+6.31	+7.56
	10	$+W\frac{N}{n^2}(n^2-2)$	+2.81	+3.33	+3.91	+4.95	+5.98
	11	$+\frac{1}{4}W\frac{N}{n^2}(3n^2-8)$	+1.91	+2.33	+2.79	+3.60	+4.40
	12	$+\frac{1}{2}W\frac{N}{n^2}(n^2-4)$	+1.00	+1.33	+1.68	+2.26	+2.82
	13	$+\frac{1}{2}W\frac{N}{n}$	+0.600	+0.575	+0.559	+0.539	+0.526
	14	$+W\frac{N}{n}$	+1.20	+1.15	+1.12	+1.08	+1.05
	15	$+\frac{3}{2}W\frac{N}{n}$	+1.80	+1.73	+1.68	+1.62	+1.58
	R_1	$W\frac{(3n^2-4)}{n^2}$	2.56	2.67	2.75	2.84	2.89
	R_2	$W\frac{N^2}{n^2}$	1.44	1.33	1.25	1.16	1.11

+ = tension — = compression

TABLE 32.—WIND STRESS COEFFICIENTS—HOWE TRUSS—10 PANELS



Case	Member	General formula	Value of n				
			3 $\theta = 33° - 41'$	$2\sqrt{3}$ $\theta = 30°$	4 $\theta = 26° - 34'$	5 $\theta = 21° - 48'$	6 $\theta = 18° - 26$
Case IV	1	$-\frac{1}{8}W\frac{(13n^2-20)}{n}$	−4.04	−4.91	−5.88	−7.63	−9.34
	2	$-\frac{1}{8}W\frac{(11n^2-12)}{n}$	−3.63	−4.33	−5.13	−6.57	−8.00
	3	$-\frac{1}{8}W\frac{(9n^2-4)}{n}$	−3.21	−3.75	−4.37	−5.52	−6.67
	4	$-\frac{1}{8}W\frac{(7n^2+4)}{n}$	−2.79	−3.18	−3.63	−4.47	−5.33
	5	$-\frac{1}{8}W\frac{(5n^2+12)}{n}$	−2.38	−2.60	−2.88	−3.42	−4.00
	6	$-\frac{5}{8}W\frac{N^2}{n}$	−2.71	−2.89	−3.13	−3.62	−4.17
	7	$-\frac{1}{4}W\frac{N^2}{n}$	−1.08	−1.16	−1.25	−1.45	−1.67
	8	$-\frac{1}{4}W\frac{N}{n}(n_2+16)^{\frac{1}{2}}$	−1.50	−1.53	−1.58	−1.73	−1.90
	9	$-\frac{1}{4}W\frac{N}{n}(n^2+36)^{\frac{1}{2}}$	−2.02	−1.97	−2.01	−2.11	−2.24
	10	$-\frac{1}{4}W\frac{N}{n}(n^2+64)^{\frac{1}{2}}$	−2.56	−2.51	−2.50	−2.54	−2.63
	11	$+\frac{1}{8}W\frac{N}{n^2}(13n^2-20)$	+4.88	+5.67	+6.56	+8.16	+9.80
	12	$+\frac{1}{8}W\frac{N}{n^2}(11n^2-20)$	+3.97	+4.67	+5.44	+6.83	+3.23
	13	$+\frac{1}{8}W\frac{N}{n^2}(9n^2-20)$	+3.07	+3.67	+4.33	+5.48	+6.65
	14	$+\frac{1}{8}W\frac{N}{n^2}(7n^2-20)$	+2.16	+2.67	+3.21	+4.14	+5.08
	15	$+\frac{5}{8}W\frac{N}{n^2}(n^2-4)$	+1.26	+1.67	+2.09	+2.80	+3.50
	16	$+\frac{1}{2}W\frac{N}{n}$	+0.600	+0.575	+0.559	+0.539	+0.526
	17	$+W\frac{N}{n}$	+1.20	+1.15	+1.12	+1.08	+1.05
	18	$+\frac{3}{2}W\frac{N}{n}$	+1.80	+1.73	+1.68	+1.62	+1.58
	19	$+2W\frac{N}{n}$	+2.40	+2.30	+2.24	+2.17	+2.10
	R_1	$\frac{5}{4}W\frac{(3n^2-4)}{n^2}$	3.20	3.34	3.44	3.55	3.61
	R_2	$\frac{5}{4}W\frac{N^2}{n^2}$	1.80	1.66	1.56	1.45	1.39

+ = tension — = compression

DETAILED DESIGN OF A WOODEN ROOF TRUSS

By W. S. Kinne

142. Conditions Assumed for the Design.—To illustrate the principles governing the design of a wooden roof truss, a complete design will be made of a truss of the type shown in Fig. 144 (*p*), p. 455. It will be assumed that the truss is supported on masonry walls which are 50 ft. apart, and that the trusses are spaced 16 ft. apart. The roof covering will be shingles on sheathing carried by rafters spaced 16 in. on centers. Purlins placed at the top chord panel points carry the roof loads to the truss. Fig. 157 shows the general arrangement of the roof and the trusses.

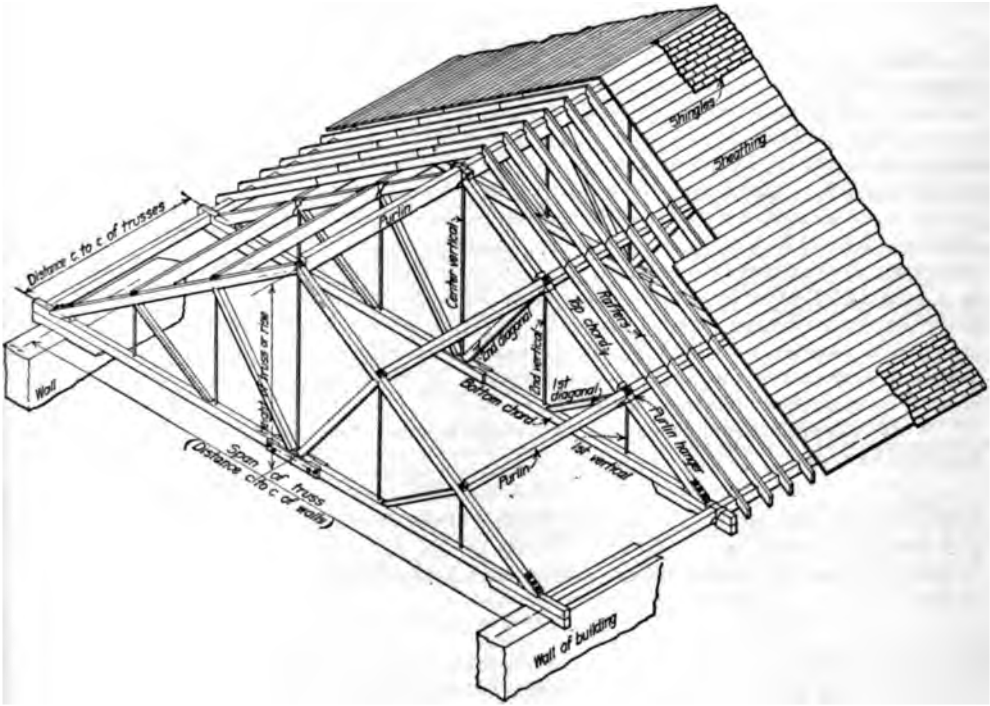


Fig. 157.—Detailed design of a wooden roof truss.

The pitch of the roof will be taken ¼, for, as stated in Art. 123, this is in general the most economical pitch. To secure members of reasonable length, the span will be divided into six panels, as shown in Fig. 158. All members will be made of wood, except the verticals, which will be steel rods. Western Hemlock will be used for all wooden truss members, and also for the purlins, rafters, and sheathing.

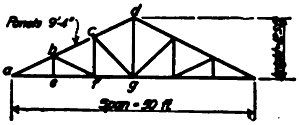


Fig. 158.

The loads to be carried by the truss will be taken in accordance with the principles stated in the chapter on Roof Trusses—General Design. Snow loads will be taken as 20 lb. per sq. ft. of roof surface, and the unit wind pressure will be taken as 30 lb. per sq. ft. of vertical surface. The unit wind pressure is to be reduced by the Duchemin formula in determining the components normal to the roof surface. Minimum snow load will be taken as one-half of the maximum, or 10 lb. per sq. ft. of roof, and the minimum wind load will be taken as one-third of the maximum.

The actual weight of the roof covering, rafters, and purlins is to be determined, assuming that Western Hemlock weighs 3 lb. per foot board measure. In estimating the weight of the truss, the formula $w = 0.04\,l + 0.000167\,l^2$ will be used, where w = weight of trusses per sq. ft. of covered area, and l = span length in feet.

Combinations of loadings for maximum fiber stresses in rafters and purlins, and for maximum stresses in truss members will be as follows:

(a) dead load and snow load.
(b) dead load, minimum snow load, and maximum wind load.
(c) dead load, maximum snow load, and minimum wind load.
(d) a minimum load of 40 lb. per sq. ft. of horizontal covered area. The object of this last loading condition is to make certain that a fairly rigid and substantial structure is obtained.

Working stresses for Western Hemlock will be taken as recommended by the American Railway Engineering Association. These values are given in Sec. 7, Art. 10. For timber used in building construction, the working stresses given in the above mentioned table are as follows: extreme fiber stress in tension or cross bending, 1650 lb. per sq. in.; shearing parallel to the grain, 240 lb. per sq. in.; shearing transverse to the grain, 150 lb. per sq. in.; compression—bearing parallel to the fibers, 1800 lb. per sq. in., bearing perpendicular to the fibers, 330 lb. per sq. in., columns under 15 diameters, 1350 lb. per sq. in., columns over 15 diameters in length, 1800 $(1 - l/60\,d)$ lb. per sq. in., where l = length of column in inches and d = least side or diameter. Bearing pressures for washers which cover only a part of the area of the member can be increased 25%—that is, to 412.5 lb. per sq. in. for bearing perpendicular to the fibers, and 2250 lb. per sq. in. for bearing parallel to the fibers. This increase in fiber stresses is allowable, for experiments have shown that the bearing pressures are indirectly distributed to the area immediately surrounding the washer, thus increasing its effective area. The allowable bearing pressure on masonry will be taken as 300 lb. per sq. in.

Where the compression acts at an angle to the member, the working stress is given by the empirical formula

$$r = q + (p - q)\,(\theta/90)^2$$

where r = allowable working stress at an angle θ to the axis of the member, as shown in Fig. 159; and p = bearing on end fibers = 1800 lb. per sq. in.; and q = bearing across the fibers = 330 lb. per sq. in. For these values the above formula becomes: $r = 330 + (1800 - 330)\,(\theta/90)^2$, or,

$$r = 330 + 0.1815\,\theta^2$$

Where pins or bolts bear on the end fibers of the material, as in the design of the built-up bottom chord member given in Art. 145, the allowable bearing values must be modified to fit the conditions shown in Fig. 159. The allowable bearing will be taken as $\frac{2}{3}$ of the usual end bearing value, or as 1200 lb. per sq. in. This working stress is considered as applied to the diametrical area of the pin or bolt.

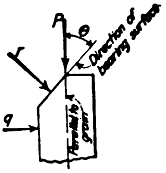


FIG. 159.

In accordance with the discussion given in the chapter on Roof Trusses—General Design, the working stresses for wind will be increased 50% over the values given above. This increase in working stresses can be accounted for by reducing the unit wind pressure so that the same working stresses can be used for all loadings. Since the working stresses for wind are $\frac{3}{2}$ of those for other loadings, if $\frac{2}{3}$ of the unit wind pressures be used, the same working stresses can be used for all loadings. The unit wind pressure on a vertical surface will then be taken as $\frac{2}{3} \times 30 = 20$ lb. per sq. ft. From the Duchemin formula, the normal pressure on a $\frac{1}{4}$ pitch roof is 14.9 lb. per sq. ft. of roof surface.

In choosing the sections of timber with which to form the members of the truss, it must be remembered that the actual size of a piece of timber should be used in the calculations. The dimensions usually given for timbers are the distances from center to center of saw cuts. These dimensions are known as the nominal dimensions of the piece; they are usually given in even inches, as for example, 2 × 4 in., 6 × 8 in., etc. Actually the timber is smaller than its nominal dimensions by the width of the saw cut, which is about ¼-in. thick. Thus a rough sawed piece, whose nominal dimensions are 4 × 6 in., is really only a 3¾ × 5¾-in. section. If this section is dressed, or planed on all sides, the section is about ½-in. scant all around from the nominal dimensions, or actually a 3½ × 5½-in. section is obtained instead of the 4 × 6-in. nominal section. The section obtained thus has an actual area of only about 80 %, and a section modulus of only 79% of the corresponding values for the nominal section. These percentages vary with the size of the timber.

The difference between the actual and the nominal sizes of timber is taken into account in the calculations by two different methods. In one method the unit stress is reduced by an amount depending upon the reduction in area or section modulus. This method, to be effective, requires the use of a sliding scale of corrections, which makes it rather undesirable. In another, and better method, the actual sizes are used and the working stresses taken as given above. This latter method will be used in the work to follow. It will be assumed that all material is dressed on four sides, and that the actual dimensions are about ½ in. scant of the nominal dimensions. In speaking of sections, however, the nominal dimensions will be used.

The working stress for steel tension rods will be taken as 16,000 lb. per sq. in. on the net section of the rod at the root of thread. In general, round rods will be used. They will be upset at the ends if the diameter required is greater than ¾-in. Bending stresses in steel bolts will be taken as 24,000 lb. per sq. in.

143. Design of Sheathing, Rafters, and Purlins.—In the chapter on the Design of Purlins for Sloping Roofs, Sect. 2, there is given a complete design of the sheathing, rafters, and purlins for conditions practically the same as assumed in the preceding article. Therefore, only the essential features of the design under consideration will be given. Wherever possible, reference will be made to the design mentioned above, and also to the design of the steel roof truss in the following chapter, for which similar conditions exist.

From Fig. 157 it can be seen that the span of the sheathing is 16 in., the distance center to center of rafters. As the loads are the same as for the above mentioned designs, it can readily be seen that 1-in. sheathing is satisfactory. The rafters are to be designed for the combinations of loading stated in Art. 142. As the roofing is quite rigid, it can be assumed that the load to be carried by the rafters is the component of loads perpendicular to the roof surface. It will be found that the loading of case (*b*) of Art. 142 gives the required maximum. The conditions are as shown in Fig. 160. (See also the design given in Art. 151.)

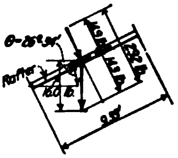


FIG. 160.

From the data given and the assumptions made in Art. 142, the minimum snow load is a vertical load of 10 lb. per sq. ft. of roof and the normal wind load is 14.9 lb. per sq. ft. of roof. Assuming that shingles weigh 3 lb. per sq. ft. of roof, and that 1-in. sheathing weighs 3 lb. per ft. board measure, it will be found from the force diagram of Fig. 160 that the total normal component is 29.2 lb. per sq. ft. of roof area.

From Fig. 157, the area carried by a rafter is (16/12) 9.33 = 12.4 sq. ft., and the uniformly distributed load is 29.2 × 12.4 = 363 lb. If a 2 × 4-in. rafter be assumed, whose weight at 3 lb. per ft. board measure is 3 × 9.3 × $\frac{8}{12}$ = 18.7 lb., the total uniformly distributed load is 363 + 19 = 382 lb. Assuming that the rafters are continuous over several purlins, the moment to be carried can be calculated from the formula $M = \frac{1}{10} wl = \frac{1}{10} \times 382 \times 9.33 \times 12 = 4270$ in.-lb For the working stress of 1650 lb. per sq. in., given in Art. 142, the required section modulus is 4270/1650 = 2.59 in.^{3} Assuming the dimensions of a dressed 2 × 4 to be 1⅝ × 3⅝ in., the section modulus furnished is $(bd^2)/6 = 3.02$ in.^{3} The assumed section will be adopted, as it is the smallest advisable section.

As shown in Fig. 161, each purlin supports 12 rafter loads. From the calculations given above, each rafter load is 382 lb. Fig. 161 shows the loads in position. The maximum moment occurs under the load next to the beam center. As the purlins usually span only the distance between trusses, simple beam conditions will be assumed, and $M = 2292 \times 5.5 - 382(1 + 2 + 3 + 4 + 5) = 6880$ ft.-lb. = 82,500 in.-lb. Assume a 6 × 8-in. purlin section. The weight of the assumed purlin is $6 \times 8 \times \frac{3}{12} = 12$ lb. per ft., and the moment due to its weight is $M = \frac{1}{8}wl^2 = \frac{1}{8} \times 12 \times 16^2 + 12 = 4600$ in.-lb. Total moment = 82,500 + 4600 = 87,100 in.-lb. Required section modulus = 87,100/1650 = 52.8 in.^{3} Section modulus furnished by a 6 × 8-in. purlin, dressed to 5½ × 7½ in., is 51.8 in.^{3} Although the assumed section is slightly under size, it will be adopted.

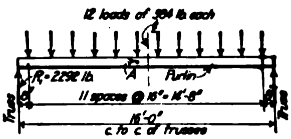


FIG. 161.

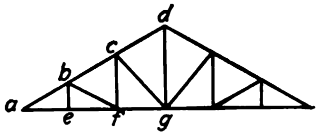
144. Determination of Stresses in Members.—The general methods of stress calculation are given in Sect. 1. Stresses can be determined by means of the graphical methods given in the above mentioned section, or by means of the tables of stress coefficients given in the chapter on Roof Trusses—Stress Data. The latter method has been used in the design under consideration. As the general methods of prócedure are given in detail in Art. 153, only the essential features are repeated here. The reader is referred to the discussion given in the following chapter, as it applies also to the design under consideration.

In Art. 142 the formula for the dead weight of the trusses is given as $w = 0.04\,l + 0.000167\,l^2$, where l = span = 50 ft., and w = weight of trusses in lb. per sq. ft. of horizontal covered area. Then $w = 0.04 \times 50 + 0.000167 \times 50^2 = 2.42$ lb. From Fig. 157, the horizontal covered area per panel is $50 \times 16/6 = 133$ sq. ft. The dead panel load due to the weight of the truss is then $2.42 \times 133 = 323$ lb. The dead load due to shingles is 3 lb. per sq. ft. of roof, and that due to the sheathing is also 3 lb., giving a total load of 6 lb. per sq. ft. of roof. From Fig. 157, the roof area per panel is $9.33 \times 16 = 149$ sq. ft. The dead panel load due to sheathing and shingles is then $149 \times 6 = 894$ lb. From Fig. 161, the weight of 12 rafters and one purlin is brought to each panel point. Each rafter weights 18.7 lb., and the purlin weighs 12 lb. per ft., as given in Art. 143. The resulting panel load is $12 \times 18.7 + 16 \times 12 = 224 + 192 = 416$ lb. The total dead panel load is then $323 + 894 + 416 = 1833$ lb.

As given in Art. 142, the snow load is 20 lb. per sq. ft. of roof, and the wind load is 14.9 lb. per sq. ft. of roof. Since the roof area per panel is 149 sq. ft., the snow panel load is a vertical load of $149 \times 20 = 2980$ lb., and the wind panel load is $14.9 \times 149 = 2220$ lb., a load which acts normal to the roof surface. In Art. 142, a minimum load of 40 lb. per sq. ft. of horizontal covered area is also specified. The panel load for this loading is $40 \times 133 = 5320$ lb., a vertical load.

The stresses due to the above panel loads are given in Table 1. Dead load stresses are given in col. 1; snow load stresses are given in col. 2; minimum, or half snow load stresses, are given in col. 3; wind stresses for wind from

TABLE 1.—STRESSES IN MEMBERS



Member	Dead load 1	Snow load 2	One-half snow load 3	Wind from left 4	Wind from right 5	One-third wind 6	D L., ½ S. L., and wind 7	D. L., ⅓ wind, and snow 8	Vertical loading 9	Maximum stress 10
ab	−10,250	−16,650	−8,325	−6,950	−4,160	−2,320	−25,525	−29,220	−29,800	**−29,800**
bc	− 8,200	−13,320	−6,660	−5,270	−4,160	−1,760	−20,130	−23,280	−23,800	**−23,800**
cd	−6,150	−10,000	−5,000	−3,610	−4,160	−1,390	−15,310	−17,540	−17,820	**−17,820**
ae-ef	+9,180	+14,900	+7,450	+7,770	+2,800	+2,590	+24,400	+26,670	+26,600	**+26,670**
fg	+7,310	+11,920	+5,960	+5,280	+2,800	+1,760	+18,580	+21,020	+21,300	**+21,300**
bf	− 2,060	− 3,340	−1,670	−2,780	0	− 930	− 6,510	− 6,430	− 5,960	**− 6,510**
cg	− 2,590	− 4,200	−2,100	−3,520	0	−1,170	− 8,210	− 7,960	− 7,510	**− 8,210**
cf	+ 916	+ 1,490	+ 745	+1,230	0	+ 410	+ 2,990	+ 2,816	+ 2,660	**+ 2,990**
dg	+ 3,670	+ 5,960	+2,980	+2,480	+2,480	+ 825	+ 9,130	+10,445	+10,640	**+10,640**
be	0	0	0	0	0	0	0	0	0	**0**

+ = *tension.* − = compression.

the left are given in col. 4, and for wind from the right, the stresses are given in col. 5; minimum, or one-third wind stresses are given in col. 6. The wind stresses are calculated on the assumption that both ends of the truss are rigidly fastened to the masonry walls, and that the reactions are parallel to the direction of the wind—that is, normal to the roof surface. The assumption of fixed ends is reasonable, for a wooden truss is not effected by temperature changes, and no provision for expansion need be made, as in the case of the steel truss.

The maximum stresses, as given by the combinations of cases (*b*), (*c*), and (*d*) of Art. 142, are given in cols. 7, 8, and 9 respectively. Stresses for col. 9 are calculated from the dead load by ratio of the panel loads for a minimum load of 40 lb. per sq. ft. of covered area, which is 5320 lb., and the dead panel load, which is 1833 lb. Col. 10 gives the greatest of these maximum values, which is the stress for which the members are to be designed.

145. Design of Members.—As stated in Art. 142, the top and bottom chord members and the diagonal web members will be made of timber, and the vertical members will be made of steel rods. The working stresses for the wooden compression members whose length exceeds 15 times the least width is given in Art. 142 as $1800\ (1 - l/60\ d)$, where l = length in inches, and d = least dimension in inches. Compression members whose length is less than 15 times the least width are to be designed for a working stress of 1350 lb. per sq. in. The working stress for wooden tension members is given as 1650 lb. per sq. in. For steel members the working stress is 16,000 lb. per sq. in. All data for the design is given in Table 2.

Sections for wooden compression members should be square, if possible, in order to secure a member of equal rigidity in planes perpendicular to the sides of the members. Single pieces are preferable to members built up of planks placed side by side and nailed or bolted together to form a single member. The excessive cost of, or difficulty in obtaining single pieces, may decide in favor of the built-up member.

Wooden tension members must contain considerable excess area in order to provide for notching at the joints. Single pieces are preferable for use as tension members. If planks are used, placed side by side to form a built-up member, considerable care must be taken in order to make certain that the proper net area is provided at all points. Further discussion of this detail will be given in connection with the design of the lower chord member.

Design of Top Chord Member.—The design of the top chord member will be determined for the conditions existing in member *a–b*, where the stress is a maximum. From Table 1 the stress in member $a-b$ is 29,800 lb. compression. Assume a 6 × 6-in. member, of which the actual size will be taken as 5½ × 5½ in. Since the length of member *a–b* is 112 in., the ratio $l/d = 112/5.5 = 20.4$. Therefore the working stress is to be determined by the formula $1800(1 - l/60\,d)$. For the assumed section the working stress is $1800\ (1 - 112/60 \times 5.5) = 1800(1 - 0.34) = 1190$ lb. per sq. in.; and the required area is $29{,}800/1190 = 25.0$ sq. in. The area provided by the assumed section is $5.5 \times 5.5 = 30.25$ sq. in. The assumed section is ample and it will be adopted.

In trusses of the size under consideration, it is usual to make the entire top chord of the same cross section. For larger trusses, the section of the upper end of the top chord is sometimes reduced in size. A butt splice is made at one of the panel points. This splice can be designed by the methods given in the chapter on Splices and Connections—Wooden Members.

If the top chord member is to be made of planks, a 2 × 6-in. piece, actual dimensions about 1⅝ × 5½ in., would probably be used in the case under consideration. To provide the proper area, three pieces will be required. For this section, $d = 3 \times 1\frac{5}{8} = 4\frac{5}{8}$ in.; $l/d = 23$; and the allowable working stress is 1120 lb. per sq. in. The area required is then $29{,}800/1120 = 26.6$ sq. in., and that provided is $3 \times 1\frac{5}{8} \times 5.5 = 26.8$ sq. in. The section is ample. To hold the several pieces together, bolts about ½ in. in diameter should be placed through the pieces at intervals such that the value of l/d for a single piece will be not greater than the value for the whole member. From the calculation given above, l/d for the whole member is 23. Since d for a single plank is 1⅝ in., the distance between bolts must be about $(23)(1\frac{5}{8}) = 37.4$ in. Bolts spaced 3 ft. apart will probably be satisfactory.

Design of Compression Web Members.—The compression diagonals *b–f* and *c–g* are designed by methods similar to those used for the top chord member. It was found that 4 × 4-in. members, actual size assumed as 3⅝ × 3⅝ in., are sufficient as far as stress conditions are concerned. It sometimes happens that the size of member as designed must be increased to provide sufficient bearing area for joint details. The actual sizes as designed are given in Table 2. If changes are required, they will be made in Art. 146 on the design of joints.

Design of Bottom Chord Tension Member.—From Table 1, the maximum stress in the bottom chord occurs in members *a–e–f*, where the stress is 26,670 lb. tension. The net area required for the allowable working stress of 1650 lb. per sq. in. is $26{,}670/1650 = 16.2$ sq.

in. In general, it will be found that in order to provide for notching at the joints, etc., the adopted section must provide an area about ⅔ greater than the required net area, or in this case, the adopted section shold provide at least 16.2 × 1⅔ = 27 sq. in. A 6 × 6-in. member, actual size 5½ × 5½ in., provides 30.25 sq. in. This section will be adopted, subject to the condition that it must provide the required net area at the joints, a point which will be definitely determined in the following article.

The lower chord member for the truss under consideration will now be designed as a built-up section. It will be assumed that 2 × 8-in. plank, actual size 1⅝ × 7½ in., are to be used. Since the rods composing the vertical members pass through the chord section, an odd number of pieces will be provided, and the center piece, which will contain the rods, will not be assumed to carry any of the chord stress. Assume a section consisting of five pieces, placed as shown in Fig. 162.

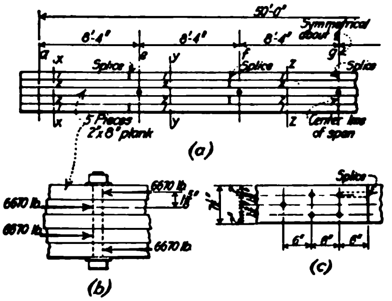


Fig. 162.

The splices in the member will be located as shown in Fig. 162; they will be placed about a foot from the panel points. For the arrangement shown, the planks can be ordered in lengths not to exceed 20 ft. It will be noted that in each panel, only two pieces are available at the splices to carry the total tension. The net area of these pieces for the member *a–e–f* must then be 26,670/1650 = 16.2 sq. in., or 8.10 sq. in. for each plank. Assuming the splices to be made with 1-in. bolts, of which there are two on the same vertical section, as shown in Fig. (*c*), the net area of a 2 × 8-in. plank is 1⅝ (7.5 − 2 × 1) = 8.95 sq. in. The assumed section is probably sufficient, as all notching for the joint at *f* can readily be made on the three inside members.

In determining the number, size, and position of the bolts connecting the several planks forming the bottom chord member, due attention must be paid to the transmission of stress across the spliced sections. Thus in Fig. 162(*a*), the total stress in member *a-e* on the section *x-x*, close to joint *a*, is carried by four planks, assuming that the center plank is inactive, as stated above. Therefore, on section *x-x* each plank has a stress of 26,670/4 = 6670 lb. At the splice just to the left of joint *e*, all of the load is carried by the planks numbered 2 in Fig. (*a*). Therefore between the sections *x-x* and joint *e*, the stresses of 6670 lb. in planks 1 have been transferred to planks 2, which are fully stressed at the splice, as calculated above.

The stress in planks 1 will be transferred to planks 2 by means of 1-in. bolts, as assumed above. The number of bolts required will be determined by the safe bearing on the end fibers of the wood, and by the safe bending stresses in the bolts. At 1200 lb. per sq. in., the safe bearing for a 1⅝-in. plank on a 1-in. bolt is 1200 × 1.625 × 1 = 1950 lb. The number required for bearing is then 6670/1950 = 3.42, or four bolts. Assuming the loading conditions on the bolts to be as shown in Fig. (*b*), the total moment to be carried by the bolts is 6670 × 1.625 = 10,820 in.-lb. From the tables of safe bending moments on bolts for a fiber stress of 24,000 lb. per sq. in., the allowable bending moment on a 1-in. bolt is 2360 in.-lb. Therefore, 10,820/2360 = 4.6, or five bolts are required for bending moment. These bolts are shown in position in Fig. 162 (*c*).

The distance from the centers of the bolts to the edge of the splice is determined by the required strength in shearing on the dotted lines shown in Fig. (*c*). Since five bolts are to be used, the load on each bolt is 6670/5 = 1335 lb. From Art. 142, the shearing value of hemlock parallel to the grain is 240 lb. per sq. in. The required distance from the center of the bolt to the edge of the plank is then 1335/2 × 1.625 × 240 = 1.72 in. The arrangement shown in Fig. 162(*c*) is convenient, and will be adopted.

At the right of the splice at joint *e*, an arrangement of bolts similar to that described above must also be used, for the stress in planks 2 must be transferred to planks 1 because of the splice in planks 2 at joint *f*. As the calculations are similar to those given above, they will not be repeated.

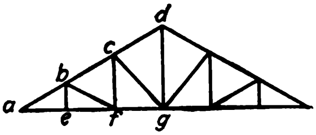
In the panel *f-g*, similar calculations must also be made. As the stresses are smaller than those in the end panels, four bolts will be found sufficient. At points between the splices, the planks are to be held together by ½-in. bolts placed about 2-ft. centers.

Design of Vertical Tension Rods.—The vertical tension members will be made of round rods threaded at the ends and provided with square nuts. As shown in Table 2, a plain ¾-in. diameter round rod provides some excess area for member *c-f*. Since this is about the smallest advisable size of rod for such members, it will be used. It is to be remembered that the area of the rod at the root of thread governs the design.

Although member *d-e* has no definite stress, a ¾-in. rod will be used.

For member *d-g* an area of 0.665 sq. in. at root of thread is required. A plain rod 1⅛ in. in diameter will furnish the required area. It will probably be better practice to use a rod of smaller diameter with an upset end. From the tables of upset ends for round rods, it will be found that a 1-in. rod with a 1⅜-in. upset end is required.

TABLE 2.—DESIGN OF MEMBERS



Member	Max. stress (lb.)	Length of member (in.)	Least width (in.)	*L/D*	Working stress (lb./in.^{2})	Area required (sq. in.)	Section	Area provided (sq. in.)
ab	−29,800	112	5½	20.4	1,190	25.0	6″ × 6″	30.25
bc	−23,800	......	....	.....	....	....	6″ × 6″	
cd	−17,820	......	....	....	....	....	6″ × 6″	
ae-ef	+26,670	......	.....	....	1,650	16.2	6″ × 6″	30.25
fg	+21,300	......	....	....	1,650	12.9	6″ × 6″	30.25
bf	− 6,510	112	3⅝	31.0	875	7.45	4″ × 4″	13.15
cg	− 8,210	141	3⅝	35.3	630	13.0	4″ × 4″	13.15
cf	+ 2,990	......	.....	.....	16,000	0.187	¾″ round rod	0.302
dg	+10,640	......	....	.....	16,000	0.665	1″ round rod upset to 1⅜″	1.05
be	0	......	....	....	....	0	¾″ round rod	0.302

+ = tension. — = compression.

146. Design of Joints.—A great variety of joint details are in use for wooden roof trusses. The general principles governing the design of joints have been given in the chapter on Roof Trusses—General Design, where typical joint details are shown. In the present article, the design methods will be given for some of the details in common use, particular attention being paid to details suitable for the type of truss under consideration.

The general principles of joint design given in the chapter on the Detailed Design of a Steel Roof Truss apply also to a wooden roof truss. Center lines of members must be made to intersect in a common point. If this can not be done, the additional stresses in the members due to the eccentric connections must be calculated and proper provision made for them.

In designing the joint details, the stresses transmitted from one member to another must be carefully determined and the bearing areas between the members proportioned to provide for the stresses to be carried. In general, simple details are desirable, and the joints should be made up with as few parts as possible. Indirect connections, and those in which the distribution of the stress to several parts is indeterminate, should be avoided. Where the stresses are small, one member can be notched into another to form the joint details. Where very large stresses are to be transmitted from one member to another, metal bearing plates or castings, side plates, or bolted connections are required. The general principles for the design of splices and similar connections are given in the chapter on Splices and Connections—Wooden Members.

Design of Joint b.—As the stress to be transmitted from member *b–f* to the top chord member is comparatively small, a notch detail of the form shown in Fig. 163 will be used. In order to make certain that the resultant pressures on the faces 1–2 and 2–3 intersect on the center line of the member at point 4, the notch will be made with faces at 90 deg., as shown in Fig. 163. In this way a central connection is made and eccentric moments are eliminated.

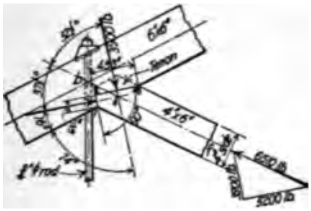


Fig. 163.

Assume a notch 1¼ in. deep on face 1–2. The dimensions and form of the resulting notch are shown in Fig. 163. These dimensions were scaled from a large scale layout of the joint. In making the layout, the actual dimensions of the members were used.

Resolving the stress in member *b–f* into its components perpendicular to the faces of the notch by means of a force diagram, the forces to be carried are as shown in Fig. 163. Since these loads act at an angle to the grain of the material, the strength of the notch depends upon the allowable bearing values on these surfaces, as determined by the formula of Art. 142, for which the conditions are shown in Fig. 159. The angles which the surfaces 1–2 and 2–3 make with the grain of the material of the chord member and of member *b–f* are as shown in Fig. 163. These angles were measured with a protractor from a large scale layout of the joint. Angles were read to the nearest half degree.

The allowable bearing values as calculated from the formula of Art. 142 are as follows:

Chord member:
surface 1–2, $330 + 0.1815(74)^2 = 1330$ lb. per sq. in.
surface 2–3, $330 + 0.1815(16)^2 = 375$ lb. per sq. in.
Member *b–f*:
surface 1–2, $330 + 0.1815(52.5)^2 = 850$ lb. per sq. in.
surface 2–3, $330 + 0.1815(37.5)^2 = 585$ lb. per sq. in.

For these allowable bearing values, the areas required are as follows:

Chord member:
surface 1–2, $5200/1330 = 3.90$ sq. in.
surface 2–3, $3900/375 = 10.4$ sq. in.
Member *b–f*:
surface 1–2, $5200/850 = 6.12$ sq. in.
surface 2–3, $3900/585 = 6.67$ sq. in.

These calculations show that the required areas are 6.12 sq. in. for surface 1–2, and 10.4 sq. in. for surface 2–3.

As the notch 1–2 is assumed to be 1¼ in. deep, the width required on this surface is $6.12/1.25 = 4.90$ in. From the design given in Art. 145, a 4 × 4-in. member is sufficient for member *b–f* as far as the column design is concerned. This member, however, does not provide the required width on surface 1–2, as given by the above calculations. The required area can be provided by one of two methods; either the notch can be made deeper, or the member can be made wider. As designed in Art. 145, the chord member is 6 in. wide and member *b–f* is 4 in. wide. It is therefore possible to increase the width of member *b–f*. In this case it does not seem advisable to make the notch deeper than assumed, because the excess area provided by the section adopted does not allow much cutting. The required area will be provided by increasing member *b–f* to a 4 × 6-in. section, actual size assumed as 3⅝ × 5½ in., placed with the 4-in. side in the plane of the truss, as shown in Fig. 163. The area provided on surface 1–2 is then $5.5 \times 1.25 = 6.875$ sq. in., which is satisfactory.

In order to prevent member *b–f* from slipping out of place due to shrinkage of the parts, it is best to provide a tenon projecting from the surface 2–3 into a slot in the chord member, as shown in Fig. 163. This tenon should be about 1 in. thick, and the slot in the chord member which receives the tenon should be about 1⅛ in wide. The net width of the surface 2–3 is then 5.5 − 1.125 = 4.375 in. From Fig. 163, the length of the surface 2–3 is 4.53 in. The area provided is then 4.53 × 4.375 = 19.8 sq. in. From the calculations given above, an area of 10.4 sq. in. is required. The detail is satisfactory and will be adopted.

Fig. 164 shows another arrangement for joint *b*. A S-shaped bent steel plate has one of its legs notched into the chord member, while the other leg forms a projection against which the member *b–f* bears. The depth of the projection 1–2 is determined by the allowable bearing on this surface, which, from the formula of Art. 142, is 330 + 0.1815(36.8)^2 = 575 lb. per sq. in. Resolving the stress in *b–f* into components parallel and perpendicular to the chord member, the loads shown in the force diagram are obtained. Therefore, the area required on surface 1–2 = 2910/575 = 4.98 sq. in. If *b–f* be taken as a 4 × 4-in. member (actual size 3⅝ in. square), the required distance 1–2 = 4.98/3.625 = 1.378 = 1⅜ in.

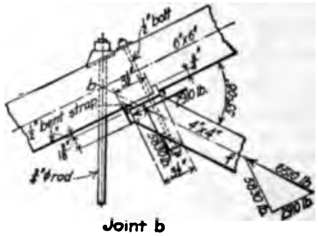


FIG. 164.

The thickness of the plate is determined by its strength as a cantilever beam of length 1⅜ in. The plate will be made the full width of the chord member, which is 5½ in. wide. Assuming the pressure to be concentrated at the center of the surface 1–2, the moment is ½ × 2910 × 1.375 = 1930 in. lb., and the thickness required for a working stress of 16,000 lb. per sq. in. is $d = (6M/fb)^{1/2}$ = (6 × 1930/16,000 × 5.5)$^{1/2}$ = 0.3625 in. A ½-in. plate will be used.

From the formula of Art. 142, the allowable bearing pressure for the 4 × 4-in. member on the surface 2–3 is 330 + 0.1815 (53.2)^2 = 840 lb. per sq. in. The bearing area required between the 4 × 4-in. member and the under side of the plate is 5830/840 = 6.95 sq. in. On the upper surface of the plate, the bearing is directly on the side of the chord member, and the allowable bearing is 330 lb. per sq. in. The bearing area required on the lower face of the chord member is 5830/330 = 17.7 sq. in. From a large scale layout of the joint, the dimensions were found to be as shown in Fig. 164. The bearing area provided between the 4 × 4-in. member and the plate is then 3½ × 3⅝ = 12.7 sq. in., and the area provided between the chord member and the plate is 5.5 × 3.5 = 19.2 sq. in., as the plate is assumed to cover the full width of the chord member.

The component of thrust parallel to the chord member is taken up by notching into the chord member. As the bearing is on the end fibers of the material, the allowable bearing is 1800 lb. per sq. in., and the area required is 2910/1800 = 1.62 sq. in. The depth of the notch required is 1.62/5.5 = 0.294 in. A ¾-in. notch will be used, for a shallower notch is not effective.

The bent plate is kept in contact with the chord member and with member *b–f* by means of lag screws, or by means of a bolt passing through the members. Fig. 164 shows the adopted detail.

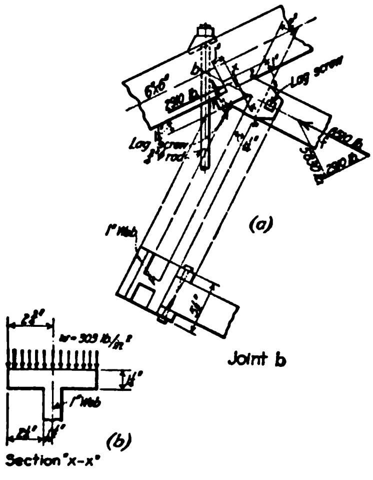


FIG. 165.

Fig. 165 shows a detail for joint *b* which makes use of a cast-iron angle block. This block is notched into the top chord by means of a lug cast on the angle block. Member *b–f* bears directly on the end of the angle block. In order to save material, and also to reduce the weight of the angle block, it will be made up of two bearing surfaces, 1–2 and 3–4, connected by a cast web.

The design of an angle block of the form shown in Fig. 165 consists in the determination of the size of the lug which notches into the top chord, and the thickness required for the cantilever beams forming the bearing surfaces 1–2 and 3–4. The force diagram shows the components of load parallel and perpendicular to the top chord member.

The depth of the lug must be sufficient to transfer to the end fibers of the top chord member a stress of 2910, as shown by the force diagram. As the allowable bearing on the end fibers of the material is 1800 lb. per sq. in., and the width of the chord member is 5½ in., the depth of notch required is only 2910/1800 × 5.5 = 0.294 in. As the required notch is too shallow to be effective, a 1-in. notch will be used. The width of the lug is determined by its strength as a cantilever beam under a moment of 2910 × 0.5 = 1455 in.-lb If the working stress for cast iron is taken as 3000 lb. per sq. in., the width required is $(6M/bf)^{1/2}$ = (6 × 1455/5.5 × 3000) = 0.727 in. A width of 1 in. will be adopted. The details of the lug are as shown in Fig. 165.

The area required on the surface 1–2 is determined by the bearing strength of the timber across the fibers, which is 330 lb. per sq. in. From the force diagram, the load to be transmitted to the chord member is 5830 lb. The area required is then 5830/330 = 17.7 sq. in. If it be assumed that the top surface of the lug does not carry compression due to imperfect workmanship, the area provided on surface 1–2 is (4.5 − 1.0) 5.5 = [illegible] in., which is ample.

The thickness of the upper bearing surface is determined by the necessary thickness [illegible] cantilever beam. Fig. (*b*) shows a vertical section *x–x* of Fig. (*a*) = 303 lb. per sq. in., acting as shown in Fig. (*b*). For the ca beam 1 in. wide is ½ × 303 × 2.25^2 = 765 in.-lb. at the edge of of 3000 lb. per sq. in. for cast iron, the required thickness is $(6M/bf)^{1/2} = (6 \times 765/3000)^{1/2} = 1.24$ in. The section will be made 1¼ in. thick.

By a similar process it will be found that the thickness of the bearing surface 3–4 can also be made 1¼ in. thick. The angle block will be fastened to the chord member by means of lag screws. To hold the member *b–f* in place, side pieces will be cast on the lower bearing surface. Lag screws through the projections thus formed will hold the member rigidly in position. All details are shown in Fig. 165.

Member *b–e*, the vertical tension rod, passes through the chord member and bears on the chord by means of a cast washer. As member *b–e* has no definite stress, a washer similar to the one designed for joint *c* will be used. Fig. 166 (*c*) shows the details of the washer.

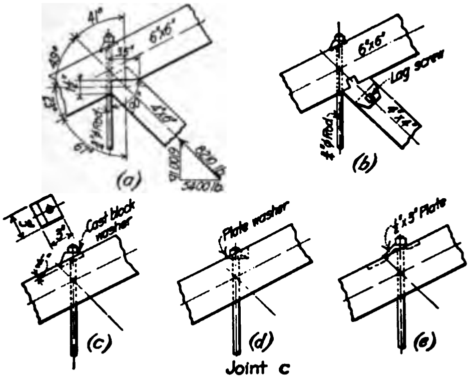


FIG. 166.

Design of Joint c.—Fig. 166 shows two designs for joint *c*. The design methods are similar to those used for joint *b*. Fig. (*a*) shows a joint made by notching, and Fig. (*b*) shows an angle block design. Due to the angle between member *c–g* and the top chord member, a solid block was used in this case.

The vertical rod *c–f* transmits to the upper chord its stress of 2990 lb. This load is brought to the top of the chord member by a washer. In this case a cast angle washer will be used, as shown in Fig. 166 (*c*). The design of this washer consists in providing a base area sufficient to transmit to the top fibers of the chord member, a stress of 2680 lb., the component of stress perpendicular to the chord member, and in providing an area at the toe of the washer which will provide for a load of 1340 lb., the component of stress parallel to the chord member. The stresses to be carried were determined from the force diagram of Fig. (*c*).

As stated in Art. 142, the bearing under washers which bear perpendicular to the grain is 412.5 lb. per sq. in. The area required on surface 1–2 of Fig. (*c*) is then 2680/412.5 = 6.5 sq. in. Since the rod composing member *c–f* is ¾ in. in diameter, the hole in the washer should be about 1 in. in diameter. As the hole in the base of the washer is elliptical in form, the area will be taken as 1.5 sq. in. The required gross area of the base is then 6.5 + 1.5 = 8.0 sq. in. A 3 × 3 in. base will be used.

To resist the component of load parallel to the chord member, the washer will be set into the chord member. As the allowable end bearing on the fibers is 1800 lb. per sq. in., and as the washer is 3 in. wide, the indentation must be at least 1340/1800 × 3 = 0.25 in. A ½-in. indentation will be used, as shown in Fig. (*c*).

Other forms of washer details in common use for sloping chords are shown in Figs. (*d*) and (*e*). In the form shown in Fig. (*d*), the top chord is notched to form a horizontal surface. A round or square washer is then used whose base area is determined for the allowable bearing, as calculated from the formula of Art. 142. Fig. (*e*) shows a bent plate washer. The design of this detail is similar to the one shown in Fig. (*c*).

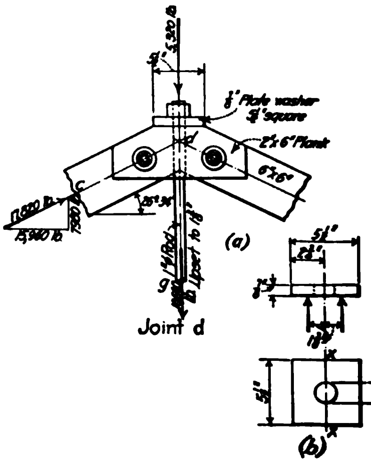


FIG. 167.

Design of Joint d.—Joint *d*, the apex joint, is a butt joint in which the members intersect at an angle. The design of this joint consists in providing the proper area between the abutting surfaces, and the provision of proper bearing under the washer on the vertical member *d–g*. Rigid fastenings are to be provided in order to hold the members in line.

Fig. 167 shows a detail of the apex joint in which the top chord members from the two sides of the truss butt against each other on a vertical line and against a plate washer on the end of member *d–g*, the vertical rod. The maximum stress in member *c–d* is 17,820 lb., as given in Table 1. This stress is due to the vertical loading of 40 lb. per sq. ft. of covered area, for which the panel load is 5320 lb. The stresses in all members, and the panel load, are shown in position.

The details of the joint depend on the method of supporting the purlin at this point. If the purlin is set on the top of the washer, the bearing area on the under side of the washer must be determined for the vertical components of the stresses in the chord members. From the force diagram, the load to be carried is 2 × 7980 = 15,960 lb. If a detail of the form shown in Fig. 178 (*b*) is adopted, where the purlin load is distributed equally to the two chord members, the load to be provided for on the under side of washer is 15,960 – 5320 = 10,640 lb., which is equal to the stress in the vertical rod. The latter detail will be adopted in this case, as shown on the general drawing, Fig. 179.

From the formula of Art. 142, the allowable bearing on the under side of the washer is $330 + 0.1815\,(26.5)^2 = 460$ lb. per sq. in., and that on the vertical bearing surface is $330 + 0.1815\,(63.5)^2 = 1060$ lb. per sq. in. The area required on the under side of the washer is then 10,640/460 = 23.1 sq. in., and on the vertical bearing surface the area required is 15,960/1060 = 15.1 sq. in Assuming the plate washer to cover the full width of the chord member, the length required is 23.1/5.5 = 4.2 in. To allow for the area taken out for the vertical rod, a 5½-in. square steel plate will be used, as shown in Fig. 167 (*a*). If the horisontial bearing area for each chord member is made 2¾ in., a layout of the joint will show that the vertical bearing surface is about 4¾ in. The area provided on the vertical bearing surface is then 4.75 × 5.5 = 26.13 sq. in., which is more than required.

The thickness of the plate washer will be determined on the assumption that it forms a double cantilever beam. Fig. (*b*) shows the assumed distribution of loading, which is approximate but accurate enough under the conditions. The moment to be carried on section *x–x* is 5320 × 1,375 = 7,315 in.-lb. For an assumed working stress of 16,000 lb. per sq. in., the thickness required is $d = (6M/bf)^{1/2} = (6 \times 7315/4 \times 16{,}000)^{1/2} = 0.83$ in. A ⅞-in. plate will be used. As shown in Fig. (*b*), a 1½-in. hole is provided in the washer for the vertical member, which leaves a net width on section *x–x* of b = 55 – 1.5 = 4.0 in.

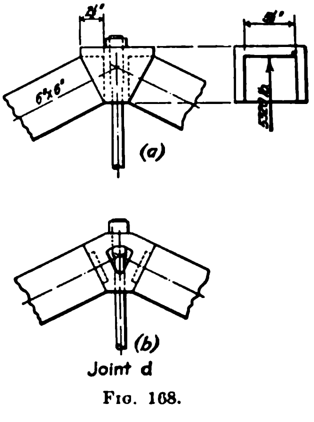


FIG. 168.

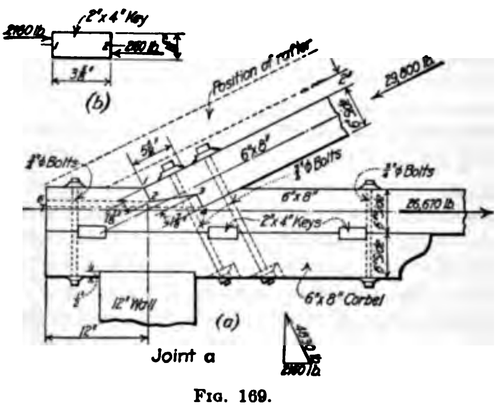


FIG. 169.

To hold the chord members in place, short pieces of 2 × 6-in. plank are fastened to the faces of the chord members by means of ¾-in. bolts. These pieces do not carry any definite stress.

Fig. 168 shows two forms of cast-iron block details for the joint at point *d* In the design of Fig. (*a*), the bearing surfaces required are determined by the same methods as used in the design of Fig. 167. The required thickness of metal can be determined by considering the upper surface to be a fixed ended beam supported by the side surfaces. The details shown in Fig. 168 are more expensive than the one shown in Fig. 167. It is doubtful if the added expense is worth while, for the detail of Fig. 167 is simple, effective, and inexpensive.

Design of Joint a.—The design of the joint at *a*, the heel of the truss, requires careful consideration. At this point the stresses to be provided for are greater than at any other point in the truss. In general the members meet at an acute angle, which adds to the difficulties encountered in the design. Designs will be worked out in detail for a joint formed by notching one member into the other; for one formed by a bent strap with lugs; for a joint consisting of steel side plates; and for a cast-iron shoe.

Fig. 169 shows an arrangement for a joint at point *a* formed by notching the top chord member into the lower chord member. The notch is so arranged that the surfaces 1–2 and 3–4 provide equal areas. The connection formed between the members is central and no eccentric moments are to be provided for.

It can be seen from Fig. 169 that the bearing value at the notches is governed by the allowable values for the horizontal member. From the formula of Art. 142, the allowable bearing is $330 + 0.1815 (63.5)^2 = 1060$ lb. per sq. in. Hence the total area to be provided on surfaces 1–2 and 3–4 is $29{,}800/1060 = 28.1$ sq. in. If the notches are made 1⅞ in. deep, as shown in Fig. 169, the width of bearing required is $\frac{1}{2} \times 28.1/1.875 = 7.5$ in. From Table 2, the stress in member $a-b$ calls for a 6 × 6-in. piece, of which the actual width is 5½ in. Since it is not advisable, and in fact impossible in this case to make the notches deeper because of the reduction in the available net area of the lower chord section, the members must be made wider if this form of joint is to be used. The calculations above show that a 6 × 8-in. member, actual width 7½ in., must be used for both the top and bottom chord members. This change will be made and the other details of the design will be worked out.

The net area of the lower chord member must now be checked up. As shown in Fig. 169, the weakest section is on a vertical section through point 4, where the net area provided is $7.5 \times 3 = 22.5$ sq. in. From Table 2, the net area required for member a-e is 16.2 sq. in. The area furnished is therefore ample, provided no further cutting is required.

The loads brought to the surfaces 1–2 and 3–4 must be resisted by the shearing resistance offered by the surfaces 2–6 and 4–7. The shearing resistance developed must be equal to the horizontal component of the stress in the top chord member, which is 26,670 lb., as shown by the force diagram. Assuming that surface 2–6 carries one half of this load, the length required on surface 2–6 is $\frac{1}{2} \times 26{,}670/240 \times 7.5 = 7.4$., when the shearing working stress is 240 lb. per sq. in., as given in Art. 142. Surface 4–7 is below surface 2–6 so that it can be counted upon to act as shear resisting area. To provide some excess area due to possible defects in the material, the bottom chord member will be extended 12 in. beyond the intersection of center lines, as shown in Fig. 169. A layout of the joint will show that the lower chord member will not project outside the roof line if the purlin is placed with its lower surface on the same level as the under side of the top chord member.

The top chord member will be held in place on the lower chord member by means of bolts passing through the members, as shown in Fig. 169. These bolts do not carry any definite stress, as they serve only to hold the parts together. Two ¾-in. bolts will be used, located as shown in Fig. 169. In order to avoid further cutting of the lower chord member to provide seats for the washers at the lower ends of the ¾-in. bolts, a 6 × 8-in. timber, known as a corbel, will be bolted to the under side of the chord member, as shown in Fig. 169.

Although the ¾-in. bolts do not carry any definite stress, it is usual to assume that the probable maximum stress in the bolt is equal to its full net strength in tension. Washer details and bearing areas are then determined for this load. As the area at the root of thread for a ¾-in. bolt is 0.302 sq. in., the probable maximum bolt stress is $16{,}000 \times 0.302 = 4830$ lb. For the conditions shown in Fig. 169, the allowable bearing value under the washers is governed by the conditions under the corbel. From the formula of Art. 142, the allowable bearing value is $330 + 0.1815 (26.5)^2 = 460$ lb. per sq. in. As stated in Art. 142, this may be increased for washers which cover only a part of the area of the bearing surface. The bearing area required is then $4830/460 \times 1.25 = 8.4$ sq. in. From the table of Standard Cast Washers given on p. 246, it will be found that the standard washer for a ¾-in. bolt provides a bearing area of about 7.9 sq. in. Under the conditions, a standard washer will be used, although the area provided is somewhat deficient. If the discrepancy in area is greater than for the case under consideration, it will be best to design a special steel plate washer similar to those used at joints d, f, and g.

Since the probable bolt stresses are inclined to the axis of the corbel, keys or wedges must be inserted between the lower chord member and the corbel to prevent any movement of the parts. If three wooden keys are provided, as shown in Fig. 169, each key must take one-third of the horizontal component of the total stress in the bolts. From a force diagram, the horizontal component of the stress in the bolts is found to be $2 \times 2{,}160 = 4320$ lb. In addition to this load, the keys must also provide for the horizontal component of the reaction due to wind. From the coefficients for wind load reactions given in the chapter on Roof Trusses—Stress Data, the maximum horizontal force to be provided for is 2.06 × 2,220 × sin 26° 34′ = 2050 lb. The total to be carried by the keys is then 4320 + 2050 = 6550 lb.

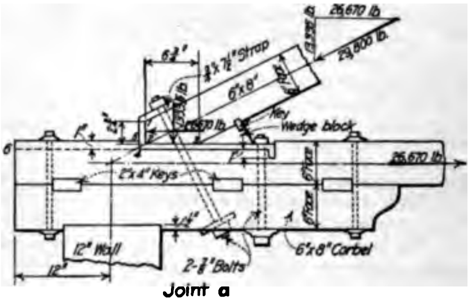
A 2 × 4-in. key, actual size 1⅝ × 3⅝ in., will be assumed. Fig. (*b*) shows the conditions for which the key is to be designed. The area required for bearing against the side fibers of each key is ⅓ × 6550/412.5 = 5.28 sq. in., assuming a working stress as for bearing under washers. The area provided by the assumed key is ½ × 1.625 × 7.5 = 6.08 sq. in., which is sufficient. The length of the key is determined by the area required to develop a shearing resistance equal to one-third of the total horizontal force to be carried, which is ⅓ × 6550 = 2183 lb. As given in Art. 142, the allowable shearing stress transverse to the grain is 150 lb. per sq. in. The area required for each key is then 2183/150 = 14.5 sq. in. As shown in Fig. (*b*) the area provided by a key on the surface 1–2 is 3.625 × 7.5 = 26.5 sq. in. The assumed key is satisfactory. To prevent the key from twisting, due to the eccentric application of the forces, a ¾-in. bolt will be placed close to each key, as shown in Fig. (*a*).

The bearing area provided between the masonry wall and the corbel is determined by the allowable bearing on the masonry, which is given in Art. 142 as 300 lb. per sq. in. From Art. 144 it will be found that the reactions at the wall are as follows: dead load, 5500 lb.; snow load, 8940 lb.; wind load, vertical component 4100 lb., horizontal component 2050 lb. The resulting reactions are then: (*a*) dead load, minimum snow load, and maximum wind load, vertical component 14,070 lb., horizontal component 2050 lb.; (*b*) dead load, maximum snow load, and minimum wind load, vertical component 14,810 lb., horizontal component 700 lb.; and (*c*) reaction due to a vertical load of 40 lb. per sq. ft. of covered area, 15,960 lb. Case (*c*) therefore determines the required bearing area, which is 15,960/300 = 53.3 sq. in. If a 12-in. wall is assumed, the arrangement shown in Fig. 169 provides a bearing area of 12 × 7.5 = 90 sq. in., which is greater than required. To prevent horizontal movement on the wall, the corbel will be notched over the wall, as shown in Fig. 169. The area required in bearing against the wall is 2050/300 = 6.83 sq. in. A 1-in. notch will provide 7.5 sq. in.

Fig. 170 shows a design made up for a bent strap with a lug notched into the lower chord. It will be assumed that all of the stress in the top chord member is transferred to the lower chord member by means of the bent strap. The bolts serve only to hold the parts together.

The bearing areas on surfaces 1–2 and 2–3 must be large enough to provide for the components of forces shown in the force diagram. From the formula of Art. 142, the allowable bearing value on the surface 1–2 is 1060 lb. per sq. in., and that on surface 2–3 is 460 lb. per sq. in. Since the fibers at the end of the top chord member are confined by the bent strap, which tends to increase the allowable bearing value, it seems reasonable to allow an increase of 25% in the working value given above. The bearing areas required are: surface 1–2, 26,700/1060 × 1.25 = 20.1 sq. in.; and surface 2–3, 13,335/460 × 1.25 = 23.2 sq. in. Since the under side of the bent strap bears directly on the side fibers of the lower chord member, the allowable bearing is 330 lb. per sq. in. If this be increased 25%, as assumed above, the area required is 13,335/330 × 1.25 = 32.4 sq. in.

In order to secure a notch of reasonable depth on line 1–2 of Fig. 170, it will be found necessary to increase the width of the chord members to 8 in., as in the case of the design of Fig. 169. A notch 2¾ in. deep will provide an area of 2.75 × 7.5 = 20.6 sq. in., which slightly exceeds the required area. On surface 2–3, an area of 6.75 × 7.5 = 50.6 sq. in. is provided, which exceeds the area required.



Joint a

FIG. 170.

The strap must be set into the chord member to a depth which will provide for the horizontal component of 26,670 lb. in bearing on the end fibers of the material. Assuming that one-half of the load is taken at the front end of the strap detail, and that the other half is taken by a lug at the rear end, the depth of notch required at each place is 26,670/2 × 1800 × 7.5 = 0.988 in. A 1-in. notch will be used, as shown in Fig. 170.

The thickness of the strap is determined by the conditions at the lug on the rear end. Considering the lug to be a cantilever beam which carries half of the horizontal component of the stress in the top chord member, and assuming that the thickness of the strap is ¾ in., the bending moment to be carried by the strap is ½ × 13,335 (1.0 + 0.75) = 11,700 in.-lb. This moment occurs on a vertical section a[illegible]t where the lug joins the horizontal portion of the strap. Assuming that the strap is made of steel [illegible]e allowable working stress is

16,000 lb. per sq. in., the required thickness is $(6M/bf)^{1/2} = (6 \times 11{,}700/7.5 \times 16{,}000)^{1/2} = 0.765$ in. A ¾-in. strap 7½ in. wide will be used, arranged as shown in Fig. 170. It is necessary also to make certain that the net area of the strap is sufficient to act as a tension member. As the tension area required is 13,335/16,000 = 0.835 sq. in., the strap furnishes excess area.

To hold the strap in place on the end of the top chord member, two ⅞-in. bolts, placed about 4 in. center to center, will be used. These bolts do not carry any definite stress, but experience has shown that the joint, to be effective, must have all of its parts held securely in position. Bolts of the size adopted will be found to be ample for trusses of the size under consideration.

The strap will be held in place on the lower chord member, partly by means of a block keyed in place, and partly by means of vertical bolts placed close to the face of the lug, as shown in Fig. 170. An exact determination of the stress in these bolts can not be made. By assuming that the moment of the stress in the bolt taken about the edge of the wedge block is equal to the moment on the lug considered as a cantilever, an approximate determination of the bolt stress can be made. On this assumption the moment of the bolt stress is 11,700 in.-lb., as calculated above. By scale from Fig. 170 the lever arm of the bolt stress about the edge of the wedge block is 1 in. The stress in the bolt is then about 11,700 lb. At 16,000 lb. per sq. in., an area of 11,700/16,000 = 0.73 sq. in. is required. Two ⅞-in. bolts will furnish the required area.

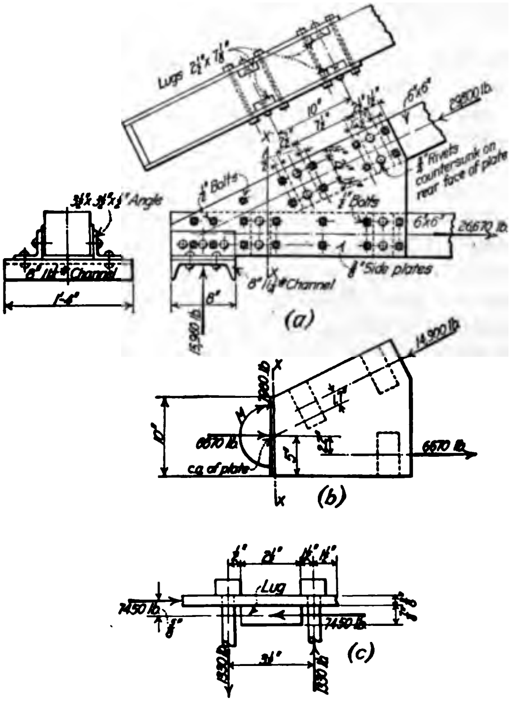


Fig. 171.

The length required on the surface 4–6 to resist in shear the load brought to surface 4–5, and all details of the corbel and keys, are calculated by the methods given for the design of Fig. 169. All details of the adopted design are shown in Fig. 170.

Fig. 171 shows a detail for joint *a* made up of structural steel plates and shapes. In this design the stresses in the top and bottom chord members are transferred to steel side plates by means of lugs riveted to the plates. The load is transferred from the side plates to the masonry walls by a shoe composed of angles riveted to a short piece of rolled channel. A detail of the form shown in Fig. 171 is especially useful for trusses in which the distance from the intersection point of the center lines of members and the end of the truss is limited, as, for example, in structures in which the walls are built up above the lower chord of the trusses. A long overhanging end detail of the form shown in Figs. 169 or 170 could not be used in such cases, for the end of the truss would project through the walls.

As shown in Fig. 171 (*a*), the stress in the top chord member is transferred to the side plates by means of four lugs. The load on each lug is then 29,800/4 = 7450 lb. Since the allowable bearing pressure on the end fibers of the material is 1800 lb. per sq. in., and since the chord member is 5.5 in. wide, the depth of notch required is 7450/1800 × 5.5 = 0.753 in. A ⅞-in. lug will be used. As the amount of cutting to provide notches on the chord members is small, the 6 × 6-in. section designed in Table 2 can be used.

The lugs will be fastened to the side plates by rivets ¾-in. in diameters. From the tables of rivet values given in the chapter on Splices and Connections—Steel Members, the value of a ¾-in. rivet in single shear is 4420 lb. Hence, 7450/4420 = 2 rivets are required in each lug, as shown in Fig. (*a*). In order to provide room for these rivets, the lugs will be made 2½ in. wide.

The distance between the lugs on the top chord member is determined by the shearing area required to resist the load on the lugs. Since the load to be carried by each lug is 7450 lb., and since the allowable shear is 240 lb. per sq. in., the area required between lugs is 7450/240 = 31.0 sq. in. As the top chord member is 5½ in. deep, the distance between the lugs must be 31.0/5.5 = 5.64 in. To allow for inequalities in material and uneven bearing on the lugs, the clear distance between lugs will be made 7½ in., as shown in Fig. (*a*). As the top chord member is in compression, the shear area must be provided to the right of the lug, or toward the apex of the truss. For the lower chord member, which is in tension, the shear area must be provided to the left of the lug—that is, between the end of the truss and the lug. The arrangement of lugs shown on Fig. (*a*) for the lower chord member provides more shear area between the lugs than is required to carry the loads. The lugs are placed as shown in order to bind the plates firmly to the chord member.

The thickness of the side plates is determined either by the limiting slenderness ratio required as a compression member at the lower end of the top chord member, or by the section required to resist the bending stresses due to the applied loads. From Fig. 171 (*a*), the maximum unsupported length of plate at the top chord member is about 8 in. If l/r is limited to 125, the minimum allowable $r = 8/125 = 0.064$ in. For a rectangle $r = 0.289\,d$. Therefore, $d = 0.064/0.289 = 0.22$ in. Since it will be necessary to countersink some of the rivets in the rear face of the plate, in order to secure a smooth face, a plate at least ⅜ in. thick must be used, as shown by the dimensions of countersunk rivet heads given in the chapter on Splices and Connections—Steel Members.

Fig. 171 (*b*) shows the forces acting on one of the side plates at a section where the depth of plate is 10 in. The forces shown on section *x-x* represent the internal stresses. These forces are a shear of 7980 lb., a thrust of 6670 lb., and a bending moment about the center of gravity of the section of 14,900 × 1.7 + 6670 × 2.2 = 50,000 in.-lb. The extreme fiber stress, which is compressive, occurs at the upper edge of the plate. The fiber stress is to be calculated from the formula given in Art. 100 for bending and direct stress, from which $f = P/A + Mc/I = 6670/10 \times 0.375 + 6 \times 50{,}000/0.375 \times 10^2 = 1780 + 8000 = 9780$ lb. per sq. in. The effect of shear can be neglected, as in the case of ordinary beam design. Other sections were investigated, but fiber stress at section *x-x* was found to be a maximum. Since the fiber stress found above is well within allowable limits, the ⅜-in. plate will be adopted.

The side plates are held in place against the chord members by means of bolts placed as shown in Fig. (*a*). Fig. (*c*) shows the forces acting on one of the lugs at the compression chord. These forces tend to cause a clockwise rotation of the lug. This rotation is resisted by bending in the side plates, by tension in bolt 1, and by compression on the side fibers of the timber at bolt 2. Neglecting the effect of the bending of the side plate, and assuming that the compression is concentrated at the bolt, the resisting forces are found to be 7450 × 0.625/3.5 = 1330 lb. Fig. (*c*) shows the conditions on which this equation is based. To carry this stress, ½-in. bolts will be used, arranged as shown in Fig. (*a*). At bolt 2 the side plate presses against the chord member with a force of 1330 lb. If the allowable bearing on the side of the chord member be assumed to be the same as for washers, the width of bearing required is 1330/412.5 × 5.5 = 0.6 in. As the side plate extends 1½ in. beyond the lug, proper provision has been made for the compression at this place. The lugs on the lower chord member are subjected to similar conditions. Fig. (*a*) shows the adopted arrangement of lugs and bolts.

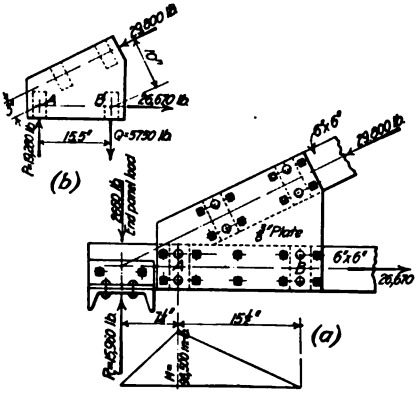


FIG. 172.

The details of the shoe are as shown in Fig. (*a*). Short pieces of 3½ × 3½ × ½-in. angle are riveted to the side plates. As the maximum vertical reaction is 15,960 lb., and the rivets are in single shear, 15,960/4420 = 4 rivets are required. In Fig. (*a*) six rivets are shown in place. The sole plate is formed by an 8-in. 11.25-lb. channel. The flanges of the channel are placed downward and provide resistance against horizontal motion, taking the place of the notch used in the design of Fig. 169.

A modified form of the joint of Fig. 171 is shown in Fig. 172. In this design the side plates do not extend far enough along the lower chord member to include the shoe, which is fastened directly to the chord member. The stresses in the chord members are transferred to the side plates from which the combined loads are transferred back to the lower chord member and thence to the wall through the shoe. This arrangement causes a bending moment at the end of the lower chord member, and also causes vertical forces to be sent up which must be resisted by the bolts at *A* and *B* of Fig. 172 (*a*). From Fig. (*a*), the moment in the chord members is (15,960 − 2660) 7.25 = 96,500 in.-lb. Fig. (*b*) shows the side plates removed with all forces in position. To hold the plate in equilibrium under the action of the stresses in the chord members, forces *P* and *Q* must act as shown. These forces can be determined subject to the conditions that moments about any point outside of the plate must be zero, and that *P*–*Q* is equal to the vertical component of the top chord stress. Fig. (*b*) shows the resulting values.

The design of this form of joint will not be carried beyond this point. Design method for the determination of the sizes of bolts required at *A* and *B* are given in the chapter on Splices and Connections—Wooden Members. The fiber stresses in the chord member can be determined by the methods given for the design of wooden beams.

The arrangement of Fig. 171 is decidedly better than the one of Fig. 172; the former detail is therefore recommended, as the latter detail leads to very heavy bending and bolt stresses in the case of large structures.

Fig. 173 shows a design for joint *a* in which a cast shoe is used. The horizontal component of the top chord stress, which is 26,670 lb., is transferred to the bottom chord member by means of lugs set into the lower chord. The vertical component of the top chord stress is transferred to the lower chord member in bearing on its upper fibers. It is the usual practice in the design of a shoe of the form shown in Fig. 173 to assume that the bearing on surface 2–4 is uniformly distributed over the area of contact between the shoe and the chord member. This assumption holds true only when ΣV, the vertical component of the top chord stress, is applied at the center of the bearing area on the chord member. In the case under consideration, which is shown in Fig. 173, ΣV intersects the surface 2–4 at a point 2.8 in. from its center. The maximum bearing pressure therefore occurs at point 2. At other points the bearing pressures are smaller than at 2, while at point 4 the direction of pressure is upward. This upward pressure must be resisted by a bolt, for upward pressures in such details can not be resisted directly by the surface 2–4. The principles of design are similar to those outlined for the design of the column footings given in the chapter on the Detailed Design of a Roof Truss with Knee-braces.

As shown in Fig. 173, the top chord member bears directly on a flat base 1 in. thick which is supported by two webs, one on each side of the casting. This base can be designed as a beam fixed at the ends by the side web plates. The adopted thickness of base is somewhat greater than required by the stresses. It was made 1⅛ in. thick in order to secure a rigid connection at this point. The top chord member is held in place on the shoe by two side plates, and by means of a short lug set into the end of the member. In this design the 6 × 6-in. pieces called for in the design given in Table 2 can be used, as the bearing area on the end of the chord member and the net area required for the lower chord member are furnished by the arrangement shown.

The vertical lug on the rear end of the shoe is made twice as deep as the one at the front end, as shown in Fig. 173. This is done in order to reduce the required shear resisting area in front of the shoe. Assuming that the rear lug takes ⅔ of the horizontal force and that the front lug takes the balance, the load at the front lug is ⅓ × 26,670 = 8890, and the load at the rear lug is 17,780 lb. Since the allowable bearing on the end fibers of the material is 1800 lb. per sq. in., and the chord member is 5½ in. wide, the depth required for the front lug is 8890/1800 × 5.5 = 0.898 in., and for the rear lug, a depth of 17,780/1800 × 5.5 = 1.80 in. is required. The front lug will be made 1 in. deep, and the rear lug will be made 2 in. deep, as shown in Fig. 173 (*a*).

The position of ΣV, the vertical component of the top chord stress, can be determined as soon as the depth of the lugs is fixed. As shown in Fig. (*a*), ΣH and ΣV intersect on the center line of the top chord member. To locate the line of action of ΣH, take moments about surface 2–4, from which

$$x = \frac{8{,}890 \times 0.5 + 17{,}780 \times 1}{8{,}890 + 17{,}780} = 0.833 \text{ in.}$$

Having given the line of action of ΣH, the position of ΣV can be determined by a layout of the joint, from which it will be found that ΣV lies 3.8 in. from the intersection of the center lines, as shown in Fig. (*a*).

The distance from the front lug to the end of the chord member is determined by the length required to develop a shearing resistance of 8890 lb. For a working shear stress of 240 lb. per sq. in., the distance required is 8890/5.5 × 240 = 6.74 in. The length provided furnishes some excess area. Since the shearing area required for the rear lug is twice as great as that for the front lug, the adopted dimensions provide excess area. As the shear area for the rear lug is below that for the front lug, the entire distance from the rear lug to the end of the chord member can be counted on as shear area if necessary.

The thickness of the lugs is determined by their strength as simple cantilever beams. It will be found best to make the casting either of cast steel, or of malleable cast iron. For these materials the fiber stress in bending can be taken as 7500 lb. per sq. in. If ordinary cast iron is used, for which the allowable bending stress is about 3000 lb. per sq. in., very wide lugs would be required, resulting in a heavy, awkward casting. The stronger material will therefore be used.

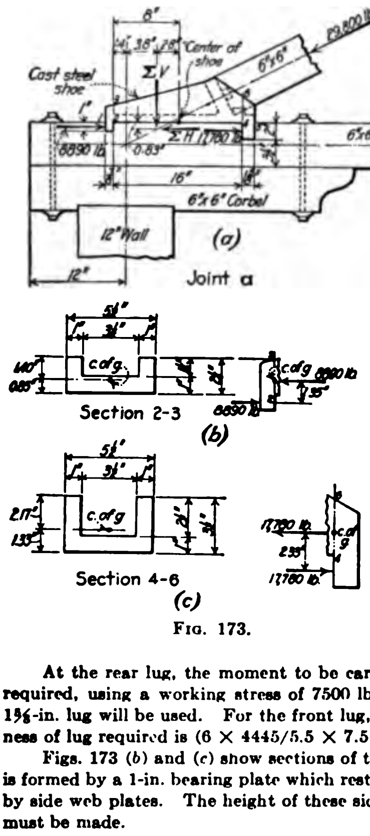


FIG. 173.

At the rear lug, the moment to be carried on the surface 4–5 is 17,780 × 1 = 17,780 in.-lb. The thickness required, using a working stress of 7500 lb. per sq. in., is $(6M/bf)^{1/2} = (6 \times 17{,}780/5.5 \times 7500)^{1/2} = 1.61$ in. A 1⅝-in. lug will be used. For the front lug, the moment to be carried is 8890 × 0.5 = 4445 in.-lb., and the thickness of lug required is $(6 \times 4445/5.5 \times 7.5)^{1/2} = 0.805$ in. A ⅞-in. lug will be used.

Figs. 173 (*b*) and (*c*) show sections of the body of the shoe. As shown by these sections, the body of the shoe is formed by a 1-in. bearing plate which rests directly on the lower chord member. This base plate is strengthened by side web plates. The height of these side web plates is varied to suit the stress conditions for which provision must be made.

Fig. (*b*) shows the conditions which determine the size of the body of the shoe on section 2–3, close to the front lug. The thickness of the bed plate can be determined by assuming that it acts as a simple beam supported by the side webs. Neglecting the supporting effect of the lug, and assuming that the load to be carried is equal to the maximum allowable bearing value of the timber, which is 330 lb. per sq. in., and that the span of the bed plate is the distance between the centers of the vertical web plates, we have for a 1-in. strip, a moment of $M = \frac{1}{8}wl^2 = \frac{1}{8} \times 330 \times 4.5^2 = 835$ in.-lb. For a fiber stress of 7500 lb. per sq. in., as assumed above, the required thickness of base plate is $d = (6M/bf)^{1/2} = (6 \times 835/7500 \times 1)^{1/2} = 0.818$ in. A 1-in. base plate will be used.

The depth of the side webs must be great enough to provide for the stresses due to the loading conditions shown in Fig. (*b*). From this sketch it can be seen that section 2–3 is subjected to a thrust of 8890 lb., and a moment of 8890 (0.85 + 0.5) = 12,130 in.-lb. This force and moment act at the center of gravity of the section, which can be located by the methods explained in Sect. 1. As this is a case of combined stresses, the formula $f = P/A \pm Mc/I$ will be used. This formula is derived and its application explained in the chapter on Bending and Direct Stress. For the conditions shown in Fig. (*b*), the fiber stress at point 2 is $f_2 = P/A + Mc/I = 8890/8 + 12{,}130 \times 0.85/2.99 = 4560$ lb. per sq. in. (comp.) and at point 3 the fiber stress is $f_3 = P/A - Mc/I = 8890/8 - 12{,}130 \times 1.40/2.99 = 4690$ lb. per sq. in. (tens.). Fig. (*c*) shows a section at 4–6, near the rear lug. For the forces and dimensions shown it will be found, by the same methods as used for section 2-3, that the fiber stress at point 4 is 6240 lb. per sq. in. compressive, and that at point 6 is 5740 lb. per sq. in., tensile. As all of these fiber stresses are within the allowable value of 7500 lb. per sq. in., the sections will be adopted.

The length of the bearing surface between the shoe and the chord member—that is, surface 2-4 of Fig. (a)—is determined by out-and-try methods. If possible, the shoe should be located so that the vertical component of the top chord stress, shown by ΣV in Fig. (a), acts at the center of the bearing surface 2-4. When this can be done, the bearing pressure over the surface 2-4 is uniform. In the truss under consideration, the angle between the chord members is small and a shoe arranged as described above would not be as compact as desired. It will be necessary, in order to secure a well proportioned shoe, to place the center of the bearing surface behind the line of action of ΣV. This will result in an uneven distribution of the bearing pressure between the shoe and the chord member. As there will probably be upward pressures near point 4, a bolt will be provided to resist the total upward force. The distance between the top chord seat and the rear lug will be made just sufficient to allow a ¾-in. bolt to be inserted, as shown in Fig. (a).

A length of bearing on line 2-4 of 16 in. will be assumed. The bearing stress on this area can be determined by the methods given in Art. 165. From eq. (3) of the article mentioned, with $P = \Sigma V = 13{,}335$ lb.; $b = 5.5$ in.; $d = 16$ in.; and $e = 2.8$ in.; we have $p_2 = P/bd\ (1 + 6e/d) = (13{,}335/5.5 \times 16)(1 + 6 \times 2.8/16) = 151.5\ (1 + 1.05) = 310$ lb. per sq. in. Since this bearing value is less than the allowable of 330 lb. per sq. in., the assumed length is sufficient.

Since the term $6e/d$ in the above equation is greater than unity, it is evident that tension exists at point 4, although, as indicated by the low value of the term $(1 - 6e/d)$, this tension is very small. From eq. (5) of the article mentioned above, the total tension in the bolt at the rear lug is $T = Pd/24e\ (6e/d - 1)^2 = (13{,}335 \times 16/24 \times 2.8)(6 \times 2.8/16 - 1)^2 = 7.95$ lb. The ¾-in. bolt is much too large, but it will be used.

A corbel similar in form to the one shown in Fig. 169 will be used with the design under consideration. All details of the casting and the corbel are as shown in Fig. 173 (a).

Design of Joint f.—Joint details for point *f* can be arranged as described for joint *b*. Fig. 174 shows three forms of joint details for joint *f*. Fig. (a) shows a design for notching, Fig. (b) shows a bent strap design, and Fig. (c) shows a cast-iron shoe. A plate washer is shown on the lower end of the vertical *c-f*. This washer is designed by the methods used for the washer at joint *d* and shown in Fig. 167.

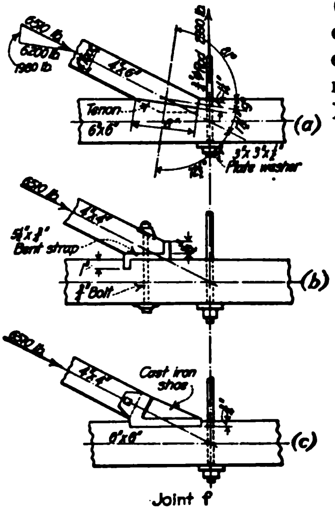


Fig. 174.

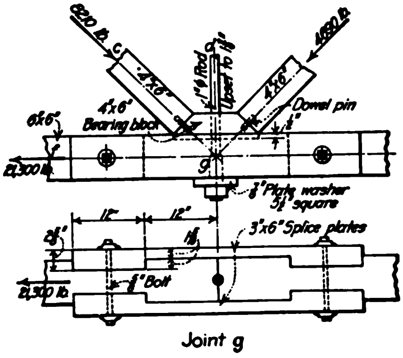


Fig. 175.

Design of Joint g.—The lower chord of a wooden roof truss is usually spliced at the center point, which, in the truss under consideration, is joint *g*. Two designs will be given in detail for the tension splice required at this point. One design will be worked out for a tabled fish plate splice constructed entirely of wood, and another will be worked out using steel side plates and bolts. Design methods for these two forms of splices are given in the chapter on Splices and Connections—Wooden Members.

Fig. 175 shows a tabled fish plate splice of wooden construction. This splice is composed of two wooden plates with lugs which fit into recesses cut into the sides of the lower chord member. The design of the splices consists in the determination of the net area required for the splice plates and for the recessed portions of the lower chord member; the determination of the bearing area required between the splice plate and the chord member; the determination of the shearing area required on the projecting portions of the splice plate and the chord member; and the provision of bolts to hold the splice plates in position.

Since there are two splice plates, and since the total load to be carried is 21,300 lb., the net area required in the body of each splice plate is 21,300/2 × 1650 = 6.45 sq. in. Assuming the width of the splice plate to be 5.5 in., the thickness required is 6.45/5.5 = 1.17 in. As the load on the splice plate and the chord member act directly on the end fibers of the material, the allowable bearing value is 1800 lb. per sq. in. The width of bearing required is then 21,300/2 × 5.5 × 1800 = 1.08 in. A 3 × 6-in. piece, actual dimensions $2\frac{5}{8}$ × $5\frac{1}{2}$ in., can be used as a splice plate. As shown in Fig. 175, the lugs will be made $1\frac{5}{16}$ in. deep, and the thickness of the splice plate at the center will also be made $1\frac{5}{16}$ in. This arrangement will provide ample net and bearing areas.

The length of the lugs required on the splice plates and on the end of the chord member is determined by the shearing area required to carry a load of $\frac{1}{2}$ × 21,300 = 10,650 lb. For a working shearing stress of 240 lb. per sq. in., the length of the lug required is 10,650/240 × 5.5 = 8.07 in. To provide for possible defects in the material, the lugs will be made 12 in. long, as shown in Fig. 175.

Since the load to be carried by the splice plate is applied $1\frac{5}{16}$ in. from the axis of the plate, a moment is set up which tends to rotate the lug from its seat on the chord member. The amount of this moment is 10,650 × 1.3125 = 14,000 in.-lb. To hold the lug in its seat, a bolt will be placed through the splice plate and the chord member, as shown in Fig. 175. An approximate estimate of the stress in this bolt can be made by dividing the moment calculated above by the distance from the point of contact between splice plate and chord member to the bolt, which in this case is 6 in. Neglecting the effect of the resisting moment developed by the body of the splice plate, the stress in the bolt is 14,000/6 = 2330 lb. For a working stress of 16,000 lb. per sq. in., the required area at the root of thread is 2330/16,000 = 0.147 sq. in., which is furnished by a $\frac{5}{8}$-in. bolt. Standard washers on the ends of this bolt will provide proper bearing area on the side fibers of the splice plate.

The net area of the chord members on the line of the bolt must be investigated. Since the depth of the cutting on each side of the main member is $1\frac{5}{16}$ in., as shown in Fig. 175, the net width of member is 5.5 − 2 × 1.3125 = 2.875 in. Assuming the hole for the bolt to be $\frac{3}{4}$ in. in diameter, the net depth of the chord member is 5.5 − 0.75 = 4.75 in. Hence the actual net area of the chord member is 4.75 × 2.875 = 13.65 sq. in. The net area required, as shown in Table 2, is 21,300/1650 = 12.9 sq. in. Therefore, as shown by the above calculations, the splice is sufficient in all of its details.

As shown in Fig. 175, two diagonal web members and a vertical tension rod enter joint *g*. The load in the tension rod is transferred to the chord member by means of a plate washer on the under side of the chord member. This washer is designed by the methods used for the washer at joint *d*, except that the allowable bearing pressure for the chord member at *g* is determined for the side fibers of the material, a value which is somewhat smaller than for joint *d*. However, it will be found that the two washers can be made of the same dimensions.

The two web members entering joint *g* are shown as seated on a wooden block set into the top of the chord member. Ample bearing area is provided by the arrangement shown in Fig. 175. Since the wind stress in one of the diagonals is 3520 lb., and that in the other is zero, as given in Table 1, the bearing block must be notched into the chord member in order to hold the diagonals in place. A force diagram will show that the component of the wind stress parallel to the chord member is 2380 lb. For an allowable bearing of 1800 lb. per sq. in., the bearing area required is 2480/1800 = 1.38 sq. in. If the bearing block is made the full width of the chord member, a notch 1.38/5.5 = 0.251 in. deep is required. As shown in Fig. 175, a $\frac{1}{2}$-in. notch is provided, for a shallower notch would not be effective.

Fig. 176 shows a design for joint *g* in which steel side plates and bolts are used. The design of this joint consists in the determination of the number and size of bolts; the determination of the size of the side plates; and the spacing of bolts required to maintain safte shearing stresses in the timber.

If the thickness of the side plates be assumed as $\frac{1}{4}$ in., the loading conditions for a bolt are as shown in Fig. 176 (*b*). The total moment to be carried by all of the bolts is 10,650 × $1\frac{1}{2}$ = 15,975 in.-lb. From the table of safe bending moments on pins for an allowable fiber stress of 24,000 lb. per sq. in., the safe bending moment is 2350 in.-lb. for a 1-in. bolt, and 3350 in.-lb. for a 1⅛-in. bolt. Therefore, seven 1-in. bolts, or five 1⅛-in. bolts are required. To secure a compact joint, five 1⅛-in. bolts will be used. Before this number of bolts is finally adopted, the bearing pressure exerted by the bolts on the timber and on the steel side plates must be examined. For an allowable working bearing value of 1200 lb. per sq. in. for bolts bearing on the timber, the area required for each bolt is 21,300/5 × 1200 = 3.53 sq. in. The bearing value provided by a 1⅛-in. bolt is 5.5 × 1.125 = 6.19 sq. in. For the side plates, the allowable bearing value on the steel plate is 24,000 lb. per sq. in., and the bearing area required for each bolt is 21,300/5 × 24,000 = 0.178 sq. in. The bearing area provided by two ¼-in. side plates on each bolt is 2 × 1.125 × 0.25 = 0.56 sq. in. As the assumed bolts are safe in bending and bearing, they will be adopted.

Fig. 176 (b) shows the arrangement of the bolts. Net areas on sections x–x and y–y must be investigated before this arrangement is adopted. At section x–x, the net area required is 21,300/1650 = 12.9 sq. in. Assuming that the bolts fit the holes exactly, the net area of the chord member at section x–x is (5.5 − 1.125) 5.5 = 24.1 sq. in. At section y–y, the stress in the chord member is 4/5 × 21,300 = 17,050 lb.; the net area required is 17,050/1650 = 10.32 sq. in., and the net area provided is (5.5 − 1.125 × 2) 5.5 = 17.9 sq. in. The net areas provided are therefore sufficient.

The distance between bolts, and the distance between the end of the chord member and a bolt is determined by the shear area required to develop a resistance equal to the load on a bolt. From Fig. 176 (a), the required distance between bolts for a shearing stress of 240 lb. per sq. in. is 21,300/5 × 5.5 × 2 × 240 = 1.61 in. As shown in Fig. 176 (a), the adopted bolt spacing exceeds the required spacing. The adopted spacing was used in order to avoid interference between the first set of bolts and the bearing block for the diagonal members. Six-inch spacing was adopted for the other bolts in order to secure a neat looking joint. All of the details of the bearing block for the diagonal members and washer for the vertical tension rod are the same as shown on Fig. 175.

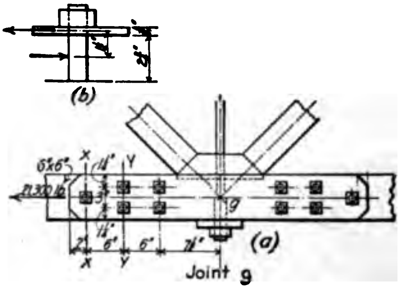


Fig. 176.

Joint Details for Trusses with Built-up Members.—In some cases truss members are made of built-up members composed of planks placed side by side and bolted together to act as a single piece, as described in Art. 145 for the top and bottom chord members of the truss under discussion in this chapter. Joint details for such members can be made up along the same lines as those given above for members composed of single sticks. In any case, it is well to provide excess bearing areas at all points in order to allow for possible defects in workmanship and in materials, due to the fact that the bearing surfaces are composed of several parts which must work together, each taking its proportion of the total load.

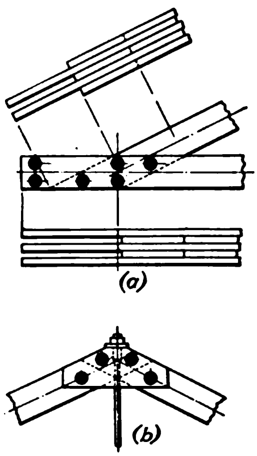
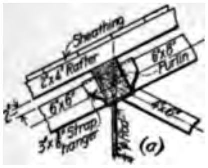


Fig. 177.

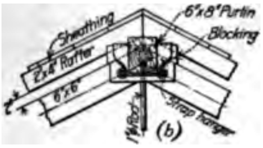
Fig. 177 shows arrangements of built-up joint details for joints a and d. In Fig. (a) is given a detail for joint a. A design is given in Art. 145 for a bottom chord member composed of five 2×8 in.-plank. A top chord section of the same size will also be used in this detail. As shown in Fig. (a), three of the top chord plank and two of the lower chord plank are cut away, and the remaining pieces are fitted together to form a joint. The parts are held together by means of bolts which can be designed by the methods given in the chapter on Splices and Connections—Wooden Members. Fig. (b) shows a form of joint for the apex of the truss.

Details of Purlin Connections.—In Art. 127 there is given a general description of the forms of purlin connections in general use. For the truss under consideration, a strap hanger of the form shown in Fig. 146 (b) of the above-mentioned article will be used. Standard sizes of strap hangers are given in trade catalogues, from which it will be found that a 3× ⅜-in. strap is required for a 6 × 8-in. purlin.

It will be assumed that the purlin is to be placed with its lower edge on the same level as the lower face of the top chord member. Since the purlin as designed in Art. 144 is a 6 × 8-in. section, actual depth 7½ in., and the top chord member, as designed in Table 2 of Art. 145, is a 6 × 6-in. section, actual depth 5 ½ in., the purlin projects 2 in. beyond the top of the chord member, as shown in Fig. 178 (a). The 3 × ⅜-in. strap hanger is held in position on the chord member by lag screws. In locating the purlin at joint *b*, it is desirable that the purlin be placed with its center at the intersection of the center lines of the truss members. It may not be possible in all cases to do this, because of interference between the washer and the strap hanger. The purlin will be placed as close to the desired position as the conditions will permit.



Joint *b*



Joint *d*

FIG. 178.

Fig. 178 (*b*) shows a detail for joint *d*, the apex of the truss. A single purlin of the same size as for joint *b* is used at joint *d*. The purlin at *d* is placed in a vertical position and is held in place by a strap hanger which is supported by blocks fastened to the chord member by means of lag screws.

The designs for joint *a* shown in Figs. 169 to 173 can be arranged without the use of a purlin. In place of a purlin the masonry can be built up between the trusses, and a wall plate provided on which the rafters are seated. If a purlin is desired at this point, a detail can be used of the form shown in Fig. 146(*d*), p. 459.

147. General Drawing and Estimated Weight.—In Fig. 179 there is shown a general drawing of the truss designed in the preceding articles. It will be noted that the joints shown on this drawing are made by notching one member into another, and that the structure is practically an all-wood construction. These details were shown because they are of the type generally used for wooden trusses, and because they are readily designed, easily constructed, and a thoroughly practical, reliable structure is obtained, when such details are used.

An approximate estimate of weight will be made for the truss shown on Fig. 179 in order to check up on the dead weight estimated by the formula of Art. 142 and used in the calculation of stresses in Art. 145. In estimating weights, it was assumed that Western Hemlock weighs 3 lb. per foot board measure, and that steel and cast iron weigh 490 lb. per cu. ft. Weights of steel rods were taken from the steel handbooks.

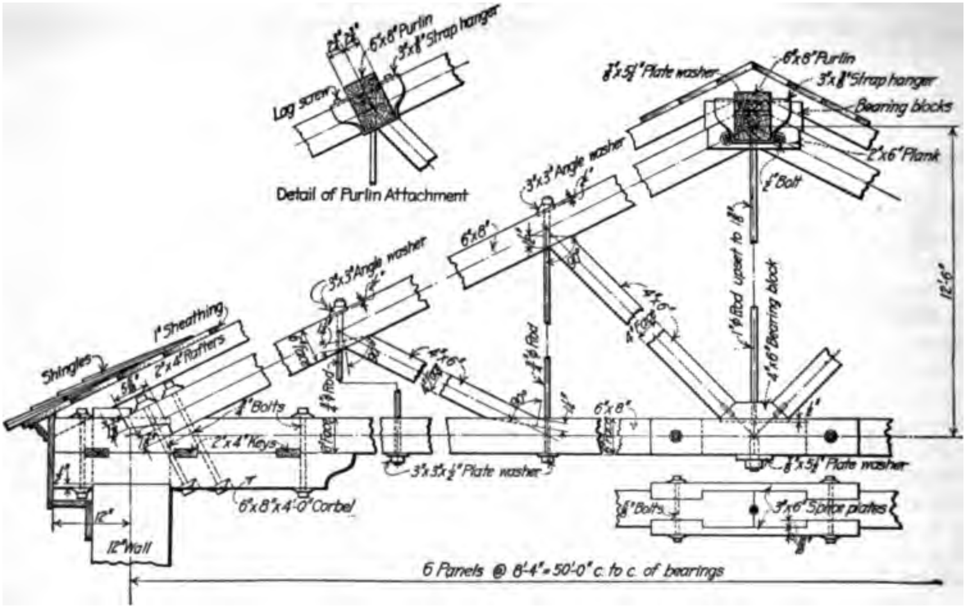


FIG. 179.

The total weight of the trusses was found to be 1695 lb., divided as follows: main members, 1350 lb.; steel rods, 80 lb.; plate and cast washers, 100 lb.; bolts and dowel pins, 75 lb.; and strap hangers, 90 lb. Since the span is 50 ft., and the distance between trusses is 16 ft., the horizontal covered area per truss is 50 × 16 = 800 sq. ft. The actual truss weight per sq. ft. of horizontal covered area is then $^{1695}\!/_{800}$ = 2.12 lb. From Art. 144 the weight as estimated by formula is 2.42 lb. per sq. ft. of covered area. The estimated weight is therefore about 14% in excess of the actual weight. However, as brought out in the discussion on dead weight formulas given in the chapter on Roof Trusses—General Design, this difference between actual and estimated weight is not great enough to warrant a recalculation of the dead load stresses. The design as given in the preceding articles will therefore be considered as final.

DETAILED DESIGN OF A STEEL ROOF TRUSS

By W. S. Kinne

148. General Conditions for the Design.—A complete design will be made of the steel roof trusses for a building with masonry side and end walls. It will be assumed that the layout of the building, as determined by other considerations, is as shown in Fig. 180. A roof covering consisting of wood shingles on plank sheathing will be used. The structure will be assumed as located in the Central States. It will be designed for a minimum load capacity of 40 lb. per sq. ft.

The general requirements governing the design of the steel work will conform to the standard practice for this type of structure. Working stresses for steel will be 16,000 lb. per sq. in. on the net section of tension members, and $16,000 - 70\ l/r$ lb. per sq. in. on the gross area of compression members (l = length of member in inches, and r = least radius of gyration of section in inches). The limiting slenderness ratio for compression members will be $l/r = 125$ for main members and $l/r = 150$ for bracing. It will be assumed that the trusses are not exposed to moisture or corrosive gases, so that the minimum thickness of material can be taken as $\frac{1}{4}$ in. All members carrying calculated stress will be made of two angles, the member and joint details to be arranged according to the discussion given in the chapter on Roof Trusses—General Design.

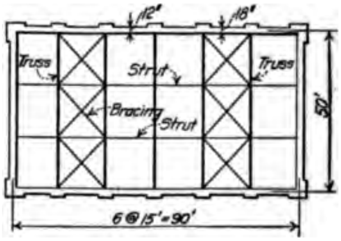


Fig. 180.

Rivets will be taken as $\frac{3}{4}$ in. in diameter, and rivet holes will be punched $\frac{1}{16}$ in. larger then the rivet diameter. In calculating net areas of tension members the diameter of rivet holes will be taken $\frac{1}{8}$ in. larger than the rivet, or $\frac{7}{8}$ in. Working values for shop rivets will be based on 10,000 lb. per sq. in. for shear, and 20,000 lb. per sq. in. for bearing; corresponding values for field rivets will be 7500 and 15,000 lb., respectively.

The smallest angle leg which will hold a $\frac{3}{4}$-in. rivet is usually taken as $2\frac{1}{2}$ in. Where an angle leg does not contain rivets, a 2-in. leg can be used. No reduction in section area will be made where angles are connected by one leg only, except the usual reduction for rivet holes.

Working stresses for wooden sheathing will be taken as 1000 lb. per sq. in. for bending. The bearing on masonry walls will be 200 lb. per sq. in. Purlins will be made of rolled steel sections. To avoid excessive deflection, the adopted section will be limited in depth to $\frac{1}{80}$ of the span.

149. Type and Form of Truss.—The type and form of truss to be used, and the spacing of the trusses will be determined by a consideration of the principles outlined in the chapter on Roof Trusses—General Design. As a shingle roof is to be used, the minimum desirable roof pitch is $\frac{1}{4}$. This is also the pitch which will result in the most economical structure. It will therefore be adopted.

From Fig. 180, the distance between walls is 49 ft. If it be assumed that the end bearing plates are to be 12 in. long, the effective span will be 50 ft. Since the adopted pitch is $\frac{1}{4}$, the height of the truss will be $^{50}\!/_{4} = 12.5$ ft., as shown in Fig. 181. The length of the top chord member is $(25^2 + 12.5^2)^{\frac{1}{2}} = 28$ ft. If the top chord members be limited in length to about 8 ft., it will be necessary to divide the top chord into four parts, each $^{28}\!/_{4} = 7$ ft. long. From Fig. 144, p. 455, a convenient form of truss is offered by the compound Fink truss of Fig. (*b*), or by the four-panel Pratt truss of Fig. (*k*). Of these two forms of trusses, it will be found that for points near the center of the span the Fink truss can be made up with shorter members than those needed for the Pratt truss. As shown by the tables of stress coefficients given in the chapter on Roof Trusses—Stress Data, the stresses in the members of the Fink truss are a little larger than those in the Pratt truss. Everything considered, however, it seems best to use the Fink type, as shown in Fig. 181.

The economical spacing of trusses, as given in Art. 124, is about ¼ of the span length, or in this case, 12.5 ft. From Fig. 180, the distance of end walls is 90 ft. If the truss spacing be made 15 ft., there will be 6 bays and 5 trusses required. Where 7 bays are used, the truss spacing will be about 13 ft. As economical conditions favor long truss spacing, the arrangement shown in Fig. 180 will be adopted.

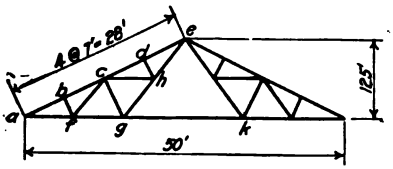


Fig. 181.

150. Loadings.—As stated in Art. 148, the structure is supposed to be located in the Central States. The snow load for this region, as given in the table in Art. 136, is 20 lb. per sq. ft. of roof surface. For this section of the country, the unit wind pressure is generally taken as 30 lb. per sq. ft. on a vertical surface. From the table of wind pressures given in Art. 135, the intensity of normal pressure on a one-quarter pitch roof is 22.4 lb. per sq. ft. of roof surface.

The dead weight of the truss will be estimated by means of one of the weight formulas given in Art. 134. From the Carnegie Handbook formula, for 40-lb. capacity, the weight is given as

$$0.2(\sqrt{50} + 0.125 \times 50) = 2.7 \text{ lb. per sq. ft. of horizontal covered area.}$$

Assuming the weight of the bracing to be 0.8 lb. per sq. ft., the total dead weight of truss and bracing will be 2.7 + 0.8 = 3.5 lb. per sq. ft. of horizontal covered area.

The weight of the roof covering can be estimated from the table given in Art. 133. Shingles weigh about 3 lb. per sq. ft. of roof, and the sheathing, which will be hemlock, will weigh about 3 lb. per sq. ft. of roof per inch of thickness.

151. Design of Sheathing.—The thickness of the sheathing can be determined from Table 2, p. 458. Thus for a roof of 40-lb. capacity, as assumed in Art. 148, Table 2 shows that for a slope of 6 in. per foot, which corresponds to one-quarter pitch, the limiting span of 1-in. sheathing is 6.84 ft. for a fiber stress of 1000 lb. per sq. in. This is but slightly less than the distance between top chord panel points, as shown in Fig. 181. The value given above is the limiting span for bending, as deflection is not limited for shingle roofs. Although material 1-in. thick can be used for sheathing as far as stress conditions are concerned, it is not considered good practice to use such thin material for long spans. It is advisable to use 2-in. material, which will be adopted.

A more exact design of the sheathing can be made by considering the combinations of loads acting on the sheathing. These combinations are similar to those mentioned in Art. 137. They are: (*a*) dead load and snow load; (*b*) dead load, minimum snow load, and maximum wind load; and (*c*) dead load, maximum snow load, and minimum wind load. The dead load is the weight of the shingles and of the sheathing, which will be assumed to be 2 in. thick. At 3 lb. per ft. B. M., the sheathing weighs 6 lb. per sq. ft. of roof. From Art. 150, the maximum wind and snow loads are respectively 22.4 and 20 lb. per sq. ft. of roof surface, the wind load acting normal to the roof and the snow load acting vertical. Minimum snow load will be taken as one-half of the maximum, and minimum wind load will be taken as one-third of the maximum.

The allowable fiber stress for the sheathing will be taken as 1000 lb. per sq. in. As mentioned in Art. 135, the wind load is an occasional loading and the working stresses can be modified accordingly. It will be assumed that the working stress for wind loading, when combined with stresses due to direct loading, is increased 50%. This can be taken into account by reducing the wind load by ⅓—that is, by using a unit wind load of 20 lb. per sq. ft. The normal load for a roof of ¼ pitch is then 14.9 lb. per sq. ft. This load can be combined with those for dead and snow load, and a working stress of 1000 lb. per sq. in. applied to the resulting moment.

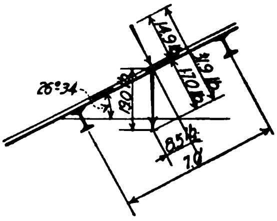


Fig. 182.

In designing the sheathing, it will be assumed to act as a beam supported by purlins placed at the top chord joints of the truss. As shown in Fig. 181, the purlins are spaced 7 ft. apart. Since the sheathing is continuous over the purlins, it will be assumed that the maximum moment is given by the formula $M = \frac{1}{10} wl^2$. The loads will be resolved into components perpendicular and parallel to the sheathing. It will be assumed that the moment to be carried by the sheathing is due to the normal loads; the effect of components parallel to the sheathing will be neglected.

The total vertical load for the combination of case (*a*) is 3 lb. for shingles, 6 lb. for sheathing, and 20 lb. for snow, a total of 29 lb. As shown in Fig. 182, the roof surface forms an angle of 26 deg. 34 min. with the horizontal. The component perpendicular to the roof is then 29 × cos 26 deg. 34 min. = 29 × 0.895 = 25.9 lb. per sq. ft. of roof. For case (*b*), which is shown in Fig. 182, the vertical load is 3 lb. for shingles, 6 lb. for sheathing, and 10 lb. for minimum snow load; a total vertical load of 19 lb., for which the component perpendicular to the roof is 19 × 0.895 = 17 lb. The wind load normal to the roof is 14.9 lb. Hence the total normal load is 17.0 + 14.9 = 31.9 lb. In the same way it will be found that the total normal load for case (*c*) is 30.9 lb. Case (*b*) therefore gives the maximum normal component.

The maximum moment to be carried by the sheathing due to the normal loads is then $M = \frac{1}{10} wl^2 = \frac{1}{10} \times 31.9 \times 7^2 \times 12 = 1875$ in.-lb. For a rectangular section the fiber stress is given by the formula $f = Mc/I = 6M/bd^2$. Considering a section of sheathing 1 ft. wide and 2 in. thick, we have

$$f = \frac{6 \times 1875}{12 \times 2 \times 2} = 234 \text{ lb. per sq. in.}$$

As the allowable fiber stress is 1000 lb. per sq. in., the sheathing is stronger than necessary. To conform to the general practice, the assumed sheathing will be used.

152. Design of Purlins.—Purlins are designed by the methods outlined in the chapter on Design of Purlins for Sloping Roofs in Sect. 2. As the sheathing is quite rigid, it will be assumed that the purlins carry only the components of loads perpendicular to the roof surface. The combinations of loading will be the same as for the design of the sheathing. From the preceding article the maximum component of normal loads is 31.9 lb. To this must be added the weight of the purlin, which will be assumed to be 1.3 lb. per sq. ft. normal to the roof. The total normal load is then 31.9 + 1.3 = 33.2 lb. Since the trusses are spaced 15 ft. apart, the area carried by a purlin is 7 × 15 = 105 sq. ft. of roof surface. The total uniformly distributed load for a purlin is then 33.2 × 105 = 3486 lb., and the moment to be carried, assuming the purlin to be a simple beam between trusses, is $M = \frac{1}{8}Wl = \frac{1}{8} \times 3486 \times 15 \times 12 = 78{,}500$ in.-lb. For an allowable working stress of 16,000 lb. per sq. in., the required $I/c = 78{,}500/16{,}000 = 4.9$ in.^{3} From the handbooks, this is furnished by a 7-in. 9¾-lb. channel. The true weight of this section, in lb. per sq. ft. normal to the roof surface, is $9.75 \times \cos 26° 34'/7 = 9.75 \times 0.895/7 = 1.25$. This is so close to the assumed value that the calculations will not be revised.

153. Determination of Stresses in Members.—The stresses in the truss members are to be determined for the same combinations of loads as used for the design of the sheathing and the purlins. Two general methods of calculation can be used. In the first method, the dead and snow loads are taken as vertical forces and the wind load is considered as acting normal to the roof on the windward side. In the second method of calculation, dead, wind, and snow loads are represented by a uniform vertical load acting over the entire roof surface. As stated in Art. 137, this second method of calculation can be applied to trusses of the Fink type. The stresses thus obtained are practically the same as those obtained by the first method of calculation. While the first method probably more nearly approximates the actual conditions, the second method results in a considerable saving of time spent in stress calculation. For the truss under consideration both methods of calculation will be carried out and the results compared.

The first step in the calculation of the stresses in the members is the determination of the panel loads. In the first method of calculation outlined above it will be found best to determine the panel loads due to dead, snow, and wind loads separately. The resulting stresses can then be determined and the proper combinations made up to determine the maximum stress.

As stated in Art. 151, the dead weight of the shingles and sheathing is a vertical load of 9 lb. per sq. ft. of roof surface. Since the purlins are spaced 7 ft. apart, and the trusses are 15 ft. apart, the roof area per panel is 7 × 15 = 105 sq. ft. The dead panel load due to the roofing is then 9 × 105 = 945 lb. To this must be added the weight of the purlin and the estimated weight of the truss. From Art. 152, the adopted purlin is a 7-in. 9¾-lb. channel. As the weight of one 15-ft. purlin is carried to each top chord panel point, the dead load due to the purlin is 9¾ × 15 = 146.3 lb. From Art. 150, the estimated weight of the truss and bracing was found to be 3.5 lb. per sq. ft. of horizontal covered area. As the span is 50 ft., and since there are 8 roof panels, the horizontal covered area per panel is $15 \times \frac{50}{8} = 93.75$ sq. ft. The panel load due to the weight of the truss and bracing is then $93.75 \times 3.5 = 328.1$ lb. Adding together these partial panel loads, the total dead panel load is: $945.0 + 146.3 + 328.1 = 1419.4$ lb. A panel load of 1420 lb. will be used in the calculation of dead load stresses.

The stresses in the truss members due to the dead panel load can be determined by the methods of stress calculation given in Sect. 1, or by means of the tables of stress coefficients given in the chapter on Roof Trusses—Stress Data. Col. 1 of Table 1 gives the calculated dead load stresses.

From Art. 150, the snow load is a vertical load of 20 lb. per sq. ft. of roof surface. Since the roof area per panel is 105 sq. ft., the snow panel load is $20 \times 105 = 2100$ lb. The stresses due to this panel load can be determined by the methods outlined above for the dead load stresses. As the panel loads for dead and snow load are both vertical and are applied at the same points, the snow load stresses can be determined by ratio from the dead load stresses as given in col. 1 of Table 1. Thus if the dead load stresses be multiplied by the ratio of snow and dead panel loads, the resulting stresses will be the required snow load stresses. For the truss under consideration, the ratio of snow and dead panel loads is $2100/1420 = 1.48$. This ratio can be set off on a slide rule and the stresses calculated with sufficient accuracy for all ordinary cases. The snow load stresses for the truss under consideration are given in col. 2 of Table 1. To assist in making up the combined stresses there is also given in col. 3 of Table 1 the stresses due to one-half of the maximum snow load.

The wind pressure on the roof surface of a one-quarter pitch roof due to a unit pressure of 30 lb. per sq. ft. is given in Art. 150 as 22.4 lb. per sq. ft. Where the working stress for wind is increased 50 % over that used for dead and snow loads, as in the case under consideration, the change can be made by a reduction in the intensity of the wind pressure corresponding to the increase in working stress. Since the working stress for wind is $\frac{3}{2}$ of that for the other loads, the intensity of the wind pressure can be taken as $\frac{2}{3}$ of the value given for a 30-lb. unit pressure. A uniform working stress of 16,000 lb. per sq. in. can then be used for all loadings.

The normal wind load per sq. ft. of roof corresponding to a working stress of 24,000 lb. per sq. in. is $\frac{2}{3} \times 22.4 = 14.9$ lb. As the area of the panel is 105 sq. ft., the wind panel load is $14.9 \times 105 = 1565$ lb. The resulting stresses are calculated by the methods of Sect. 1, or by means of the wind stress coefficients given in the chapter on Roof Trusses—Stress Data. In calculating the wind stresses it will be assumed that one end of the truss is fixed and that the other end is supported on a smooth plate on which it is free to slide. As it is generally assumed that the frictional resistance between smooth plates is zero, the reaction at the free end is vertical. The assumed end conditions are covered by Cases I and II of the wind stress coefficients for the Fink truss. The calculated wind stresses for wind on the left side of the truss are given in col. 4 of Table 1. In col. 5 the stresses for one-third wind load are given.

The combinations of dead, snow, and wind load stresses for maximum stresses in the truss members are the same as given in Art. 151 for the design of the sheathing. These combinations are: (*a*) dead load, one-half snow load, and maximum wind load, and (*b*) dead load, maximum snow load, and one-third wind load. The maximum stresses for case (*a*) are given in col. 7 of Table 1. They are obtained by adding the values given in cols, 1, 3, and 4. Values for case (*b*) are given in col. 8. They are obtained by adding values given in cols. 1, 2, and 5.

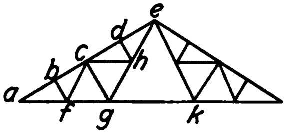
Maximum stresses as determined by the second method of calculation outlined above are given in col. 9 of Table 1. The vertical uniform load which is to represent the combined effect of wind and snow can be taken from Table 9, p. 469. For a roof of one-quarter pitch located in the Central States, the load is given as 25 lb. per sq. ft. of roof surface. The equivalent load can also be estimated from the values for wind and snow given in Art. 150. To estimate this load, assume that the vertical component of the wind is combined with the snow load in the same manner as for maximum stresses in the first method of calculation. The vertical component of the wind load is $14.9 \times \cos 26° 34' = 13.4$ lb. per sq. ft. of roof. If one-half of the snow load of 20 lb. per sq. ft. of roof be added to this load, there is obtained an equivalent load of 23.4 lb. For maximum snow and one-third wind the combined load is $\frac{1}{3} \times 13.4 + 20 = 24.4$ lb. These values compare very well with the load of 25 lb. taken from the above mentioned table.

The panel load for equivalent vertical loading is determined by adding to the panel load for the above load, the dead panel load as given above. As the area of the roof panel is 105 sq. ft., the panel load for combined wind and snow is 25 × 105 = 2625 lb. The dead panel load, as given above, is 1420 lb., and the total panel load is 1420 + 2625 = 4045 lb. Col. 9 of Table 1 gives the resulting stresses, which were calculated from the dead load stresses of col. 1 by means of the ratio of panel loads, 4045/1420 = 2.845, which was set off on a slide rule and the stresses read directly.

In some cases it is also specified that the roof shall be designed for a load capacity of not less than 40 lb. per sq. ft. of covered area. The specified capacity depends upon the service conditions and with the location of the structure, varying from 30 to 60 lb. For the truss under consideration, the panel load will be 40 × 93.75 = 3750 lb. Since this panel load is less than the one used for the calculation of the stresses given in col. 9 of Table 1, the resulting stresses will be smaller than those given in col. 9. In some cases these stresses may exceed the others, in which case they will determine the design.

Comparing the stresses obtained by the two methods of calculation, as given by cols. 7 and 8 for the first method, and by col. 9 for the second method, it will be found that, for top and bottom chord members, the stresses given by col. 9 are a little larger than those given in either col. 7 or 8, and that the stresses in the web members are almost identical in cols. 7, 8, and 9. The second method of calculation therefore gives practically the same results as the more exact first mehtod. The stresses given in col. 9 will be used as the maximum stresses for the design under consideration.

TABLE 1.—STRESSES IN MEMBERS



Member	Dead load	Snow load	$\frac{\text{S. L.}}{2}$	Wind from left	$W/3$	Wind from right	D. L., $\frac{\text{S. L.}}{2}$ & max. W	D. L., maximum S. L. & $W/3$	Uniform vertical loading
	1	2	3	4	5	6	7	8	9
ab	−11,120	−16,450	−8,225	−7,050	−2,350	−3,920	−26,395	−29,920	−31,660
bc	−10,490	−15,500	−7,750	−7,050	−2,350	−3,920	−25,290	−28,340	−29,850
cd	− 9,840	−14,550	−7,275	−7,050	−2,350	−3,920	−24,165	−26,740	−28,040
de	− 9,210	−13,640	−6,820	−7,050	−2,350	−3,920	−23,080	−25,200	−26,230
bf *dh*	− 1,270	− 1,880	− 940	−1,565	− 522	0	− 3,775	− 3,672	− 3,620
cg	− 2,540	− 3,760	−1,880	−3,130	−1,043	0	− 7,550	− 7,348	− 7,240
af	+ 9,940	+14,700	+7,350	+8,750	+2,920	+ 688	+26,040	+27,560	+28,315
fg	+ 8,520	+12,600	+6,300	+7,000	+2,334	+ 688	+21,820	+23,454	+24,270
gk	+ 5,680	+ 8,410	+4,205	+3,500	+1,167	+ 688	+13,385	+15,257	+16,180
fc *ch*	+ 1,420	+ 2,100	+1,050	+1,750	+ 583	0	+ 4,220	+ 4,103	+ 4,045
gh	+ 2,840	+ 4,200	+2,100	+3,500	+1,167	0	+ 8,440	+ 8,207	+ 8,090
he	+ 4,260	+ 6,300	+3,150	+5,250	+1,750	0	+12,660	+12,310	+12,135

+ = tension. − = compression.

154. Design of Members.—The conditions for the design, as stated in Art. 148, contain the following references to working stresses: tension, 16 000 lb. per sq. in. on the net section; compression, (16,000 − $70l/r$) lb. per sq. in. on the gross section, l/r not to exceed 125. The minimum thickness of material is given as ¼ in. All members carrying calculated stress are to be made up of two angles. Design methods for tension and compression members are given in Sect. 2.

In making up truss members such as the top and bottom chord, which are continuous over several panels, it is the usual practice to design the member for the section of maximum stress, and to use the same section for the entire member. This is good practice, for it will probably be found that if the sections are changed to fit the stresses and splices made at each joint, the cost of the shop work on these splices will exceed the cost of the excess material required for continuous members.

Trusses of small size can generally be shipped in one piece. All joints can be riveted up in the shop and the truss erected as a unit in the field. The limiting dimensions of fully riveted trusses are governed by the methods of transportation. It is generally specified that a truss or girder, which is to be shipped by train, must have one dimension not exceeding from 10 to 12 ft. Trusses with a greater least dimension than that mentioned must be broken up into smaller parts. The truss under consideration in this design will have a total height, which is its least dimension, of about 13 ft. It must then be broken up into smaller parts. For trusses of the type under consideration, it is usual to provide field splices at joints *g*, *e*, and *k* of the truss diagram of Fig. 181. The least width of the pieces thus formed will be the distance along member *c–g*, which is about 8 ft. Continuous members will then be used for the top chord member *a* to *e*; the bottom chord from *a* to *g*; and the diagonal from *g* to *e*. Member *g–k* will be shipped as a single piece.

Design of Tension Members.—The maximum stress in the bottom chord member from *a* to *g* occurs in the section *a–f*, where the stress is 28,315 lb For a working stress of 16,000 lb. per sq. in., the required net area is 28,315/16,000 = 1.77 sq. in. An angle must now be selected whose net area—that is, the area of the section minus the area of the rivet holes—will provide the required area. As stated in Art. 148, the rivets are to be ¾ in. in diameter, and the rivet holes are to be made ⅛ in. larger, or ⅞ in. The area to be subtracted from the gross area of the section in determining net area is then the thickness of the material multiplied by ⅞. The number of rivet holes to be subtracted from each angle in the determination of the net areas depends on the type of end connection used for the member in question. When an angle is connected by both legs, the area of two rivet holes should be deducted from each leg so connected, or the distance between the rivets in the two legs of the angle should be made such that it will be necessary to deduct but one rivet hole. Tables of limiting spacing for this condition are given in the chapter on Splices and Connections—Steel Members.

Fig. 189 shows the details of joint *a* as adopted for this design. The bottom chord member is shown as connected by one leg. One rivet hole will then be deducted from each angle. Assuming two 2½ × 2½ × ¼-in. angles, whose gross area as given by the handbooks is 2 × 1.19 = 2.38 sq. in., and deducting one rivet hole from each angle, or a total of 2 × ⅞ × ¼ = 0.44 sq. in., the net area of the two angles is 2.38 − 0.44 = 1.94 sq. in. As given above, the required area is 1.77 sq. in. The assumed section is therefore ample, and will be adopted. To assist in the determination of the net area of members, tables of areas to be deducted for various rivet sizes and thicknesses of material are given in Sect. 2.

Member *f–g* will be made the same as *a–f*. From Fig. 188, it will be noted that the member is connected by both legs. Assuming two rivet holes deducted from each angle, the net area of the section is 2.38 − 4 × 0.22 = 1.50 sq. in. As shown in Table 2, the required net area is 24,270/16,000 = 1.52 sq. in. Since the net area for two rivets deducted from each angle is practically the same as the required area, the rivets can be spaced as desired. If the proper area is not provided in any case, either larger angles must be assumed, or the distance between the rivets in the two legs of the angles must be such that only one rivet hole need be deducted from each angle in determining net areas.

Fig. 190 shows another design for the joint at *a*. It will be noted that member *a–f* has rivets in both legs. Deducting four rivet holes from the assumed section, the net area is found to be 2.38 − 0.88 = 1.50 sq. in. The assumed section is too small. It will be found that a 2½ × 2½ × 5/16-in. angle will provide the required area. However, this section is somewhat heavier than the lightest of the 3-in. sections. If a 3 × 2½ × ¼-in. angle be assumed, it will be found that the net area with two holes deducted from each angle is 2 (1.31 − 2 × 0.22) = 1.74 sq. in., which is sufficient. This section would be adopted if the design of Fig. 190 were used.

Members *g–h* and *h–e* are made continuous. Table 2 shows that 2½ × 2½ × ¼-in. angles are used. These angles provide considerable excess area, but from the conditions of the design, as given in Art. 148, they are the minimum allowable angles. The remaining tension members are designed by the methods explained above. Table 2 contains all data in convenient form.

Design of Compression Members.—Compression members are designed by cut-and-try methods. That is, a section is assumed, the allowable working stress calculated from the column formula, the required area determined, and the required and provided areas compared. The assumed section is adopted if the area provided is equal to that required. It is not always possible to obtain an exact fit, but the two areas should not differ any more than is necessary.

f the assumed section is insufficient, or if it provides excess area, the process must be epeated until the desired agreement is obtained. Gross or total section areas are used in he design of compression members; rivet holes are not deducted, as in the case of tension nembers.

The top chord will be made continuous from *a* to *e*. As shown in Table 2, the maximum stress, which is 1,660 lb., occurs in member *a–b*. Assume two 3½ × 3 × 5⁄16-in. angles, placed as shown in Fig. 183. Since the llowable working stress depends on the ratio of length to least radius of gyration, the angles should be so placed hat the radii of gyration for the axes *OX* and *OY* of Fig. 183 will be as large as possible, and also, the radii for he two axes should be as nearly equal as the conditions will permit. In this way a member is secured which has he same rigidity in all directions. This condition can best be realized by the use of angles with unequal legs placed rith the longer legs back to back. In Fig. 183 the angles are shown separated by small space. This is done to make room for the gusset plates at the joints, as xplained in the chapter on Roof Trusses—General Design. For trusses of the ize under consideration, a ⅜-in. space is ample.

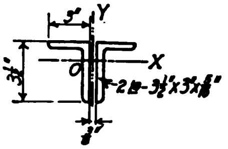


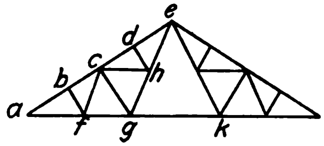
Fig. 183.

The radii of gyration for angles placed as shown in Fig. 183 can be found in ables given in the steel handbooks. From such tables it will be found that the adii are 1.10 in. for axis *OX* and 1.35 in. for axis *OY*. From Table 2 the length f member *a-b* is 84 in. Hence the ratio of length to least radius of gyration is $/r = 84/110 = 76.5$. Substituting this value of l/r in the column formula of rt. 148, the allowable working stress is $16{,}000 - 70\,l/r = 16{,}000 - 70 \times 76.5 =$ 0,650 lb. per sq. in. The area required is $31{,}660/10{,}650 = 2.97$ sq. in. From the steel handbooks, the area of he assumed angles is $2 \times 1.93 = 3.86$ sq. in. The assumed section is a little too large, but no other section of ss weight per foot could be found that would bring a closer agreement between required and provided areas. t was therefore adopted.

The top chord design as given above applies to members carrying compression only. If the purlins are placed etween the panel points, the top chord acts as a beam as well as a compression member. Design methods for this ondition are given in Art. 158.

Table 2 gives the design data for the other compression members. The design methods used are exactly the me as those given above for member *a-b*. Sections of minimum size were adopted, consisting of two 2½ × 2 (¼-in. angles with the longer legs separated by a ⅜-in. space.

TABLE 2.—DESIGN OF MEMBERS



lember	Stress (lb.)	l (in.)	r (in.)	l/r	f (lb. per sq. in.)	Area required (sq. in.)	Section (in.)	Area provided Gross (sq. in.)	Area provided Net (sq. in.)
ab	−31,660	84	1.10	76.5	10,650	2.970	2 ∟s 3½ × 3 × 5⁄16	3.86	
bc	−29,850	84	1.10	76.5	10,650	....	2 ∟s 3½ × 3 × 5⁄16	3.86	
cd	−28,040	84	1.10	76.5	10,650	....	2 ∟s 3½ × 3 × 5⁄16	3.86	
de	−26,230	84	1.10	76.5	10,650	....	2 ∟s 3½ × 3 × 5⁄16	3.86	
bf–dh	− 3,620	42	0.78	53.9	12,230	0.296	2 ∟s 2½ × 2 × ¼	2.12	
cg	− 7,240	84	0.78	107.8	8,460	0.837	2 ∟s 2½ × 2 × ¼	2.12	
af	+28,315	....	....	.....	16,000	1.77	2 ∟s 2½ × 2½ × ¼	2.38	1.94
fg	+24,270	....	....	.....	16,000	1.52	2 ∟s 2½ × 2½ × ¼	2.38	1.50
gk	+16,180	....	....	.....	16,000	1.01	2 ∟s 2½ × 2½ × ¼	2.38	1.50
fc-ch	+ 4,045	....	....	.....	16,000	0.252	2 ∟s 2½ × 2 × ¼	2.12	1.68
gh	+ 8,090	....	....	.....	16,000	0.504	2 ∟s 2½ × 2 × ¼	2.12	1.68
he	+12,135	....	....	.....	16,000	0.759	2 ∟s 2½ × 2 × ¼	2.12	1.68

+ = tension. − = compression.

155. Design of Joints.—The general principles of joint design are given in the chapters on Roof Trusses—General Design, and Splices and Connections—Steel Members. Well designed joints are just as important as well designed members. To secure good joint design, a few fundamental principles of design must be observed. The center lines of all members entering a joint must intersect at a common point. If the conditions are such that this can not be done provision must be made for the additional stresses due to joint eccentricity. All stresses should be traced through the joint, and proper connections made between all parts. Typical joint details are given in the chapter on Roof Trusses—General Design.

In trusses of the size under consideration in this design, the angles are usually connected to the gusset plates by means of rivets through one leg only, as shown in Figs. 184 to 190 inclusive. Theoretically, this is not good practice, for all of the stress is transferred to the gusset plate through one angle leg, resulting in excess local stresses. However, in small trusses the members generally contain more area than required for stress conditions, which assists in carrying the excess stresses. In larger trusses lug angles are riveted to the gusset plate and to the outstanding legs of the angles, thereby transferring the stresses from both legs of the angles into the gusset plate and avoiding excessive local stresses.

The number of rivets required in the end connection of any member depends on the working stresses for the rivets and on the method of making the connection to the gusset plate. The principles governing the design of riveted joints are given in the chapter on Splices and Connections—Steel Members.

As stated in Art. 148, the working stresses for shop rivets are 10,000 lb. per sq. in. for shear and 20,000 lb. per sq. in. for bearing. Corresponding values for field rivets are given as 7500 and 15,000 lb. per sq. in. respectively. Tables of rivet values are given in the chapter on Splices and Connections—Steel Members, and also in the steel handbooks. From these tables the single shear values of ¾-in. shop and field rivets are 4420 and 3310 lb. respectively. The bearing value of a rivet depends on the thickness of the gusset plate. For trusses of the size under consideration, a ⅜-in. plate is usually ample. In any case the adopted thickness should be such that large gusset plates can be avoided. For a ⅜-in. plate, the bearing of a ¾-in. shop rivet is 5625 lb., and the corresponding value for a field rivet is 4220 lb. The design of the several joints will now be considered in detail.

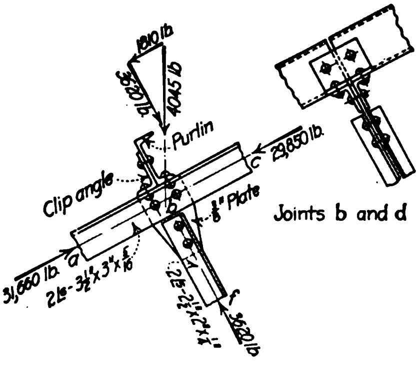


FIG. 184.

Joint b.—Fig. 184 shows the details of joint *b*. The stresses in the members and the panel load at joint *b* are shown in position. As shown by the force diagram, the stress in member *b-f* is balanced by the component of the joint load perpendicular to the top chord, and the difference between the stresses in the top chord members *a-b* and *b-c* is balanced by the component of the joint load parallel to the top chord. The complete design of the joint therefore consists in transferring the stress in member *b-f* to the gusset plate and thence to the top chord angles; and also in equalizing the difference in stress between members *a-b* and *b-f* by means of a purlin connection.

Member *b-f*, whose stress is 3620 lb., is connected to the gusset plate by shop rivets in bearing on the ⅜-in. plate. The value of these rivets, as given above, is 5625 lb. per rivet, and the number required to connect *b-f* to the gusset plate is 3620/5625 = 1 rivet. Since a rigid connection can not be made with a single rivet, it is the general practice to use not less than two rivets in any connection. Two rivets have therefore been used in the connection shown in Fig. 184.

The load to be transferred from the gusset plate to the top chord angles is equal to the stress in member *b-f*. Since the conditions are the same as for the connection between *b-f* and the gusset plate, two rivets will be used, as shown in Fig. 184.

Member *a-b-c*, the top chord, is continuous across joint *b*. As shown by the force diagram, the difference in stress between members *a-b* and *b-c*, which is 31,660 − 29,850 = 1,810 lb., is balanced by the component of the joint load parallel to the top chord. To equalize the stresses in a-b and b-c, rivets capable of transferring 1810 lb. from the purlin to the top chord must be placed in position. These rivets will be placed in the outstanding leg of the clip angle and in the flange of the channel, as shown in Fig. 184. The value of the connecting rivets is determined either by their single shear value as shop rivets, which is 4420 lb., or by the bearing value on the leg of the $\frac{5}{16}$-in. clip angle, which is 4690 lb. The single shear value governs, and only one rivet is required in the purlin connection. In order to make a rigid connection, it will be necessary to use two rivets in the clip angle and two more in the flange of the channel. Fig. 184 shows the complete details. Joint d is similar to joint b; the same details will be used.

Joint c.—Fig. 185 shows the details of joint c. The design of this joint is carried out by the same methods as used for joint b. In this case the stresses in members f-c, g-c, and h-c, are transferred to the gusset plate, and the resultant of these stresses, which can be seen from Fig. 185 to be 7240 − 2 × 1810 = 3620 lb., is to be transferred to the top chord angles.

As before, the rivets connecting the angles to the gusset plate are in bearing on a $\frac{3}{8}$-in. plate and have a value of 5625 lb. per rivet. One rivet is required for members f–c and h–c, and two rivets are required for g-c. Two rivets are used in each member, as shown in Fig. 185. The stress of 3620 lb., which is to be transferred from the gusset plate to the top chord, will require only one rivet, as at joint b. To secure a rigid connection, 5 rivets have been used, spaced about 4 in. apart, as shown in Fig. 185.

The load to be transferred by the purlin connection to the top chord angles is the same as for joint b, as shown by the force diagram. Details similar to those at joint b will be used, as shown in Fig. 185.

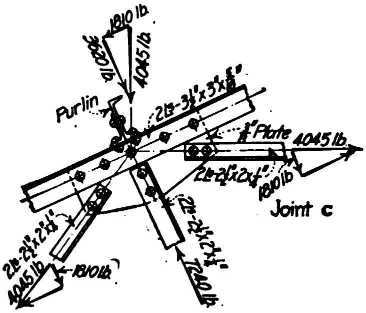


FIG. 185.

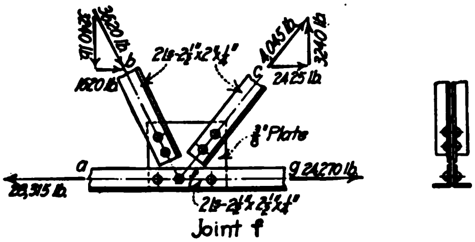


FIG. 186.

Joint f.—The conditions at joint f are shown in Fig. 186. As before, the chord members are continuous across the joint. The design of the joint consists in transferring the stresses in the members c–f and b–f to the gusset plate and thence to the chord angles, and in equalizing the stresses in members a–f and f–g. Since double angles are used for all members, and the gusset plate is $\frac{3}{8}$-in. thick, the rivet value is 5625 lb., as before. A single rivet is sufficient to transfer the stresses from members b–f and c–f to the gusset plate. Two rivets have been used in each member, in order to make a rigid connection.

As shown by the force diagram of Fig. 186, the stresses in b–f and c–f have components perpendicular to the chord member which balance each other, and have components parallel to the chord member whose sum is equal to the difference in stresses in the chord members. The rivets connecting the gusset plate to the chord angles must then be capable of transferring a load of 28,315 − 24,270 = 4045 lb. A single rivet is sufficient, but the general practice is to use the detail shown in Fig. 186. One rivet in placed at the intersection of the center lines of the members, and other rivets are placed near the edges of the plate, as shown in Fig. 186. Joint h is similar to joint f. The same details will be used.

Joint e.—Fig. 187 shows the conditions at joint e. The purlin load at this joint can be considered either as a single vertical load, as shown by the full line arrow of Fig. 187, or as two loads, shown by the dotted arrows, whose resultant is equal to the single load. The design methods are the same in the two cases.

As noted early in this article, a field splice will be located at joint e. One side of the joint will be riveted up in the shop, and the rivets or bolts in the other side of the joint will be placed in position when the truss is assembled in the field. In order that a symmetrical joint may be made, the rivet values will be determined as for field rivets, and the same number will be used for both shop and field rivets. The connection will then be made with field rivets in bearing on a ⅜-in. plate. These rivets have a value of 4220 lb., as given above.

The design of this joint consists in transferring to the gusset plate, the stresses in the several members, and in the provision of a purlin connection. Member *d–e*, whose stress is 26,230 lb., requires 26,230/4220 = 7 rivets. For member *h–e*, whose stress is 12,135 lb., 12,135/4220 = 3 rivets are required; they are shown in position in Fig. 187. The load brought to the joint by the purlin will be provided for by means of a connection similar to that used at the other joints. If a single vertical purlin is used, a suitable bearing plate, or shelf angles attached to the gusset plate forms a satisfactory connection. Where two purlins are used at the apex of the truss, connections similar to those shown for joints *b* and *c* can be used. General details of purlin connections are shown in Art. 127.

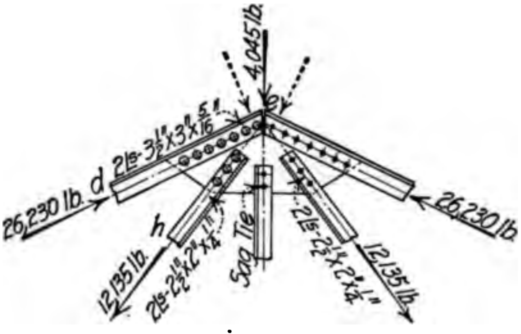


Fig. 187.

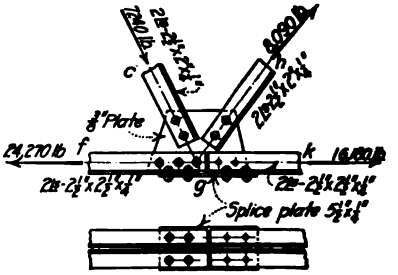


Fig. 188.

Joint g.—Fig. 188 shows the details of joint *g*. Member *g–k* is field spliced at this point; all other members entering the joint are shop riveted. The splice in the bottom chord member can be made in two ways. In one case, the stresses in the members are transferred directly to the gusset plate by means of rivets in the vertical legs of the angles. This method is satisfactory where the stresses in the members are small. Where large stresses are to be transferred to the gusset plates, the joint is likely to be quite large if this method is used. To avoid large plates, the joint detail shown in Fig. 188 is generally used. This joint consists of a splice plate on the horizontal legs of the angles in addition to the rivets placed in the vertical legs. In this way part of the stress is carried by the splice plate, thereby reducing the stresses to be transferred by the vertical legs of the angles to the gusset plate.

The design of joint *g* consists in transferring to the gusset plate the stresses in members *g–h* and *g–c*, and in the provision of a partially continuous bottom chord member in which part of the stress is carried around the joint by a splice plate and the balance of the stress is transferred directly to the gusset plate. As shown in Fig. 188, the rivets in members *g–c* and *g h* are shop rivets in bearing on a ⅜-in. plate. These rivets have a value of 5625 lb. per rivet. Member *c–g* requires 7240/5625 = 2 rivets, and *g–h* requires 8090/5625 = 2 rivets; they are shown in position in Fig. 188. In determining the amount of stress to be transferred across the joint by the splice plate on the horizontal legs of the bottom chord angles, certain assumptions must be made regarding the distribution of the stresses. A common and reasonable assumption is that the stress in member *g–k* is uniformly distributed over the area of the member, and hence in this case the stresses in the two legs of the angle are equal, since the angle has equal legs. It is then assumed that the stress in the horizontal legs of the angles is transferred to the splice plate, and thence around the joint, while the stress in the vertical legs of the angles is carried directly to the gusset plate. Member *f–g* is assumed to have transferred to the splice plate a portion of its stress which is equal to the stress transferred to the splice plate by the horizontal legs of member *g-k*. The balance of the stress in member *f–g* is assumed to be transferred to the gusset plate through the vertical legs of the angles of member *f-g*. Since the stress in *f–g* is always greater than that in *g–k*, it follows that there will usually be an uneven distribution of stress to the legs of the angles of member *f–g*, unless the member is made up of unequal legged angles in which the distribution of area happens to be correct. In the present case equal legged angles are used, and unequal stress distribution results. However, in small trusses where it is permissible to connect angles by one leg, the conditions are more favorable than where the splice plate is not used.

On the assumptions made above, the stress in the vertical and horizontal legs of the angles of member *g–k* is 16,180/2 = 8090 lb. Since member *g k* is field spliced at this point, the rivets in the vertical legs are field rivets in bearing on a ⅜-in. plate; they have a value of 4220 lb. per rivet. The number required is 8090/4220 = 2, which are shown in position in Fig. 188. The stress of 8090 lb. in the horizontal legs of the angles is transferred o the splice plate by field rivets which are either in single shear or in bearing on the ¼-in. material composing the ngles and the splice plate. From the tables of rivet values, the field shearing value of a rivet is 3310 lb., and the eld bearing value for a ¼-in. plate is 2810 lb. The latter value governs and the number required is 8090/2810 = rivets. As shown in Fig. 188, four are used, two in each angle.

The stress in member f–g is 24,270 lb., of which 8090 lb. is taken up by the splice plate, as assumed above. here is then left 24,270 − 8090 = 16,180 lb. to be transferred from the vertical legs of the angles to the gusset late. The connecting rivets are shop rivets in bearing on a ⅜-in. plate, and have a value of 5625 lb. per rivet. he number required is 16,180/5625 = 3, which are shown in position in Fig. 188.

The splice plate on the horizontal legs of the chord angles must have sufficient net area to provide for the tress to be carried across the joint. This stress is 8090 lb., and the required net area is 8090/16,000 = 0.505 sq. 1. Assuming a plate ¼-in. thick and 5½ in. wide, which is slightly in excess of the spread of the lower chord ngles, the net area, deducting two rivet holes, is (5.5 − 2 × ⅞) ¼ = 3.75 sq. in. The assumed plate provides large excess area, but it is the smallest plate that can be used under the conditions for the design stated in Art. 48.

Joint a.—Two designs will be given for joint *a*, the heel of the truss. Fig. 189 shows a esign in which the stresses in the chord members and the shoe are brought directly to the gusset late. In the design shown in Fig. 190, he bottom chord member is prolonged nd acts as a support for the shoe. The ivets must then carry the vertical end eaction and the horizontal tension in he chord member. These designs will e carried out in detail.

In the design shown in Fig. 189, all members re connected to the gusset plate by shop rivets bearing on a ⅜-in. plate. The rivet value is hen 5625 lb. Member b–a requires 31,660/5625 = 6 rivets, and member a–f requires 28,315/5625 = 5 rivets; these are shown in place in Fig. 189. he vertical end reaction is carried to the gusset late by means of a pair of short angles which re connected to the plate by shop rivets in earing. As the gusset plate does not bear irectly on the sole plate, the rivets must carry he entire reaction to the gusset plate. From rt. 153, the panel load for the loading giving aximum stresses in the members is 4045 lb., and the end reaction is 4 × 4045 = 16,180 lb. The number of ivets required to connect the shoe angles to the gusset plate is 16,180/5625 = 3. Fig. 157 shows four rivets in lace. The number was increased to four in order to bind the shoe angles more firmly to the gusset plate, as the ngles were assumed to be 12 in. long.

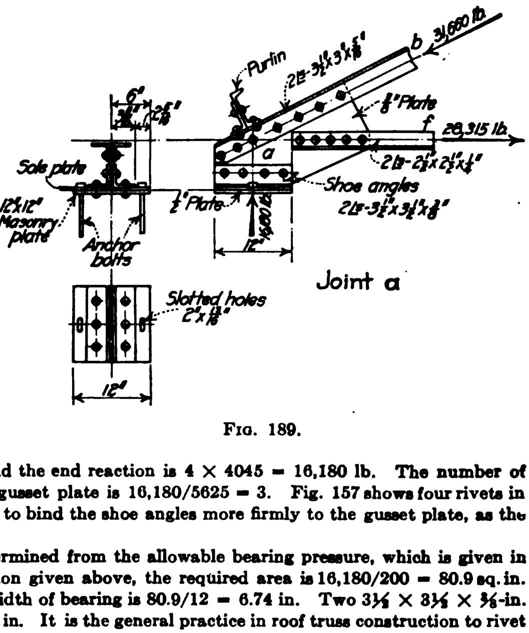


Fig. 189.

The bearing area on the masonry walls is determined from the allowable bearing pressure, which is given in rt. 148 as 200 lb. per sq. in. For the end reaction given above, the required area is 16,180/200 = 80.9 sq. in. ince the shoe angles are 12 in. long, the required width of bearing is 80.9/12 = 6.74 in. Two 3½ × 3½ × ⅜-in. ngles will be used, which will furnish a width of 7 in. It is the general practice in roof truss construction to rivet sole plate to the under side of the shoe angles, and also to place a masonry plate on the wall. These plates are ade wider than the shoe angles, in order to provide holes for the anchor bolts which are located outside the ngles, as shown in Fig. 189. A plate about 12 in. wide will allow sufficient room in the case under consideration. he thickness of the sole and masonry plates must be such that they will not be overstressed due to the upward ressure on the portion of the plates which overhang the shoe angles. If this overhanging portion be considered s a cantilever beam acted on by a uniform load equal to the reaction divided by the total area of the sole plate, he required thickness is readily determined. In this case, the upward pressure is carried by a 12 × 12-in. plate, nd the unit pressure is 16,180/144 = 112.2 lb. per sq. in. As shown in Fig. 189, the overhang is 2⁹⁄₁₆ in. The ending moment at the edge of the angle is then ½(2⁹⁄₁₆ × 112.2) 2⁹⁄₁₆ = 300 in.-lb. per inch of plate. As there re two plates under the shoe angles, it will be assumed that each plate carries one-half of the moment. The equired thickness for each plate can be determined from the formula $d = (6M/bf)^{\frac{1}{2}}$, where d = thickness of plate; f = bending moment per plate, which is 150 in.-lb.; b = width of plate under consideration, which is one inch; nd f = allowable working stress, which is 16,000 lb. per sq. in. Then

$$d = (6 \times 150/16{,}000)^{\frac{1}{2}} = 0.237 \text{ in.}$$

ach plate will be made ½ in. thick, as this is the thickness of plate generally used in practice.

The design of the joint shown in Fig. 190 (*a*) differs from the one given for the arrangement shown in Fig. 189 nly in the design of the bottom chord attachment. As shown in Fig. 190 (*a*), the stress in the bottom chord ember and the end reaction are brought to the gusset plate by the same group of rivets. Since the reaction and he chord stress do not have the same line of action, the rivets must be designed to carry the resultant of these forces. This resultant is $(16{,}180^2 + 28{,}315^2)^{1/2} = 32{,}600$ lb. The rivets are in bearing on a ⅝-in. plate, and their value is 5625 lb. per rivet; the number required is $32{,}600/5625 = 6$ rivets. Fig. 190 (a) shows the required number in place. It is desirable that these rivets be placed symmetrically with respect to the intersection of the center lines of the members. This is not always possible, due to insufficient room at the end of the chord member. The connection is therefore eccentric, and the rivets are subjected to additional stresses due to the induced moments. In general, the eccentricity, if unavoidable, should be kept as small as possible.

The stresses due to eccentricity are usually not calculated in practice. If desired, they can be calculated by the methods given on page 289. These methods will now be applied to the arrangement shown in Fig. 190 (a). The rivets are subjected to a horizontal load due to the stress in the bottom chord member, which is considered to be equally divided among the rivets, and to a vertical load which can be divided into parts. One part is due to the vertical reaction, assumed to be uniformly distributed over the rivets, and a second part due to the eccentric moment. Fig. (b) shows the assumed distribution of this latter part of the stress. It can be shown that the stress on the end rivets a and f, due to the eccentric moment, is given by the formula, $r = Mc/\Sigma x^2$, where r = stress on rivet, M = moment due to eccentricity, c = distance from center of gravity of rivet group to end rivet, and x = distance from center of gravity of rivet group to any rivet. From Fig 190, it can be seen that the eccentricity of the connection is one-half of a rivet space, or 1⅛ in. The eccentric moment is then, $M = 16{,}180 \times 1\frac{1}{8} = 18{,}200$ in.-lb. If the rivet spacing be taken as the unit distance, $c = 2.5$, and

$$\Sigma x^2 = 2(0.5^2 + 1.5^2 + 2.5^2) = 17.5$$

With these values we have, $r = 18{,}200 \times 2.5/17.5 = 2600$ lb. This load acts upward on rivet a and downward on rivet f, as shown in Fig. (b). The vertical load on rivet a due to the reaction is also an upward load, and its amount is $16{,}180/6 = 2700$ lb., giving a total vertical load of $2700 + 2600 = 5300$ lb. on rivet a. All other rivets have smaller loads, that on rivet f being the difference of the above values, or 100 lb. These values are to be combined with the loads brought to the rivets by the stress in the chord member, which is $28{,}315/6 = 4720$ lb. per rivet. The resultant stress on rivet a is $(5300^2 + 4720^2)^{1/2} = 7070$ lb., and that on rivet f is $(4720^2 + 100^2)^{1/2} = 4730$ lb. Values for other rivets vary between these two extreme values.

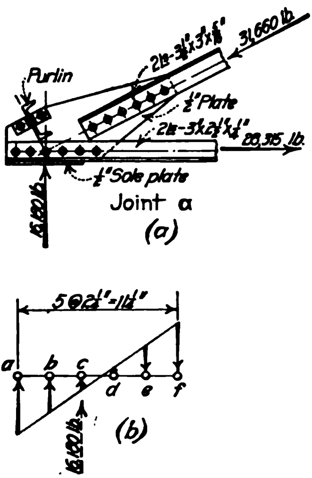


FIG. 190.

Since the allowable stress on a rivet for a ⅝-in. gusset plate is 5625 lb., the end rivet is overstressed. This can be relieved, either by reducing the eccentricity, which is not possible in this case, or by increasing the thickness of the gusset plate. From the tables of rivet values, it will be found that if the thickness of the gusset plate be increased to ½ in., the bearing value of the rivet will be 7500 lb. The rivets are then not overstressed, and the design is satisfactory. Other features of the design are the same as for Fig. 189.

The purlin connection for the design of Fig. 189 is the same as that for joints b and c. In the design of Fig. 190, the top chord angles do not provide proper support for the purlin. If a purlin is used at this point, a convenient method of support is provided by enlarging the gusset plate so that it will carry a standard channel connection, as shown in Fig. (a).

156. Minor Details.—In Art. 154, the compression members were designed on the assumption that the two angles forming the member act as a single piece. In order that this condition may be realized the angles must be riveted together at short intervals. The distance between the connecting rivets, which are known as stitch rivets, can be determined from the condition that for equal rigidity in all directions, the ratio of unsupported length to radius of gyration for a single angle must not exceed that for the composite member, as given in Table 2 of Art. 154. Thus, if L and R be respectively the unsupported length and the radius of gyration for the composite section, and l and r be the corresponding values for a single angle, we have

$$l = Lr/R$$

The value of L/R for member a–b is given in Table 2 of Art 154 as 76.5. From the steel handbooks the value of the least r for a $3\frac{1}{2} \times 3 \times \frac{5}{16}$-in. angle is 0.66 in. Substituting these values in the above equation, we have, $l = 76.5 \times 0.66 = 50.5$ in. Again, for member b–f, $L/R = 53.9$, $r = 0.42$, and therefore $l = 53.9 \times 0.42 = 22.6$ in. By the same method it will be found for member c–g that $l = 107.8 \times 0.42 = 45.3$ in. In practice, these connecting rivets are spaced from 2 to about 2½ ft. apart in compression members, and, although not required for tension members, they are generally provided, and are spaced from 3 to 3½ ft. apart. The space between the angles is maintained by means of ring fills, or washers, through which the rivets pass.

The ends of the truss are fastened to the masonry walls by means of anchor bolts. For trusses of the size under consideration in this design, anchor bolts $\frac{3}{4}$ in. in diameter and about 2 ft. long are used. Two bolts are placed at each end of the truss, as shown in Fig. 189.

To provide for the expansion of the truss due to temperature changes, it is the general practice to assume that the maximum range of temperature is 150 deg. With a coefficient of expansion for steel of 0.0000065, the change in length of a 50-ft. truss is $50 \times 150 \times 0.0000065 \times 12 = 0.585$ in., or nearly $\frac{5}{8}$ in. To allow for this movement, the anchor bolts at one end of the truss are usually set in slotted holes. Allowing $\frac{1}{16}$-in. clearance all around the anchor bolt, the required length of slot is $2 \times \frac{1}{16} + \frac{3}{4} + \frac{5}{8} = 1\frac{1}{2}$ in. In practice, a $\frac{13}{16} \times 2$-in. slotted hole would probably be provided.

The purlin connection for joint *c*, and for the other top chord joints, has been designed in Art. 155, and is shown in Fig. 184. As shown in Fig. 184, the clip angle consists of a short piece of $5 \times 3\frac{1}{2} \times \frac{5}{16}$-in. angle shop riveted to the top chord angles. The vertical leg of the clip angle should be long enough to extend well up on the flange of the channel, thus providing a means of support which will prevent overturning.

A sag tie is sometimes provided where the length of the bottom chord member *g–k* is such that excessive deflection is likely to occur due to the weight of the member. Sag ties are generally made of a single angle of the smallest size allowable under the specifications. Where the pitch of the truss is $\frac{1}{4}$, or less, the use of a sag tie is advisable.

157. Estimated Weight.—The truss members were designed for dead load stresses determined from an assumed weight of truss which was calculated from an empirical formula. It is generally taken for granted that the assumed weight is correct, and no attempt is made to calculate the weight of the truss as designed. This procedure is allowable, for, as pointed out in Art. 134, the dead weight of trusses of the size considered in this design is a comparatively small part of the total load to be carried by the truss. A considerable error can then be made in estimating the dead load without causing any appreciable error in the maximum stresses.

In order to check the correctness of the dead weight formula used in Art. 150, an estimate has been made of the truss as designed in the preceding articles. Layout drawings were made of the several joints and the sizes of plates and lengths of members determined from these sketches. Weights of members and plates were taken as given in the steel handbooks. The several items, as estimated, were: main members, 1700 lb.; gusset plates, 170 lb.; clip angles, rivet heads, and ring fills, 120 lb.; a total of 1990 lb. for one truss. As the horizontal covered area for one truss is $15 \times 50 = 750$ sq. ft., the true weight of the truss is $1990/750 = 2.65$ lb. per sq. ft. of horizontal covered area. In Art. 150 the weight of the truss, as estimated by the formula, is given as 2.7 lb. per sq. ft. The assumed and calculated weights agree so closely that no revision of stresses is necessary.

158. Design of Top Chord for Bending and Direct Stress.—In certain cases the limiting span of the roof covering is such that purlins must be placed between the panel points of the top chord. The top chord member is then subjected to bending as well as direct stress, and must be designed as a combination beam and column. To illustrate the design methods for such cases, the design of the preceding articles will be modified by placing a purlin at the center point of each top chord panel in addition to those placed at the panel points. Working conditions, loadings, and allowable stresses will be taken as assumed in Art. 148.

Proceeding as in Art. 152, using the same type of roof covering, but with purlins spaced 3.5 ft. apart, it will be found that the required purlin section is a 6-in. 8-lb. channel, which is the minimum section allowed under the conditions of Art. 148. This change in the purlin arrangement will cause a slight increase in the dead load stresses. However, for the purposes of this design, it will be assumed that the stresses in the members are unchanged, and that the values given in Table 1 of Art. 153 can be used in the subsequent calculations.

The chord section is to be designed for the same combinations of loading as used in Art. 151 for the design of the sheathing. Moments and simultaneous stresses are to be calculated for these combinations of loading, and a section chosen which will provide the area required by the maximum of these conditions of loading. In calculating the moments due to the applied loading, the chord sections may be considered as beams fixed at the ends, and the length may be taken as one panel. Based on these assumptions, Fig. 191 gives bending moment diagrams and moment coefficients for several loading conditions. These values were determined by the methods given in the chapter on Restrained and Continuous Beams in Sect. 1.

Fig. 192 shows the loading conditions for the several combinations of loading given in Art. 183. These loads can be resolved into components parallel and perpendicular to the chord members. It can readily be seen that the component perpendicular to the chord member will cause bending moments whose amounts can be determined by means of the coefficients given in Fig. 191, and that the components parallel to top chord tend to add to the compression in the member. The values given in Fig. 192 are in lb. per sq. ft. of roof surface.

Fig. 192 (*a*) shows the conditions for combined dead, snow, and wind load expressed as a uniform vertical load. Since the purlins are to be spaced 3.5 ft. apart, the roof area per purlin is 3.5 × 15 = 52.5 sq. ft. The normal load is then 52.5 × 26 = 1365 lb., and the component parallel to the chord member is 52.5 × 13 = 682 lb. To these loads must be added the corresponding components due to the weight of the purlin. As stated above, the adopted purlin is a 6-in. 8-lb. section. The end reaction at each truss, due to the weight of a purlin is 8 × 15 = 120 lb.; the normal component of the purlin load is 120 × cos 26° 34′ = 107 lb., and the component parallel to the top chord is 120 × sin 26° 34′ = 54 lb. This gives a total normal load of 1365 + 107 = 1472 lb., and a component parallel to the top chord of 682 + 54 = 736 lb. From col. 9 of Table 1, Art. 153, the stress in member *a*–*b* for combined vertical loading is 31,660 lb. Adding to this stress the component of load parallel to the chord member, the total stress in member *a*–*b* is 31,660 + 736 = 32,396 lb. From Fig. 191 the moments at the ends and at the center of a beam fixed at the ends and loaded with a single load placed at the beam center are equal to $Wl/8$, positive moment at the beam center, and negative moment at the ends. With W = 1472 lb., as calculated above, and l = 7 ft., the top chord panel length, the moments are, M = 1472 × 7 × 12/8 = 15,480 in.-lb.

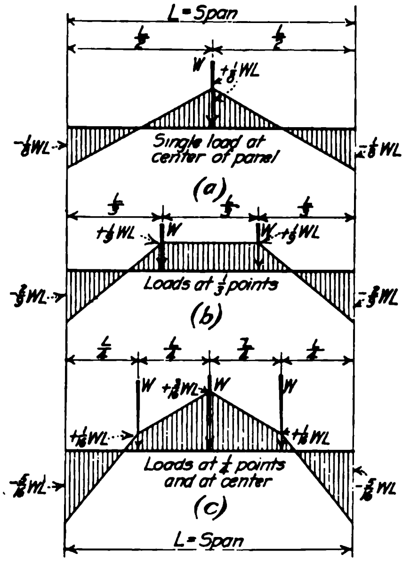


FIG. 191.

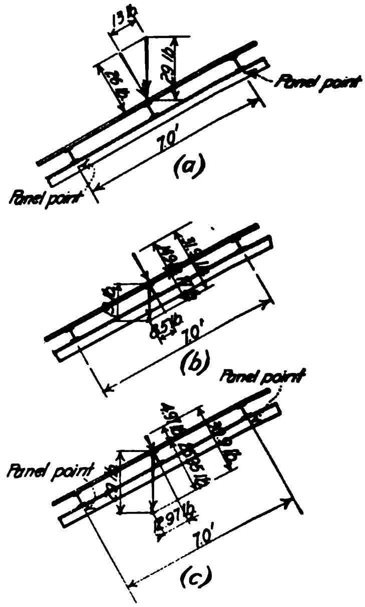


FIG. 192.

Fig. 192 (*b*) shows the components for dead load, one-half snow load, and maximum wind load, and Fig. (*c*) shows corresponding values for dead load, maximum snow load, and one-third wind load. These combinations correspond to cases (*b*) and (*c*) of Art. 151. By the same methods as used above, the moments and the simultaneous compression for the three conditions of loading shown in Fig. 192 are:

Condition of loading	Maximum moment	Simultaneous compression
Fig. (*a*)	15,480 in.-lb.	32,396 lb.
Fig. (*b*)	18,700 in.-lb.	26,895 lb.
Fig. (*c*)	18,120 in.-lb.	30,654 lb.

The required chord section can be determined by the methods given in the chapter on Bending and Direct Stress in Sect. 1. The method there given is applied to the cause under consideration by assuming a chord section and calculating the maximum fiber stresses due to the combinations of loadings given above. If the calculated fiber stresses agree closely with the allowable working values, the assumed section is accepted. If the calculated values are too small or too large, another trial must be made, until finally an agreement is reached between actual and allowable fiber stresses.

A method which leads more directly to the desired section is obtained from the following analysis. Consider first the case of a column acted upon by an axial load P. The maximum stress on the extreme fibers of the section is given by the expression, $f = P/A + Pec/I$, where P = axial load; A = area of section; e = eccentricity of load application due to imperfect centering of the load and to imperfections in column construction; c = distance from column center to extreme fiber; and, I = moment of inertia of the column section. If Ar^2 be substituted for I, where r is the radius of gyration of the section, the above equation can be written in the form, $f = P/A(1 + ec/r^2)$. Solving for the required area, we have,

$$A = P(1 + ec/r^2)/f \tag{1}$$

As stated by eq. (1), the area of the column section for a given load P is found by increasing the load by a certain percentage, and dividing this increased load by the maximum allowable fiber stress The general practice in column design is to use the column load without increase, and to allow for the term ec/r^2 of eq. (1) by reducing the allowable working stress. This reduction in working stress is made by means of a selected column formula. Eq. (1) is then changed to read

$$A = P/f_c \tag{2}$$

where f_c is the working stress as given by the column formula.

Consider now the case of a column subjected to a moment M in addition to the axial load P The total stress on the extreme fibers of the section will be

$$f = P/A + Pe/I + Mc/I = P/A(1 + ec/r^2) + Mc/Ar^2$$

Solving for A, the required area, we have

$$A = P(1 + ec/r^2)f + Mc/fr^2$$

It will be noted that the first term of this expression is the same as eq. (1). Replacing this term by one of the form of eq. (2), we have

$$A = P/f_c + Mc/fr^2 \tag{3}$$

That is the area required for a column subjected to bending and direct stress is equal to the area required as a beam plus the area required as a column; the fiber stress for bending is the maximum allowable, in this case 16,000 lb. per sq. in., and the fiber stress for column action is that given by the column formula, which in this case in $16{,}000 - 70\,l/r$. The value of r is to be taken for the entire section.

In applying eq. (3) to the determination of the section required for the several combinations of moment and direct stress given above, it will probably be found best to make a rough calculation of area, using moments and loads which are the average of the given values. Next assume that an angle with a certain width of leg is to be used. Approximate values of c and r can be used in this calculation. From the handbooks it will be found that for unequal angles with the longer legs placed back to back, the values of c and r are practically equal for an axis parallel to the shorter legs, and that they are approximately equal to ⅓ of the length of the longer legs. On comparing the area determined by the substitution of these approximate quantities in eq. (3) with the areas given in the handbooks for angles of the assumed width, it is possible to tell whether a wider or narrower angle should be used.

For the case under consideration, a rough average of the moments and direct loads is M = 18,000 in.-lb., and P = 30,000 lb. Assume that a 4-in. angle is to be used. The approximate values of c and r will be $\frac{1}{3} \times 4 = 1.33$ in. In applying eq. (3), substitutions must be made for points at the center and at the end of the member. This is due to the fact that column action is present at the center of the member, while at the ends of the member simple compression exists. Again, at the center of the member the moment is positive and at the ends the moment is negative. The compression fiber is then at the top of the member at its center point, and $c = \frac{1}{3}$ width of member; at the end points the compression fiber is on the side of the member, and $c = \frac{2}{3}$ width of member. The greater of the areas thus obtained determines the area required for the member.

The length of the member under consideration is given in Table 2 of Art. 154 as 84 in. Then with $r = 1.33$, we have $f_c = 16{,}000 - 70\,l/r = 16{,}000 - 70 \times 84/1.33 = 11{,}670$ lb. per sq. in. The calculated areas are as follows:

At center of member,

$$A_c = \frac{30{,}000}{11{,}670} + \frac{18{,}000 \times 1.33}{16{,}000 \times (1.33)^2} = 2.57 + 0.85 = 3.42 \text{ sq. in.}$$

At end of member,

$$A_e = \frac{30{,}000}{16{,}000} + \frac{18{,}000 \times 2.66}{16{,}000 \times (1.33)^2} = 1.87 + 1.70 = 3.57 \text{ sq. in.}$$

From the steel handbooks, it will be found that the area of the smallest 4-in. angle is 4.18 sq. in. Similar trials made for 3 and 5-in. angles showed that the former was probably too small, and the latter too large. More exact calculations will therefore be made for the 4-in. angles.

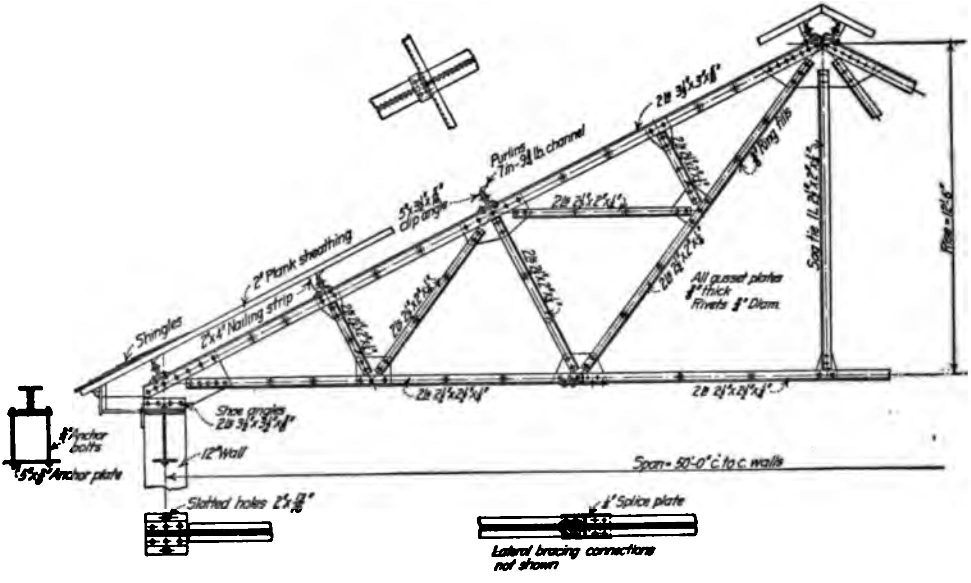


FIG. 193.—General drawing of 50-ft. steel roof truss.

The chord section will be assumed as made up of two 4 × 3 × $\frac{5}{16}$ in. angles with the 4-in. legs separated by a $\frac{3}{8}$-in. space. Since the chord member is supported laterally at its center point by the purlins, the greatest unsupported length is in a vertical plane. From the steel handbooks, $r = 1.27$ in., and $c = 1.26$ in. at the center of the member and $c = 4.0 - 1.27 = 2.64$ in. at the end of the member. From the column formula, $f_c = 16{,}000 - 70 \times 84/1.27 = 11{,}370$ lb. per sq. in. Proceeding as above, it will be found that the values given for the conditions of Fig. (c) require the greatest area. These calculations follow.

Area required for condition of loading shown in Fig. 193 (c):

At center of member

$$A_c = \frac{30{,}654}{11{,}370} + \frac{18{,}120 \times 1.26}{16{,}000 \times 1.27^2} = 3.59 \text{ sq. in.}$$

At end of member

$$A_e = \frac{30{,}654}{16{,}000} + \frac{18{,}120 \times 2.74}{16{,}000 \times 1.27^2} = 3.85 \text{ sq. in.}$$

For the conditions of loading shown in Figs. (a) and (b), the results obtained were as follows: (a) $A_c = 3.60$ sq. in., $A_e = 3.66$ sq. in.; and (b) $A_c = 3.28$ sq. in., $A_e = 3.66$ sq. in. Since the calculated areas are all less than that furnished by the assumed angles, whose area is 4.18 sq. in., and since the agreement between required and provided areas was as close as could be obtained, using standard angles, the assumed section will be adopted.

The design of the top chord section, as given above, is based on the assumption that the chord members act as beams fixed at the ends. At panels points where the member is continuous across the joint, as at *b*, *c*, etc., this assumption is probably realized. At joint *a* the chord member is riveted to the gusset plate. In order to fix this point, an external moment must be applied which will be equal to the moment brought to the joint due to the end moment in the fixed beam. The lower chord member and the bearing of the shoe on the masonry will offer some resistance to the moment, but as the lower chord member is not as rigid as the top chord, it can not be depended upon to provide fixed end conditions at the joint.

An external moment of the desired amount can be produced at joint *a* by making the center line of the reaction eccentric with respect to the intersection of the center lines of the members. Thus, for the conditions governing the chord design, the end moment is 18,120 in.-lb., and the end reaction is 16,180 lb. The required eccentricity is then $18{,}120/16{,}180 = 1.12$ in. Since the end moment is negative, it tends to cause a clockwise rotation of the joint. If the reaction line be moved 1.2 in. to the right of the position shown in Fig. 189, the desired eccentric moment will be produced. A similar result can be obtained for the design shown in Fig. 190.

159. Design of Bracing.—A general discussion of the bracing of roof trusses is given in Art. 129. Bracing for roof trusses of the type considered in this chapter is generally placed only in the plane of the lower chord of the truss. It is usually assumed that the sheathing and purlins, when placed in position, will provide sufficient bracing for the plane of the top chords. In some cases a ridge strut running the full length of the building is placed at the apex of the truss. This ridge strut serves also as erection bracing before the purlins are placed in position. Where the roof covering is corrugated steel, bracing is generally placed in the plane of the top chord, as the corrugated steel is not rigid enough to provide the necessary lateral support.

Bracing of the type mentioned above is not subjected to any definite loads; a rigid analysis of stresses can not be made. The designer must rely upon his judgment and experience in determining the type and position of the bracing, and the size of the members to be used in any structure.

Fig. 180 shows the arrangement of bracing which will be adopted for the truss under consideration. Pairs of trusses near the ends of the building will be provided with diagonal bracing placed in the plane of the bottom chord. The other trusses will be connected to the braced trusses by means of a continuous line of struts placed in the plane of the bottom chord. These struts are located at joints *g* and *k*. In addition to this bracing a ridge strut, located at joint *e*, will be run the full length of the building.

The diagonal members of the bracing in the plane of the lower chord will be made of single angles of minimum size. As the angles are to be connected by one leg only, a $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle will be used. The struts will be considered as compression members; their size will be determined subject to the condition that l/r must not exceed 150, which is the limiting value set for such members in Art. 148. As the trusses are 15 ft. apart, the angles must have a radius of gyration of at least $r = l/150 = 12 \times 15/150 = 1.2$ in. From the steel handbooks it will be found that the standard angles of least weight which will answer the requirements are two $4 \times 3 \times \frac{5}{16}$-in. angles placed with the 4-in. legs vertical and separated by at least a $\frac{1}{4}$-in. space. These angles will therefore be used for the struts between trusses, and also for the ridge struts.

The bracing in the plane of the lower chord of the truss is attached to plates riveted to the truss, as shown in Fig. 193. At joint *g* the splice plate on the horizontal legs of the bottom chord angles is enlarged to include the connecting rivets in addition to those required for the splice. An exact determination of the number of rivets required in the ends of the bracing angles can not be made, as these members have no definite stress. Some designers assume that the connections are to be designed for the full strength of the member. On this assumption the $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angles would require $16{,}000(1.06 - 0.22)/2810 = 5$ field rivets. Experience shows that for small trusses, two rivets are sufficient.

160. The General Drawing.—Fig. 193 shows a general drawing of the truss designed in the preceding articles. On this drawing is shown the sizes of members, thickness of gusset plates, number of rivets in the members at each joint, arrangement of bracing, and all other details determined in the preceding calculations. It will be noted that only the general features of the design are shown on this drawing. This is the type of drawing turned out by the average designing office.

Before the truss can be constructed in the shop, a drawing must be made showing in greater detail the dimensions of the members and plates and the spacing of the rivets. A drawing of this nature is known as a shop drawing. The principles governing the making of shop drawings are given in the chapter on Structural Steel Detailing. The reader is referred to p. 319 for a complete shop drawing of a truss quite similar to the one designed in the preceding articles.

DETAILED DESIGN OF A TRUSS WITH KNEE-BRACES

By W. S. Kinne

161. General Considerations and Form of Trusses.—The discussion of the preceding chapter was confined to roof trusses supported on rigid masonry walls. This type of structure is shown in Fig. 194 (*a*). The truss is not called upon to assist in carrying lateral forces. Resistance to lateral forces is provided by the walls on which the truss is simply supported.

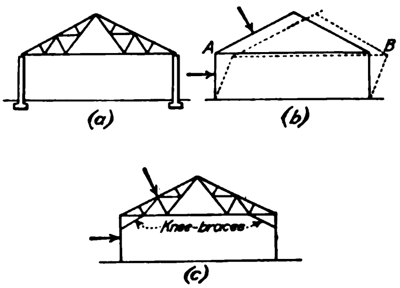


Fig. 194.

In certain types of structures, particularly mill buildings and storage sheds, the trusses are supported on steel columns, as shown in Fig. (*b*). The outside walls are formed either by a curtain wall of brick, or by sheathing or corrugated steel siding which is supported by the columns. In either case these walls act merely as partitions, and do not assist in carrying lateral forces, as in the case of the rigid walls of Fig. (*a*). If lateral forces are applied to a truss resting on columns, as shown in Fig. (*b*), the structure tends to collapse, as shown by the dotted lines. This distortion must be prevented by bracing capable of resisting horizontal forces.

The bracing provided to resist horizontal forces must answer two conditions. It must not obstruct the clear space between the walls and the lower chord of the trusses, and it must provide a means of joining the trusses and the columns into a rigid frame work. In small structures the required resistance to distortion is sometimes provided by means of riveted joints at A and B of Fig. (*b*). This method is not economical, even for trusses of moderate size. Fig. 194 (*c*) shows a simple means of providing the required bracing. Short members known as knee-braces, are connected to the column and to a lower chord panel point. The structure thus formed answers the above requirements, and the stresses in the members are readily determined.

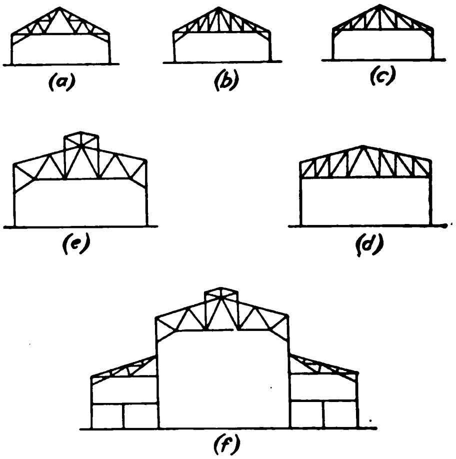


Fig. 195.

Fig. 195 shows a few of the forms of knee-braced bents in common use. Fig. (*a*) shows a Fink truss with knee-braces, and Figs. (*b*) and (*c*) show trusses of the Pratt type. Fig. (*d*) shows a flat Pratt truss with the end members prolonged to form a column. Other forms of trusses can be arranged in a similar manner. Figs. (*e*) and (*f*) show trusses provided with a monitor at the apex. In the form shown in Fig. (*f*), side trusses are also provided.

162. General Methods of Stress Determination.—Fig. 196 shows a knee-braced bent acted on by wind loads W_1 perpendicular to the side walls, and loads W_2 normal to the roof surface. General methods of stress determination will be developed for the conditions shown in Fig. 196. Assume first that the truss is simply supported at points A and B by hinges, or by some method which will prevent horizontal movement under the action of the applied loads. Let R of Fig. (*a*) represent the resultant of the loads W_1 and W_2. The reactions at A and B are to be determined for the force R.

For the conditions shown in Fig. 196, it will be noted that there are four unknowns to be determined; a vertical and a horizontal force at A and B. The problem is therefore *indeterminate,* for, as stated in the chapter on Principles of Statics in Sect. 1, only three unknowns can be determined in any system of non-concurrent forces. Some assumption must then be made regarding the relation between certain of these forces before a solution can be made. It will be convenient in this case to consider the relation between the horizontal components of the forces at A and B. The desired relation can be obtained from a principle brought out in the analysis of statically indeterminate structures which states that where there is more than one path over which the stresses due to a given load may pass in order to reach the abutments or points of support, the load will be divided over these paths in proportion to their relative rigidities. It is reasonable to assume in this case that the loads are transmitted from the truss to the columns and thence to the points of support. As the columns are generally made alike, and are therefore of equal rigidity, it is usually assumed that the horizontal components of the applied loads are equally divided between the two points of support. Thus, if H be the horizontal component of R, we have

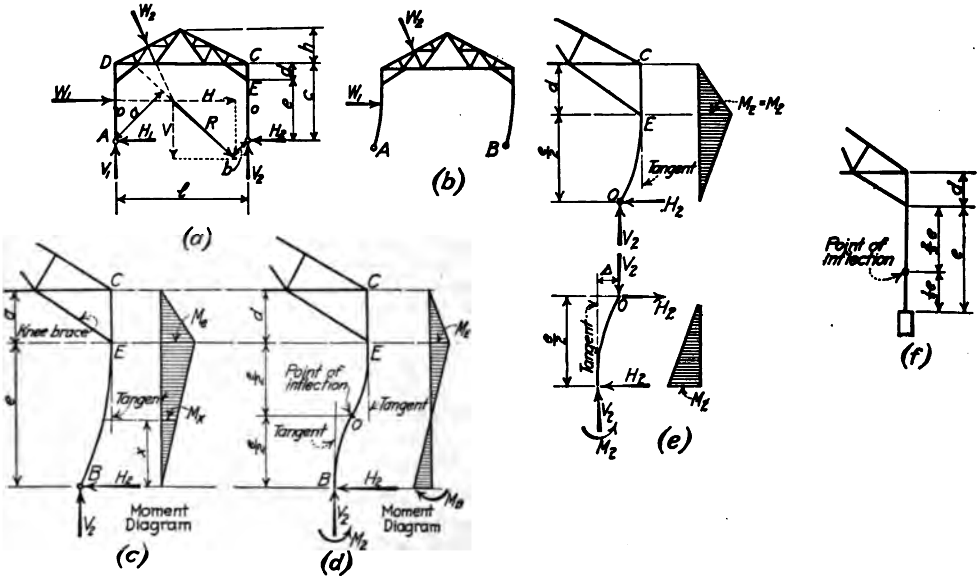


FIG. 196.

$$H_1 = H_2 = H/2 \qquad (1)$$

where H_1 and H_2 represent the horizontal components of the reactions at A and B, Fig. 196 (*a*). The vertical components of the reactions, shown by V_1 and V_2 in Fig. (*a*), can be determined by moments. Thus in general terms, we have from moments about B

$$V_1 = Rb/l \qquad (2)$$

and from moments about A

$$V_2 = Ra/l \qquad (3)$$

The reactions are thus completely determined.

Before proceeding to the determination of the stresses in the truss members, it will be necessary to consider the conditions existing in the columns. As shown in Fig. 196 (*a*), the horizontal forces are carried to the points of support by means of a vertical member. As the loads act at right angles to the member, it is subjected to bending as well as direct stress. The distortion of the structure as a whole is of the nature shown in Fig. (*b*). In Fig. (*c*) is shown, to an enlarged scale, one of the distorted columns. Since the column is riveted to the truss at point C, and to the knee-brace at point E, it seems reasonable to assume that E–C remains vertical, and that the distortion of E–B greatly magnified, is as shown in Fig. (*c*). The column is then a three force piece, as it is subjected to bending moment, shear, and direct stress at all points. If M_x, V_x, S_x represent these quantities at any section a distance x above the base of the column, we have for member B–E of Fig. 196.

$$M_x = H_2x \qquad V_x = H_2 \qquad S_x = V_2 \qquad (4)$$

The moment, as given by the first of these expressions, is a maximum at point E, the foot of the knee-brace, varying uniformly to zero at the foot of the column, as shown by the moment diagram of Fig. (c). Values of the shear and direct stress for member C–E depend on the stress in the knee-brace, which is as yet unknown.

In general the columns are rigidly fastened to the foundations by a detail of the type shown in Fig. 210. The distortion of the column is then of the nature shown in Fig. 196 (d). When the base is fixed, the tangent to the curve at point B can be assumed to be vertical. As the tangent at E is also vertical, the curvature between the two points can be assumed to be a reversed curve, with the point of inflection, or change in curvature, at point O, halfway between E and B. Since a point of inflection is also a point of zero moment, the variation in moment for member B–C is as shown in Fig. (d). The moment at O is zero, and the moments at points equal distances above and below O are equal in amount, but opposite in kind. It will be noted that the portion O–E of the deformed column of Fig. (d) is similar to the portion B–E of Fig. (c). Since the moment at O is zero, this point can be regarded as a hinged joint. In the determination of stresses the column can be separated into two parts at point O, as shown in Fig. (e). The reactions, as given by eqs. (1), (2), and (3), are to be calculated for a knee-braced bent consisting of that part of the structure above points O of Fig. (a). The moment at the base of the column can be determined from the conditions shown in Fig. (e) for the lower portion of the column.

The position of the point of inflection has an important bearing on the stresses in the members. It can be seen from eqs (1), (2), and)3) and from Fig. (a), that the values of the reactions depend upon the effective height of the bent. A fixed end bent, considered as hinged at O, midway between the knee-brace and the base, will in general have smaller stresses in its members than one with simply supported ends, considered as hinged at A and B. However, unless the connections at E and C of Fig. (d) are absolutely rigid, and the base of the column is fixed, the point of inflection, O, can not be assumed as located halfway between the base of the column and the foot of the knee-brace. Any tendency of the tangents to deviate from the vertical will cause the point of inflection to be lowered, the limit being points A and B, or a hinged connection at the base of the columns. Since the base of the column is usually rather wide in the plane of the truss, it can always be considered as partially fixed due to the action of the dead load. In most cases the column is firmly attached to the foundations by means of anchor bolts which are screwed up tight. As long as these bolts remain tight, the base of the column can be considered as fixed. But experience shows that this can not be relied upon. It seems best, therefore, to assume that the point of inflection is somewhat below the mid-point between the knee-brace and the base of the column. This assumption is on the safe side, as the stresses in the truss members are increased thereby, and the moment to be carried by the columns is also increased.

In the calculations to follow, it will be assumed that the distance from the base of the column to the point of inflection is one-third of the distance from the base of the column to the foot of the knee-brace, as shown in Fig. (f). There is considerable difference of opinion among designers and writers on this point. The recommendation made above seems to be reasonable and to be founded on conditions which actually exist in the structure; it will therefore be adopted.

Methods of stress calculation are best explained by means of a problem. For this purpose, a truss of the form considered in the preceding chapter will be placed on columns and provided with knee-braces. Fig. 197 shows the dimensions of the knee-braced bent thus formed. The wind pressure on a vertical surface will be taken as 20 lb. per sq. ft., and that on an inclined surface will be 20 lb. reduced by the Duchemin formula, which is given in Art. 135. Since the assumed conditions are the same as for the design given in the preceding chapter, the wind panel load normal to the roof surface is 1565 lb., as calculated in Art. 153. The total horizontal load on the side of the structure above the point of inflection is 15 × 15 × 20 = 4500 lb. This load is distributed to the vertical panel points as shown in Fig. 198(a). It will be assumed that the bases of the columns are partially fixed, and that the point of inflection is located at a point above the base of the column equal to one-third of the distance between the base and the foot of the knee-brace, as shown in Fig. 197. Figs. 197 and 198 (a) show the portion of the bent above the assumed points of inflection, with the applied loads in position.

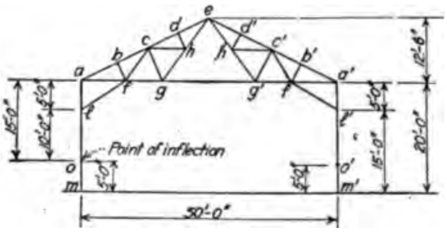


FIG. 197.

The reactions at the points of inflection, O and O' of Fig. 197, assumed to be points of support for a hinged knee-braced bent, can be calculated by the methods given in Sect. 1. From Fig. 198 (a), the total horizontal component of applied loads is $4500 + 6260 \sin 26° 34' = 4500 + 6260 \times 0.447 = 4500 + 2800 = 7300$ lb. The horizontal components of the reactions, as determined from eq. (2), are

$$H_1 = H_2 = H/2 = 7300/2 = 3650 \text{ lb.}$$

The forces act as shown in Fig. 198 (*a*). The vertical reactions are determined from moments about the bases of the columns, using eqs. (2) and (3). Thus for R_2, from moments about O with dimensions and loads as shown on Fig. 198 (*a*), we have

$$R_2 = \frac{6260 \times 20.71 + 4500 \times 7.5}{50} = 3260 \text{ lb.}$$

and

$$R_1 = \frac{6260 \times 23.99 - 4500 \times 7.5}{50} = 2340 \text{ lb.}$$

These forces are shown in position on Fig. 198 (*a*). All external are thus completely determined.

The next step in the calculations is the determination of the stresses in the members of the truss. In general it will be found that graphical methods of stress determination are preferable for this purpose. Algebraic methods of stress calculation are somewhat more precise than graphical methods, but in the application of algebraic methods considerable time is consumed in the calculation of lever arms of loads and members. This is avoided by the use of graphical methods, and the results obtained are accurate enough for all practical purposes.

In the application of graphical methods to a knee-braced bent a little difficulty is encountered in the case of the columns. These members are subjected to shear, moment, and direct stress, thus forming three force pieces. The graphical methods of Sect. 1 are applicable only to one force pieces—that is, members subjected either to tension or compression. Two methods can be employed for the graphical solution of the case under consideration: (*a*) The columns can be removed and in their place can be substituted a system of forces whose effect on the structure as a whole will be the same as that of the columns, and (*b*) since a moment can be considered as a force times a distance, a temporary framework can be added to the truss system, arranged so that the moment at the foot of the knee-brace will cause stress in the members of the auxiliary framework. After the stresses in all members of the truss have been determined, the temporary framework can be removed and the true stresses in the columns determined. This method is quite similar in principle to the one given in Sect. 1, Art. 84, for the determination of the stresses in certain members of the Fink truss. The methods described above will now be applied to the knee-braced bent of Fig. 198 (*a*).

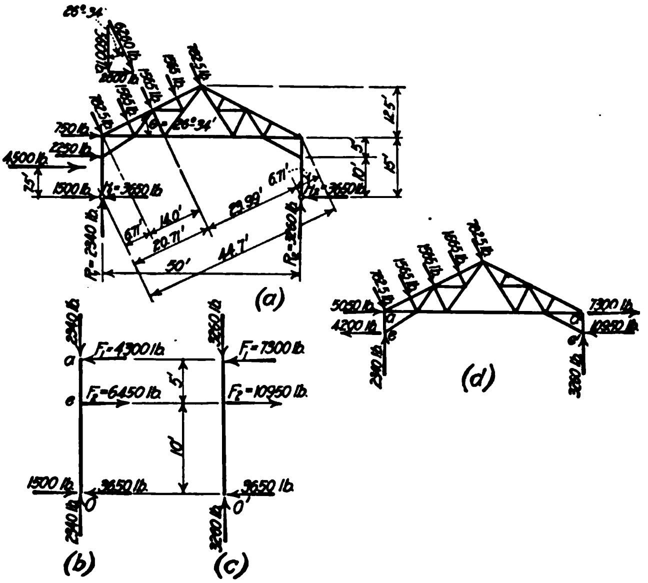


Fig. 198.

The application of the first method outlined above is shown in Figs. 198 (*b*), (*c*), and (*d*). Figs. (*b*) and (*c*) show the columns removed with all forces acting. Forces F_1 and F_2 show the action of the column on the truss. These forces are determined by the methods of statics, subject to the condition that the column is in complete equilibrium. From Fig. (*b*), which shows the conditions for the windward column, moments about point l give

$$F_1 = (3650 - 1500)10/5 = 4300 \text{ lb.}$$

and moments about point a give

$$F_2 = (3650 - 1500)15/5 = 6450 \text{ lb.}$$

For the leeward column, shown in Fig. (c)

$$F_1 = 3650 \times 10/5 = 7300 \text{ lb.}$$

and

$$F_2 = 3650 \times 15/5 = 10{,}950 \text{ lb.}$$

All forces are shown in position in Figs. (b) and (c).

Since action and reaction are equal in amount but opposite in direction, forces F_1 and F_2 are to be applied to the truss in directions opposite to those shown in Figs. (b) and (c). They appear directly on the leeward side, but on the windward side they are to be combined with the loads shown at a and e of Fig. (a). At a the applied load is 4300 + 750 = 5050 lb., and at e the load is 6450 − 2250 = 4200 lb. These forces are shown in position and direction on Fig. 198 (d). At the foot of the knee-brace, vertical forces equal to the reaction at the foot of the column are applied, as shown in Fig. (d). The resulting forces hold the structure in equilibrium.

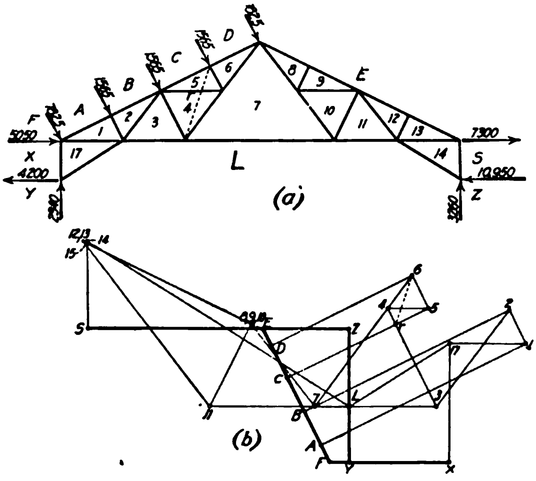


FIG. 199.—Knee-braced trusses.

Fig. 199 (b) shows the stress diagram for the forces shown on Fig. 198 (d) and repeated on Fig. 199 (a). This stress diagram is constructed by the methods given in Sect. 1. The stresses in the members, as scaled from the diagram, are recorded in col. 4 and 6 of Table 1, Art. 164. The stresses in the upper portion of the columns are given directly in the stress diagram. In the lower portions of the columns, the stress is equal to the reaction at the point in question, as given in Fig. 198 (d).

The temporary framework for the second method of stress determination outlined above is shown in Fig. 200 (a). Any convenient arrangement can be used. In this case the top chord member was prolonged to an intersection with a horizontal through the foot of the knee-brace. This point was then connected to the foot of the column by a temporary member. These members are shown by dashed lines in Fig. 200 (a). The loads applied to the windward side of the building are considered as acting at the joints of the auxiliary framework, as shown in Fig. (a). With the auxiliary framework in place, it is possible to draw the stress diagrams for all joints. Fig. 200 (b) shows the complete stress diagram.

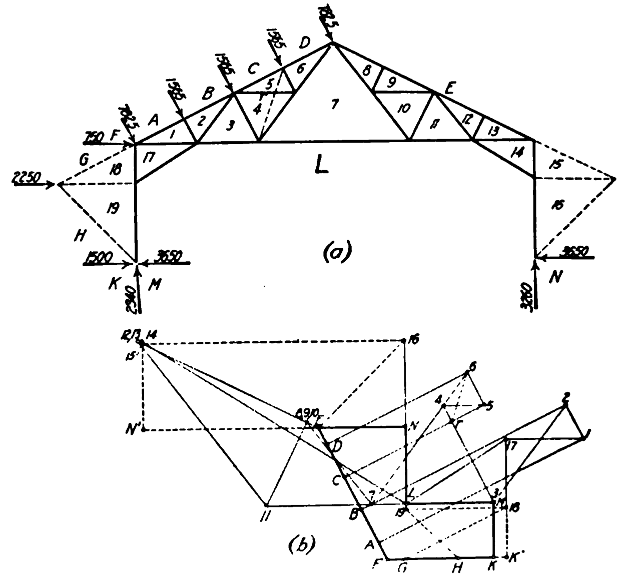


FIG. 200.—Knee-braced trusses.

The stresses for the columns, as given by the stress diagram of Fig. (b), are not the true stresses for these members, for the addition of the auxiliary frames has effected the stresses in the columns; all other stresses are the true stresses in the members in question. To determine the true stresses in these members, the auxiliary frames must be removed and the column stresses redetermined, subject to conditions which will be discussed later. Thus for the windward column it can be seen by inspection that as soon as the framework is removed, the stress in the lower section of the column is a compression which is directly equal to the reaction at the foot of the column, which in this case is 2340 lb. Consider the upper portion of the column. It is quite evident that the stress in this member must be of such magnitude that it will hold in equilibrium the stress in the lower portion of the column plus the vertical component of the stress in the windward knee-brace The desired stress can be determined from Fig. (b) by locating the forces mentioned and adding them graphically. In Fig. 200 (*b*), *K–M* represents the reaction at the foot of the column, and *L*–17 represents the stress in the knee-brace. If these forces be projected on a vertical line drawn through point 17, we have as the sum of these forces the component *K′*–17, which represents the amount of the desired stress in the upper portion of the column; the stress as scaled from the stress diagram is 5000 lb., and the kind of stress is compression. Similar methods are to be used for the leeward column. As before, the stress in the lower portion of the column is compression, and it is equal to the reaction at the foot of the column. Since the stress in the leeward knee-brace is compression, its vertical component acts downward. Therefore the stress in the upper portion of the column must balance the difference between the stress in the lower portion of the column and the vertical component of the stress in the knee-brace. The desired stress can be determined from Fig. 200 (*b*). The force *L–N* represents the reaction at the foot of the column, and *L*-14 represents the stress in the leeward knee-brace. If these forces be projected on a vertical line through point 14, the required difference in stress components will be represented by the force *N′*–14. The required stress scales 3700 lb., and the kind of stress is tension.

On comparing the two methods given above, it will be found that the construction of the auxiliary frames required by the second method involves less time and is a simpler process than the calculation of the external forces required for the first method. The stress diagrams constructed for the two methods lead to exactly the same results, if the operations are correctly performed. However, it will be found that the stress diagram for the first method can be more accurately constructed than the one for the second method. This is partly due to the fact that the stress diagram of the first method contains four less joints than the one for the second method, and also to the fact that it is difficult to arrange an auxiliary framework which will provide good intersections for the lines of action of the resulting stresses. Again, the stresses in the columns are given directly by the stress diagram for the first method, but, from the discussion given above, it can be seen that the determination of the column stresses by the second method requires considerable care and study. Everything considered, the first method of calculation, as shown in Fig. 199, is preferable, and it is recommended as the best method of stress determination for problems of the nature here considered.

163. Conditions for the Design of a Knee-braced Bent.—To illustrate the principles of design for a knee-braced bent, a truss of the span length and type designed in the preceding chapter will be placed on columns and provided with knee-braces. The columns will be made 20 ft. high, and the knee-brace will intersect the column at a point 5 ft. below the top of the column. Fig. 197 shows the structure thus formed. The distance between the trusses will be taken as 15 ft., and the roof covering will be made the same as used in the design of the preceding chapter. In this way much of the material of the preceding design can be used for the structure under consideration. It is not probable that a shingle roof would be used in practice for a structure of this type. A corrugated steel or a slate or tile is a more practical type of roofing. However, the general principles of design are the same for all cases, and the discussion given in this chapter can readily be modified for any type of roof covering.

Loadings and working stresses will be the same as given in Arts. 148 and 150 of the preceding chapter, with the exception of the dead load of the trusses, which will be determined by the Ketchum formula given in the chapter on Roof Trusses—General Design. This formula is $w = P/45\ (1 + L/5\sqrt{A})$, where P = capacity of truss, which will be taken as 40 lb. per sq. ft. of horizontal covered area; L = span in feet; A = distance between trusses, which will be 15 ft.; and w = weight of truss per sq. ft. of horizontal covered area. With the above values, $w = 3.18$ lb. To allow for that part of the bracing carried by the trusses, this weight will be increased to 4.25 lb. per sq. ft. of horizontal covered area. The snow load will be taken 20 lb. per sq. ft. of roof surface, and the wind loads on the sides and the roof will be based on a unit pressure of 30 lb. per sq. ft. on a vertical surface. This unit pressure will provide for all possible wind stress conditions for a structure in an exposed position. If the structure is in a sheltered location, a unit pressure of 15 or 20 lb. per sq. ft. would be sufficient. The wind pressure will be assumed to act normal to the roof surface and perpendicular to the sides of the building.

Working stresses for steel in tension will be 16,000 lb. per sq. in. on the net section of the member. For compression the working stress will be given by the formula $16{,}000 - 70l/r$, where l = greatest unsupported length of member, and r = least radius of gyration of the section. Gross areas are used, and l/r is limited to 125 for main members and to 50 for bracing.

Corresponding working stresses for wind loadings will be based on 24,000 lb. per sq. in., as in the preceding chapter. Rivet values for shop rivets are to be based on an allowable shearing value of 10,000 lb. per sq. in., and an allowable bearing value of 20,000 lb. per sq. in.; corresponding values for field rivets are 7500 lb. for shear and 15,000 for bearing. Rivets ¾ in. in diameter will be used. The minimum thickness of material will be ¼ in.

Members and connections subjected to a reversal of stress will be designed for each kind of stress. This assumption is reasonable, for the reversal in stress is due to a change in the direction of the wind. This can not occur suddenly, so that there will be a time interval between the two kinds of stress.

As stated in Art. 162, there is considerable uncertainty regarding the exact conditions at the bases of the columns. In many cases it is assumed that the point of inflection, shown in Figs. 197 and 198, is located half way between the base of the column and the foot of the knee-brace. This assumption requires rigid connections between the column and the knee-brace and a rigid connection between the column and the truss. Also, the base of the column must be rigidly attached to the foundations, which must be immovable. All of these conditions must be realized before the above assumption can be made. As it is practically impossible to secure all of these conditions, it does not seem advisable to assume that fixed end conditions exist. However, the end detail of the base of the column, as shown in Fig. 202, is so arranged that it is probable that the assumption of hinged ends is not justified, as the base is flat, and is fixed to some extent by the dead load. It therefore seems best to assume that the base is partially fixed, and that the point of inflection is somewhat below the mid-point of the column. In an excellent article on Wind Stresses in Steel Mill Buildings,[1] R. Fleming recommends that the point of inflection be taken at a point one-third of the distance between the foot of the column and the knee-brace. This recommendation has been followed in the solution of the problem of Art. 195, and will be adopted for the design to be made.

164. Determination of Stresses in Members.—The stresses in the members are to be determined for the same general conditions as in the design of the preceding chapter. In this case, however, it is not possible to use an equivalent uniform load to represent the effect of wind and snow combined. The stresses for these loadings must be determined separately and combined with the dead load for the following conditions: (*a*) dead load and snow load; (*b*) dead load and wind load; (*c*) dead load, minimum (one-half) snow load, and maximum wind load; and, (*d*) dead load, maximum snow load, and minimum (one-third) wind load. In making up these combinations, the greater of the wind stresses given in cols. 4 or 6 of Table 1 is to be used. This will provide for all possible conditions. The maximum stress determined from these combinations is to be used in the design of the member. It will be noted that condition (*b*) often results in a reversal of stress in the member.

Since the adopted roof covering, the loading conditions, and the working stresses are the same as for the design of the preceding chapter, the dead panel load due to the roof covering and the purlins will be the same as given in Art. 153 of the preceding chapter. The panel load due to the roofing is then 945 lb., and that due to the purlin is 146.3 lb. As given above in Art. 163, the weight of the truss and bracing is 4.25 lb. per sq. ft. of horizontal covered area. From the preceding chapter, the horizontal covered area per panel is $15 \times \frac{50}{8} = 93.75$ sq. ft. The panel load due to the weight of the truss is then $93.75 \times 4.25 = 398.4$ lb. The total dead panel load is then $945.0 + 146.3 + 398.4 = 1489.7$ lb.; a load of 1490 lb. will be used in the calculations to follow.

In the calculation of the stresses in the members of the knee-braced bent shown in Fig. 164, it is the usual practice to assume that the knee-braces are not stressed by the action of vertical loads. This assumption is not strictly correct, for the deflection of points f and f' is resisted by the knee-brace, which is thus subjected to a small stress. At the same time, a small bending moment is set up in the column. These stresses and moments are so small compared to the other stresses and moments that the stresses due to the deflection of points f and f' can be neglected. This is equivalent to removing the knee-braces and calculating the stresses in the remaining members. The stresses can then be determined by the methods used in Art. 153 of the preceding chapter. These stresses are given in col. 1 of Table 1.

The panel load due to snow will be the same as for the preceding design. As the area of the roof panel is $7 \times 15 = 105$ sq. ft., and the snow load is 20 lb. per sq. ft., the panel load is $20 \times 105 = 2100$ lb. The snow load stresses are given in col. 2 of Table 1. These stresses can be calculated from the dead load stresses by multiplying by the ratio of panel loads, which in this case is $2100/1490 = 1.41$. Since the conditions are the same as for the preceding design, the stresses in this case can be taken from Table 1 of Art. 153 of the preceding chapter. In col. 3 the stresses for minimum, or one-half snow load, are given.

The wind load stresses for the structure under consideration have been worked out in the problem given in Art. 162. As stated in Art. 163, the unit wind pressure is to be taken as 30 lb. per sq. ft. and the allowable working stress for wind loading is to be based on 24,000 lb. per sq. in. Since this working stress is $\frac{3}{2}$ that allowed for dead and snow loads, the wind pressure can be reduced by ⅓, which gives a unit pressure of 20 lb. per sq. ft. A uniform

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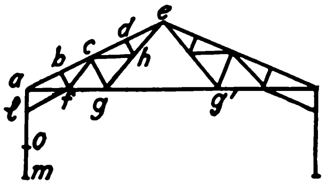
[1] *Eng. News*, vol. 73, No. 5, p. 210, Feb. 4, 1915.

TABLE 1.—STRESSES IN MEMBERS



Member	Dead load (1)	Snow load (2)	Minimum Snow load (½ S. L.) (3)	Wind left to right (4)	Minimum wind left (½ W.L.) (5)	Wind right to left (6)	Minimum wind right (½ W. R.) (7)	D. L. + S. L. (8)	D. L. + wind (9)	D. L. + ½ S. L. + wind (10)	D. L. + S. L. + ½ wind (11)	Maximum stress (12)
ab	−11,660	−16,450	−8,225	− 9,600	−3,200	+ 8,400	+ 2,800	−28,110	−21,260	−29,485	−31,310	−31,310
bc	−11,000	−15,500	−7,750	− 9,600	−3,200	+ 8,400	+ 2,800	−26,500	−20,600	−28,350	−29,700	−29,700
cd	−10,330	−14,550	−7,275	− 6,600	−2,200	+ 550	+ 185	−24,880	−16,930	−24,205	−27,080	−27,080
de	− 9,660	−13,640	−6,820	− 6,600	−2,200	+ 550	+ 185	−23,300	−16,260	−23,080	−25,500	−25,500
bf − dh	− 1,335	− 1,880	− 940	− 1,565	− 525	0	0	− 3,215	− 2,900	− 3,840	− 3,730	− 3,840
cg	− 2,670	− 3,760	−1,880	− 4,580	−1,530	+ 3,900	+ 1,300	− 6,430	+ 1,230 − 7,250	− 9,130	− 7,960	− 9,130 + 1,230
lf	0	0	0	+ 4,950	+1,650	−13,000	− 4,340	0	+ 4,950 −13,000	+ 4,950 −13,000	+ 1,650 − 4,340	+ 4,950 −13,000
af	+10,430	+14,700	+7,350	+ 3,220	+1,075	− 220	− 75	+25,130	+13,650	+21,000	+26,205	+26,205
fg	+ 8,940	+12,600	+6,300	+ 3,650	+1,220	− 5,850	− 1,950	+21,540	+12,590	+18,890	+22,760	+22,760
gg	+ 5,940	+ 8,410	+4,205	− 1,450	− 485	− 1,450	− 485	+14,350	+ 4,490	+ 8,695	+13,865	+14,350
fc	+ 1,490	+ 2,100	+1,050	+ 5,050	+1,685	− 8,750	− 2,920	+ 3,590	+ 6,540 − 7,260	+ 7,590 − 6,210	+ 5,275	+ 7,590 − 7,260
ch	+ 1,490	+ 2,100	+1,050	+ 1,700	+ 570	0	0	+ 3,590	+ 3,190	+ 4,240	+ 4,160	+ 4,240
gh	+ 2,980	+ 4,200	+2,100	+ 5,120	+ 1,375	− 4,350	− 1,450	+ 7,180	+ 8,100 − 1,370	+10,200	+ 8,555	+10,200 − 1,370
he	+ 4,470	+ 6,300	+3,150	+ 6,850	+2,285	− 4,350	− 1,450	+10,770	+11,320	+14,470	+13,055	+14,470
al	− 5,960	− 8,400	−4,200	− 5,000	−1,670	+ 3,700	+ 1,235	−14,360	−10,960	−15,160	−16,030	−16,030
lo	− 5,960	− 8,400	−4,200	− 2,340	− 780	− 3,260	− 1,090	−14,360	− 8,300	−13,420	−15,450	−15,450
M_o ft.-lb.	0	0	0	21,500	7,170	36,500	12,170	0	36,500	36,500	12,170	36,500
M_m ft.-lb.	0	0	0	10,750	3,585	18,250	6,085	0	18,250	18,250	6,085	18,250

+ = tension − = compression. All stresses given in pounds.

allowable working stress of 16,000 lb. per sq. in. can then be used for all loadings. The wind pressure on the sides of the structure will be taken as 20 lb. per sq. ft., and that on the roof surface will be taken as calculated from the Duchemin formula which is given in Art. 135. As the slope of the roof surface is 26 deg. 34 min. and the unit pressure is 20 lb. per sq. ft., the normal wind pressure is found to be 14.9 lb. per sq. ft of roof surface. Since a complete solution of this problem is given in Art. 162, the work will not be repeated.

The wind stresses in the members, as determined in Fig. 199 or 200 of Art. 162, are given in cols. 4 and 5 of Table 1. Minimum, or one-third wind stresses are given in cols. 6 and 7. Table 1 also gives the values of the moments at the foot of the knee-braces. These moments are calculated from eq. (4) of Art. 162. For point *e* of the windward column, it can be seen from Figs. 197 and 198(*a*) that the moment is (3650 − 1500) × 10 = 21,500 ft.-lb., and for the leeward column, the moment at point *l'* is 3650 × 10 = 36,500 ft.-lb. Moments at the base of the column are also given. These moments are equal to the horizontal component of the reaction multiplied by the distance to the assumed point of inflection.

The combined stresses for the combinations of cases (*a*), (*b*), (*c*), and (*d*), as outlined above, are given in cols. 8, 9, 10, and 11 respectively. In col. 12 the greatest of these maximum values are tabulated.

165. Design of Members and Columns.—The general principles governing the design of the members of a knee-braced bent are the same as those used in the design of the preceding chapter Table 2 gives all data required for the design. In the truss under consideration, a few of the members are subjected to a reversal of stress. Such members are to be designed to carry each of these stresses. The section will therefore be determined for the stress which requires the greater area. One member, *g–h*, is subjected to a small compression under certain conditions. The area required is determined by the tension in the member. However, since the member is likely to be called upon to carry compression, the limiting *l/r* conditions must be met, which will probably determine the make-up of the section. Where a member is subjected to a large compression and a smaller tension, the compression area determines the required section. It is necessary, however, to examine the net area, in order to make certain that proper provision has been made for the tensile stress. The detailed design of a few of the members will now be taken up, and new points involved in the design will be discussed.

Member *e–f*, the knee-brace, is subjected to a tension of 4950 lb., and to a compression of 13,000 lb.; the length of the member is 111.5 in. Try two 3½ × 3 × ¼-in. angles, placed with the 3½-in. legs separated by a ⅜-in. space. The least radius of gyration of these angles is 1.10 in.; the slenderness ratio is l/r = 111.5/1.10 = 101.5; the allowable working stress in compression is 8900 lb. per sq. in.; and the area required is 13,000/8900 = 1.46 sq. in. Since the working stress in tension is 16,000 lb. per sq. in., the net area required for the tension is 4950/16,000 = 0.309 sq. in. The gross area of the assumed angles is 3.86 sq. in., and the net area, deducting one rivet hole from each angle, is 3.32 sq. in. These areas are considerably in excess of the required areas, but the value of the ratio l/r for the assumed angles is 101.5, which is close to the maximum allowable. The section must therefore be used.

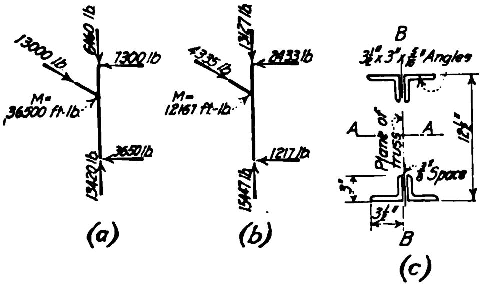


Fig. 201.

Member *g-h* is subjected to a tension of 10,200 lb., or to a compression of 1370 lb. The area required for tension, which is 10,200/16,000 = 0.638 sq. in., will determine the design, but the member selected must conform to the limiting slenderness ratio conditions required for compression members. In this case it will be found that a section made up of the minimum angles will answer all requirements. Assume two 2½ × 2 × ¼-in. angles, the minimum allowable, for which the least r = 0.78 in. For a length of member of 94 in., we find that l/r = 94/0.78 = 120.5, a value slightly less than the maximum allowable, but acceptable in this case. The net area of the assumed angles, deducting one rivet hole from each angle, is 1.68 sq. in. Although the area provided is somewhat in excess of that required, the section must be used in order to answer the l/r conditions.

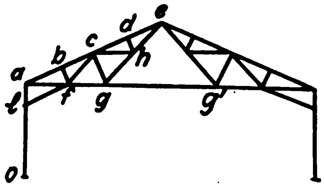
The design of the column and its base presents some new problems, which will be discussed in detail. As stated in Art. 163, the columns are three-force pieces, which are to be designed for moment, shear, and direct stress. From Fig. 196 (*a*) and Table 1, it can be seen that the maximum moment conditions occur at the foot of the leeward knee-brace. Fig. 201 shows the forces acting on the column for two conditions of loading. Fig. (*a*) shows the combined forces due to dead load, one-half snow load, and maximum wind load, and Fig. (*b*) shows the conditions for dead load, snow load, and one-third wind load. Design methods similar to those developed in the preceding chapter for the design of the top chord will be used for the design of the columns.

The area of the section will be determined by the moment and the direct stress, and the design of the details, such as the lacing and the riveting of the main angles, will be determined by the shear. The area of the section will be determined after which the details will be designed.

The loading conditions for which the column is to be designed are: (*a*) compression, 13,420 lb.; moment 36,500 ft.-lb.; shear, 3650 lb.; and (*b*) compression 15,447 lb.; moment, 12,167 ft.-lb.; shear, 1217 lb. In this case it will be best to assume a section, and then compare the area required as determined from eq. (3) of Art. 158 of the preceding chapter with the area furnished by the assumed section.

Assume a column section composed of four angles connected by lacing, arranged as shown in Fig. 201 (*c*). This section must be made quite wide in the plane of the truss, in order to resist the bending moments. It must have a width along the axis *A-A* such that the allowable ratio $l/r = 125$ will not be exceeded, where l = one-half the total height of the column. This is founded on the assumption that the base of the column is flat and that it is rigidly

Table 2.—Design of Members



Member	Stress (lb.)	Length (in.)	Radius of gyration (in.)	$\frac{l}{r}$	Unit stress (lb. per sq. in.)	Area required (sq. in.)	Section, (sq. in.)	Area provided (sq. in.)	
								Gross	Net
ab	−31,310	84	1.10	76.5	10,650	2.94	2 ∟s 3½ × 3 × 5/16	3.86	
bc	−29,700	.....	.....	.....	.....	....	2 ∟s 3½ × 3 × 5/16		
cd	−27,080	.....	.....	.....	.....	....	2 ∟s 3½ × 3 × 5/16		
de	−25,500	.....	.....	.....	.....	....	2 ∟s 3½ × 3× 5/16		
bf − *dh*	− 3,840	42	0.78	53.9	12,230	0.314	2 ∟s 2½ × 2 × ¼	2.12	
cg	− 9,130 + 1,230	84	0.78	107.8	8,460 16,000	1.08 0.078	2 ∟s 2½ × 2 × ¼	2.12	1.68
lf	+ 4,950 −13,000	111.5	1.10	101.5	16,000 8,900	0.309 1.46	2 ∟s 3½ × 3 × 5/16	3.86	3.32
af	+26,205	.....	.....	......	16,000	1.64	2 ∟s 2½ × 2½ × ¼	2.38	1.94
fg	+22,760	.....	.....	.....	16,000	1.42	2 ∟s 2½ × 2½ × ¼	2.38	1.50
gg'	+14,350	.....	.....	.....	16,000	0.896	2 ∟s 2½ × 2½ × ¼	2.38	1.50
fc	+ 7,590 − 7,260	94	0.78	120.5	16,000 7,570	0.475 0.957	2 ∟s 2½ × 2 × ¼	2.12	1.68
ch	+ 4,240	.....	.....	.....	16,000	0.259	2 ∟s 2½ × 2 × ¼	2.12	1.68
gh	+10,200 − 1,370	.....	.....	.....	16,000 7,570	0.638 0.181	2 ∟s 2½ × 2 × ¼	2.12	1.68
he	+14,470	.....	.....	.....	16,000	0.905	2 ∟s 2½ × 2 × ¼	2.12	1.68

+ = tension. − = compression.

fastened to the foundations. It is also assumed that the top of the column is held in line by an eave strut, as shown in Fig. 210. If these conditions are not realised the full height of the column must be used. On the above assumption, the least allowable $r = \frac{1}{2} \times 20 \times 12/125 = 0.96$ in. Assume four $3\frac{1}{2} \times 3 \times \frac{5}{16}$-in. angles placed as shown in Fig. 201 (*c*). The radius of gyration for the axis *A-A* is found to be 5.53 in., and that for the axis *B-B* is 1.66 in. From eq. (3), Art. 158 of the preceding chapter, using the loadings given above, dimensions as given on Fig. 201(*c*), and $f_s = 16{,}000 - 70\ l/r = 16{,}000 - 70 \times 15 \times 12/5.53 = 13{,}720$ lb.,

Case (*a*)

$$A = \frac{13{,}420}{13{,}720} + \frac{36{,}500 \times 12 \times 6.25}{16{,}000 \times 5.53^2} = 0.98 + 5.60 = 6.58 \text{ sq. in.}$$

Case (*b*)

$$A = \frac{15{,}447}{13{,}720} + \frac{12{,}167 \times 12 \times 6.25}{16{,}000 \times 5.53^2} = 1.13 + 1.87 = 3.00 \text{ sq. in.}$$

The section must also be investigated for column action in the plane of the axis *A-A*. Since $r = 1.66$ in., and $l = 20$ ft. $= 240$ in., $f_s = 16{,}000 - 70 \times 240/1.66 = 10{,}940$ lb. per sq. in., and the area required $= 15{,}447/10{,}940 = 1.42$ sq. in. The section is therefore ample, as the area provided is $4 \times 1.93 = 7.72$ sq. in. As the assumed section answers all conditions, it will be adopted.

The arrangement of the lacing, or other connection, between the angles composing the column section, will depend upon the amount of shear to be carried. As shown in Fig. 201 (*a*), the maximum shear to be carried on the portion of the column below the knee-brace is 3650 lb., and above the knee-brace, the shear is 7300 lb. Assume that single lacing of the form shown in Fig. 202 (*a*) is to be used. Below the knee-brace, where the shear is 3650 lb., the stress on a lacing bar is 3650 × sec. 45° = 5710 lb. The rivets will be shop rivets in bearing. In order to meet the requirements for bearing, the lacing bar must be ⅜ in. thick; the rivet value will then be 5625 lb., which is satisfactory.

The size of the lacing bar is determined by its strength as a column and as a tension member. Since the bar is held rigidly between the angles, the unsupported length, l, may be taken as half of the total length, or, as shown in Fig. 202 (*a*), $l = \frac{1}{2} \times 9 \times \sec. 45° = 6.36$ in. Assuming the lacing bar to be a $2\frac{1}{2} \times \frac{3}{8}$-in. section, the least radius of gyration is $r = d/12 = 0.289\ d = 0.108$ in., and $l/r = 58.8$. The allowable working stress is $16{,}000 - 70 \times 58.8 = 11{,}780$ lb. per sq. in., and the area required is $5710/11{,}780 = 0.49$ sq. in. The assumed section provides $2 \times 0.375 = 0.75$ sq. in. For a working stress of 16,000 lb. per sq. in. in tension, the area required is $5710/16{,}000 = 0.258$ sq. in. Deducting one rivet hole from the area of the section, the net area is $0.75 - 0.33 = 0.42$ sq. in. Since the assumed section is standard it will be adopted, although it is a little larger than required.

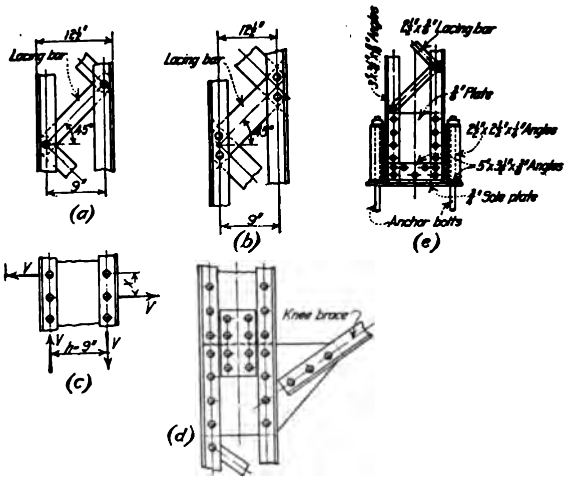


Fig. 202.

The stress in the lacing bars above the knee-brace will be 7300 × sec. 45° = 10,340 lb. Two rivets will be required in the end of each lacing bar, as shown in Fig. 202 (*b*). In some cases a plate is used in place of the lacing bars. This is often done when more than one rivet is required in the end of each bar. Fig. (*c*) shows an arrangement of this kind. The plate is to be connected to the angles at intervals determined from the conditions shown in Fig. (*c*), where V_s = shear on the section, which is 7300 lb.; r = rivet value; and x = distance between rivets. Taking moments about a rivet, we have $rh = Vx$, from which, $x = rh/V$. Assuming a ⅜-in. plate, the rivets will be in bearing and will have a value of 5625 lb. per rivet. Substituting these values in the above equation, $x = 5625 \times 9.0/7300 = 6.93$ in. In practice a spacing of about 4.5 in. would be used. Where the detail shown in Fig. (*d*) is used, the web plate and the gusset plate should be connected as shown. As the web plate is assumed to carry shear only, two rows of rivets in the splice are sufficient. If the splice is to be designed for moment as well for shear, the principles given in the chapter on Splices and Connections—Steel Members must be used.

Fig. 202 (*e*) shows a common detail for the base of a column where fixed or partially fixed end conditions are assumed. A sole plate, generally about ¾ in. thick, is riveted to angles fastened to the main angles of the column. Anchor bolts imbedded in the concrete or masonry foundations are placed between pairs of anchor angles. These bolts are tightened up against plate washers resting on top of the anchor angles. The anchor bolts are placed in the plane of the moment to be resisted. If the stresses are small, one bolt on each side of the base of the column is sufficient, but where large stresses are to be resisted, two bolts are used on each side.

The conditions for which anchor bolts are usually designed are shown in Fig. 203. Forces P and H are determined from Fig. 196 (e), which shows the portion of the column below the assumed point of inflection. The deflection Δ is so small compared to the other distances that it can be neglected. As shown in Fig. 203, the forces tend to tip the column about point A. Taking moments about A

$$M_o = Hh - Pd/2, \text{ where } M_o = \text{overturning moment.}$$

Anchor bolts are usually designed on the assumption that they resist all of the overturning moment. If t = distance from point A to the anchor bolt,

$$\text{Stress in anchor bolt} = M_o/t \tag{1}$$

In some cases t is taken as the distance between anchor bolts. No calculation of the compressive stress in the concrete or masonry under the base is made in this method. It is assumed that if the compressive stresses foun by dividing the load to be carried by the area of the base is kept small, the added stresses due to overturning will not exceed allowable limits.

In Fig. 204 there is shown the conditions for an approximate analysis of the stresses in the anchor bolts and the compression on the foundations. The general principles upon which the method is based and the assumptions made are similar to those used in determining the bearing pressures on the base of a retaining wall, as given in the chapter on Retaining Walls. In the case under consideration the additional assumption is made that when the overturning moment is such as to cause tension on any part of the base, that tension is taken up by the anchor bolts.

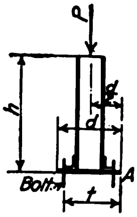
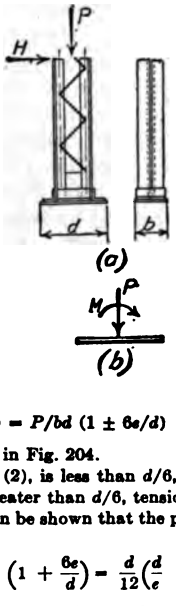


FIG. 203.



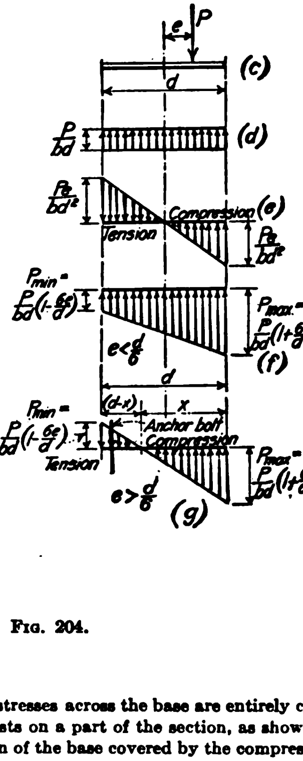


FIG. 204.

Fig. 204 (a) shows the lower portion of the column with forces in position as determined from Fig. 196 (e). The action of these forces on the base of the column can be represented by a moment M and a force P, as shown in Fig. 204 (b). These can be represented by the load P placed at a distance e from the center of the base, where

$$e = M/P \tag{2}$$

The stresses on the base can be divided into two parts; one part due to the effect of P, and the other due to M. These stresses are shown in Figs. (d) and (e) respectively. The resultant stress on the base is the sum of these stresses, and is given by the expression

$$p = P/bd\,(1 \pm 6e/d) \tag{3}$$

where the several terms have the values shown in Fig. 204.

It can be shown that if e, as given by eq. (2), is less than $d/6$, the stresses across the base are entirely compression, as shown in Fig. (f), and where e is greater than $d/6$, tension exists on a part of the section, as shown in Fig. (g). From similar triangles in Fig. (g) it can be shown that the portion of the base covered by the compressive stresses is

$$x = \frac{d^2}{12}\left(1 + \frac{6e}{d}\right) = \frac{d}{12}\left(\frac{d}{e} + 6\right) \tag{4}$$

The unit compressive stress on the foundations is given directly by eq. (3). To determine the total tension in the anchor bolts, assume the total tension is taken by the anchor bolt. This tension, T, is represented by the volume of the tension stress diagram, which is

$$T = \frac{1}{2}P_{min} \times (d - x)b = \frac{p}{2d}\left(6\frac{e}{d} - 1\right)(d - x)$$

$$T = \frac{pd}{24e}\left(\frac{6e}{d} - 1\right)^2 \tag{5}$$

For the case under consideration, it will be found from Table 1 and from Fig. 198 that P = 13,420 lb. and M = 3650 × 5 = 18,250 ft.-lb. = 219,000 in.-lb. These values occur in the leeward column.

The details of the column base are shown in Fig. 202. For a column section of the dimensions shown in Fig.

201, a sole plate 9 in. wide and 20 in. long will be required. These dimensions will be assumed for a trial section. From eq. (2), $e = 219{,}000/13{,}420 = 16.3$ in.; and from eq. (4), with $b = 9$ in., and $d = 20$ in.,

$$x = \frac{20^2}{12 \times 16.3}\left(1 + \frac{6 \times 16.3}{20}\right) = 12.05 \text{ in.}$$

The maximum compressive stress on the foundation is given by eq. (3) as

$$p = \frac{P}{bd}\left(1 + \frac{6e}{d}\right) = \frac{13{,}420}{9 \times 20}\left(1 + \frac{6 \times 16.3}{20}\right) = 442 \text{ lb. per sq. in.}$$

Assuming a concrete foundation, this fiber stress is allowable, for the working compressive stress in concrete is usually given as 650 lb. per sq. in. The stress in the anchor bolt is given by eq. (5) as

$$T = \frac{Pd}{24e}\left(\frac{6e}{d} - 1\right)^2 = \frac{13{,}420 \times 20}{24 \times 16.3}\left(\frac{6 \times 16.3}{20} - 1\right)^2 = 10{,}480 \text{ lb.}$$

Since there is considerable initial tension in the anchor bolts due to the fact that they are screwed up tight when the structure is erected, and since the overturning of the column tends to add to the initial tension, it is best to specify low working stresses for anchor bolts. An allowable stress of 10,000 lb. per sq. in. will therefore be used. The required area of anchor bolt is then 10,480/10,000 = 1.05 sq. in. From the handbooks a 1⅜-in. round rod provides an area of 1.054 sq. in. at the root of thread.

Anchor bolts should be imbedded in the concrete to a depth such that the bond stress developed will equal the strength of the bolt. In this case 20 diameters of the bolt, or 27½ in., will be required. If a plate is used connecting the ends of the bolts, as shown in Fig. 210, the imbedment need not be as great as calculated above. All details of the column base and anchorage are shown on the general drawing of Fig. 210.

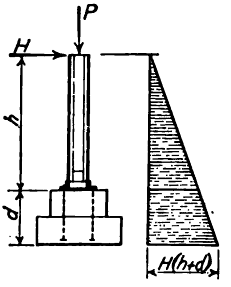


Fig. 205.

The method of analysis given above, while not exact, is accurate enough for all practical purposes. A more exact analysis can be made by taking into account the relative deformations of the steel anchor bolt and the masonry foundation. If the foundation is made of concrete, the methods of analysis given for Bending and Direct Stress in Sect. 1 can be used. By this method the stresses in the concrete will be found to be a little greater than those given above, and the stress in the anchor bolt will be slightly less than before.

The foundations for the columns are designed by the methods given in the chapter on Retaining Walls. The total moment to be carried at the base of the foundation is $H(h + d)$ as shown in Fig. 205. Maximum pressures on the soil can be determined by the same principles as explained above for the case shown in Fig. 204. Eq. (3) will give the desired pressures. By trial the width of base can be made of the width required to give the desired stresses.

166. Design of Joints.—The principles governing the design of the joints are the same as used in the preceding chapter. Field splices will be provided at joints g and e of Fig. 197. The columns will be field spliced to the truss at joint a, and the knee-brace will be field spliced at both ends. Field splices will also be placed at corresponding points on the right-hand side of the truss. From the shearing and bearing values given in Art. 163, the single shear value of a shop rivet is 4420 lb., and the bearing value on a ⅜-in. plate is 5625 lb. Corresponding values for field rivets are 3310 and 4420 lb., respectively. Where a member is subjected to tension and compression, the connecting rivets are to be determined for the greater stress.

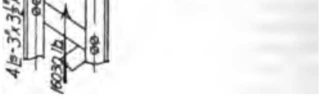


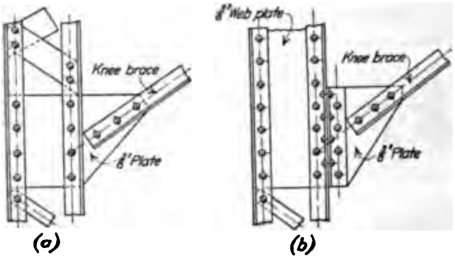
Fig. 206.

All joints will be practically the same as for the truss designed in the preceding chapter, except joints f and a. At joint f the knee-brace must be connected to the gusset plate. As a field splice is to be provided and since the rivets are in bearing on a ⅜-in. plate, the rivet value is 4220 lb. The maximum stress in the knee-brace is 13,000 lb. compression, and 13,000/4420 = 3.08 rivets are required; three will be used. To provide for these rivets the gusset plate at f will be enlarged, as shown on the general drawing, Fig. 210.

Fig. 206 shows the conditions at joint *a*. Members *a–b* and *a–f* are connected by shop rivets, and the column is connected by field rivets. From Table 1, the maximum stress in the upper end of the column is 16,030 lb. Hence 16,030/4420 = 4 rivets are required. Fig. 206 shows 6 in place.

The conditions at the foot of the knee-brace, where it is connected to the column, are shown in Fig. 207. Three field rivets are required in the end of the knee-brace, the same number as calculated for this member at joint *f*. Two forms of connections to the column are shown in Fig. 207. In Fig. (*a*) is shown a form used when the column is laced above and below the knee-brace. Extra rivets are used in the connection between the gusset plate and the column in order to secure a central connection for the knee-brace, thus avoiding excess stresses due to eccentric moments.

Fig. 207 (*b*) shows a detail in which a plate is used above the knee-brace because of heavy shears which cannot be provided for by means of lacing. In this detail the knee-brace is connected to the column by means of a pair of short angles riveted to the column angles. When the knee-brace is in tension, these rivets are subjected to a direct pull, and are in tension. From Table 1, the maximum tension in the knee-brace is 4950 lb. As shown, 8 rivets are provided to take the component of the tension perpendicular to the column, which is 4950 × 94/111.5 = 4160 lb. The direct tension on each rivet is 4160/8 = 520 lb., which can safely be carried by the rivets. Where large stresses in tension are to be carried by the rivets, turned bolts should be substituted for the rivets.



(*a*) (*b*)

FIG. 207.

Fig. 202 (*d*) shows another detail for this joint. It is a combination of the forms shown in Figs. 202 (*a*) and (*b*) As shown in Fig. 202 (*d*) the gusset plate and the web plate are connected by a small plate, by means of which the shear is transmitted across the joint. Where a web plate is used in Fig. 206 in place of lacing, a similar plate must be provided. In the case under consideration, the web plate is supposed to provide only for the shearing stresses. For large columns the web plate is often designed to carry moment as well as shear. The connection between web and gusset plate must then be designed for shear and moment, as explained in the chapter on Splices and Connections—Steel Members.

167. Design of Girts.—It will be assumed that the sides and ends of the building are to be covered with corrugated steel backed with a suitable anti-condensation lining. The siding will be spported by girts composed of rolled sections. As stated in Art. 163, the unit wind pressure will be taken as 20 lb. per sq. ft., and the working stress in the girts will be 16,000 lb. per sq. in.

The principles governing the design of the girts are similar to those given for the design of purlins in the chapter on Design of Purlins for Sloping Roofs in Sect. 2. The girts are to be designed for a vertical load due to the weight of the girt and the siding and its lining, and a horizontal load due to the wind pressure. Corrugated steel of No. 24 gage will be used for the siding. From the data given in the chapter on Roof Trusses—General Design, the siding weighs 1.3 lb. per sq. ft., and the allowable safe span is 4.5 ft. It will be convenient in this case to divide the height of the building into six spaces, placing the girts $\frac{20}{6}$ = 3 ft. 4 in. apart. On the sides of the building the columns are spaced 15 ft. apart, and the wall area carried by each girt is 15 × $3\frac{1}{3}$ = 50 sq. ft. Assuming that the anti-condensation lining is composed of two layers of $\frac{1}{16}$-in. asbestos paper and two layers of tar paper backed by poultry netting, all of which weighs about 1.3 lb. per sq. ft., the weight of siding and lining is 1.3 + 1.3 = 2.6 lb. per sq. ft., and the total load per foot of girt is 2.6 × 3.33 = 8.66 lb. The wind load per foot of girt is 20 × 3.33 = 66.7 lb.

As shown in the chapter on Roof Trusses--General Design and in Fig. 210, girts are often made from channel sections placed with the web perpendicular to the siding, and they are attached to the columns by rivets in the flanges of the channel. When so placed, the discussion given in the chapter on Unsymmetrical Bending in Sect. 1 shows that the channel presents its axis of least moment carrying capacity to the action of the vertical loads. To relieve the heavy bending stresses thus induced, tie rods can be used extending vertically to the eave strut, or running diagonally from the top girt to the upper ends of the columns. It is not always possible to use tie rods due to interference with openings in the walls for doors and windows. When tie rods are used it is reasonable to assume that the girt takes the horizontal load, and that the tie rods provide for the vertical loads. Two designs will be made, one with tie rods, and the other without tie rods, assuming the girt to be a beam under unsymmetrical loading.

Assuming that tie rods are used, and that the girt takes only the horizontal wind pressure, the total uniformly distributed load to be carried by a girt is 50 × 20 = 1000 lb. The moment to be carried, assuming simple beam conditions, is $M = \frac{1}{8} Wl = 1000 \times 15 \times \frac{12}{8} = 22{,}500$ in.-lb. For a working stress of 16,000 lb. per sq. in., the section modulus required is $I/c = M/f = 22{,}500/16{,}000 = 1.41$ in.^{3} If the least width of the section be limited to $\frac{1}{40}$ of the span in order to avoid excessive deflection, the minimum allowable girt section is a 5-in. 6.5-lb. channel section. The size of the tie rod can be determined by the methods given in the chapter on Design of Purlins for Sloping Roofs in Sect. 2.

Consider now the case where tie rods are not used and the girt is subjected to unsymmetrical bending. Assume a 6-in., 8-lb. channel section as a girt. The total vertical weight of siding, lining, and girt is then 8.66 + 8.00 = 16.66 lb. per foot for each girt. As given above, the horizontal wind load per foot of girt is 66.7 lb. The resultant of these loads, as shown by the force diagram of Fig. 208, is 69.0 lb. Two cases will be considered, (a) moment due to resultant load of 69.0 lb. per ft. of girt, and (b) moment due to vertical loading. For case (a) the moment to be carried is $69 \times 15 \times \frac{12}{8} = 23{,}280$ in.-lb., and $S_1 = M/f = 23{,}250/16{,}000 = 1.45$ in.^{3}, and for case (b) $M = 16.66 \times 15^2 \times \frac{12}{8} = 5{,}630$ in.-lb., and $S_2 = 5630/16{,}000 = 0.352$ in.^{3} These values of S_1 and S_2 are plotted in amount and direction to scale in Fig. 208 (b). In the same figure, the S-Polygon of a 6-in., 8-lb. channel is shown, constructed by the methods explained in the chapter on Unsymmetrical Bending in Sect. 1. Since the plotted values fall inside the S-line for the assumed channel, the section is satisfactory, and it will be adopted.

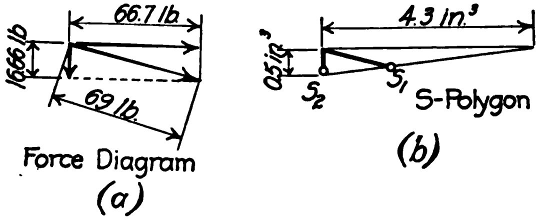


FIG. 208.

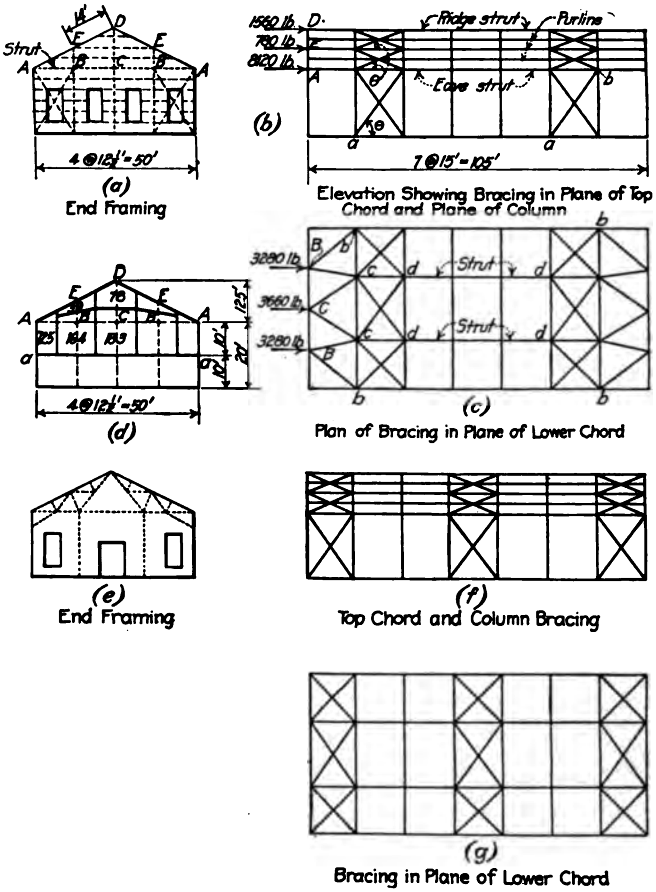
In practice, girt sections are used which are considerably smaller than the section arrived at in this design. Where theory and practice differ, as they do in the case under consideration, the designer must rely upon his experience and judgment in making a choice of the sections to be used for the girts. In this case, theory will be assumed to govern, and the adopted details will be as shown in Fig. 210.

168. Design of Bracing.—The design of the bracing will be governed by the adopted arrangement, which in turn is governed by the layout of the building. A general discussion of the form of bracing for buildings composed of knee-braced bents has been given in Art. 129.

To illustrate the general methods for the design of the bracing of a knee-braced building, it will be assumed that the structure under consideration in this chapter consists of 7 bays of 15 ft. each, as shown in Fig. 209. Two arrangements of bracing are shown in Fig. 209. In Fig. (a) (b), and (c), the framing for the end of the building consists of vertical posts to which the girts are attached. Bracing in the plane of the top chord, the bottom chord, and the planes of the columns is provided for two pairs of trusses. Wind loads from the ends of the building are brought to the lateral trusses by means of rigid bracing. Unbraced bents are connected by means of a line of struts at points g and g' of Fig. 197, by struts at the eaves, and by a line of struts at the ridge.

Figs. 209 (e), (f), and (g) show an arrangement wherein knee-braced bents are placed at the ends of the building. These end bents are made the same as the others, so that future extensions in the length of the building are readily made. The figures show the position of the other bracing. As the design methods for the two arrangements are similar, detailed calculations will be given only for the arrangement of Figs. (a) to (d) inclusive. Both of the arrangements for end bracing shown in Fig. 209 are used in practice. The arrangement of Figs. (a) to (c) is probably cheaper than the one shown in Figs. (e) to (g), for in the first arrangement all of the members are simple beams composed of rolled sections, such as I-beams or channels. Very little shop work is required on these members. In the second arrangement, the same amount of shop work is required as for the other knee-braced bents, for all are made alike. This shop work costs several times as much as that for the first arrangement. The case with which the building can be extended is about the same in both cases. When the entire end of the building is to be opened at certain times, the second arrangement is preferable.

In general the design of the bracing for a structure composed of knee-braced bents consists in the determination of the wind loads applied to the sides and ends of the building, and in the provision of bracing of suitable size so located as to transmit the applied loads to the foundations of the structure. The knee-braced bents provide the proper resistance to wind on the sides and roof of the structure. Provision for these loads has already been made in the design of the preceding articles. In the first arrangement shown in Fig. 209, diagonals placed in the plane of the ends of the structure provide for the loads not carried directly by the knee-braced bents. All wind loads applied to the ends of the building are provided for by the bracing shown in Figs. (*b*) and (*c*), or in Figs. (*f*) and (*g*).



(*a*) End Framing

(*b*) Elevation Showing Bracing in Plane of Top Chord and Plane of Column

(*c*) Plan of Bracing in Plane of Lower Chord

(*d*)

(*e*) End Framing

(*f*) Top Chord and Column Bracing

(*g*) Bracing in Plane of Lower Chord

FIG. 209.

In the arrangement of end framing shown in Fig. 209 (*a*), the siding and girts are carried by vertical I-beams supported by the foundation at the base; by a member running across the end of the building at the height of the eaves, shown by the dashed line from *A* to *A*; and by a rafter at the roof line. These beams are to be designed to carry the wind loads brought to them by the siding. The dead load effect, which is a vertical load, is small and can be neglected. As shown in Fig. (*a*), the end of the building is divided into four equal parts of 12.5 ft. each by vertical beams. Considering each vertical member as a simple beam supported by the foundation and the strut *A*-*A*, the effective span is 20 ft. If the reduced wind loading of 20 lb. per sq. ft. is used, the load to be carried per foot of vertical height is 20 × 12.5 = 250 lb., and the bending moment is $M = \frac{1}{8} wl^2 = \frac{1}{8} \times 250 \times 20^2 \times 12 = 150{,}000$ in.-lb. For a unit stress of 16,000 lb. per sq. in., which corresponds to the reduced wind load of 20 lb., as stated in Art. 163, the section modulus required is $I/c = M/f = 150{,}000/16{,}000 = 9.38$ in.^{3} From the steel hand books, a 7-in., 15-lb. I-beam is required. The same section will be used for all members. The rafter *A*-*E*-*D* is designed by similar methods, using the total load to be carried by the roof.

The exact distribution of the wind load brought to the end of the building between the bracing in the plane of the roof and the plane of the lower chord is indeterminate. It will be assumed that the load on the lower half of the building is carried directly to the foundations. In Fig. (*d*), the area under consideration is that below the line *a*-*a*. The balance of the loads will be assumed as carried at points *A*, *B*, *C*, *D*, and *E* in proportion to the areas tributary to these points. Fig. (*d*) shows the assumed distribution of areas. The numbers show the areas tributary to the several points. At 20 lb. per sq. ft., the loads brought to the several points are as shown on Figs. (*b*) and (*c*). The load of 1560 lb. at the apex of the truss is assumed to be carried along the ridge strut to the two sets of bracing in the plane of the top chord. If this bracing be assumed to be composed of members capable of carrying tension only, there are four members in position to take the load. The stress in each member is then 1500 × sec $\theta/4$, where θ = angle which the member makes with the direction of the wind. In this case the panels of bracing extend over two panels of the top chord, or 14 ft., and the trusses are 15 ft. apart. Therefore, $\sec \theta = (14^2 + 15^2)^{\frac{1}{2}}/15 = 1.37$. The stress in the members of the upper panel of bracing is then 1560 × 1.37/4 = 535 lb.

The bracing in the lower panels of the top chord bracing must carry the loads at points *E* and *D* of Fig. (*a*), or 1560 + 780 + 780 = 3120 lb. As before, four members carry this load, and the stress in each member is 3120 × 1.37/4 = 1070 lb.

The stresses in the bracing, as calculated above, are all very small. A single $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle, the minimum allowable under the conditions of Art. 161, is sufficient for all members. The details of the bracing are shown in Fig. 210.

The loads acting on the bracing in the plane of the lower chord are shown in Fig. (*c*). These loads are distributed to the bracing by means of struts connecting points *B*, *C* and *b*, *c*. As the loads are small, the size of the struts will be determined by l/r conditions. The length of strut *Bb* is $(12.5^2 + 15^2)^{\frac{1}{2}} = 19.5$ ft. As the stresses are very small it is reasonable to allow a maximum value of $l/r = 175$. Then $r = 19.5 \times 12/175 = 1.34$ in. From the steel handbooks two $4 \times 3 \times \frac{5}{16}$-in. angles with the 4-in. legs separated by a $\frac{3}{8}$-in. space have an r of 1.3 in. This section is considerably larger than the one used in practice. For the same reasons as given at the close of Art. 167, the above design will be adopted, as shown in Fig. 210.

The load at points *c* of Fig. (c) is brought to this point from joints *B* and *C* by the struts *Cc* and *Bc*. From the conditions at points *C*, it can be seen that the two struts *Cc* each have a component of stress parallel to the load which is equal to one-half of the load. Similar conditions hold for struts *Bb* and *Bc* at joint *B*. Therefore the load brought to point *C* is $\frac{1}{2}$ (3660 + 3,280) = 3970 lb. Assuming that the diagonals carry tension only, and that the loads are carried by the diagonals in both sets of bracing, the stress in members *b*-*d* is $\frac{1}{2} \times 3970 \times \sec \theta = 3180$ lb. The minimum section, which is a $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle, will furnish sufficient area. The lines of struts connecting the two panels of bracing in the plane of the lower chord will be made of the section as used for struts *Cc*, etc.

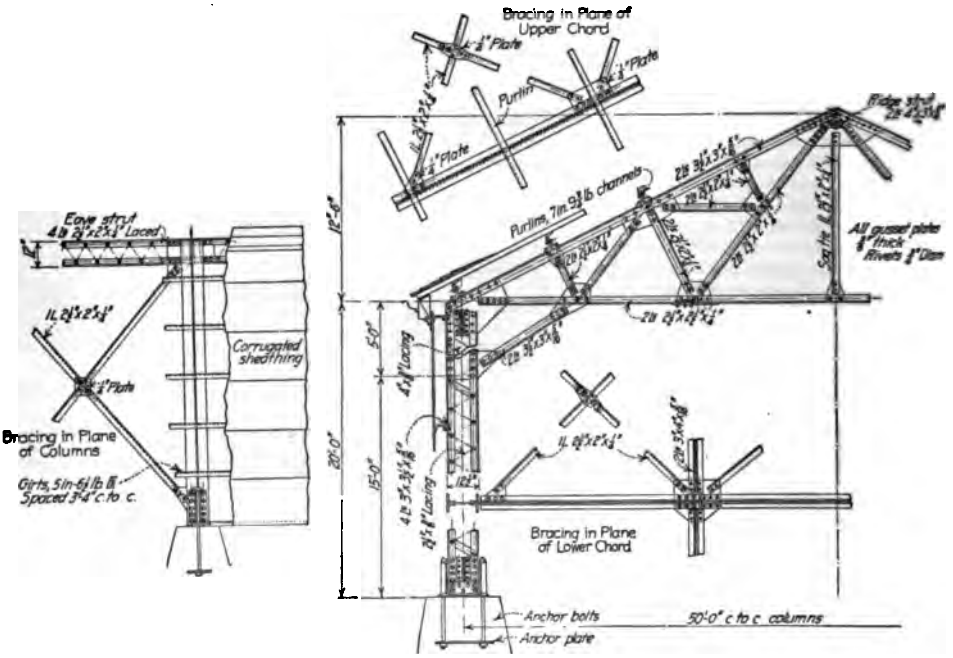

Bracing in Plane of Upper Chord
$\frac{1}{4}$" Plate
$\frac{1}{4}$" Plate
$\frac{1}{4}$" Plate
Purlin
Ridge strut
Purlins, 7 in. $9\frac{3}{4}$ lb channels
All gusset plates $\frac{1}{4}$" thick Rivets $\frac{3}{4}$" Diam
12'-6"
5'-0"
15'-0"
20'-0"
Eave strut 4 Ls $2\frac{1}{2}$"x2"x$\frac{1}{4}$" Laced
1L $2\frac{1}{2}$"x2"x$\frac{1}{4}$"
$\frac{1}{4}$" Plate
Corrugated sheathing
Bracing in Plane of Columns
Girts, 5 in. $6\frac{1}{2}$ lb channels Spaced 3'-4" c. to c.
2Ls $2\frac{1}{2}$"x$2\frac{1}{2}$"x$\frac{1}{4}$"
2Ls $3\frac{1}{2}$"x3"x$\frac{5}{16}$"
1L $2\frac{1}{2}$"x2"x$\frac{1}{4}$"
$12\frac{1}{2}$"
Bracing in Plane of Lower Chord
Anchor bolts
Anchor plate
50'-0" c to c columns

Fig. 210.—General drawing of knee-braced roof truss.

Fig. 209 (*b*) shows the bracing in the plane of the columns. All of the wind load above the line *a*-*a* of Fig. (*d*) must be carried to points *A*, and thence by the eave strut to the two panels of bracing. As shown in Fig. (*b*), the load to be carried by each set of column bracing is 8120 lb. Assuming that members take tension only, members *a*-*b* each have a stress of $\frac{1}{2} \times 8120 \times \sec \theta = 7650$ lb. A $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle will provide sufficient area. In some cases rods are used in place of rolled sections. When rods are used they are fastened to a gusset plate by means of a clevis. Some designers consider rods preferable to rolled shapes because the erection in the field is somewhat simplier than for riveted joints.

The eave strut, shown in Fig. 210, is composed of four angles laced to form a rigid member. As a rule these members are not designed for any definite stress, but are made up to answer l/r conditions.

Complete details of the structure designed in the preceding articles are given on the general drawing of Fig. 210.

ARCHED ROOF TRUSSES

By W. S. Kinne

169. Form of Arch Trusses.—Roof trusses of the type designed in the preceding chapters do not in general provide an economical structure for spans exceeding 100 ft. A more economical type of roof truss for long span trusses is provided by the arch type. As stated in Art. 121 of the chapter on Roof Trusses–General Design, an arch is a type of framed structure in which the reactions at the supports are inclined to the vertical for all conditions of loading.

Arches used for roof trusses are usually classified according to the method of supporting the structure, and according to the type of framing. As arches are commonly supported at the abutments by means of pins, which are known as hinges, the method of supporting the arch is designated by the number of hinges used. In Fig. 211 (*a*) is shown a type of arch which is rigidly fastened to the abutments without the use of hinges. This is known as a hingless arch. Fig. 211 (*b*) shows a type in which two hinges are used, one at each abutment. This is known as a two-hinged arch. In many cases a third hinge is provided at the crown of the arch, as shown in Fig. 211 (*c*). This is known as a three-hinged arch.

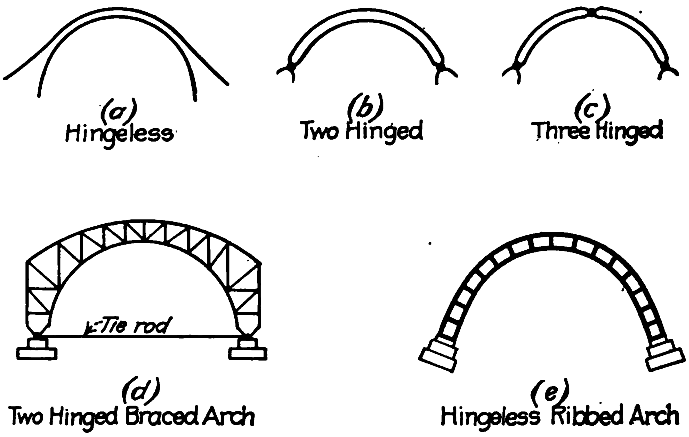


Fig. 211.

In general, two types of framing are used for arched roof trusses. A very common type consists of a trussed frame work of the form shown in Fig. 211 (*d*). This type is known as a braced arch. The type shown in Fig. 211 (*e*) is a plate girder form, which is known as a ribbed arch.

An arched roof truss is generally designated by a combination of the two classifications given above. Thus Fig. 211 (*d*) shows a *two-hinged braced arch*. Other classifications are in use, but the one described above is widely used, and is comparatively simple.

A great variety of arch trusses have been used in building construction. Many of these structures are described in architectural and engineering periodicals. Examples of arches of the several types given above will be shown and the relative advantages of the several types will be discussed. In general it can be said that an arch truss requires rigid and practically unyielding abutments, since arches, with the exception of the three-hinged type, are statically indeterminate, and any yielding of the supports will result in large changes in the stresses in the members.

Hingeless arches supported directly on the abutments, as shown in Fig. 211 (*e*), are seldom used in building construction. This type of arch requires absolutely rigid supports, a condition which is difficult to realize in practice. In framing the roofs for some of the recent large terminal railway stations, arch trusses are used which are riveted to heavy columns. As the columns are very heavy, they form practically a rigid support for the arch, which can therefore be assumed as a hingless arch.

The two-hinged type of arch is used to great advantage where a comparative rigid structure is desired—as, for example, where floors are to be supported over a large drill hall or auditorium. This type of construction is used in the Armory and Gymnasium of the University of Wisconsin. Fig. 212 shows a cross section of the building and the general outline of the arch trusses.

Two-hinged arches require rigid supports, but, due to the fact that hinges are supplied at the supports, the moment at these points is zero. Hence the abutments can be designed for direct thrust only. If the foundation conditions are uncertain, or if the points of support are considerably above the ground level, as shown in Fig. 212, the horizontal components of the reactions can be taken by means of a tie rod which connects the two end hinges. In Fig. 212, this tie rod is placed just under the floor. Where tie rods are used, it is usual to anchor one end of the arch to the abutments, and to place the other end on sliding plates or on rollers. In this way the abutments can be designed to take up the vertical loads, and the tie rod can be designed to take up the horizontal forces.

Three-hinged arches are somewhat more flexible than arches of the other types, and are used advantageously for structures in which only a roof load is to be carried. Arches of the three-hinged type are statically determinate—that is, all stresses can readily be determined by the methods of simple statics. In this respect they have a great advantage over the other types, as the work required in stress calculation is greatly simplified.

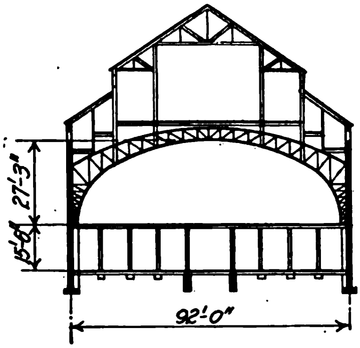


FIG. 212.—Section of gym and armory, University of Wisconsin.

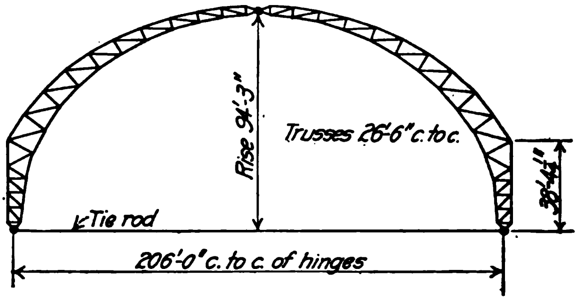


FIG. 213.

Many three-hinged arches of long span have been constructed in recent years for use in drill halls, auditoriums, and exposition buildings. A typical three-hinged arch construction is used in the drill hall at the University of Illinois. This structure is described in the *Engr. News* for Dec. 11, 1913, p. 1182. Fig. 213 shows the form and general dimensions of the arches.

In buildings in which a large floor is surrounded by galleries, the members of the arch frame interfere with free passage along the gallery, as shown in Fig. 214. This difficulty has been avoided in certain structures by placing the arch on cantilever brackets above the gallery level. A structure arranged in this manner is described in *Engr. News*, vol. 63, No. 18.

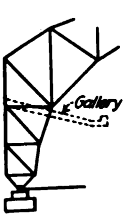


FIG. 214.

The spacing of arch trusses to be adopted in a given structure should be rather wide. Since in general the trusses are quite heavy, and since considerable shop work is required, the cost of the trusses per square foot of covered area is large. Therefore, to obtain economical conditions a wide spacing of trusses must be used, as shown by the discussion given in the chapter on Roof Trusses—General Design. In general, a truss spacing of from 25 to 40 ft. is used. This spacing requires the use of framed trusses between the arches. These trusses act as purlins, and also form part of the bracing required for the arches. The design of the purlins and the roof covering is carried out by the methods used in the preceding chapters.

The shape of an arch truss is generally determined by the architectural features of the structure. From the standpoint of the structural designer, it is desirable that the adopted form of the arch be one that can readily be laid out. This assists greatly in the preparation of the stress diagrams and the working drawings. A form of arch whose outline is composed of circles, or a combination of circles, is desirable from this standpoint.

Suppose that in a given case it has been decided that an arch composed of circles is to be formed to pass through the points A, B, C, D, and E of Fig. 215. Suppose further, that AB is a single arc, and that EC is composed of two arcs which are tangent at D. Formulas for the determination of the required radii will now be given. These formulas are all based on propositions given in plane geometry, to which the reader is referred for proofs.

From plane geometry, the formula for the radius of a segment of a circle, for which the chord and the rise or mid-ordinate are known, is

$$\text{Radius} = \frac{(\tfrac{1}{2}\ \text{chord})^2 + (\text{rise})^2}{2 \times \text{rise}}$$

As stated above, AB is the arc of a circle. Fig. 215 shows that ½ chord $= AK$, and rise $= BK$. These distances can be scaled from a layout of the arch, or calculated from given data. Hence,

$$R = \frac{(AK)^2 + (BK)^2}{2BK}$$

In the same way, the radius of the arc DC is

$$R_1 = \frac{(DL)^2 + (CL)^2}{2CL}$$

Since arcs DC and DE are tangent, the center for arc DE lies at G, a point on radius DF. The value of R_2 can be calculated by methods similar to those used above. In general, the rise of the arc ED is so small that it can not be scaled with sufficient accuracy. However, by measuring the vertical and horizontal projections of the arc DE and the angle α included between the radius DF and the vertical, easily measured distances are obtained. For the distances given in Fig. 215, it can be shown that

$$R_2 = \tfrac{1}{2}\,\frac{(EM)^2 + (MD)^2}{MD\cos\alpha - EM\sin\alpha}$$

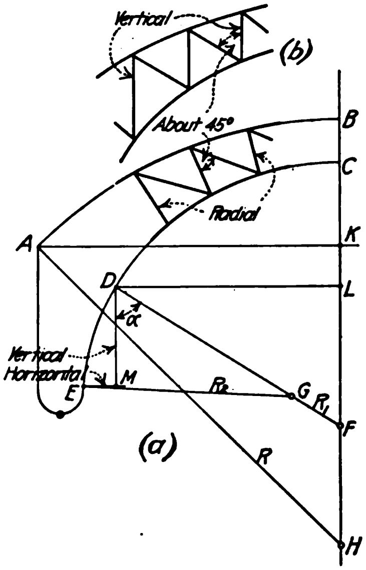


FIG. 215.

Many different arrangements of web members are used in framing a braced arch. Two common methods are shown in Fig. 215. In Fig. (*a*) the web struts are placed on the radii of the chord members. In some cases the radii of the top chord are used; in others the radii of the lower chord are used; and in a third case the radii of an arc half way between the two chords are used. Fig. (*b*) shows a case in which these members are placed in a vertical position. In Figs. (*a*) and (*b*), the other web members are placed at about 45 deg. to the struts. The panel lengths are usually arranged so that this is possible.

The adopted arrangement of truss members will depend to some extent on the type of roof framing which is to be used. If the purlins are seated on the top of the upper chord members, either arrangement can be used. In general this implies comparatively close truss spacing so that rolled shapes can be used as purlins. If deep trussed purlins are used, it is desirable that they be placed in a vertical position. Hence a framing with vertical members is best adapted to this construction.

170. General Methods for Determination of Reactions and Stresses.—The several types of arch trusses will be considered in the order determined by the difficulties encountered in the determination of the reactions. This order is (*a*) three-hinged arches, (*b*) two-hinged arches, and (*c*) hingeless arches.

The calculation of reactions and stresses in arch structures can be made either by algebraic or by graphical methods. In general, graphical methods will be found preferable, for the calculation of the lever arms of members and forces in the algebraic method requires considerable time. However, in many cases these lever arms can be scaled with sufficient accuracy from a large scale drawing of the truss. Under such conditions, the two methods require about the same amount of time. In the work to follow, algebraic and graphical methods will be given for the solution of reactions and stresses.

170(*a*) Three-Hinged Arches—*Algebraic Solution for Reactions.*—Let Fig. 216 represent a three-hinged arch acted upon by loads P_1, P_2, and P_3. It will be assumed that the points of support, *A* and *B*, are on the same level. The reactions at *A* and *B* can be represented by two forces at each point. Let H_1, V_1, and H_2. V_2 represent these forces, assumed to act as shown.

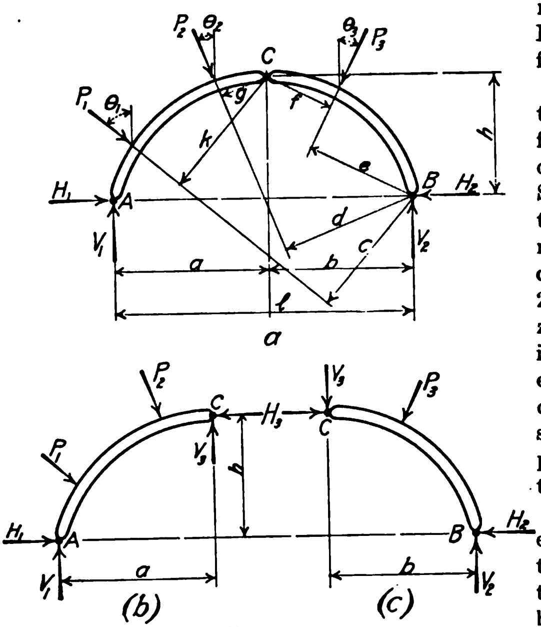


FIG. 216.

At first sight, the problem is indeterminate, for there are four unknown forces present, and as stated in the chapter on "Principles of Statics" in Sect. 1, only three unknowns can be determined in any system of non-concurrent forces. However, the introduction of a hinge at the crown, point *C* of Fig. 216, reduces the moment at this point to zero. This can be made the basis of an independent moment equation. This equation, together with three equations derived from the conditions of equilibrium stated in Sect. 1, gives rise to four independent equations from which the reactions can be completely determined.

In applying the four independent equilibrium conditions stated above to the determination of the reactions for the conditions shown in Fig. 215, it will be found convenient to use moment equations about *A* and *B*, considering the structure as a whole. Thus from moments about *B* equal zero, we have

$$V_1l - P_1c - P_2d - P_3e = 0$$

from which

$$V_1 = \frac{P_1c + P_2d + P_3e}{l}$$

In general terms, this can be written

$$V_1 = \frac{\Sigma Px_B}{l} \qquad (1)$$

where P = any load, x_B = distance from moment center B to this load, and l = span length. The value of V_2 is given by a similar moment equation about point *A*, from which

$$V_2 = \frac{\Sigma Px_A}{l} \qquad (2)$$

where x_A is the distance moment center *A* to any force *P*.

On separating the structure at the crown, as shown in Figs. 216 (*c*) and (*b*), and writing a moment equation about point *C* for the forces on the left of the point, as shown in Fig. (*b*), we have

$$+ V_1a - P_1k - P_2g - H_1h = 0$$

from which

$$H_1 = \frac{V_1a - P_1k - P_2g}{h} \qquad (3)$$

In the same way, moments about *C* for loads on the right side of the crown, as shown in Fig. (*c*) gives

$$+ V_2b - P_3f - H_2h$$

from which

$$H_2 = \frac{V_2 b - P_3 f}{h} \tag{4}$$

If a check on the calculated values is desired, it can be obtained by summation of vertical and horizontal forces for the structure as a whole, from which

$$V_1 + V_2 = \Sigma P \cos \theta$$

and

$$H_1 - H_2 = \Sigma P \sin \theta$$

where P is any load and θ is the angle between the line of action of this load and the vertical. Eqs. (1) to (4) are general, and can be applied to any loading conditions.

In calculating the stresses in the members of the arch, the forces acting on the crown hinge must also be known. These forces can readily be calculated for the conditions shown in Figs. (b) and (c) as soon as the reactions at A and B are known.

Graphical Solution for Reactions.—Graphical solutions are based on the fact that zero moment at any point indicates that the resultant of the forces on either side of the point must pass through the point in question. Since the equilibrium polygon for any set of forces represents the action line of resultants on either side of a point, and since hinges are assumed to be points of zero moment, it follows that the equilibrium polygon drawn for the loads on any three-hinged arch must be made to pass through the three hinges. The solution of this problem therefore consists in passing an equilibrium polygon through three given points. Several typical cases will now be considered in detail.

The work which follows is based on the principles of graphic statics given in the chapter on "Principles of Statics" in Sect. 1. Therefore, construction methods for the several cases will be explained, but, in general, proofs will not be given for these methods.

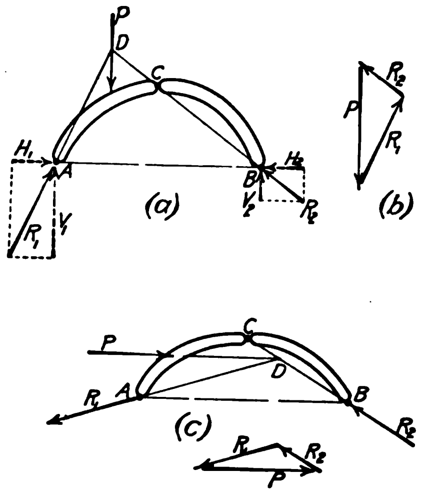


FIG. 217.

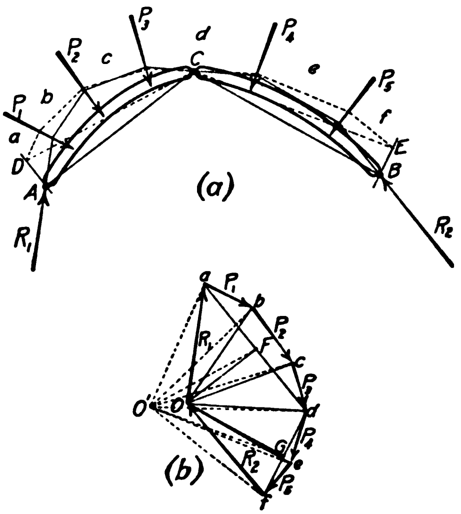


FIG. 218.

Single Load on One Arm of Arch.—Fig. 217 (a) shows a single vertical load on one arm of a three-hinged arch. Since there is no load on the right-hand arm of the arch, and since, as stated above, the line of the resultant forces passes through the hinges, it is evident that the reaction R_2 acts along a line connecting hinges B and C, as shown in Fig. (a). Also, since the structure under consideration is in equilibrium, the resultant of the forces on either side of load P must meet at a point on the action line of the load. Therefore, to find the direction and position of the action line of R_1, produce CB to an intersection with P at point D, and connect A and D. The position and direction of R_1 and R_2 are then completely determined.

To determine the amount of R_1 and R_2, construct a force diagram, as shown in Fig. (b). Lay off force P in amount and direction to any scale. By the methods given in Sect. 1, resolve P into components parallel to the action lines of R_1 and R_2 as given in Fig. (a). The resulting forces give the amount of the reactions, which are thus completely determined. If values corresponding to H_1, H_2, V_1, and V_2 of the algebraic solution are required, they can be determined by resolving R_1 and R_2 of Fig. (*b*) into their vertical and horizontal components. Fig. (*c*) shows the construction for a single horizontal load.

Any Set of Loads.—Fig. 218 (*a*) shows a three-hinged arch supported by hinges at *A*, *B*, and *C* and carrying a set of inclined loads on both arms. The complete solution for the reactions at *A* and *B* requires that an equilibrium polygon for the applied loads be passed through points *A*, *B*, and *C*.

Construct a force diagram for the applied loads, as shown in Fig. (*b*). As the location of the pole for an equilibrium polygon which will pass through the three given points is not known as yet, it must be determined by cut-and-try methods. Assume any pole, as *O′* and construct the corresponding equilibrium polygon. All lines for this construction are shown dotted in Figs. (*a*) and (*b*). In constructing this equilibrium polygon begin with the string which passes through the point *C*. For the case under consideration, this is a line prallel to *O′d* of Fig. (*b*).

Assume for the purpose of this discussion that the applied loads are divided into two groups composed of the loads on either side of point *C*—that is, loads P_1, P_2, and P_3 in one group, and P_4 and P_5 in another group. Determine the direction of the resultants of these two groups. The line *a-d* of Fig. (*b*) shows the direction of the resultant for P_1, P_2, and P_3, and *d-f* shows the direction of the resultant of P_4 and P_5. In Fig. (*a*) draw through points *A* and *B* lines *A-D* and *B-E* parallel respectively to *a-d* and *d-f* of Fig. (*b*). Draw the closing lines *D-C* and *C-E* of Fig. (*a*) for the equilibrium polygons for the two groups of loads, pole at *O′*. In Fig. (*b*) draw lines *O′F* and *O′G* parallel respectively to *D-C* and *C-E* of Fig. (*a*). This operation is equivalent to assuming that the two groups of loads are supported at points *A* and *C* for the left-hand group and *C* and *B* for the right-hand group by forces parallel respectively to the resultants of the two groups.

From the principles of graphic statics it can be shown that while an infinite number of equilibrium polygons can be drawn through point *C* for the conditions shown in Fig. (*a*), in all of these polygons the last string for each group and its closing line will always intersect on the lines *A-D* and *B-E* produced. Also, points *F* and *G* of Fig. (*b*) locate the points of load divide for *A* and *C* and for *C* and *B*. The position of these points will always be the same, regardless of the assumed location of the pole *O′*. Hence these statements also hold true for the equilibrium polygon for points *A*, *B*, and *C*, in which case the intersection of last strings and closing lines is at points *A* and *B* of Fig. (*a*). Therefore *A-C* and *C-B* are the closing lines for the required equilibrium polygon.

To locate the pole of the required equilibrium polygon, in Fig. (*b*) draw *F-O* and *G-O* parallel respectively to *A-C* and *C-B* of Fig. (*a*). Point *O* of Fig. (*b*), the intersection of *F-O* and *G-O*, is the required pole, and the full line equilibrium polygon of Fig. (*a*) passing through points *A*, *B*, and *C* is the required polygon. The direction of the reactions at *A* and *B* is given by the last strings of the true equilibrium polygon, produced, as shown in Fig. (*a*), and the amount of the reactions is given to scale by the corresponding forces in Fig. (*b*). Thus R_1 is given by *O-a* and R_2 is given by *O-f*.

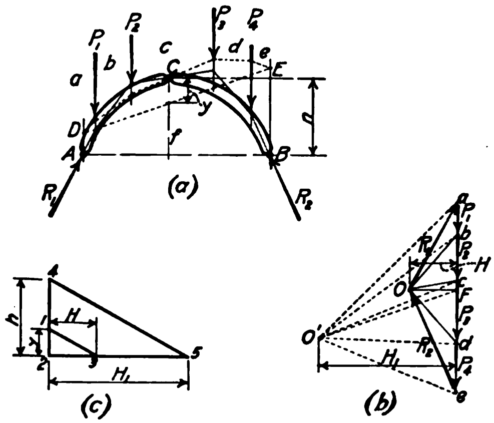


Fig. 219.

Where the applied loads consist of a set of parallel vertical forces, all of which are unequal in amount, the construction of Fig. 218 can also be used. A somewhat simpler solution for this case is shown in Fig. 219. Again assume any pole, as *O′* of Fig. (*b*), with a pole distance H_1. Construct the corresponding equilibrium polygon, which is shown by the dotted lines of Fig. (*a*). Measure the vertical intercept, *y* of Fig. (*a*), between the string of the equilibrium polygon which passes through *C* and the closing line *D-E*.

From the principles of graphic statics, the moment at *C* due to vertical forces to the right or left of the point is $M_c = H_1y$, where H_1 = pole distance, and *y* = the intercept described above. Consider the corresponding value for the equilibrium polygon through points *A*, *B*, and *C*, as shown in Fig. (*a*). The closing line is *A-B*, the equilibrium polygon passes through point *C*, and the vertical intercept is *h*, the height of the crown hinge above hinges *A* and *B*. If *H* be the true pole distance, $M_c = Hh$. But the moment about *C* is a constant and hence the two expressions for M_c given above are equal. Therefore on equating the above expressions, the value of the true pole distance *H* can be determined. On equating these expressions for M_c we have, $H_1y = Hh$, from which, $H = H_1\,y/h$.

A graphical solution of this equation is shown in Fig. (*c*). To obtain the value of *H*, draw a set of rectangular axes 2–4 and 2–5. On the horizontal axis lay off the value of H_1, represented to scale by 2–5, and on the vertical axis lay off *y* = 1–2 and *h* = 2–4. Connect points 4 and 5, and through 1 draw 1–3 parallel to 4–5. Then *H* = 2–3 to the same scale as H_1.

To locate the true pole *O* in Fig. (*b*) draw through *O′* a line *O′-F* parallel to *D-E*, the closing line of the dotted equilibrium polygon of Fig. (*a*). Then *F* of Fig. (*b*) is the load divide point of the vertical forces. Since the closing lines for all poles intersect at point *F*, and since the closing line for the true polygon is a horizontal line, draw from point *F* a horizontal line. Lay off on this line *F-O* = *H* of Fig. (*c*). Point *O* of Fig. (*b*) is the required pole. The full line equilibrium polygon of Fig. (*a*) shows the required polygon. Fig. (*a*) shows the direction of the reactions R_1 and R_2. Their amount is shown in the force polygon of Fig. (*b*).

A special case of vertical loading, in which equal loads are symmetrically placed with respect to the crown hinge, is shown in Fig. 220. Since the loads are symmetrically placed with respect to the crown hinge, only half of the force diagram and the equilibrium polygon need be drawn, since it is known that the string of the equilibrium passing through point C is horizontal, as shown in Fig. (*a*). Draw the force polygon for the loads to the left of the center, as shown in Fig. (*b*). Choose a pole O' and draw an equilibrium polygon, shown by the dotted lines of Fig. (*a*). Since the loads are symmetrical about the center hinge, the closing line of the trial equilibrium polygon will always be horizontal. Therefore, O' is to be located on a horizontal line through point d of Fig. (*b*).

Produce A–E and D–E, the first and last strings of the equilibrium polygon, to an intersection at point E of Fig. (*a*). This locates a point on the line of action of the resultant of the group of loads to the left of the crown hinge. This resultant is shown by R in Fig. (*a*). Since the first and last strings of the equilibrium polygons drawn for any pole will meet on the line of action of R, the true pole can be located as follows: Through hinge C draw a horizontal line C–F intersecting R at F. This line is the last string of the equilibrium polygon through points A, B, and C. Connect A and F. The resulting line is the first string of the required equilibrium polygon. To locate the true pole in Fig. (*b*), draw from point a a line a–O parallel to A–F of Fig. (*a*). Then O of Fig. (*b*) is the required pole. The true equilibrium polygon is shown by the full lines of Fig. (*a*).

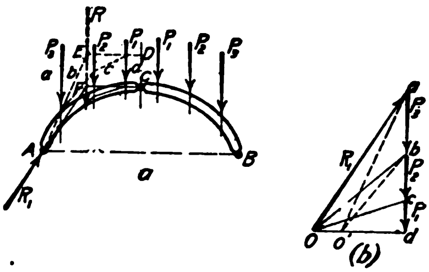


FIG. 220.

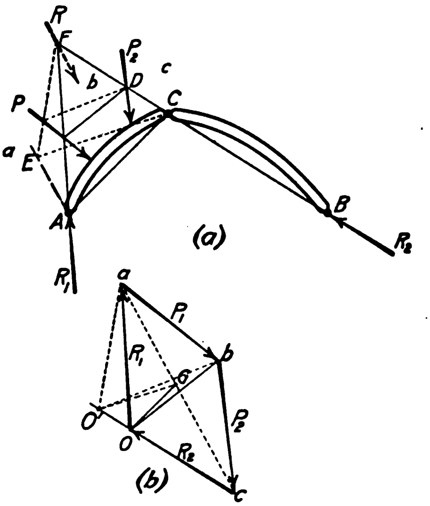


FIG. 221.

Fig. 221 shows a three-hinged arch supporting loads on one arm only. Since there are no loads on the right-hand side of the arch, the direction of R_2 is given at once, as shown in Fig. (*a*). The construction is the same as for Fig. 217. Construct the force polygon of Fig. 221(*b*) and choose a pole O'. Since the last string of the equilibrium polygon must pass through C and B of Fig. (*a*), the pole O' of Fig. (*b*) should lie on a line c–O' which is parallel to B–C of Fig. (*a*). Construct an equilibrium polygon for pole O'. This polygon is shown by the dotted lines. Begin the construction at point D, and close on a line A–E, which is parallel to the resultant of the applied loads. Line a–c of Fig. (*b*) shows the direction of this resultant. The closing line of the polygon is E–C of Fig. (*a*). In Fig. (*b*) locate the load divide point G by drawing through O' a line O'–G parallel to the closing line E–C of Fig. (*a*). To locate the true pole for an equilibrium polygon through A, B, and C, draw from point G of Fig. (*b*) a line G–O parallel to A–C of Fig. (*a*). Point O of Fig. (*b*) is the required pole. Fig. 221 shows the required construction.

This problem can also be solved by assuming that the applied loads are replaced by their resultant R. Assume a pole O' as before and locate the position of R. The construction is shown by the dotted lines of Fig. 221 (*a*). By applying the same principle as used in Fig. 217 for a single load, the direction of R_1 can be determined at once, for the action line of R_1 meets the resultant R at F, a point on B–C produced.

Temperature Stresses.—The changes in the reactions and stresses in three-hinged arches due to changes in temperature are so small compared to the stresses due to direct loading that they are usually neglected. It will be found that the effect of temperature changes on a three-hinged arch is to increase or decrease the dimensions of the structure, depending on the character of the change. If the abutments are rigid, the change in dimensions results in a rise or fall of the crown hinge. If a tie rod is used, so placed as to be protected from sudden changes of temperature, a similar effect is produced. When the tie rod is exposed to the same conditions as the truss, both crown and abutment hinges change position. However, it can be shown that assuming very severe conditions, the changes in dimensions will not exceed 0.1% of the principal dimensions of the structure. Hence temperature changes can be neglected.

170(*b*). Two-Hinged Arches.—The reactions at the points of support for any two-hinged arch can be represented by four unknown forces, as shown in Fig. 222 for a braced arch. Since there are four unknowns to be determined and only three independent equilibrium equations are available, another independent condition must be at hand from which a fourth equation can be formed. In structures of the two-hinged type, the fourth condition equation is made to depend upon the elastic deformation of the arch. This elastic deformation is therefore dependent upon the form of the arch, the sizes of all members, and the conditions of the end supports. Where rigid supports are provided, an equation is formed which states that the horizontal movement of one support with respect to the other is zero. If the resistance to horizontal forces is provided by a tie rod connecting the two supports, it is usual to anchor one end of the arch truss to the foundations and to place the other end on rollers or a sliding plate. For this construction the movement of one support with respect to the other is placed equal to the extension of the tie rod. The method outlined above will be applied to two-hinged arches of the braced and ribbed type.

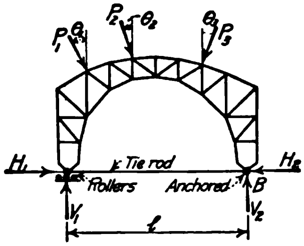


Fig. 222.

Reactions for a Two-Hinged Braced Arch.—Fig. 222 shows a two-hinged braced arch with a tie rod connecting the hinged points of support. It will be assumed that support B is anchored to the foundations and that support A is placed on rollers. Assume that the structure carries the loads P_1, P_2, and P_3, acting as shown. Applying the three conditions of static equilibrium to the structure of Fig. 222, we have

$$\left.\begin{aligned} V_1 &= \Sigma P x_B / l \\ V_2 &= \Sigma P x_A / l \end{aligned}\right\} \tag{5}$$

and

$$H_1 - H_2 = \Sigma P \sin \theta \tag{6}$$

In these equations P = any load, x_A and x_B = perpendicular distance from any load to A and B respectively, θ = angle which any load makes with the vertical, and l = span between hinges.

The fourth independent equation is made to depend upon the elastic deformation of the arch. As stated above, the movement of point A with respect to point B is to be placed equal to the extension of the tie rod. This movement can be calculated by methods for the determination of the deflection of framed structures given in standard works on bridge stresses.[1] From these works, the deflection of any point in a framed structure is given by the formula

$$D = \sum \frac{Sl}{AE} u \tag{7}$$

where D = deflection of any point; S = stress in any member due to the applied loads; u = a ratio which is equal to the stress in any member due to a 1-lb. load applied at the point whose deflection is desired and in the direction of the desired deflection; l = length of any member; A = its area; and E = modulus of elasticity of the material of which the structure is built.

In the case under consideration, the tie rod is a tension member. Hence the movement of point A is to the left. The 1-lb. load used for the determination of values of u is to be applied horizontally at point A and acting to the left. It is assumed that the tie rod is removed when values of u are calculated.

Let H_1 = stress in the tie rod, and let A_t, l_t, and E_t = respectively, the area, length, and the modulus of elasticity of the material for the tie rod. The extension of the tie rod under a stress H_1 is then $H_1 l_t A_t E_t$. Placing the extension of the tie rod equal to the horizontal movement of point A, as given by the general equation for deflection, we have

$$\sum \frac{Sl}{AE} u = H_1 \frac{l_t}{A_t E_t} \tag{8}$$

In this formula, S is the stress in any member of Fig. 222. This stress can not be determined until H_1 is known. However, S can be expressed in terms of H_1 and the stress in any member of the arch of Fig. 222 with the tie rod removed. This can be done in the following manner: Remove the tie rod and calculate the stresses in all members of the statically determinate arch truss thus formed. Let S' denote this stress for any member. Since H_1 and u have the same

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[1] See Modern Framed Structures, by Johnson, Bryan, and Turneaure, Parts I and II.

line of action, it is evident from the definition of u given above that the effect of H_1 on the stress in any member can be expressed by a term of the form $-H_1u$. The minus sign is used because by definition the 1-lb. load acts to the left with respect to point A, while H_1 is a tension and therefore acts to the right with respect to point A. This difference in direction can be accounted for by the use of a minus sign. We then have

$$S = S' - H_1u \tag{9}$$

Substituting this value of S in eq. (8),

$$\sum\left(\frac{S'l}{AE}u - H_1\frac{l}{AE}u^2\right) = \frac{H_1l_t}{A_tE_t}$$

Solving this equation for H_1, the stress in the tie rod is found to be

$$H_1 = \frac{\sum\frac{S'l}{AE}u}{\sum\frac{l}{AE}u^2 + \frac{l_t}{A_tE_t}} \tag{10}$$

In substituting in eq. (10), close attention must be paid to the signs of the stresses S' and u. It will be best to use plus for tension and minus for compression. When S' and u are multiplied, like signs result in plus values, and unlike signs result in minus values. If the signs have been correctly handled, the sign of the result will indicate the direction of H_1. A plus sign indicates that the arrow in Fig. 222 acts as shown, and a minus sign indicates that H_1 acts in the opposite direction.

With eq. (10), and eqs. (5) and (6) given above, the reactions can be determined for an arch with a tie rod. If the hinges are supported by rigid abutments, the effect is equivalent to a tie rod of infinite area. For this condition, the term l_t/A_tE_t is zero, and eq. (10) becomes

$$H_1 = \frac{\sum\frac{S'l}{AE}u}{\sum\frac{l^2}{AE}u^2}$$

Again, if no tie rod is provided, and if the abutments do not provide lateral support, A_t can be taken equal to zero. For this condition the denominator of eq. (10) becomes infinite and hence $H_1 = 0$, or, Fig. 222 is a simple span.

It will be noted in eq. (10) that the value of H_1 is dependent upon the form of the arch truss, as indicated by S', u, and l, and also upon the size of the members, as indicated by A. Therefore, before H_1 can be determined for a given arch, the areas of the members must be known, or they must be assumed. If the structure to be designed is similar in size and loading conditions to an existing structure, it is possible to draw some conclusions regarding the probable size of members for the proposed structure. When this information is not available, a preliminary design can be made, using a value of H_1 determined on the assumption that all members have the same area. Stresses in all members can then be determined by methods to be presented later in this article. After the stresses have been determined, members can be designed to fit these stresses. Using the areas thus determined, another calculation for H_1 can be made, the stresses in the members recalculated, and the members redesigned, if necessary. Usually it will be found necessary to make only one complete design following the preliminary design.

Effect of Temperature Changes on a Two-Hinged Braced Arch. The reactions at the points of support of the two-hinged arch of Fig. 222 due to changes in temperature can be determined by substituting in place of the term $\sum\frac{S'l}{AE}u$ of eq. (10) an expression for the change in the distance between points of support due to the given temperature change. Assume that the structure of Fig. 222 is supported by rigid abutments at A and B. Suppose that the temperature rises t degrees. If the coefficient of linear expansion of the material of which the arch is constructed is c per unit of length, the change in the distance from A to B is $+ ctL$. If H_t denote the horizontal reaction at A, we have from eq. (10),

$$H_t = \frac{\pm ctL}{\sum\frac{l}{AE}u^2} \tag{11}$$

The plus sign is to be used for a rise in temperature, and the minus sign is to be used for a fall in temperature. For a rise in temperature H_1 and H_2 act as shown in Fig. 222; for a fall in temperature they act in opposite directions. It is to be noted that for temperature changes, $V_1 = V_2 = 0$, and that $H_1 = H_2$.

Where a tie rod is used which is protected from changes in temperature due to the fact that it is under ground in a special trough, the methods for the calculation of the reactions are the same as given above. In this case the temperature change t must be based on the known or assumed difference in temperature between truss and tie rod. The denominator of eq. (11) must include the term $\frac{l_t}{A_t E_t}$ of eq. (10).

When A and B of Fig. 222 are connected by an exposed tie rod, for which temperature changes are exactly the same as for the rest of the structure, it can readily be seen that $H_t = 0$, for a temperature reaction exists only when resistance is offered to the tendency of the framework between A and B to expand. Rigid supports, or a tie rod which does not expand as much as the frame work will cause a temperature reaction, while a tie rod whose expansion is equal to that of the frame work will not cause a temperature reaction.

The temperature change to be used in the calculation of H_t of eq. (11) varies with the conditions. For a building which is heated and is not subjected to sudden changes in temperature, 15 to 20 deg. above and below the normal, or a range of 30 to 40 deg. is sufficient. If severe conditions are to be expected, with sudden changes of temperature, 50 or 60 deg. above and below normal, or a range of 100 to 120 deg. should be specified.

Ribbed Arches of Two Hinges.—Hinged arches of two hinges are seldom used in building construction. For methods of calculation for structures of this type the reader is referred to standard text books on the subject of arches.[1]

170c. Hingeless Arches.—Hingeless braced arches of the type mentioned in Art. 169 have been used to some extent in building construction. Arches of the hingeless type are used extensively in bridge work, particularly in the form of steel or reinforced concrete ribs. Since the essential difference in the bridge and roof arch of the hingeless type lies in the applied loading, the reader is referred to standard works on the subject of steel and concrete arches.[2]

170d. General Methods for Determination of Stresses in Braced and Ribbed Arches.—Stresses in the members of a braced arch, or in the web and flanges of a ribbed arch, are best determined by graphical or semigraphical methods. Algebraic methods can also be used, but in general such methods require considerable time for the solution of the problem. The accuracy of the results obtained by the algebraic methods is probably somewhat greater than is possible by the use of graphical methods. However, graphical methods give results which are accurate enough for all practical purposes, and since much time can be saved thereby, especial attention will be given to graphical methods in the work to follow.

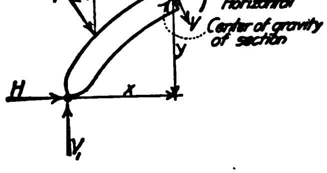

FIG. 223.

In Art. 172 is given a complete solution for stresses in a three-hinged arch. A detailed discussion of the methods employed is given in connection with this solution.

The stresses in an arch of the two or three-hinged type can be determined as soon as the applied loads and the reactions at the supports are known. In general the principles of stress determination are similar to those given in Sect. 1, although the presence of inclined reactions and the curvature of the arch rib causes slight modifications in the methods of calculation. While the arch rib is essentially a curved beam, in most cases the depth of the arch rib is so small

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[1] Modern Framed Structures, Part II. By Johnson, Bryan, and Turneaure.

[2] Modern Framed Structures, Part II. By Johnson, Bryan, and Turneaure. Principles of Reinforced Construction, By Turneaure and Maurer. Reinforced Concrete, Part III. By G. A. Hool. Concrete Engineers' Handbook by Hool and Johnson. Steel Roof Trusses Designed as Elastic Arches, By W. S. Tait, *Engr. News-Record, Apr. 18, 1918.*

compared to its radius of curvature that the internal stresses can be determined without appreciable error by the methods given in the chapter on Bending and Direct Stress in Sect. 1.

An algebraic solution will be given for the conditions shown in Fig. 223, which represents a portion of an arch hinged at A with all forces in position. The internal stresses are represented by a moment, M; a thrust, T; and a shear, V. These internal stresses can be determined by summations of moments and of vertical and horizontal forces taken about the center of gravity of the section, including all external applied loads and reactions. Thus from Fig. 223

$$M = +V_1x - H_1y - P_1a - P_2b = \Sigma M \tag{12}$$

If $\Sigma V = V_1 - P_1 \cos \theta_1 - P_2 \cos \theta_2$ and $\Sigma H = H_1 + P_1 \sin \theta_1 + P_2 \sin \theta_2$, which are respectively the summations of vertical and horizontal external forces, we have

$$T = (\Sigma V) \sin \alpha + (\Sigma H) \cos \alpha \tag{13}$$

and

$$V = (\Sigma V) \cos \alpha - (\Sigma H) \sin \alpha \tag{14}$$

where α is the angle which the tangent to the arch axis makes with the horizontal.

Having given the internal forces acting on any section, the fiber stresses can be determined from the expressions

$$\left.\begin{aligned} f_1 &= \frac{T}{A} + M\frac{c_1}{I} \\ \text{and} \qquad f_2 &= \frac{T}{A} - M\frac{c_2}{I} \end{aligned}\right\} \tag{15}$$

where T and M are as given above; f_1 and f_2 = the fiber stress on the extreme upper and lower fibers, respectively; c_1 and c_2 = the corresponding distances from the extreme fibers to the center of gravity of the section; and A and I = area and moment of inertia of the section respectively. The derivation of these equations is explained in the chapter on Bending and Direct Stress in Sect. 1. For the conditions shown in Fig. 223, the fiber stresses given in eqs. (15) are compressive. If on substituting in these equations the sign is reversed, the resulting stresses are tensile.

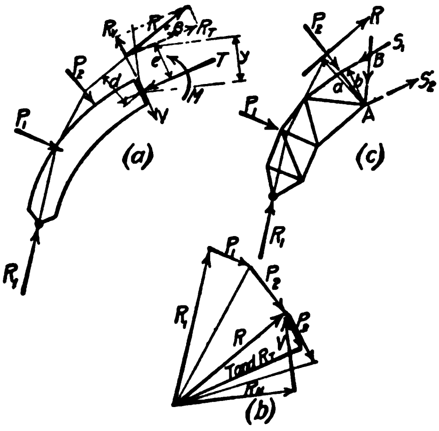

FIG. 224.

A graphical solution for internal stresses is shown in Fig. 224. This solution requires the construction of the force and equilibrium polygons. Fig. 224 shows these polygons in part for certain assumed loads and reactions. Since the string R of the equilibrium polygon is the resultant of all fasces on either side of the section, we have

$$M = Rd \tag{16}$$

where d is the perpendicular distance from R to the center of gravity of the section under consideration. This moment can also be expressed in other terms. If e of Fig. (a) represent the distance from the center of gravity of the section to the intersection of the plane of the section produced and the line of action of R, and if R_T = component of R parallel to a tangent to the arch axis at the section in question, then

$$M = R_Te \tag{17}$$

Again, if R_H = horizontal component of R, and y = vertical distance from center of gravity of section to line of action of R, as shown in Fig. (a), then

$$M = R_Hy \tag{18}$$

The values of R_T and R_H are readily determined from the force polygon of Fig. (b) by resolving R into the required components. Values of T and V are obtained from the force polygon by resolving R into components parallel and perpendicular to the tangent to the arch axis at the section in question, as shown in Fig. (*b*).

Fiber stresses can be determined by the use of eqs. (15), substituting values of M and T as determined above. These equations can be modified some what and the fiber stresses can be determined from the values of T and e of Fig. (*a*). From eq. (17) and Fig. (*a*), $R_T = T$, and hence, $M = Te$. Substituting this value of M in eq. (15) and also noting that $I = Ar^2$, where A = area of the section, r = its radius of gyration, these equations can be written in the form

$$\left.\begin{aligned} f_1 &= \frac{T}{A}\left(1 + \frac{ec_1}{r^2}\right) \\ \text{and} \qquad f_2 &= \frac{T}{A}\left(1 - \frac{ec_2}{r^2}\right) \end{aligned}\right\} \tag{19}$$

In some cases the desired results are obtained more directly by the use of eq. (19) than by the use of eq. (15).

The graphical methods of calculation given above are general and apply to all types of arches. However, the distances d, e, and y shown in Fig. 224 (*a*) are often so small that they can not be determined with the desired degree of precision. Under such conditions, the moments should be calculated by algebraic methods, using eqs. (12).

Methods of stress calculation similar to those outlined above can also be applied to the braced arch. Fig. 224 (*c*) shows a section cut through any panel of a braced arch. To determine the stress S_1 in a chord member, take moments about point A, the intersection of the other members cut by the section. Since R is the resultant of all external forces to the left of the section, we have

$$S_1 = Ra/b$$

where a and b, respectively, are the lever arms of R and S_1, as scaled from the drawing. The stress in S_2 can be obtained from a similar equation about B. If members S_1 and S_2 intersect within the limits of the drawing, the stress in S_3 can be determined by moments taken about the intersection point. If they do not intersect within the limits of the drawing, a resolution equation can be taken for an axis perpendicular to one of the chord members.

171. Loading Conditions for Arch Trusses.—The loads to be carried by an arch roof truss can be determined from the data given in the chapter on Roof Trusses—General Design by methods similar to those used in the preceding chapters on the design of wooden and steel roof trusses. In most cases the slope of the roof surface is not uniform, as in the cases considered in the preceding chapters, for it is made to conform to the contour of the top chord of the arch. As the wind and snow loads depend for their value on the roof slope, the wind and snow panel loads for arch trusses will vary with the location of the panel point. An application of the methods of calculation is given in the problem of Art. 172.

Formulas for the weight of arch trusses which will apply to all types of arch structures are not available, as structures of this type vary so widely in form and in class of service that sufficient consistent and reliable information has never been collected on which to base a formula. In general, the designer must draw conclusions regarding the probable weight of the arch to be designed, either from existing structures of the same size, or from his judgment based on passed experience. After a design has been made, based on an assumed dead weight, the true weight of the structure should be calculated and the assumed weight revised, if found necessary. From an examination of the weights of existing arches, it was found that the weight per square foot of covered area may be anywhere from 10 to 25 lb., depending upon the span length, spacing of trusses, and the specified loading conditions.

Maximum stresses in the members of arch trusses are to be determined for loading conditions similar to those used for simple roof trusses. In general the following loading conditions are used: (*a*) dead load, (*b*) snow load on left side of roof, (c) snow load on right side of roof, (*d*) snow load on whole roof, (*e*) wind load on left side of roof, and (f) wind load on right side of roof.

In combining the stresses due to these loads in order to obtain maximum stresses, most designers assume that snow and wind loads do not act on the roof at the same time. Other-assume conditions similar to those used in the preceding chapters. This is a matter on which the designer must use his judgment. In making up the maximum stresses in the members, the dead load stresses should be combined with the snow or wind load stress which will produce greatest tension and greatest compression in the members. It must be remembered, in this connection, that the wind and snow load stresses may be of the same character as the dead load stresses, or they may differ in character. In the latter case, if they exceed the dead load stresses, a reversal of stress will occur. This information must be at hand before a correct design of members can be made.

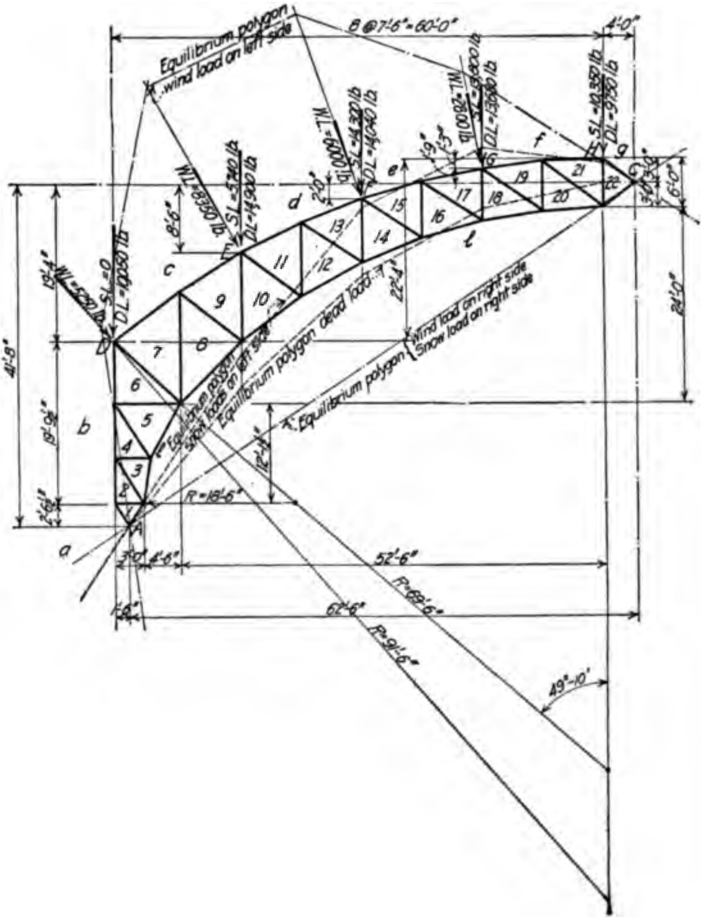

Fig. 225.—Truss diagram—typical three-hinged arch.

172. Determination of Stresses in a Typical Three-Hinged Arch Truss.—The methods of stress calculation outlined in Art. 170*d* will now be applied to a typical three-hinged arch of the dimensions shown in Fig. 225. This arch has a span of 125 ft., *c.* to *c.* of end pins, and a rise of 41⅔ ft. The type of framing adopted divides the truss into panels of 7.5 ft., as shown in Fig. 225. Purlins will be placed at alternate panel points. The distance between trusses will be taken as 30 ft. It will be assumed that the sides of the building consist of self-supporting masonry walls. No part of the weight of the walls will be assumed as carried by the trusses. It will be assumed, however, that the roof load at point *D* of Fig. 225 is carried by the trusses.

Dead Load Stresses.—The dead load stresses are to be determined for the weight of the roof covering and the weight of the trusses. It will be assumed that the roof covering consists of tile or slate laid on 2-in. plank, which are supported by rafters. These rafters will be assumed to be placed parallel to the trusses, and will be assumed to be supported by purlins of the type described in Art. 174. Design methods for the roofing and the rafters are given in the chapter on Roof Trusses—General Design. A roof covering of the assumed type will be found to weigh about 20 lb. per sq. ft. of roof surface. The weight of the trusses is determined by methods outlined in Art. 171. It will be assumed, as a basis for a preliminary design, that the weight of the trusses and purlins is 10 lb. per sq. ft of horizontal covered area.

The panel loads due to the roof covering and the dead weight of the arch will be assumed to be concentrated at the point of attachment of the purlins. As the roof load is given in pounds per sq. ft. of roof surface, and since the roof area tributary to the purlins depends upon the slope of the roof, the panel loads due to the roofing will vary. Since the dead weight is given in pounds per sq. ft. of horizontal covered area, the part of the panel load due

to the weight of the trusses will be the same at all points, for the horizontal spacing of the purlins is taken as 15 ft., as shown in Fig. 225.

To illustrate the methods used in calculating panel loads from the above data, the dead panel load for point *F* of Fig. 225 will be determined. In calculating the roof area tributary to point *F*, it will be assumed that points *E*, *F*, and *G* are joined by straight lines. For the dimensions shown on Fig. 225, *E-F* = 16.3 ft., and *F-G* = 15.5 ft. As stated above, the roofing weighs 20 lb. per sq. ft., and the trusses are spaced 30 ft. apart. The roofing panel load at *F* is then ½ (16.3 + 15.5) × 30 × 20 = 9540 lb. By similar methods, the roofing panel loads at other points are as follows: *D*, 5550 lb.; *E*, 10,400 lb.; *G*, 9180 lb.; and *H*, 6300 lb. Assuming that the trusses and purlins weigh 10 lb. per sq. ft. of horizontal covered area, as stated above, the dead panel load due to trusses and purlins is 10 × 15 × 30 = 4500 lb. At point *H*, where the horizontal projection is 11.5 ft., the panel load is 3450 lb. As the weight of several members is probably transferred to joint *D*, it will be assumed that a full panel of truss weight is carried at this point. Adding the loads due to the roofing and the truss dead weight, the total panel loads at the several joints are as follows: *D*, 10,050 lb.; *E*, 14,900 lb.; *F*, 14,040 lb.; *G*, 13,680 lb.; and *H*, 9750 lb. These panel loads are shown in position on Fig. 225.

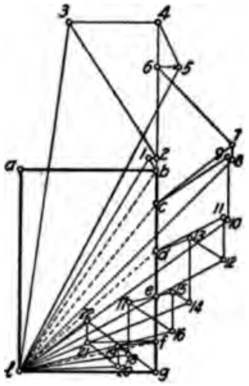

Fig. 226.—Dead load stress diagram—stresses in members of left half of arch.

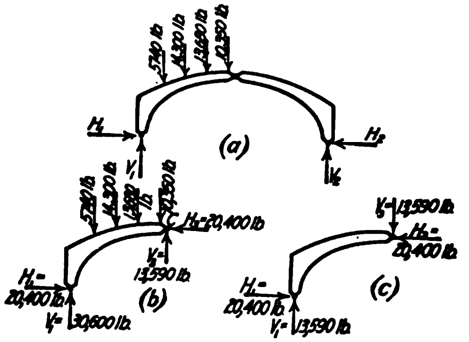

Fig. 227.—Loading diagrams—snow load stresses.

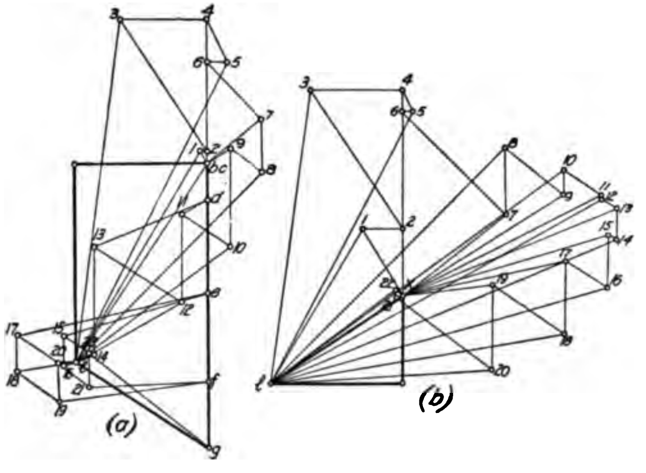

Fig. 228.—Snow load stress diagram.

The reactions at the hinges *A* and *C* due to dead load are calculated by the methods given in Art. 170*a*. Since the dead panel loads are all vertical, and are symmetrically placed with respect to the center hinge, the vertical component of the reaction at *A* is evidently equal to the sum of the panel loads on one side of the center of the arch, or, V_1 = 62,420 lb. The horizontal component of the reaction at *A* is equal to the moment about *C* divided by the rise of the arch. For the loads and dimensions shown in Fig. 225,

$$H_1 = \frac{62{,}420 \times 62.5 - 9750 \times 4 - 13{,}680 \times 19 - 14{,}040 \times 34 - 14{,}900 \times 49 - 10{,}050 \times 64}{41.67} = 42{,}000 \text{ lb.}$$

Since all of the loads are vertical the reaction at hinge *C* is horizontal and equal to H_1.

In the case under consideration, algebraic methods are readily applied to the determination of the reactions as all of the lever arms can be obtained from Fig. 225 without further calculation, except simple addition. While graphical methods can be applied to this case, little is to be gained thereby. The algebraic method of calculation is therefore recommended.

The stresses in the members of the arch due to the applied loads shown on Fig. 225 and the reactions calculated above are readily determined by the graphical methods of stress analysis given in Sect. 1. Fig. 226 shows the stress diagram as drawn for the left side of the arch.

In constructing stress diagrams of the kind shown in Figs. 226 to 229, great care must be used in drawing the diagrams, for, to be correct, the diagram must close. That is, suppose that the diagram is begun at point *A* of Fig. 225, and carried forward to point *C*. If the diagram is accurately drawn, the resultant of the stresses in members *g*-22 and *l*-22 at joint *C* will be equal to R_2, the hinge reaction at *C*. In Fig. 226, exact closure of the stress diagram is obtained when the horizontal components of *l*-22 and *g*-22 are equal to *l*-*g*, and when point 22 is directly over point 21. The effect of cumulative errors on the closure of the diagram can be reduced by starting the diagram at point *A* and carrying it about half way across the framework. Another start can then be made at point *C*, and closure made on the part of the diagram already drawn. It will usually be found that closing errors can be reduced by this method.

Accurate construction of stress diagrams is greatly facilitated if the truss diagram, shown by Fig. 225, is drawn to a large scale. This results in long lines, from which the slope of the members can readily be obtained. If a small size truss diagram is used, the lines are so short that an accurate determination of the true slopes is impossible. The stress diagrams should be drawn to a scale which will result in lines which can be drawn with triangles not exceeding about the 12-in. size. This avoids inaccuracies resulting from lines drawn by several shifts of the triangle. Also, the stress diagram should be located as close to the truss diagram as possible, in order to avoid transferring lines for a long distance, which is certain to result in inaccurate work.

It is best to make frequent checks on the graphical work by means of stresses calculated by the algebraic method explained in Art. 170*a*. Stresses in chord members are readily calculated by the method shown in Fig. 224(*c*), and form a convenient check. If the graphical and algebraic methods do not check, it is well to revise the graphical work before proceeding with the construction of the diagram.

Snow Load Stresses.—Stresses due to snow load are to be determined for three conditions of loading, as stated in Art. 171. These conditions are (*a*) snow load on left side of roof, (*b*) snow load on right side of roof, and (*c*) snow load on whole roof.

The panel loads due to snow are to be determined from the data given in Table 8, p. 467. Since the roof slope varies, the unit snow load will depend upon the location of the panel point. Several different assumptions can be made regarding the variation in the snow load. For the case under consideration, it will be assumed that the outside roof surface is an arc of a circle, and that the unit snow load for the area tributary to any panel point is equal to the load for a plane tangent to the roof surface at the panel point.

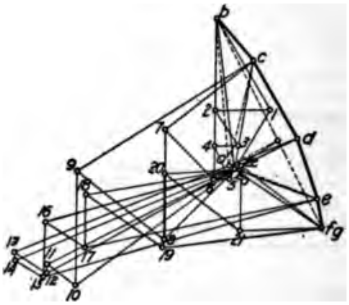

Fig. 229.—Wind load stress diagram.

Thus at point *F* of Fig. 225, a plane tangent to the roof surface makes an angle of about 18 deg. 30 min. with the horizontal. It can be shown that this angle corresponds closely to a pitch of ⅙, as defined in the chapter on Roof Trusses—General Design. From the table of snow loads referred to above, the snow load per sq. ft. of roof surface for a tile roof of ⅙ pitch located in the Central States is 30 lb. By methods similar to those used above for the dead panel load due to roofing, it will be found that the snow panel load for point *F* is ½ (16.3 + 15.5) × 30 × 30 = 14,300 lb. Panel loads at other points are as follows: *D* = 0 (slope 45 deg., unit snow load = 0); *E* = 5740 lb. (slope = 30 deg., unit snow load = 11 lb.); *G* = 13,800 lb. (slope practically flat, unit snow load = 30 lb.); *H* = 10,350 lb. (slope = flat, unit snow load = 30 lb.).

In tabulating the stresses in a symmetrical three-hinged arch, it is usual to make a table containing the members of the left half of the arch. Table 1, in which the stresses for the arch of Fig. 225 are tabulated, contains the members of the left half of the arch. All stresses required in Table 1 for the three snow loading conditions can be determined from stress diagrams drawn for all members of the arch due to snow loads on one arm of the arch, no load on the other arm, as shown in Fig. 227(*a*).

The reactions at the points of support and at the crown hinge due to the loading shown on Fig. 227(*a*) can be determined by the methods given in Art. 170*a*. These reactions are as follows, using the notation shown on Fig. 227: V_1 = 30,600 lb.; H_1 = 20,400 lb.; V_2 = 13,590 lb.; H_2 = 20,400 lb.; V_3 = 13,590 lb.; and H_3 = 20,400 lb. All forces act as shown in Fig. 227. A graphical solution of the reactions can be made by the method shown in Fig. 221.

The stresses in the members of the left half of the arch for case (*a*), loads on the left half of the arch, are given by a stress diagram drawn for the loading conditions of Fig. 227(*b*). This stress diagram is shown in Fig. 228(*a*). The stresses scaled from this diagram are recorded in col. 2 of Table 1. Stresses in the members of the left half of the arch for case (*b*), loads on the right half of the arch, are given by the stress diagram of Fig. 228(*b*), which is drawn for the loading conditions shown in Fig. 227(*c*). It will be noted that the loading conditions shown in Fig. (*c*) are opposite hand of those for the right-hand half of the arch, loads on the left half, as shown in Fig. (*a*). Stresses scaled from the stress diagram of Fig. 228(*b*) are recorded in col. 3 of Table 1. The stresses for members of the left half of the arch for case (*c*), loads on the whole arch, can be obtained by adding the stresses given in Figs. 228(*a*) and (*b*) for the member in question. These stresses are recorded in col. 4 of Table 1.

Wind Load Stresses.—As in the case of the wooden and steel simple roof trusses designed in the preceding chapters, it will be assumed that the working stresses for wind loads are 50% larger than those for dead and snow loads. Assuming, as before, that the working wind load is 30 lb. per sq. ft., and that the working stress for wind loading is 24,000 lb. per sq. in., the working wind load to be used for a 16,000 lb. unit stress is 20 lb. per sq. ft. Wind panel loads will therefore be determined for a unit wind pressure of 20 lb. per sq. ft.

In determining the normal wind pressure to be used at the several panel points, the same assumptions will be made as for snow panel loads. Thus at point F where the slope of the tangent to the roof surface corresponds to a, ⅙ pitch, the normal wind pressure, as given by Table 7, p. 467, is 13.9 lb. per sq. ft. of roof surface. The resulting panel load is ½(16.3 + 15.5) × 30 × 13.9 = 6000 lb., acting normal to the roof. By methods similar to those used for the snow panels loads, it will be found that the wind panel loads at the other points are as follows: D = 5250 lb. (slope = 45 deg., unit wind load = 18.9 lb.); E = 8350 lb. (slope = 30 deg., unit wind load = 16 lb.); G = 2800 lb. (slope = about 9 deg., unit wind load = 6.1 lb.); and H = O (slope flat). These loads are shown in position on Fig. 225. Since the side walls are assumed to be self-supporting, it will be assumed that the wind loads in these walls are carried directly to the foundations without causing any stress in the members of the arch trusses. If the construction is such that the arch carries the horizontal wind load, the wind panel loads can be calculated by methods similar to those used in the chapter on the Detailed Design of a Truss with Knee-braces.

The reactions due to wind loads will be determined by graphical methods, for the work required by a graphical solution will be found to be considerably less than that required by an algebraic solution. Using the method given in Fig. 221 of Art. 170a, the final equilibrium polygon is shown in position in Fig. 225. The resulting reactions are shown to scale on the force polygon of Fig. 229.

Table 1.—Stresses in a Three-hinged Arch Roof Truss

(Fig. 225)

Member		Dead load (1)	Snow load Left side loaded (2)	Snow-load Right side loaded (3)	Snow load Both sides loaded (4)	Wind load Left side loaded (5)	Wind load Right side loaded (6)	Maximum tension (7)	Maximum compression (8)
Top chord	*b*-1	+ 4,500	+2,250	+12,250	+14,500	−10,000	+ 4,930	19,000	5,500
	b-2	+ 4,000	+1,900	+10,500	+12,400	−8,600	+ 4,220	16,400	4,600
	b-4	+45,900	+22,300	+32,100	+54,400	−11,900	+12,900	100,300	
	b-6	+32,000	+15,600	+28,600	+44,200	−14,900	+11,500	76,200	
	c-7	+29,600	+10,800	+20,200	+31,000	−10,300	+ 8,200	60,600	
	c-9	+26,200	+4,200	+29,500	+33,700	−19,200	+11,850	59,900	
	d-11	+23,500	− 4,500	+34,500	+30,000	−25,900	+13,900	53,500	2,400
	d-13	+11,200	−18,900	+36,000	+17,100	−28,200	+14,500	47,200	17,000
	e-15	+ 5,000	−23,100	+33,100	+10,000	−26,400	+13,300	38,100	21,400
	e-17	− 8,500	−30,100	+25,600	− 4,500	−22,100	+10,300	17,100	38,600
	f-19	− 11,800	−23,100	+14,000	− 9,100	−15,000	+ 5,630	2,200	34,900
	f-21	− 21,200	−18,200	− 900	−19,100	− 7,750	− 360		40,300
	g-22	− 26,000	−23,800	− 1,500	−25,300	− 9,700	−600	....	51,300
Bottom chord	*l*-1	− 77,000	−37,900	−28,000	−65,900	− 6,500	−11,300	......	142,900
	l-3	−108,900	−53,600	−46,000	−99,600	− 2,350	−18,500	......	208,500
	l-5	−106,000	−52,000	−47,600	−99,600	+ 900	−19,200	......	205,600
	l-8	− 92,500	−41,000	−51,500	−92,500	+ 9,100	−20,700	......	185,000
	l-10	− 79,500	−29,700	−56,200	−85,900	+18,100	−22,600	......	165,400
	l-12	− 71,800	−18,700	−58,800	−77,500	+19,800	−23,600	......	149,300
	l-14	−56,700	− 2,900	−58,000	−60,900	+22,000	−23,400	......	117,600

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Verticals and horizontals	*l*-16	− 48,800	+ 2,000	−54,300	−52,300	+18,300	−21,800	......	103,100
	l-18	−34,000	+ 9,300	−46,200	−36,900	+14,100	−18,600	......	80,200
	l-20	−30,300	+ 2,600	−34,300	−31,700	+ 6,650	−13,800	......	62,000
	l-22	−26,000	− 2,200	−24,000	−26,200	− 600	− 9,650	......	52,200
	1-2	− 2,500	− 1,100	− 6,200	− 7,300	+ 5,150	− 2,500	2,650	9,800
	3-4	− 27,200	−13,300	−14,200	−27,500	+ 2,200	− 5,700	......	54,700
	5-6	+ 6,600	+ 3,100	+1,600	+ 4,700	+ 1,420	+ 850	11,300	
	7-8	− 4,500	− 8,200	+10,300	+ 2,100	−10,300	+ 4,150	5,800	14,800
	9-10	− 19,600	−15,300	+3,800	−11,500	−10,700	+ 1,500	......	34,900
	11-12	− 13,000	−13,800	− 600	−13,200	− 800	− 240	......	26,800
	13-14	− 20,000	−16,800	− 4,900	−21,700	− 700	− 1,970	......	41,700
	15-16	− 12,000	− 4,600	− 8,200	−12,800	+5,180	− 3,300	......	24,800
	17-18	− 16,000	− 5,700	−11,200	−16,900	+5,050	− 4,500	......	32,900
	19-20	− 5,900	+ 5,900	−13,100	− 7,200	+ 6,850	− 5,260	950	19,000
	21-22	+ 6,500	+ 5,200	+ 1,000	+ 6,200	+ 6,200	+ 400	12,700	
Diagonals	2-3	+ 50,000	+24,500	+25,800	+50,300	− 3,900	+10,400	100,300	
	4-5	− 15,200	− 7,300	−3,800	−11,100	−3,350	− 1,530	......	26,300
	6-7	− 33,200	−12,200	−22,500	−34,700	+ 6,500	− 9,050	......	67,900
	8-9	+ 1,500	+ 6,000	−11,600	− 5,600	+10,400	− 4,660	11,900	10,100
	10-11	+ 1,500	+ 9,100	− 7,000	+ 2,100	+ 3,300	− 2,820	10,600	5,500
	12-13	+ 12,500	+16,000	− 2,800	−13,200	+ 3,500	− 1,130	28,500	700
	14-15	+ 6,200	+ 5,500	+ 1,400	+ 6,900	− 3,150	+ 570	13,100	
	16-17	+ 16,000	+ 8,500	+ 7,800	+16,300	− 4,500	+ 3,140	32,300	
	18-19	+ 4,100	− 8,000	+13,500	+ 5,500	− 8,700	+ 5,430	17,600	4,600
	20-21	+ 11,800	− 6,000	+18,600	+12,600	− 8,950	+ 7,500	30,400	
Reactions	V_1	+ 62,420	+30,600	+13,590	+44,190	+14,200			
	H_1	+ 42,000	+20,400	+20,400	+40,800	− 1,850	+ 5,450		
	V_2	0	+13,590	−13,590	0	+ 5,450	+ 8,250		
	H_2	+ 42,000	+20,400	+20,400	+40,800	+ 8,250	− 5,450		
	R_2	42,000	24,500	24,500	40,800	9,850	9,850		

Stress Notation;— + = tension — = compression

Reaction Notation:—Positive reactions act as shown in Fig. 227 (*b*).

As stated in Art. 171, wind stresses are to be determined for wind load on either half of the arch. The stress diagram of Fig. 229 is drawn for stresses in the members of the left half of the arch due to loads on the left side of the crown hinge. These stresses are recorded in col. 5 of Table 1. Stresses in the members of the left half of the arch due to wind loads on the right side of the crown hinge can be determined by ratio from the snow load stresses for the corresponding condition of loading. This short cut is possible because for loads on the right side of the arch, stresses in members of the left half of the arch are due to the action of the right half against the left half. As shown in Figs. 221 and 227 (*a*), this action can be represented by a force acting on a line connecting the crown and abutment hinges. Therefore the wind stresses required for col. 6 of Table 1 can be obtained by multiplying the stresses given in col. 3 by the ratio of the reactions at the supporting hinges for the two cases. From Fig. 228 (*b*), the reaction at *A* for snow load on the right half of the arch is 24,500 lb. The reaction at *A* for wind loads on the right half of the arch is the same as that given in Fig. 229 for the right-hand support, which is found to be 9850 lb. Hence, if the stresses in col. 3 are multiplied by 9850/24,500 = 0.402, the resulting stresses will be the values required for members of the left half of the arch due to wind loads on the right half. These stresses are shown in col. 6 of Table 1.

Maximum Stresses in Members. The maximum stresses in the members of the arch under consideration will be calculated on the assumption that wind and snow loads do not act at the same time. Table 1 gives the possible combinations of the dead load stresses and the snow or wind stresses which will result in the greatest tension and compression in the several members.

173. Design of Members and Joints for a Typical Three-hinged Arch.—The principles governing the selection of the form of members for arch trusses, and the design of these members are the same as for the trusses designed in the preceding chapters. These principles are given in the chapter on Roof Trusses-General Design. The application of these principles to the design of arch trusses will be illustrated by a partial design of members and joint details for the three-hinged arch for which the stresses have been calculated in Art. 172.

The form of the members of an arch truss will depend on the amount of stress to be carried. For the truss under consideration in Art. 172, it will be found from a study of the stresses given in Table 1, that the stresses in all members, except a few of the lower chord members, can be provided for by sections composed of two angles. The bottom chord members in which large stresses exist can be made of angles and plates. Truss members for large arches, in which very heavy stresses exist, can be made of the same form as those used in bridge truss work. The trusses for the drill hall of the University of Illinois, described in *Engr. News* for Dec. 11, 1913, are composed of I and H beams. The *Engr. Rec.* for Oct. 7, 1916 contains a description of an arched roof truss whose members are composed of angles and plates.

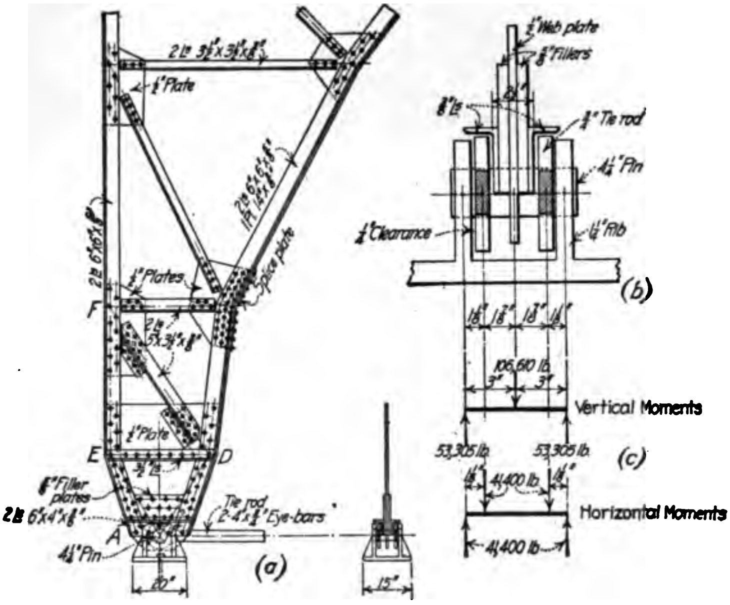

Fig. 230.

By methods similar to those used in the designs of the preceding chapters, it will be found that the members listed as top chord members in Table 1 of Art. 172 can be made of two 6 × 6 × ½-in. angles, separated by a ½-in. space for gusset plates. This section furnishes excess area for some of the members, but since it meets the requirements of most members, it will be adopted throughout. The bottom chord members are subjected to somewhat greater variations in stress than the top chord members. Adequate provision for all stresses will be provided by the following sections: members *l*-12 to *l*-14, two 6 × 6 × ½-in. angles; members *l*-12 to *l*-10, two 6 × 6 × ⅝-in. angles; and members *l*-8 to *l*-1, two 6 × 6 × ⅝-in. angles and a 14 × ⅜-in. plate. All web members, except a few near the end of the arch, can be made of two 3½ × 3 × ⅜-in. angles. For the other web members, two 5 × 3½ × ⅜-in. angles will answer. Figs. 230 and 232 show the general arrangement of members.

Joint details for the three-hinged arch under consideration in this chapter are designed by the methods outlined in the chapter on Roof Trusses—General Design. With the exception of the hinged joints at *A* and *C*, the application of these principles is exactly the same as for the simple trusses designed in the preceding chapters.

Fig. 230 shows the adopted details for the hinge joint at *A* and a portion of the lower end of the arch truss. As shown on Fig. 230, the members at the lower end of the truss are connected to a large gusset plate which includes several joints and members. This is necessary because the members are short and the stresses are large, thus requiring large joint details. A single plate greatly strengthens the end detail and makes possible a very compact joint.

It will be assumed that the rivets used in the design under consideration are $\frac{7}{8}$-in. in diameter, and that the allowable bearing and shearing values are 24,000 and 12,000 lb. per sq. in. respectively. From Fig. 230 it can be seen that the rivets connecting the members to the plates are in bearing on a $\frac{1}{2}$-in. plate. For the allowable values given above, the rivet value is 10,500 lb. All of the end details shown in Fig. 230 provide sufficient rivets to connect the members to the gusset plates. It will be noted that lug angles are used on member *D-F*. These lugs are used in order to reduce the size of the end connection, and also to provide a connection between both legs of the angles and the gusset plate. This is advisable where the stresses in the members are large. The design of lug angle details is considered in the chapter on Splices and Connections—Steel Members in Sect. 2.

The top and bottom chord members are usually spliced at frequent intervals in trusses with curved chords. When the chord section consists of two angles, an effective splice is furnished by a detail similar to that used at joint *g* of the steel roof truss designed in the chapter on the Detailed Design of a Steel Roof Truss. By using this detail, the stress in the horizontal legs of the angles is transferred across the splice by means of the splice plate, leaving only the stress in the vertical legs of the angles to be transferred to the gusset plate, thus securing compact joint details. A similar detail can be used where the chord section consists of angles and plates. If the joints are milled so that a bearing fit is assured, only enough rivets need be provided to hold the members in contact. Figs. 230 and 232 show the details adopted for the design under consideration.

The design methods to be used for the shoe and the pin at joint *A* depend upon the assumptions made regarding the action of the supporting forces at the abutments. If it be assumed that the horizontal component of the reaction is taken by a tie rod, the shoe and the supporting foundation can be designed for vertical forces only. Fig. 230 shows a shoe designed on this assumption. If it be assumed that the foundations can resist vertical and horizontal forces, the shoe must be placed at an angle to the vertical, as shown in Fig. 231. Designs based on these two assumptions will be considered in detail.

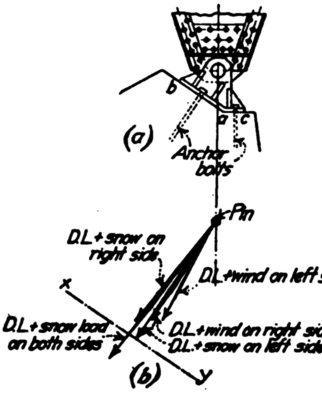

Fig. 231.

Consider first the tie rod design shown in Fig. 230. In this design it is assumed that the horizontal and vertical components of the reaction are taken respectively by the tie rod and the shoe. Table 1 of Art. 172 shows that these reactions are a maximum for dead load and snow load on both arms of the arch. The horizontal component of the reaction is found to be 42,000 + 40,800 = 82,800 lb., and the vertical component is found to be 62,420 + 44,190 = 106,610 lb.

Assuming that the working stress in the tie rod is 16,000 lb. per sq. in., the area required is 82,800/16,000 = 5.27 sq. in. Two 4 × $\frac{3}{4}$-in. eye-bars furnish 6.0 sq. in. If the allowable bearing on a concrete foundation is taken as 400 lb. per sq. in., the area of the base of the shoe must be 106,610/400 = 266 sq. in. The shoe shown in Fig. 230 provides a base area of 15 × 20 = 300 sq. in.

Design methods for the pin connecting the shoe, tie rod, and truss are given in the chapter on Splices and Connections—Steel Members. The size of the pin is determined subject to the following conditions: the bearing areas between the members and the pin must be sufficient to keep the bearing pressures within the allowable limits, which will be taken as 24,000 lb. per sq. in., and, the extreme fiber stress due to bending, considering the pin as a simple beam, must be within the allowable limits, which will be taken as 25,000 lb. per sq. in.

The design of the pin is carried out by assuming the size of pin. Having given the maximum load to be carried by the pin, the bearing areas required for the several parts are determined. If the parts butting on the pin do not furnish the required area, they must be increased by the addition of pin plates until the proper area is provided. Assuming the centers of pressure to be located at the centers of the bearing areas, the bending moments due to the applied loads are calculated and compared with the resisting moment provided by the assumed pin. If the assumed pin is found to be inadequate, the calculations must be revised.

For the case under consideration, a $4\frac{1}{4}$-in. pin will be assumed. Fig. 230 shows the adopted arrangement of the joint details. The load brought by the pin to the shoe is equal to the vertical component of the reaction, which is 106,610 lb. At 24,000 lb. per sq. in., the width of bearing required on the webs of the shoe is $106{,}610/4\frac{1}{4} \times 24{,}000 \times 2 = 0.518$ in. for each web. Assuming that a cast-steel shoe is used, the webs will be made 1 in. thick, as the use of thinner material is not advisable.

The load brought by the arch to the pin is equal to the resultant of the horizontal and vertical components of the maximum reaction, which is due to dead load and snow load on both arms of the arch. For the components given above, this load is $(\overline{82{,}800}^2 + \overline{106{,}610}^2)^{\frac{1}{2}} = 135{,}000$ lb. The width of bearing required at the lower end of the arch truss is $135{,}000/4\frac{1}{4} \times 24{,}000 = 1.32$ in. Since the main gusset plate at joint *A* is $\frac{1}{2}$ in. thick, the width of bearing must be increased by the addition of pin plates. Fig. 230(*a*) shows the adopted detail. The main angles are spread somewhat, and the space between the angles is filled by means of ⅝-in. plates placed on both sides of the gusset plate. To stiffen the plates, and also to tie the main angles together, a 6 × 4 × ⅜-in. angle is riveted on each side of the plates. The total thickness of bearing provided by this detail is 2½ in., which is in excess of that required, but as a rigid detail is desired, it is not advisable to use a smaller number of plates.

The bending moment on the pin can be determined by calculating the moments due to the vertical and horizontal forces, and finding their resultant. Fig. 230(c) shows the components of forces and the lever arms. These lever arms are determined for the packing shown in Fig. 230(b). A clear space of ¼ in. is provided between the several members. From Fig. (c), the vertical component of moment is 53,305 × 3.0 = 166,500 in.-lb., and the horizontal component of moment is 41,400 × 1.125 = 46,600 in.-lb. The resultant moment is then $(166{,}500^2 + 46{,}600^2)^{1/2} = 173{,}000$ in.-lb. From the tables of bending moments on pins, it will be found that the safe moment on a 4¼-in. pin for an allowable fiber stress of 25,000 lb. per sq. in. is 188,410 in.-lb. The assumed pin will be adopted.

The pin plates which were added to the gusset plate at point *A*, in order to increase the width of bearing on the pin, must be fastened to the gusset plate so that all plates will act as a unit. Assuming that the load carried by each plate is proportional to its thickness, the load carried by each ⅜-in. angle is 135,000 × 0.375/2.5 = 20,600 lb., and the load carried by each ⅝-in. filler plate is 135,000 × 0.625/2.5 = 33,800 lb. As shown in Fig. 230(a), the rivets connecting the 6 × 4 × ⅜-in. angles to the plates are in double shear, when both angles are assumed to act together. For the allowable shearing value given above, the double shear value of a rivet is 14,400 lb. Assuming that the two angles act together, the total load to be carried is 2 × 20,600 = 41,200 lb., and the number of rivets required is 41,200/14,400 = 3 rivets. The detail of Fig. 230(a) shows three rivets close to the pin and four others at the ends of the angles. Assuming that the ⅝-in. filler plates and the angles on each side of the gusset plate act together, the total load to be carried is 2(33,800 + 20,600) = 108,800 lb. As shown in Fig. 230(a), the connecting rivets are in bearing on the ½-in. gusset plate, and hence the number of rivets required is 108,800/10,500 = 11 rivets. Fig. 230(a) shows 14 rivets in place in the filler plates and the angles.

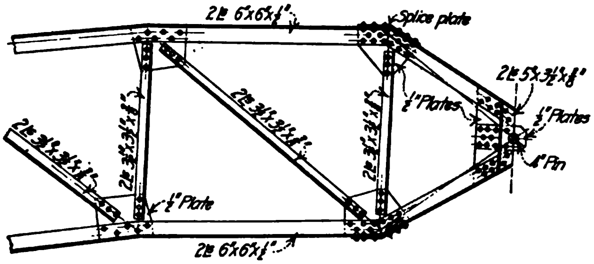

FIG. 232.

Fig. 231 shows the details of a shoe designed to carry the vertical and horizontal components of the reactions. The slope of the base of the shoe is determined by the condition that it should be perpendicular to the resultant of the maximum reactions. Fig. 231(b) shows the amount and direction of the resultant reactions due to all possible combinations of dead and snow or wind load reactions. These resultants were plotted from the values given in Table 1. It will be noted from Fig. (b) that the reactions lie close together, and that a plane x-y at a slope of 8 in. in 12 in. is normal to the average direction of these resultants.

The base area required on the line a-b must be sufficient to provide for the maximum reaction of 135,000 lb. which occurs for dead load and snow load on both sides of the arch. It is usual to provide a short horizontal base area, shown by a-c of Fig. 231(b). All details are as shown on Fig. 231. The design methods are similar to those used for Fig. 230.

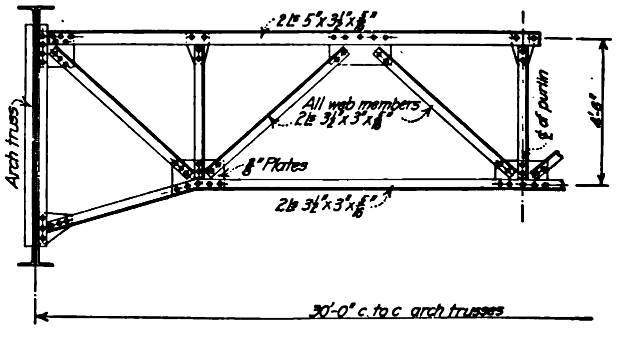

FIG. 233.

Fig. 232 shows the details of the pin joint at the crown hinge, and a portion of the truss. The design methods for the pin and the pin plates, and for the end connections of the members, are the same as for the detail of Fig. 230.

174. Bracing for Arch Trusses.—The general plan of the bracing for an arch truss is quite similar to the one designed in the chapter on the Detailed Design of a Truss With Knee-Braces. Since the trusses are large and must be rigidly braced, lateral systems are generally placed between every other pair of trusses. In the plane of the vertical side walls, bracing is placed in every bay. A very good idea of the form and arrangement of the required bracing can be obtained from the description of the University of Illinois drill hall, which is given in the *Engr. News* for Dec. 11, 1913, and from the description of the Springfield Coliseum given in *Engr. Rec.* for Oct. 7, 1916, to which the reader is referred.

The trussed purlins which connect the trusses at alternate panel points, form part of the bracing as well as acting as purlins. Fig. 233 shows the details of these purlins, which are connected to the vertical truss members at the points shown in Fig. 225. The purlins are designed to carry the roof load and the maximum snow or wind loads. Fig. 233 shows the adopted sections. The lower chord members of the end panels are sloped so that the lower chord member of the purlin is connected to the vertical members of the arch near the foot of these members.

ORNAMENTAL ROOF TRUSSES

By W. S. Kinne

175. Architectural Timber Work[1].—Architectural timber work is an important element of interior design, especially in churches. The roof structure is frequently of wood, using the hammer beam truss where the roof is high. In buildings with low pitched roofs the braced arch is most common. This form of construction brings some thrust upon the walls, which must be counteracted by buttresses or extra heavy masonry. The roof design concerns not only the trusses, but the purlins, rafters and sheathing as well, all of which may be decorated to a greater or less degree. Structural considerations must be modified and supplemented to meet architectural requirements. Members of no structural value may be introduced; stresses must be provided for without too great insistence on economy of materials. As a general rule, horizontal and vertical members are satisfactory, together with arched members. Large diagonal members are usually disappointing in perspective. The timbering is sometimes covered with "boxing" of more expensive wood, but the effect is usually poor as compared with actual beams. Laminated beams are frequently used. The laminatons may be masked by mouldings and decorative elements. The advantage lies in the good connections and masked joinings secured. Steel rods should not be exposed. A few examples of ornamental trusses are shown.

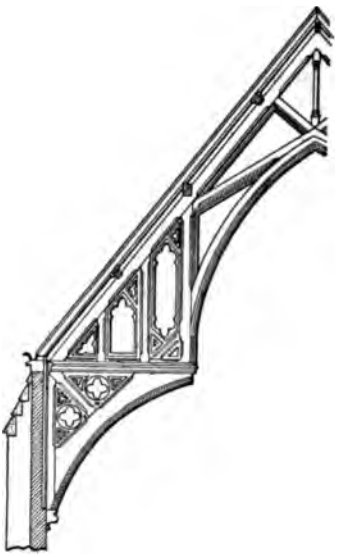

Fig. 234.—Hammer beam with scissors truss above.

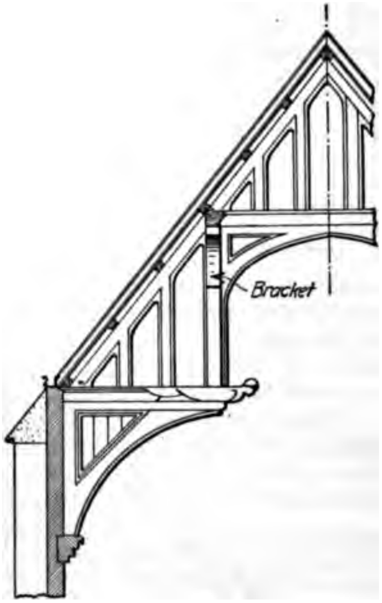

Fig. 235.—Hammer beam with A-truss.

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[1] This article contributed by Arthur Peabody, State Architect, Madison, Wis.

Figs. 234 and 235 show hammer beam trusses of the usual form. In the first a scissors truss is used over the hammer beam. In the second a rafter and tie beam are used. Fig. 236 shows an approximation to the hammer beam truss, but depends for its strengh partly on the rigidity of the members. This truss should be built of seasoned lumber and should be gone over and the bolts tightened up after being in service for about a year.

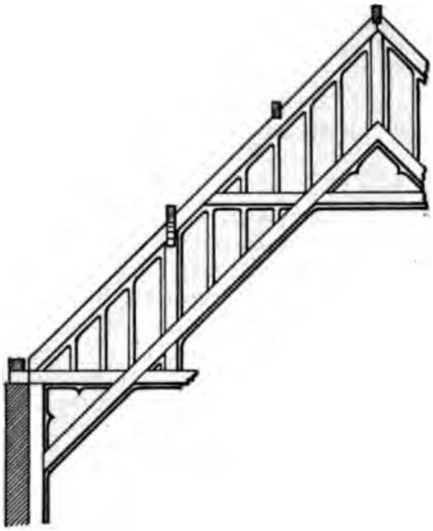


FIG. 236.—Laminated truss.

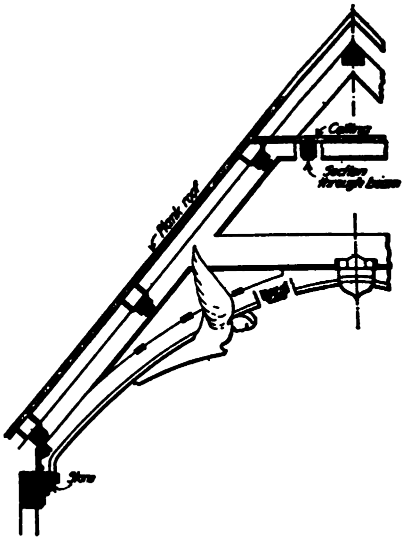


FIG. 237.—Braced arch (St. John's College, Oxford).

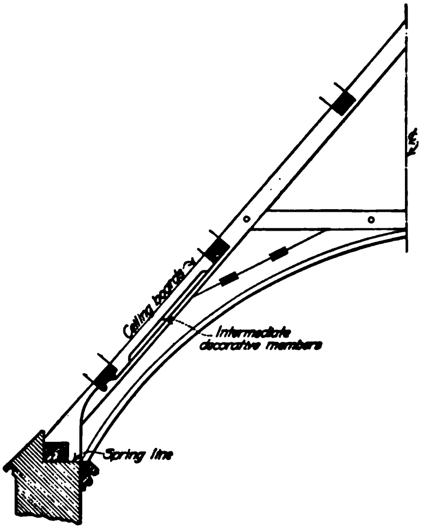


FIG. 238.—Braced arch and rafter.

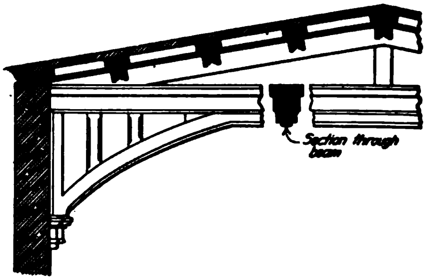


FIG. 239.—King post truss and bracket. (Bodlean Library.)

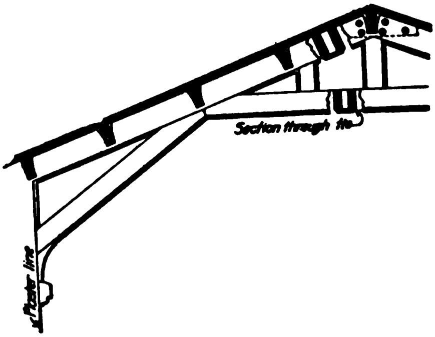


FIG. 240.—Braced rafter.

Fig. 236 and 237 show high pitched roofs supported by a timber arch The arched members add something to the rigidity of the structure and a great deal to appearance. Fig. 239 shows a low pitched roof supported by a king post truss with a timber arch below. The construction of this truss will be entirely masked by the decoration. Figs. 237, 238 and 239 are from buildings near Oxford, England.

Fig. 240 is a modification of the low pitched truss type, formed of doubled timbers and a few false members. This truss should be supported on quite rigid posts built into the wall. The action of the post and bracket is that of a cantilever, to which the upper chord is fastened.

Fig. 241 shows a scissors truss. This form of support is less meritorious architecturally and structurally, but is much used on cheap work. Its principal merit is the arched effect of the slanting members.

The span of all the above trusses is taken, for convenience, at 28 ft. Spans of much greater width may require an attic space with concealed trusses. In this event the interior will show the ceiling only, which will be supported from above.

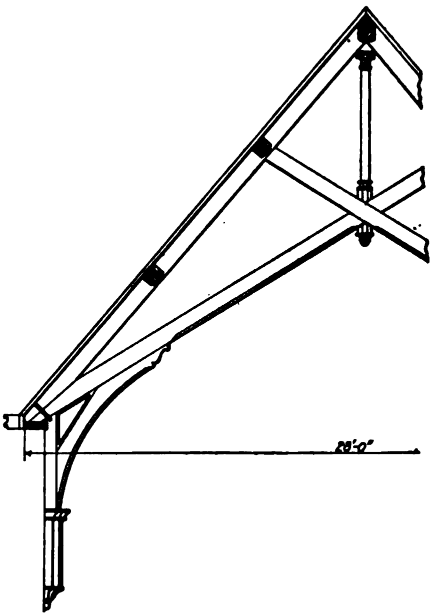


Fig. 241.—Scissors truss.

176. Analysis of Stresses in a Scissors Truss.—The stresses in a truss of the Scissors type, shown in Fig. 241 of Art. 175 are readily determined by the methods of stress analysis given in Sect. 1. Panel loads due to dead and wind loads are determined by the methods used in the preceding chapters on roof truss design. As the roof slope is generally quite steep, snow loads need not be considered.

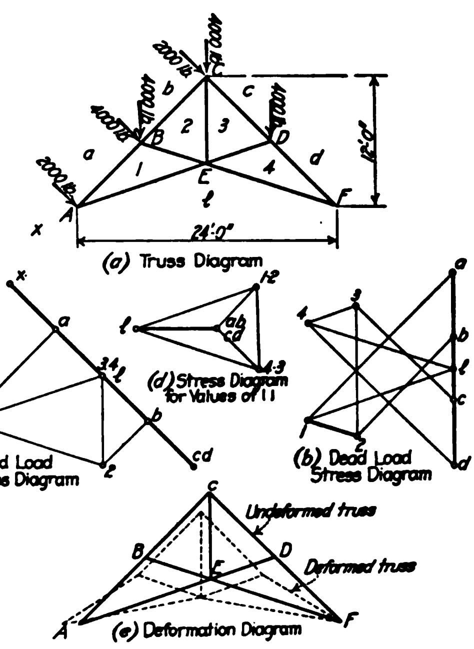


Fig. 242.

To illustrate the methods of stress analysis for trusses of this type, the stresses in the truss of Fig. 242 will be determined for dead and wind loads. Panel loads for dead and wind load, determined by the usual methods, are shown in position on Fig. 242(*a*). The dead load stress diagram is shown in Fig. (*b*), and the wind load stress diagram is shown in Fig. (*c*). Table 1 gives the resulting stresses for dead and wind loads, and also the maximum stresses due to combined dead and wind loads.

Roof trusses of the scissors type are usually constructed of wood, with the exception of the vertical member *C–E* of Fig. 242 (*a*), for which a steel rod is used. Experience has shown that the elastic deformation of the members of a scissors truss results in a considerable horizontal movement of the points of support. To reduce the amount of this movement, it is the general practice to use excess area in the top and bottom chord members. For the truss of Fig. 242 (*a*) it will probably be advisable to use 6 × 10-in. wooden pieces for all members except the middle vertical, which will be made of a 1½-in. round steel rod. Typical joint details applicable to the truss under consideration are shown in Art. 179.

The horizontal movement of the points of support of the truss of Fig. 242 (*a*) can be calculated by means of eq. (7), p. 566. This equation is

$$D = \sum \frac{Sl}{AE} u \qquad (1)$$

where D = deflection of any point; S = stress in any member; A = area of any member; l = length of any member; E = modulus of elasticity of the material composing the members; and u = a ratio which is equal to the stress in any member due a 1-lb. load applied at the point whose deflection is desired and acting in the direction of the desired deflection.

TABLE 1.—STRESSES IN A SCISSORS TRUSS

(Fig. 242)

Member	Dead load	Wind right	Wind left	**Max. stress**
AB	−12,750	−4,000	−4,000	**−16,750**
BC	− 8,600	−2,000	−4,000	**−12,600**
AE	+ 9,600	+4,500	0	**+14,100**
BE	− 3,120	−4,500	0	**− 7,620**
CE	+ 8,250	+2,800	+2,800	**+11,050**

+ = tension. − = compression.

TABLE 2.—HORIZONTAL DEFLECTION OF POINTS OF SUPPORT

CALCULATION OF THRUST ON WALLS

SCISSORS TRUSS

(Fig 242)

Member	Stress 1	l 2	A 3	$\frac{l}{AE}$ 4	$\frac{Sl}{AE}$ 5	u 6	$\frac{Sl}{AE}u$ 7	$\frac{l}{AE}u^2$ 8	$-Hu$ (H = 6,510 lb.) 9	S 10
AB	−16,750	102	52.2	0.000001955	−0.0328	−0.707	+0.0233	0.000000977	+ 4,610	**−12,140**
BC	−10,600	102	52.2	0.000001955	−0.0208	−0.707	+0.0148	0.000000977	+ 4,610	**− 5,990**
CD	−12,600	102	52.2	0.000001955	−0.0246	−0.707	+0.0175	0.000000977	+ 4,610	**− 7,990**
DF	−16,750	102	52.2	0.000001955	−0.0328	−0.707	+0.0233	0.000000977	+ 4,610	**−12,140**
AE	+14,100	152	52.2	0.000002905	+0.0410	+1.58	+0.0648	0.00000725	−10,300	**+ 2,800**
EF	+ 9,600	152	52.2	0.000002905	+0.0279	+1.58	+0.0441	0.00000725	−10,300	**− 700**
BE	− 7,620	76	52.2	0.000001403	−0.0111	0	0	0	0	**− 7,620**
DE	− 3,120	76	52.2	0.000001403	−0.00455	0	0	0	0	**− 3,120**
CE	+11,050	96	1.77	0.000001810	+0.0200	+1.00	+0.0200	0.00000181	− 6,510	**+ 4,540**
							+0.2078	0.00002023		

For the truss under consideration, the deflection of the left end, A of Fig. (a), will be determined with respect to the right end, point F, which will be assumed to stand fast. This deflection will be determined for the maximum stresses in all members due to the dead and wind load stresses, as given in Table 1. These maximum stresses are recorded in Table 2. The lengths and areas of the several members are also given in Table 2. Lengths of members are given in inches, and areas are given in square inches. As assumed above, the main members are composed of a 6 × 10-in. piece. Assuming that dressed lumber is used, the area is calculated as for a 5½ × 9½-in. section to conform to the methods used in the chapter on Detailed Design of a Wooden Roof Truss. The modulii of elasticity of wood and steel are taken respectively as 1,000,000 and 30,000,000 lb. per sq. in.

Since the horizontal motion of point A is desired with respect to point F, the values of u as defined above, are to be calculated for a 1-lb. load applied at A and acting horizontally. It will be assumed that the 1-lb. load acts to the left. A positive sign for the resultant deflection will indicate that the direction of the deflection was correctly assumed. If the sign is negative, the true deflection is to the right. Values of u were calculated by means of the stress diagram of Fig. (d), and the stresses are recorded in Table 2.

The desired deflection is determined by calculating the value of the term $\frac{Sl}{AE}u$ for each member, and adding all such terms, paying particular attention to the sign of each result. It is to be noted that for stress, plus indicates tension and minus indicates compression. In multiplying the several values, like signs result in plus signs, and unlike signs result in minus signs. The resulting values are given in Table 2 under the proper heading, and at the foot of the column is given the sum of all terms, which is the desired deflection. The result, +0.2078, indicates that point A moves to the left, 0.2078 in.

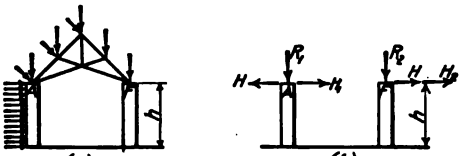
A study of the values of $\frac{Sl}{AE}u$ given in Table 2, col. 7, shows that about 80 % of the total deflection calculated above is due to the elastic distortion of members A–B and D–F, the lower ends of the top chord member, and A–E and E–F, the lower chord member. Since the deflection contributed by any member is inversely proportional to the area of that member, it follows, as stated above, that large members with considerable excess area should be provided for the chord members in order to reduce the horizontal movement of the supports.

By calculations similar to those given in Table 2, the vertical and horizontal components of the deflection of all points of the structure have been calculated. The dotted lines of Fig. 242 (e) show the distorted position of the truss, and the full lines show the undeformed truss. In plotting the movement of the several points, a scale was used which shows these movements at about 150 times their value to the scale of the truss. Hence, as plotted, the actual movement of the joints is greatly exaggerated. This is done in order to show the relative rather than the actual movement of the joints.

The diagram of the deformed truss brings out some points which should be considered in selecting the form of the members for trusses of this type. It will be noted that members A–B–C and C–D–F are bent out of line due to the deformation of the structure. If these members are made continuous, which is the usual practice, heavy secondary bending moments are et up at the middle points of the members. Since the fiber stresses in the members due to these moments are proportional to the depth of the member, it follows that the depth of the member in the direction of the bending should be as small as possible, in order to avoid excessive fiber stresses. In the case of the 6 × 10-in. members adopted for the design under consideration, the 6-in. face should be placed in the vertical direction and the 10-in. face should be placed horizontal. This would probably not fit in with the architectural features of the design. However, since considerable excess area is provided in these members, the total combined fiber stress with the 10-in. face placed vertical will probably be within the allowable limits. Everything considered, square sections are preferable for trusses of this type.

The ends of trusses of the scissors type are generally rigidly fastened to the supporting walls by means of anchor bolts or by a base plate bedded in the masonry. After the trusses have been erected, the roofing and other applied loads are added as the construction proceeds. On the removal of the erection false work or other temporary construction supports, the full loads are applied to the trusses, which tend to deform, causing the points of support to move horizontally, as calculated above. Since the trusses are generally rigidly fastened to the walls, as stated above, the walls are forced outward due to the resistance offered to the horizontal motion of the ends of the truss. Horizontal forces are therefore set up which cause bending moments in the walls. These moments, and the resulting fiber stresses, are a maximum at the foot of the walls. If the fiber stresses are excessive, the walls will be cracked at the base. To avoid failure of the walls due to this cause, the bending moments and fiber stresses must be estimated and a wall thickness adopted which will offer the required resistance. If one end of the truss is allowed to move freely as the loads are applied, the walls will be relieved of the greater part of the bending moment mentioned above. However, this is not the usual practice. In view of this fact, methods will be given for the determination of the horizontal forces which must be resisted by the walls.

The methods of calculation for the determination of the thrusts at the tops of the walls due to the deformation of a scissors truss are similar to those used in Art. 170 *b* for the determination of the reactions for a two-hinged arch. Let Fig. 243 (*a*) show a scissors truss, or any other type of truss in which the elastic deformation of the members produces thrusts on the supporting walls. To make the solution general in nature, vertical and inclined applied loads are shown in position. Consider the truss removed from the walls, and represent the action of the trusses on the walls by the forces shown in Fig. 243 (*b*). The forces H represent the thrusts at A and F due to the deflection of the truss. Evidently these forces are equal in amount and act in opposite directions, as shown in Fig. (*b*). The forces H_1, H_2, R_1, and R_2 represent the action of the applied vertical and inclined loads, and are calculated by the methods of Statics given in Sect. 1, considering the truss as a free body removed from its supports.



(*a*) (*b*)

FIG. 243.

The forces H_1 and H_2 include the effect of the wind on the vertical sidewalls. This effect is indeterminate, but it is sufficiently accurate to assume that the moment due to the horizontal wind load is equally divided between the two walls. It will therefore be assumed that the truss, acting as a strut between the two walls, transfers to the top of the right-hand wall, a load which will produce the assumed moment at the base of the wall. If w = wind load per foot of wall, and h = height of wall, the moment to be carried by each wall is $M = \frac{1}{4} wh^2$. On the assumption made above, the load at the top of each wall is $P = M/h = \frac{1}{4} wh$.

Assuming that the truss is rigidly fastened to the walls, it is evident that the horizontal movement of points A and F of the truss is equal to the horizontal movement of the tops of the walls, points A and F of Fig. (*b*). For the determination of H, the thrust of the trusses on the walls, an equation of elastic equilibrium can be established by equating the deflection of the truss, as calculated by eq. 1, to the combined deflection of the walls for the forces shown in Fig. (*b*).

The values of S to be used in eq. (1) for the determination of the horizontal motion of points A and F of the truss are the actual stresses in the members. These stresses include the effect of the thrust H and the effect of the applied loads. As stated in Art. 170 in connection with the derivation of eqs. (8) and 10), these stresses can be expressed in the form

$$S = S' - Hu \tag{2}$$

where S = actual stress in any member; S' = stress in any member due to the applied loads for the truss considered as removed from the walls and considered as a simple truss; H = thrust on the walls; and u = a ratio defined above for eq. (1). Substituting this value of S in eq. (1), the horizontal movement of point A of the truss with respect to point F is

$$\Delta = \sum \frac{S'l}{AE} u - H \sum \frac{l}{AE} u^2 \tag{3}$$

The deflection of the walls due to the applied loading shown in Fig. (*b*) depends on the form of the walls. If they are of uniform cross section for the full height, they form simple cantilever beams acted upon by the horizontal forces shown in Fig. (*b*). The effect of the vertical loads R_1 and R_2 on his horizontal deflection is so small that it will be neglected. From Sect. 1, the deflection of a simple cantilever beam due to a load P is given by the expression $\Delta = Pl^3/3EI$. To reduce this value to a general expression adaptable to all forms of walls, the term $l^3/3EI$ will be called the deflection coefficient of the wall. In the work to follow, this coefficient will be denoted by k, using subscripts 1 and 2 respectively to indicate the left and right-hand walls. With this notation, the total movement of points A and F of Fig. 243 (*b*) for the forces shown, is given by the expression

$$\Delta = (H - H_1)k_1 + (H + H_2)k_2$$

from which

$$\Delta = H(k_1 + k_2) - H_1k_1 + H_2k_2 \tag{4}$$

Equating eqs. (3) and (4) and solving for H, we have

$$H = \frac{\sum \frac{S'l}{AE}u + H_1k_1 - H_2k_2}{\sum \frac{l}{AE}u^2 + (k_1 + k_2)} \tag{5}$$

which is a general expression for the thrust on the walls due to a rigidly attached truss of the type shown in Fig. 242.

To illustrate the application of eq. (5) to a given set of conditions, certain assumptions will be made regarding the walls supporting the truss of Fig. 242 and the resulting thrust on these walls will be calculated. Suppose that the truss under consideration is rigidly attached to a masonry wall 18 in. thick and 15 ft. high, and assume that because of window openings, a section of wall 8 ft. long is available to resist the thrust of the trusses, which will be assumed to be 16 ft. apart.

For the applied dead and wind panel loads shown in position on Fig. 242(*a*), it can be shown that $H_1 = H_2 = 2{,}800$ lb. To this load must be added the effect of wind on the side walls. As stated above, this effect will be assumed to be due to a load $wh/4$, where w = load per foot of wall. For a 30-lb. wind load acting on a 15-ft. wall, trusses 16 ft. apart, $wh/4 = \frac{1}{4} \times 30 \times 16 \times 15 = 1800$ lb. The total horizontal load is then $H_1 = H_2 = 2800 + 1800 = 4600$ lb. Since the walls are alike, and are simple cantilever beams of height h, the value of the deflection constant, as defined above, is

$$k_1 = k_2 = \frac{h^3}{3EI}$$

where E = modulus of elasticity of the material composing the wall, which will be assumed to be 3,500,000 lb. per sq. in.; and I = moment of inertia of the wall section, which is given by the formula $I = \frac{1}{12}bd^3$. For the assumed conditions, h = 15 ft. = 180 in.; b = effective width of wall = 8 ft. = 96 in.; and d = thickness of wall = 18 in.; and

$$k = \frac{(180)^3}{(3)(3{,}500{,}000)(\frac{1}{12})(96)(18)^3} = 0.0000119$$

The term $H_1k_1 - H_2k_2$ of eq. (5) can readily be seen to be equal to zero for the assumed conditions. Table 2 gives directly the term $\sum \frac{S'l}{AE}u$, for the stresses S' are exactly the same as given by Table 1. The term $\sum \frac{l}{AE}u^2$ is readily calculated from the values given in Table 2. Col. 8 gives the several values and the required summation. The value of $k_1 + k_2 = 2k$ can be determined from the calculations given above. Substituting these values in eq. (5), we have

$$H = \frac{0.2078}{0.00002023 + 0.00002380} = 4710 \text{ lb.}$$

which is the thrust of the trusses on the walls for the assumed conditions.

The combined fiber stress in the walls due to the bending moments induced by the total horizontal loads at the tops of the walls must be investigated. From Fig. 243(*b*), it can be seen that the maximum fiber stress will occur at the inside lower edge of the right-hand wall. This fiber stress is to be determined for bending due to horizontal forces and compression due to the weight of the wall and the truss reactions at the wall. As stated above, H_2 = 4600 lb. Hence the total horizontal force is $H + H_2 = 4710 + 4600 = 9310$ lb., and the bending moment at the foot of the 15-ft. wall is $9310 \times 180 = 167{,}500$ in.-lb. Since the wall section is rectangular, the fiber stress due to bending is $f_b = 6M/bd^2$, where b = effective width of wall = 96 in., and d = thickness of wall = 18 in. Hence,

$$f_b = \frac{(6)(167{,}500)}{(96)(18)^2} = 324 \text{ lb. per sq. in.}$$

This fiber stress is tensile on the inside edge of the wall. The compression at the same point due to the weight of the wall and the truss reaction is equal to the total load divided by the effective area. Assuming that the material composing the walls weighs 160 lb. per cu. ft., the weight of the wall is $8 \times 1.5 \times 15 \times 160 = 28{,}800$ lb. From

Figs. 234 and 235 show hammer beam trusses of the usual form. In the first a scissors truss is used over the hammer beam. In the second a rafter and tie beam are used. Fig. 236 shows an approximation to the hammer beam truss, but depends for its strengh partly on the rigidity of the members. This truss should be built of seasoned lumber and should be gone over and *the bolts* tightened up after being in service for about a year.

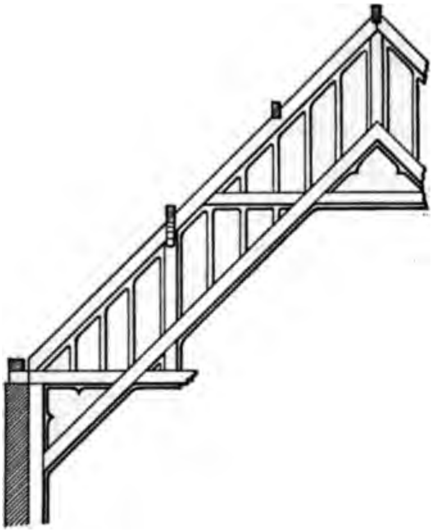


Fig. 236.—Laminated truss.

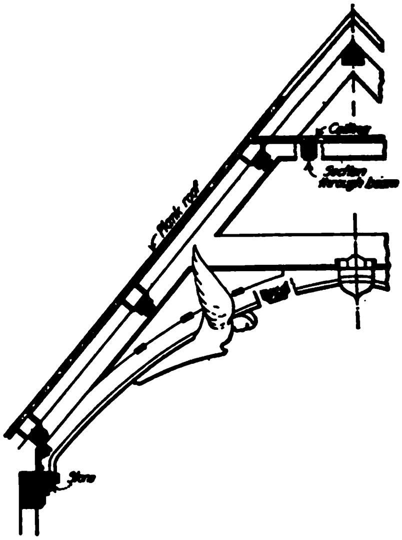


Fig. 237.—Braced arch (St. John's College, Oxford).

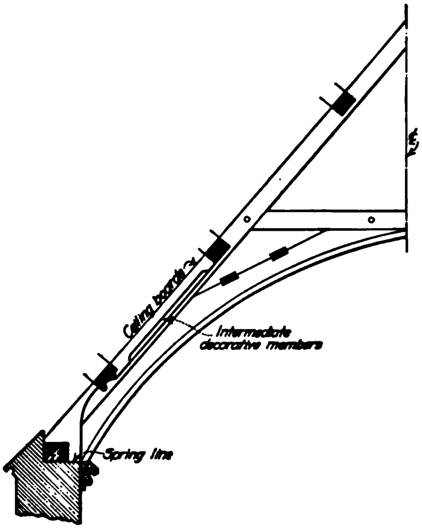


Fig. 238.—Braced arch and rafter.

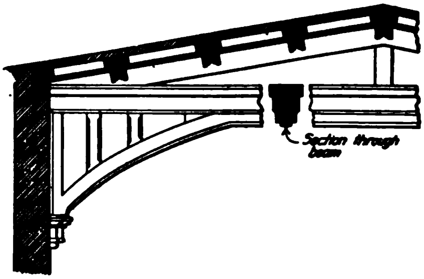


Fig. 239.—King post truss and bracket. (Bodlean Library.)

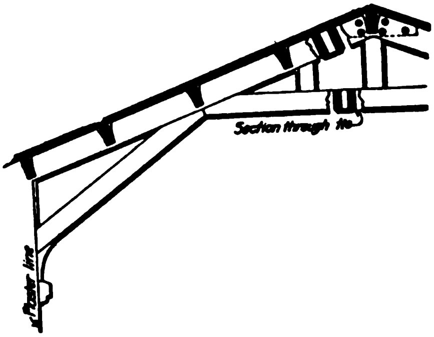


Fig. 240.—Braced rafter.

Fig. 236 and 237 show high pitched roofs supported by a timber arch The arched members add something to the rigidity of the structure and a great deal to appearance. Fig. 239 shows a low pitched roof supported by a king post truss with a timber arch below. The construction of this truss will be entirely masked by the decoration. Figs. 237, 238 and 239 are from buildings near Oxford, England.

Fig. 240 is a modification of the low pitched truss type, formed of doubled timbers and a few false members. This truss should be supported on quite rigid posts built into the wall. The action of the post and bracket is that of a cantilever, to which the upper chord is fastened.

Fig. 241 shows a scissors truss. This form of support is less meritorious architecturally and structurally, but is much used on cheap work. Its principal merit is the arched effect of the slanting members.

The span of all the above trusses is taken, for convenience, at 28 ft. Spans of much greater width may require an attic space with concealed trusses. In this event the interior will show the ceiling only, which will be supported from above.

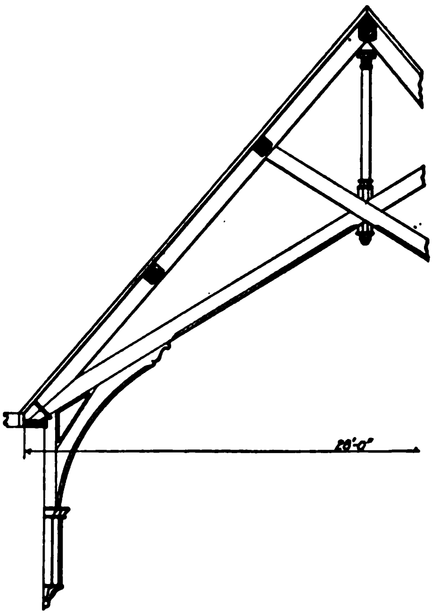


Fig. 241.—Scissors truss.

176. Analysis of Stresses in a Scissors Truss.—The stresses in a truss of the Scissors type, shown in Fig. 241 of Art. 175 are readily determined by the methods of stress analysis given in Sect. 1. Panel loads due to dead and wind loads are determined by the methods used in the preceding chapters on roof truss design. As the roof slope is generally quite steep, snow loads need not be considered.

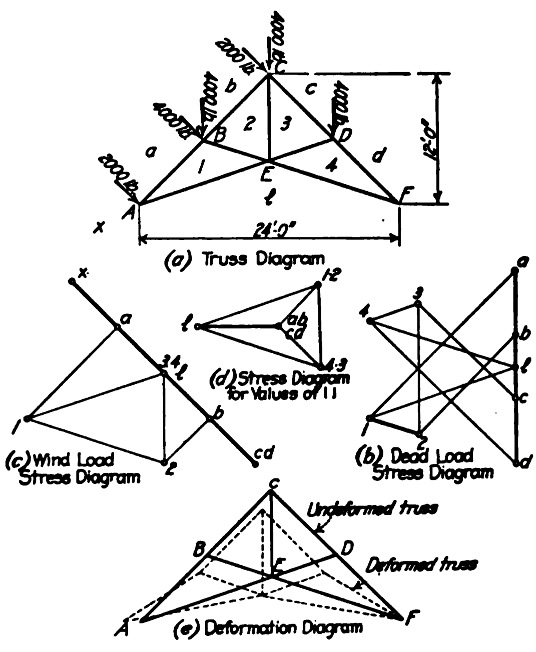


Fig. 242.

To illustrate the methods of stress analysis for trusses of this type, the stresses in the truss of Fig. 242 will be determined for dead and wind loads. Panel loads for dead and wind load, determined by the usual methods, are shown in position on Fig. 242(a). The dead load stress diagram is shown in Fig. (b), and the wind load stress diagram is shown in Fig. (c). Table 1 gives the resulting stresses for dead and wind loads, and also the maximum stresses due to combined dead and wind loads.

Roof trusses of the scissors type are usually constructed of wood, with the exception of the vertical member *C–E* of Fig. 242 (*a*), for which a steel rod is used. Experience has shown that the elastic deformation of the members of a scissors truss results in a considerable horizontal movement of the points of support. To reduce the amount of this movement, it is the general practice to use excess area in the top and

Figs. 234 and 235 show hammer beam trusses of the usual form. In the first a scissors truss is used over the hammer beam. In the second a rafter and tie beam are used. Fig. 236 shows an approximation to the hammer beam truss, but depends for its strengh partly on the rigidity of the members. This truss should be built of seasoned lumber and should be gone over and *the bolts* tightened up after being in service for about a year.

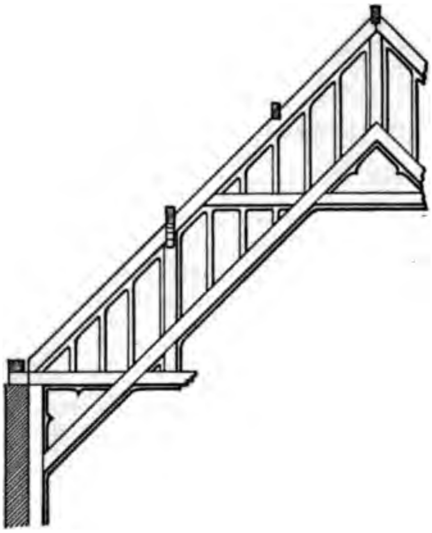


Fig. 236.—Laminated truss.

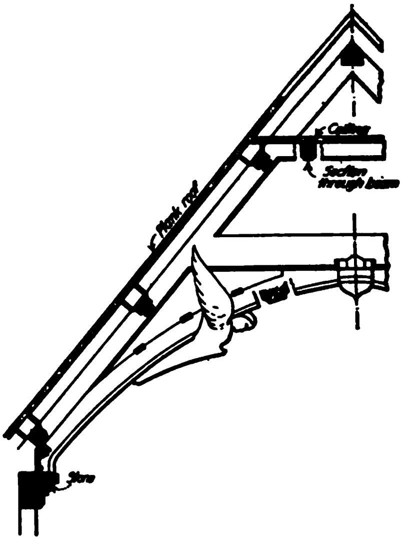


Fig. 237.—Braced arch (St. John's College, Oxford).

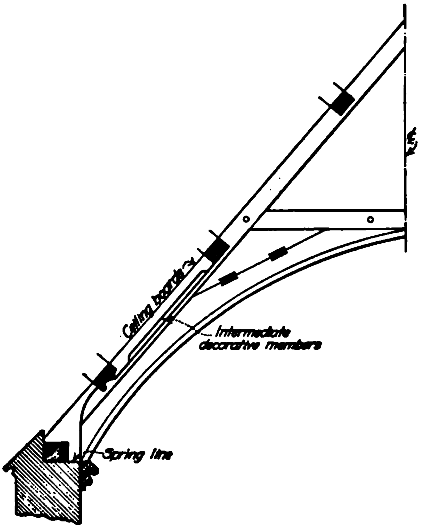


Fig. 238.—Braced arch and rafter.

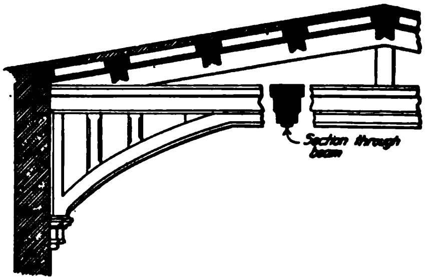


Fig. 239.—King post truss and bracket. (Bodlean Library.)

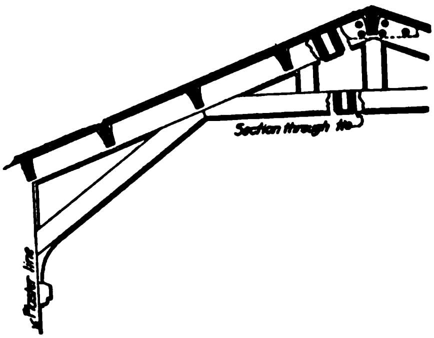


Fig. 240.—Braced rafter.

oof); (4) the probable weather conditions during the roof construction; and (5) the ease of placng the roof materials.

181. Conditions to be Considered in Roof Design.

181*a*. Climatic Conditions.—The climatic agencies which tend to affect the inegrity of a roof are the following: rain, snow, ice, high winds, salt air (along the sea coast), ıeat, and cold.

Rain.—To provide for rain, the roof must be tight and have proper drainage. By proper drainage is meant fair slope for the roof surface, so that water will not remain in puddles, and also a proper distribution of good sized utters and leaders to carry the rain water to the ground. In determining the size and distribution of the gutters nd leaders exceptionally heavy rains must be taken into account since, in case the downtakes are too far apart, uch rains will produce a good sized current of water tending to abrade the gutter surface as well as causing damage y overflow. The accumulation of leaves, twigs, and rubbish of various kinds necessitates strainers at all downtakes nd a periodical inspection of the roof.

Snow.—Snow sometimes causes exceptionally heavy loads on roofs having a slight slope, or on roofs with high arapet walls as is sometimes found around tower roofs for ornate purposes. Drifting snow may bank up by lowing down from high-level roofs on to roofs at lower levels, filling up "pockets" where it will remain until it melts way. On roofs consisting of a series of secondary roofs as, for example, on saw-tooth roofs or common monitor oofs, the snow often is found banked up deep in the valley gutters. Dry snow, driven by a high wind, will drift hrough small crevices, which will prevent the use of certain roofs over dynamos and electrical work generally. now prevents the use of skylights with small inclination for shops that are not heated, as in such cases the snow ıay remain for weeks and prevent daylight from coming through.

Ice.—Ice is likely to cause trouble on account of its expansive action and its tendency to accumulate when once tarted. On account of this it is necessary (1) to have perfect roof drainage, meaning a proper slope of surface and utters, and capacious downtakes; (2) to make a periodic inspection of the roof to remove rubbish accumulations round strainers; (3) when outside downtakes (leaders) are used, to select the corrugated or expansion type, in hich the material has a fair chance to avoid disruption due to ice action; (4) to make wide and shallow gutters ıstead of deep and narrow ones; and (5) to use wide flashings from eaves and valley gutters under the roofing ıaterial. In gutters where ice is apt to form in spite of precautions taken in planning the building, a steam pipe unning under the full length of the gutter will be found to do good service.

Wind.—Wind pressure on the roof adds an appreciable amount of load on a steep surface. The influence of igh wind on the roof and roof covering becomes most evident (1) in its driving action on snow and rain, as referred o above; (2) in its tendency to raise up light roofing units, as slate shingles and light flat tile; and (3) in its tenency to raise up and dislodge thin roofing materials, like sheet metal, corrugated steel, and prepared felt roofings—articularly along overhangs and eaves, where the fastenings are most exposed and the wind pressure most active.

Salt Air.—Salt air along the sea coast has a greater corroding influence on roofing metals than moisture alone. n such locations metallic roofs require more frequent repairs and painting. Generally, acid-laden air tends to estroy metals quite rapidly, and this action becomes much greater when two metals touch, as zinc and copper, roducing a galvanic action.

Heat and Cold.—Heat and cold act on roofs in various ways. Variation in temperature causes expansion nd contraction, which in some roofing materials must be taken special care of by expansion joints. Great heat ill dry out some felt and tar coverings so that they will crack and give opportunity for frost to destroy the covering. ttention should be given to the composition of such coverings, avoiding volatile tar compounds which flow at a omparatively low temperature. Where a metal roofing is protected by paint, a clean surface and a heat esisting paint is essential. The action of cold is felt through the agency of ice formation described above.

181*b*. Uses to Which the Structure is Put.—In dwellings, from the small house o the large public building or hotel, the roof is generally in keeping with the balance of the uilding as regards fireproof or non-fireproof construction—the particular type (whether plank, oncrete, tile, or gypsum-composition) depending upon climatic conditions, fire risk and xterior loads. In manufacturing plants, however, in addition to the above-mentioned conitions must be considered the kind of roof most suitable for the particular activity to be carried n in the building. In steel and iron works and in any plant where the fire risk is great, a fireroof roof is essential. In manufacturing establishments using strong acids or alkalies, metallic oofs or roofings will corrode rapidly. It is not good practice to use a plank roof on steel purins and trusses unless the risk of the plank catching fire is negligible. Many cases are on record f total destruction of steel frame buildings, trusses and columns, by burning of the wooden oof plank.

Another condition to look out for is condensation on the under side of roof, due to rapid cooling and lack of orosity of roof materials. To overcome this in the case of a corrugated steel roof, an asbestos lining is placed under he roof. Asbestos protected metal roofing has been used in similar cases, also asbestos corrugated roofing. The ypsum, insulated concrete roof and the plank roof—the latter sometimes coated on the underside with a fireproof ompound—are good nonconductors.

181c. Fire Risk.—Fire risk is, necessarily, a consideration of vital importance. Mention has already been made of the advisability of using fireproof roofs unless the fire risk from the inside is negligible. The surface, however, should always be fireproof to avoid a fire starting from sparks or burning embers carried by a high wind. Parapet walls afford more protection for combustible roof beams and plank than a sheet metal cornice. Fire walls projecting well above the roof prevent a fire from running along a roof. All roof houses and bulk heads should be fireproof throughout. Skylights should be screened and also have wire glass. Standpipes should be conveniently located and long skylights or monitors broken up for easy access to any part of the roof.

181d. Special Imposed Loads.—Special imposed loads may be crowds of people as, for example, when the roof is used (1) for a school or other playground; (2) for entertainment, as hotels, theatres, and restaurants; or (3) for manufacturing processes in certain industries. Such roofs must have a wearing surface in addition to standard roofing requirements.

181e. Least Cost.—In reviewing least cost the following points should be considered:

1. Least cost must not under any circumstances mean inferior materials or workmanship.
2. Best value often received by not using patented devices which may bring a royalty into the cost.
3. Time required in placing the roof.
4. Well known materials and standardised construction methods.
5. Cost of upkeep including insurance.

182. Precautions in the Design and Erection of Roofs.—Roofs that have to be constructed in the winter months must be protected from the destroying influence of frost which may permeate the roof slab and render it weak.

Concrete slabs, especially cinder concrete slabs, must be protected from frost during set and followed up quickly by the roofer.

Gypsum-composition slabs are quite porous and must be covered at the earliest possible moment with the roofing to prevent snow, rain, and frost from breaking up the slab and causing sags. Gypsum-composition roofs depend for their integrity more on the suspension than the bond principle, and may be considered to rest on the imbedded steel wire cables. The cables are stretched for considerable distances ahead of the slab, and ice or snow may lodge on them, preventing wholly or in part the bonding action. Before pouring the slabs, the snow and ice should be removed from the cables, and the roofer should follow immediately with his protection. End bays should be braced securely with angle struts and diagonals to prevent sideways movement of purlins with resulting sag of slabs.

On all but the so-called "flat roofs" (pitch 1 in. per foot) the roof material will cause the supporting purlins to bend sideways toward the eaves unless prevented by sag ties anchored securely to a braced top panel or heavy member at the peak.

Where a choice has to be made between several suitable roofing materials, the fact that the roof has to be placed during cold or inclement weather will probably cause the choice of a roof easily and quickly placed, and offering least opportunity to be injured by snow and ice.

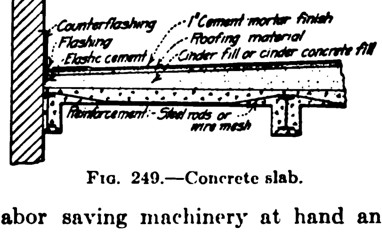


FIG. 249.—Concrete slab.

183. Roof Decks.

183a. Concrete.—A reinforced concrete slab deck is (see Fig. 249) probably more durable and fire resisting than any other type of roof construction. The economy of a concrete slab depends upon the amount of concrete used on the job. If the floors are of concrete, or if concrete is used extensively on the job, the contractor will have labor saving machinery at hand and be in a position to construct the roof at a low cost. Concrete roofs are used extensively on fireproof buildings, such as theatres, hotels, office and loft buildings, factories, etc. Cinder concrete being lighter in weight than stone concrete is generally used. Piping, shafting, lighting and other fixtures may be fastened directly to the under side of the slab by means of rods, dowels or expansion bolts. A concrete roof should not be used where condensation will take place unless properly insulated.

Cinder concrete weighs 108 lb. per cu. ft. Only clean steam boiler cinders should be used. Stone concrete weighs 144 lb. per cu. ft. Reinforcement may be steel rods, wire mesh, or expanded metal.

183b. Hollow Tile.—Terra cotta hollow tile (see Figs. 250, 251, and 252), both porous and semi-porous, are used for roof decks in fireproof construction. Either flat or segmental arches are used in main roofs. For flat roofs of pent houses and bulk heads, and for steep slopes as in mansard roofs, book tile are used, supported on tees. Hollow tile gives a comparatively light roof and may be used where concrete is found suitable. Where the roofing material is to be applied directly to the tile, porous tile should be used, as it will receive the nails. The porous tile will prevent condensation in ordinary cases.

Book tile is laid between tees, spaced 1 in. farther apart than the length of tile. Book tile for roofs comes in various lengths from 16 to 24 in., 12 in. wide and 3 to 4 in. thick. The 24-in. tile is generally used. Book tile weighs 20 lb. per sq. ft. for 3-in. thickness and 24 lb. per sq. ft. for 4-in. thickness. Roof tile weighs 26 lb. per sq. ft. for 6-in. tile; 29 lb. for 7-in., 32 lb. for 8-in., 36 lb. for 9-in., 38 lb. for 10-in., 44 lb. for 12-in., 50 lb. for 14-in., 54 lb. for 15-in., and 55 lb. for 16-in.

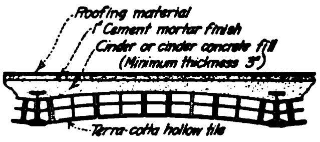


Fig. 250.—Segmental arch.

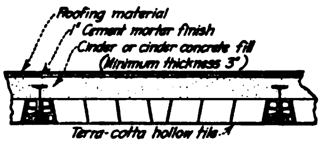


Fig. 251.—Flat arch end construction.

183c. Reinforced Gypsum.—The use of gypsum for roof slabs (see Fig. 253) is a comparatively modern development. The first type used was tile, 3 in. thick, 2½ ft. long. Later on, tile up to 6 ft. were used, followed by gypsum T-beams, spanning from truss to truss, generally of 10-ft. maximum length. The method used at the present time is to build a centering that produces a 4-in. slab and a T-beam of a total depth of 6 in. These T-beams are spaced 6 in. on centers. In calculating strength, no part of the web is considered as taking compression, meaning by web the part of the steam below the slab itself. Reinforcement is placed at the bottom of the T-beam; and wire mesh, needed principally for expansion or contraction, is placed at the bottom of slab.[1]

Ordinary concrete formulas are used with the following working stresses: Compression in extreme fiber, 350 lb. per sq. in.; shear, 20 lb. per sq. in.; bond stress, 30 lb. per sq. in.; bearing, 300 lb. per sq. in.; tension in steel, 16,000 lb. per sq. in. Ratio between moduli of steel and gypsum, 30.

The gypsum sets quickly and allows the speedy removal of forms. As there is some heat developed when the gypsum hardens, this property is useful in cold weather. The form work is executed to a greater finish than for those used for concrete.

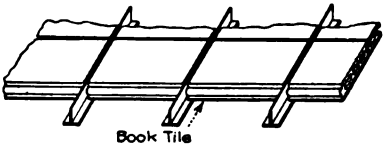


Fig. 252.

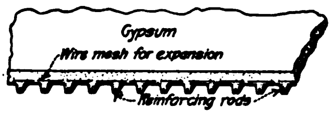


Fig. 253.—Reinforced gypsum slab.

183d. Gypsum Composition.—Gypsum has a low conductivity for heat and is a good material to use where much moisture is present in the air, as in power houses, textile mills, and similar manufacturing plants. The suspended system consists of two No. 12 galvanized cold drawn steel wires twisted together, spaced from 1 to 3 in. apart and securely anchored at the end purlins by means of hooks (see Fig. 254). This system with a 3-in. slab will span 10 ft. for a light roof load. A 4-in. thickness is preferable for heavier loads. The supporting medium in this type is the series of wire cables, the slab acting as a covering. An equalizing bar is placed at the middle of the span to assure an equal deflection of the cables. The slab is porous, as there is present with the gypsum other substances as cocoanut fiber, shavings, or even as-

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[1] *Eng. Rec.*, Dec. 16, 1916, by Virgil G. Marani, Cons. Engr., Cleveland, O.

bestos chips. In selecting this roof slab, inquiry should be made as to whether the admixtures are apt to cause discoloration or flaking on the underside of the slab. The slab should be promptly protected from snow and ice which quickly injure a porous slab. The lightness of the material, about 4 lb. per in. of thickness, causes economy in the supporting trusses and purlins.

183e. Wood.—Wooden roofs are used in mill construction and on frame buildings, and also on steel structures where the fire hazard is negligible (see Figs. 255, 256, and 257). In frame construction, the rafters are generally spaced 16 in. on centers, covered with ⅞-in. matched sheathing. Where shingles, tile, or slate is to be used, roofing slats may be used, omitting the plank—thus allowing a space of 2 to 3 in between the slats. In mill construction, heavy roof timbers are used with purlins spaced 5 to 6 ft. apart with a 3-in. plank sheathing. With steel construction, nailing pieces must be bolted to the purlins. Either a single thickness of plank heavy enough to sustain the loading may be used, or two thicknesses of plank, the second layer applied diagonally. If wooden purlins are used, clips are provided on the trusses for attaching the purlins.

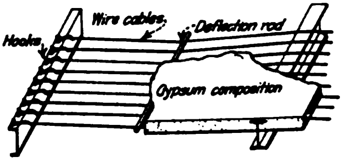


Fig. 254.—Suspended gypsum composition slab.

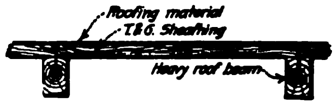


Fig. 255.—Mill construction.

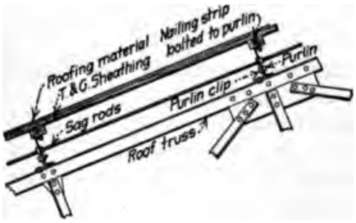


Fig. 256.—Steel construction.

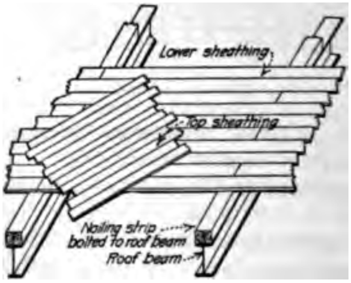


Fig. 257.—Double sheathing.

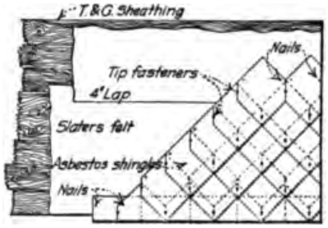


Fig. 258.—"French" or diagonal method of laying asbestos shingles.

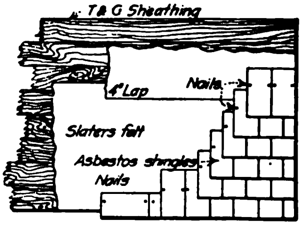


Fig. 259.—"American" or straight method of laying asbestos shingles.

184. Roof Coverings.

184a. Shingles.—Shingles are made of asbestos, wood, or metal. ***Asbestos Shingles.***—Several makes of asbestos shingles are on the market. They are made of about 15% asbestos fiber and 85% Portland or hydraulic cement, formed under a pressure of 700 tons per sq. ft. Asbestos shingles are very durable and suffer very little from the climatic conditions. They are also fireproof, affording protection against sparks. These shingles can be cut with a saw. They should be applied on matched sheathing covered with slaters' felt or waterproof paper (see Figs. 258 and 259). Galvanized iron or copper nails should be used for fastening. Weight of asbestos shingles, 2½ to 4½ lb.per sq. ft.

Wooden Shingles.—Wooden shingles are made of cypress, cedar, redwood, white and yellow pine, and spruce—the lasting qualities in the order given. White cypress shingles are the most durable. Redwood shingles are the least inflammable, and are used extensively along the Pacific Coast. A shingle roof should have a slope of 6 in. to the foot, except for less important roofs where 4½ in. to the foot may be used. Shingles may be nailed to slats, or a plank sheathing may be used covered with waterproof paper or felt (see Figs. 260 and 261). Standard size of wooden shingles: 20 in. long, 2½ to 16 in. wide, 9/16 in. thick at butt end. 1000 shingles 4 in. wide will lay 111 sq. ft. of roof surface with 4-in. gage (exposure to weather), 125 sq. ft. with 4½-in. gage, and 139 sq. ft. with 5-in. gage. It will take 900 shingles to cover 1000 sq. ft. with a 4-in. gage, 800 with a 4½-in. gage, and 720 with a 5-in. gage. Five pounds of three-penny nails or 7½ lb. of four-penny nails should be provided for 1000 shingles. A man will lay from 1000 to 1500, 4-in. shingles per day according to the class of work. For hip and valley roofs 5 % should be added for cutting, and irregular roofs with dormers, 10 % should be added.

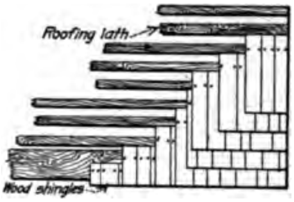


FIG. 260.—Slat method of laying wooden shingles.

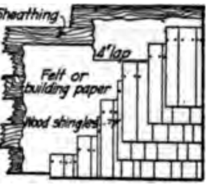


FIG. 261.—Sheathing method of laying wooden shingles.

When the space under the shingles is to be occupied, the sheathing method is the one to be preferred on account of protection from heat and cold. The open slat method gives longer life on account of more ventilation. The life of shingles may be prolonged by dipping them in linseed oil or creosote.

Metal Shingles.—Metal shingles are made of tin, galvanized steel, galvanized iron, zinc, or copper. They are generally made interlocking and have stiffener ribs, and are made in many shapes and sizes. At present they are not much used, having no great advantage over wooden shingles.

184*b*. Slate.—Slate comes in sizes from 7 × 9 in. to 24 × 44 in., and from ⅛ to ¾ in. thick. The common roofing sizes used are 12 × 16 in., 12 × 18 in., 12 × 20 in., and 14 × 24 in. Common thicknesses are 3/16 in. and ¼ in. The 3/16-in. thickness weighs 6½ lb. laid, and the ¼ in. weighs 8 lb. Slate should be laid with a lap of 3 in. over the second course below (see Fig. 262). The top course along the ridge, 2 to 4 ft. from gutters and 1 ft. from the hips and valleys, should be laid in elastic cement. A man can lay 2½ squares of slate per day. The slope of roof should be 6 in. per ft. for 14 × 24-in. slate and 8 in. per ft. for smaller slate. For small sizes 3 penny nails should be used, and for 12 × 20 in. and over, 4 penny nails. All holes should be drilled. A hard slate should be selected of the tough and springy variety. If slate is too soft, holes become enlarged; if too brittle, the slate breaks when squaring and in shipment. Slate should be laid on slats or sheathing with a paper or felt base.

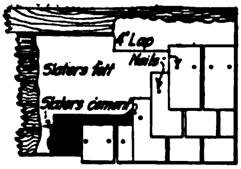


FIG. 262.—Slate roof.

184*c*. Tin.—Tin has been used extensively on dwellings, public buildings and factories. If kept continually and thoroughly covered with red lead or oxide, with pure linseed oil, a tin roof properly laid will last, in a dry climate, from 30 to 50 yr. Much depends on the quality of the iron and method of coating with tin. The pure iron plates recently brought out, such as the Armco iron, appear very good. As with all metal roofs, salt air shortens the life. Tar paint or tar paper should never be used for tin roofs. The I. C. grade of tin should be used for roofs as it does not expand as much as the heavier I. X. grade. Sheets come in sizes of 10 × 14 in. and multiples, and weigh 50 lb. per square before the tin is applied. General sizes used, are 14 × 20 in., and 20 × 28 in. The 20 × 28-in. sheets are easier to apply but the smaller, having more seams, make a stiffer roof. Tin must not be used on roofs where people are apt to walk. Roofs with a slope of less than 4 in. to the foot should have flat seams (soldered); steeper slopes may use standing seams (not soldered). Flat seams should have edges turned ½ in. and locked. Standing seams should have one edge turned 1¼ in. and the other edge turned 1½ in. perpendicular to the sheet. After placing high and low standing edges together, the edges should be bent over and curled (see Fig. 263). Standing seams need not be soldered. The cross seams are, of course, flat soldered seams. Long strips are made up in the shops, the side seams formed on the roof. All flat seams should be locked and soldered, sweating the solder into the seams. Cleats should be folded into the seams and spaced 8 in. apart for flat seams and 12 in. apart for standing seams. Each cleat should be nailed into the roof with two 1-in. barbed tinned wire nails. 14 × 20-in sheets should be used for flat seams and 20 × 28 in. for standing seams. Acid should never be used as a flux for soldering tin. Rosin is much to be preferred. Felt or waterproof paper may be used under the tin but never tar or tarred paper. With flat seams a box of 112, 14 × 20-in. sheets will lay 180 sq. ft., or 625 sheets per 1000 sq. ft. With standing seams a box of 112, 20 × 28-in. sheets will lay 356 sq. ft. or 312 sheets per 1000 sq. ft.

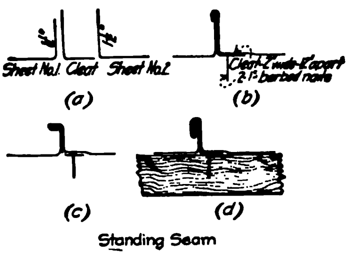


FIG. 263.—Tin roofs.

184d. Copper.—Copper is used extensively on buildings of the better class for ornamental purposes, and also on domes, mansards, etc., where a durable and light roof is required. Its first cost is high, but it requires no paint and the upkeep is low.

In hot climates copper is not so durable as in the temperate zone and will oxidize; great heat, generally, causing oxidation and buckling. In moderate climates the metal takes on a coating of carbonate of copper and turns green, and this action prevents the deterioration from going deeper. As compared with lead, it will not creep on steep roofs from expansion. It is ductile, tenacious, and malleable, thus easily worked. It has less expansion and is more durable than zinc, and presents a fine appearance. Owing to recent high cost, zinc, and at times lead, has been used instead of copper. Lap seams should be avoided wherever possible, using instead trough or roll seams (see Fig. 264). Copper sheets come in sizes 24 × 48 in. to 72 × 48 in. Soldering should be avoided as much as possible. When soldering is necessary rosin should be used for the flux. The usual sheet for roofing weighs 12 to 14 ounces per sq. ft.

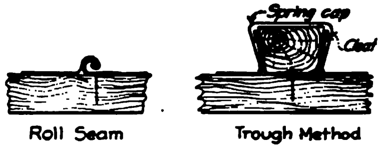


FIG. 264.—Copper roofs.

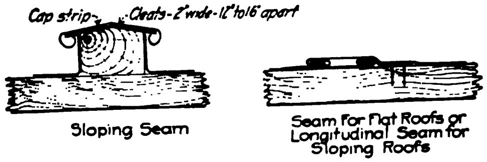

Cap strip
Cleats-2" wide-12" to 16" apart
Sloping Seam
Seam for Flat Roofs or Longitudinal Seam for Sloping Roofs

FIG. 265.—Zinc roofs.

184e. Zinc.—As a roofing material zinc is gaining in use in the United States, and has been used very extensively in Europe. Usually 16-ounce zinc sheets are specified. Zinc must not be used in contact with other metals, except iron, on account of the setting up of galvanic action due to the almost universal presence of moisture. When used on wood containing some acid, a layer of building paper or felt should be interposed. Zinc is soluble in diluted acids, and is attacked to some extent by salt air, soot, and acids in some lumber with which it may come in contact. In a dry clean air, zinc is very durable; it can not be bent and twisted like lead, all sharp bends requiring cutting and soldering. Zinc may be laid like tin with standing joints.

but it must be remembered that zinc has a much greater coefficient of expansion, which is the basic idea in all details for zinc construction (see Fig. 265). The expansion "roll cap" is recommended for all seams running up and down the roof. In Europe corrugated zinc sheets are used.

184f. Lead.—Lead is used for roofing on small curved surfaces, and on roofs where there are a number of corners and projections to cover. It is easily bossed and stretched and can be made to fit warped surfaces without cutting or soldering. While heavier than zinc or tin, the reduction in labor may overcome the handicap of more weight and greater cost. Lead has a large coefficient of expansion and will creep on steep roofs. It should not be used for a greater stretch than 10 to 12 ft. without a joint roll or drip. It comes in cast sheets 6 ft. wide and 16 to 18 ft. long, and in rolled sheets 6½ to 7 ft. wide and 25 to 35 ft. long. Roofing lead should weigh 7 lb. per sq. ft. A greater pitch than 1 in. per foot should not be used unless creeping is amply provided for. Narrow thick plank should be used to prevent warping, so that raised edges will not cut the lead. Lead should not be nailed or soldered. Locks and welts should be used. If possible, horizontal joints should be made by providing drips (see Fig. 266). Joints from ridge to eaves should be made on a 2 to 3-in. round. All sharp corners should be avoided.

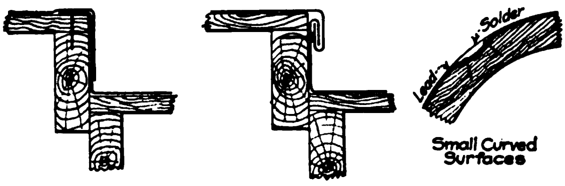


Fig. 266.—Lead roofs.

184g. Corrugated Steel.—Corrugated steel roofing is generally laid directly on purlins, but sheathing may also be used. It offers a rapid means of roofing at a low first cost. Corrugated steel is extensively used for mill buildings, train sheds, foundries, wharves, skip bridges, mine buildings, sheds, etc. It should not be used for a smaller slope than 4 in. per ft. unless a longer lap is used. For long life the sheets should be kept painted, particular attention being paid to the sheets along the eaves and gables, and around the stacks or other openings. Corrugated sheets come in 26-in. widths with 2½ × ⅝-in. corrugation as a standard. Sheets are generally laid on the roof with the end lap 6 in. and side lap two corrugations, the net covering width 21½ in., the usual thickness No. 20 or No. 22 gage. The sheets are fastened to the purlins with straps or clips (see Fig. 267). Clips are made of No. 16 steel, 1½ in. wide × 2½ in. long crimped one end to go over the edge of beam or channel flange. Straps make a better roof. Straps are made of No. 18 steel, ¾ in. wide, passed around the purlins and bolted to sheets with ⁵⁄₁₆-in. stove bolts, one strap to the linear foot. One bundle of hoop steel weighs 50 lb. and contains 400 ft.

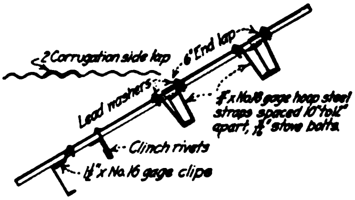


Fig. 267.—Corrugated steel.

To avoid condensation, an asbestos lining (anti-condensation lining) should be placed under sheets, or plank sheathing should be used. Sheets are either galvanized or not-galvanized (black). Black sheets must always be painted, preferably with red lead or iron oxide with pure linseed oil. Where corrosive gases attack the sheets, as in smelters where sulphurous gases are produced, asphalt, graphite, or tar paints (pure) should be used, as they provide a more inert paint body.

Corrugated steel is nailed to wooden sheathing with barbed wire nails, 8 penny size spaced 12 in. apart. 96 nails weigh about 1 lb. 20% excess should be added for waste—No. 22 gage corrugated sheets weigh 170 lb. per square, black, and 190 lb. galvanized. No. 20 gage sheets weigh 205 lb. and 225 lb. respectively, laid, including 2 corrugations for side lap, 6-in. end lap, sheet 8 ft. long × 26 in. wide.

184h. Asbestos Protected Metal.—Asbestos protected metal consists of a steel core encased in successive layers of asphalt, asbestos, and a heavy waterproofing envelop. Corrugated sheets come in 28-in. widths, 2½-in. corrugations and 5 to 12-ft. lengths. Net covered space, when laid, with 1½-in corrugation lap is 24 in. This roofing is corrosion proof against acid fumes, corrosive gases, salt air, moisture, and alkalies. Having small conductivity for heat and electricity, it is well fitted for many uses where plain steel sheets are not suitable. Thus, it is an excellent material for conditions of high humidity and large difference in temperature, inside and outside of building. It is light, and is applied in the same way as corrugated steel; or aluminum, galvanized iron, or copper hangers may be used. Purlins should be spaced from 3 ft. 10 in. for No. 26 gage up to 7 ft. 10 in. for No. 18 gage, on a slope of 4 in. or more in 12 in. Colors are terra cotta, dark grey, and white. Special mansard roof sheets 28 in. wide × 5 to 10 ft. long are made, beads ½ in. high, 1¾ in. wide, spaced 6½ in. on centers (see Fig. 268). These sheets lay 26 in. to the weather.

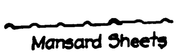


Fig. 268.

184*i*. Asbestos Corrugated Sheathing.—Asbestos corrugated sheathing consists of asbestos fiber and hydraulic or Portland cement mixed with water and subjected to a pressure of 9000 lb. per sq. in. These sheets have a hard, smooth surface, and make a light, permanent, fireproof roof. They are not affected by acid fumes, moisture, or other corrosive agencies and are insulators of heat and electricity. Purlins may be spaced 3 ft. apart; aluminum wire with lead washers are used for fastening the purlins (see Fig. 269). The asbestos sheets are manufactured in lengths from 4 to 10 ft., 27½ in. wide, 1 in. deep, and on the average 3/16 in. thick.

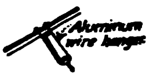


Fig. 269.

184*j*. Slag or Gravel Roofing.—Slag or gravel roofing may be laid on concrete or gypsum slab, or on plank roofing. With plank sheathing the roof should first be covered with dry felt. Then two-ply felt (tarred) is laid and mopped with pitch. Then on top of this three-ply tarred felt is laid and mopped on top with pitch. While the pitch is soft, it is covered with 3 lb. per sq. ft. of crushed slag or 4 lb. per sq. ft. of clean gravel, well screened, of ¼ to ⅝-in. size. With a concrete or gypsum slab the felt should be omitted and the slab mopped with pitch before laying the tarred felt. If the slab has a pitch of more than 1 in. in 12 in., provision should be made for nailing. Asphaltic felt and pitch may be substituted for coal tar felt and pitch. A good gravel or slag roof should last for 20 to 25 yr. and is more fireproof than tin. Oils of asphalt do not evaporate as quickly as those of coal tar; hence the life and flexibility of the asphalt gravel roof is the greater.

184*k*. Prepared Roofing.—There are several brands of prepared roofing on the market. Such roofings are composed of either paper, felt, or asbestos paper and saturated with different brands of waterproofing compounds, and are generally laid on a plank sheathing of matched boards. They are lapped at the edges and nailed to the roof with galvanized iron nails and tin washers, and the seams are thoroughly cemented together (see Fig. 270). With some brands the entire surface is covered with a water-proof cement and powdered asbestos sprinkled on the surface. On sloping surfaces of 4 in. or more in 12 in., it is not necessary to cement the seams if the roofing is laid parallel to the eaves and there is enough lap to prevent the rain from driving in.

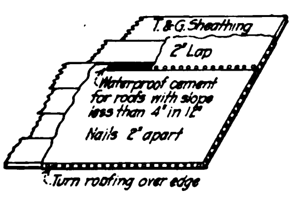


Fig. 270.—Prepared roofing.

184*l*. Clay Tile.—Clay tile for roofing is made in several different forms—Spanish tile, Pan tile, Ludowici tile, plain tile, and several others. Plain tile come in sizes 6¼ × 10½ × ⅝ in. and are laid the same as slate, with one-half the length to the weather. Spanish tile, Pan tile, and Ludowici tile, are of the interlocking type, and may be laid on angle sub-purlins, plank sheathing, or book tile. When laid on angle sub-purlins, the tile is fastened with copper wire. The underside of the joints should be pointed to prevent dust and dry snow from drifting in. A porous, non-sweating tile, glazed on the top surface only, should be used where there is danger of condensation. With book tile or plank sheathing, felt should be used and the tile nailed on with copper nails. Clay tile weighs from 750 to 1400 lb. per 100 sq. ft.

184*m*. Cement Tile.—On buildings where a permanent, rapidly constructed roof is essential, cement tile serve the purpose admirably. These tile are made of clean sharp sand and Portland cement, reinforced with steel. They are made in two styles, interlocking tile for sloping roofs and flat tile for flat roofs. The interlocking tile comes in various sizes; the most common are 26 × 52 × ⅞ in., lay 24 × 48 in. to the weather, and weigh about 14 lb. per sq. ft. They have a projection along the upper edge which hooks over the purlin. One side has a roll, and the other side a rabbet. Tiles are interlocked by placing the roll of one tile over the rabbet of another (see Fig. 271). Horizontal joints are made by lapping one tile over the tile below. No fastening is necessary. Flat tiles are used for roofs with a pitch of less than 4¾ in. in 12 in. These are 1½ in. thick and are laid on I-beam purlins, spaced 5 ft. on centers. The joints are pointed and the surface is covered with composition roofing.

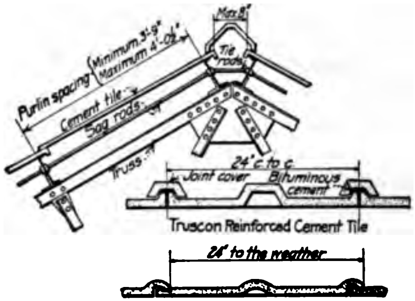


Fig. 271.—Cement tile.

184n. Metal Tile.—Metal tiles are stamped out of sheet steel, copper, tin, and zinc, to imitate clay tile. They are very light, and the first cost is less than clay tile. They are made in different patterns and sizes, and are interlocking. As a rule they are nailed to wood sheathing covered with felt. Metal tiles are not as durable as clay tile, and require frequent painting.

184o. Glass.—Glass roofs are used on domes, green houses, and public buildings, and on factories and mill buildings where daylight is essential. For green houses, flat, plain glass is generally used. Wire glass, however, is used where strength is required. Ribbed or other glass with a rough surface should not be used for this purpose as they diffuse the light rays. On domes, a heavy wire glass with a surface having ribs or prisms on one side is required, as there it is necessary to diffuse the light rays as well as the heat rays. On factories and mill buildings the usual practice is to have glass inserts, although a few buildings have been constructed with the entire roof made of glass. Glass inserts may be cast in cement tile slabs,or corrugated glass sheets may be used, reinforced with wire, in conjunction with corrugated steel, asbestos, or asbestos protected metal sheets.

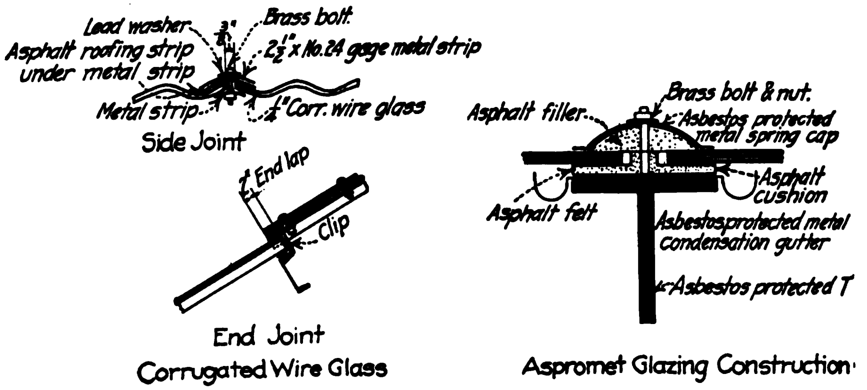


Fig. 272.—Glass roofs.

Corrugated glass sheets are 5½ ft. long 26 in. wide and ¼ in. thick; other lengths, however, may be obtained. The corrugations are made to fit standard corrugated steel sheets. The sheets are fastened to the purlins by means of clips. They should have no side lap but should be fastened together by placing a 3-in. strip of asphalt felt along the joint and a 3-in. strip of No. 24 gage under the joint. Bolts, ¼-in. diameter, passing between the glass sheets and spaced about 10 or 12 in. apart should be used to clamp the whole joint together (see Fig. 272). End joints should be made by lapping the sheets 2 in., preferably over a purlin. 2-in. strips of asphalt felt should be used on top of the purlins and between the sheets.

Flat glass sheets have end laps, and the side joints are made water tight by means of a spring cap. No putty is used. Flat glass weighs about 3½ lb. per sq. ft. and corrugated glass about 4¾, for ¼-in. thickness.

185. Condensation on Roofs.—Condensation takes place when the temperature inside the building is much higher than the outside and when there is enough moisture in the air to reach the dewpoint. The best of ventilation is necessary to prevent condensation. In buildings where there is little or no heat, condensation can be wholly avoided by proper ventilation.

Tar and gravel roofing is a poor insulator and, when used on plank sheathing, there is danger of decay of the wood where such roofs are subject to heat and moisture. The warm air goes through the plank quite readily and strikes the cold under surface of the roofing causing condensation. During the heating season the upper surface of the plank is continually moist. This may occur near the peaks where the hot vapors abound.

To prevent condensation forming under concrete slabs they must be insulated. This may be done by insulating the outer surface from cold or the inner from heat radiation. In the latter method the slab will not only be insulated on the inner surface but will also be insulated to a certain degree by the roofing material on the outside.

185*a*. Methods of Insulating Roofs on the Outside.—There are several methods of insulating roofs on the outside.

A cinder fill is probably the most extensively used for insulating a concrete roof slab, as it serves the double purpose of insulation and drainage. This provides an efficient insulation for buildings except where there is excessive moisture present as in paper mills, power houses, etc.

A cinder concrete fill also makes a good insulation for a concrete slab, but is not quite as efficient as cinder fill, and is more costly.

A 3 or 4-in. soft clay partition type hollow tile laid end to end, to provide a continuous air space, makes an excellent insulation for all types of buildings. Plastic cement should be laid at the walls to take care of the expansion. Hollow tile can only be used on sloping roofs as it does not provide for drainage.

A combination of hollow tile and cinder fill probably gives the best insulation that can be constructed without the use of cork. It combines the advantages of both the cinder fill and the hollow tile, and provides a drainage for the flat slab.

A double roof construction on concrete slabs, consisting of the usual slab and a thin auxiliary slab supported on a wood frame construction, gives very good results, but is expensive and non-fireproof.

Roofing blankets, consisting of felt or heavy tar or building paper placed under roofing material, will give a sufficient insulation for buildings used for light manufacturing purposes, warehouses, etc., where very little moisture is present. A blanket of one or two layers of cork 1 in. thick gives excellent results but is expensive. Cork in conjunction with hollow tile gives an insulation that is practically perfect.

185*b*. Methods of Insulating Roofs on the Inside.—Roofs insulated on the inside by means of suspended ceilings give good results for all classes of buildings, paper mills, textile mills, power houses, etc. This forms a dead air space which prevents radiation of heat. Metal lath is hung below the slab and covered with plaster (1 part hydrated lime, 5 parts Portland cement and 12 parts sand, mixed before water is added, and containing long cow hair). There is danger of the metal lath rusting and it will not stand a hot fire.

Gypsum is a fine material to use for slabs where condensation is feared. It requires no other insulation and has given good satisfaction on many buildings.

Asbestos provides another means of insulation and is used in the form of asbestos corrugated sheathing and asbestos protected metal.

When corrugated steel sheets are used in mill buildings, an effective insulation consists of one or two layers of asbestos paper, followed by two layers of building paper, placed under the corrugated steel sheets, and prevented from sag by a wire netting stretched over the steel purlins. This is the simplest form for an inexpensive roof.

186. Parapet Walls.—Buildings with exterior and division walls of masonry should have parapet walls formed by building the walls above the roof, except in detached buildings with overhanging eaves where a cornice is used. For residence buildings parapet walls should be 8 in. thick and extend 2 ft. above the roof for exterior walls and 8 in. for division walls. For public and business buildings they should be 12 in. thick and extend 3 ft. above the roof. Parapet walls are coped with terra cotta, stone, concrete, or cast iron. Parapet walls are a protection against fire (see Art. 209 for details).

187. Cornices.—Cornices made of sheet metal are often used instead of parapet walls. Better architectural effects may thus be obtained and the cornices may be worked in with the gutter. Brackets of sufficient strength must be provided for the cornices (see Art. 208 for details).

ROOF DRAINAGE

By John S. Branne

When the designer has determined upon the best roof for a building, in the sense of the most suitable roof at the least cost, he must also have solved, generally, the problems of getting rid of the roof water. A carefully planned roof drainage has much influence on the life of the roof and roof covering, and contributes, although to a lesser degree, to the sightliness of the structure and to the convenience of tenants.

188. Provisions for Proper Drainage.

188*a*. Pitch.—A roof, in order to be watertight, must have sufficient pitch or slope to shed the water and prevent it from blowing or backing in under the roofing. With a sealed roof covering only enough slope to enable the water to flow off is necessary, but with a shingle, tile, corrugated steel, or slate roof more slope must be provided to prevent the water from backing up and running into the building at the horizontal laps. The following slopes are the minimum that should be used for various roof coverings: wood shingles, 6 in. vertical to 12 in. horizontal; slate, 6 in.; tile, 4 to 7 in.; corrugated sheathing, 4 in.; metal flat seams, ½ in; metal standing seams, 8 in.; ready roofing, 1 in.; slag, ½ in,; and gravel ½ in.

188*b*. Flashing.—One of the most important things about a roof is the flashing. Flashing may be of Ix tin, 16-oz. copper, 14-oz. zinc, or composition. It should be high enough to prevent the water from backing up or flowing over the top (see Fig. 273*a*). Narrow flashings are frequently used with a mistaken idea of economy, and always are a source of trouble. Along a wall, the flashing should extend 8 to 10 in., or higher if there is danger of the water backing up, due to the clogging of roof leaders, causing water pockets. With corrugated sheets, flashing is used with one wing corrugated to match the sheets, covered with a two corrugation lap (see Fig. 273*b*). In valleys and around stacks, the flashing should extend in 12 in. (or more) up the slope (see Fig. 273 *c*). On the ridge it is customary to use flashing, a ridge roll, or a cap. Flashing along high-class brick and stone walls may be counter flashed with 4-lb. lead extending 1 to 2 in. into the wall, and down to within 1 in. of the roofing. Lead wedges should be used in the joints to secure the counter flashing. All seams must be riveted, or locked and soldered. With a composition roofing the felt should be turned up the wall, well mopped with tar or asphalt, and counter flashed. If there is danger of breaking the felt, a metal flashing should be used, extending 12 in. under the felt and sealed to the felt with tar or asphalt.

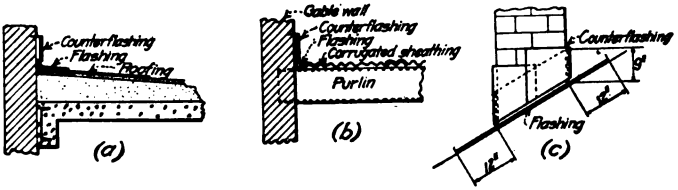


Fig. 273.—Flashing.

188*c*. Gutters.—Great care must be taken in selecting the type of gutter to be used. On flat roofs having projecting eaves a gutter should never be placed at the edge except in warm climates where there is no frost. With a roof of this type, the snow will melt on the portion of the building that is heated and run down on the colder projection, and form ice. As the ice grows thicker the water will back up on the roof and find its way over the flashing and under the roofing material. A gutter should be formed behind the wall line by flattening out a 5-in. single bead eaves trough and bending up the beaded edge 3½ in. perpendicular to the roof, the remainder laying flat on the roof. This should be placed so that it will drain into inside leaders. Wherever eaves troughs are used, snow guards should be placed to prevent the snow from sliding down the roof and bending or breaking the gutter. In designing gutters, the size and location of leaders must be taken into account. Gutters are generally made the same sizeas the leaders unless the leaders are spaced more than 50 ft. apart, then the size of gutters must be increased 1 in. for every additional 20 ft. of leader spacing for sloping roofs

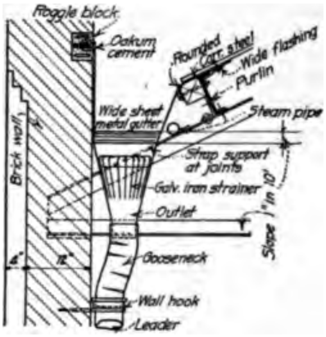


FIG. 274.—Gutter of parapet wall, corrugated steel roof.

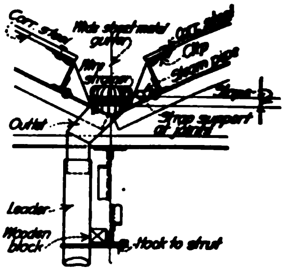


FIG. 275.—Valley gutter, corrugated steel roof.

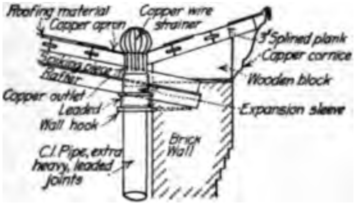


FIG. 276.—Eaves gutter, plank roof composition flooring.

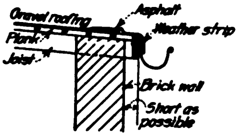


FIG. 277.—Eaves gutter, gravel roofing.

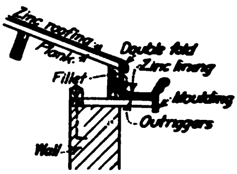


FIG. 278.—Eaves gutter, zinc roof. (Lining must move freely on account of large expansion and contraction.)

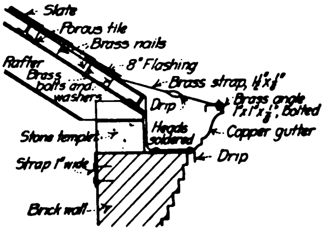


FIG. 279.—Eaves gutter, slate and porous tile roof.

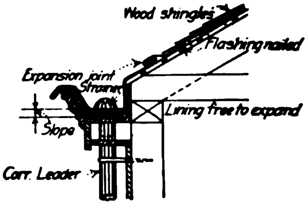


FIG. 280.—Eaves gutter, shingle roof zinc lining.

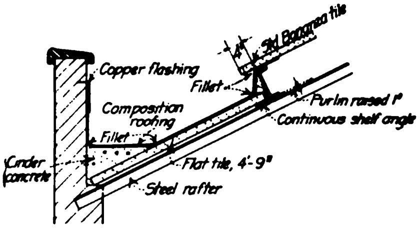


FIG. 281.—Eaves gutter, bonanza tile.

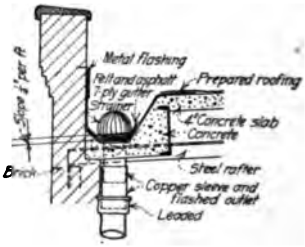


FIG. 282.—Eaves gutter, concrete roof.

and for every additional 30 ft. of leader spacing for flat roofs. Gutters smaller than 5 in. are difficult to solder and had better not be used. Gutters have generally a height of 1¾ times the bottom diameter. If box gutters are used, they should have an equivalent area. Gutters should slope 1 in. in 15 ft.

188*d*. Leaders.—The size of leaders depends on the rate of rainfall and the number used. A sufficient size of leader must be provided to keep the roof free from water. The rate of rainfall varies greatly in different localities, but provisions for handling a rainfall of 5 in. per hour willl do for practically all purposes. A good rule is to provide 1 sq. in. of leader area for every 150 sq. ft. of roof surface. Leaders should be spaced not more than 50 ft. apart for peaked roofs and not more than 75 ft. apart for flat roofs. The leaders should not be less than 4 in. in diameter for main roofs and 3 in. for porch roofs and sheds. Inside leaders should be made of extra heavy cast-iron or galvanized wrought-iron pipe with a trap wherever they open at the roof near dormers, chimneys, and ventilating shafts. Outside leaders should be made of galvanized iron or copper. All roof connections should be made watertight with copper ferrules. It is well to bear in mind the advantage of using the expansion type of outside leader, consisting generally of a sheet, bent in the form of a square, with an expanding joint, and with the sheet painted with red lead on the inside before being bent into the leader shape. A durable metal is necessary. Since copper is very expensive, although also very lasting, a pure iron may be used, galvanized

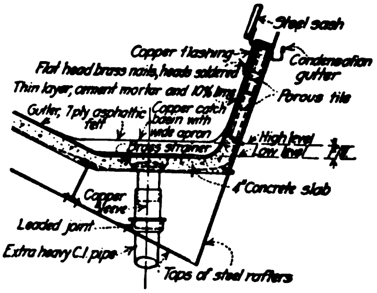


FIG. 283.—Saw-tooth gutter, concrete roof.

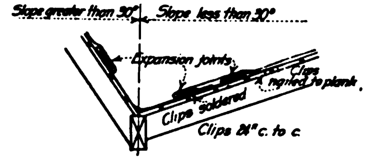


FIG. 284.—Valley gutter, zinc on plank roof. (Note expansion methods, depending on slope.)

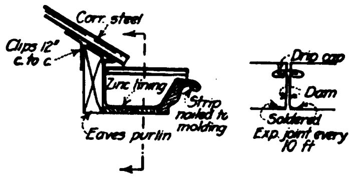


FIG. 285.—Zinc gutter, corrugated steel roofing. (Note expansion arrangement.)

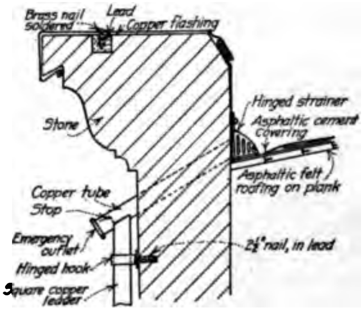


FIG. 286.—Flashed parapet. (Arrangement for leader in stone wall.)

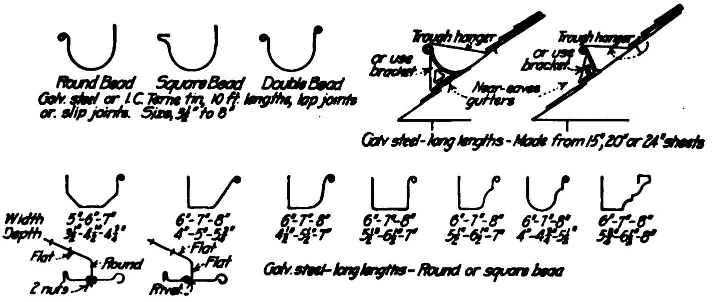


FIG. 287.—Various types of eaves, troughs, and hangers (from Catalogue Souther Iron Co., St. Louis).

—as, for example, the Armco iron. At the leader basket, strainers should be placed at a leader entrance to keep out leaves and twigs.

188e. Catch Basins.—Catch basins should be made of copper, 8 in. square, 4 in. deep and with a 4-in. flange at the roof. The edge should be raised ⅜ in. to prevent pitch from running in when the last coat is applied.

188f. Methods of Obtaining Drainage Slopes on Flat Slabs.—Concrete roof slabs are generally made level to decrease the cost of the form work. Some means for obtaining the necessary slope for drainage must be provided. This is generally done by placing a cinder fill or a cinder concrete fill on top of the slab, or by placing a thin slab supported by wood above the main slab. The latter method is but little used as it is expensive, and falls in the non-fireproof class. A cinder fill is lighter and cheaper than a cinder concrete fill. A good grade of steam boiler cinders should be used. They should be graded to give the proper slope, should have a minimum thickness of 3 in., and be well tamped and sprinkled. A cement mortar finish, 1 in. thick (composition: 1 cement to 3 sand) must be floated on before the cinders dry out. The mortar finish must be kept from 1 to 2 in. away from walls, and joints should be filled with plastic cement. Cinder fill weighs from 50 to 60 lb. per cu. ft. Cinder concrete fill is similar to cinder fill, the difference being that 1 part of cement is added to 8 parts of cinders and the finish is made ¾ in. thick instead of the 1 in. for the cinder fill.

189. Drainage Schemes.—In order to get the best service from a drainage scheme it is necessary to consider usefulness, durability, materials, workmanship, and fitness.

189a. Usefulness.—The water must be drained from the roof as quickly as possible, and at the ground level it must be provided with a suitable drain to run it to the sewer, street gutter, or to the rain water cistern, far enough from the building to be sure that it will not find its way into the cellar. The rain water cistern is a large hole in the ground, lined with stone or brick laid in cement mortar, and filled with graded stone. In the smaller cisterns the lining is often omitted. When the lined type is used, the water is available for the tenants for household use; with the unlined variety the object is to make the water seep into the subsoil. The slope of the roof gutter must not be too steep as this will cause a rapid current, causing backing-up of water, overflow, and abrasion of the gutter surface, which is most objectionable with roofings with a sanded or pebbled surface. Where open valley gutters shed a stream on a lower roof surface, the latter must be protected against abrasion and leakage by properly distributing the flow through a spreader, which discharges on a specially reinforced roofing surface. The better way is to carry such masses of water in their own leaders direct to catch-basin, and terminate such leaders so as to throw the flow of water in the direction wanted, and avoid the possibility of water rushing up under flashings.

In buildings with overhanging eaves the water is frequently allowed to drip on the ground. When such a building, which may be used for a mill or a factory, has a series of transverse saw-tooth skylights, with their gutters shedding water on the main roof a little distance below, the water will pour over the eaves in a mass just where it leaves the transverse gutter, or very near this point. This condition seriously interferes with opening windows below, especially when the windows turn on a horizontal pivot, and the roof overhang is small, as in that case the water pours directly on the inclined window surface. Such conditions can be avoided, in part, by a large eaves overhang, and better yet, by a parapet wall and inside eaves gutter. This latter method also avoids the annoyance of eaves water coming down on entrance stairs, into material bins, or on other articles placed close to the building wall.

Where the buildings have several roof levels, and the lower roofs drain into the main leader from high levels, it becomes necessary to provide a trap at the junction of the main leader and low-roof leader. If this is not done, the water rushing down from the high roof will sometimes back up on the low roof, especially if the low-roof leader is short and a large amount of water is passing down the main roof leader. During heavy thunder showers it has been noticed that when this precaution is not taken the water around the low-roof catchbasin will spout up several feet in the air and flood the low roof.

Whenever the roof water is carried to the ground by leaders, provision must be made to drain the water away from the building for reasons of sanitation, sightliness and life of foundation walls. Where storm sewers are not available, and the building lies lower than the street, a rain water cistern should be dug at a distance from the building of not less than 50 ft. The subsoil drain should be placed well under the frost line and have a slope of about *1 in. in 10 ft.*

The greatest demand on the roof drainage system occurs during a heavy rain storm of short duration, say for 5 or 10 minutes, during which time the rain may amount to 1 in., although such downpour seldom occurs. This shows the necessity of inspecting the drainage at least twice a year, spring and autumn, to remove rubbish and repair damage done by ice and rust.

189*b*. Durability.—Inspection, mentioned above, is necessary for durability. Metal work may require painting or soldering or even renewal, fastenings of the metal to roof or walls may have worked loose and strainers may need to be renewed. Tar and felt roofing may need to be coated with tar or asphalt to fill cracks and to soften the entire surface. Sand, pebbles, leaves, and twigs should be removed, leaders flushed, and subsoil pipe looked after. It is important to attend to these things so as to avoid rot and decay setting in along the eaves and walls where the damage is not always seen until it assumes proportions calling for expensive repairs.

189*c*. Materials and Workmanship.—Materials and workmanship should be of the best. If iron is used, the pure varieties should be secured which in the end are more economical than ordinary grades. Although black painted iron does very well for steep roof material, it does not measure up for gutters, leaders, and other parts where the water remains much longer; here the iron must be tinned or galvanized. If zinc or copper is used, painting may not be necessary except for securing a harmonious tint. For leaders, in all localities that have frost, the corrugated or expansion type should be used. When gutters are built up of tarred felts, all sharp bends should be avoided and sharp corners filled with wooden or mortar fillets, of large radius, so that the felt may have a secure base and support. Lead, copper, zinc, galvanized iron, and tinned iron have lasting qualities in the order given.

189*d*. Fitness.—With buildings of the better class, the eaves gutters may be incorporated with the cornice and made quite ornate. Leaders must look well and be placed as much out of the way as possible, in the first place for appearances, and in the second place to avoid mechanical damage from the ground level up to say 4 ft. above the ground. For the lower 4 ft. double strength cast-iron pipe should be used, which will stand the impact of iron ash cans, etc., taken out of all residences once or more during the week. Where leaders are so located that repairs are costly, the most durable materials must be used. Where there are no eaves gutters, as on the simpler types of sheds, or manufacturing buildings, there must nevertheless be short sections of eaves trough placed over main entrance stairs to prevent drip and ice formation on the steps. Piazza roofs should have gutters that will drain readily, preferably having the high level over the main entrance steps. In the case of small piazza gutters, almost level, an overflow is often found directly over the main entrance steps due to a settling in the shallow piazza foundations.

SKYLIGHTS AND VENTILATORS

By John S. Branne

190. Skylights and Ventilators in General.—For buildings occupying large areas, it is often impossible to provide sufficient daylight for the interior by means of windows in the exterior walls. In large buildings several stories high, light courts are introduced, and in smaller buildings where this can not be done, light shafts are used, the daylight coming through a skylight placed above the roof level where it is diffused into the interior of the building by windows in the sides of the lightshaft.

In all large private and public buildings the roof has one or more skylights which give light to the upper story, and sometimes so arranged as to help the illumination all the way down in buildings of moderate height. In such cases the skylight is often very large and is placed over an open light well which is guarded by a railing, and contains the main stairway.

In one-story buildings requiring an exceptional amount of light, as green houses and horticultural buildings, the entire roof is made of glass. In one story shop and factory buildings train sheds, etc., daylight is provided for the interior by one of the following methods of providing a glass surface:

1. Light through glass placed in the plane of the roof.
   - *a.* Glass tile.
   - *b.* Glass inserts in concrete tile.
   - *c.* Glass inserts in concrete slab.
   - *d.* Corrugated glass sheets.
   - *e.* Flat glass skylights.
   - *f.* Translucent fabric, taking the place of glass.
2. Light through glazed surfaces not in the plane of the roof.
   - *a.* Common box skylights.
   - *b.* Longitudinal monitors.
   - *t.* Transverse monitors.
   - *d.* Saw-tooth construction.

In planning for light, the designer at the same time must keep ventilation in mind, because most special skylight devices placed above the plane of the main roof surface are also well adapted for securing ventilation. A glazed surface may be made wholly or in part movable. The vertical (or *nearly* vertical) sides of monitor and saw-tooth roofs may be made part glass and part louvres. Louvres may also be provided on the vertical sides of box skylights.

The designer must gather all the knowledge available as to light requirements, based on the occupation of the tenants of the building, and on the more or less favorable location of his building as regards height and location of surrounding structures.

The necessity of the best available light and ventilation for the efficiency of all the workers, of whatever grade and responsibility, is now a well known economic fact, taken into account by every employer of labor. The nearer the glazed surface approaches the working floor, the better the light; but if too near, the heat rays in summer will be very uncomfortable.

North light is the best as there are no direct sun rays. Where direct sunlight will strike the glazed surface of the skylight, glass must be selected that will diffuse the sunlight; that is, scatter or break the direct rays so as to reach the condition of light without glare. Such glass is ribbed or contains small prisms, of various styles as to depth and spacing of ribs and prisms. When there is no objection to the loss of a little light, rough glass is used. The ribbed and prismatic types gather dirt very quickly, and require frequent cleaning; rough glass to a lesser degree. When the glass is placed, due consideration must be given as to which side is most accessible to the window cleaner, the inside or outside face.

The amount of glass required for mill and factory buildings depends entirely on occupation of tenants or workers, and no general rule can be given. 30 % of the side walls used for windows is often found, and again the entire side wall may be glass except for the space occupied by wall pilasters.

The roof light must be studied with regard to location of machinery or desks, etc., and also from the standpoint of possible leaks, and breakage of glass. Care must be taken in placing skylights so as not to place them too near valleys or other depressions which may cause snow to cover them.

It costs more, of course, to heat buildings with large glass surfaces during the winter months; but it should also be remembered that there is a saving of artificial light all the year around.

As regards fire protection, the following is taken from the 1909 code of the National Board of Fire Underwriters, p. 103:

> All openings in roof for the admission of light, other than elsewhere provided in this code, over elevator, stair, dumb waiter shafts, and theatre stage roofs, shall have metal frames and sash, glazed with wired glass not less than ¼ in. thick, or with glass protected above and below with wire screens, of not less than No. 12 galvanized wire, and not more than 1 in. mesh.

The consistent use of wire glass in a building may save as much as 10 % on the fire insurance.

In all large dwellings, and in many small ones, and in all public buildings, means are provided for carrying off foul air by ventilating shafts or ducts placed in the walls. Those in the walls are carried up to the top of the parapet or higher. When ventilating shafts are *used, they are* sometimes made large and provide light for interior rooms. Such shafts must be fireproof and be carried not less than 2 ft. above the roof when covered with ventilating skylight, nor less than 3 ft. above the roof when open, terminating in a tile or cement coping.

Machine shops, factories, shops, manufacturing establishments of the many types found, often provide ventilation through the verticle sides of box skylights, through round metal ventilators placed along the ridge, or through the vertical or slightly inclined sides of monitors and saw-tooth roofs.

191. Notes on Glass.—Glass used in skylights of all kinds may be plain or reinforced; the latter type has wire mesh imbedded in it. This wire mesh may be placed between two plates of glass which are then rolled together; or rolled into one plate of glass. The first type is made by the "sandwich process;" the second by the "solid process," also called the "Pennsylvania continuous process." The "solid process" produces a stronger glass.

Single strength glass is 1/16 in. thick—extreme size 24 x 60 in., 30 x 54 in., or 36 x 50 in. Double strength glass is 1/8 in. thick—extreme size 30 x 90 in., 38 x 86 in., or 48 x 80 in.

	Thickness	Extreme size
Polished wire glass	about 5/16	48 × 130 in.
Polished glass	1/4 – 5/16, standard	48 × 130 in.
Rough wire glass	1/4 – 3/8	48 × 130 in.
Rough glass	1/8 – 3/16 – 1/4 – 3/8 – 1/2	48 × 130 in.
Ribbed wire glass	1/4 – 3/8	48 × 130 in.
Ribbed glass	1/8 – 3/16 – 1/4 – 3/8 – 1/2	48 × 130 in.
Factrolite wire glass	1/4 – 3/8	48 × 130 in.
Factrolite glass	1/8 – 3/16 – 1/4	48 × 130 in.

The ribbed variety diffuses light well; the factrolite variety has a still greater diffusion, and creates a very uniform light. The "Aquaduct" glass is a ribbed glass with deep and narrow grooves. The manufacturers claim that the capillary attraction will retain and carry off condensation at a slope as low as 10 deg. with the horizontal. Plain glass or wire glass, sandblasted to give it a frosted appearance, is sometimes used for skylights.

Stock sizes of wire glass run from 14 to 40 in. wide and from 50 to 100 in. long. The unsupported width should not exceed 24 in. If ribbed glass is used, the ribs should run parallel to the slope, or stand vertical for side windows. When windows are double glazed, place the ribbed surfaces towards each otner and cross them.

In vertical or slightly inclined windows, with small danger of breakage, double or single strength glass may be used if not interfering with fire protection policy.

192. Skylights in Plane of Roof.

192*a*. Glass Tile.—Glass tiles are often used on roofs in conjunction with clay tiles, and are made of the shape and size of the clay tile so as to match laps, thus requiring no further attention than laying them as decided by the designer (see Fig. 288). Sometimes they are laid in large units, forming several large roof lights, or in rows extending the length or part of the length of the building; more rarely scattered all over with the clay tile. The most economical way is probably to lay them in large units or long rows so as not to be constantly watching a certain pattern or design scattered all over the roof.

192*b*. Glass Inserts in Concrete Tile.—Glass inserts are used to some extent in concrete tile and are very efficient. The interlocking "Bonanza" tile, size to weather 24 x 48 in., has 1/4-in. ribbed wire glass inserts, 14 x 26 in. (see Fig. 289). The tile with inserts may be laid in continuous rows or arranged to meet special conditions. The glass is laid into the form when the concrete is poured, and the finished tile is shipped to the building site like the all-concrete tile.

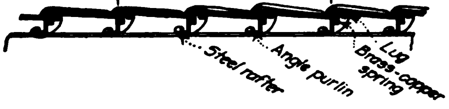


FIG. 288.—Imperial tile with glass tile.

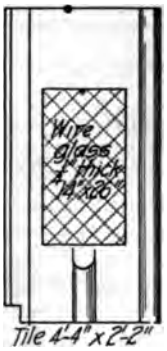


FIG. 289.—Reinforced cement tile with glass inserts.

192*c*. Glass Inserts in Concrete Slabs.—Glass inserts to be used in concrete slabs come in sizes from 6 to 6½ in. square, and from 1¼ to 1¾ in. thick. Light concrete ribs, reinforced, are poured between the inserts (see Fig. 290). The "units", made of many small inserts, can be made in sizes to suit the beam or girder spacing, or purlin spacing, and each unit is surrounded by a border of concrete. For tightness and to take up expansion and contraction, the units are separated by a thin joint of oakum packing covered with elastic cement.

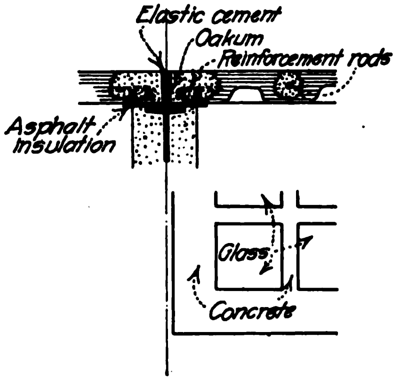
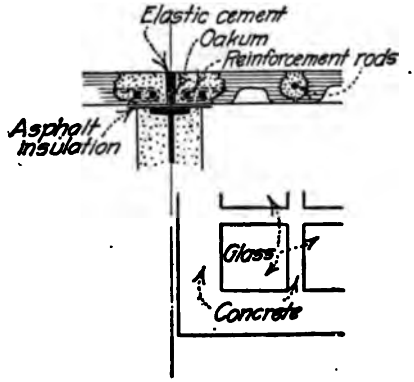


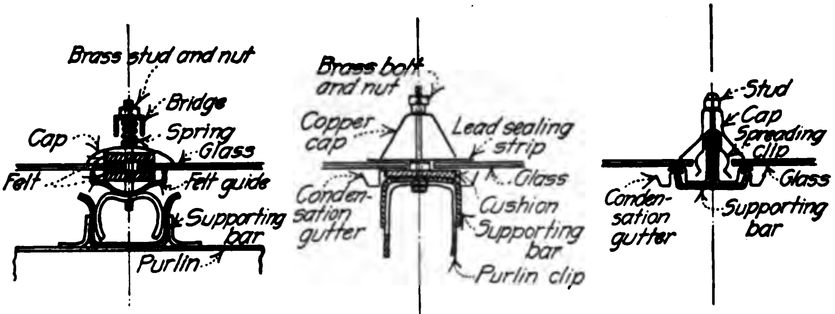
Fig. 290.—Glass inserts in concrete slab—Keppler type.

192d. Corrugated Glass Sheets.—Corrugated glass sheets are 26 in. wide, 66 in. long, and ¼ in. thick, and have standard 2½-in. corrugations (see Fig. 272, p. 597). They are used with corrugated steel, corrugated asbestos, protected corrugated steel. The corrugations diffuse the light and heat rays, preventing glare, and the manufacturers claim that a building covered with this glass is no warmer in summer than the same building would be if covered with corrugated steel sheets.

192e. Flat Glass Skylights.—Flat glass skylights are often used in the plane of the roof but unless there is sufficient slope of roof to shed the snow as it falls, the light will be shut off and the purpose of the skylight defeated. These skylights must be particularly well flashed, to prevent leaks. Flat skylights should at least have a slope of 2 in. per foot.

192f. Translucent Fabric.—Translucent fabric is manufactured by dipping a wire mesh into an oil composition which hardens into an amber colored, translucent sheet. It is well adapted to buildings where the vibrations of running machinery are so great as to break glass. Also it may well be considered in locations where the foundations are apt to settle, as *in filled-in ground,* throwing purlins out of line, and straining all rigid materials. This fabric





Anti Pluvius Landon's Improved Multiunit

Fig. 291.—Skylight bars.

withstands ordinary heat, but when exposed to fire burns readily. The fabric softens a little when exposed to very high temperatures. It collects some dirt which should be washed off.

193. Skylights Not in Plane of Roof.

193*a*. Common Box Skylights.—Common box skylights are better than the flat ones on account of the greater ease of thorough flashing up along the high curb to prevent leakage. The top may be of the same slope as the roof, or may be arranged with a ridge to cause the snow to slide off. One advantage of the high curb is the possibility of arranging ventilating louvres all around the curb. When the slope of glass top is made 7 to 8 in. per foot, the snow will slide off.

193*b*. Longitudinal Monitors.—The object of longitudinal monitors is to provide light as well as ventilation. For the right amount of light in a mill building, shop, or factory, no set rules can be given, but each class of building must be considered by itself. In a general way, for buildings with a height to eaves of 16 to 20 ft., with ample side windows, say about 30% of wall surface, no monitor is required when the width of building is not over 40 ft. This refers to shops where the work is done principally along the walls, and the central portion of building is used for an aisle. When the width becomes greater, the monitor is placed along the ridge of roof, and is made about ¼ of the width between walls.

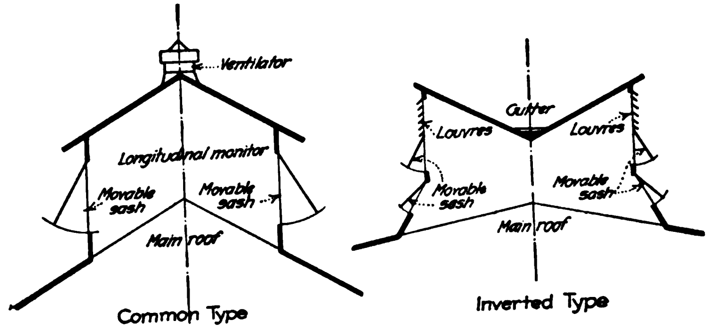


Fig. 292.—Longitudinal monitors.

The monitor roof is made of the same roofing material as the main roof; the monitor sides are glazed; and the sash is either wholly or in part movable. A wide monitor having its ridge in the same vertical plane as that of the main roof, does not ventilate efficiently under all circumstances, and under such conditions there should be a series of round sheet metal or asbestos ventilators placed along the monitor ridge.

To overcome this condition an inverted monitor type has been placed on the market, with its valley gutter in the center and discharging hot air, smoke, fumes, and dust very efficiently to the highest parts of monitor and out through louvres or movable sash (see Fig. 292).

The monitor roof may be made of glass, if slope is made sufficiently steep to shed snow; and the higher part can be made to swing up for ventilation.

193*c*. Transverse Monitors.—Transverse monitors (Fig. 293) are most adapted for flat roofs, or for roofs with a slight slope. If used for steep roofs, the sash along the sides becomes irregular and difficult to operate. When the slope is slight, they are practical in construction and look well. These monitors start as near the wall as is necessary to get good light, and have glazed or louvred sides, the same as the longitudinal monitor. With this type of monitor, there is an easy access from one side of building to the other, and they should always be set back from the building side sufficiently to provide a comfortable walk for inspection and cleaning of roof and sash. With a truss spacing of 16 ft. they should be placed in every third bay, which will place glazed sides about 30 ft. apart. This type of monitor avoids the valley gutter which often causes trouble in the saw-tooth construction by leaking.

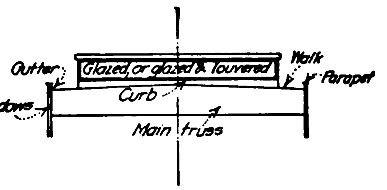


Fig. 293.—Transverse monitor.

193*d*. Saw-tooth Construction.—Saw-tooth construction is used to get a very strong north light. To accomplish this every bay has a saw-tooth, the steep side is glazed and the gently sloping side has solid roofing. A very even lighting is thus obtained.

Ventilation is secured by making the upper part of the sash movable (see Fig. 294). Sometimes round sheet metal ventilators are placed along the saw-tooth ridge, and louvres are provided on the two gable ends. When the glazed (steep) side faces due north, the glass can be perfectly clear, if placed vertically or very steep, so that the sun even at noon cannot shine through. This steepness, in the northern part of the United States, should be such that the angle with the horizontal is not less than 72 deg., and in the southern part, not less than 78 deg. If the angle is smaller, there will be direct sunlight at noon, and this may necessitate ribbed or rough glass. When the glass is inclined, more light comes through.

The saw-tooth type of skylight sometimes gives trouble by leaks developing along the valley gutters. To overcome this trouble the following precautions must be taken:

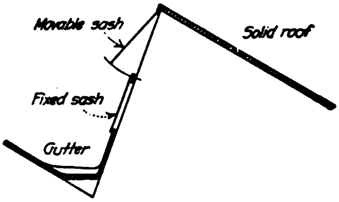


Fig. 294.—Saw-tooth type.

(1) The gutter should be made wide, and all sharp corners avoided by providing liberal fillets and a perfect bearing surface under the gutter body. A narrow gutter invites the expansive action of ice, banks-up the snow which accumulates by direct fall and by sliding off the glass, and makes it very difficult for window cleaners to stand in it. As the gutters are used frequently for thoroughfare across the roof, the gutter surface must be protected either by a special wearing surface or by placing a plank walk along the gutter. This walk must not block the flow of water. It is better to spend money for a good wearing surface, as the plank rots, and twigs and leaves may block the water.

(2) Flashings on both sides of the gutter should be made wide, and the supports for the gutter strong so that no deflection may set in and form water pockets in the gutters. Sometimes much snow and ice form in saw-tooth gutters. If the gutters are long, it will be better to use interior downtakes which can be brought down along the columns.

194. Miscellaneous Notes on Skylights.—Wherever glass is used, some provision has to be made for carrying off condensation, such as, small gutters in buildings where machinery or product would receive serious injury from water. There are several types of skylight bars on the market (see Fig. 291), all aiming to collect and carry off condensation. Unless copper is selected, a closed bar section must not be used, as it can not be painted.

All glass except expensive plate glass, has an uneven surface and a cushion has to be provided between metal sash bars and glass by using putty, cement, asphaltic compounds, or felt. The glass on the better class of modern sash is held by copper spring caps covering the joints and fastened to the bars with brass nuts and bolts.

195. Ventilators.—As described in Art. 193, light and ventilation are often provided by the same bulkhead, or skylight, whether this be a small box skylight or a large monitor. In the section on "Heating, Ventilation and Power," in Part III, the questions of fresh air requirements are fully discussed, and it will be seen that they vary according to the uses and character of the building.

Box skylights may be used as ventilators by having high curbs filled with louvres or movable sash, small hinged doors, etc. This will prove enough where small amounts of air have to be expelled.

Longitudinal monitors of the common or inverted type give excellent ventilation by using louvres, shutters, or movable sash along the sides. Louvres are made of black or galvanized steel or iron, asbestos, or asbestos protected metal, all according to durability required and care given after placing. Shutters are made of sheet iron or steel, black or galvanized. Movable sash is the most useful arrangement, giving both light and ventilation, and can be operated in large sections by hand or even driven by small motor.

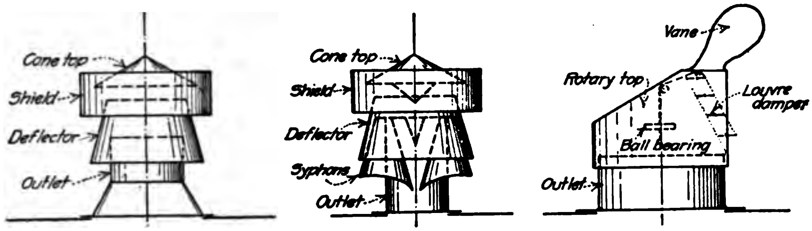
Transverse monitors are used for ventilation just as described for longitudinal monitors. This type has been used considerably, as the light distribution is very good, and while not so perfect as in the saw-tooth type, yet has not the disadvantage of the saw-tooth gutter.

Saw-tooth construction is well adapted to ventilation, on account of its shape, resembling one-half of the inverted type monitor. The light, as stated, is also perfect. The disadvantages are: a slightly higher cost than common transverse monitors, and the gutter.

Open roof ventilation is used largely for rolling mills and smelters where the heat is intense and the air is burdened with smoke, fumes, and gases. The method commonly used is to provide two planes of purlins and by laying the lower end of roofing sheets on high purlins and the upper end on low purlins an effect is produced like a large louvre laid on the roof slope. The only protection asked here is to keep out to a large extent snow and rain, whence the lower ends of each set of sheets overlap upper end of sheets below. In addition to this, sides of building may not have any walls.

Sheet metal ventilators, asbestos ventilators, etc.—The use of these has been referred to already. Several types are on the market, both as regards materials and method of operating (see Fig. 295).

The suction of air is taken care of in various ways. One type is entirely stationary, and relies on the motion of the outside air against the curved surfaces of the ventilator to suck the air out. Another type allows the upper par to move with the wind, so as to draw the air out. A third type has a rotary cap with spiral blades both on top and on the underside of the cap and is either wind propelled or power driven. All ventilators must keep out rain. Some have glass tops and admit light. Dampers should be provided, and a type chosen that will prevent back draft. Another type of draft regulation is a sliding sleeve, and with this type a glass top is used. This sleeve can be raised or lowered by means of a cord running over a pulley.



Aspromet Type Fixed

Arex Type Fixed

Swartwout Rotary Top

FIG. 295.—Types of ventilators.

WALLS

By Frederick Johnck

196. Masonry Walls Below Grade.—Concrete is used perhaps more extensively than any other material for walls below grade. The forms are made of 1 or 2-in. lumber reinforced with 2 or 4-in. scantling as the case may require. Safe allowable bearing pressures on walls for the concrete mixtures commonly used are as follows, assuming Portland cement concrete:

1-2-4 concrete	350 lb. per sq. in.
1-3-5 concrete	300 lb. per sq. in.
1-3-6 concrete	250 lb. per sq. in.

The common construction is to employ concrete curtain walls 12 in. thick between the wall columns and in addition to reinforcing them vertically, to take the earth pressure, to place rods near the bottom of the wall so as to make the wall carry itself as a beam from footing to footing.

For buildings of moderate height, stone is often used for walls. This is very economical when a local stone can be obtained. Stones should be laid with cement or lime and cement mortar, carefuly bedded in a full bed of mortar and worked around until a full solid bearing is obtained.

The use of brick for exterior walls below grade is gradually becoming less on account of the additional cost over that of a concrete wall. Brick used for walls are hard-burned common brick, laid up in lime and cement mortar. Brick walls should not be less than 12 in. thick.

In small residence construction, a hollow, vitrified, salt glazed tile has come into use for basement walls. These tile are 8 in. wide 16¼ in. long and 8 in. thick,and are laid with broken joints like stone ashlar. Special tile laid vertically are used for corners. If they can be obtained at the local yard, they are more economical than brick or concrete.

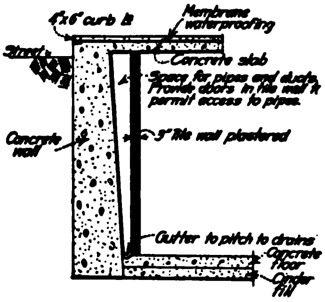


FIG. 296.

The question of waterproofing walls below grade against moisture and dampness is a very important one. A description of the various methods is given in Sect. 5, Art. 29.

If the walls below grade form the sides of rooms that are to be decorated, an inner tile wall should be built, leaving an air space between that and the outer wall, as shown in Fig. 296. At the bottom of this space a gutter should be formed pitched to drain, so as to carry off any moisture that might pass through the outer wall. In erecting these tile walls the lower two courses of the tile should be laid on an asphalt bed to prevent moisture passing up by capillary attraction and causing the tile to disintegrate.

197. Masonry Walls Above Grade.

197a. Concrete Walls.—The use of solid concrete for walls above grade is not generally considered advisable on account of the cost of form work, the tendency of concrete to absorb moisture and cause damp walls on the inside, and also on account of the difficulty of treating them in an architectural manner. To overcome these objections many forms and shapes of hollow cement blocks have been made. These are usually laid up like cut stone.

197b. Brick Walls.—The use of brick for walls above grade is considered the best and most economical for masonry walls. On street fronts and on exposed sides where an architectural effect is desired, the exterior surface of the wall should be faced with a pressed brick. In residence, church, or other work where large wall surfaces can be treated, a variety of effects can be secured by the use of tapestry brick, pavers, and bricks varying in shade; also by using color in the mortar for the joints. Other effects may be produced by laying the brick in various bonds, such as the Cross Bond, Flemish Bond, etc., as shown in Figs. 297, 298, 299, and 300, also by laying alternate courses of wide and narrow brick as shown in Fig. 301. When this is done the narrow course should be a darker brick. Effects can also be secured by using full, raked, pointed, and tool joints as shown in Fig. 302. In raking out a joint it is customary to rake the horizontal joints only. Brick work is also sometimes laid up with very wide joints and gravel used in the mortar, as shown in Fig. 303. When this is done, wood blocks or metal clips must be set in to prevent the load from crushing out the mortar as the work progresses.

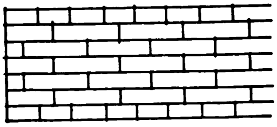


Fig. 297.—Common bond.

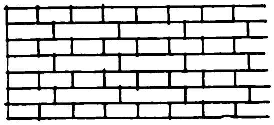


Fig. 298.—English bond.

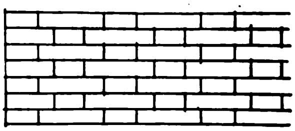


Fig. 299.—Flemish bond.

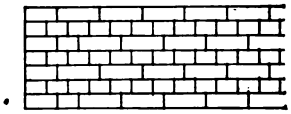


Fig. 300.—English cross bond.

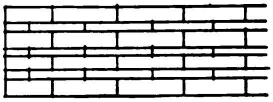


Fig. 301.—Alternate wide and narrow brick.

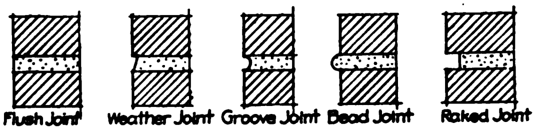


Fig. 302.—Joints in brick work.

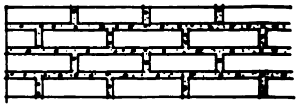


Fig. 303.—Brick laid in wide gravel mortar joints.

A great deal of care and judgment should be used in the selection of brick for the purpose intended. For instance, in a locality that is free from smoke and soot, a brick with varying shades can be used effectively; while in dirty, smoky places it is better to use a paver or some smooth-faced brick that the rain will wash. Again, in courts or in alleys a white enamel brick is desirable to reflect light into the building. White enamel beick should always be laid with a very narrow full joint. The advantage of this brick is that it can be washed when it becomes dirty. Enamel brick should be burnt in one fire so as to make the chemical change in the body and the glase simultaneous. In the dry process where the brick is first burned and the enamel is applied and then fired again, the bond is weak and a pulling or chipping of the enamel occurs. Enamel brick are best cleaned with an alkaline solution, such as caustic soda or sodium carbonate. This cleans the enamel and does not effect the cement or lime mortar in the joints.

Pier Construction.—Since the introduction of the skeleton type of construction and also in the pier type of building, the elevations are often designed to produce a Gothic effect, which is a natural manner to express this type of construction. In doing this the brick work follows closely the form of the column, and the spandrels or spaces between the columns are treated either in plain brick or in pattern brick panels. In this type of wall construction the use of steel shelf angles on the columns at the floor levels is recommended (see Fig. 304). This not alone prevents wall cracks but on large work enables the builders to run two crews of brick layers, one at the bottom and one half way up on the structure. In this construction of the spandrel a steel angle is necessary on which to carry the face brick. This angle can be left exposed on the bottom in slow burning and mill buildings, as shown in Fig. 305, but should be covered with a fireproof material in fireproof buildings (see Fig. 306).

Corbels and Ledges.—In slow-burning and mill constructed buildings, and often in ordinary construction, it is well to corbel out and form ledges to support the joist or floor construction. This not alone allow sthe joist to fall out without tearing down the wall in case of a fire, but also prevents smoke and small fires from traveling into the next story above by passing between the wall and the floor construction. Corbels and ledges should project at least 4 in. out from the face of the wall as shown in Fig. 307.

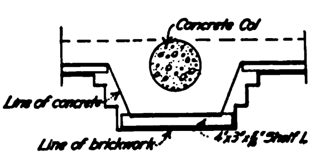


Fig. 304.—Detail showing self angle on concrete column.

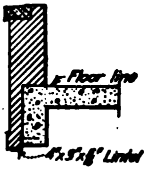


Fig. 305.—Spandrel details.

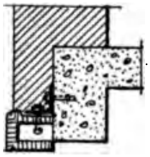


Fig. 306.—Spandrel detail for terra cotta lintel.

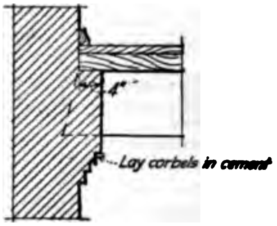


Fig. 307.

Erection of Brick Walls.—In the erection of masonry walls, no wall should at any time be carried up more than two stories above another wall of the same building on account of the danger of an uneven loading on the building foundations, the lack of a continuous bond around the entire structure and also the danger of a heavy wind storm throwing the wall out of line.

Bond in Brick Walls.—In laying common brick in walls, every fifth course should be laid as a header to form a proper tie through the wall. In face brick two headers and a stretcher or their equivalent should be laid in every sixth course to form a proper bond between the face brick and the common brick.

Brick Sills.—Bricks are often used for window sills in brick walls in place of stone or other material, in order to produce the desired architectural effect and sometimes to save time and money. Brick used for sills should be vitrified brick laid in cement mortar and laid as a header course.

Parapet Walls.—Parapet walls should be erected around all flat roof buildings as a fire stop to prevent fires from traveling from one roof to another; also to prevent water from the snow from running down and ruining the building walls and from falling down on people passing on the walks below. Parapet walls should be at least 18 in. high on the street fronts, and 36 in. high on the lot line and for dividing walls, It is a good practice to face the inside of all walls with a vitrified brick to prevent disintegration from moisture absorbed from the snow, which lies banked against it during the winter months. Sections through parapet walls are illustrated in the chapter on "Cornices and Parapet Walls."

Mortar for Brick Walls.—Mortar to be used for brick walls is usually determined by the load to be carried.

Stress Allowed on Brick Work.—The follwing table taken from the Chicago Building Ordinance gives the safe load per square inch allowed on brick work:

Paving brick—1 part Portland cement to 3 parts sand	350 lb. per sq. in.
Pressed brick—1 part Portland cement to 3 parts sand	250 lb. per sq. in.
Hard common select—1 part Portland cement to 3 parts sand	200 lb. per sq. in.
Common brick—All grades—Portland cement mortar	175 lb. per sq. in.
Good lime and cement mortar	125 lb. per sq. in.
Good lime mortar	100 lb. per sq. in.

Weight of Brick Work in Common Brick Walls:

9-in. brick wall	83 lb. per sq. ft.
13-in. brick wall	120 lb. per sq. ft.
17-in. brick wall	160 lb. per sq. ft.
21-in. brick wall	195 lb. per sq. ft.

Wall Thicknesses.—Although wall thicknesses for brick walls are determined by the safe stress allowed per square inch on the brick work, yet, from common practice, certain, safe, definite rules have been fixed upon. The table and rules given below do not recognize enclosing walls less than 12 in. thick. Walls 8 in. thick have been erected and have stood up for a number of years, but it is not recommended that they be used in general practice.

TABLE SHOWING WALL THICKNESSES IN INCHES FOR ENCLOSING BRICK WALLS.

	Bsmt.	1	2	3	4	5	6	7	8
One story	12								
Two story	16	12	12						
Three story	16	16	12	12					
Four story	20	20	16	16	12				
Five story	24	20	20	16	16	16			
Six story	24	20	20	20	16	16	16		
Seven story	24	20	20	20	20	16	16	16	
Eight story	24	24	24	20	20	20	16	16	16

Walls less than 50 ft. long can be built 4 in. less in thickness than called for by the above table, except that in no case should brick walls be built less than 12 in. thick. Brick walls in elevator or stair shafts need not exceed 16 in. in thickness nor its upper 50 ft. exceed 12 in. in thickness. Where masonry buttresses or piers or pilasters occur, walls may be reduced in thickness by one-half of the projection of the buttress or pier, but no wall should be reduced to less than 12 in. in thickness and no 12-in. wall should be less than 30 ft., and no 16-in. wall higher than 50 ft. Buttresses or piers should be at least ⅒ as wide as the space between them. Buttresses and piers and pilasters should be so placed as to receive the principal girders and trusses.

197c. Brick Walls Faced with Ashlar.—In the case of brick walls faced with stone, granite, terra cotta, or other ashlar, this facing should be considered as part of the wall for the purpose of carrying weight, unless every second course is a bond course extending back into the wall a distance of at least 8 in. In addition to this it is well to tie each piece of ashlar back with two galvanized iron anchors. No ashlar should be less than 4 in. in thickness, nor should the height of any piece of ashlar be more than 20 in. As a general rule the brick backing for ashlar should be laid in a cement, or lime and cement, mortar. Where terra cotta is used for ashlar, it is made as a hollow block formed with inside webs to gain strength and prevent warping while it is being burned. The hollow space in terra cotta ashlar also allows an opportunity for the brick to form a bond by extending into these spaces.

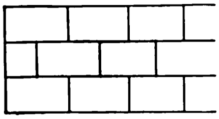


FIG. 308.—Coursed ashlar with same size blocks.

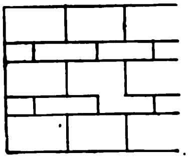


FIG. 309.—Coursed ashlar with wide and narrow courses.

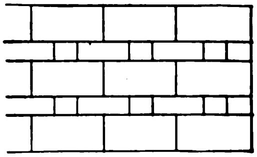


FIG. 310.—Coursed ashlar with header blocks.

Ashlar Jointing.—Of the many ways of jointing granite, stone, or terra cotta ashlar, the *coursed ashlar* as shown in Fig. 308 is perhaps the cheapest and most common, as the blocks can be made or quarried all of the same size. Another form of *coursed ashlar* is shown in Fig. 309. In this method the courses alternate with a wide and narrow course. This can also be varied by the use of a small header course as illustrated in Fig. 310. When a stone of uniform size cannot be obtained from the local quarry or when it is necessary to produce a varied or more interested form of jointing, what is known as *broken ashlar* is used. This form costs more and also requires more time to lay. It is made up of 4, 6, 8, 10, 12 and 14-in. pieces, as shown in Fig. 311, or in 4, 8, and 12-in pieces, as shown in Fig. 312. Another form of ashlar often used is what is known as *random coursed ashlar*, shown in Fig. 313. In this type the joints A, B, and C carry through in a straight line.

Ashlar Finish for Stone Work.—Perhaps the first step in stone work finish is the *rock face* (Fig. 314), the face of the stone being left rough as it came from the quarry. Next comes the rock face with the margin line finished with a chisel (Fig. 314). Then the stone is given the *broached finish* (Fig. 314)—that is, the surface is dressed level and continuous grooves are left in it; this might be called the first step toward the tooled finish. The *tooled finish* is done with a wide flat chisel. This is a very common finish for sandstone and limestone. Tooling is done in 6, 8, or 10 cut, measuring 6, 8, or 10 grooves to the inch. For finer work than the tooled surface a *rubbed finish* is used. This is done by taking a stone when first sawed and placing it on a revolving bed, then rubbing the face with a soft stone, water, and sand.

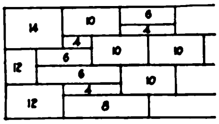


Fig. 311.—Broken ashlar made up of 4-6-8-10-12- and 14-in. pieces.

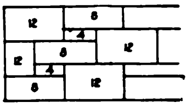


Fig. 312.—Broken ashlar made up of 4-8-12-in. pieces.

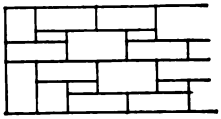
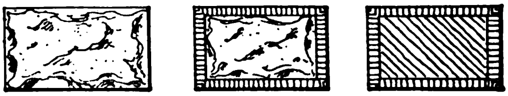


Fig. 313.—Random coursed ashlar.

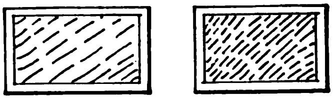


Rock face.

Rock face with tooled margin.

Broached with tooled margin.

Fig. 314.



Rough pointed with dressed margin.

Fine pointed with dressed margin.

Fig. 315.

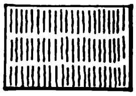
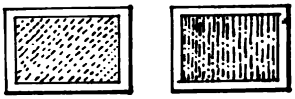


Fig. 316.—Drove.



Crandalled.

Patent-hammered.

Fig. 317.

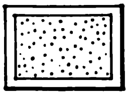


Fig. 318.—Bush-hammered.

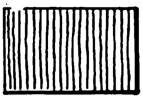


Fig. 319.—Tooled face, 6 to 10 cut.

Other forms of surface finish for stone ashlar are rough pointed (Fig. 315), fine pointed (Fig. 315), drove work (Fig. 316), crandalled (Fig. 317), patent hammered (Fig. 317), bush hammered (Fig. 318), etc.

Ashlar Finish for Concrete Blocks.—As concrete blocks are a cast product, they can have the face finished in almost any of the surface finishes used for stone work. Herein is one of the great objections to cast concrete as ashlar. In stone work an individuality and interest in the wall surface comes in that no two stones are alike, while in concrete each piece is like its neighbor making a rather monotonous effect.

Finish on Terra Cotta Ashlar.—In the making of terra cotta, a variety of finishes can be had in the surface itself and also in the glaze and color. At first terra cotta was only made in one color, which was the natural red color of the burnt clay; now it can be secured in almost any color or combination of colors and effects that may be desired.

Painting of Ashlar Work.—When stone or granite is used for ashlar or for trimmings, it should be painted on the back and on the edges to within 1 in. of the face with a black waterproof paint to prevent discoloration from cement and moisture.

Brick Walls Faced with Cement Blocks.—In addition to the use of stone, granite, or terra cotta for ashlar, a cast cement block in imitation of stone is also often used. It has the advantage over stone in that molded and ornamented pieces can be produced at a less expense than the same work could be cut in stone. It does not, however, make as interesting a wall from an architectural standpoint as stone, granite, or terra cotta.

197*d*. Damp Proofing of Walls.—All masonry walls above grade that are to be plastered on the inside should be given a coat of damp proofing, so that the moisture will not come through and stain the plaster. This precaution is not so necessary if the walls are to be furred and lathed on the inside before being plastered.

197*e*. Furring.—Furring for interior walls to be plastered can be done by ⅞ × 2-in. wood furring strips set vertically to which the wood lath are nailed to receive the plaster; or by a 2-in. tile furring scored for plaster; or by V-shaped metal furring to which the metal lath are wired.

197*f*. Brick and Tile Walls.—In late years walls have been erected in residences and country clubs made of hollow burnt clay tile with a brick veneer facing. This gives a light wall with an air space and an inside surface that can be plastered on direct. In this type of construction a narrow course of tile should be used about every third course so as to permit the brick to enter tnto the wall and form a bond.

197*g*. Tile and Plaster Walls.—Perhaps one of the cheapest masonry walls that can be built for small buildings is a tile wall plastered. The tile should be scored both sides so that both the exterior and interior plaster will form a good bond. Buildings of this type, two stories or more in height, should be erected in the skeleton form of construction so that the tile will be used only as a filler. Tile for such walls should be at least 12 in. thick and laid vertically so as to develop its full strength. Lintels over windows and door openings can be formed by means of tile arches, or the tile work can be carried on steel lintel angles. A variety of effects in color and texture can be obtained in the plastering of the outside walls. Tile in walls to be plastered should be laid with broken joints similar to brick work so as to avoid long vertical cracks forming in the plaster. If the wall is to have box frame windows, care must be taken to secure special tile shapes to receive the weight box and also to form a 1-in. wind break at the head of the openings. The inside trim can be secured by nailing into the joints between the tile.

197*h*. Frame Walls.—The most common form of wall throughout this country is the wood frame wall constructed with 2-in. studs, sheathing, and clapboard or shingles, and plastered on the inside. The studs are 2 × 4, 2 × 6, or 2 × 8 in., depending upon their length and the load to be carried. These studs are spaced either 12 or 16 in. on centers which is determined by the length of the lath. On the outside of the studs is nailed the sheathing which is ⅞ in. thick, matched and dressed on one side; then a layer of paper is put on; and finally the clapboards or shingles. On the inside are the lath and over this the plaster. A 2-in. plate, the width of the studs, is nailed to the top to provide bearing for the rafters. At the bottom a plate is required on top of the joist to form a bearing for the studs (see Fig. 320). Sometimes, *however, the studs are* extended down to the sill under the joist as shown in Fig. 321.

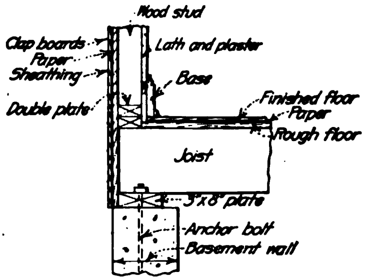


FIG. 320.—Detail showing studs resting on plate on top of joist.

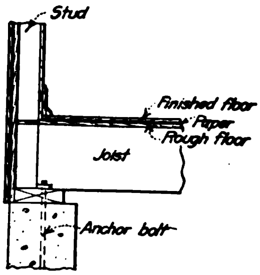


FIG. 321.—Detail showing studs resting on wall plate.

Studding.—Formerly a great deal of pine was used for studding, but owing to the scarcity and high cost of pine, hemlock and spruce have taken its place. Material used for studding should be clear and free from shakes and large knots.

Sheathing.—Sheathing is now made entirely from hemlock or spruce. Sheathing should be nailed to each stud with two eight penny nails. To give additional bracing to the house, sheathing is very often nailed on diagonally.

Building Paper.—The use of building paper between the sheathing and the clapboards or shingles is very desirable as the wood in the wall shrinks which forms cracks through which the wind finds its way. Building or sheathing paper should be tough, elastic, and impenetrable to moisture or air. A tar paper is not recommended as the oil in the paper soon evaporates and leaves the paper very brittle and soft. Paper is usually put on horizontally with at least a 2 or 3 -in. lap. If additional protection is required, a sheathing quilt can be used. This is somewhat more expensive.

Clapboard or Siding.—Siding is usually of two kinds—beveled and drop siding (see Fig. 322). Drop siding is often molded as shown. As beveled siding is cut with a saw from the circumference to the center it is a quarter-sawed piece of lumber and hence shrinks very little after it is in use. Drop siding is a plain sawed material and hence will shrink. The most durable material for siding or clapboard is cypress or redwood. Soft pine has been used a great deal but owing to the scarcity of the material it has gone almost out of use. Clear spruce is also used, but it is not so good as pine or cypress. Siding is sometimes nailed directly to the stud without a sheathing, but this is not desirable as it does not give the building secure enough bracing nor does it make it warm enough in the winter. A priming coat of paint should always be given the siding as soon as it is finished, as this will keep the sun from warping it and in a measure prevent shrinkage.

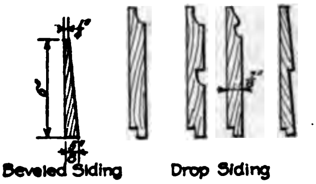


Fig. 322.

Wall Shingles.—Shingles are often used on vertical exterior walls, sometimes as a matter of economy but generally to produce an architectural effect. Shingles make a warmer wall covering than siding as they are three thicknesses, while siding is only one. Shingles on wall surfaces are laid the same as for roof surfaces. Shingles should always be dipped in creosote stain before they are used. To produce a rustic effect a long hand-made shingle called a ***shake*** is used. These can only be obtained in certain localities.

197*i* Wood and Plaster Walls.—In wood and plaster walls the studs, sheathing, and paper are used the same as above described for frame walls. The walls are then prepared for plastering by the use of furring and lath. If wood furring strips are used, they are generally made of ⅞ × 2-in. material, 12 or 16 in. on centers, and nailed on vertically. The wood lath are nailed over this furring, the same as for interior plastering, and then the surface is plastered.

197*j*. Brick Veneer Walls.—Wood and brick walls, or brick veneer walls as they are called, are quite common for dwellings. They have an advantage in that they give the appearance of a brick building at a very small expense. A lower rate of insurance can also be secured on this type of construction. If properly constructed, they make a very warm building. The brick is laid as a 4-in. facing 1 in. away from the sheathing, so as to produce an air space. The brick in veneered buildings are held to the frame work by means of metal ties placed on every other brick in every fourth or fifth course. Brick work over window or door openings should be carried by means of small lintel angles.

197*k*. Sheet Metal Walls.—For sheet metal walls, what is known as corrugated siding is used. This siding is made in sheets with ⅝, 1¼, 2, 2½, 3, and 5-in. size corrugations and in length of 5 to 12 ft. This siding is set vertically with a 1-in. lap at the bottom and one corrugation at the side. Siding can be secured in black, painted, or galvanized, and for special work a rustless siding is made by immersing the metal in an asphaltic compound and then covering the surface with a covering of pure asbestos felt laid over the hot asphalt and forced into it under pressure. This forms a sheet that is gas and fume proof. Corrugated metal siding can be used over a wood or steel frame work as the case may require. If nailed to wood, the nails should be driven in the trough of each alternate corrugation about 2 in. above the lower end of the sheet which will be 1 in. above the top end of the under sheet. The side lap, unless very long sheets are used, need not be nailed. If the siding is attached to a sheet frame work, then special clips are used and the siding screwed or bolted to these clips.

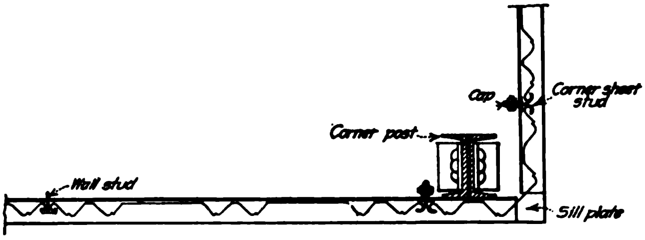


Fig. 323.—Corner plan showing patent molded steel walls.

A patent interlocking molded siding manufactured by the C. D. Pruden Company of Baltimore is also used extensively for quick and light factory and shop building. This siding is made of standard gage galvanized steel sheets 2 ft. wide by 8, 9, 10, and 12 ft. long. Fig. 323 shows a detail plan giving a general idea of this type of construction.

198. Party Walls.—A party wall is a dividing wall used or intended to be used by both of the adjoining property owners. It is generally centered on the lot line. Before a party wall is constructed, a definite written agreement should be made between the two property owners defining very clearly the rights of each to the use of the wall; the thickness, height, and depth that the wall is to be constructed; and the right to underpin and to increase its height. It is customary for the owner who builds first to pay for the entire cost of the wall and then when the adjoining property owner decides to build, to have him pay the first owner one-half of the cost of the wall, this cost being based on the cost of labor and material at the time the second owner decided to make use of the wall. Party walls are made about the same thickness as the enclosing walls. Some city ordinances require these walls to be 4 in. thicker than enclosing walls, while others permit them to be constructed 4 in. thinner. The party wall has the advantage over the line wall in that it permits of a balanced footing, saves ground space, and is more economical, as both parties share the cost of same. Openings in party walls should have thorough fire protection to prevent the fire from going from one building into the other. It is customary to have self-closing fire doors on each side of the wall. These doors should have fusible links and close by gravity or by weight.

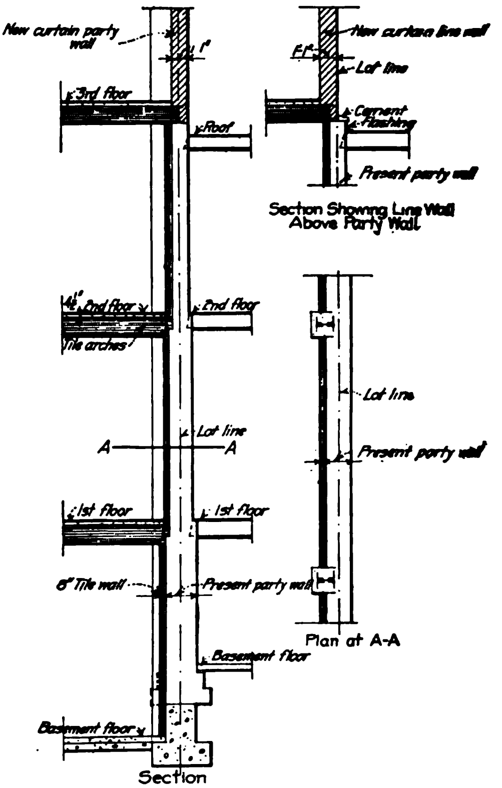


Fig. 324.—Treatment of existing party wall.

In the case of an existing party wall in which the new building is to have the same or less basement level, and in which the height of the new building is not to exceed the one on the other side of the party wall, the problem is a very simple one. If the party wall is comparatively new, it may not need anything more than patching up in places, so that the new plastering can be done directly on the wall; or if the wall be a trifle uneven it can be furred, lathed, and plastered; or a new tile wall can be erected against the old wall to receive the plastering. Frequently the basement of the new building is at a lower depth than the wall, in which case it is necessary to underpin the party wall and carry it down to the necessary level. If the new skeleton building is to extend up above the present building, it may be necessary to cut chases in the old wall to receive the wall columns; then the wall may remain as it stands and a new tile partition is built parallel to the old wall to receive the plaster. The additional height may then be cared for as a curtain wall, either as a line or party wall (see Fig. 324).

199. Curtain Walls.—In buildings of the skeleton type of construction the outer masonry walls are supported in each story by means of spandrel girders and therefore only carry their own weight.

On alley and lot line exposures the curtain walls should be constructed of 12 in. of brick to secure the proper fire protection. In street walls where large windows occur the spandrel below the window may be constructed of 12 in. of brick, or 4-in. brick facing backed with 8-in. fire clay tile, or 4-in. terra cotta backed with 8 in. of brick or tile. Spandrels below windows are also constructed of reinforced concrete. In such cases a minimum thickness of 8 in. of concrete should be used. These spandrels are often reinforced to act as the upper part of the wall beam, but the usual method is to consider this portion separate from the beam and merely reinforce with small rods or wire fabric so as to prevent cracks. If this is done, the spandrels may be put in after the main structural parts have been cast, which saves time in the erection of the building and allows the use of more care in obtaining a neat finish on the spandrel walls. Reinforced concrete is well adapted to construction of walls that require considerable strength but for ordinary curtain walls and for spandrels below windows they are more expensive than brick on account of the cost of forms.

200. Walls for Cold Storage Buildings.—In the construction of walls for cold storage buildings, the ability to resist moisture and the transmission of heat is of the greatest importance. The insulating value of the structural wall need not be considered, as this is taken care of by cork or lith linings. If permitted by the city ordinances, perhaps the best method for constructing exterior walls is with brick and hollow tile, as shown in Fig. 325. A hard vitrified brick is recommended on account of its ability to resist moisture. These brick should be bonded into the tile as shown. It will be noted that the exterior wall is constructed entirely separate from the interior frame work, and are tied together by means of galvanized anchors. In wall-bearing types of buildings, an insulation can be effected by carrying the insulating materials around the ends of the girders (see Fig. 326). In constructions of this type the flooring should stop against the wall insulation as shown. Another method of masonry wall construction is a double brick wall with the space between filled with granulated cork (see Fig. 327). In this case, wall ties are also necessary to hold the structure together.

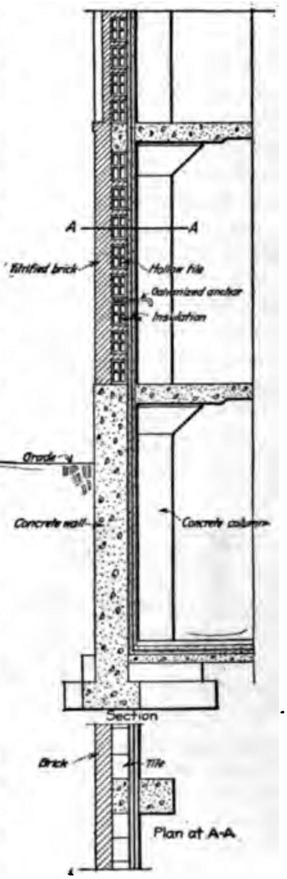


Fig. 325.—Details of brick and tile cold storage walls.

201. Wall Insulation and Partition Deadening.—For the purpose of insulating walls to keep out the cold, and for the deadening of partitions between apartments or studios, the best material now in use that can be secured at a reasonable price is a quilt made of cured eel grass stitched between two layers of tough paper. This quilt is manufactured by the Samuel Cabot Co., Boston, Mass. As the blades of grass cross each other at every angle, they form small dead air cells which prevent the air from circulating so that heat conduction is prevented and sound waves deadened. This quilt is made 3 ft. wide and in rolls of 250 sq. ft. each. It is made in single, double, and triple-ply. The single-ply is sufficient for lining houses, double-ply is used for sound deadening, and triple-ply is used for cold storage and other work where unusual conditions prevail. This quilt is also made with waterproof, and with asbestos paper. Figs. 328 to 333 inclusive show various methods of using quilt as deadener and for insulation.

202. Vault Construction.

202a. Vaults in Fireproof Buildings.—In modern fireproof buildings of the skeleton type, the vaults act as additional fire protection only and the walls are made of but a single thickness and at other times of two thicknesses with an air space between. These walls should start on the floor construction and extend to the ceiling.

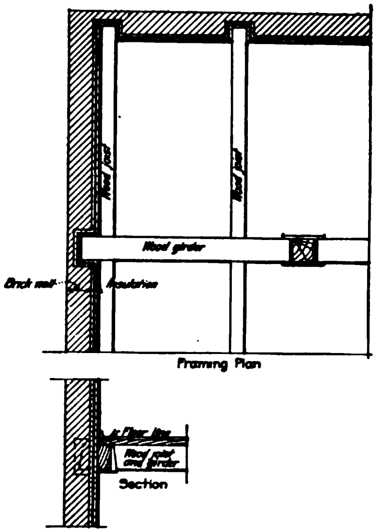


FIG. 326.—Wall bearing type of construction showing beams insulated.

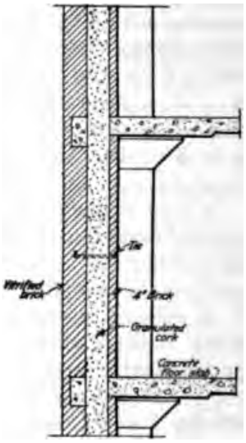


FIG. 327.—Double brick wall with space filled with granulated cork.

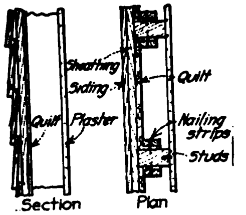


FIG. 328.—Wall insulation with one layer of quilt.

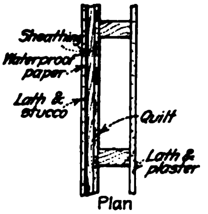


FIG. 329.—Wall insulation with one layer of quilt on studs for outside plaster walls.

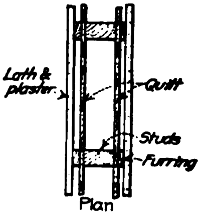


FIG. 330.—Partition deadening with two layers of quilt-wood construction.

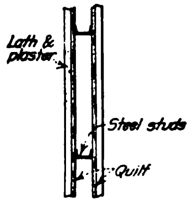


FIG. 331.—Partitions deadened with two layers of quilt-fireproof construction.

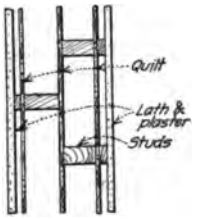


FIG. 332.—Partition deadened with three layers of quilt.

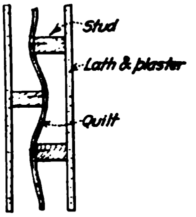


FIG. 333.—Partition deadened—quilt nailed to staggered studs.

202b. Vaults in Mill, Slow-burning, and Ordinary Constructed Buildings.—As the fire hazard increases it becomes more necessary to protect the contents of the vault. Thus in buildings of this class, the walls, floors, and ceilings of the vault are made of heavy masonry, and the vault walls rest on foundations independent of the building, so that in case the building is destroyed by fire the vault will remain standing intact. Walls for vaults of this type should be constructed of either brick or concrete, built so as to form an air space, or the walls and ceiling should be lined on the inside with hollow tile. It is very necessary to have a strong ceiling over these vaults to withstand any damage that may be caused by falling timbers or adjoining brick walls.

In recent years a great many vaults have been built to store small quantities of oils, varnishes, etc. These vaults should have self-closing fire doors and have the door sills at least 6 in. above the floor so that in case of a leak in a barrel the varnish or oil will not run out and permit the fire to travel back into the vault. Vaults of this kind should also have vents when possible; care must be taken to protect these vents with self-closing louvres.

202c. Bank and Safety Deposit Vaults.—Vaults in banks and safety deposit companies should have burglar proof features as well as being constructed to withstand fire. When possible it is well to have the vault stand free from adjoining walls so that when the watchman makes his rounds he can inspect all sides of it. The walls should be constructed of brick with steel linings or of concrete heavily reinforced with steel. In some cases, walls are not alone constructed of reinforced concrete but also have steel linings. Steel linings for vaults are made of two or more thicknesses of chrome steel about ¼ in. thick and erected with lap joints. Walls for ordinary small banks are now usually made of 12 in. of concrete reinforced with ¼-in. round steel wires, 2-in. mesh, one mesh set 1½ in. from the inside of the wall, and another mesh 1½ in. from the outer surface of the wall. The floor and ceiling of the vault should also be reinforced in a similar manner. A wall of this kind will require about 8 hr. to penetrate, which is the usual length of time set on the door time clock. Special 1-in. square bar reinforcements should be set in the wall at the hinge side of the vault door to properly carry the weight of the steel door. This reinforcement should be carried up and through the vault roof slab and turned down on the other side. To protect the contents of a vault from dampness, the walls are often lined with 4 in. of brick having an air space between the lining and the vault wall. This air space should be carefully ventilated.

PARTITIONS

By Frederick Johnck

203. Partitions in Mill, Slow-burning, and Fireproof Constructed Buildings.—Partitions or dividing walls in mill, slow-burning, and fireproof-constructed buildings are not generally required to support a load, but to serve the purpose of dividing a space into rooms. Therefore, such partitions need have only sufficient strength to carry their own weight and be rigid enough to withstand ordinary horizontal thrusts. The materials employed should be light, incombustible, and poor conductors of heat. If the space to be enclosed is to be fireproof, the doors and windows in the partitions should be self-closing and be made of incombustible material, glazed with wire glass. For ordinary office partitions, dividing the office from the corridor or the reception room, the lower 3½ ft. is usually made of an incombustible material and the upper part of a fixed wood and glass partition, with movable transoms to permit ventilation of the rooms.

203a. Brick Partitions.—Partitions around elevators and stair shafts in slow-burning and mill constructed buildings, and partitions around boiler room and coal storage space in all commercial types of buildings, are usually constructed of brick. When walls of this material are used to enclose the elevator shaft in ordinary mill and slow-burning buildings, they form a means of support for the overhead elevator machinery. When used to enclose stairways in a building of the slow-burning type, they form a safe means of exit in case of fire. All openings in these partitions should be protected with incombustible doors or windows. Brick partitions around boiler rooms and cold storage spaces prevent the spreading of fires that often occur in such places. Partitions constructed of brick are also used for dividing large buildings into small areas to reduce fire risks, also round shipping platforms to withstand the hard usage from trucks and boxes. Openings in walls enclosing shipping platforms and in walls dividing the building into smaller areas should be carefully protected with steel jamb guards. Partitions constructed of brick should be at least 12 in. thick. Brick for partition work should be good, hard-burned, kiln-run common brick, laid in lime and cement mortar.

203b. Concrete Partitions.—Partitions of stone concrete of the same thickness as those of brick are sometimes used in place of brick, but the cost of form work often brings the cost of the wall above that of brick. Concrete for partitions should be mixed in the proportion of 1 part cement, 3 parts sand, and 5 parts stone—stone to be no larger than will pass through a ¾-in. ring. If concrete is used for partitions around very large coal storage spaces, it is often necessary to reinforce same with the proper amount of steel. In certain localities a hollow cast-concrete block is used which makes a fairly satisfactory wall. These blocks are generally made by a local company, so that in competition with other materials, they can be sold for less money on account of the saving in freight. They have the advantage over solid concrete walls in that they can be taken down and changes made in the arrangement of the room with less difficulty.

Solid concrete partition walls may be made 3 or 4 in. thick if reinforced. Extra rods should be placed near the edges of all openings, and rods should project into the floor and ceiling for anchorage. It is usually convenient to pour the concrete after the floor is laid, and, where partitions are not located under beams, this may be done by leaving a slot in the floor at the proper place. A solid concrete wall 4 in. in thickness makes a very efficient fire resisting partition, but is heavy and difficult to install. For this reason metal lath and plaster, tile, and plaster blocks are generally used in preference to concrete.

203c. Tile Partitions.—Partitions of hollow tile made of burnt clay are generally used around offices and rooms in slow-burning and mill constructed buildings, and also around stairs and elevator shafts in fireproof buildings. Hollow tile for partition work of this kind is very desirable and no better material can be had. The tile block is usually 12 x 12 in. square and 3, 4, 6, 8, or 12 in. thick. Tile to be used in partitions to be plastered is scored. The 3-in. tile is used in office and room partitions up to 12 ft. in height. Partitions more than 12 ft. high, and partitions around stairs and elevator shafts, are usually 4 or 6 in. in thickness. The larger tile are generally used in long dividing walls. Tile for partition work should be a good hard-burned clay tile, laid vertically so as to develop full strength and carefully wedged in at the ceiling. For partitions that are to be plastered a tile should be selected that has not been warped in burning, so as to permit of an even coat of plaster over the entire surface. Care should also be taken in selecting tile that will not cause plaster stains or pop marks. To avoid this it is well to secure a material from a plant that has been in operation for some time and observing the material after it has been in use a year or more. On account of changes in offices, tile partitions are now often laid directly on top of the wood floor. Wood bucks at doors and other openings are required. These bucks are sometimes nailed into the joints or wood strips bedded in the joints, or they are made wider than the partitions and channeled out to receive the tile, as shown in Fig. 334. Necessary furring strips nailed into the joints to receive the wood base, picture mold, and chair rail should be set before the plastering is applied.

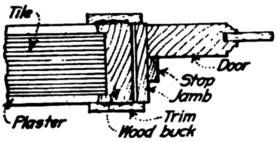


Fig. 334.

The weights per square foot of standard tile partitions are given in the accompanying table.

Weight of Tile Partitions

Size of tile (in.)	Weight per square foot (pounds)	Weight per square foot plastered both sides (pounds)
3	13	21
4	15	23
6	22	30
8	28	36
10	34	42
12	35	43

As a general rule, a hard-burned tile weighs less than a porous or semi-porous tile, as the thickness of the material can be made less. Mortar for tile work should be composed of 1 part Portland cement to 3 parts clean, sharp sand —lime not to exceed 10% by volume.

203d. Gypsum Block Partitions.—In recent years a partition made of calcined gypsum mixed with fiber and molded into a block shape has come greatly into use. These blocks are made solid or hollow, 12 in. wide, 30 in. long, and 3, 4, 5, 6, and 8 in. thick. They are laid in regular courses breaking joints as in brick work and are set in lime mortar. The gypsum block partition is not as fireproof nor will it stand as great a horizontal thrust as a tile partition, but it has an advantage of being lighter in weight and also an advantage in that openings can be cut in the partition with a saw. The cost of this partition is also a trifle less than tile. The usual wood bucks at openings and grounds for trim are required the same as for tile partitions.

The weight per square foot of gypsum block partitions is given in the following table.

WEIGHT OF GYPSUM BLOCK PARTITIONS

Size of block	Weight per square foot (pounds)	Weight per square foot plastered both sides (pounds)
3 in. hollow	9.9	17.9
3 in. solid	12.4	20.4
4 in. hollow	13.0	21.0
5 in. hollow	15.6	23.6
6 in. hollow	16.6	24.6
8 in. hollow	22.4	30.4

203e. Expanded Metal and Plaster Partitions.—A thin partition of plaster applied to metal lath, making a solid partition about 2 in. thick, is often used around small offices and toilet rooms in factories of slow-burning or mill construction. This type of partition is light in weight and a trifle less expensive than any form of tile. The difficulty of cutting openings makes them rather undesirable in partitions that need to be changed often. The metal and lath partition is usually constructed of vertical 1-in. steel channels set 12 or 16 in. on centers, bent and punched at the ends for nailing to floor and at ceiling. At the openings a 1 × 1-in. angle, punched so that the wood buck can be screwed on, is used. Over these studs a metal lath is stretched and wired to the studding with galvanized wire. Grounds are secured to the lath by means of staples. Plastering is first a scratch coat on one side, a brown coat on each side, and then the white coat on each side for finishing. The weight of this partition is about 17 lb. per sq. ft.

204. Partitions in Non-fireproof Buildings.—Partitions or dividing walls in non-fireproof buildings, are often required to support a light load, so as to reduce the span of the joists above.

204a. Wood and Plaster Partitions.—For such buildings as residences and small stores, hotels, offices, etc., where the question of fire risks is not a strong factor, the most common form of partition is the wood stud, lath, and plaster partition. The studs are either 2 × 4 in. or 2 × 6 in., spaced 12 or 16 in. on centers. On these studs are nailed wood lath, and over the lath the plaster is applied. Lath made of pine, spruce, or hemlock are used. They should be straight grained and well seasoned. The regular size of lath is ¼ × 1½ in. and 4 ft. long. This length regulates the spacing of the studs. The lath are nailed on in parallel rows about ¼ in. apart with 3 penny nails to enable the plaster to form a key. To prevent cracking the lath are laid with broken joints at every seventh or tenth lath. Over the lath the plaster is applied either in two or three coats, as may be required. The necessary grounds to receive the trim should be nailed on before the plastering is done. The weight per square foot of wood and plaster partitions is given in the table on p. 622.

WEIGHT OF WOOD AND PLASTER PARTITIONS

Size of studs (inches)	Spacing of studs (inches)	Weight of partition per square foot, plastered both sides (pounds)
2 × 4	12	18
2 × 4	16	17
2 × 6	12	19
2 × 6	16	18

204*b*. Expanded Metal and Plaster Partitions.—Expanded metal and plaster partitions are sometimes used in non-fireproof buildings, constructed as described in Art. 203*e*. Metal lath over wood studs are also sometimes used. It is seldom that any special advantage is gained by the use of such partitions in non-fireproof buildings.

204*c*. Sound Deadeners for Partitions.—To prevent the sounds from passing through the building by the full contact of the partitions with the floor construction, metal saddles with felt cushions are made to carry the partitions. In the case of wood partitions the bottom plate rests in the cradle, but with tile partitions a wood buck is first laid to receive the tile.

204*d*. Wall Board Partitions.—Wall board for partition work is a built-up wood fiber, bonded together with a moisture-resisting cement. It is approximately 5/16 in. thick, 32 and 48 in. in width and comes in lengths from 6 to 12 ft. It can be painted or treated with calcimine, but it cannot be papered.

204*e*. Plaster Board.—Plaster board is a fire resisting material, composed of alternate layers of calcined gypsum and fibrous felts. It is nailed direct to the stud and plastered over. It comes in ¼, ⅜, and ½ in. thickness and in sheets 32 × 36 in. It can also be used in constructing 2-in. solid plaster partitions in place of metal lath.

204*f*. Lith Partitions.—A thin sound-proof partition can be made of 2 × 4-in. wood studding, set sideways, and the space between built up with lith. On each side of this core, the metal lath and plaster are applied. Lith board is made 18 in. wide and 48 in. long. It contains 80 % of rock wool and 20 % of flax fibers, two materials of high insulating value.

205. Partitions in Cold Storage Buildings.—The essential thing to be considered in the construction of partitions in cold storage buildings is insulation. The construction is, therefore, usually determined by the amount of insulation required.

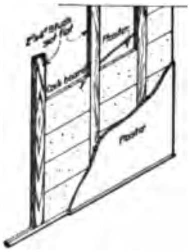


FIG. 335.—2 × 4-in. stud partition with cork filler.

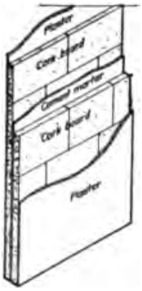


FIG. 336.—Double cork board partition.

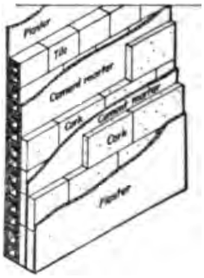


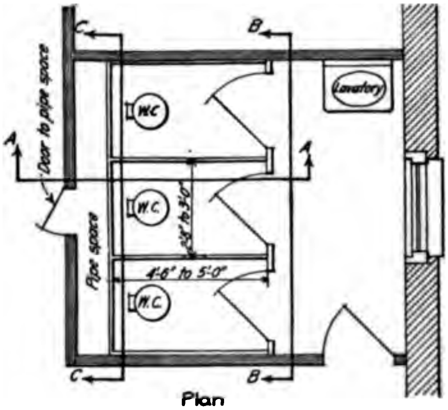
FIG. 337.—Tile and cork board partition.

Fig. 335 shows a partition constructed of 2 × 4-in. wood studs set flat, the space from stud to stud being filled with 2-in. cork boards. Both sides of this core are lathed with galvanized wire lath, and plastered. If the plastering is not desired, matched and dressed boards can be used; in which case a waterproof paper should be used between the cork and the boards. The cork boards should also have an asphalt joint at each stud to prevent the passage of air. Fig. 336 shows a double cork-board partition, the boards cemented together with cement mortar.

ides of this partition are also lathed and plastered. In cheaper types of construction the l lath is omitted and the plastering is applied direct on the cork. These partitions can be ed to a height of 12 to 14 ft.

'hen tile is used for partitions, it is customary to · one side and on the other side to use a cement · to hold the cork boards. Over the cork another f plaster is applied. Often it is necessary to use yers of 2-in. cork, as shown in Fig. 337. This on is recommended when fireproof construction iired. Portland cement mortar should be used to ie cork to the tile.

ı the erection of partitions in cold storage buildiat are to receive salt meats, care must be taken as little iron as possible, as the salt will soon rust t it away. Copper nails, anchors, etc., and bronze is hardware should be used for this kind of work.



Plan

FIG. 338.

!06. Partition Finishes.—The most com- and satisfactory finish for partitions is ·r finished with either two or three coats, ɜ case may require. Patent paster is now ieral use and instructions for applying this ven by all manufacturers.

· wainscot work in public halls, corridors, and toilet rooms, no better material can be secured than marble, thick. Marble should be set with fine plaster of Paris joints and securely anchored into the partitions ietal anchors. For wainscot in kitchens, bath rooms, etc., a white glazed tile is used a great deal. These

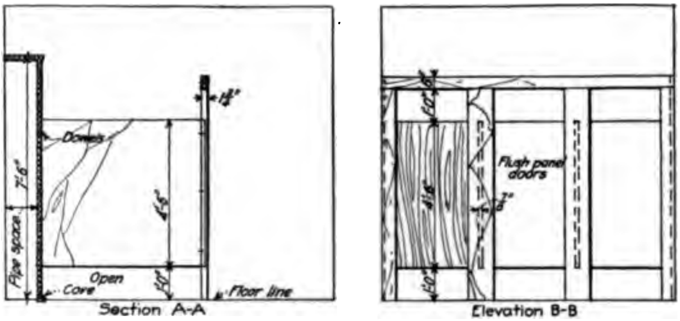


FIG. 339.—Details of marble and slate toilet stall partitions.

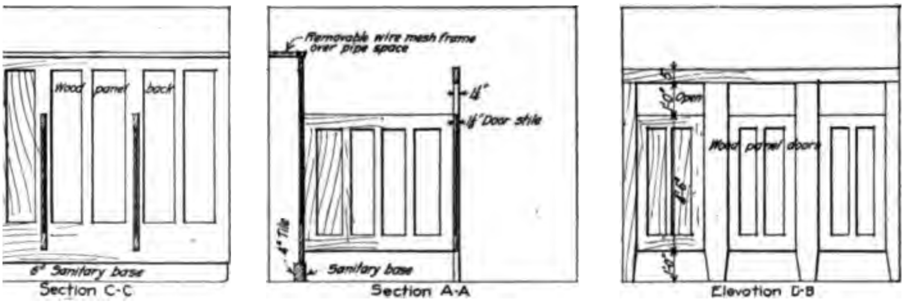


FIG. 340.—Details of wood panel toilet room stall partition.

ɜ rectangular in shape. Special shapers for caps, corners, angles, and cove base are made for this work. ɜ economical material for wainscot to take the place of tile is Keene's cement. This cement can be jointed inted with an enamel finish so as to produce a very serviceable surface. In places that require the walls to be scrubbed, an elastic sanitary composition similar to that used for floors is often used. As this material does not require paint, it can be cleaned with a scrub brush and washing powder.

207. Toilet Room Partitions.—The main consideration in the construction of toilet room partitions is to secure a serviceable material, and to so design the partitions as to make them as sanitary as possible. The most desirable material and also the most expensive is marble. For this purpose the white Italian or the Tennessee grey is more generally used. A more economical material, and one used a great deal in industrial work, is black slate. Slate can be secured in the same thickness and size slabs as marble.

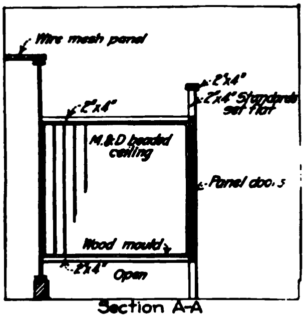


Fig. 341.—Detail of 2 × 4 and beaded ceiling partition for toilet room stalls.

In the construction of marble and slate toilet room partitions, the front stiles (1¼ in. thick) should extend to the floor. The back and end partitions should also extend to the floor and have a cove marble base so as to make the corners easy to clean. The dividing partition should be set 10 or 12 in. above the floor and should not be as high as the front or back. The backs for water closet stalls should be set away from the wall so as to allow ample pipe space, and should extend up at least 7 ft. 6 in., so as to conceal the flush tanks (see Fig. 338). Over the pipe space should be set a removable shelf, ⅞ in. thick, so that the space can be closed up and kept clean. The marble and slate for partitions should be held together with dowels so as to avoid as much metal work as possible. In certain classes of industrial work, the front doors and stiles are omitted and the dividing partitions are made very low so as to give the attendant complete supervision of the room. In detail of this kind, pipe standards are necessary as a frame work to hold the marble or slate together. Wood paneled partitions made of oak or birch, and varnished, make a good partition for less expensive grades of buildings. Where wood is used for partition work, the backs should be set on a hollow-tile base—the hollow tile to form a back for the sanitary cove base.

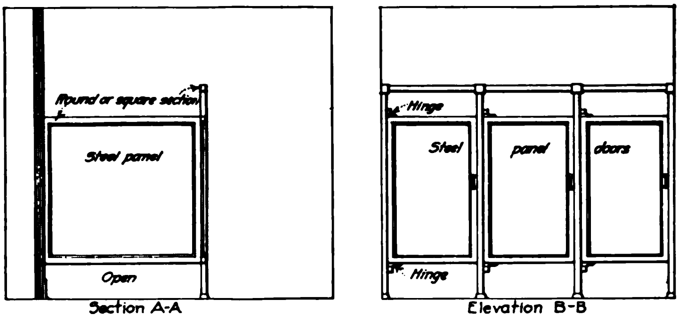


Fig. 342.—Details of metal toilet room stall partition.

In recent years a partition has been made of sheet steel and used a great deal in factory work. This type of partition should always be carefully painted so that it will not rust. The cheapest partition for toilet room stalls is the 2 × 4-in. stud partition filled with matched and beaded ceiling. Details of toilet room partitions are given in Figs. 338 to 342 inclusive.

CORNICES AND PARAPET WALLS

By Frederick Johnck

208. Cornices.—After the main walls of a building are erected, about the first item that receives the finished treatment is the cornice. The details given here are not so much to illustrate architectural design as to show the construction features of the various types of cornices and the manner of providing supports for the material used.

Fig. 343 illustrates an ordinary wood box cornice and the manner in which this type is constructed. The rafters are continued out over the building and lookouts are nailed to these so as to form nailing pieces to carry the wood soffits. In this type of work the sheet metal lining is carried up under the shingles as shown on the drawing.

In Fig. 344 is shown another form of wood cornice with a sheet metal hanging gutter. In this case the wood lookouts are cut in some ornamental form and nailed to the side of the roof rafters. The hanging gutter has the advantage over the box type in that it can be more easily replaced when it is rusted out.

Figs. 345 and 346 illustrate wood cornices on masonry walls. The rafters rest on and are nailed to a wood plate which is firmly anchored into the wall. Wood lookouts are built into the masonry and secured to the end of the rafters to form nailing blocks for the wood soffit. In Fig. 345 is shown a standing gutter, a type of gutter used a great deal in early colonial work. Nailing blocks should be built into the masonry so that the lower sections of the cornice or freeze can be properly secured in place. Wood for cornices should be white pine or cypress, and should be carefully painted with a priming coat as soon as the wood work is in place.

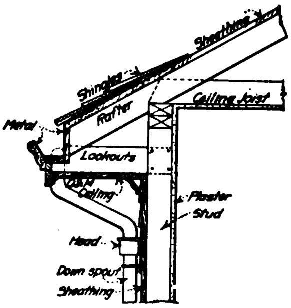


Fig. 343.—Wood cornice detail.

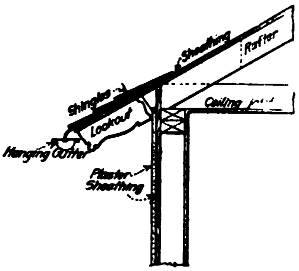


Fig. 344.—Wood cornice with hanging gutter.

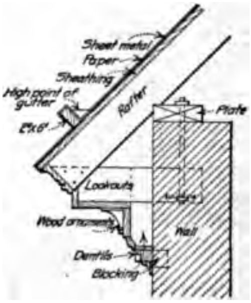


Fig. 345.—Detail of wood cornice with standing gutter on roof. (Cornice on residence at Roxborough, Pa.)

When it is not possible to afford a stone or terra cotta cornice, a sheet metal one is often used as illustrated in Fig. 347. These cornices are supported on wood lookouts built into the masonry. The top and end of the lookouts are sheathed as shown in the illustration to form a straight edge and also to secure proper nailing surface for the sheet metal. Additional reinforcements back of the moldings are sometimes necessary; these are made with galvanized or wrought iron strips as the case may require.

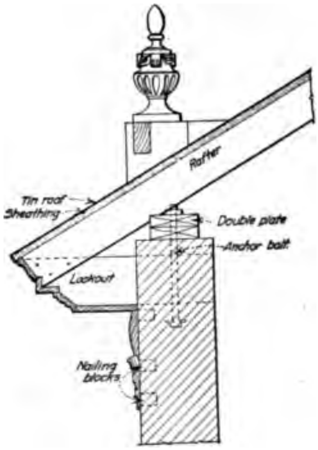


Fig. 346.—Detail of wood cornice on brick wall. (Cornice on Independence Hall, Philadelphia, Pa.)

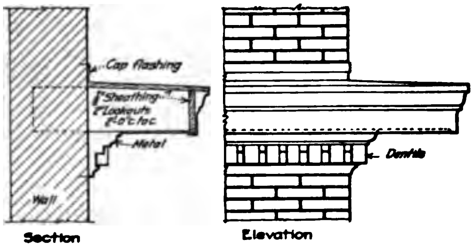


Fig. 347.—Details of sheet metal cornice.

Fig. 348 is an illustration of a terra cotta cornice used on reinforced concrete buildings and shows the means of anchorage of the terra cotta to the masonry and the lintel over the window to the shelf angle which is bolted into the concrete work. In terra cotta cornice work the brick should be built into the voids of the blocks as indicated in the details.

In Fig. 349 is shown a stone cornice and the manner in which the various blocks are secured in place by galvanized wrought-iron anchors. The back of all stone work should be painted to within 1 in. of the exposed edge with black waterproof paint to prevent moisture from the wall entering and discoloring the stone work.

When a terra cotta cornice has a greater projection than can be properly balanced on the wall, it should be carried by means of steel brackets or lookouts properly anchored into the masonry, as shown in Fig. 350. This figure also illustrates the method of securing terra cotta balusters in place. In the use of terra cotta for cornices, care must be taken in detailing the top joint so that the water will not enter the joint and freeze, causing the terra cotta to break.

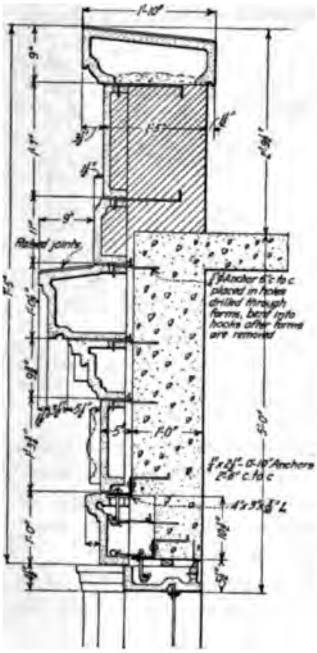

1'-10"
9"
1'-1"
1'-5"
1'-8"
11"
9"
Raked joints
1'-0½"
9½"
1'-3½"
5"
1'-0"
1'-0"
1'-9½"
5'-0"
⅝" ⌀ Anchor 6" c to c placed in holes drilled through forms, bent into hooks after forms are removed
⅛"x2½"-0-10" Anchors 2'-8" c. to c
4"x3½"x⅜" L
10½"
5½"

Fig. 348.—Terra cotta cornice on reinforced concrete building.

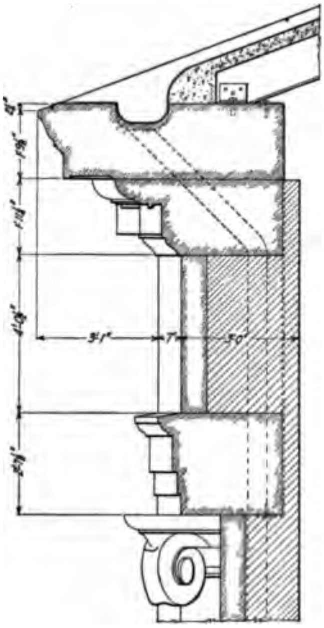


Fig. 349.

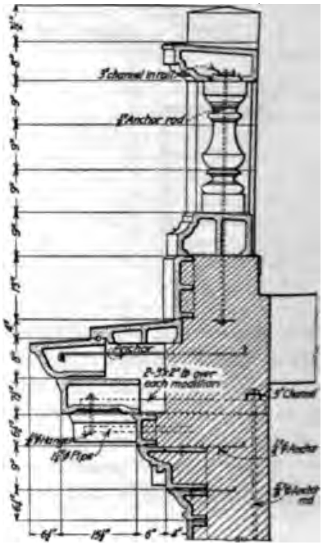


Fig. 350.

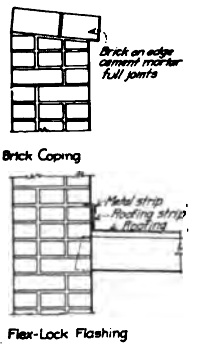


Fig. 351.

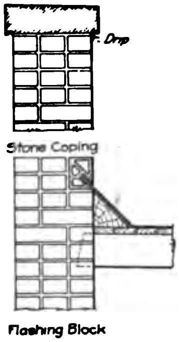


Fig. 352.

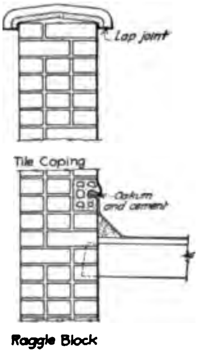


Fig. 353.

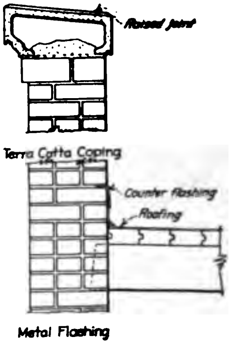


Fig. 354.

209. Parapet Walls.—The main points to be considered in the treatment of parapet walls are (1) the top finish or coping, (2) the treatment on roof side, and (3) the flashing. Fig. 351 shows a simple brick parapet wall with a brick coping and a metal strip for flashing. The brick for coping should be a hard vitrified brick and be laid in a full cement mortar joint. The metal strip, used for flashing just above the roof line, consists of a roofing-felt strip folded into a metal board and set into the brick joint. These metal strips are also secured into the brick work with galvanized bent hooks. The roofing is brought up under the roofing strip the same as under a regular cap flashing.

Fig. 352 illustrates a parapet wall with a stone coping and a raggle or flashing block above the roof to receive the flashing. The stone coping extends over the brick wall and is cut with a drip on the inside and outside. The flashing or raggle block is a hard burned clay block with a slot to receive the cap flashing, as illustrated. This detail also shows a splay block at the roof line so as to prevent the sharp turn of the roofing in the corner.

In Fig. 353 is shown a parapet wall with a salt glaze tile coping, and another form of raggle or flashing block. The tile coping is made with a hub so as to form a lap joint.

Fig. 354 illustrates a terra cotta coping for parapet walls and the ordinary cap flashing over the roofing. Cap flashing should be carefully painted on both sides before it is put in place.

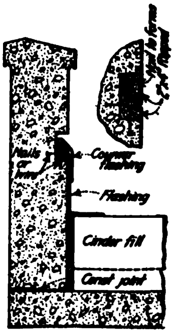


Fig. 355.

For the treatment of parapet walls on the roof side the best system is the use of vitrified brick, as common brick often disintegrates due to the moisture from snow being banked against it in winter. Parapet walls are also often treated on the roof side with a coat of asphaltum when the roof is laid. If this is done, they should receive a new coat every 5 yr. or so.

Concrete is also used for parapet wall construction in factory work. They may be constructed of 8 in. of reinforced concrete, or of 12 in. of plain concrete. For the proper flashing of concrete parapet walls the detail shown in Fig. 355 has proven satisfactory. A 2 × 4-in. piece of lumber is ripped on the diagonal and then placed in the forms at the desired height, the upper strip being securely nailed thereto, so as to insure its removal when the forms are taken down. The lower piece is just tacked to forms (from outside) with wire nails driven into it to anchor it to the concrete. The flashing and counterflashing are then placed in the same manner as for brick walls.

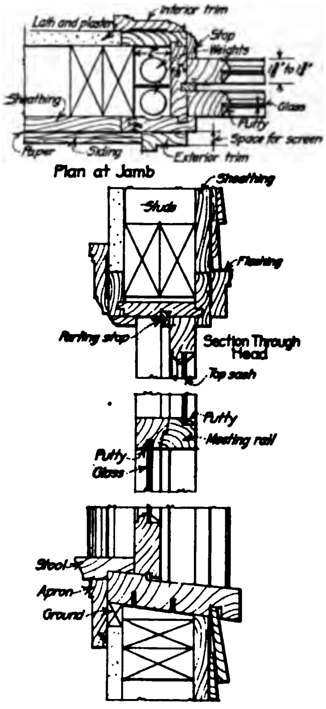


Fig. 356.—Detail of box frame window for frame walls.

WINDOWS

By Frederick Johnck

210. Wood Windows.—In Fig. 356 is illustrated a box frame for double hung sash to be used in frame buildings. The depth of the wall studs determines the width of the box. In this detail the exterior wall surface is shown as siding; if plaster is used it may be necessary to increase the width of the trim to receive the furring, lath, and plaster. In the construction of double hung windows, the pulley stile should be made of straight grained yellow pine, and the other parts of the frame of white pine or cypress. The sash vary in thickness from 1⅜ to 1¾ in. depending on the width of the window and the glass used in glazing. If plate glass is used, it is better to have the 1¾-in. thickness in the sash to carry the weight. The exterior trim over the top of the window should be flashed with metal flashing extending up under the siding as illustrated. At the bottom, the sill should be undercut to receive the siding or exterior covering so as to form a tight joint.

211. Casement Windows in Frame Walls.—In Fig. 357 is illustrated a detail of casement window with the sash arranged to swing out. When this detail is used the screens must be placed on the inside and the sash operated with hardware so designed that the sash can be opened without opening the screens. This detail also shows the inside of the jamb veneered to match the trim of the room. In Fig. 357 is also shown a sash detailed to swing in. This permits the screen to be placed on the outside, but requires the curtains to be secured directly to the sash instead of the trim as is the usual way. In detailing the sash for casement windows, it is better to set the glass in wood stops so that the glass will not shake out if the wind should slam the window shut.

212. Basement Windows in Masonry Walls.—This type of frame is often called a plank frame, and is perhaps the simplest type used in building construction. The jamb is made of 1¾ in. thick lumber, and the sash 1⅜ or 1¾ in. as may be required. The usual method to operate these sash is to hinge them at the top to swing in (see Fig. 358).

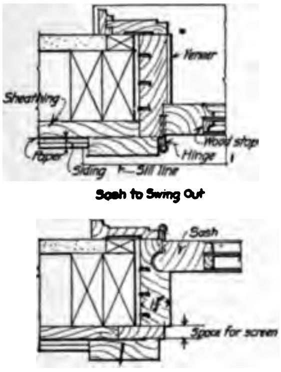
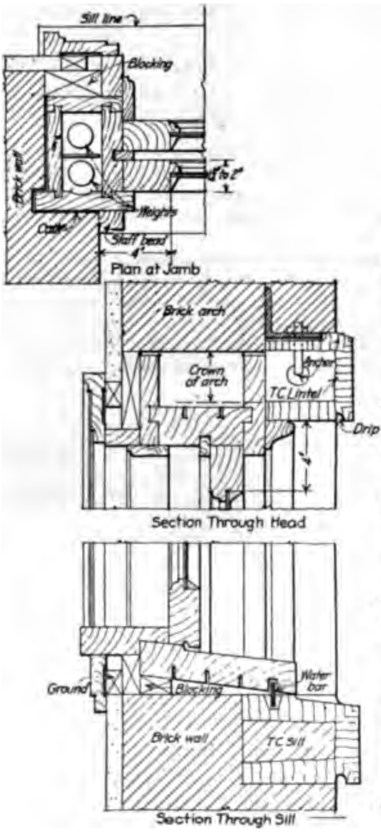
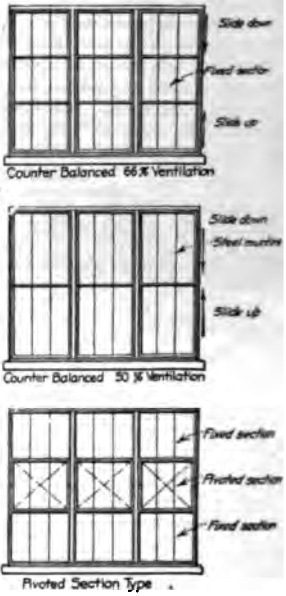


FIG. 357.—Casement window detail—frame wall.





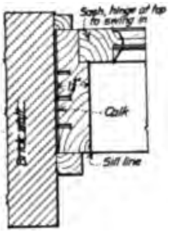


FIG. 358.—Jamb of basement window in brick wall.

FIG. 359.—Details of box frame windows in masonry walls.

FIG. 360.—Details of steel windows.

213. Box Frames in Masonry Walls.—This frame differs in construction from the box frame in frame walls in that it is a complete unit set into a masonry wall and built in as the wall is constructed. These frames should be carefully calked with oakum so as to make a good air-tight job. A water bar is used in the sill so that the rain will not drive in. This water bar should be cemented into the raggle of the stone or terra cotta sill. On the inside it is necessary to block out the frame to the full thickness of the wall so as to form a nailing support for the trim (see Fig. 359).

214. Steel Windows.—Windows made of rolled steel sections have come into great use for factory and warehouse work. As the sash sections are very small, these windows permit the maximum amount of light to pass through. They are made in the counterbalanced vertically gliding types, permitting 50% ventilation; in the triple sash, permitting 66% ventilation; and in the pivoted type which is the most common. The question of being able to wash the sash on the outside should be given great consideration in the selection of the type to be used. It is also well to use the glass in as large a section as possible so as to reduce the labor of washing the windows. When it is required to use wire glass in steel sash in walls exposed to fire risks, the glass should be set in special approved glazing angles as required by the Insurance Underwriters.

215. Hollow Metal Windows.—Hollow metal windows are used to secure proper fire protection on alley or lot line walls (see Fig. 361). They are made of 22 and 24-gage galvanized iron, or of 20 oz. copper, and glazed with wire glass. The glass rabbets should be ¾ in. deep. The frame and the sash should be made with as few parts as possible, and should comply with all the rules of the Insurance Underwriters. When mullions are required, they can be made with a 5-in. I-beam enclosed with at least 2 in. of concrete or other fireproof material. These I-beams should be securely fastened into the masonry at the top and bottom, but proper allowance should be made for expansion and contraction when heated. Hollow metal windows are made double hung, both sash pivoted at sides, and top sash pivoted and bottom sash fixed. Fig. 361 shows the method for trimming hollow metal windows on the inside of the wall.

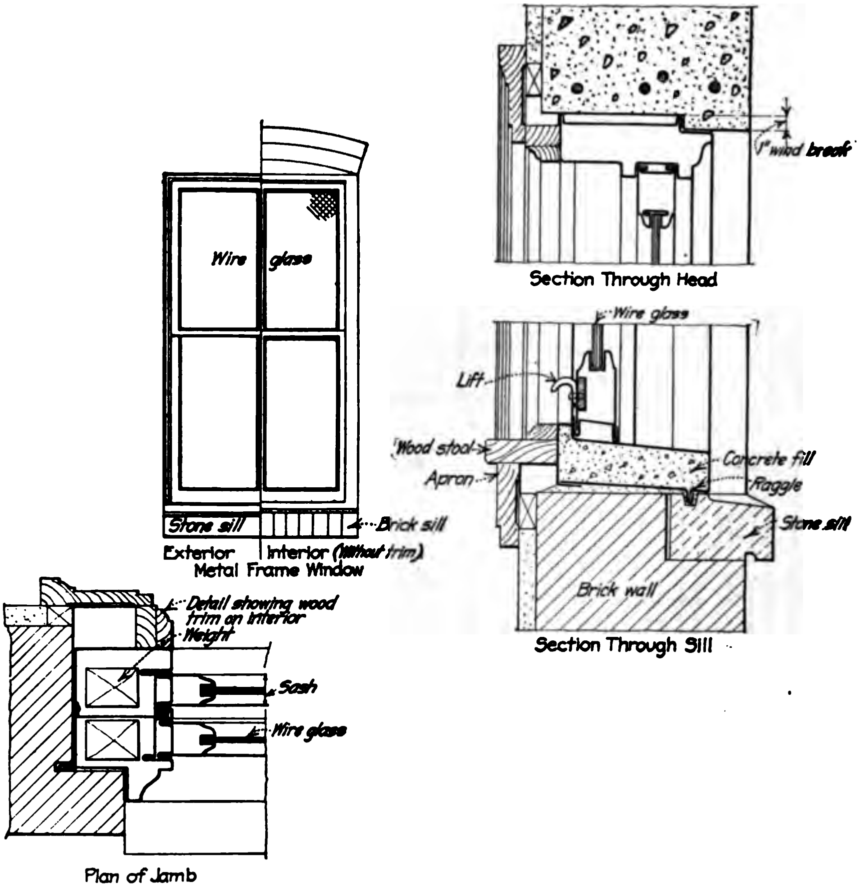


Fig. 361.—Details of hollow metal windows.

DOORS

By Frederick Johnck

216. Doors in Residences.—For residence work certain types and sizes of doors have come into general use. Fig. 362 shows the general arrangement of panels now in common use. Doors for residences are made 1⅜ and 1¾ in. thick for interior work and 2 and 2¼ in. thick for entrance doors (see Fig. 363). Entrance doors are usually made 3 ft. wide so that furniture can be taken in. Bedroom doors can be 2 ft. 8 in. wide and closet doors 2 ft. 2 in. wide. For bath rooms it is customary to make doors 2 ft. 6 in. wide. These doors are made 6 ft. 8 in. to 7 ft. in height depending on the height of the ceiling in the room. In bed room closets, a full length mirror is sometimes used. These mirrors should be set so that a small space is allowed between the mirror and the wood back. Interior doors generally should be of the veneer type, while outside doors are better if made of solid wood as the moisture has a tendency to raise the veneer.

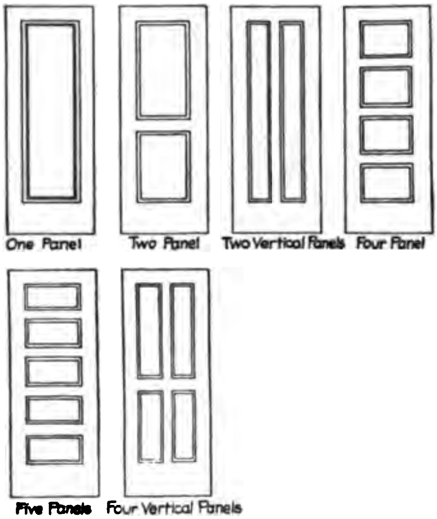


Fig. 362.—Various types of doors for residence work.

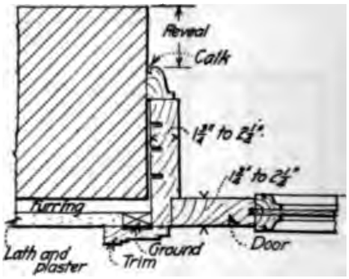


Fig. 363.—Door and frame detail for exterior brick walls.

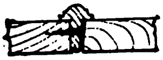


Fig. 364.—Single astragal.

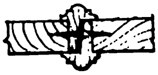


Fig. 365.—Double astragal.

The veneer for inside doors is glued to a built-up core or over a two or three-ply material for panels. If double or French doors are used, a single, or double astragal is very necessary to form a tight joint (see Figs. 364 and 365). Fig. 366 shows the detail of a door and trim for wood and plaster partitions. The studs are double and the finished jamb is set away from the stud so as to have room to wedge the door up plumb. This detail shows a two-piece trim; the molded section is called the back band. In order to have the doors swing so as to clear the carpets or rugs, a threshold is used, as shown in Fig. 367.

217. Office Building Doors.—Wood doors for office buildings may be divided into two general types—communicating doors and corridor doors (see Fig. 368). They are made with either single or double panels. The two-panel type is perhaps the most common and serviceable. Both panels in communicating doors between offices are made of wood. These doors are usually 3 ft. wide and 7 ft. high. Corridor doors are made 4 in. wider to permit large desks and other pieces of furniture to be taken into the room. The upper panel in corridor doors should be of maze glass so that the corridor will have the proper amount of daylight. Transoms are also used over these doors so that the office can be ventilated.

Very often in office building work, the doors are made with split jambs, as shown in Fig.

369. This permits the trim to be secured to the jamb and the door to be fitted in the factory so as not to cause any delay at the building.

218. Hospital and Hotel Doors.—Hospital and hotel doors are often made flush panel, with a line of inlay of some other kind of wood to make them more attractive (see Fig. 370). The flush panel makes a very sanitary door for such work, as there are no moldings to catch the dust and dirt. These doors are made 1¾ in. thick the same as for doors in office buildings.

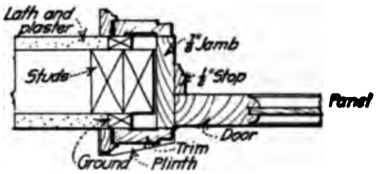


Fig. 366.—Door detail for wood and plaster partition.

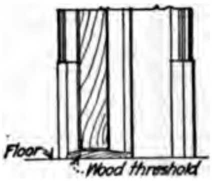


Fig. 367.—Sill section.

219. Refrigerator Doors in Cold Storage Buildings.—Refrigerator doors for cold storage buildings are made of wood and insulated either with cork or lith (see Fig. 371). The wood frame or buck is first erected similar to that used for ordinary doors in office buildings. The jamb is so detailed as to form a continuous air space entirely around the door. This is usually done with a felt filler which forms two seals of contact between the door and frame. At the bottom of the door another piece of felt is used which fits against the cement or wood sill as the case may be. The frame for these doors should be very carefully anchored into the wall so as to properly carry the weight of the door. On account of the salt air, in meat storage buildings it is well to use only bronze, brass, or white metal hardware so as not to have trouble with rust:

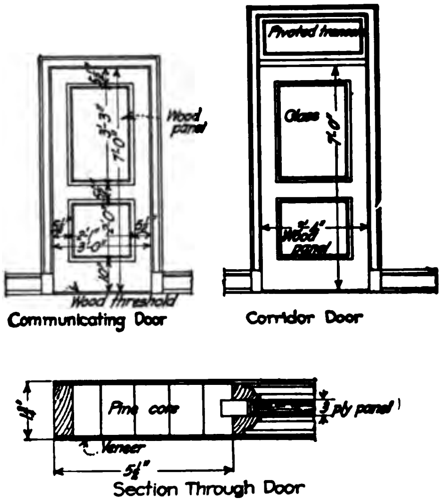


Fig. 368.—Detail of doors for office buildings.

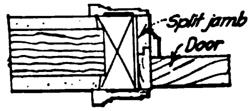


Fig. 369.—Plan of door in tile partition showing split jamb.

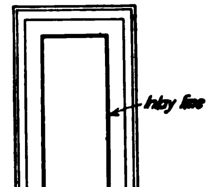
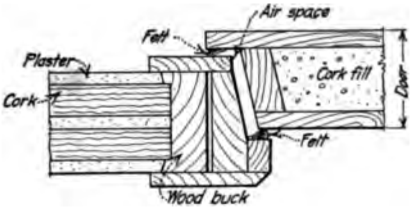


Fig. 370.—Flush panel door for hotels and hospitals.

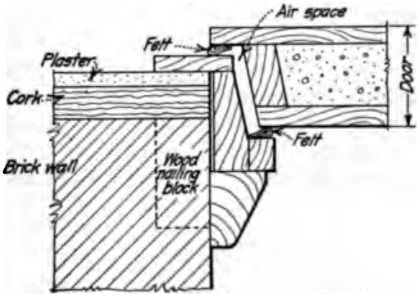
220. Cross Horizontal Folding Doors.—For shipping room doors the cross horizontal folding type has proven very satisfactory. Doors of this type are made of wood, sheet steel, or corrugated steel and are hinged above the center line so as to fold up like a jack knife (see Fig. 372). They can be operated with a lift on the bottom rail or by means of a chain, and also by a chain gear if they are very large. The doors are counterbalanced with iron weights which slide up and down in the metal weight pocket. If light is desired, it is best to use wire glass in the upper panels, as ordinary glass would break if the door is not operated with care.

221. Steel Doors.—Doors made of plate steel reinforced with angles (see Fig. 373) are used a great deal for boiler rooms, coal storage rooms, pent houses, and for stair doors in factory and warehouse construction. The thickness of the plate varies in order to comply with the Underwriters', union trade conditions, and city ordinances. For certain openings, door checks to close the doors are required to reduce fire risks. Doors of a similar character for this purpose are also made of corrugated sheets of steel with non-combustible materials between.

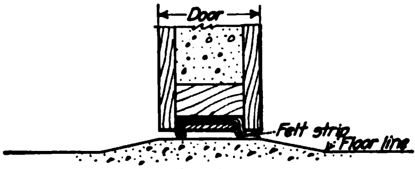
For large openings, doors of this type are also made to slide on gravity tracks, and are used on both sides of fire walls. When this is done they should be counterweighted so as to stand open and be equipped with fusible links so as to be self-closing in case of fire. In the use of steel fire doors, care should be taken to see that they comply with all insurance and building laws of the locality in which they are to be used.



Refrigerator Door and Jamb Detail for Cork Partitions



Refrigerator Door and Jamb Detail for Brick Wall Partitions



Section Through Sill for Refrigerator Door

FIG. 371.—Details of refrigerator doors in cold storage buildings.

222. Kalameined Doors.—The kalameined door (Fig. 374) is made by drawing a thin sheet of metal over a wood core. This door is used a great deal for wire shafts, passenger elevator doors, etc. The trim should also be Kalameined so as to afford full fire protection. As these doors can be hung by the carpenter, they are erected on wood bucks as shown in the illustration.

223. Hollow Metal Doors.—Hollow metal doors (Fig. 375) complete with jamb, trim door buck, etc., are commonly used as doors to wire shafts, pipe spaces, passenger elevators, etc. These can be furnished with shop coat of paint or can be supplied with a baked enameled finish. When light is required, the glass used should be wire glass so as to resist fire. Panels in these doors are often made with ¼-in. asbestos board.

224. Freight Elevator Doors.—To prevent accidents and to provide a door that could be easily operated by the man on the elevator, a standard door divided horizontally in the center so that one-half could slide up and the other half could go down has been adopted (see Fig. 376). The two best known doors of this type are the Meeker and the Pellee. These doors are made of steel sheets, or corrugated iron sheets, reinforced with steel angles and tees. They are made semi-automatic which are closed by the car as it leaves the landing, or full automatic which open when the car reaches the landing and closes as it passes the landing. In the semi-automatic type it is well to provide a steel gate in addition to the door, so as to prevent accidents if the car door should be left open. These gates should slide up and be counterbalanced. Doors for elevator shafts should bear the Board of Underwriters' labels, and the gates should be approved by the Casualty Insurance Companies.

225. Pyrona Doors.—To secure a wood veneer surface over a fireproof material the Pyrona Process Company manufactures a door which has a fireproof sheathing bonded into the wood core over which the wood veneer is applied. This door gives all the appearances of a wood door and can be hung by the carpenter. It is used for wire and pipe shafts in residences and apartment buildings. The trim for these doors can be treated in the same manner as the door. *Fig. 377 shows* a pyrona door detail complete with trim, etc.

226. Metal Clad Doors.—The metal clad door for use in fire walls is a wood flush panel door covered with sheet metal. It is a cheaper door than a steel one but will not stand the hard usage from trucks, etc., running into them. The wood also has a tendency to dry rot due to the lack of ventilation.

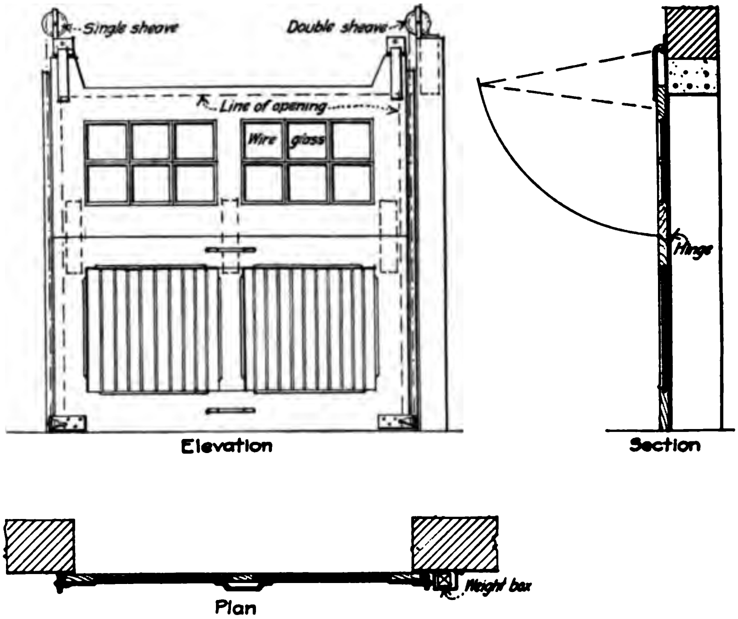


Fig. 372.—Details of cross horizontal folding doors for shipping platforms.

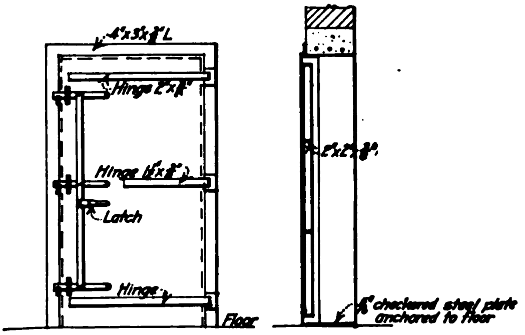


Fig. 373.—Details for steel doors for boiler room, coal rooms, and warehouse stair shafts.

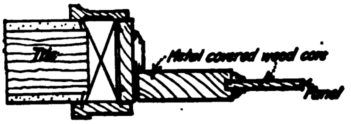


Fig. 374.—Kalameined door.

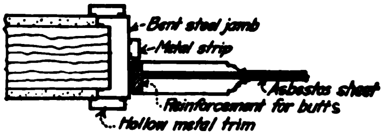
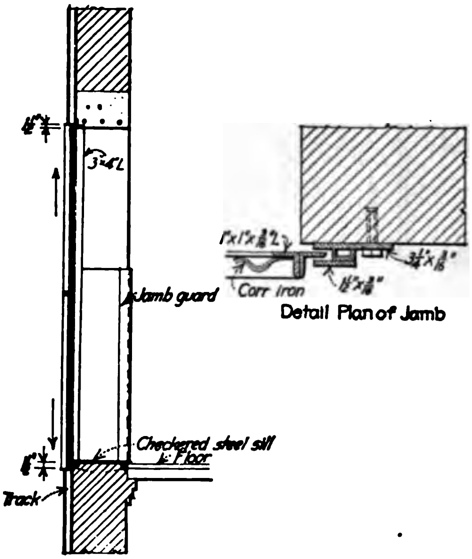


Fig. 375.—Hollow metal door.

227. Alignum Fireproof Doors.—Alignum is manufactured in slab form from fireproof mineral components, amalgamated under hydraulic pressure. It is worked the same as wood and can be finished with practically the same materials. The slab can be reinforced with wire mesh for extra strength and then secured to both sides of vertical ribs which make a hollow fireproof door. This product is manufactured by the Alignum Fireproof Products Company, Inc.

228. Revolving Doors.—For store purposes and entrances to public and semi-public buildings, the revolving door is very efficient. These doors are made with three or four wings and should be provided with automatic releasing fire exit devices so that they can collapse and give a full width door opening in case of fire. This type of door complete with vestibule will permit people to enter freely and yet allow a minimum amount of cold air to come in during the winter months.



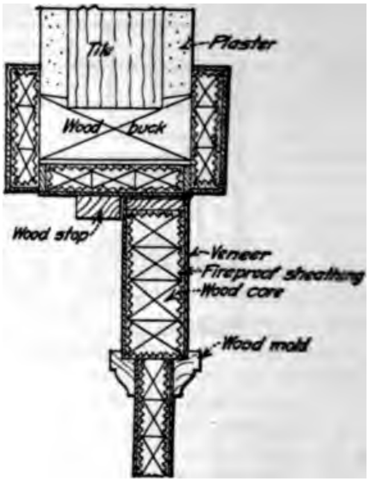


Fig. 376.—Freight elevator door.

Fig. 377.—Pyrono process door and trim.

STAIRS

By Corydon T. Purdy

229. Definitions.—Stairs are variously classified. A *newel* stair is one in which the stair rail or balustrade is constructed with newel posts at its angles, or turning points, while a *geometrical* stair is one in which the newel posts are not used in making turns. It follows that newel stairs are in straight runs, ordinarily broken by landings between floors, and that the geometrical stairs are curved and continuous.

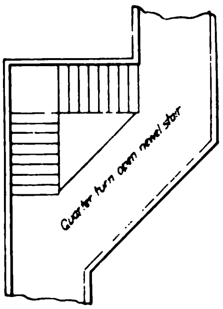


Fig. 378.

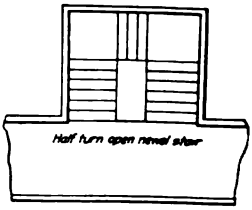


Fig. 379.

Judged by their horizontal lines, stairs are *straight*, *quarter-turn*, or *half-turn*, and geometrical stairs are more commonly termed *curved* stairs, *circular* stairs, *elliptical* stairs, *winding* stairs, or *spiral* stairs, as the case may be.

Most stairs are constructed with an opening in the floor larger than the stairs, so that there is an open vertical space from floor to floor. A newel stair returning on itself without such an open space—that is, with the balustrade of one flight in the same vertical plane with that immediately above or below—is called a *dog-legged* stair.

In dwelling houses the *front* stairs are the ones made to be seen and generally used, and the *back* stairs are made for domestic use and ordinarily out of sight.

Stairs are *open* or *closed* when they are open or enclosed by walls.

A *tread* is the horizontal part of a step.

A *riser* is the vertical part of a step.

A *step* is the combination of a tread and a riser.

A *winder* is a step in which one end of the tread is wider than the other.

A *stair* may be a step, a series of steps, or a continuity of steps from floor to floor, or the word in its singular form may apply to all the stairs in one continuous stairway. In many ways, the singular and plural form of the word can be used interchangeably.

A *flight of stairs*, technically, is a continuous series of steps without a break, but in ordinary conversation it is generally taken to mean the entire height of stair from one floor to the next, including landings.

A *stair case* is an expression that properly applies to the whole stair construction, including the place it occupies and its enclosing walls. In common usage, it is almost synonymous with the word "stairs", but improperly so.

The *run* of a flight of stairs is its horizontal length.

The *rise* of a flight of stairs is its vertical height.

The *pitch* of a flight of stairs is the angle of its ascent.

A *landing* is a platform in the stairs between floors.

The *nosing* of a tread is the projection of the tread in front of the riser.

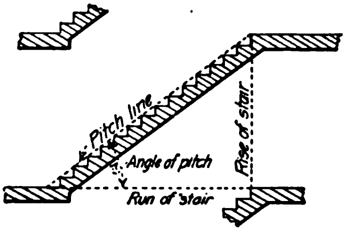


FIG. 380.—Flight of stairs.

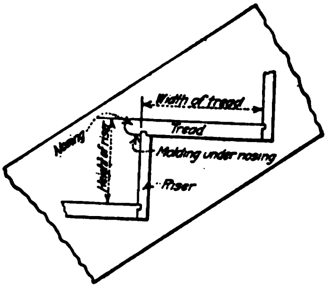


FIG. 381.—Step in wood stair

A *stringer* is a longitudinal member of the stair construction. It may support the stairs, or it may only appear to do so.

A *wall stringer* is the one that adjoins the wall.

A *front stringer* is the one on the open side of the stairway.

A *baluster* is a small column or post supporting a rail.

A *balustrade* is a series of balusters joined by a rail to form an enclosure. This word properly applies to massive work in stone or its imitation, but now it is much used by architects for the lighter work in wood and iron employed in modern stair construction.

A *newel* is a principal or more important post supporting a hand rail. Newels are used at the beginning and at the end of a balustrade, and also at turning points on landings.

230. Risers and Treads.—The importance of stair construction, the character of the work to be employed, and the difficulties involved, vary widely with different types of buildings. There are, however, a few things regarding the design of stairs that have general application and one of them relates to the risers and treads.

The height of risers should be exactly the same from one floor to the next, even if it figures out an odd fraction of an inch to make it so, and there is no exception to this requirement. The treads should have a uniform width, except where winders are used. In high buildings where the heights of stories vary, the height of the riser will ordinarily change when the story height changes. In such a case, the change in the height of the riser should be made as little as possible. To get this height in any staircase, determine the exact height of the story from finished floor to finished floor, and divide it by some number that will give for an answer the approximate height of riser desired. The divisor will be the number of steps required, and at the most, two or three trys should indicate the combination that is most desirable. The best practice in America is to make risers in ordinary stairs from 7 to 7½ in. high.

The relation of the riser to the tread depends upon the use of the stairway. Treads 10 in. wide are most commonly required with 7 to 7½ in. in height of riser, and this makes a standard pitch that should be widely used. These proportions make the most satisfactory stairs in dwelling houses, tenements, apartment houses, hotels, office buildings, and factories, and particularly where the stairs are in constant use. Such stairs are easy of ascent for ordinary persons. If the height of the riser is reduced, the width of the tread should be increased; and, vice versa, if the height of the riser is increased, the width of the tread should be made less. Generally speaking, stairs in public buildings should have wider treads and less height of riser. The same is true of most stairs in which the architectural features are particularly important. A 6½-in. riser and 11-in. tread make a pitch to the stairway that is more attractive and inviting. The following is a rule of French origin which fixes the relation of the riser to the tread: The sum of the width of the tread and twice the height of the riser equals not less than 24 in., nor more than 25. Stairs in the United States conform generally to this rule. In England there is a rule that the product of the height of the riser in inches and the width of the tread shall be 66 in., but it is not much in use in this country. The New York Building Law requires the application of this English rule; but fixes the product at not less than 70 in., nor more than 75. It also limits the height of riser to 7¾ in. and the width of tread, without nosing, to 9½ in.

In designing stairs, the first thing is always to determine the number of steps and height of riser, and the next thing is to fix the width of the tread and the run of the stairs. Beyond this part, the problem varies with the character of the building and the purpose of the stairway.

231. Width of Stairs, Number, and General Design.—Dwellings, both in the city and country, should have two stairs, the front, or principal stairs, for general use, and a back stairs for the service of the house. The former should be at least 3 ft. 6 in. wide. In most dwellings such stairs are in constant use, and they should have a standard pitch and two or more flights between floors, so that the labor of passing from floor to floor will be reduced to a minimum. This consideration is more important than any other, for the stairs are used day and night, by old and young, and if going up and down stairs becomes a burden anywhere, it is in the home. It is common practice to make the front stairs in the first story of dwellings the attractive feature of the house. In the construction of such buildings, any expenditure allowable for a purely architectural feature, is properly put in these stairs, and in many homes where the character of the construction will warrant it, the stair work is elaborate and ornate. The old Colonial staircases, still to be found in many houses of New England and Virginia, have served as a national model for stair work in dwellings. Some of these staircases are more than 150 yr. old. The symmetry and directness of their design is their chief characteristic. Some of them are very ornamental and beautiful, and some of the workmanship in their construction is not excelled in this generation.

In buildings for the service of the public—such as post office buildings, capitols, libraries, and railway stations—stairways should always be wide enough to meet all requirements of the most exacting condition. Where practicable they should be as wide as the entrances, passageways, and concourses which they serve. It is also equally important that such stairs should be constructed with short flights and commodious landings. All of these provisions serve to prevent overcrowding, confusion, and accidents. The most unsatisfactory and unfortunate feature of our Metropolitan Subway Railway construction is the narrow difficult stairways which street conditions have required in many places.

Schools and college buildings are usually classified as public buildings, but they have a different stair problem. In such buildings most of the travel ebbs and flows according to a program, and the travelers are known to each other This means less confusion and less chance of accident. The requirements for stairways in such buildings can therefore be made correspondingly easier than for stairways open to the general public and in constant use both ways.

Theatres, assembly halls, and dance halls are also public buildings, but they have still another stair problem, chiefly one of quick exit. The width of the stairs and number should be sufficient to empty the building in three or four minutes at the most. Each floor or balcony should have its own separate stairway, and in large theatres, each division of a floor or balcony should have a separate exit.

Stairs in high buildings, office buildings, and hotels are not much used, and are constructed to meet an emergency rather than for every day use. Perfected elevator systems take the travel; but both legal requirement and good judgment call for stairways large enough and in sufficient numbers to afford a satisfactory exit for the entire population of a building within the space of a few minutes. The new Commodore Hotel in New York, with its 2000 bed rooms, has five stairways, each 3 ft. 8 in. wide, and the Equitable Office Building has four stairways each 4 ft. 2 in. wide. Each stairway is continuous from the roof downward through all typical stories and the same exit area is made good to the street.

It is not enough that these buildings are absolutely fireproof, that their floors, doors, windows, and trim are all made of metal or wood that will not burn. There is hardly one chance in a thousand that a fire would spread beyond the room in which it started in either building. Nevertheless, their enormous population makes the construction of stairways in such buildings mandatory, whether special laws require it or not. They should be designed as simple in construction as possible, with easy flights and a standard pitch.

If any stairs in a hotel are in general use, they are those connecting the main floors, ordinarily the lower floors, where the same conditions practically prevail as those in public buildings. Here the stairways may properly be fewer in number and wider, with less than standard pitch, and more expensive. Almost the same conditions occur in some office buildings, particularly where banks or other rooms of a public character are located on the second floor. In both hotels and office buildings, such stairways are sometimes made elaborate in architectural design and ornamentation, but such an expenditure would be worse than wasted in the upper stories particularly if it in any degree lessened their value as an exit. Similar conditions prevail in apartment houses, and stairways in such buildings should be designed on the same basis as in hotels

Mill and factory buildings present still another problem, particularly where they are not served with elevators. In such buildings the stairs are used to their full limit, both up and down, at certain hours in the day, and it is this use of the stairs, rather than their need as a safe exit in case of fire, that should control the design. All such buildings should have at least two lines of stairways from roof to street, and this rule should hold regardless of the size of the building. In such buildings the possibility of a temporary obstruction of a stairway is greater than in other buildings, and the two stairways serve also to meet that difficulty.

Factory stairs should be standard pitch, more commodious than stairs in office buildings, and as simple and substantial in construction as possible. Stairways in loft buildings should properly be treated the same as in factories, for such buildings are particularly available for the making of clothing and other light manufacturing. It is not sufficient that the owner of a loft building intends it for some other use, for buildings stay, and owners and conditions change.

In large cities, the number and width of stairs for most buildings are fixed by the building laws, and they must be known and followed; but in some places building laws are wanting and in others they are incomplete. In any case, the design of the stairways of an important building should be based on its population, whether legal requirements compel it or not. For the determination of populations of different floors of fireproof buildings, the areas considered should be rooms enclosed by walls or partitions of fireproof materials; and corridors, halls, entrances and other areas unusuable for the purposes of the building should not be included. The New York law provides that the population in any one floor of a fireproof building shall be taken as being one person for every 10 sq. ft. in places of assembly, every 15 sq. ft. in schools and courthouses, 25 sq. ft. in stores, 32 sq. ft. in factories, 50 sq. ft. in office buildings, and every 100 sq. ft. in hotels. This is probably the best authority obtainable and it is the best practice in present construction. The population of single floor areas of fireproof buildings of different types and sizes on this basis is as follows:

POPULATION PER FLOOR FOR THE DIFFERENT AREAS PER INDIVIDUAL

Usable floor areas, (sq. ft.)	Public assembly, 10 sq. ft.	Schools, courthouses 15 sq. ft.	Stores 25 sq. ft.	Factories, work rooms 32 sq. ft.	Offices, 50 sq. ft.	Hotels 100 sq. ft.
3,000	300	200	120	94	60	30
4,000	400	266	160	125	80	40
5,000	500	333	200	156	100	50
6,000	600	400	240	187	120	60
7,000	700	...	280	219	140	70
8,000	800	...	320	250	160	80
9,000	900	...	...	281	180	90
10,000	1000	...	...	312	200	100
11,000	....	...	...	...	220	110
12,000	....	...	...	...	240	120
13,000	....	...	...	...	260	130

No stairway should be less than 3 ft. 6 in. wide, nor less than the stairway in the story above. In general, it is better to have two stairways 3 ft. 6 in. wide than one 7 ft. wide. No building having 3000 sq. ft. of usable floor area on one floor should have less than two separate stairways. The stairways of most buildings should be sufficient in number and width to provide standing space for the population of the floor which they immediately serve, or nearly so, when occupied to their full capacity.

In a building of ordinary ceiling height, an enclosed stairway 3 ft. 6 in. wide with one half-turn landing and a hallway at the floor level of moderate size will afford standing space for 45 people, and each additional 6 in. in width of stairway will afford standing space for 10 additional people. Accordingly, a stairway 5 ft. wide will provide standing space for 75 people, and one 7 ft. wide for 115 people. New York regulations allow not more than one person for each 22 in. of stair width, and 1½ treads on the stair proper, and not more than one person for each 3½ sq. ft. on landings and halls within the stairway space; and the floor served can not be occupied by more persons than this requirement will permit. The two methods of determining the capacity of stairs give substantially the same results.

On the basis of 45 people for a stairway 3 ft. 6 in. wide and 10 additional people for each 6 in. additional width, and the general provisions and limitations, the number and widths of stairways for different sizes and types of buildings may properly be made as given in the following tabulations:

Number of Stairways and Width of Each

Usable floor area (sq. ft.)	Schools, courthouses	Stores	Factories, work rooms	Office buildings	Hotels
3,000	2–6'6"	2–4'6"	2–4'0"	2–3'6"	2–3'6"
4,000	3–6'0"	2–5'6"	2–4'6"	2–3'6"	2–3'6"
5,000	4–5'6"	2–6'6"	2–5 0"	2–4'0"	2–3'6"
6,000	4–6 0"	3–5'6"	3–4'6"	2–4'6"	2–3'6"
7,000	......	3–6'0"	3–5'0"	3–3 6"	2–3'6"
8,000	....	3–6'6"	4–4'6"	3–4'0"	2–3'6"
9,000	....	......	4–5'0'	3–4'6"	2–3'6"
10,000	......	......	4–5'6"	4–4'0"	2–4'0"
11,000	......	......	......	4–4 0"	2–4'0"
12,000	......	......	......	4–4'6"	3–3'6"
13,000	......	......	......	4–4'6"	3–3'6"

Practice differs as regards fixing the width of stairs in places of public assembly, and is not so exacting as for other buildings. The New York requirements call for a stairway 4 ft. wide in the clear between railings or walls for 50 people, and allow 50 additional people for every additional 6 in. width of stairway.

This difference is reasonable for most places of public assembly are designed so that the stairways serve only one level, or, at the most, only two levels; whereas the stairways of the other types of buildings serve many levels, and if their stairways are not sufficient to accommodate the entire population of the building at one time, or nearly so, in case of great emergency, disaster would be certain.

Where sprinkler systems are installed in fireproof buildings, the stairway requirements may properly be reduced, and it is so provided under the New York Building Law. On the other hand, if the buildings are not fireproof, the stairway requirements should be increased. The amount of reduction to be permitted in one case, and the amount of increased requirements in the other case, depend upon the conditions, and whether those conditions are likely to be permanent.

232. Locations of Stairways.—In dwellings, the main stairway ordinarily occupies a central and prominent place in the house. In buildings of the old Colonial type, the main floor is divided into two parts by the hall, and the main stairway is located in this room, or it is directly connected to it. In most government buildings, school houses, churches, theatres, railway stations, and other buildings of a public character, the locations of the stairways are fixed by the design of the building. To change the location would mean to re-design the building, or, at least, to make material changes in other important parts of it. To make ingress and egress easy, and travel in public buildings convenient and comfortable, is one of the most important considerations in the design of such buildings, and the arrangement of stairways and passageways must be worked out as a part of the general design. This is not true of all buildings. The general scheme of a hotel or an office building can often be arranged without much regard to the location of the stairs—that is to say, they can be figured into the design in various ways without materially altering the general scheme of the building.

Where two stairways are required, they should not be near each other, and if there are more than two, they should be well separated and placed so as to afford the easiest and quickest service possible to the building as a whole. The distribution of stairways is particularly important in the design of large factory buildings. It may be materially to the advantage or to the disadvantage of the business in the building. Such stairways should be located so that there will be little or no interference in passage from work to stairs, from work to locker or wash rooms, and from such rooms to stairs. Stairways should never be located around or adjacent to elevator shafts without solid walls between them.

A double or interlocking staircase has been devised that makes a very ingenious economy of space. The two stairways occupy the same space that either of them alone would require. The arrangement can not be used unless the floors are 16 or 17 ft. or more above each other, and it is particularly adaptable for exits for theatres, school houses, and other public buildings, when ceilings are high. Fig. 382 shows how this stair is constructed. The arrangement increases the fire risk, and in some places might be prohibited, but if the enclosure walls are properly made and particularly if the entrances are protected by intermediate corridors, or otherwise, the danger of smoke might be sufficiently eliminated to remove this difficulty.

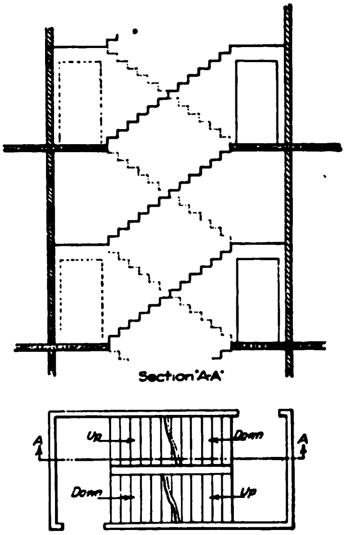


Fig. 382.—Double or interlocking stair.

233. Landings and Winders.—Winding steps should never be used in newel stairs, and in some cities they are prohibited by law, except in ornamental construction where the use of the stair is not very important. Winders have been used in American practice a very great deal in dwelling house construction, in order to economize space and to save expense in construction, but it is a very bad practice. It is more difficult to go up and down such stairs, and the danger of falling on the stairs is very greatly increased.

Winding steps are a necessary part of curved stairs, and in such construction the width of the tread should be limited. It should be the same width as the treads of other steps, about 2 ft. out from the hand rail, or the inside of the stair, which is about the ordinary line of travel. The average width, if the stairs are not too wide, should be not greater than would be used if the stair were straight, and the minimum width should be not less than 6 in.

Landings should be separated by 4 or 5 steps. Square landings serve to prevent accidents, and they also serve as resting points going up and down stairs. No straight flight of stairs should be more than 10 or 12 ft. in height without a landing. It is very desirable to have at least one landing in every ordinary story, as buildings are constructed in our American cities.

234. Balustrades and Hand Rails.—Balustrades and hand rails are necessities in the construction of stairways. Even if the stairway is entirely enclosed by walls on both sides, the hand rail is an important part of the construction. Without it the danger of injury to people using the stairway would be greatly increased.

The balustrade offers an exceptional opportunity for decorative work. A great deal of very beautiful work in the construction of balusters and newel posts has been worked into some of the old Colonial staircases. In the lower stories of office buildings and hotels, and particularly in public buildings, the balustrades are often made of stone, marble, or bronze, massive and sometimes very rich in design. In all buildings, balustrades and hand rails should be made substantial and strong enough to maintain their position under any kind of a strain. Wide stairways should have a hand rail on both sides, either as a part of the balustrade or fastened to the wall, and in public places where the stairs are in constant use by large numbers of people, very wide stairs should have an intermediate hand rail.

235. Stairway Enclosures.—In the early history of high building construction in our American cities, it was considered quite the proper thing to build the stairways around elevator shafts, with nothing between them but a light iron screen. The folly of this construction, however, became quickly apparent. The openings from floor to floor, which they afforded, became the flues for smoke and rapid spread of any fire in the building. The next step in this evolution was the separation of the stairs from the elevators. They were placed in or adjoining the corridors of the building. This was better, but the well hole in the stairway was still an element of danger in case of fire. The only construction of stairs which can be depended upon to make them a safe exit, reasonably free from smoke, is their construction within enclosing walls. Our best building laws require the enclosure walls in all high buildings. The construction of such shafts is treated in the following chapter.

236. Materials, Details, and Methods of Construction.—In most cities the building laws require stairs to be constructed entirely of incombustible material, except in frame buildings and in non-fireproof buildings of moderate size. All such stairs are supported by iron strings, or they are made of reinforced concrete construction. If they are supported by iron strings, the treads should be made of solid steel or cast-iron plates. Marble, slate, or other stone should not be used for finish treads without such plates under them. The reason for this is obvious; in case of fire the stone treads are likely to crack or break from heat or water. In the most economical construction of this character, the treads and risers are made of stamped steel plates in different forms, some of which are arranged to carry cement treads.

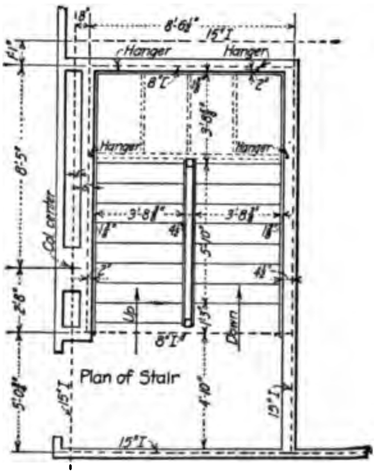
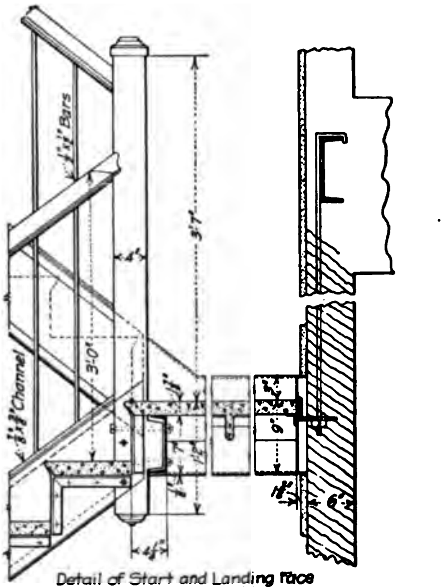


Fig. 383.—Typical stairway in the Commodore Hotel, New York City.



Detail of Start and Landing Face Strings at Platforms

Fig. 384.

Figs. 383 to 387 inclusive show the plan, section and details of the construction of a typical stairway in the Commodore Hotel in New York City. These figures give the actual measurements that are used, the enclosing walls, the structural iron that supported them, and the support of the stairs. It is given as an exceptionally good exampl of a very economical construction; but thoroughly substantial and fully meeting all the requirements of the building laws.

The stair comes very near being a dogged-leg stair. The open space between the hand rails, as shown on Fig. 387, is only about 1 in., and between the iron strings about 3 in. One newel post serves both the upward and the downward flights of the stairs. It is carried on the 8-in. beam at the floor and on an 8-in. channel at the landing, and held in place by bolts directly through the post and the webs of the structural members.

The height of the stair from floor to floor is 10 ft. 6 in.; there are 17 risers, each 7.41 in. high. The treads are 10 in. wide. The treads and risers and the landing are made of sheet steel stamped to form, and covered with cement.

These stairways are in the middle of the building, artificially lighted day and night. As the elevator service in the building is ample, both for the guests and for the service of the building, these stairways are not likely to be much used, except in some possible emergency.

Reinforced concrete stairways are particularly adaptable to buildings made of reinforced concrete construction, and are often more economical than iron stairs.[1] When all the materials and equipment are at hand and in use in the construction of the floors and walls of the building, the additional concrete in the stair construction can be put in

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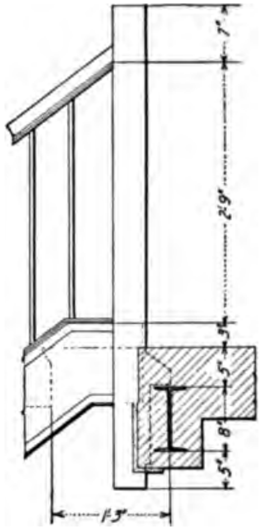
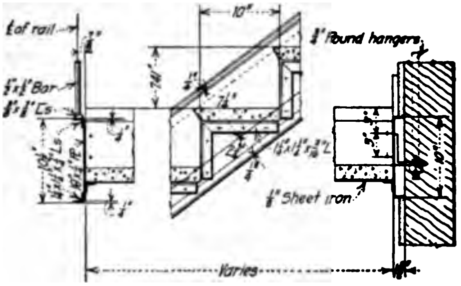
[1] *For the design of reinforced* concrete stairs, see Sect. 2, Art. 43.

place for the actual cost of the material and labor required, without overhead charges. Moreover, in a building of reinforced concrete construction, stairways of the same material can be designed so that they will become an integral part of the structure. The common method of construction is an inclined slab of concrete with the form of the stair molded on the upper side, the thinnest part of the slab made thick enough and the reinforcement made sufficient to meet all requirements of strength. Reinforced concrete stairways can be adapted to difficult conditions often times quite as easily as to simple ones, which would not be the case in iron construction. The slab can be made to include landings, and special wall or column construction in any way that may be desired without adding materially to the cost. Almost any combination of constructions desired is practicable with this material.



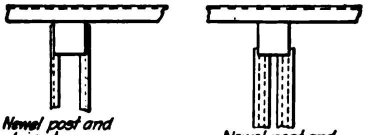
Floor Landing at Top of Flight

Fig. 385.



Detail of Strings and Risers

Fig. 386.

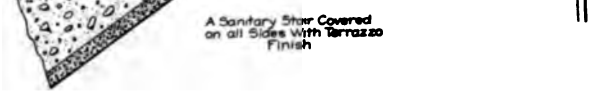


Newel post and stair stringer connection

Newel post and hand rail connection

Fig. 387.

Fig. 388 shows a sanitary stair of reinforced concrete construction, with all parts covered with terrazzo. Such a stair is particularly desirable in a hospital. The terrazzo work can be carried up the wall, if desired, to form a wainscotting. In the finish, the entire stair is one piece without a crack, and, if wanted, without a square corner to catch and hold the dirt. The same thing can be made with a cement finish for factories or other buildings where the terrazzo is too expensive. It is a form of construction that can not be adapted to iron stairs. A stream of water can be turned on such a stair without any disadvantage.



A Sanitary Stair Covered on all Sides With Terrazzo Finish

Fig. 388.

The reinforced concrete part of the stair is poured in wood forms after the reinforcing rods have been put in place, and left in the rough. After the forms are removed the finish lines are carefully determined, and the terrazzo is molded in place with tools made to fit the corners and projections as may be required.

Stairways in dwellings are generally made of wood, and their construction requires the most skillful joinery known. Indeed, so great is the demand for skill in such work that most of it is done by men who do no other kind of work.

Except in massive work where the balustrate is made of stone, hand rails are mostly made of wood. In factories, hospitals, and other buildings where the appearance and finish of the work will permit, the structural work is exposed, and in reinforced concrete stair work the risers and treads can be finished in cement or terrazzo. In finer work, iron or steel strings are covered with cast-iron facia, and treads should properly finish with a wall string of the same material. The balustrades of stairs made of incombustible material, excepting the hand rail, are usually made of iron or bronze.

Stairs should be calculated to carry 100 lb. per sq. ft. of live load, and all details of their construction should be developed with the same care that is given to floor construction.

SHAFTS IN BUILDINGS

By Corydon T. Purdy

The importance of enclosing stairways and elevators with fireproof walls has been evolved along with the other features of modern construction, but more slowly than most of them. When we had only five story buildings, no point was made of it. For years afterward the stairway around an open elevator was considered the proper construction by the best architects, and it is only a few years since we stopped building elevators fronted with open grilles, and stairways in open corridors. Now enclosures are required in many places, and should be everywhere.

The one thing that has forced this evolution, step by step, is the growing appreciation of the necessity of enclosure walls for the preservation of life. The open elevator or stairway, in case of fire, became a flue that drew the fire to itself, making it the worst place for travel instead of the best. If it did not get the fire, it did get the smoke, and in one fire in a New York hotel, several lives were lost in a few minutes on this account, when practically no damage was done to the building.

All openings in floors should be enclosed with walls, forming vertical shafts, except (1) small openings for ducts and flues for which requirements vary, (2) openings for stairways in the first story of city buildings, and (3) stairways in dwellings. There should be very few other exceptions.

237. Kinds of Shafts.—Shafts are open and closed. Open shafts are open to the air—that is, they are not covered with a roof or any other kind of covering. Closed shafts are roofed in and completely covered at the top.

In general, there are five kinds of shafts: light shafts, vent shafts, dumb-waiter shafts, elevator shafts, and shafts formed by stair enclosures. Light and vent shafts are constructed both open and closed, the others being always closed.

238. Open Shafts.—Open shafts are made for purposes of ventilation and light. They should be enclosed with walls similar to those required for the exterior construction of the building, except if the shaft is small, in which case some reduction in thickness of walls may be allowed provided that by so doing there is no depreciation in the strength of the structure as a whole. All openings in such shafts should be protected from fire, whether the building be fireproof or not, and windows should have fireproof construction, wire glass, and fire shutters.

239. Closed Shafts.—Small vent and dumb-waiter shafts should be enclosed with walls made the same as partitions ordinarily required in fireproof buildings. Vent shafts should have no openings, except for ventilation purposes, including windows, and dumb-waiter shafts should have no openings except the doors for the dumb-waiter service. These openings should have iron or concrete frames, and fireproof doors and windows. Such shafts should also have fireproof construction at the top and bottom. This fireproof construction works both ways. It prevents the fire from getting into the shaft, and then if the fire does enter the shaft, it holds it in and prevents the spread of the fire on the floors above. The complete enclosure of the shaft at the bottom prevents the entrance of the fire at the most dangerous point, and the enclosure at the top stops the draft which would otherwise be established.

240. Stairway Enclosures.—The kind of an enclosure required for a stairway depends upon the size and construction of the building, its use, and to some extent on outside conditions. In high buildings serving a large population, they should be of the best type of construction. This is true of most buildings in our large cities, but in buildings 3 or 4 stories high, of ordinary construction, with brick exterior walls and with floors and roof supported by wood joists, any slow-burning enclosure wall answers the purpose as well as one made of fireproof materials. In a case of fire, the people will pass out of the building before the enclosure is burned. It is impossible to make a rule that will apply to all cases, determining under what conditions the cheaper enclosure is applicable, but open stairways should not extend through more than three stories in any kind of a building, in city or country. In New York City not more than two stories in any building can be connected by an open well or unenclosed stairway.

Slow-burning enclosures can be made in various ways—with wood studding and wire lath and plaster, or of solid wood several layers thick, or otherwise.

Fireproof enclosure walls should be made better than the ordinary partitions of so-called fireproof buildings. In buildings that are not fireproof, they should be self-supporting from the foundation upward, the same as exterior walls, and made of materials that will meet all requirements of strength, as well as of fire resistance. In fireproof buildings, enclosure walls can be carried from floor to floor on the fireproof floor construction, or on the steel or reinforced concrete framing. Under the New York building law, such enclosing walls must be 8 in. thick if made of brick; 6 in. thick if made of solid concrete or of hollow blocks of terra cotta, concrete, or gypsum; and 4 in. thick if made of reinforced concrete. Such walls can also be made of metal studding covered with wire lath and plastered with cement mortar, but they must be solid at least 2½ in. thick.

Enclosure walls in fireproof buildings should also be well constructed. All mortar used in making them should be cement mortar. Their support and connection at floors and ceilings should be substantial and sufficient to resist any destructive force that the wall itself will resist. Metal studding should project into both floor and ceiling, and be cemented in place; the work should be so designed that beams or other steel construction will not project through the enclosure walls. At all points, the metal of the steel frame should be covered by at least 1½ in. of fireproofing material.

Openings in such enclosure walls should be made with corresponding care. The edges of the openings should be reinforced with steel to insure the strength of the wall against the weakening effect of the opening. Door and window frames should be made of metal, of wood covered with metal, of fireproofed wood, or of their equal as a fire resisting material. The doors and sash should likewise be made of fire resisting materials. The windows should be provided with iron shutters. Glass, wherever it is used, should be wire glass, and if windows are badly exposed, the glass should be in two thicknesses, separated by at least 1 in. of air space. Sash should be fitted with automatic self-closing devices. Doors should open outwardly and should be self-closing. They should not be locked when the building is inhabited. Each story in such an enclosure should be provided with artificial light, which should be as independent as possible of the other lighting in the building, and as fully protected as possible from injury by any fire likely to occur in the building, from within or without.

The above specification is for the best construction, but these enclosures are a small part of the entire construction of a building, and the additional cost that they incur is not a large part of the total cost of the building. The evolution of stair construction has now reached the stage in which the public demands the best in these particulars.

The construction demanded for stair enclosures in factories and loft buildings and other places where workmen and workwomen congregate is, in all its essential elements, the same as required for hotels and office buildings, and as complete as herein specified. The finish may be omitted, and the work may be left in the rough, but the construction should be equally substantial and the prevention of smoke equally certain.

Some building laws require "fire towers." A "fire tower" is an enclosed stairway, as above specified, with both its doors and windows opening to the outside of the building, and at a point that is not badly exposed by a fire in another building. Fire towers should be connected at each floor to a nearby exit doorway from the building. The balconies required to make the connection should be made of substantial fireproof construction, and as wide as the corridors or stairs which they serve.

The complete enclosure of stairway shafts in city buildings should continue to the ground floor, with an exit leading as directly as possible to the outside of the building. Such stairways should also continue to the roof, where they should be enclosed with a substantial fireproof construction with a skylight or windows.

241. Elevator Shafts.—The walls of elevator shafts and the fireproofing of surrounding and supporting structural members should be made with the same care and good workmanship called for in the construction of stairway enclosures. One is quite as important as the other. If there are only two elevators in a building, they should have separate shafts. New York City does not permit more than two elevators in one shaft, and whether there is any regulation in regard to it or not, the separation of elevators in large city buildings into two or three or more shafts is very desirable.

The size of elevators, as well as their number, depends upon the service required. These factors must be determined or assumed before the number and size of the shafts can be fixed. The horizontal clearance in the shafts, at the sides of the elevators, depends upon the size or character of the guides or rails which are used and the construction of the car, and the clearance required behind the car for the counterweight depends upon the size of the counterweight. A clearance of 3½ in. on each side of the car is the least allowance for iron rails and a recessed car. If the rails are extra heavy or their supports unusually difficult, this clearance must be increased. Wood rails require more clearance than iron rails. If the pilaster effect in a car on account of making a recess for the guides is objectionable, and the side of the car is made straight, a 6-in. clearance is the least that should be allowed, even with iron guides.

The space required for counterweights is never less than 8½ in., and a greater allowance is desirable. A clearance of ¾ to 1½ in. in front of the car should also be allowed. New York City does not permit more than 1¼ in. If the threshold of the doorway is constructed to project into the area of the shaft to make this clearance satisfactory, the under side of the projection should be beveled to the line of the shaft as a measure of safety.

The above clearances are on the basis of elevator guides on the sides of the car and counterweights in the same shaft. Corner guides are very undesirable, and counterweights in separate shafts where they can not be readily seen are also objectionable. The simplest arrangement of these details is the best and ordinarily the most economical in construction. If an elevator shaft is constructed with given clearances for a proposed size of car, it is necessary that the erection of the shaft construction be perfectly plumb to permit the size of car as proposed. If the shaft is not plumb, the size of the car will have to be reduced, for the guides must be vertical whether the walls of the shaft are or not.

The clearance required overhead for the car depends upon the speed of the elevator. The New York regulations call for 2 ft. when the speed is not over 100 ft. per minute, and 5 ft. if the speed exceeds 350 ft. per minute, and these regulations represent the best practice. The clearance is measured from the top of the car, when it is in position at the top floor of the building, to the under part of the lowest overhead construction. The clearance overhead for the counterweights depends upon the type of the elevator. The New York regulations require not less than 6 in. for traction and hydraulic elevators, and not less than 3 ft. for drum type elevators, when the pit buffer is completely compressed. If the shaft is covered with a floor under the construction supporting the machinery, these clearance measurements would be to the under part of the floor. They should in any case be ample, and the extra expense for making them so is ordinarily not worth considering.

Most building laws require a grating or floor construction under the overhead sheaves and their supports. Whether this is required or not, such construction is desirable and it should be made substantial. The best method is to make a concrete floor provided with grated openings under the lowest sheaves and under the lowest supporting sheave beams, covering the entire area of the shaft. The grating is desirable to permit of the exit of smoke that might find its way into the shaft in spite of all efforts to prevent it. The grating should be sufficiently close to prevent ordinary tools from falling through.

Ordinarily 8-ft. head room above this overhead floor will afford ample room for the sheaves and their supports and for taking care of them. If the machinery is over the elevators, this space should be increased 3 or 4 ft., and a separate floor should be constructed immediately under the machinery. If the machines are over the elevators, the room containing the machines should be incorporated into the shaft construction, and in either case, all the overhead construction should be thoroughly fireproof and substantial, and should be well lighted with skylights or windows.

No rules can be made for the framing around elevator shafts in either steel or reinforced concrete construction. Nearly every building is a new problem. Guide rails should be supported every 12 or 15 ft., and where story heights are greater, the framing of an intermediate support is necessary.

In designing a large building, it is important to obtain a preliminary layout for the elevators from some manufacturer of elevators before completing the design. From such a layout the elevator loads, taken into the columns, can be determined and provided for, and any change in the layout made afterwards is not likely to materially alter *the distribution of the loads so determined.*

When the elevator machinery is in the basement, the total load for each elevator is equal to the weight of the car, plus a live load of 75 lb. per sq. ft. of car floor, multiplied by 2, plus the weight of the cables. The weight of the car and its live load is multiplied by 2 to cover the counterbalance and lifting load. The total load to be taken care of in the construction of the building is two times this result. The second multiplication by 2 covers impact and other minor factors.

When the elevator machinery is at the top of the building, the load is somewhat reduced so far as the lifting is concerned, but the weight of the machinery itself, which is considerable, is added. The framework provided for the support of the beams which carry the sheaves, is regarded as a part of the construction of the building. Very heavy beams are sometimes required for this purpose. The requirements must be determined from the layout of the elevators, and if the original calculation is made from a preliminary layout, the design should be re-examined when the final layout is provided. The beams that carry the sheaves, ordinarily termed "sheave beams," are included as a part of the elevator contract, and not a part of the construction of the building.

All elevators have buffers and must be constructed with pits, or with extensions of the elevator shaft below the lowest level to which the elevator is to descend. If the elevator is to stop at the first floor, and there is a basement in the building, and it is desirable, it will be sufficient to extend the elevator shaft to the basement floor, and to construct the walls with a doorway from the basement into the shaft. Two or more shafts of this character adjoining each other should be connected in the basement by doorways. If the machinery is in the basement, the machine room should be of fireproof construction adjoining the shafts, and connected to them by doorways in the basement.

If it is desired to have one or more elevators run to the basement, the shafts should be constructed with the pits below the basement floor the full size of the shaft. These pits should be made of masonry, waterproof, and not less than 4 ft. in depth. If the speed of the car exceeds 400 ft. per minute, the pit should be 6 ft. deep. There are two things that may make the construction of these pits difficult: (1) the possible effect they may have upon the design of the foundations of the building, and (2) the waterproofing of the walls and floor so that the pit shall be perfectly dry. The best way to meet the foundation difficulty is to keep the pit away from the foundations, though that may involve the whole scheme of the elevator arrangement. The pit should always be waterproofed, but sometimes the work must be especially well done to keep the pit dry.

TANKS

By H. J. Burt

242. Sprinkler Tanks.—For the highest grade service, two types of tanks are used jointly—a pressure tank and a gravity tank. The *pressure* tank provides the high pressure which is needed at the beginning of the fire. (In very large installations, it is advisable to make two units of the pressure tank.) The *gravity* tank when used with the pressure tank, furnishes the reserve supply, and comes into action when the pressure in the pressure tank has dropped to a point where the water will flow from the gravity tank into the pressure tank. The gravity tank is set at such an elevation that it will give an effective pressure at the highest sprinkler head, though not as great as given by the pressure tank.

In a less efficient installation, the gravity tank alone may be used. In cases where only a few heads are installed, the house tank may be used as a supply, but the practice should not be followed to any extent, and if used, the house tank should be increased in size and arranged so that the sprinkler supply is constant and cannot be reduced by the house service demands.

The all essential thing about pressure tanks is to have them air-tight. All tanks must be water-tight.

242a. Location of Sprinkler Tanks.—If ground space is available, and particularly if several buildings are to be served from the gravity tank, it is desirable to make the tank structure independent of the building. Steel water-towers, which have been highly developed and standardized by a number of manufacturers, are best suited for this purpose.

If, as is usually the case in cities, space outside the building is not available for this structure, the tanks must be supported on the building. The structural problem of carrying the weight will usually govern the location, although in some instances appearance will have an influence. The following cases illustrate locations and methods of support:

On narrow buildings, say 50 ft. or less in width, having masonry supporting walls, trusses may be used, spanning from wall to wall. The position selected for the trusses will be governed by any other features, such as chimneys, pent houses, etc., that need to be cleared. The walls, as normally built, will most likely have the necessary strength to carry the load, and to distribute it over a considerable length of foundation. Fig. 389 illustrates a structure of this description.

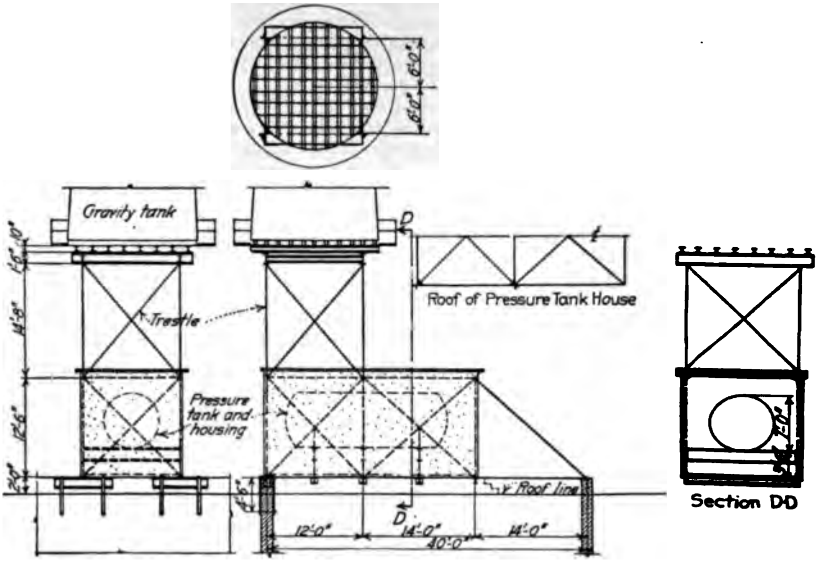


FIG. 389.—Sprinkler tanks supported by trusses spanning from wall to wall.

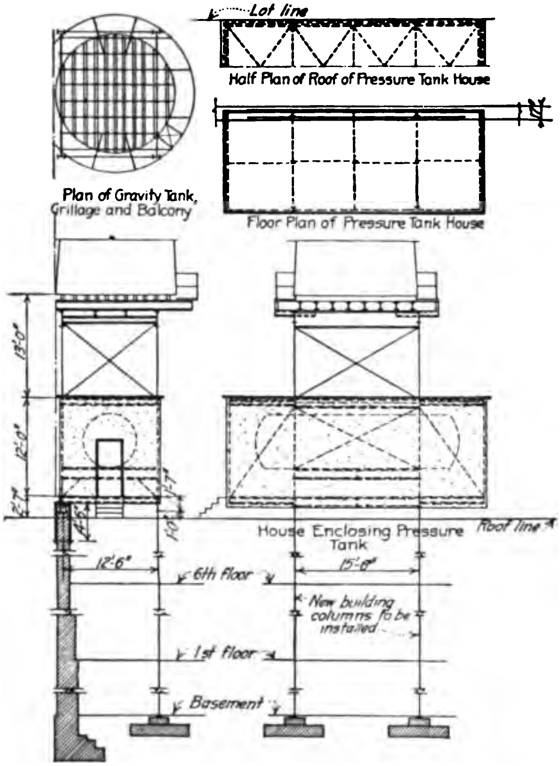

Lot line
Half Plan of Roof of Pressure Tank House
Plan of Gravity Tank, Grillage and Balcony
Floor Plan of Pressure Tank House
13'-0"
12'-0"
House Enclosing Pressure Tank
Roof line
12'-6"
15'-0"
6th floor
New building columns to be installed
1st floor
Basement

FIG. 390.—Sprinkler tank supports, using one wall of building and two new building columns.

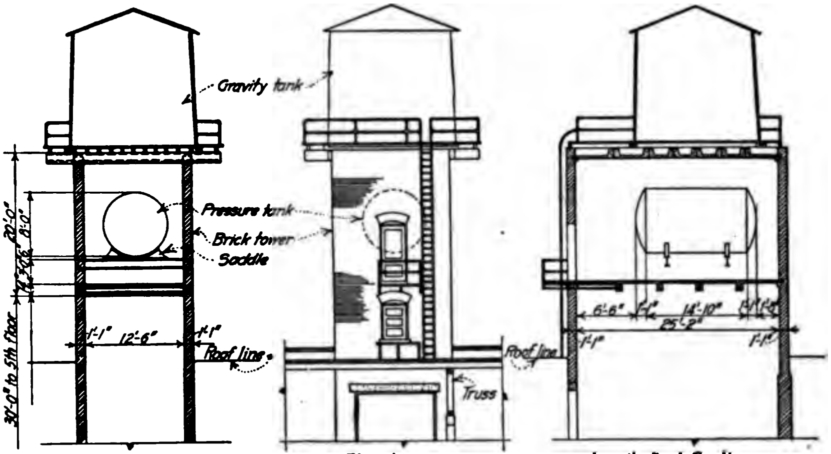
Four of the building columns, if of fireproofed steel or concrete, may be selected to support the tank, and be designed to carry the additional weight. The weight of the tank structure and the water should be treated as dead load in its effect on the foundations.

Fig. 390 illustrates a case when the wall of the building furnishes support for one side of the tank structure and two new columns have been inserted in the building to support the other side.

If there are masonry walls enclosing an elevator or stair shaft, they may provide the support for the tank. They may, if desired, be extended upward to form a tower enclosing the tanks. Fig. 391 illustrates such a case.

The pressure tank may be placed in the basement or put underground outside the building. In such cases it must operate under greater air pressure. Such location makes the piping more complicated if a gravity tank also is used. It is not recommended if it can be avoided.

242*b*. Supports for Gravity Tanks.—The design of the supports for gravity tanks involves gravity loads and wind loads. Gravity tanks are treated as dead loads, the tanks being filled to capacity. No deductions are made as is done for floor loads. Tanks and tank structures are usually in exposed situations, and it is recommended that they be designed to resist a wind pressure of 30 lb. per sq. ft. on the projected area of tank and supports.



Cross Section Elevation Longitudinal Section

FIG. 391.—Sprinkler tank supported by brick tower which houses pressure tank over elevator shaft.

The gravity and wind stresses are concurrent. The supports will be designed for the maximum combinations of stress. If with an empty tank, the wind produces an uplift at any bearing, suitable anchorage must be supplied.

The wood tank must be supported on chime joints so cut as to clear the ends of the staves and thus receive the whole load from the tank bottom. It will generally be advantageous to specify the standard sizes of local wood tank manufacturers.

It is desirable that supports within the building be fireproof.

242*c*. Pressure Tanks.—The pressure tank is a steel cylinder placed horizontally with segmental ends. The usual working pressure when placed on top of the building is 75 lb. per sq. in. The tank should be designed for a greater pressure, say 100 lb. per sq. in.

The stress on a longitudinal joint per linear inch is $P \times r$, P being the pressure in pounds per square inch and r the radius in inches. The stress on a circumferential joint per linear inch is $P \times \frac{r}{2}$. This is the stress on the joint connecting the segmental ends to the cylinder, and is also the stress in the head if the head is a full hemisphere. $P = 100$ lb.

Assume a tank of 6-ft. diameter, or $r = 36$ in., joint efficiency 50%, and unit stress 16,000 lb. per sq. in. Then the stress on longitudinal joint $= 100 \times 36 = 3600$ lb. per linear inch, and the thickness of plate required $= \frac{3600}{(16{,}000)(0.50)} = 0.45$ in. Use $\frac{7}{16}$-in. plate.

The stress in the joining of the segmental end to the cylinder is $100 \times 18 = 1800$ lb. per lin. in., and the thickness of plate required for the segmental end $= \frac{1800}{(16{,}000)(0.50)} = 0.225$ in., say $\frac{1}{4}$ in. On account of the work required in shaping the head, it is desirable to make it thicker than the computed amount, and to adopt $\frac{3}{8}$ in. as a minimum.

Careful designing of the riveting of the joints may give an efficiency greater than 50% and thus reduce the thickness of plate.

Under normal working conditions, the pressure tank is ⅔ full of water, the other third being air space. In giving the capacity, the water space only is indicated. Given the volume of water, multiply it by 1½ to get total volume of the tank.

The tanks are set in two saddles made of wood or cast iron, as shown in Fig. 391. These supports should be at approximately the quarter points. The supporting beams should be so designed that they will be capable of supporting the tank when full of water.

The appurtenances, such as manholes, gages, pipe connections, and enclosure, must be as required by the regulations of local authorities or the insurance representatives.

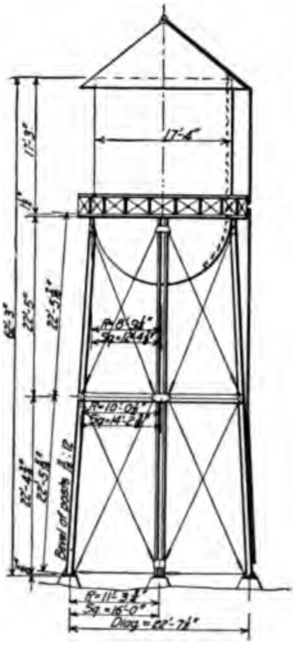


Fig. 392.—General plan of 40,000-gallon tank and tower.

242d. Gravity Tanks.—The gravity tank is usually a cylindrical tank and may be constructed of steel, concrete, or wood.

The steel tank with a hemispherical bottom is the most satisfactory type if conditions permit its use. This type has been standardized by a number of manufacturers. Their designs can be checked or new designs made as explained in Ketchum's "Structural Engineers' Handbook," p. 365. This form of tank may be used whether set on an independent tower outside the building or on a special tower on top of the building (see Fig. 392).

Concrete tanks can be made but are not much used. The expense of forms and of constructing the small yardage of concrete at such a height, makes them uneconomical. Concrete tanks can best be made with flat bottoms.

Wood tanks are cheapest and least durable, but will give good service if well built and well maintained.

242e. Design of a Cylindrical Gravity Tank.—The stress on the longitudinal seam, or section, of a cylindrical tank is Pr per linear inch as given on p. 647. If the cylinder is vertical, the pressure P at any level is $0.434H$, H being the depth in feet below the surface of the water.

Assume for example a tank 16 ft. in diameter, and 20 ft. high; then the maximum stress on the cylinder, *i.e.*, just above the bottom, $= 0.434 \times 20 \times 8 \times 12 = 833$ lb. per sq. in. This stress must be resisted by the plate of a steel tank, the reinforcing rods of a concrete tank, or the hoops of a wood tank.

For the *steel tank*, a unit stress of 16,000 lb. per sq. in. will be used, with 50% efficiency of joint, or 8000 lb. per sq. in. on the gross area. The sectional area required $= \frac{833}{8000} =$ 0.104 sq. in. This being a section 1 in. high, the thickness required is 0.104 in. A ⅛-in. plate will be sufficient for the stress, but for surety of calking and durability, 3⁄16 or even ¼ in. may be considered the desirable minimum.

For the *concrete tank*, a steel stress of 12,000 lb. per sq. in. will be used. Thus the steel required per inch of height is $\frac{833}{12,000} = 0.07$ sq. in. Round rods ½ in. in diameter have an area of 0.1963 sq. in. and are to be spaced 2¾ in. apart at the bottom. Likewise, the spacing and size are computed at successive elevations up the sides of the tank. Low unit stresses are used for the rods to avoid stretch that might produce minute cracks.

For the *wood tank*, a steel stress of 12,000 lb. per sq. in. will be used for the hoops. Then the steel required per inch of height is $\frac{833}{12,000} = 0.07$ sq. in. Round rods ¾ in. in diameter, having a net area (in the thread) of 0.30 sq. in. can be spaced 4¼ in. centers near the bottom, and at wider distances upward toward the top. Round rods must be used; flat bars are not permissible on account of rapid corrosion. Low unit stresses are used for the rods to allow for initial stress.

The flat tank bottom can be considered as a series of beams 1 in. wide and designed according to the weight of the water and the spacing of the supports. The bottom of a steel tank will not be less in thickness than the lowest course of side plates. The bottom of a concrete tank will not be less than 3 in. and may be cast integral with the supporting beams.

The bottom of a wood tank will not be less than 1⅝ in. net thickness.

For details of the design of steel tanks, see Ketchum's "Structural Engineers' Handbook," p. 365.

For details of the design of concrete tanks, see Hool and Johnson's "Concrete Engineers' Handbook," p. 765.

For details of the design of wood tanks, see "Regulations of the National Board of Fire Underwriters for the Installations of Gravity and Pressure Tanks."

243. House Tanks.—In important buildings it is generally necessary to provide one or more tanks for water supply. Various local conditions require their use. The pressure of the public supply may not be sufficient to deliver water to the upper floors, or the public supply may be unreliable as to pressure, and it is always subject to accident or to heavy draft for fire purposes. Accordingly, the tank is designed to secure the proper pressure for the upper floors to which the city supply will not reach, also to act as an equalizer between the pump discharge and the building demand and provide a supply for a short period of time in the event of the shutting down of the service. The lower floors should be taken care of by the service pressure if such does not complicate the piping system.

The supply may come from a private well; or, treated water may be used for drinking or culinary purposes, thus making a tank necessary.

243*a*. Capacity of House Tanks.—The capacity required varies with the uses and conditions. No very definite rules can be given. If the pumping plant is automatic, the storage need be only enough for two or three hours of maximum use. If the plant requires manual operation, two or more pumpings a day may be planned. For very small buildings, 1000-gal. capacity is ample, increasing from this size to 2000 or 2500 gal. Beyond this size, it is generally advisable to install two tanks, cross connected and valved so that either may be thrown out of service for cleaning purposes.

It is advisable to make the tank as small as practicable, so that the water may be changed frequently and remain fresh. In large important buildings, such as hotels, etc., it is advisable to provide two services from two street fronts if practicable, to avoid interruption in the service to the house tank supply. The available space for the tank and the cost of installation may have an influence in deciding the capacity.

243*b*. Location of House Tanks.—The storage must be of course above the highest fixture to be served. The usual location is in the attic space or in a pent-house above the roof. In the latter case, it is desirable to locate it adjacent to the elevator pent-house, to avoid the building of a separate house. In some cases it may be enclosed in a stairway pent-house. It should be enclosed for protection against dirt and against freezing. Heating may be necessary.

243*c*. Construction Materials.—The tank may be constructed of either steel, concrete, or wood. Steel is preferable, as it can be readily made water-tight and with reasonable maintanance will be permanent. Its cost will be greater than concrete or wood. Concrete may be used but will require special care in construction to make it water-tight, especially at pipe connections. Its use would be appropriate only in a concrete building.

Wood is the cheapest material, and can be made tight if sufficient care is used in construction. It cannot be considered permanent. Greater security against leakage in rectangular tanks can be secured by lining with sheet lead or with tin plate having soldered joints. The wood is more likely to rot if the tank is lined than if it is unlined.

243*d*. Details of House Tanks.—The supply pipe should be run over the top of the tank, or its outlet placed at the level of the overflow; otherwise, any failure of its supply or leakage through the pump will drain the tank. Connection of the supply pipe to the distributing system is objectionable for the above reason and the added reason that it transmits vibrations throughout the distributing system. The outlet should be 2 or 3 in. above the bottom to allow for the deposit of sediment, but a drain should be taken from the bottom to secure thorough cleaning when necessary.

An overflow outlet shall be provided at least 6 in. below the top of the tank. The pipe should be at least as large as the supply pipe and should not be connected to the drainage or plumbing system of the building, but should discharge on to the roof.

It is desirable to set the tank in a steel pan, the pan provided with a drain pipe discharging in a conspicuous place so that any leakage or overflow will be quickly discovered. This pan is essential for steel tanks on account of condensation. The pan should be about 3 in. deep and about 1 ft. larger diameter than the tank.

A reliable tell-tale or gage must be used with its index in a conspicuous place near the pump or place of control

243e. House Tank Design—*Rectangular Wooden Tanks.*—Assume tank 12 ft. long, 6 ft. wide and 6 ft. deep (Fig. 393). The unsupported length of side plank is 72 in. Maximum pressure near bottom of tank is $0.44H = 2.64$ lb. per sq. in., or 380 lb. per sq. ft. The bending moment on a strip 1 in. high (as simple beam) $= \frac{2.64 \times 72 \times 72}{8} = 1710$ in.-lb. The appropriate thickness, t, of plank can be determined from this bending moment. Allowing a fiber stress of 1400 lb. per sq. in., we have $\frac{1}{6} \times 1400t^2 = 1710$. $t = 2.7$ in. Use 3-in. plank (net thickness dressed $2\frac{5}{8}$ in.). This thickness is suitable for sides, ends, and bottom.

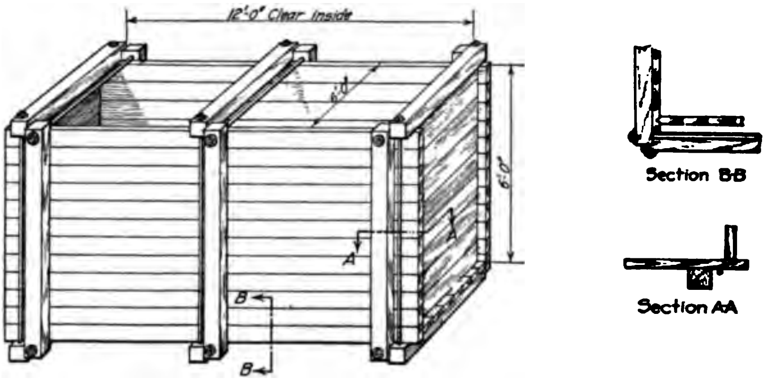


Fig. 393.—Rectangular wooden tank.

The buck stay is designed as follows:

$$\text{Total load} = 6 \times 6 \times \frac{380}{2} = 6840 \text{ lb.}$$

$$\text{Stress in top rod } (\tfrac{1}{3}) = 2280 \text{ lb.}$$

$$\text{Stress in bottom rod } (\tfrac{2}{3}) = 4560 \text{ lb.}$$

$$M. \text{ (approx.)} = \tfrac{1}{8} \times 6840 \times 6 = 5130 \text{ ft.-lb.} = 61{,}560 \text{ in.-lb.}$$

This requires a 6 × 8-in. timber.

The maximum rod stress given above requires 0.28 sq. in. net section computed at 16,000 lb. per sq. in. but as this rod will have an initial stress due to cinching up the tank and may have additional stress from swelling of the wood it is considered expedient to use $\frac{7}{8}$-in. round rod having a net area of 0.41 sq. in.

The vertical rods have no stress from water pressure but have the cinching and swelling stresses referred to above. For simplicity of design $\frac{7}{8}$-in. round rods will be used throughout.

Cypress, red wood, fir and long leaf pine are suitable for tank construction. No nails, screws, or bolts should be used, the tank being held together with timbers and rods as shown. Sills are used to allow circulation of air under the tank, to avoid decay. The sills must be notched if necessary so that the tank bottom will bear directly thereon. No painting is permissible on the planks. They are left free to absorb the water, thus preventing shrinkage and resulting leaks, also preventing decay. The tie rods and fittings should be heavily painted with red lead or asphalt. All joints should be grooved or splined and set in a paste of white lead and oil.

Rectangular Concrete Tanks.—The pressures and their application are the same for concrete tanks as described for wood tanks. Two sets of rods must be used in each slab, placed at right angles to each other, whether required by the stresses or not. This is to prevent cracks. The vertical rods of the sides and ends should be continuous with the bottom rods, *i.e.*, the rod should extend down one side, across the bottom, and up the other side. The horizontal rods in the sides and ends should be continuous around the perimeter of the tank and spliced.

The concrete must be of a very dense mixture to meet both the structural and waterproof requirements. The concrete may be made waterproof as explained in Sec. 5, Art. 29.

The pan for a concrete tank may be made by forming it of a membrane waterproofing laid directly on the concrete floor, and covering it carefully with at least 3 in. of concrete.

Cylindrical Tanks.—The sizes of cylindrical tanks for house supply are so small that minimum sections will generally be used.

For steel tanks ¼-in. plate should be used throughout.

For concrete tanks, the walls and bottom should be 3 in. thick. The circumferential rods should be ⅜-in. rounds spaced 3 in. on centers, and the vertical rods should be of the same diameter spaced 1 ft. on centers. For the bottom, ⅜-in. rods should be used, both directions, spaced 4 in. on centers with the ends bent up into the walls 6 to 8 in.

For wooden tanks, staves and bottom should be not less than 1⅛ in. thick, net. The rods should be ¾-in. rounds spaced 6 in. on centers near the bottom and 12 in. maximum near the top.

If the tank is over 10,000 gal. capacity, it should be designed as illustrated in Art. 242*e*.

244. Gasolene Tanks.—Local building regulations should be consulted in regard to gasolene tanks Good practice requires gasolene tanks to be buried in the ground and covered with not less than 5 ft. of earth; and to be placed outside the walls of the building. Before being placed, tanks should be given a heavy coat of asphalt paint. After being set in place with all fittings attached, and before being covered, they should be tested with a pressure of 75 lb. per sq. in.

Gasolene tanks and their fittings are standardised by the manufacturers, and their standards should be followed. The thickness of shell and the riveting can be checked on the basis of the test pressure of 75 lb. per sq. in.

The size of tank may be limited by municipal regulation. The quantity to be stored can best be determined from the needs of the industry served. The ordinary tank-car holds about 10,000 gal. If purchased by the car-load, the storage provided should be about 50% in excess of the car-load.

If no local regulations govern the construction and placing of the tank, it should conform to the regulations of the National Board of Fire Underwriters for the Installation of Containers of Hazardous Liquids.

WIND BRACING OF BUILDINGS

By H. J. Burt

It is assumed that wind pressure acts horizontally and exerts a uniform pressure over the entire surface of the windward side of the building. Although in certain localities, as along the Gulf Coast, the severe storms come from one direction, it is customary to assume that the maximum pressure may be in any direction. In designing wind bracing it is not considered necessary to take into account the suction on the leeward side, the greater pressure at the corners of the building, or the variation of pressure with height. It is, of course, permissible to take advantage of the protection afforded by adjacent permanent buildings.

245. Wind Pressure.—The formula commonly used for expressing the relation between wind velocity and pressure is: $P = 0.004V^2$, in which V is the velocity in miles per hour, and P the pressure in pounds per square foot. This formula is of little practical use because of the uncertainty of the velocity to be provided for. For 80 miles per hour, it gives a pressure of 25.6 lb. per sq. ft.

The pressure most commonly used is 20 lb. per sq. ft. of projected area. This is required by building codes of some cities. The City of New York Building Code of 1917 requires 30 lb. per sq. ft.

Where legal requirements do not govern, it may be permissible to use 15 lb. per sq. ft. on low mill buildings where storm conditions are not likely to be severe. There are other situations where 30 lb. or even 40 lb. per sq. ft. are justified, such as for very high buildings and for buildings having large open spaces with few partitions and floors. A high wind pressure should also be used in the design of towers and signs, and for buildings in localities subject to hurricanes.

246. Effects of Wind Pressure.—The effects of wind pressure are: (a) a tendency to overturn the building as a unit, which must be resisted either by the dead weight of the building or by anchorage; and (b) a tendency to collapse the building, which must be resisted by the structural parts of the building.

247. Path of Stress.—The wind pressure must ultimately be resisted by the foundations of the building. It is applied to the wall surfaces, including windows; it is then transmitted to the floors or columns; and thence through the structural framing or cross-walls to the foundations. The path must be continuous and as direct as possible, and all members along the path must be capable of transmitting the stress in addition to their other functions. Several alternate systems of bracing may be devised for a given building. The one to be preferred structurally is that which is most direct from the exposed surface to the foundations, but the architectural requirements may compel a more devious routing. Wherever possible, advantage is taken of the members required by the gravity loads.

248. Unit Stresses.—As maximum wind stresses occur only at long intervals it is allowable to use a higher unit stress than for gravity loads. It is well established practice to specify that for stresses produced by wind alone or combined with gravity stresses, the units may be increased 50%; but the section must be not less than required for the gravity loads.

249. Resistance to Overturning.—The wind pressure on a building tends to rotate it about a horizontal axis at the ground level or at the foundation level on the leeward side.

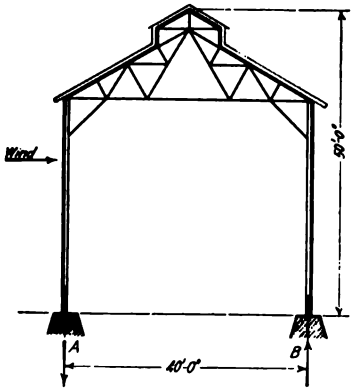


FIG. 394.—Section through mill building to illustrate overturning moment of wind load.

Assume a masonry building 40 × 100 ft. in plan, and 120 ft. in height. The maximum overturning moment about this axis is:

100 (length) × 120 (height) × 20 (pressure) × 60 (moment arm) = 14,400,000 ft.-lb.

To determine the resisting moment, the dead weight must be computed, but for purposes of illustration it is assumed in this case to be 6,000,000 lb. The resistance to overturning is:

Weight × ½ width = 6,000,000 × 20 = 120,000,000 ft.-lb.

This gives a wide margin of safety. The ratio of resistance to overturning should be not less than 1½ to 1.

Assume a steel mill building shown in section, Fig. 394. Assume panel lengths of 20 ft., and that each panel is fully braced transversely. Then the overturning moment is:

20 × 50 × 20 × 25 = 500,000 ft.-lb.

Assume that the computed weight of one panel of the building is 16,000 lb., then its resisting moment is:

16,000 × 20 = 320,000 ft.-lb.

The required resistance is 1½ × 500,000 = 750,000 ft.-lb. Thus, anchorage must be provided for 750,000 − 320,000 = 430,000 ft.-lb. The anchorage and weight of footing required at *A* (and *B*) is represented by the couple *AB*, Fig. 394. $\frac{430,000}{40} = 10,750$ lb.

250. Resistance to Collapse.—In order to prevent collapse from wind pressure, the wind bracing must transmit the horizontal wind pressure to the foundations. This can be accomplished by two types of frame work: (1) triangular, Fig. 395, having axial stresses, and (2) rectangular or portal framing, Fig. 396, having bending stresses.

251. Triangular Bracing.—The analysis of a single panel of triangular bracing is shown in Fig. 397. The wind load is assumed to be concentrated and is represented by W. The horizontal reaction at the foundation is $R = W$. The vertical reaction is $V = V' = W\frac{H}{L}$. The stress diagram gives the stresses in *a*, *b*, and *c*. The stresses are all axial.

The system of triangular bracing may be extended horizontally and vertically by additional panels, as in Fig. 398. The wind pressure is computed for each story and applied at each floor and the roof levels, as represented by W_R, W_3, etc.

Beginning at the top, the stresses in the top story members are determined. The horizontal shear W_R is divided equally between the panels of the third story, and the stresses in the members of the third story are determined as described above. If the panels are equal, the stresses of corresponding members will be equal. Each intermediate column has two equal and opposite values of V, which cancel. The diagonal stresses are $\frac{W_R}{3} \times \sec. \alpha$.

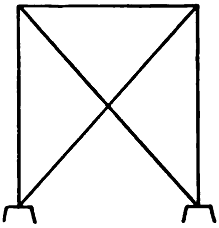


Fig. 395.

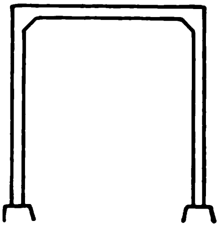


Fig. 396.

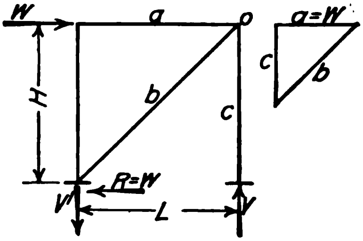


Fig. 397.

The loads of the third story are transmitted to the next lower story at the third floor, by the antireactions V_1 and V_4 at columns 1 and 4 and by the horizontal shear $\frac{W_R}{3}$ at columns 1, 2, and 3. To these are added $\frac{W_3}{3}$ and the second story stresses are determined as before. The diagonal stresses in this story are $\frac{W_R + W_3}{3} \times \sec. \beta$.

The horizontal load or shear to be resisted in any story or tier, is the sum of all the horizontal loads above that tier.

If the panels are unequal in length, each must be analyzed, and the values of V for the intermediate columns will not fully cancel. However, these values, which are column stresses, will rarely require any additions to the column section of the intermediate columns.

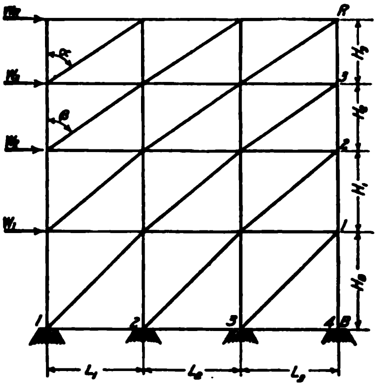


Fig. 398.—Diagram of triangular framing extending over a building.

Having determined the stresses, the sections are designed using unit stresses according to

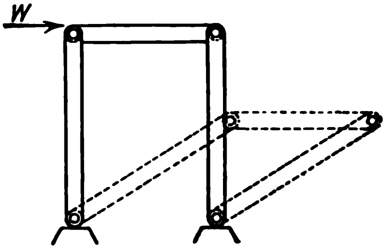


Fig. 399.

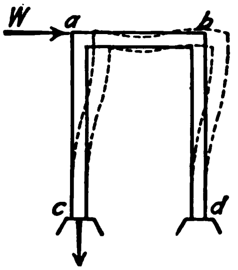


Fig. 400.

Art. 248. The diagonals carry wind stresses only. The verticals which are the building columns, and the horizontals which are girders or joists, will not usually need to be increased to carry the wind stresses.

252. Rectangular Bracing.—A rectangular frame with hinged joints offers no resistan a horizontal force, but will collapse as indicated in Fig. 399. A rectangular frame rigid joints will resist a horizontal force and tends to distort as shown in Fig. 400. In so torting, the members take the form of re curves with points of contraflexure at length.

In Fig. 401, assume hinges at the poin contraflexure e, f, and g. The bending mom at a, b, c, and d, in the verticals and at a and the horizontal, are equal, with a value of ½

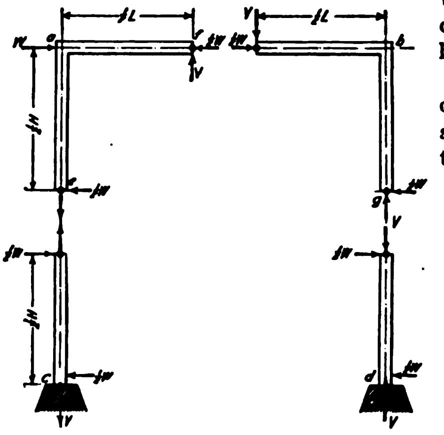


Fig. 401.—Illustrating wind load and reactions on a stiff bent.

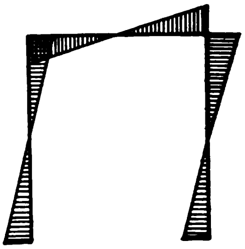


Fig. 402.

In addition to the bending stresses, the direct stresses are: ½W (compression) in ab, V = ½ (compression) in bd, and $V = \frac{1}{2}W\frac{H}{L}$ (tension) in ac. Fig. 402 is a graphical representatio the bending moments.

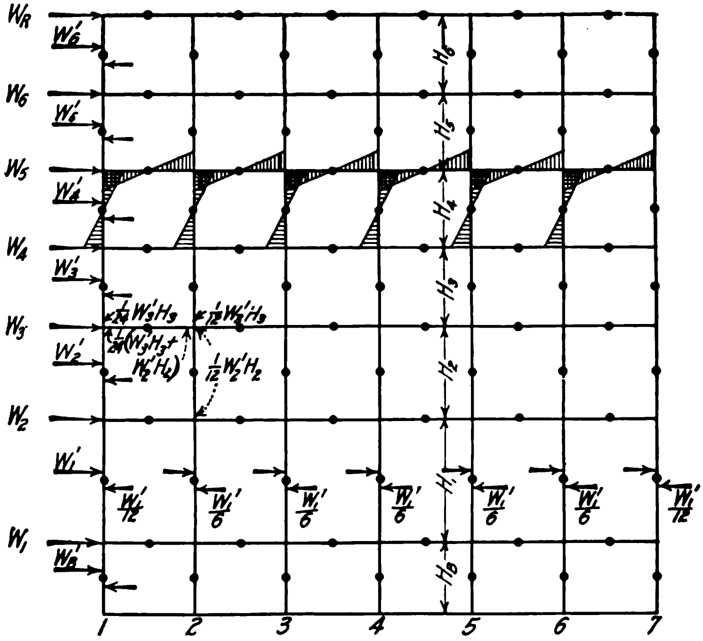


Fig. 403.

This analysis may be extended to any number of panels, and any number of stories. T is illustrated in Fig. 403. W_1, W_2 . . . . . W_R represent the wind loads at the several fl and roof levels. W_B', W_1', W_2', etc., represent the shears to be resisted by the columns in the successive stories, and in each case, is the summation of all the wind loads above that level. H_B, H_1, etc., represent the story heights.

It is necessary to assume the distribution of the shear among the columns. The assumption here made is $\frac{W'}{n}$ for the shear at the intermediate columns, and $\frac{W'}{2n}$ at the outside columns, n being the number of panels.

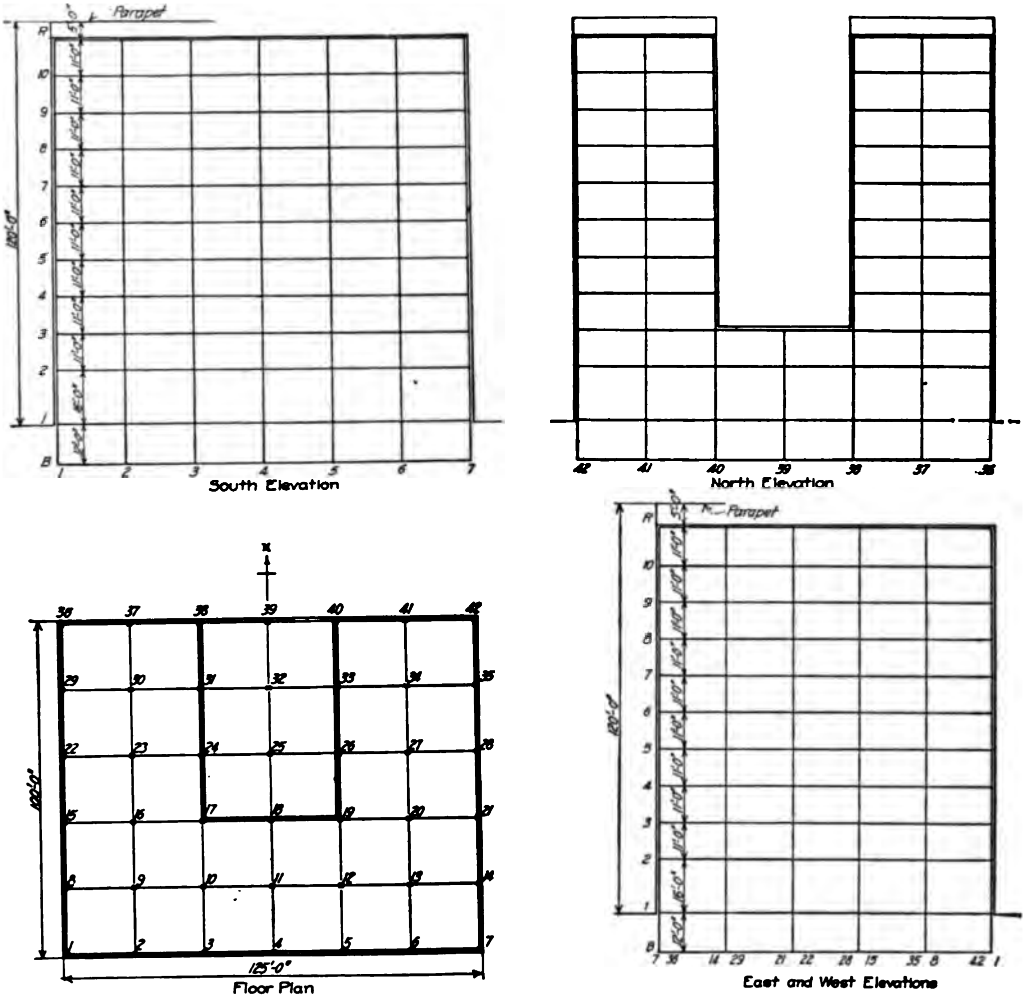


Fig. 404.—Diagram of plan and elevations for computing wind load moments and shears.

The bending moment in an intermediate column in any story equals the total shear in that story multiplied by half the story height, and the product divided by the number of panels. This is expresed by the formula

$$M = \frac{W'H}{2n}$$

The bending moment in an outside column is one-half that in an intermediate column, or

$$M = \frac{W'H}{4n}$$

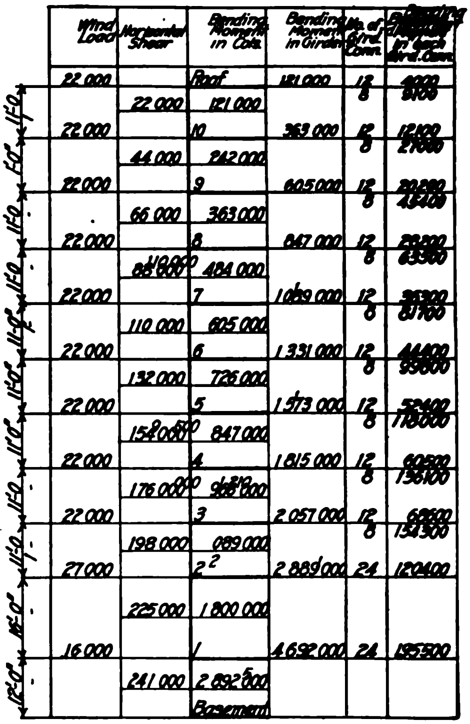
The bending moment in each girder connection at an intermediate column is the mean between the bending moments in the column above and below the girder. It is expressed by the formula

$$M = \frac{1}{2}\left(\frac{W'_aH_a}{2n} + \frac{W'_bH_b}{2n}\right) = \frac{1}{4n}(W'_aH_a + W'_bH_b)$$

The bending moment in a girder connection at the outside column is the same in amount as at intermediate columns.

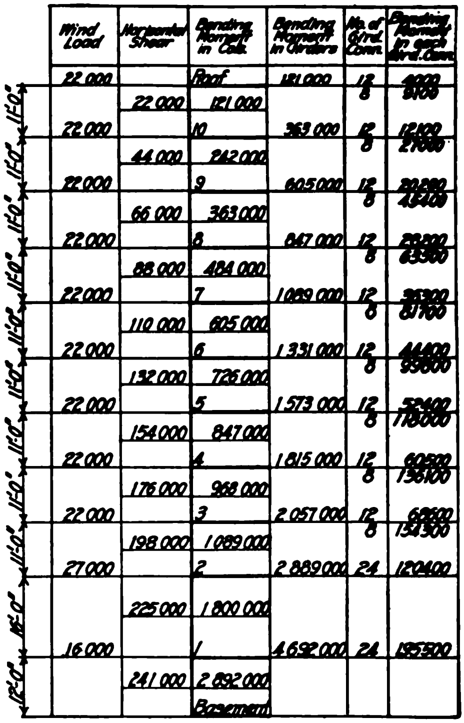
In the above formulas, a and b refer to two adjacent stories, as the third and fourth. The panel length does not affect the value of the bending moments.

In computing the shears and bending moments, the totals may be computed for each story of the entire building and these totals divided among the girder connections and the columns which resist them.



	Wind Load	Horizontal Shear	Bending Moment in Cols.	Bending Moment in Girders	No. of Gird. Conn.	Bending Moment in each Gird. Conn.
	22 000		Roof	121 000	12 8	4000 9100
11'0"		22 000	121 000			
	22 000		10	363 000	12 8	12100 27000
11'0"		44 000	242 000			
	22 000		9	605 000	12 8	20200 45400
11'0"		66 000	363 000			
	22 000		8	847 000	12 8	28200 63500
11'0"		110 000 88 000	484 000			
	22 000		7	1 089 000	12 8	36300 81700
11'0"		110 000	605 000			
	22 000		6	1 331 000	12 8	44400 99800
11'0"		132 000	726 000			
	22 000		5	1 573 000	12 8	52400 118000
11'0"		154 000	847 000			
	22 000		4	1 815 000	12 8	60500 136100
11'0"		176 000	968 000			
	22 000		3	2 057 000	12 8	68600 154300
11'0"		198 000	089 000			
	27 000		2²	2 889 000	24	120400
15'0"		225 000	1 800 000			
	16 000		1	4 692 000	24	195500
12'0"		241 000	2 892 000			
			Basement			

Tabulation of wind loads, and resulting bending moments. Wind from North or South.



	Wind Load	Horizontal Shear	Bending Moment in Cols.	Bending Moment in Girders	No. of Gird. Conn.	Bending Moment in each Gird. Conn.
	22 000		Roof	121 000	12 8	4000 9100
11'0"		22 000	121 000			
	22 000		10	363 000	12 8	12100 27000
11'0"		44 000	242 000			
	22 000		9	605 000	12 8	20200 45400
11'0"		66 000	363 000			
	22 000		8	847 000	12 8	28200 63500
11'0"		88 000	484 000			
	22 000		7	1 089 000	12 8	36300 81700
11'0"		110 000	605 000			
	22 000		6	1 331 000	12 8	44400 99800
11'0"		132 000	726 000			
	22 000		5	1 573 000	12 8	52400 118000
11'0"		154 000	847 000			
	22 000		4	1 815 000	12 8	60500 136100
11'0"		176 000	968 000			
	22 000		3	2 057 000	12 8	68600 154300
11'0"		198 000	1 089 000			
	27 000		2	2 889 000	24	120400
15'0"		225 000	1 800 000			
	16 000		1	4 692 000	24	195500
12'0"		241 000	2 892 000			
			Basement			

Tabulation of wind loads, and resulting bending moments. Wind from East or West.

FIG. 405.

In addition to the bending stresses, there are axial stresses in the horizontal and vertical members. The stresses in the horizontal members are compressive and result from the assumed distribution of the shear to the successive columns. Thus, at the third floor level the compressive stresses in girders are:

1st panel, $11/12W'_3$; 2nd panel, $9/12W'_3$; 3rd panel, $7/12W'_3$, etc.

The axial stresses in the intermediate columns are zero if the panels are of equal length, and in any case are so small relative to the gravity stresses that they can be disregarded.

The axial stresses in the outside columns can best be determined by treating the structure as a unit, for overturning, as shown in Art. 249. The resulting values of V are stresses that must be taken into account in designing the columns.

Illustration of the Computation of Wind Bending Moments.—Assume the building illustrated in Fig. 404. The exposed area is from the ground level to the top of the parapet wall, 120 ft. The parapet is assumed in this case to be 5 ft. above the roof level and gives a load area at the roof line equal to the load area at the typical floor. The wind pressure will be taken at 20 lb. per sq. ft.

It is assumed that the wall construction is strong enough to carry the wind load to the floor levels and that the floor construction is capable of distributing the load into the steel framing at the points where the resistance is provided. The computations are tabulated in Fig. 405.

Consider first the wind from north or south. The load at the roof level = 11 × 125 × 20 = 27,500 lb. (Fig. 405). Similarly, the loads at the successive floors are computed. The accumulated shears in the successive stories beginning at the top are 27,500, 55,000, etc.

The total bending moment in the columns of any story is the shear in that story multiplied by half the story height. Thus, in the tenth story, $M = 27{,}500 \times 5\frac{1}{2} = 151{,}250$ ft.-lb.; in the ninth story, 302,500 ft.-lb. The bending moments here given occur at the top and at the bottom of the column section, equal in amount and opposite in direction. In the basement, the moment arm is the story height, it being assumed that the base of the column is not fixed, to resist bending, but is fixed against sliding.

The total bending moment in the roof girders is the same as the total in the tenth story columns, 151, 250 ft.-lb.; in the tenth story girders it is the sum of the bending moments in the tenth-story and ninth-story columns, *i.e.*, 453,700 ft.-lb.; and so on at the successive floor levels. These moments are the totals to be resisted by the girder connections to the columns.

The next step is to fix the number and location of the girder connections that will be provided to resist the bending moment. In the north and south direction, provide for wind bracing along the column lines 1–36, 17−42, 17−38, and 19−40, Fig. 404, and make all connections of equal strength. This gives 32 girder connections, among which to divide the total bending moment at the successive floors.

Considering next the wind from the east or the west, the shears and moments are computed in the same manner as described above and are recorded in Fig. 405. In the east and west directions, wind bracing girders can be used along column lines 1−7, 17−19, and 40−42 (or 36−38), at the floor levels from the third to the roof; and along column lines 1−7 and 36−42 at the first and second floors. In the upper floors (third to roof) in order to use the shortest route for the stress, 40% will be taken along the column lines 1−7, and 60% divided equally along the column lines 17−19 and 80−42. Thus, the number of connections available in the first group is 12, and in the second group is 8. On this basis, the bending moments to be resisted by the girder connections are computed and tabulated. At the first and second floors the bending moment may be divided equally between the 24 girder connections along the column lines 1−7 and 46−42, and are so tabulated.

If the interior construction permits, it is desirable to use winding bracing along columns 17−19 in the first and second floors. In this case, the same percentage of burden will be assigned to them as in the upper floors—*i.e.*, 30%—and 30% will be carried along columns 36−42.

The architectural requirements may permit the interior floor girders to be utilised as wind bracing. In such cases, the distributions of the total bending moment will be made according to the conditions.

If the basement story columns are embedded in masonry walls capable of developing the bending resistance in the columns, the first floor girders will be omitted.

253. Combined Gravity and Wind Bending Moments in Girders.

253*a*. Shear.—The vertical shear in a girder, resulting from the wind load, is a function of the horizontal shears transmitted by an intermediate column section above and below the girder, of the story heights, and of the panel lengths. The shear can be expressed by the formula (Fig. 403).

$$\text{Shear} = \frac{W'_a H_a + W'_b H_b}{2nL}$$

in which a and b are subscripts indicating two adjacent stories, as the third and fourth, n = number of panels, and L = panel length.

To the shear thus determined must be added the shear from the gravity load. The resulting total shear is small compared with the bending stresses in the girder and it is not usually necessary to take it into account in designing the riveting of the girder connections. It will appear in the design of these connections that certain rivets near the axis of the girder get small stresses from the bending moment. These rivets can be assumed, or in extreme cases, designed to take the shear.

253*b*. Bending Stresses.—The typical moment diagrams for bending moments from wind loads and gravity loads are shown in Figs. 406(*a*) and 406(*b*), respectively. On the assumption that the end connections are of equal strength and take equal but opposite inclinations, they will resist equal and opposite bending moments. With these conditions, the combined moment diagram will be as shown in Fig. 407. Hence, both the end connections of a girder are designed to resist the bending moments resulting from wind pressure.

The bending moment at the center of the span equals the maximum resulting from the gravity load only. The maximum bending moment occurs some distance to one side of the center as shown in the diagram and must be computed. This moment does not necessarily govern the design of the girder section, because it is made up of the gravity and the wind moments, the latter being resisted at higher unit stresses than the gravity moment.

The girder section will be governed by: (a) the bending moment at the center of span from the gravity load and designed with normal unit stresses; (b) the bending moment at point of maximum, from gravity and wind loads, and designed with unit stresses 50% higher than the normal (see Art. 248); or (c) the bending moment at the splice of the girder section to the gusset plate, from wind load, and designed with unit stresses 50% higher than normal. This case (c) will occur only where there is no gravity load.

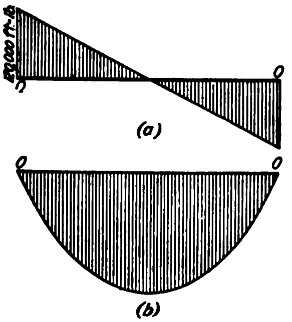


FIG. 406.—Moment diagrams. (a) For wind load. (b) For uniform load on simple beam.

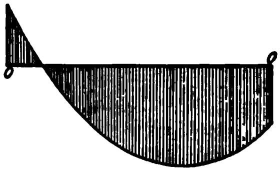


FIG. 407.—Moment diagram for combined loads. Maximum bending moment diagram.

254. Design of Wind-bracing Girders and Their Connections to Columns.—The girder section is designed in the usual manner to resist the maximum bending moment. The make-up of the section may be influenced by architectural conditions, such as vertical space available, character of masonry to be supported, etc. To illustrate the design of the connections, assume an example as follows (Fig. 408):[1]

The maximum bending moment is 400,000 ft.-lb. or 4,800,000 in.-lb.; the depth of girder is 3 ft. 0¼ in. back to back of angles; the unit stresses to be used are 50% in excess of those allowed for gravity loads.

Rivets Connecting Girder to Column.—The rivets through the end angles and column webs are field driven, ¾-in. diameter, and on the tension side of the girder (above the neutral axis in this case) are in tension. As in a beam, the unit fiber stress varies from zero at the neutral axis to a maximum at the extreme fiber; so the unit stress in these rivets varies from zero at the neutral axis to the maximum allowable amount at the farthest rivet.

Then, if the rivets are equally spaced, the average stress is one-half the maximum. The total resistance of the rivets is the average value of one rivet multiplied by the number of rivets in the tension (or compression) group represented by t (or c); the centers of gravity of the groups are at the points t and c. The moment arm is the distance a between t and c and the resisting moment is $a \times t$ (or c).[2] The number of rivets required is determined by trial. The full value of a ¾-in. rivet, field driven, in tension is 1½ times 4400 lb., or 6600 lb. Several trials lead to the use of 28 rivets on each side of the neutral axis. The value of t is $\frac{6600 \times 28}{2} = 92,400$ lb. The moment arm a is 54 in. and the resisting moment of the joint is 92,400 × 54 or 4,989,600 in.-lb., which is slightly in excess of the bending moment.

Rivets Connecting End Angles to Gusset Plate.—Now consider the rivets connecting the end angles to the gusset plate. The method is the same as that for the connections of the

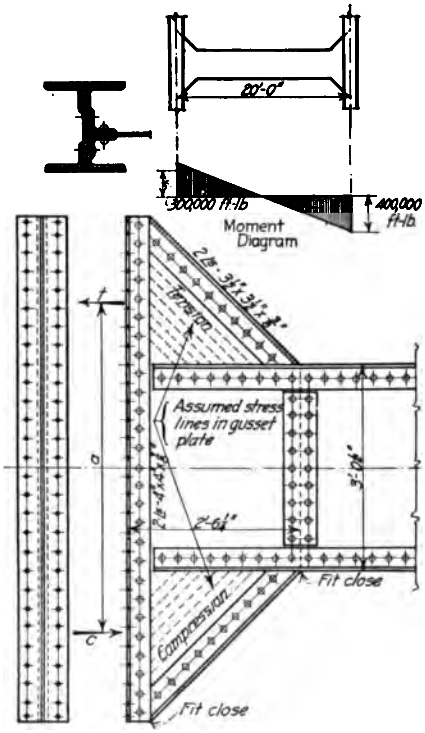


FIG. 408.—Design of a wind bracing girder.

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[1] Taken from Burt's "Steel Construction."

[2] This is not exact, for the rivets on the compression side do not act, the compression being resisted by the direct bearing of the end of the girder against the column. The error is on the safe side.

end angles to the column, except that the rivets are shop driven in double shear. The required results can easily be obtained by comparison with field-driven rivets. With one row of rivets there will be one-half as many (less one). One shop rivet in double shear is good for 15,840 lb. This is greater than the value of two rivets in tension (13,200 lb.), hence the proposed arrangement is satisfactory. It gives greater strength than is required.

The thickness of gusset plate required to develop the full shearing value of the rivets is 11/16 in. The thickness required for the actual stress is 9/16 in., which will be used.

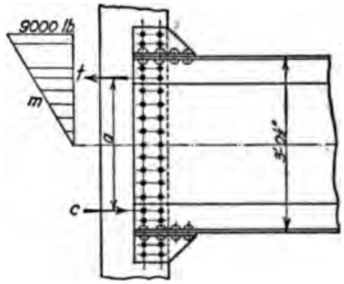

Fig. 409.

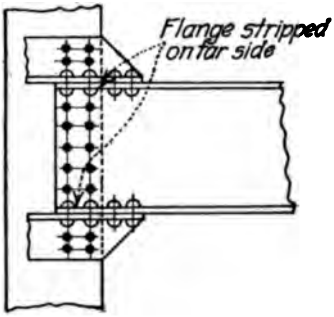

Fig. 410.

Bending Stresses in Connecting Angles.—No accurate determination can be made of bending stresses in connecting angles, so thickness must be adopted arbitrarily. If the gage line of the rivets is not more than 2½ in. from the back of the angle, the thickness should be ⅝ in. In many cases wide angles with large gage distance must be used in order to match the gage lines in the column. A thickness of 1 in. seems to be safe to a gage distance of 4 in. Intermediate values may be interpolated.

Gusset Plate.—The slope of the gusset plate should be about 45 deg., but may vary to suit conditions, such as clearance from windows, etc. Stresses in the gusset plate may be imagined to act along the dotted lines shown in the figure (Fig. 408). On the tension side of the girder, the plate is in tension, and on the compression side in compression. The thickness of plate required for rivet bearing is sufficient to give the necessary strength on the tension side, but on the compression side, stiffener angles may be required. These angles can be designed according to rules similar to those given for the stiffeners of plate girder webs. They should be used when the length of the diagonal edge of the plate is more than 30 times the thickness. The leg of the angle against the plate should be of suitable width for one row of rivets, say 3, 3½, or 4 in. The outstanding leg may vary from 3 to 6 in. A thickness of ⅜ in. is usually suitable; it may be made more or less to be consistent with size and thickness of the main members of the girder. For the case illustrated, two 3½ × 3½ × ⅜-in. angles will be used.

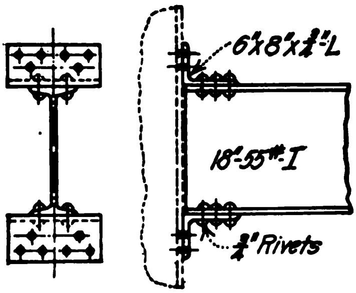

Fig. 411.

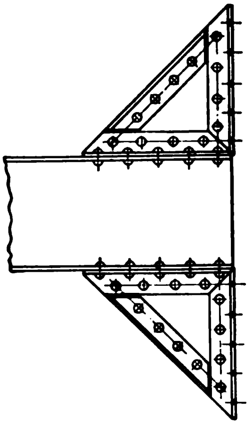

Fig. 412.

The splice of the gusset to the girder should be in accordance with the usual practice in designing plate girders, the splice being made to transmit the bending and shear at this point.

In Fig. 409, the web of the girder connects directly to the flange of the column. This form of connection is suitable for girders which are deep in proportion to the bending moment which they must resist. The method of designing the connection is the same as that explained for Fig. 408, except that the rivets are in single shear instead of tension, and that the rivets are not evenly spaced, hence the average resistance may not be one-half the maximum. The value of each rivet can be measured from the diagram at *m* in the figure. Having the values of the several rivets, the center of gravity of each group, *i.e.*, the positions of the resultants *t* and *c* can be found in the usual way.

When the form of connection shown in Fig. 409 is not adequate, a gusset plate can be used connecting directly to the flange of the column. It involves no principles or methods different from those already explained.

End Connections for I-beam Girders.—I-beam connections for resisting bending are illustrated in Figs. 410, 411, and 412.

The detail in Fig. 410 is similar to the connection shown in Fig. 409. It can develop only a small part of the capacity of the beam.

The detail in Fig. 411 also can develop only a part of the capacity of the beam, but it is available for making use of the floor girders in the upper part of the building for resisting wind stresses. The strength of this connection is limited by the bending resistance of the connecting angles or the strength of the rivets.

Bracket Connection.—The connection in Fig. 412 can be made to develop the entire net bending resistance of the beam (deducting for rivet holes in the flanges). The connection of the brackets to the column is designed in the same manner as described for the gusset plate connection. The average value of the rivets is determined from the diagram as at *m*, Fig. 409. In the connection of the brackets to the beam, all the rivets are figured at the maximum value. Their resisting moment is their total shear value multiplied by the depth of the beam.

255. Effect of Wind Stresses on Columns.[1]

255a. Combined Direct and Bending Stresses.—The bending moment on the column due to wind loads produces the same sort of stresses as result from the bending moment due to eccentric loads or any other cause producing flexure. The extreme fiber stress is computed from the formula

$$f = \frac{Mc}{I}$$

This stress is added to the stresses resulting from the direct and eccentric gravity loads on the column to give the maximum fiber stress.

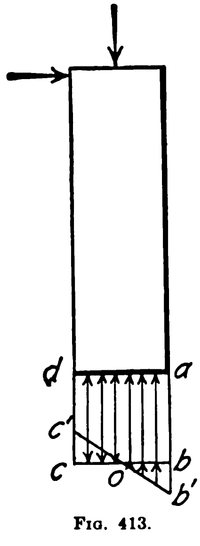

Fig. 413.

The combination of the direct and the bending stress is illustrated in Fig. 413. The stress from the direct load is represented by the rectangle *abcd* and the unit stress by *ab*. The stress from bending is represented by the triangles *bb'o* and *cc'o*, the extreme fiber stress being *bb'* in compression and *cc'* in tension. Then the maximum fiber stress is on the compression side and is *ab* + *bb'*. Thus, *bb'* represents the increase in stress due to the wind load. If, as is usually the case, *bb'* amounts to less than half *ab*, the column section required for the direct load need not be increased on account of the wind stress, because of the increased units allowed for combined stress. But if *bb'* exceeds one-half of *ab* the combined stress will govern the design using the increased unit stress.

On the tension side of the column, the wind stress will very rarely be great enough to overcome the direct compression. And if there should be a reversal of stress, there cannot be tension enough to require any addition to the section. It frequently occurs that the wind bracing girder connects to the column in such a position that one side of the column must resist practically all the wind stress. With these conditions, only one-half the column section should be used in computing the resulting extreme fiber stress.

255b. Design of Column for Combined Stresses.—The procedure in designing the column section, when the combined wind and gravity loads govern, is the same as for columns with eccentric loads. The equivalent concentric load is given by the formula

$$W'_w = \frac{W'ec}{r^2}$$

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[1] From Burt's "Steel Construction," published by American Technical Society, Chicago.

As applied to wind load (refer to Fig. 414), W'_w is the equivalent concentric load, *i. e.*, the direct load that would produce the same unit stress; W' is the horizontal shear which is assumed to be carried by the column under consideration and is assumed to be applied at the point of contraflexure of the column; e is the moment arm expressed in inches, hence $W'e$ is the bending moment in inch-pounds at the section under consideration; c is the distance from the neutral axis of the column to the extreme fiber on the compression side; r is the radius of gyration of the column in the direction under consideration. The critical section of the column is at the top of the bracket, as the bracket has the effect of enlarging the column section, so the distance e is measured to that point.

To illustrate the use of the formula, assume the following data: direct or gravity load on column is 480,000 lb; W' is 13,000 lb.; e is 30 in.; c is 7 in.; and r is 3.5 in. Then

$$W'_W = \frac{(13{,}000)(30)(7)}{(3.5)(3.5)} = 223{,}000 \text{ lb.}$$

As this is less than half the gravity load, no additional section is required on account of the wind loads. This will usually be the case except possibly at corner colunns.

256. Masonry Buildings.—Brick buildings with fireproof floors or even with wood floors do not ordinarily require wind bracing. The floors, acting as horizontal girders, will carry the loads to the end walls which will transmit them to the foundations. Nevertheless, the wind loads on such cases should be figured to determine whether any strengthening is required at special points.

257. Wood Frame Buildings.—Ordinary wood frame dwellings and similar buildings are sufficiently braced by the sheathing and plastering of the walls and by the partitions. However, if the building is unusually large or subject to unusual exposure, the case should be studied, and bracing added if any doubt exists. Diagonal members can be introduced into the walls and partitions, particularly at the corners. If such buildings are high compared with their width, the overturning resistance should be investigated.

Large frame structures, such as temporary auditoriums, should be provided with a definite system of wind bracing designed in accordance with the methods described for mill buildings, or the principles previously described.

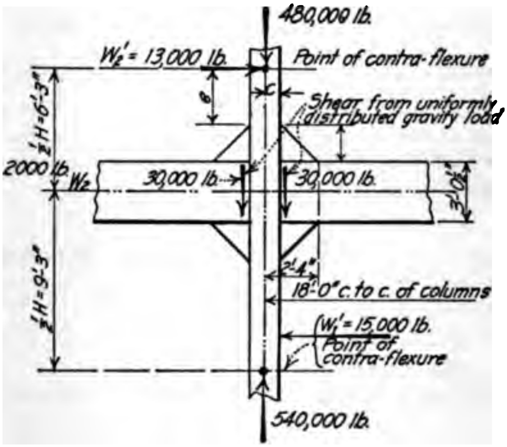


Fig. 414.

258. Mill Buildings.—A type of building much used for storage and manufacturing purposes is a one-story structure of steel frame construction with one or more wide aisles, spanned by roof trusses. The weight of the structure is usually small compared with wind pressure. The bracing of such a building is illustrated in Fig. 415.

If the sides are covered by corrugated steel or other light sheathing, the covering will be attached to horizontal girts extending from column to column. They will be designed as simple beams to resist the wind pressure.

258a. Wind Pressure on the End of the Building.—The intermediate end posts extend from the ground level to the underside of the truss in the case illustrated, but may extend to the roof, the end truss being omitted. These posts are designed as beams to resist the wind loads carried to them by the girts.

The reactions at the tops of the posts and wind load on the lower half area of the gable are carried into the horizontal truss, whose chords are the bottom chords of the roof trusses and whose web members are as shown in the bottom chord plan. This truss delivers its load into the eaves strut which may be a combination of roof purlin, girt, and strut.

The wind pressure on the top half area of the gable is carried in the truss in the roof plane. This truss is made up of the top chords of the roof trusses and the web members between. The strut at the ridge may be made of the ridge purlins suitably stiffened to resist compression. This truss also delivers its load to the eaves strut.

From the eaves strut the load is carried to the foundation by the diagonals shown in the end panels of the side elevation.

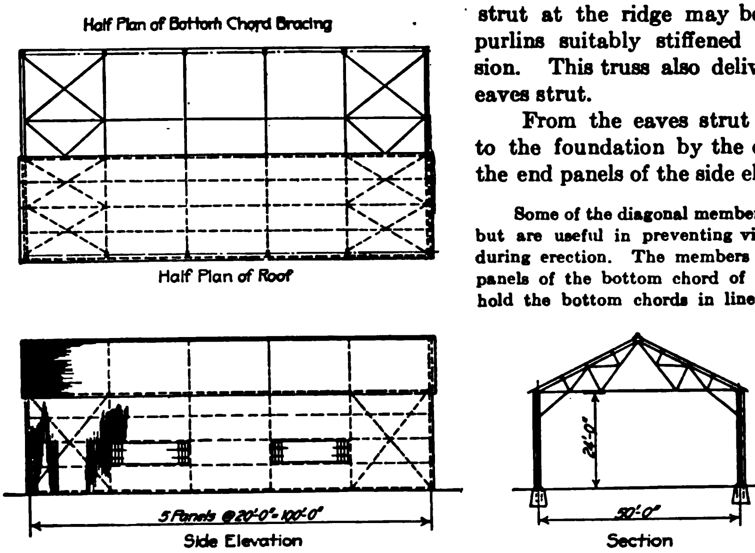

Half Plan of Bottom Chord Bracing
Half Plan of Roof
5 Panels @ 20'-0" = 100'-0"
Side Elevation
24'-0"
50'-0"
Section

FIG. 415.—Bracing for typical mill building.

Some of the diagonal members shown are redundant, but are useful in preventing vibration and for bracing during erection. The members shown in the unbraced panels of the bottom chord of the roof trusses serve to hold the bottom chords in line and prevent buckling should the wind pressure on the sides produce reversal of stress in the bottom chord. The diagonal members may be either adjustable rods, or structural shapes, the latter being generally preferred. The arrangement of the bracing may be varied from that shown to suit conditions. The important consideration is to provide a continuous path for the stress from the point of application of the load to the foundations.

258b. Wind Pressure on the Side of the Building.—For resisting the wind pressure on the side of the building, each bent is treated as a separate self-supporting unit. For method of determining the resulting stresses, see chapter on "Detailed Design of a Truss with Knee-Braces."

BALCONIES

BY H. J. BURT

The essential structural feature of a balcony is a cantilever beam or a bracket.

259. Cantilevers.—Fig. 416 shows a beam resting on the supports *A* and *B*. The overhanging end forms a cantilever for carrying the balcony load. The maximum bending moment of the cantilever is at the support *B*, likewise the maximum shear. The bending moments and shears must be computed also for the portion of the beam between *A* and *B*. After computing the bending moments and shears, the beam section can be designed in the usual manner. The moments and shears are diagrammed in Fig. 416.

For a steel or wood beam of uniform cross section, the bending moments at *O* (Fig. 416) will govern. For a concrete beam or slab the reinforcement is arranged to correspond with the bending moments throughout the length of the beam.

The span, the overhang, and the conditions of loading may be such that the maximum bending moment occurs at *B*. There may be no negative bending moment between *A* and *B*, in which case there will be an uplift at *A*.

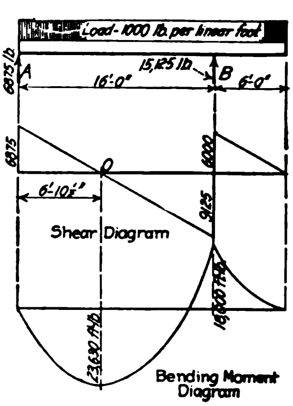


FIG. 416.—Stresses in a cantilever beam.

In case it is necessary to have a cantilever steel beam flush on top with the girder, as shown in Fig. 417, the cantilever must be spliced to transmit the bending moment. The top flange being in tension is spliced with a strap designed to transmit the top flange stress. The bottom flange being in compression, maybe spliced by two angles or bent plates as shown, which will also transmit the shear into the girder.

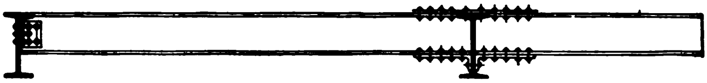


Fig. 417.—Splice in cantilever beam (steel).

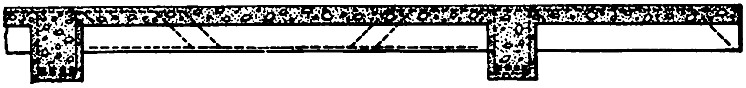


Fig. 418.—Concrete cantilever, monolithic with supporting girder.

A wood cantilever can be spliced in the same manner, but such a detail is not satisfactory.

In the similar case with concrete construction, the girder and cantilever are cast monolithic, the rods of the cantilever running through the girder (Fig. 418).

If the projection of the balcony is large, a cantilever truss is required. This condition occurs in theatres. The governing lines usually allow ample depth for an economical truss. Fig. 419 is a diagram of a truss for this purpose.

260. Brackets.—A projecting member whose moment is balanced by being connected to some rigid member as a column or a wall, is here designated as a bracket, in contra-distinction to the cantilever beam previously described where the moment of the projecting arm is balanced by the portion of the beam between the supports *A* and *B* (Fig. 416).

Fig. 420 illustrates three types of brackets: (*a*) is a beam section rigidly attached to the supporting member, (*b*) is a triangular bracket whose members are subject to axial stress, and (*c*) is a truss. The bending moments and shears for various conditions of loading are the same as for cantilever beams. These moments and shears govern the connections of the brackets to the columns or other supporting members. The connection to the supporting member is of vital importance for type (*a*), as the small depth of the bracket makes it more difficult to design the necessary bending resistance for this type, than for types (*b*) and (*c*).

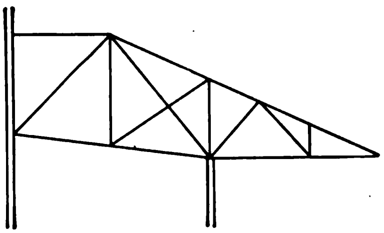


Fig. 419.—Cantilever truss for a theatre.

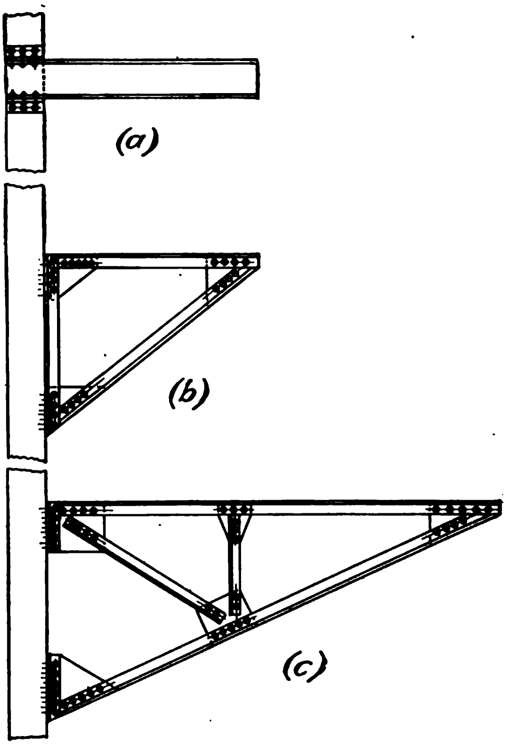


Fig. 420.—Three types of brackets.

Fig. 421 shows the connection of an I-beam bracket to the face of a column by means of top and bottom connecting angles. The bending moments of the bracket load must be balanced by the resisting couple of the rivets through the flanges of the beam acting in shear. It must also be balanced by the resisting couple of the rivets connecting the angles to the face of the column, the rivets in the top angle being in tension, and an equal compressive value being taken at the rivets in the bottom angle. These latter rivets are not actually stressed from the bending moment, but should be designed to carry the direct shear from the load on the bracket. The depth of beam used will generally be such as will give sufficient moment arm for the resisting couples. Its section will be greater than is required for the bending moment of the bracket, as it is not practicable to devise a connection that will develop the full bending resistance of the beam.

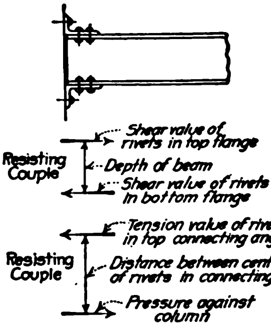

Shear value of rivets in top flange
Resisting Couple
Depth of beam
Shear value of rivets in bottom flange
Tension value of rivets in top connecting angle
Resisting Couple
Distance between centers of rivets in connecting angles
Pressure against column

Fig. 421.—Connection of I-beam bracket to face of column.

In Fig. 422 a channel bracket is riveted to the face of the column. The resisting moment of the rivets should be computed as a polar moment about the point *p*, the rivets having the longest radius being taken at their maximum shear value and the others proportionately less. The portion of shear value of the inner rivets not effective in computing the resisting moment can be utilized in resisting the direct shear of the bracket load.

The foregoing principles will apply in detailing other forms of connections of steel beams and channels to columns (see Figs. 423*A* and 423*B*).

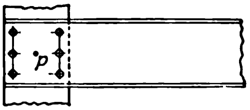


Fig. 422.—Channel bracket riveted to face of column.

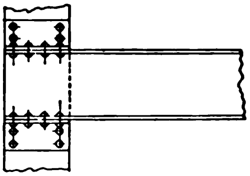


Fig. 423*A*.—Channel bracket connected to face of column.

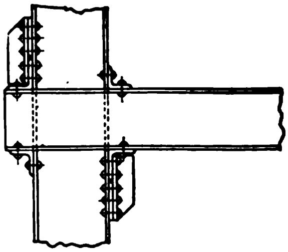


Fig. 423*B*.—I-beam bracket on side of column.

Wood beams are not well suited for use as brackets, but where employed the connections are detailed in a similar manner.

Concrete beams used as brackets are cast integrally with the columns. These can advantageously be made of variable cross section in order to easily develop the necessary shearing and bending resistance at the connection to the column, and to meet architectural requirements. Fig. 424 illustrates a concrete bracket. Being cast integral with the column, the entire strength of the section adjacent to the column is available and is designed in the same manner as a concrete beam.

The triangular bracket, type (*b*) Fig. 420, gives a greater effective depth than the beam bracket and correspondingly less stress on the connections. In Fig. 425 assume the load applied at the end of the bracket. The resisting couple is formed by *T* and *C*, and the vertical shear at the column connection is *V*. The stresses in the members *m* and *n* are determined by the stress diagram, and are axial stresses. From the stresses and reactions, the members *m* and *n*, and the connections, are designed in the usual manner. The case illustrated is steel construction.

The load may be so applied that the top chord is subjected to bending as well as direct stress, and it must be so designed. In this case there will be vertical shear to be resisted at both the upper and lower connections (Fig. 426).

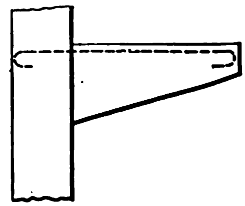


Fig. 424.—Concrete bracket.

The triangular bracket can be made of wood using details similar to those used in wood trusses. The connections at *T* and at the outer end of the bracket require careful attention.

Concrete may be used for triangular brackets, but there is little need to do so as its advantages can be secured in the beam type previously described.

The trussed bracket is a development of the triangular bracket. A stress diagram is required to determine the stresses in the truss members. The members and connections can then be designed.

This type is especially adapted to steel construction. It can be built of wood or concrete if the conditions warrant.

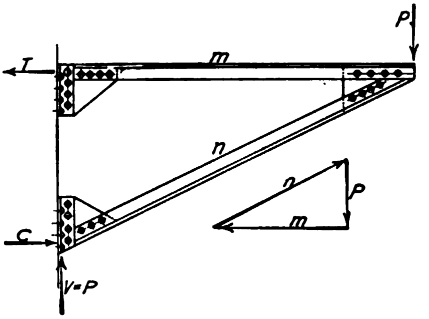


FIG. 425.—Triangular bracket stresses from end load.

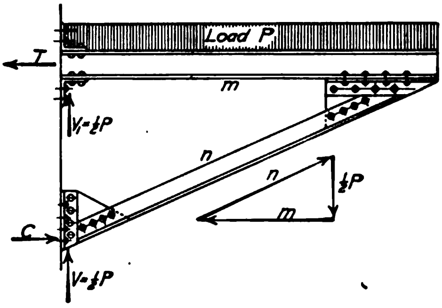


FIG. 426.—Triangular bracket stresses from distributed load.

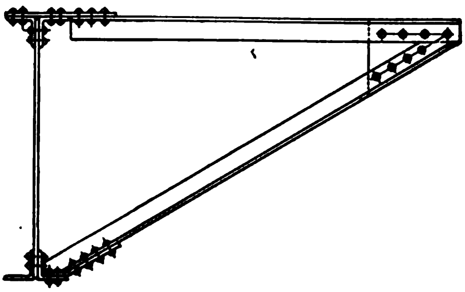


FIG. 427.—Bracket on side of plate girder.

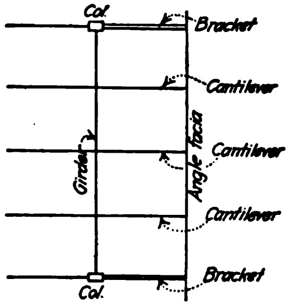


FIG. 428.—Floor framing of balcony.

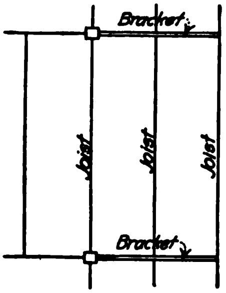


FIG. 429.—Floor framing of balcony.

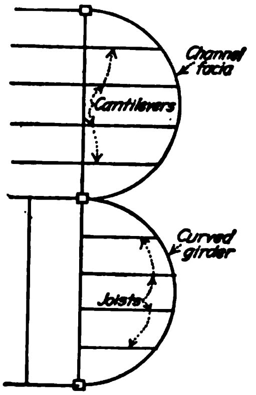


FIG. 430.—Framing for curved balcony.

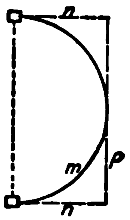


FIG. 431.—Approximate computation for curved balcony.

260a. Effect on Column.—A bracket attached to a column produces a bending moment in the column equal to the bending moment of the bracket loads. The column section must be designed accordingly by the methods given in the chapters on "Bending and Direct Stress" in Sect. 1. It may be counteracted by a beam or girder connection on the opposite side of the column, so designed as to resist the moment of the bracket.

260b. Effect of a Bracket on the Side of a Girder.—It is sometimes necessary to attach a bracket to the side of a plate girder (Fig. 427). This produces a torsional moment in the section of the girder. While the girder may have ample strength to resist the torsional stresses, it may, nevertheless, deflect laterally beyond permissible limits. It is therefore, desirable to provide a more direct resistance. This can be accomplished by anchorage into the floor construction, by suitable connections of joists, or by beams or brackets extending back to an anchorage. Either of these devices acting with the bracket, produces the equivalent of a cantilever beam giving a vertical reaction only at the supporting girder.

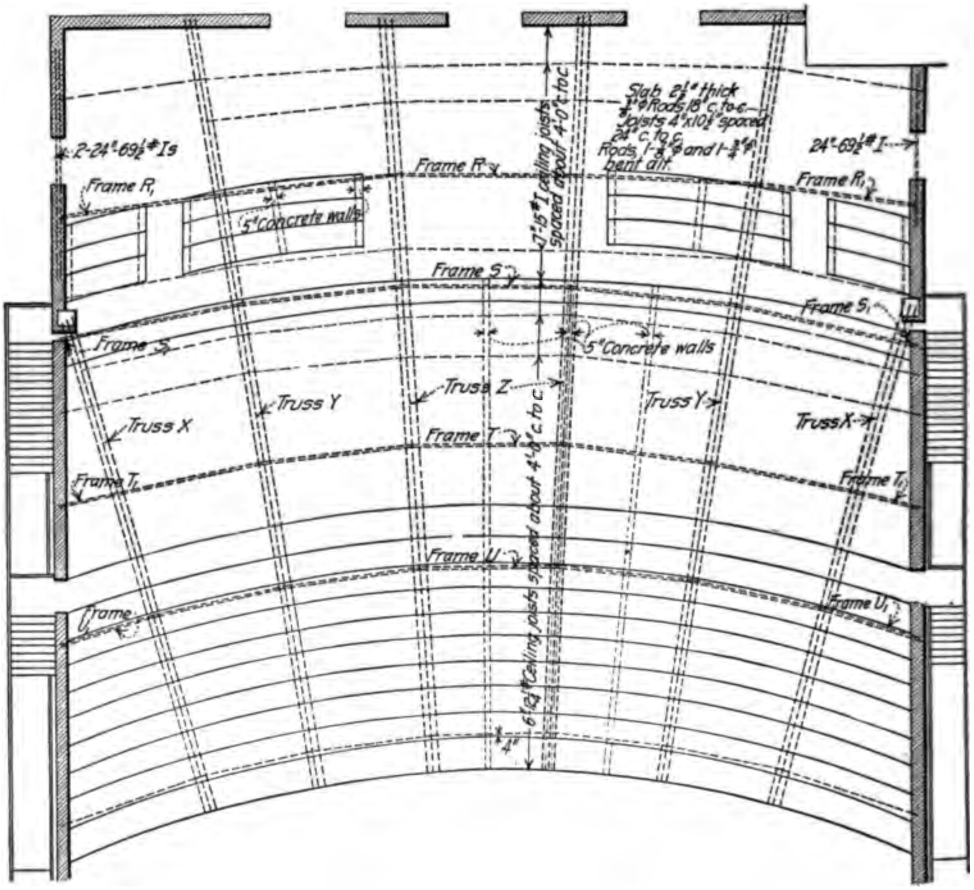


FIG. 432.—Balcony framing plan.

261. Floor Framing of Balcony.—The cantilevers or brackets serve as the main supporting members of a balcony. They may be close enough together to serve as the joists, the floor construction spanning from one to another (Fig. 428). This is usually the condition when cantilever beams are used. In other cases, the brackets may be equivalent to girders, and joists be required to support the floor (Fig. 429). The outer joist or the ends of the bracket may have to support some special load, such as a railing.

The floor framing presents no problems essentially different from those discussed under the subject of floors. The materials of construction of the cantilevers, brackets, and floors of balconies will usually be governed by the materials of the main structure.

262. Curved Balconies.—Fig. 430 illustrates a curved balcony. The upper panel is shown having cantilever beams for the supporting members. This form is preferable for irregular shaped balconies.

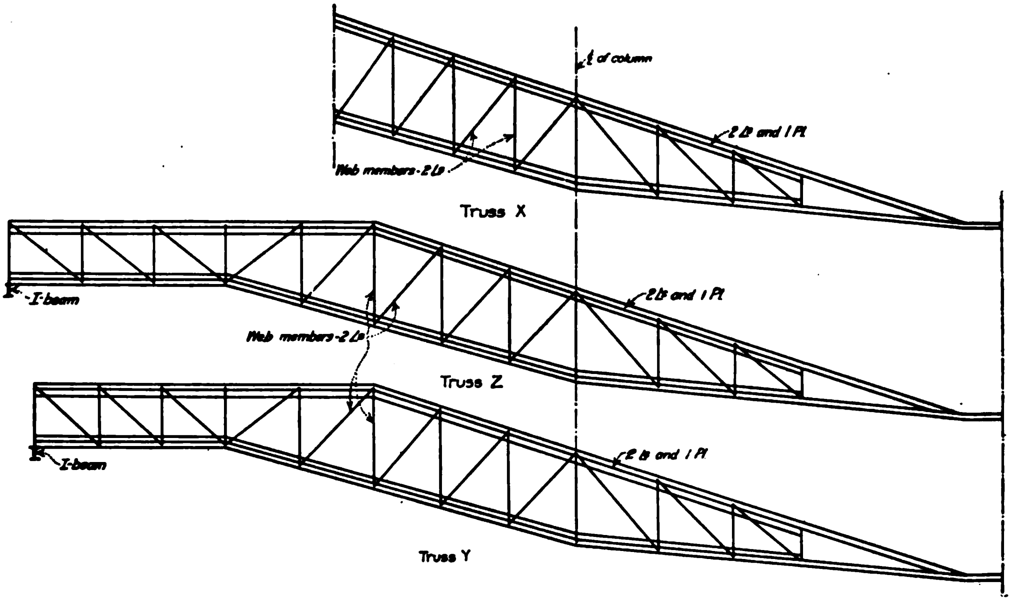


FIG. 433.—Cantilever trusses.

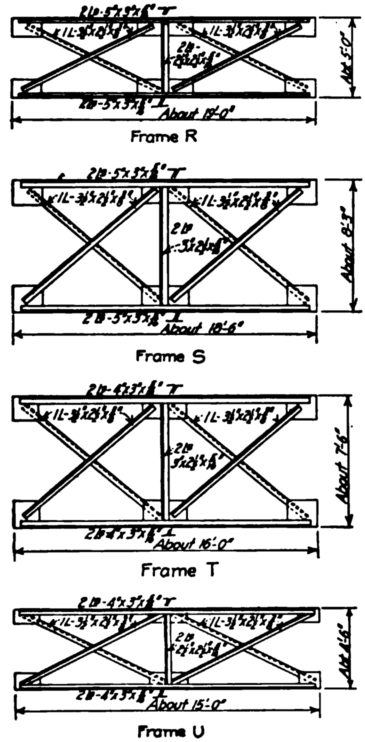


FIG. 434.—Cross frames between cantilevers.

If the conditions preclude the use of cantilevers, the curved member must serve as a support, as shown in the lower panel of Fig. 430. An accurate determination of the stresses in the curved member is not practicable but a safe approximation is as follows:

In Fig. 431, let *m* be the curved member, *n* and *p* the sides of a rectangular balcony circumscribing the curved balcony. Then *n* represents the bracket of a rectangular balcony. Determine the total load on the curved balcony and from this load compute the connections required as if supported by brackets *n*. Use these connections for the curved beam. Make the section of the curved beam not less than would be required for the member *p* of a rectangular balcony. Anchor the curved beam to the floor construction of the balcony so that the top and bottom flanges cannot buckle laterally.

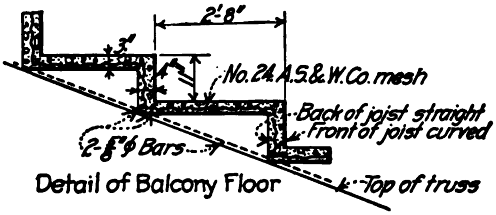


FIG. 435.

263. Theatre Balcony Framing.—Reference has been made to the form of cantilever truss used for theatre balconies. A typical truss is shown in Fig. 419. In Fig. 432 is shown the framing plan of a theatre balcony.

The cantilever trusses *X*, *Y*, and *Z* are set radially. They are braced for lateral stiffness by the cross frames

R, *S*, *T*, and *U*. The outlines and members of the cantilever trusses and the cross frames are shown in Figs. 433 and 434.

The shape of the top chord of the truss is governed by the slope of the bank of seats and the floor level back of the seats. At the front is a shallow projecting member to support the aisle along the balcony rail. The construction at this place must be as thin as it can be made, because of sight lines for the seats below the balcony. The shape of the bottom chord is con-

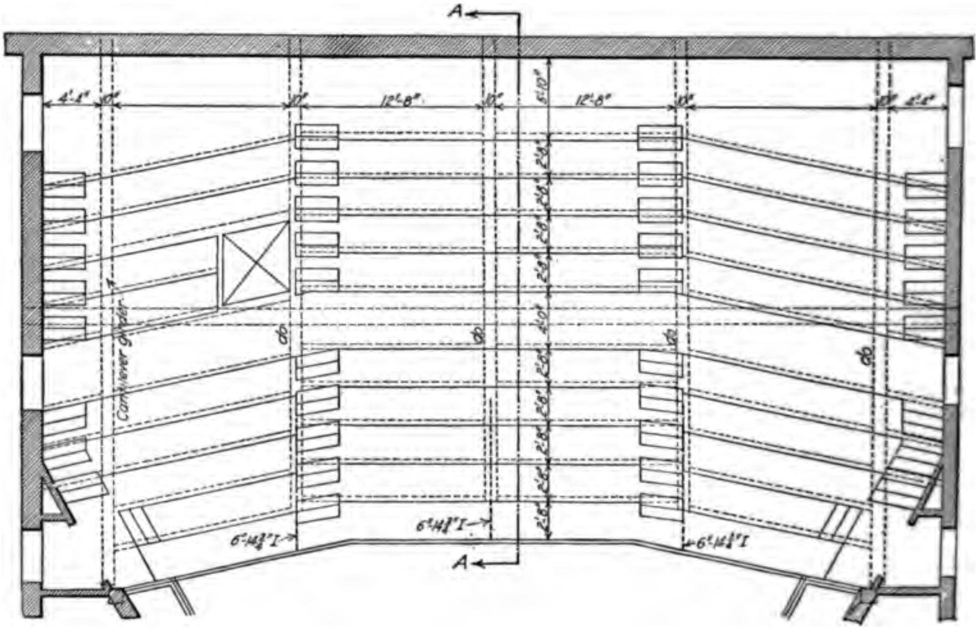
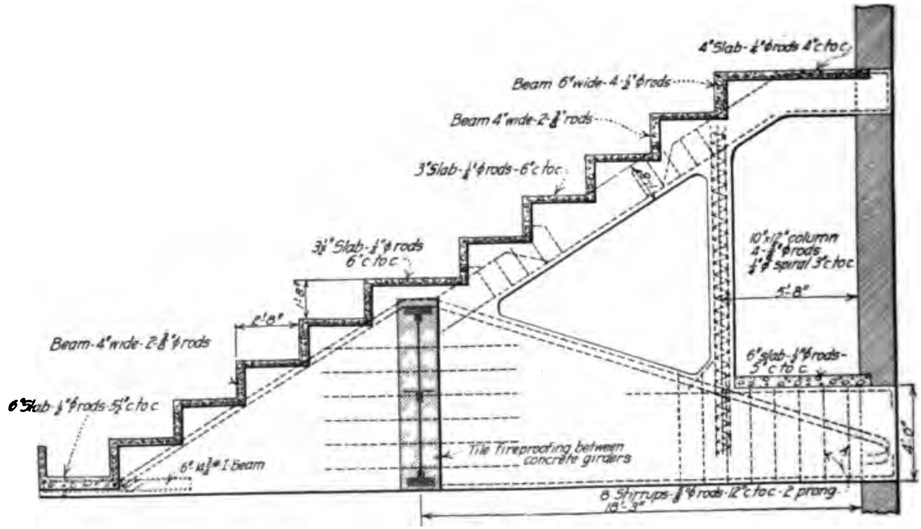


Fig. 436.—Plan of balcony.



Section "AA"

Fig. 437.—Concrete cantilever.

trolled by the lower sight lines and clearance for passages and stairways. It is sometimes necessary to provide a passage through one or more of the trusses.

Fig. 435 shows the consrruction of the floor or banks of the balcony.

A balcony built of reinforced concrete is shown in Figs. 436 and 437. The cantilevers in this case are supported by a steel girder which spans the entire width of the theatre. At the rear is a passageway through the cantilever; in front of this is an opening which serves to reduce the weight, and which may be used as a passage for air ducts of the ventilating system. The drawings show the conditions of the problem with sufficient clearness so that no detailed explanation is required.

LONG SPAN CONSTRUCTION FOR OBTAINING LARGE UNOBSTRUCTED FLOOR AREAS

By H. J. Burt

For certain purposes it is necessary to have large clear floor areas free from columns. Such spaces are required for ball rooms, dining rooms, lobbies, auditoriums, and various special situations.

If the clear space is on the top floor of the building with only the roof to be supported over it, trusses or arches can be used. This case does not come into the purview of this chapter. The cases to be considered here are those in which the clear area is in the lower part of the building so that large weights must be supported overhead.

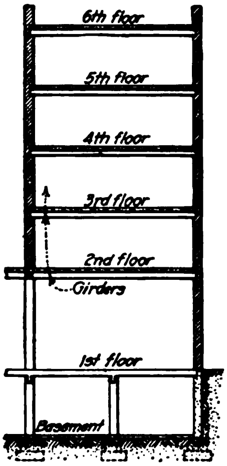


Fig. 438.—Clear space with column omitted full height of building.

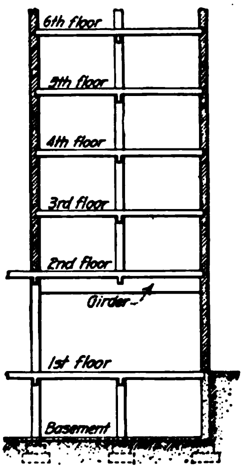


Fig. 439.—Clear space with girder over

264. The General Problem.—The predominant condition is the support of very heavy loads. Every case is a special one, so there can be no approach to standarization. The depth, span, and load conditions are such that the shearing stresses, deflections, secondary stresses, and details of construction may require special attention.

265. Examples.—A simple case is the omission of an intermediate column in a lower story. There are two solutions of this case shown in Figs. 438 and 439.

The scheme shown in Fig. 438 requires long-span shallow girders with relatively light loads. The depth of these girders will be greater than the short span girders of Fig. 439 and may encroach unduly on the headroom of the typical stories. It will be used where there is sufficient headroom and where there is not sufficient depth for the heavy girder required in the scheme shown in Fig. 439. Deflection may be an important consideration.

The second scheme requires a long-span girder, usually of limited depth with a heavy concentrated load at or near the center of the span. This is usually more economical than the scheme shown in Fig. 438 and is used where there is available space for the depth of the girders.

Fig. 440 gives the details of a girder supporting an offset column and Fig. 441 is a diagram showing the position of the column above and the supporting columns below.

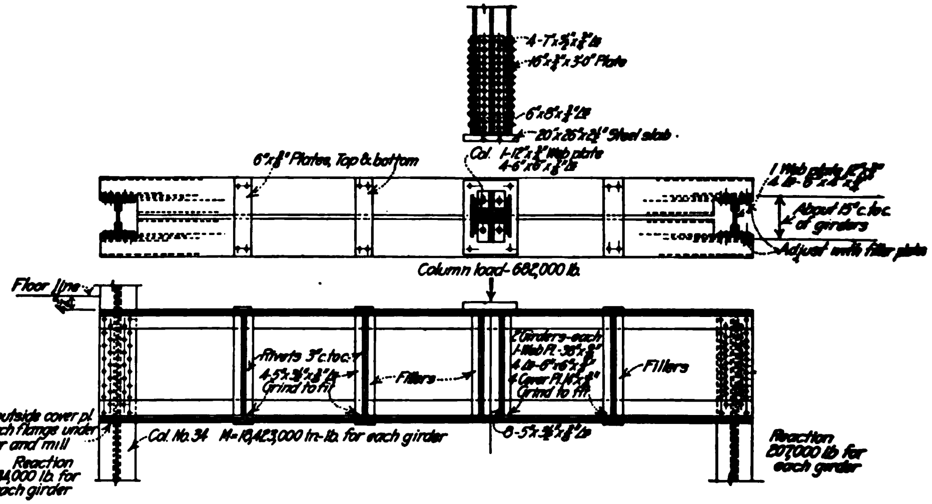


Fig. 440.—Details of a girder carrying an offset column.

This arrangement occurs at the fourth floor of a 17-story hotel building.[1] The upper segment of columns 33 carries the court wall and floors of the upper stories. The girder section consists of two plate girders tied together with batten plates. The use of two girders permits simple connections to the supporting columns without eccentricity. The two webs are needed to carry the shear. The details requiring special attention are the bearing plate

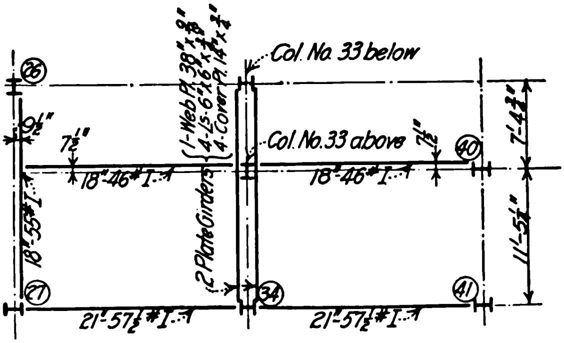


Fig. 441.—Part plan fourth floor framing showing position of offset column, Fort Dearborn Hotel, Chicago, Ill.

and stiffeners of the supported column, the stiffeners at the loaded point designed to carry the load into the girder webs, the connections to the supporting columns, and the spacing of rivets connecting flange angles to web.

Figs. 442, 443, 444[2] illustrate a special situation which occurs in hotel buildings. The typical floor layout governs the placing of the columns in the upper stories—*i.e.*, they must

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[1] Fort Dearborn Hotel, Chicago, Ill.

[2] Deming Hotel, Lafayette, Ind.

be on one or both sides of the corridor. In the lower stories in this case, two columns are not permissible and the single column which is permitted must be under the center of the corridor of the upper stories. Hence, there must be an offset at the second floor level. Two considerations lead to the use of twin columns above: (1) the resulting symmetry, shorter span, and lighter floor construction of the upper floors; and (2) the smaller shear in the girder carrying the offset. This latter item is quite important in this case as the headroom allowed is very limited. Even with the twin columns it was necessary in the design shown to use the concrete casing of the steel girder to assist in carrying the load (Fig. 444). In cases of this kind, if either of columns *A* or *B* (Fig. 442) can be extended through the lower stories, it will be better to use only the one row of columns and avoid the girder at the second floor. The girder is usually more expensive and objectionable than the unsymmetrical construction above (Fig. 445 is an illustration of this arrangement). If both *A* and *B* can be extended through the lower stories, it is advantageous to do so and avoid the girders.

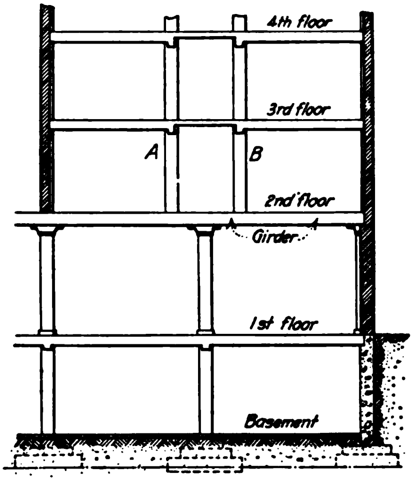


FIG. 442.—Part sectional elevation showing twin columns above and single columns below.

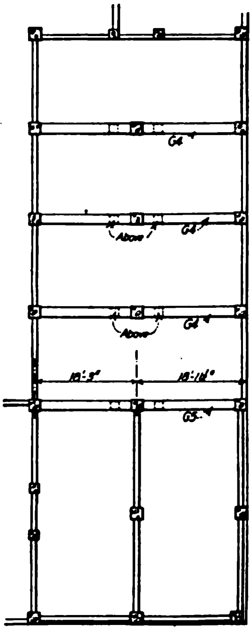


FIG. 443.—Part second floor framing plan showing position of offset columns.

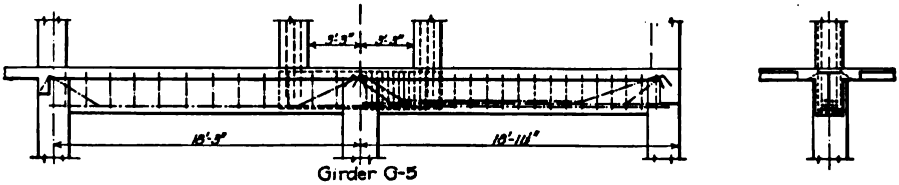


FIG. 444.—Detail of girders supporting offset columns.

The situation at the corners of the building is illustrated in Fig. 446. Columns *A* and *B* are supported on girder shown in section *V-V*. The loads of the upper columns are nearly balanced over the lower column, but girder extends to the corner column which takes whatever reaction is required to balance the loads.

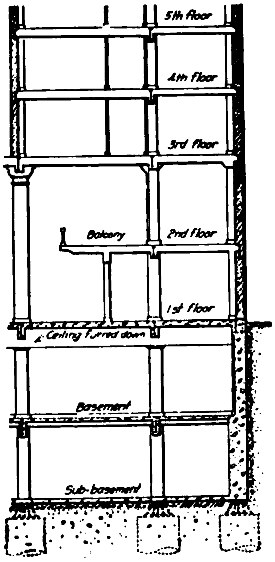


FIG. 445.—Showing method of avoiding offset columns and resulting heavy girders by using unequal panel lengths.

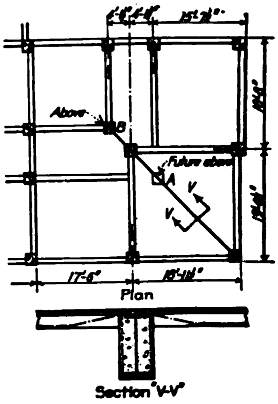


FIG. 446.—Offset columns at corner of building.

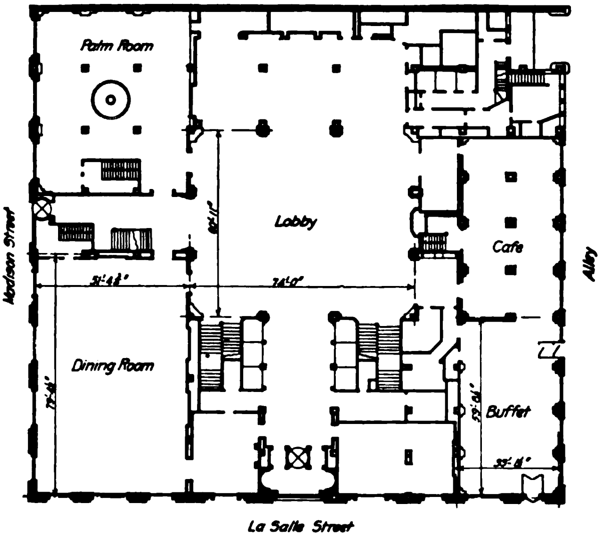


FIG. 447.—La Salle Hotel, Chicago, Ill.

The Hotel LaSalle, Chicago, Ill., presents a number of examples of clear space requiremen

Fig. 447 is a plan of the first floor, which shows a Lobby about 61 × 74 ft., a Dining Room about 51 × 80 f and a Buffet about 33 × 60 ft. Over the Buffet is a room on the mezzanine floor having the same dimensions

The Lobby is under the light court of the building so that the framing over it carries only the roof, but the conditions are such that ordinary roof trusses could not be used. The framing used is shown on Fig. 448. There are eight brackets projecting from the side columns. These brackets support a rectangle of plate girders, which in turn carry the minor framing members.

The Dining Room is so proportioned that it requires the full height of the first and mezzanine stories, so that no space is available below the second floor for the girders. Very heavy girders are required to support the 18 floors above. The entire depth of the second story is used for these girders. In this way an overall depth of about 14 ft. is available for the girders having 50-ft. span. In order to obstruct the second floor span as little as possible and to make the span between girders available for use, an opening is provided through each girder for the corridor There are three of these girders spanning between columns 1-2, 3-4, and 5-6 (Fig. 448). Each supports two main building columns as well as the direct loads from the second and third floors. The positions of these girders are shown on Fig. 448 and the design on Fig. 449(*c*).

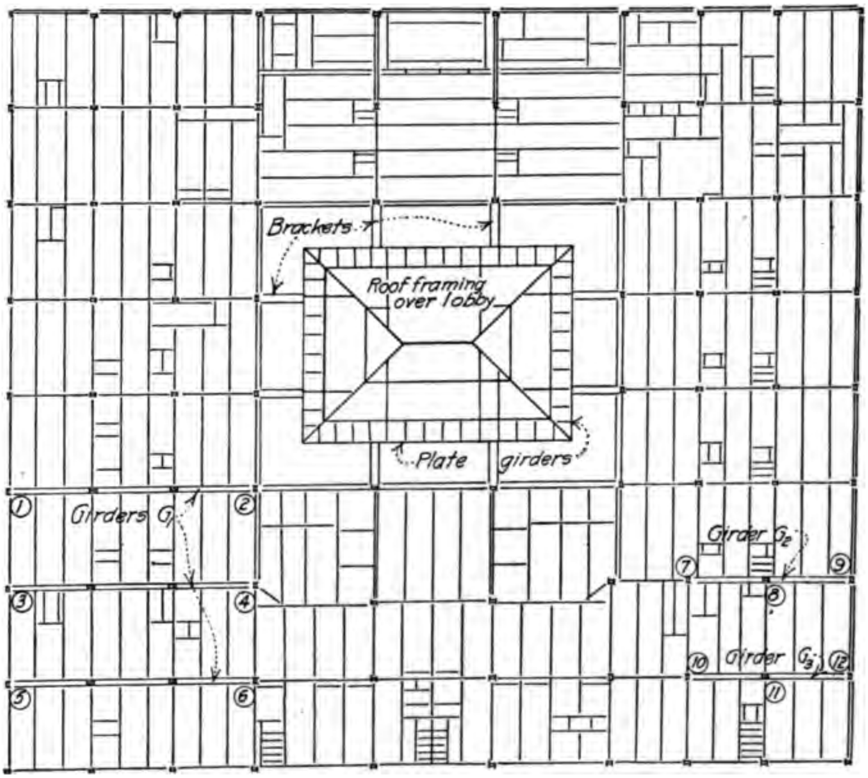


Fig. 448.—Second floor plan, La Salle Hotel, Chicago, Ill.

The floor over the Buffet is supported by plate girders spanning between columns 7-9 and 10-12 at the mezzanine floor level. As there is a corresponding clear space on the mezzanine floor, these girders carry only the mezzanine floor load.

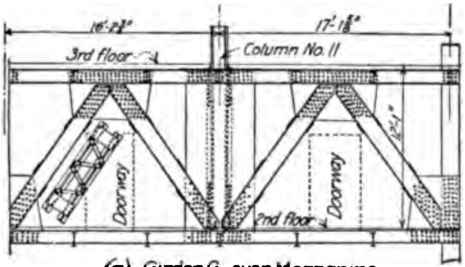
Over the clear space of the mezzanine story, columns 8 and 11 have to be supported (Fig. 448). Column 8 is carried by a pair of plate girders (Fig. 449*b*) extending below the second floor, but not above it, no obstruction above the floor being permissible at this place. Column 11 is carried by a truss whose depth is that of the second story. It is arranged so that two doorways can be cut through (Fig. 449*a*).

The Grand Banquet Hall of the hotel is on the top floor and has only a roof over. Fig. 450 shows the special arched truss designed for this purpose.

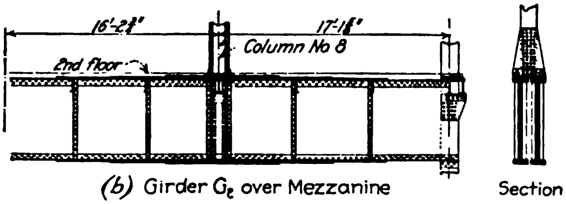
The University Club of Chicago offers several illustrations of large clear spaces. In this building they are arranged one above the other as far as practicable. This arrangement was made in order to have the best rooms face on Michigan Avenue, but it serves to reduce the concentration of loads that must be supported by individual girders. The frontispiece shows the building in question. The architectural treatment marks the location of the Main Dining Hall on the ninth floor and the Lounge on the second floor.

In the basement is a swimming pool for which a clear space 30 × 65 ft. is provided. A similar space in the first story is clear of columns so that the first and second floors are each carried by double I-beam girders spanning approximately 30 ft.

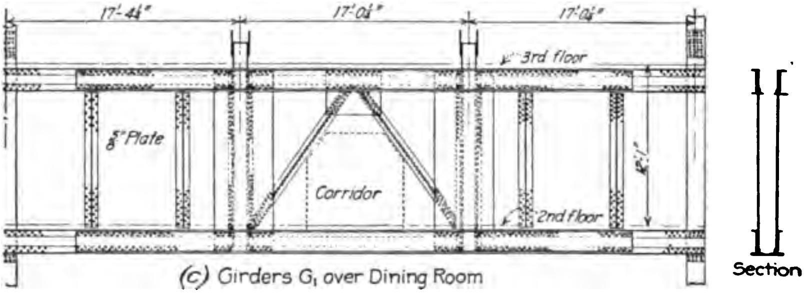
On the second floor is the Lounge, approximately 45 × 65 ft. This story is 26 ft. high, enough to allow space for girders. The arrangement of the framing over this room is shown in Fig. 451. Two double plate girders and one truss are used. The truss extends into the third story and has to provide an opening for the corridor. It is used because of the greater load which comes on it.



(a) Girder G₃ over Mezzanine



(b) Girder G₂ over Mezzanine



(c) Girders G₁ over Dining Room

FIG. 449.—Details of girders, La Salle Hotel.

The next clear space is the Billiard Room on the seventh floor. Adjoining it is a Cafe. Both of these rooms are 30 ft. wide and as the load over these rooms is only one floor, pairs of I-beams serve as girders for this space (Fig. 452).

The Library is located on the eighth floor, across the end of the building, occupying about 30 × 65 ft. (Fig. 453). Banquet Rooms are located on the same floor between columns 5-6-3-2, and College Hall is on the same floor between columns 4-5-2-1. All the girder spans over these spaces are approximately 30 ft. (Fig. 453). The loading conditions vary so that some are plate girders and others double I-beams.

The Main Dining Hall occupies approximately 45 × 90 ft. on the ninth floor. The height from floor to floor is 45 ft. 6 in., which allows space above the ceiling for the girders. The framing over this room is shown in Fig. 454. The loads above are one floor and roof and some walls. The arrangement of these loads is such as to make a number of special features in the framing as indicated.

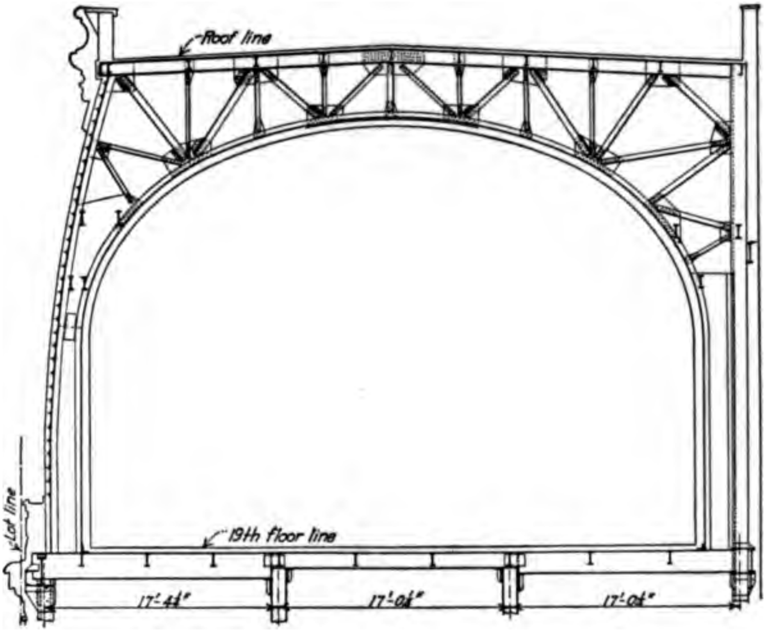


Fig. 450.—Trusses for roof over Grand Banquet Hall, La Salle Hotel.

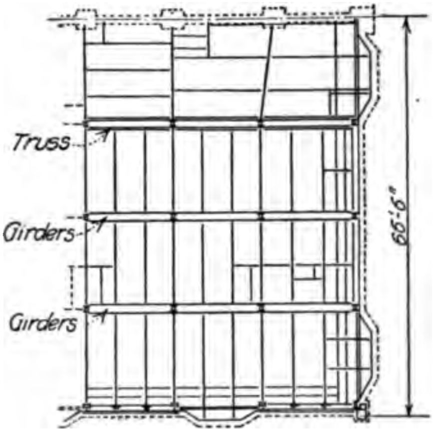


Fig. 451.—Part third floor framing plan, University Club of Chicago.

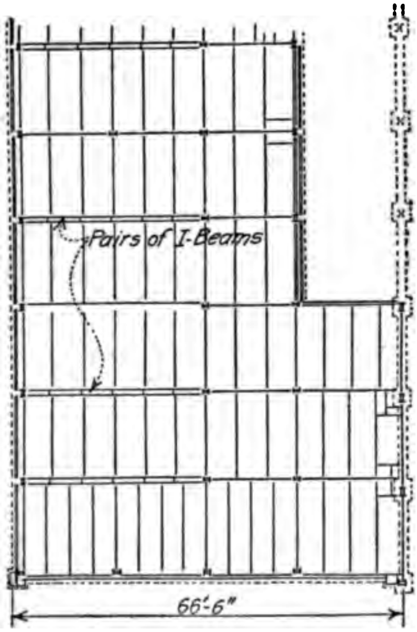


Fig. 452.—Part eighth floor framing plan, University Club of Chicago.

The foregoing illustrations and discussions show that large clear spaces can be provid where needed, but the designer should bear in mind that the special construction involved ma be very expensive. Whenever practicable, these large spaces should be planned on the to floor or under light courts so that the loads to be carried on the long spans will be relativel small.

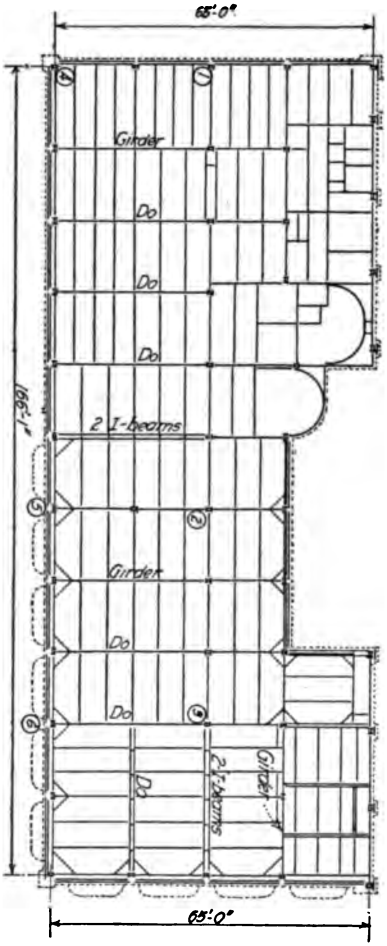


Fig. 453.—Ninth floor framing plan, University Club of Chicago.

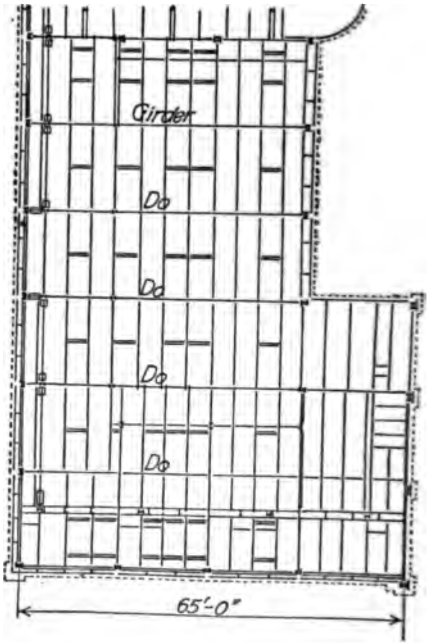


Fig. 454.—Racquet court floor framing, University Club of Chicago.

SWIMMING POOLS

By Arthur Peabody

Swimming pools, which formerly were found only in gymnasiums, have become a common feature of club houses and the Y. M. C. A., schools, and civic centers.

266. Location of Pools.—The swimming pool should be in a well lighted and ventilated room. Where possible, direct sunlight should be secured. The greater number of existing pools are located in the basement of buildings, evidently because of the expense involved in supporting the great weight of the water anywhere else. In cities, however, there are advantages in ıcing the pool in an upper story where light and air may be secured. This leaves the basement e for the power plant and other necessary equipments. In a few instances, pools are conucted in separate buildings under a glass roof which is, of course, the ideal arrangement.

267. Dimensions.—The minimum dimensions of a swimming pool, as prescribed by the tercollegiate Rules for athletic contests are: width 20 ft., length 60 ft. These have been opted as standard for Y. M. C. A. ildings. Pools should measure multiples of 5 ft. of width and ft. of length. Typical pools ꞉refore are:

20 × 60 ft.
25 × 60 ft.
30 × 60 ft.
20 × 75 ft.
25 × 75 ft.
30 × 75 ft.

few pools are 100 ft. long. The ɔth of the water acording to the ne rules shall be not less than 3 at the shallow end and 7 ft. at ꞉ deep end. The majority of ɔls have 7½ ft. of depth. For ·ing contests, pools are 8 to 8½ ft. deep with a maximum of 10 ft.

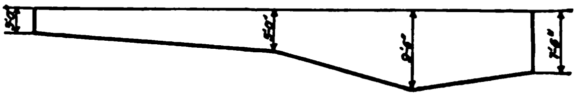


FIG. 455.

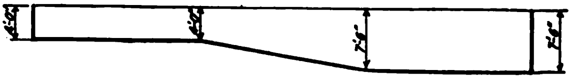


FIG. 456.

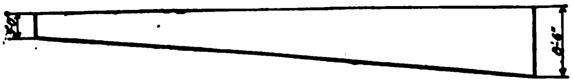
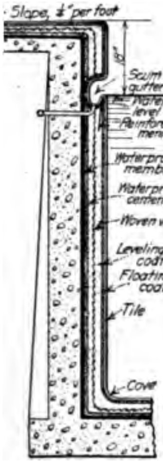


FIG. 457.

268. Shape of Bottom.—The so-called spoon-shaped bottom is considered the most service.e. This has a gradual slope to the middle of the length after which it is sloped both ways as to give a maximum depth at a point 15 ft. from the deep end of the pool (see Fig. 455). ols intended for miscellaneous use for swimmers and non-swimmers or children, sometimes divided into sections, may have a regularly increasing depth from the shallow to the deep end (see Fig. 456). An older form of bottom is sloped gently for one-third the length, more sharply over the middle third, and left practically flat the remainder of the length. All parts of the bottom are pitched sufficiently to drain the water to the outlet (see Fig. 457).

269. Construction.—The pool is constructed of reinforced concrete or of steel. The computation of strength will not be discussed here, but the pool construction must be sufficient to resist the loads, which will be considerable. The steel tank is necessary where excessive ground water may be encountered and for most pools in the upper stories of buildings. In this case, the tank which is supported on adequate columns and girders, is lined with dense concrete, inside of which a waterproof lining of lead is placed. Upon this asphalted felt is laid. An inside layer of concrete reinforced with steel fabric is then placed as a base for the tile lining. A 4-in. course of brick work ·y be substituted for the inner concrete lining.



'IG. 458.—Typical cross ;ion of reinforced con;e retaining wall for mming pool, showing ıctural and waterproof- factors in diagramtical form.

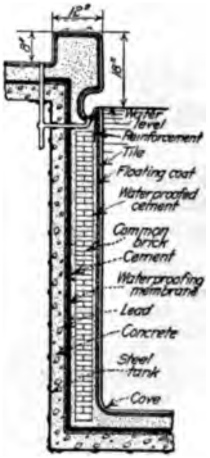


FIG. 459.—Typical cross section of wall built into steel tank, showing structural and waterproofing factors in diagrammatical form.

In the new building of the Athletic Club at Omaha, Nebraska, a concrete pool is located on ꞉ third story. The problem of its construction is similar to other concrete work of equal ɔortance.

Concrete pools resting in the ground require provision against leakage. The tank must be protected against ›olation from the outside as well as the inside. Integral waterproofing of the concrete walls and floor is necessary. Such waterproofing compounds are well known and should be used in the most effective way. The cement gun would be useful in grouting the inside and outside of the pool. Beside this, the inside of the pool should be waterproofed by membranes of burlap and asphalt or asphalted felts, cemented together with pitch or asphalt. It is found in practice that where asphalt will not adhere to the concrete, a preliminary coating of pitch will overcome the difficulty. Where ground water is present in quantity, the exterior of the concrete walls must be waterproofed as well. This is done in the same manner as on the inside, but not usually as thick. The same preparation for the tile finish of the inside is necessary as in the case of the steel tank, except that a trivial percolation would probably not create so much damage.

Figs. 458 and 459 show typical cross sections of ordinary pools.

270. Tile Finish.—In all cases, the pool must be tested and made absolutely waterproof efore any attempt is made to set the tile lining. Special care must be taken to make the work tight about the inlet and outlet connections.

271. Linings.—The linings of the walls are of marble, ceramic mosaic, or large tiles. The floor of the pool is frequently paved with hexagon floor tile. In this material the lane lines and distance numerals are shown in colored tiles, as well as any design fixed upon by the architect.

272. Overflow Troughs, Ladders, and Curbs.—The overflow trough or scum gutter is a device extending along the sides of the pool for removing the dust and other floating substances from the surface of the water. It acts also as an overflow, preventing the rise of the water above the desired level. Finally it serves as a life rail or catch-hold, taking place of the metal railing or life rope of old-fashioned pools.

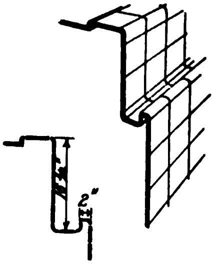


Fig. 460.—Open scum gutter of 6 × 6-in. wall tile and trimmers, suitable for private and outdoor pools.

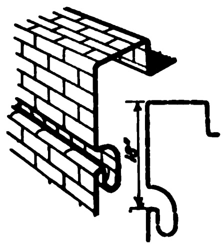


Fig. 461.—Design for wall tile gutter and curb. The water level is 18 in. below the top of the curb, the proper take-off distance.

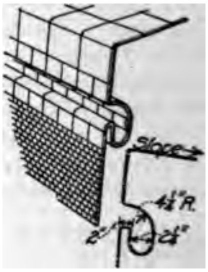


Fig. 462.—A combination of ceramic mosaic and wall tile. No curb being provided, the gangway floor should slope away from the pool.

The scum gutter should be entirely recessed in the surface of the wall. It is formed of glazed terra cotta of the same color as the tile work, or may be formed in the concrete and the mosaic tile (Figs. 460, 461, and 462).

Metal ladders and steps to pools have been replaced in new work by recessed tile-covered ladders or recessed footholds formed of glazed terra cotta or of steel covered with mosaic tile. The curb around the pool should be 12 to 16 in. wide, for comfortable standing, and at least 2 or 3 in. high; 6 in. is a common height. The object of the curb is to prevent water from flowing into the pool from the surrounding spaces. This curb is used as the take-off in athletic contests and should be 18 in. above the water.

273. Lines and Markings.—Distance numerals, depth numerals, swimming and safety lines are indicated by colored tiles. Figures are used at 5-ft. intervals and the intervening foot marks by colored lines. Distance marks begin at the deep end, and must be accurate. Swimming lanes extend the length of the pool along the bottom. The lines are 3 in. wide and should be distinct. The lanes are 5 ft. wide. Safety lines are extended across the pool and up the sides. At 5 ft. from the ends, similar lines, called turning lines, are extended across the bottom and sides. Besides these are the jack knife limits which are similar lines, 6 ft. from the end of the diving board, crossing the curb and extending a short distance below the water level, as required by the rules, for the assistance of the judges of athletic contests (see Fig. 463).

274. Diving Board.—The official diving board is not less than 12 ft nor more than 13 ft. long, by 20 in. wide. The end projects not more than 2 ft. over the pool and the fulcrum is placed at ⅓ the length from the free end. The height above the water is not less than 2½ ft. nor more than 4 ft. Provision for fastening the board should be made in the floor structure.

275. Swimming Cable.—Where swimming lessons are given, a wire cable is extended the length of the pool to support a swimming belt. Anchorage for this should be made in the walls.

276. Special Pools.—Besides the ordinary swimming pool, special pools are sometimes built for sports, such as water polo and water basketball.

The water polo pool should be 60 to 70 ft. long, 20 to 40 ft. wide, and 6 ft. deep. These games may be placed in the ordinary pool by placing the necessary marks. The playing and goal lines are as follows:

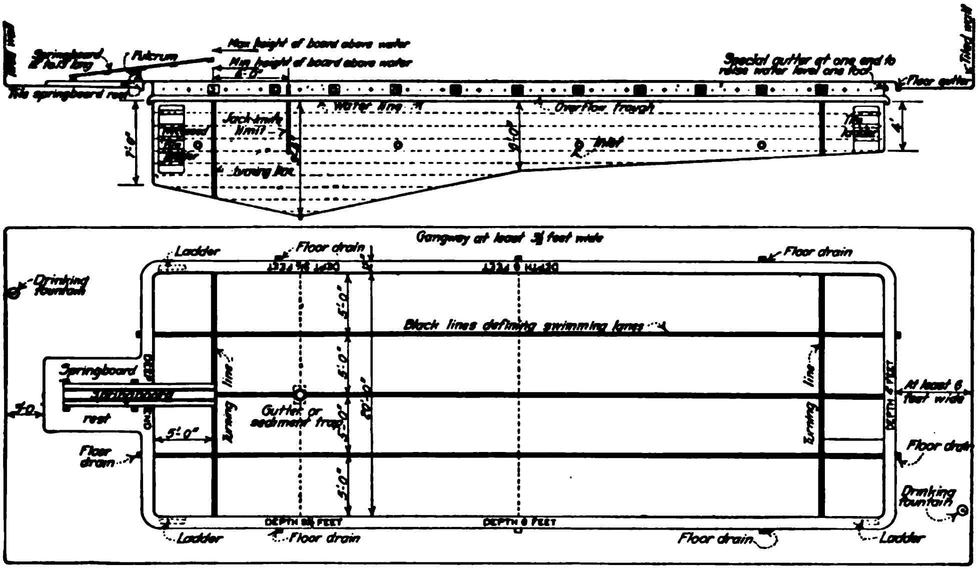


FIG. 463.—Plan and elevation of a typical swimming pool.

Center line, across the length of the pool.

Goal lines, 4 ft. from the ends.

Free throw line, 15 ft. from the ends.

Twenty-foot lines, 20 ft. from the ends.

For water basketball, a pool not over 2500 sq. ft. in area may be used. The center line and the 15-ft. lines only are required for this game.

All markings should be formed in the tile lining of the pool as before described. They may be worked into the decorative scheme of the tile work.

The foregoing description applies to interior pools. Beside these, outside pools for swimming or wading are common. The large size of out-of-door pools, as ordinarily designed, leads to less decoration and in many cases, plain concrete surfaces are employed. The structure and waterproofing of these pools require the same care as with interior pools, and the sanitation will need to be given attention. As the pools are not warmed, however, except by the sun, the water may be kept clean by frequent renewal.

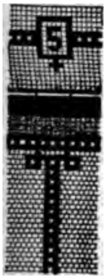


FIG. 464.—Detail of distance numeral along coping.

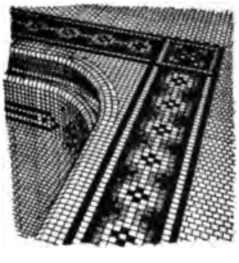


FIG. 465.—Detail of scum gutter Racine College, Racine, Wis.

277. Spaces About the Pool.—The entire area about the pool should be paved with tile or marble. The walls should be wainscoted with the same material to a height of 6 to 7 ft., or to the ceiling. The walk or gangway about the pool should be 3 to 4½ ft. wide along the sides, and at least 6 ft. at the ends. Some space should also be provided for spectators.

For athletic contests, temporary bleachers will be set as close to the pool as permissible so that the spectators can watch the games closely. It is useless to provide large and deep galleries, generally, as the swimmers or players cannot be watched satisfactorily except from the first row of chairs. Shower baths should never be placed in the pool room on account of the steam thrown off by them which will condense on the walls and ceiling and create annoyance.

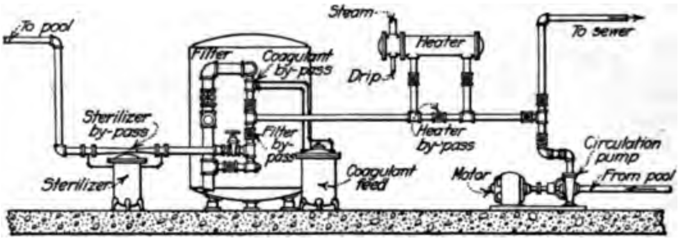


Fig. 466.

278. Water Supply and Sanitation.—The water supply pipe should be of sufficient size to fill the pool in 24 hr. The water, though it may be pure upon first being admitted, soon becomes unfit and must be cleansed and disinfected. With such treatment, however, it may be used continuously for a considerable time, in certain instances extending over more than a year. In many cases the available water supply must be treated before using.

A commercial filter, containing quartz, sand, charcoal, and other filtering agents removes the mechanical impurities after which the use of alum completes the clearing. For destroying bacteria the ultra violet ray is employed. This consists of a mercury vapor lamp suspended in a water-tight protecting glass tube held within a cast-iron chamber. The water is passed by the lamp in such a way as to secure the action of the ray sufficiently to destroy all bacteria.

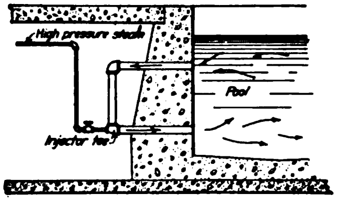


Fig. 467.

An ozone apparatus is also used for this purpose. The ozone apparatus consists of a steel tower through which the water is passed and subjected to contact with ozone. The method is undoubtedly effective and where space can be afforded and conditions warrant the installation, it will perhaps excel the ultra violet ray process. Information can be obtained as to the ozone apparatus from the U. S. public health reports, Washington, D. C.

The water is drawn from the pool by a circulating pump, forced through the heater, filter, and sterilizer, after which it returns to the pool. The pump should be of sufficient capacity to change the water once in 10 hr.

These measures secure clean water, but the walls and floor of the pool will require frequent cleansing and scrubbing to remove accumulated dust, silt, etc., from time to time.

279. Heating.—The heater should be the closed type of feed water heater with copper or brass tubes through which the water passes (see Fig. 466). The temperature of the water should be controlled by a special thermostat which will maintain a constant degree of heat, usually about 75 deg. F.

In some cases the water is heated by injecting steam directly (see Fig. 467). In the ordinary case this method will carry in water impurities, oil, rust, and scale from the boilers. It is, however, a quick and cheap method of heating and when properly done will be free from noise.

MAIL CHUTES

By Arthur Peabody

280. Requirements.—Public buildings, office buildings, apartment buildings, and hotels are usually provided with mailing chutes for first-class mail only. Where these deliver directly to public mail boxes, the regulations of the United States Post Office Department must be observed in the location and construction of the chutes and boxes. These regulations are as follows:

The mail box must not be placed more than 50 ft. from the main entrance of the building.

The mail chute must run through a public hall or premises that are freely accessible to the public and the Post Office authorities.

Every mail chute must be so constructed that its interior is quickly and easily accessible to authorized persons, but not to others.

It must not run behind a partition or elevator screen.

All contracts covering mail chute installations must be upon the form prescribed by the Post Office Department with the regulations printed upon and made part of the contract.

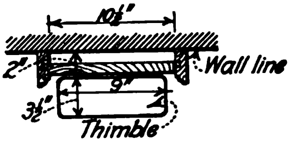


Fig. 468.—With wood backing.

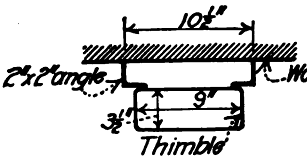


Fig. 469.—Steel angle backing.

A bond of the Post Office Department is required of the contractor. Copies of these regulations will be furnished upon request.

Other requirements are that the chutes must be absolutely vertical, without bends or offsets, to avoid possible clogging. Rough openings in the floors to permit the installation of mail chutes must be 6 x 12 in. in the clear for each chute, plumbed down through the building, located 2 in. away from the wall against which the support of the chute is fastened. Metal thimbles for floor openings are furnished by makers of mail chutes. Where the backing or support of the chute is furnished separately from the mail chute contract it must consist of a flat vertical continuous surface not less than 10½ in. wide extending from the ground floor surface to a point 4½ ft. above the floor of the highest story from which mail is delivered. The backing may be of wood, as in Fig. 468, or of steel angles 2 x 2-in. size, as in Figs. 469 and 470. Fig. 471 shows the backing in place, ready to receive the chute. It is advisable to include the backing in the contract for mail chutes to insure a satisfactory piece of work. Where the chute is in connection with an elevator screen, it must be self-supporting between floor and ceiling.

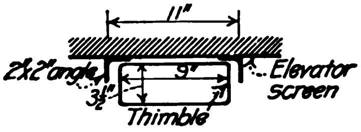


Fig. 470.—Reversed backing against elevator screen.

281. Details.—The details of this device are so specialized and patented and the regulations surrounding installations are so strict that the usual practice is to make use of one of the principal types now on the market.

Single and double chutes into one mail box are furnished as circumstance require. Openings in floors must then be made in accordance.

The chutes are formed of metal, with removable or hinged plate glass panels exposing the chutes throughout their length, and giving access to the interior at all points. The usual finish of the chutes is a dull black enamel.

The mail boxes are of standard pattern and capacity. The finish may be black or of electro-bronze (slightly oxidized or "statuary") with bronze trimmings. Special designs are available for important work following the architectural style of the building, which may be executed in real bronze. The space required for a standard mail box is 36 in. high, 21½ in. wide, by 11¼ in. deep over all. Special boxes will vary in dimensions.

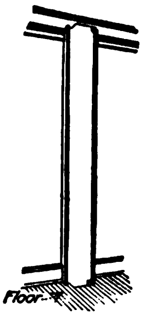


Fig. 471.—Backing ready for the chute.

RETAINING WALLS

By Allan F. Owen

Retaining walls are walls that support the lateral pressure of earth or of other material naving more or less frictional stability. They are used in buildings as basement and sub-basement walls and as walls of tanks, swimming pools, coal bins, etc. In some cases, retaining walls must be designed to support loads coming upon railroad tracks and driveways built on top of the backfill parallel with the wall.

Where possible, the earth back of retaining walls must be drained so that actual water pressure will be avoided. A thin film of water, held between a retaining wall and the fill behind it, exerts the same pressure against the wall as a body of water of the same depth. However, a small amount of water may be led away by drains so that it will never stand deep enough to harm the wall.

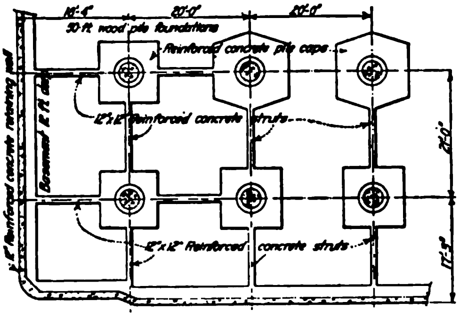

16'-5"
20'-0"
20'-0"
50 ft. wood pile foundations
Reinforced concrete pile caps
12"x12" Reinforced concrete struts
12"x12" Reinforced concrete struts
27'-0"
17'-5"
Basement 12 ft. deep
12" Reinforced concrete retaining wall

Fig. 472.—Part plan of retaining walls and foundations showing concrete struts from footings to retaining wall, Union Special Machine Company building, Chicago, Ill.

In water bearing soil the back of the wall must be waterproofed, or the wall made of waterproof concrete, and must be built heavy enough to withstand water pressure.

282. Stability of a Retaining Wall.—Two motions of the wall tend to result due to the action of the earth thrust: (1) a tendency to slide forward; and (2) a tendency to tip forward about some point on the base.

The thrust of the earth back of a retaining wall is counteracted by the friction between the base of the wall and the soil on which it rests, by the pressure of the soil at the toe of the wall, and by the pressure of the soil against key walls (if any) constructed below the plane of the base of the wall proper. Concrete struts or heavy concrete floor construction is usually necessary in deep basements to take care of the greater part of the earth thrust (see Fig. 472).

The resistance to overturning the wall is afforded by a distributed reaction of the bearing soil upward against the base of wall. The center of the resultant force acting upon the base must strike within the middle third of the base plane if the entire base is to bear on the soil. The soil pressure under the toe of a retaining wall should not be greater than the allowable (see table on p. 351).

The frictional resistance along the horizontal base of a wall may be taken as the total vertical load on the base multiplied by the coefficient of friction of the wall material upon the supporting soil. The coefficients of friction between earth and other materials are given in Table 1.

Table 1.—Coefficient of Friction Between Earth and Other Materials

Material	Coefficient
Masonry upon masonry	0.65
Masonry on dry clay	0.50
Masonry on wet clay	0.33
Masonry on sand	0.40
Masonry on gravel	0.60

When the material back of the wall is a fluid, the intensity of the horizontal pressure at any depth is equal to the weight of a cubic unit of the fluid multiplied by the given depth. Thus

for water, at a depth of one foot, the horizontal (and also the vertical) pressure is 62½ lb. per sq. ft.; at a depth of 10 ft. it is 625 lb. per sq. ft. For any material not a fluid, the horizontal pressure is less than the vertical pressure but the variation of pressure due to depth follows the same law. Thus the term "equivalent fluid pressure" for a given material is taken to mean the horizontal pressure per square foot at a depth of one foot. The equivalent fluid pressure varies with the "angle of repose" and weight of the material.

TABLE 2.—ANGLES OF REPOSE AND WEIGHT PER CUBIC FOOT FOR VARIOUS EARTHS

Material	Weight (pounds per cubic foot)	Angle of repose (degrees)
Sand, dry	90 to 110	20 to 35
Sand, moist	100 to 110	30 to 45
Sand, wet	110 to 120	20 to 40
Earth, dry	80 to 100	20 to 45
Earth, moist	80 to 100	25 to 45
Earth, wet	100 to 120	25 to 30
Gravel, round to angular	100 to 135	30 to 48
Gravel, sand and clay	100 to 115	20 to 37

TABLE 3.—EQUIVALENT FLUID PRESSURE

Angle of repose (degrees)	Coefficient	Weight (pounds per cubic foot)	Equivalent fluid pressure, (pounds)
20	0.49	80 100 120	39 49 59
25	0.406	80 100 120	32 40 49
30	0.333	80 100 120	27 33 43
35	0.271	90 110 130	24 30 35
40	0.217	90 110 130	19 24 28
45	0.172	90 110 130	15 19 22
48	0.147	100 120 135	15 18 20

From Tables 2 and 3 it will be seen that the equivalent fluid pressure may be taken at from 15 to 59 lb. according to soil conditions. Recommended values are given in Table 4.

TABLE 4.—RECOMMENDED VALUES OF EQUIVALENT FLUID PRESSURE

Well drained gravel	20
Average earth	33
Wet sand	50
Water bearing soil	62½
Fluid mud	80

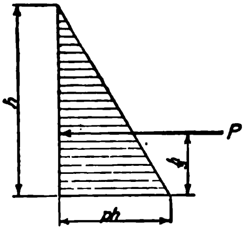


FIG. 473.—Distribution of horizontal pressure on back of wall with level back fill.

The following notation will be used:

p = equivalent fluid pressure of soil back of wall.
P = total pressure on back of wall.
h = height of wall.
b = width of base.
c = distance from back of wall to center of gravity of weight of wall and backing.
x = distance from back of wall to center of vertical reaction.
e = eccentricity of vertical reaction.
W_1 = weight of wall.
W_2 = weight of backing carried on wall.
R_1 = vertical reaction.
R_2 = horizontal reaction.

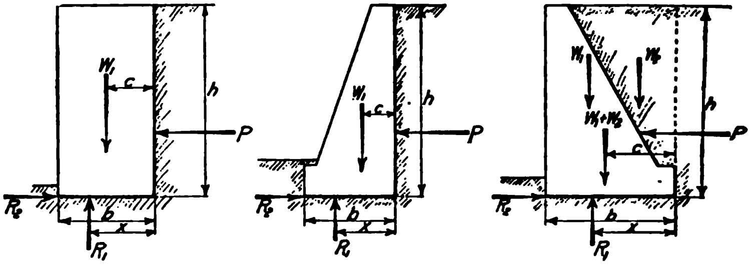
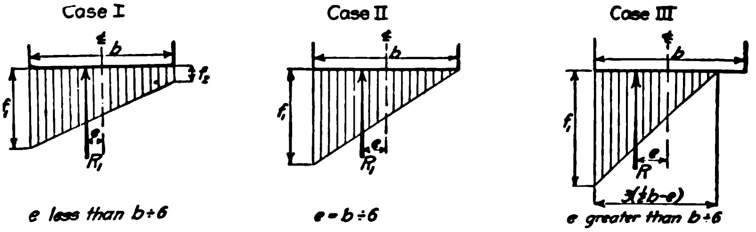


FIG. 474.—Types of masonry retaining walls.



Case I — e less than b÷6

Case II — e = b÷6

Case III — e greater than b÷6

FIG. 475.—Distribution of stress on foundations eccentrically loaded.

The horizontal pressure at the top of the wall is zero, and the pressure at the bottom of the wall = ph. The pressure varies uniformly between these limits and the total $P = \frac{ph^2}{2}$. The center of this pressure is at $\frac{h}{3}$ above the base (see Fig. 473). Referring to Fig. 474

$$R_1 = W_1 + W_2$$
$$R_2 = P$$
$$x = \frac{\frac{1}{3}Ph + (W_1 + W_2)c}{R_1}$$
$$e = x - \frac{1}{2}b$$

When $x = \frac{1}{2}b$, the soil pressure is uniform over the whole base. When $x = \frac{2}{3}b$, the pressure varies from nothing at the heel to twice the average at the toe (see Case II, Fig. 475). In Fig. 475

$$\text{Case I: } f_1 = \frac{R_1}{b}\left(1 + 6\frac{e}{b}\right)$$
$$f_2 = \frac{R_1}{b}\left(1 - 6\frac{e}{b}\right)$$
$$\text{Case II: } f_1 = 2R_1 \div b$$
$$\text{Case III: } f_1 = 2R_1 \div 3(\tfrac{1}{2}b - e)$$

283. Masonry Retaining Walls.—Masonry walls of brick, stone, or concrete may be used for low retaining walls, where the weight to be supported is small and no great thickness is required, or for high walls where consideration of space and cost will permit the great thicknesses required.

For a rectangular retaining wall of masonry weighing 150 lb. per cu. ft., the width of base given in Table 5 in terms of the height will make $e = \frac{1}{6}b$. The soil pressures will be $f_1 = 300h$ (where f_1 is in pounds and h is in feet), and $f_2 = 0$.

For a retaining wall of triangular cross section, back vertical, front battered, of masonry weighing 150 lb. per cu. ft. the same width of base as given in Table 5 will make $e = \frac{1}{6}b$. The soil pressures will be $f_1 = 150h$, and $f_2 = 0$.

For a retaining wall of triangular cross section, front vertical, back battered, of masonry weighing 150 lb. per cu. ft., supporting a fill weighing 100 lb. per cu. ft., the width of base given in Table 6 will make $e = \frac{1}{6}b$. The soil pressures will be $f_1 = 250h$, and $f_2 = 0$.

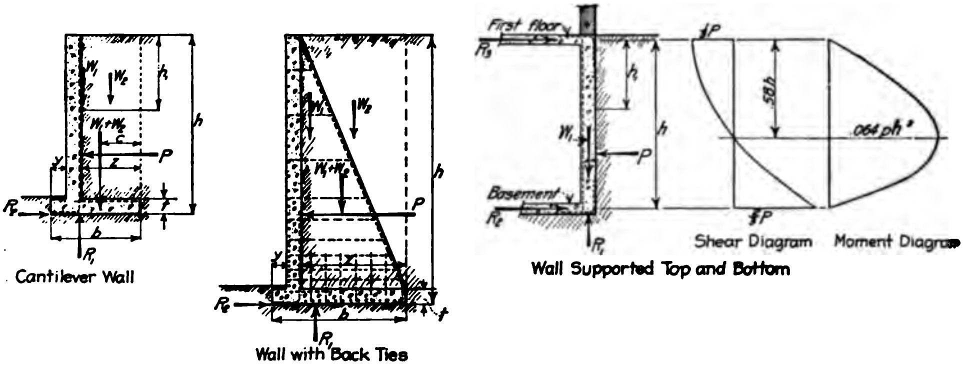
TABLE 5

p	$b \div h$
20	0.37
33	0.47
50	0.58
62½	0.65
80	0.73

TABLE 6

p	$b \div h$
20	0.45
33	0.575
50	0.707
62½	0.79
80	0.895

284. Reinforced Concrete Retaining Walls.—Reinforced concrete is the most suitable material for many retaining walls because of the possibility of making it moisture proof or water-proof as may be required, and because the weight of the backing can be utilized to advantage to prevent overturning; also the sections may be made thin and the tensile stresses resisted by steel reinforcement. Types of reinforced concrete retaining walls are shown in Fig. 476.



Cantilever Wall

Wall with Back Ties

Wall Supported Top and Bottom

FIG. 476.—Types of reinforced concrete retaining walls.

284a. Cantilever Wall.—The upright portion of the wall must be figured as a cantilever slab. At any depth h_1 (see Fig. 476)

$$M = \tfrac{1}{6}ph_1^3$$

The maximum moment occurs at the junction of wall and base, or

$$M_{max.} = \tfrac{1}{6}p(h - t)^3$$

The total upward pressure on the toe of the wall, y, may be found from the formulas and diagrams for the distribution of soil pressure (see Fig. 475). Let this pressure equal F. The distance from the front face of the vertical slab to the center of gravity of the "trapezoidal pressure" may be computed and the maximum moment in the toe slab at the face of wall will be this distance times F. Usually it will be near enough to take $M = \frac{1}{2}Fy$.

The maximum moment in the heel slab, z, may be taken at $\frac{1}{2}\ W_2z$. Care must be taken to have the reinforcing rods long enough beyond points of maximum stress to develop their strength in bond. Each of the cantilever arms of this wall may be tapered toward the free ends.

The horizontal portion, or floor slab, is usually poured before the forms for the vertical portion, or wall slab, are completed. It would be very inconvenient to handle the upright rods if they extended from the bottom of the floor slab to the top of the wall slab. Consequently, the rods in the floor slab should be cut so they will extend into the wall slab only far enough to develop their strength in bond. The bars in the vertical slab should then start at the top of the horizontal slab and may be alternately long and short to provide the steel required at the bottom and less steel at the top. Rods crossing the main reinforcement must be used to prevent cracks and these may amount to $\frac{1}{10}$ to $\frac{1}{2}$ % of the sectional area.

In designing a cantilever wall for a given height, it is necessary to assume wall and floor thicknesses and width of base. Table 7 may be used to assist in making these assumptions. Concrete is taken at 150 lb. per cu. ft., and back fill at 100 lb. per cu. ft. The width of base in each case will make $e = \frac{b}{6}$. Wall thickness assumed $\frac{b}{6}$. Floor thickness assumed $\frac{h}{12}$. f_1 is given in pounds when h is height in feet.

TABLE 7

$y \div b$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$
$z \div b$	$\frac{5}{6}$	$\frac{4}{6}$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	0
p	Values of $b \div h$					
20	0.465	0.401	0.379	0.380	0.402	0.591
33	0.597	0.516	0.487	0.489	0.517	0.760
50	0.734	0.635	0.600	0.601	0.637	0.935
62½	0.821	0.710	0.670	0.672	0.711	1.047
80	0.929	0.802	0.758	0.760	0.805	1.182
f_1	$224h$	$193h$	$162h$	$131h$	$101h$	$71h$

Illustrative Problem.—Given the following data: h = 24 ft. 6 in., p = 33 lb., b = 12 ft. 0 in., y = 1 ft. 10 in. z = 8 ft. 0 in.

Then

$$P = \frac{(33)(24.5)(24.5)}{2} = 9904 \text{ lb.}$$
$$W_1 = (63.58)(150) = 9537 \text{ lb.}$$
$$W_2 = (180)(100) = 18{,}000 \text{ lb.}$$
$$c = \frac{(9537)(8.07) + (18{,}000)(3.97)}{27{,}537} = 5.4 \text{ ft.}$$
$$R_1 = 27{,}537 \text{ lb.}$$
$$x = \frac{80{,}883 + 148{,}422}{27{,}537} = 8.33 \text{ ft.}$$
$$e = 8.33 - \frac{12}{2} = 2.33 \text{ ft., is greater than } \tfrac{1}{6}b.$$
$$f_1 = 55{,}074 \div 11 = 5007 \text{ lb. per sq. ft.}$$

Bending moments in upright cantilever at various depths are figured and plotted from the formula $M = \frac{1}{6}ph_1^3 = 5.5h_1^3$ (see Fig. 477). Moment at 22-ft. depth = 58,564 ft.-lb.

By the principles of reinforced concrete the thickness of wall is determined to be 26 in. and the required area of steel at this point 2.14 sq. in. A curve is plotted for the required area of steel as shown in the steel diagram. Stub rods ¾ in. square and 3 in. on centers are placed in the footing slab to project into the wall slab the required bond length, or 30 in. The value of these rods is represented by the triangle abc. Rods in the wall start at the top of the footing slab—one 21 ft. 9 in. long, one 8 ft. 9 in. long, and one 5 ft. 0 in. long being used in each foot length of the wall. The available area of these rods is represented by the polygon indicated, the taper top and bottom being due to the bond length requirement.

The construction joint must take a bearing of

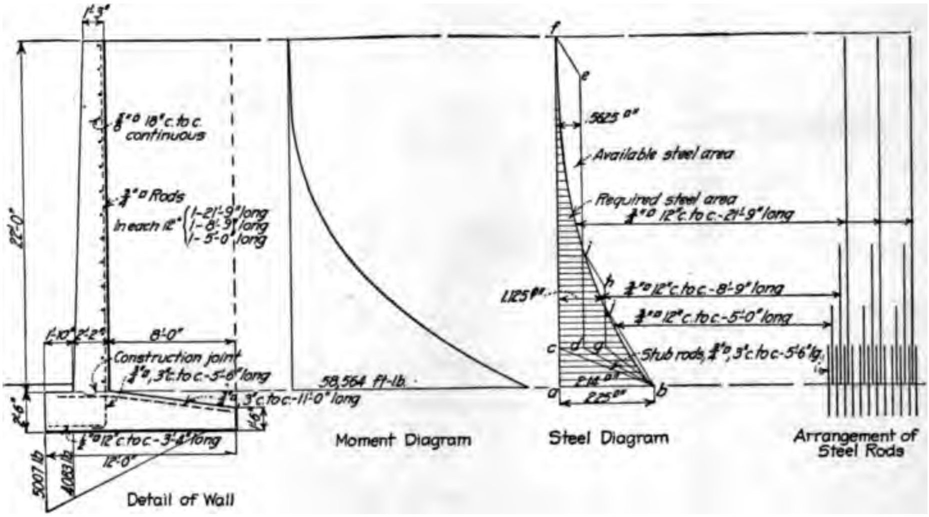
$$\frac{(33)(22)(22)}{2} = 7986 \text{ lb.}$$

With an allowable bearing of 400 lb. per sq. in. the required area is 20 sq. in. A 2 × 8-in. plank laid in the top of the slab and removed before the wall is poured will give a bearing area of 1¾ × 12 = 21 sq. in. The minimum section in shear will be 7½ × 12 = 90 sq. ft. $\frac{7986}{90} = 89$ lb. per sq. in., which is allowable for such a heavily reinforced section.

The soil pressure on the toe slab averages 4545 lb. per sq. ft. $M = (1.83)(4545)(0.92) = 7640$ ft.-lb. Steel required = 0.24 sq. in. Rods, ½ in. square, will be used spaced 12 in. on centers.

The load on the heel slab is 18,000 lb. and $M = (18{,}000)(4) = 72{,}000$ ft.-lb. The depth required is 30 in. and the steel area, 2.25 sq. in. Rods, ¾ in. square, will be used spaced 3 in. on centers.

To prevent cracks in the wall, rods ⅝ in. square, will be used spaced 18 in. on centers. This amount of steel equals $\frac{1}{10}$% of the wall area.



1'-3"
⅝"□ 18" c. to c. continuous
¾"□ Rods
In each 12" (1-21'-9" long, 1-8'-9" long, 1-5'-0" long)
22'-0"
1'-10" 2'-2"
8'-0"
Construction joint
¾"□, 3" c. to c.-5'-6" long
¾"□, 3" c. to c.-11'-0" long
2'-6"
½"□ 12" c. to c.-3'-4" long
12'-0"
5007 lb.
4083 lb.
Detail of Wall
58,564 ft-lb.
Moment Diagram
f
e
.5625 □"
Available steel area
Required steel area
¾"□ 12" c. to c.-21'-9" long
1.125 □"
h
¾"□ 12" c. to c.-8'-9" long
¾"□ 12" c. to c.-5'-0" long
c
d
g
Stub rods, ¾"□, 3" c. to c.-5'-6" lg.
2.14 □"
a
2.25 □"
b
Steel Diagram
Arrangement of Steel Rods

FIG. 477.—Design of cantilever wall.

284b. Wall with Back Ties.—In designing a wall with back ties, the vertical part of the wall is figured as a slab loaded on its back and supported by the tie counterforts (see Fig. 476). The floor *z* is figured as a slab supported by the counterforts. Reinforcement must be placed in the ties to take the tension produced and also to hold the tie to the floor and wall.

284c. Walls Supported Top and Bottom.—The most common form of retaining wall in building construction is the wall supported at the top by the first floor construction and at the bottom by the basement floor. This wall must be reinforced as a slab loaded at its back and supported top and bottom. Referring to Fig. 476

$$R_2 = \frac{2P}{3} \qquad R_1 = \frac{P}{3}$$

Moment at any depth h_1

$$M = R_2 h_1 - \frac{p h_1^3}{6}$$

The maximum moment is at the depth $0.58h$ and is

$$M = 0.064 p h^3$$

Retaining walls in buildings may be supported by heavy wall columns, and in such cases the wall is figured as a slab loaded on its back and supported on two sides, or two sides and bottom, or two sides and top and bottom. In each case the column must be investigated to see that the bending due to the earth pressure on the wall does not over-stress the column, a column section made heavy enough to take such bending stresses.

285. Structural Steel Frame Walls.—In steel frame buildings steel I-beams are som provided to take the thrust of the earth on the retaining walls and reinforced concrete sla used spanning from beam to beam and enclosing such beams (see Fig. 478).

286. Steel Sheet Piling.—Where one or more sub-basements are to be built adjoi heavy building, and the earth under its foundations must not be disturbed, steel sheet

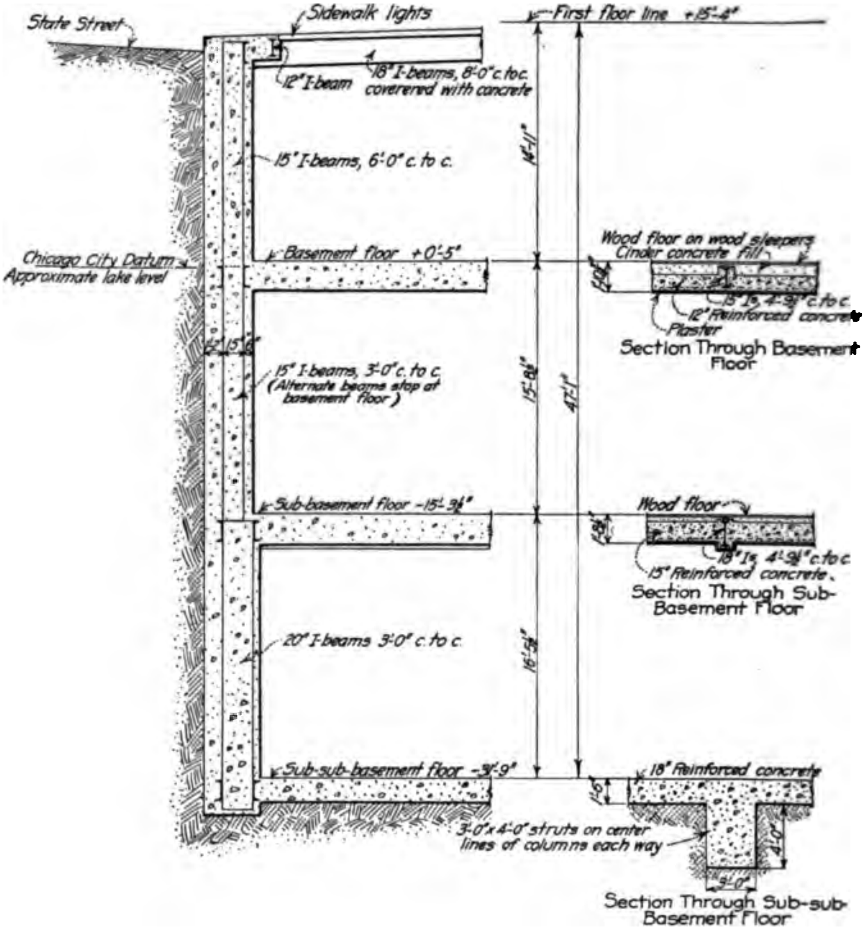

State Street
Sidewalk lights
First floor line +15'-4"
12" I-beam
18" I-beams, 8'-0" c. to c. covered with concrete
15" I-beams, 6'-0" c. to c.
14'-11"
Chicago City Datum
Approximate lake level
Basement floor +0'-5"
Wood floor on wood sleepers
Cinder concrete fill
15" I's, 4'-9½" c. to c.
12" Reinforced concrete
Plaster
Section Through Basement Floor
12' 15' 6"
15" I-beams, 3'-0" c. to c.
(Alternate beams stop at basement floor)
15'-8½"
47'-1"
Sub-basement floor -15'-3½"
Wood floor
18" I's 4'-9½" c. to c.
15" Reinforced concrete.
Section Through Sub-Basement Floor
20" I-beams 3'-0" c. to c.
16'-5½"
Sub-sub-basement floor -31'-9"
18" Reinforced concrete
1'-6"
3'-0" x 4'-0" struts on center lines of columns each way
4'-0"
3'-0"
Section Through Sub-sub-Basement Floor

FIG. 478.—Structural steel and concrete retaining wall, Mandel Bros. Store, Chicago, Ill.

is useful. The piling is driven at the wall line of the new basements before the deep excavat is made. As this excavation proceeds, the framework for the floor construction at each le is set in place and the utmost care is used to prevent the sheet piling from being forced inw by the pressure from the adjoining building. Temporary shores are used where necessary the permanent concrete floors and concrete covering for the sheet piling is placed without de (see Fig. 479).

287. Retaining Walls with Sloping Back Fill.—Where the fill slopes up from the back the wall, the direction of the earth pressure is usually considered as parallel to the surface of fill (see Fig. 480).

288. Retaining Walls with Surcharge.—When the earth behind a wall is loaded in any way—for example, when the embankment is used as a storage of material—the additonal pressure may be provided for by replacing the load by an equivalent surcharge of earth. The height of this surcharge may be determined by dividing the extra load per square foot by the weight of a cubic foot of earth. This height is shown in Figs. 481 and 482 as h_1. Let $h + h_1 = H$. Then the resultant pressure on a vertical plane for a wall with height H will be

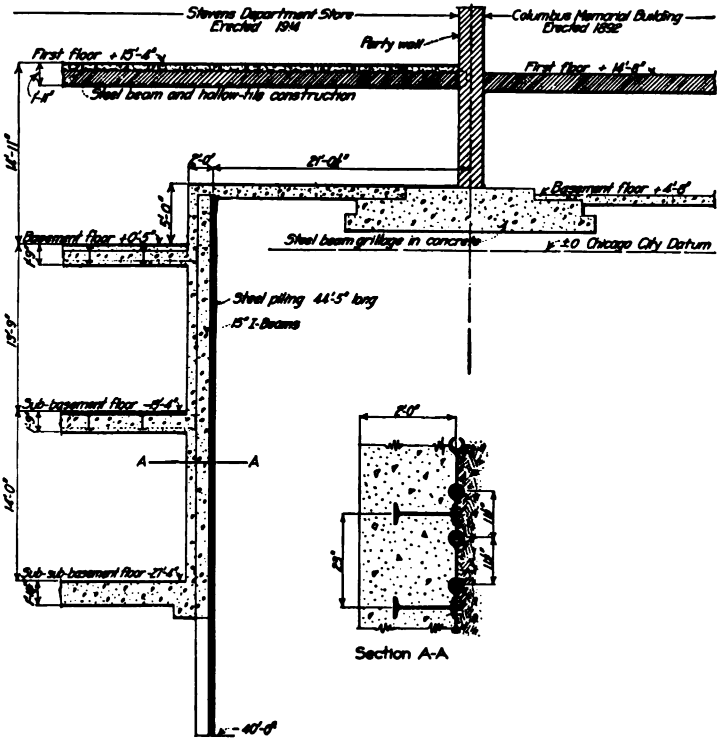


FIG. 479.—Steel sheet piling retaining wall between Stevens store and Columbus Memorial building, State St., Chicago, Ill.

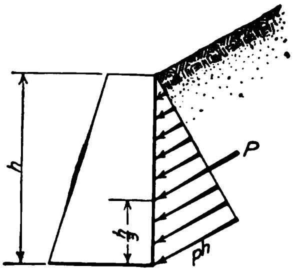


FIG. 480.

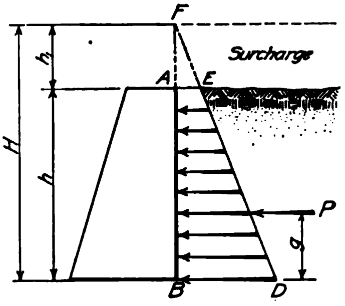


FIG. 481.

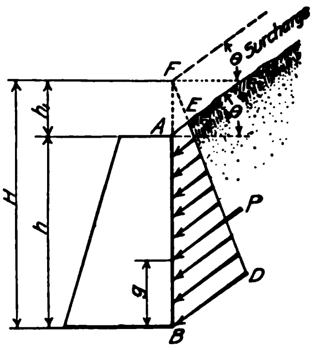


FIG. 482.

$$P_1 = \tfrac{1}{2}pH^2$$

and the resultant pressure for a wall with weight h_1 will be

$$P_1 = \frac{1}{2}ph_1^2$$

The pressure on the vertical wall AB is the difference of these, or

$$P = P_2 - P_1 = \frac{1}{2}p(H^2 - h_1^2)$$
$$= \frac{1}{2}ph(h + 2h_1)$$

and the distance of the point of application of this force from the base of wall

$$g = \frac{h^2 + 3hh_1}{3(h + 2h_1)}$$

P acts through the center of gravity of $ABDE$.

289. Retaining Wall Supporting Railroad Track.—A retaining wall adjoining a track needs special strength to support the weight of locomotives and trains standing track or passing by. When the track is close to the wall, the additional earth pressure may taken as $\frac{1}{6}$ the maximum train load per linear foot of track divided by the distance from the center of the track to the wall. Thus, for Cooper's E-50 loading and a distance of 5 ft. 6 in. from center of track to wall, t = 300 lb. approximately (see Fig. 483).

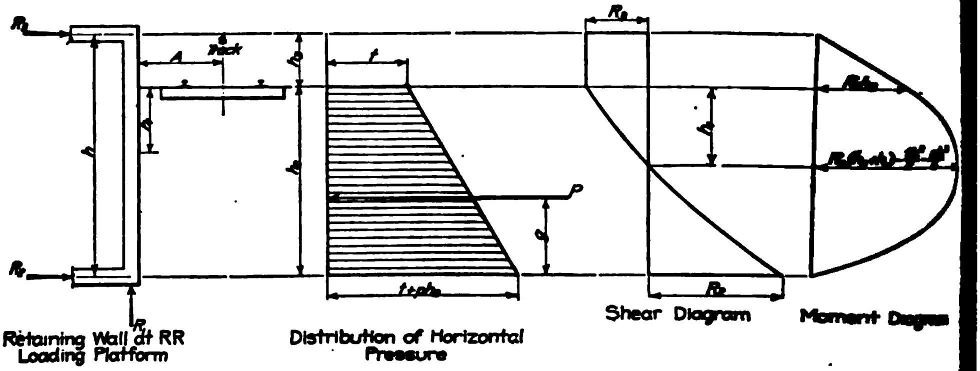


FIG. 483.

The pressure at the bottom of the wall is $t + ph_2$ and the total pressure

$$P = th_2 + \frac{ph_2^2}{2}$$

The center of this pressure is

$$g = \frac{h_2}{3} \times \frac{4t + ph_2}{3t + ph_2}$$

The reactions are

$$R_3 = \frac{Pg}{h}$$
$$R_2 = P - R_3$$

Moment at the top of fill

$$M = R_3h_3$$

Moment at any depth h_1

$$M = R_3(h_3 + h_1) - \frac{th_1^2}{2} - \frac{ph_1^3}{6}$$

The maximum moment occurs where

$$th_1 + \frac{ph_1^2}{2} = R_3$$

For a track at some distance from the wall, the effect is less than stated above and the additional pressure is applied on the lower portion of the wall only. When the nearest rail is more than 0.6h from the wall, the effect of the railroad load may be neglected.

CHIMNEYS

By W. Stuart Tait

Chimneys serve two purposes. One purpose is to create the required draft for proper combustion of fuel; the other purpose is to provide a means of discharging the gases carried by the chimney at a sufficient height above the ground that they may not be harmful to people living in the vicinity of the chimney.

Very high chimneys are more expensive than lower chimneys producing the same draft. Chimneys, therefore, over 150 ft. in height, need only be used at smelters, chemical works, and other industrial plants where noxious gases are produced.

290. Shape of Chimneys.—Chimneys of any magnitude are built circular. A round chimney is better even for an ordinary house than a square or rectangular one. For the sake of economy in construction, however, flues and chimneys of small dimensions are usually built square. Large chimneys are usually built with a slight taper. The taper does not add materially to the chimney cost while it improves its appearance vastly. A taper which is quite generally used in concrete chimneys is 1 in 72.

291. Small Chimney Construction.—The Chicago Building Code requires that small chimneys or flues be constructed as follows:

Flues having area less than 144 sq. in.	8 in. brick, or 4 in. brick with flue liner.
Flues having area between 144 and 300 sq. in.	13 in. brick, or 9 in. brick with flue liner.
Flues having area between 300 and 600 sq. in.	17 in. brick, or 13 in. brick with flue liner.

A much better chimney is obtained by using a brick wall surrounding a flue liner than can be obtained with a brick wall alone.

292. Linings for Large Chimneys.—Large chimneys must always be built with an interior wall of firebrick or other material which will withstand high temperatures. This lining must be free to expand independently from the outer shell or main chimney structure. It must be carried to such a height that the heat of the gases where the lining ends will not be great enough to damage the chimney. In concrete chimneys the lining is usually carried to a point one-third of the chimney height above the breech opening. The Chicago Code requires that the lining in a concrete chimney be carried to height equal to ten times the inside diameter of the chimney above the breech opening. Where high temperature gases occur, it may be necessary to continue the lining to the top. A firebrick lining is usually made 8 in. in thickness for the first 50 ft. of its height and 4 in. for the next 50 ft. An insulating cavity of at least 3 in. in width should be provided between the fire brick lining and the outer shell.

Designers must keep in mind that the lining will expand vertically to a considerably greater extent than the chimney proper. In addition all chimneys sway to some extent in the wind. The construction at the top of the lining must consequently be such that the lining may be free to move vertically relative to the outer shell. The lining must be corbelled out at the top of the insulating cavity closing off the cavity from the flue opening.

293. Temperature Reinforcement in Reinforced Concrete Chimneys.—In reinforced concrete chimneys, special additional temperature reinforcement should be provided at any region where a decided change in section occurs. It is also necessary to introduce extra heavy temperature steel in the top of the stack and at the top of the lining.

294. Size of Breech Opening.—The mechanical engineer will usually give the chimney designer the dimension of the stack and the size and locations of the breech opening and clean out door. The breech opening is usually made 20% greater in area than the minimum internal cross section of the chimney. For structural reasons the width of the breech opening should be held down to as small as dimension as possible. A width equal to two-thirds of the width of the chimney at the top is the maximum whih the structural engineer should endeavor to have used. This will give a flue whose height is 2⅛ times its width.

295. Size and Height of Chimneys.—Assuming an average consumption of 5 lb. of coal per horsepower per hour and taking the effective diameter of the chimney as 4 in. less than its internal diameter, we have the following formulas for the size and height of a chimney:

$$E = \frac{0.3H}{\sqrt{h}}$$

$$D = 13.54\sqrt{E} + 4$$

where E is the effective chimney area; H is the horsepower to be provided for; h is the height of the chimney in feet; and D is the internal diameter of the chimney in inches.

For steam heating plants in small buildings the following sizes of chimney flues should be used:

Direct radiation in square feet	Size of flue
200 to 400	8 × 8
450 to 900	8 × 12
1000 to 1600	12 × 12
1600 to 3000	16 × 16

If indirect radiation is used, 50% should be added to the amount of radiation to be installed in choosing the flue size from the above table. For a kitchen range an 8 × 8 flue is satisfactory.

296. Design of Chimneys.—Large chimneys are of three main types: (1) Reinforced concrete, (2) steel, and (3) brick. The chimney shaft is so porportioned and designed that the stresses developed in the material used, when the chimney is subjected to a horizontal wind pressure, are within the unit stresses recognized in engineering practice. In reinforced concrete and steel chimneys the design may be such as to produce tension in the cross section. In brick chimneys, on the other hand, no tension must occur under the combined bending due to wind pressure and the direct load of the chimney. Since practically all chimneys of these types are circular, analyses will be worked out only for this form.

In the case of a circular stack the kern or circle outside which the center of pressure may not fall, if there is to be no tension on the section, has a radius

$$r = \tfrac{1}{4}r_1[1 + (r_2/r_1)^2]$$

where r_1 is the outside and r_2 the inside diameter of the chimney.

Steel or concrete stacks may be designed by applying the formula combining direct load and bending to sections about 25 ft. apart down the shaft. Thus

$$f(\text{max.}) = \frac{W}{A} + \frac{M}{S}$$

$$f(\text{min.}) = \frac{W}{A} - \frac{M}{S}$$

where W = weight of chimney above the section considered, A = area of section, M = moment of the wind pressure above the section, and S = section modulus. Since the wind pressure may cause either tension or compression at any point around the steel or concrete stack, designers must use values of f_s such that the sum of the tensile and compressive stresses does not exceed the unit stress allowed.

The wind pressure on flat surfaces is generally specified in American building codes at 30 lb. per sq. ft. From the experiments carried out by the National Physical Laboratory of England, 32 lb. per sq. ft. is the pressure produced by a gale of 100 miles per hour velocity. In the design of circular chimneys it is customary to take a pressure intensity on the projected surface of $^2/_3$ that applying on flat surfaces. The city of Chicago requires a wind pressure of 22 lb. per sq. ft. to be used in the design of circular chinneys. Some designers use a unit pressure equal to one-half that applying on a flat surface and there are many authorities who endorse this. Designers would do well to carefully consider the wind conditions of the locality where the chimney is to be erected before deciding upon the wind pressure to be used. A circular chimney to be erected in a region subject to tornadoes should be designed for at least 25 lb. wind pressure, while a similar stack in a region where no high winds occur might be designed for a wind pressure of 15 lb. Both of the pressures refer to the projected area of the stack.

296a. Brick Stacks.—Brick stacks are usually built of specially molded hollow radial bricks. A firebrick independent lining is installed and the chimney is capped with a cast-iron ring fitting on top of the brickwork protecting the joints from the action of the weather. At the breech opening the wall must usually be buttressed. In brick stack design there must be no tension. Therefore

$$\frac{W}{A} = \frac{M}{S} \tag{1}$$

With a wind pressure of 20 lb. on the projected area and brickwork weighing 120 lb. per cu. ft., and assuming the bottom cross section of the stack to be 1.9 the mean cross section of the brickwork we have

$$(D_1^4 - D_2^4) = D_a \times H \times D_1 \times 1.60 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

where D_1 and D_2 are the exterior and interior diameters at the base, H is the height, and D_a is the average exterior diameter of the chimney. By trial, D_1 and D_2 may be found.

The chimney may be then approximately laid out, using a wall thickness at the top as follows:

8 in. for chimney up to 8 ft. inside diameter at top.

12 in. for chimney from 8 to 18 ft.

In equation (2) the weight of the stack is taken as

$$w = \frac{120 \times H \times 0.784(D_1^2 - D_2^2)}{1.9}$$

After laying out the stack, check the weight of same against the assumed weight and, if they do not agree, make adjustments. Then apply formula (1) at each point just above where the wall increases in thickness. At the base it is advisable to check the maximum unit compression.

In case the weight of the brickwork is not 120 lb. per cu. ft., adjust equation (2) by multiplying the right-hand side by 120 and dividing by the weight of the brickwork. Also, if another wind pressure than 20 lb. is to be used, multiply the right-hand side of equation (2) by the revised wind load and divide by 20. The foundation design will be similar to that given for the concrete stack.

Brickwork in hollow brick stacks weighs approximately 90 lb. per cu. ft., so equation (1) becomes

$$D_1^4 - D_2^4 = D_a \times H \times D_1 \times 2.15$$

296b. Example of Design of Concrete Stack.—Following are the computations for the design of a concrete chimney (see design on p. 696).

Height = 175 ft. 0 in. Inside diameter = 7 ft. 6 in.

f_s = 16,000. f_c = 400. n = 15. Wind pressure 20 lb. per sq. ft. on projected area.

Breech opening = 5 ft. 0 in. × 10 ft. 6 in. Top of opening = 25 ft. 0 in. above ground. Flue lining extends 75 ft. above the flue, *i.e.*, 100 ft. above the ground.

Inside diameter at top = 7 ft. 6 in. Thickness = 4 in.

Outside diameter at top = 8 ft. 2 in.

Inside diameter at top of lining = 7 ft. 6 in.
Thickness of lining (4 × 2). = 0 ft. 8 in.
Insulating cavity (3 × 2) = 0 ft. 6 in.
Assume thickness of outer shell (7 × 2) = 1 ft. 2 in.

Outside diameter 75 ft. from top = 9 ft. 10 in.

Taper on one side is 10 in. in 75 ft., or 1 in 90.

Outside diameter at base = 8 ft. 2 in. + 175/45 = 12 ft. 0½ in.

Assume an increase in the shell thickness of 1 in. in 25 ft. This gives a bottom thickness of 11 in.

It will not be necessary to analyse a section 25 ft. from the top. In this section we used only a minimum amount of vertical steel. Round bars, ½-in. diameter, spaced 18 in. apart, is a reasonable minimum. Use 17 – ½-in. round bars.

Section 50 ft. From Top:

$$M = H \times D_a \times P \times \frac{H}{2} = (50)(8.7)(20)(25)(12) = 2{,}610{,}000 \text{ in.-lb.}$$

$$W = H \times \frac{\pi}{4} \times (D_1^2 - D_2^2) \times 150 = (50)(0.785)[(8.67)^2 - (7.83)^2](150) = 82{,}500 \text{ lb.}$$

$$A = \frac{\pi}{4}(D_1^2 - D_2^2) = 14 \text{ sq. ft.}, = 2016 \text{ sq. in.}$$

$$S = 0.098\left(D_1^3 - \frac{D_2^4}{D_1}\right) = 21.4 \text{ ft.}^3$$

$$f_c(\text{max.}) = \frac{W}{A} + \frac{M}{S} = \frac{82{,}500}{2016} + \frac{2{,}610{,}000}{(21.4)(12)(12)(12)} = 41 + 70 = 111 \text{ lb. per sq. in. (compression).}$$

f_s (compression) = (15)(111) = 1500 lb., approximately. Allowable f_s (tension) = 16,000 – 1500 = 14,500 lb.

f_c (min.) = 41 – 70 = 29 lb. per sq. in. (tension).

$$A_s = \frac{(2016)(29)}{14{,}500} = 4.02 \text{ sq. in.} = 21 - \tfrac{1}{2}\text{-in. round bars.}$$

Detailed calculations will not be given for the sections 75 ft., 100 ft., and 125 ft. below the top. The [illegible] are as follows:

Section 75 ft.—compression max. = 156, tension max. = 34—steel = 29 – $\frac{1}{2}$-in. round bars.
Section 100 ft.—compression max. = 208, tension max. = 44—steel = 28 – $\frac{5}{8}$-in. round bars.
Section 125 ft.—compression max. = 281, tension max. = 77—steel = 42 – $\frac{3}{4}$-in. round bars.

Section at 150 *ft. From Top:*

$M = (150)(9.83)(20)(75)(12) = 26{,}500{,}000$ in.-lb.
$W = (150)(0.785)[(9.83)^2 - (8.67)^2](150) = 380{,}000$ lb.
$A = 21.5$ sq. ft. $= 3100$ sq. in.

$$S = \left(\overline{11.4}^3 - \frac{\overline{9.73}^4}{11.4}\right)(0.098) = 68 \text{ ft.}^3$$

f_c (max.) $= 123 + 225 = 348$ lb. (compression). f_c (min.) $= 102$ lb. (tension)
f_s (compression) $= 4500$ lb. (approx.). f_s (tension) $= 11{,}500$ lb.

$$A_s \text{ (tension)} = \frac{(102)(3100)}{11{,}500} = 27.5 \text{ sq. in.} = 46 - \tfrac{7}{8}\text{-in. round bars.}$$

The section 150 ft. from the top is at the upper side of the breech opening. We must consider a section at the lower side of this opening in order to provide the necessary strength at this opening.

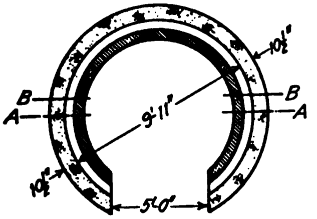


FIG. 484.

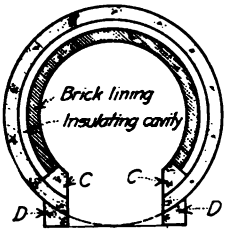


FIG. 485.

Section at 160 *ft. From Top:*

$M = (160)(9.9)(20)(80)(12) = 30{,}300{,}000$ in.-lb.

$W = (160)(0.785)[(9.9)^2 - (8.65)^2](150) = 433{,}000$ lb.

If no breech opening were cut, we would have

$A = 23$ sq. ft. $= 3310$ sq. in. $S = 75.$
f_c (max.) $= 366$ lb. (compression). f_c (min.) $= 104$ lb. (tension).
A_s (tension) $= 30$ sq. in.

For the sake of economy it is desirable to avoid introducing buttresses at the edge of the breech opening. We will, therefore, proceed to find the section modulus of the chimney section, Fig. 484.

I of complete section without breech about axis A–A $= 0.0491\,(d_1^4 - d_2^4) = 438$ ft.^{4}
I of portion removed for breeching about A–A $= (5)(0.9)(5.1)^2$ approx. $= 117$ ft.^{4}

Then

I of chimney section at breech opening about A–A $= 438 - 117 = 321$ ft.^{4}

Now find the center of gravity of the section by trial. It will be found to be about 1 ft. 0 in. from A–A. Then I of section about BB (axis through center of gravity) $= 321 + (18.5)(1.0)^2 = 339.5$ ft.^{4}

$$S = \frac{339.5}{6.7} = 50.7$$

$$f_c \text{ (max.)} = \frac{433{,}000}{(18.5)(144)} + \frac{30{,}300{,}000}{(50.7)(12)(144)} = 162 + 346 = 508 \text{ lb. (compression)} \quad f_c \text{ (min.)} = 184 \text{ lb. (tension)}$$

$$f_c + P(n - 1)f_c = f_c(\text{max.})$$
$$400 + (P14)(400) = 508$$
$$(P14)(400) = 508 - 400$$
$$P = \frac{108}{5600} = 0.0193 \text{ or } 1.93\%.$$

A_s (compression) $= (18.5)(144)(1.93) = 52$ sq. in. $= 66$—1-in. round bars.
f_s (compression) $= (400)(15) = 6000$ lb.
f_s (tension) $= 10{,}000$ lb.

$$A_s \text{ (tension)} = \frac{(184)(18.5)(144)}{10.000} = 49 \text{ sq. in.}$$

The amount of compression steel required, namely, 52 sq. in., is greater than the amount of tension steel, and we will therefore use 66–1-in. round bars. Had the width of the breech opening been a greater proportion of the width of the stack we might have found that the concrete stress developed was too high to permit of our introducing sufficient compression reinforcement to keep the actual concrete stress within the stress specified.

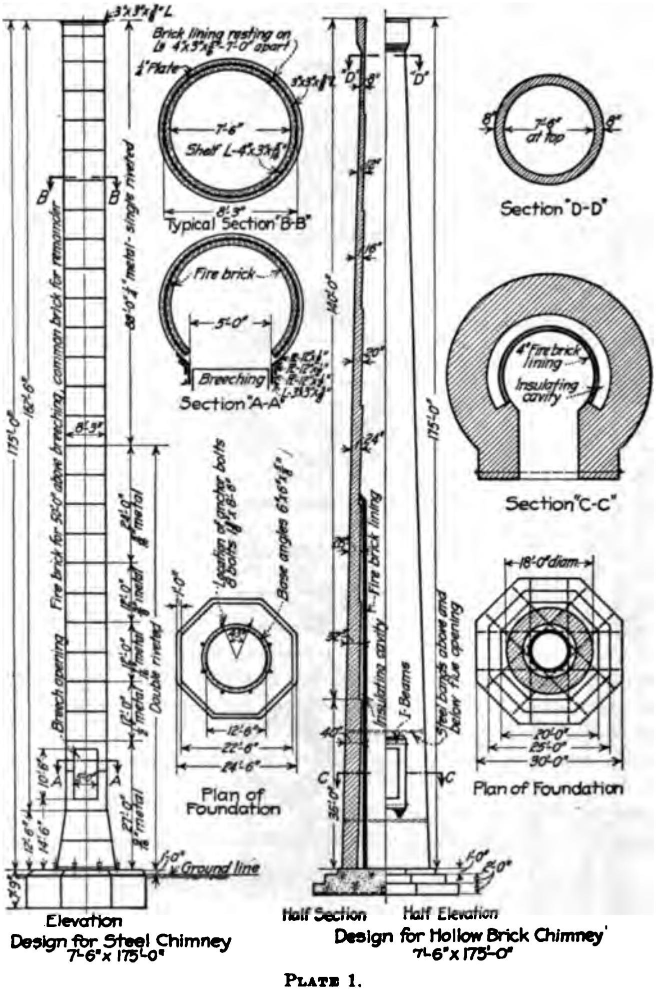
In Fig. 485 are illustrated methods of increasing the section modulus at the breech opening. The first thing to be done would be to increase the thickness of the outer shell by an amount of from 1 to 3 in. This thickening should be carried about 5 ft. above and below the breech opening. If increasing the outer shell thickness by a maximum of about 30% is not sufficient, the buttresses marked C should next be added and, in case even this is inadequate, the buttresses marked D should be added. Where buttresses are added, the designer should distribute the reinforcing steel throughout the section so that in each portion the same percentage of steel is used.

Section at 175 *ft. From Top:*

$M = 37{,}000{,}000$ in.-lb.

Compression max. = 402 lb., tension max. = 116 lb. Steel 49–1-in. round bars.

Temperature Reinforcement.—The design of the temperature reinforcement is at present left more or less to experience. The use of either rings of reinforcing bars or mesh is usual. In this design, and in fact for any ordinary concrete stack, a mesh weighing $\frac{7}{10}$ lb. per sq. ft. is satisfactory. In addition to this, $\frac{3}{8}$-in. bars, 4 or 5 in. on centers, should be used for a distance 5 ft. below the top, placed horizontally. We should also have some similar rods where there is any material change in the section. In this stack the taper is straight from top to bottom, but some are built cylindrical, with an offset. We should also introduce three extra horizontal bars of the same size as the vertical bars above and below the breech opening, and in addition ⅜-in. bars, 4 or 5 in. on centers, for a distance of 5 ft. above and below the opening. If these rings are made in two parts, the ends of the rods should be lapped a distance sufficient to develop their strength in tension. The laps should be staggered around the chimney.


Elevation
Design for Steel Chimney 7'-6" x 175'-0"

Half Section Half Elevation
Design for Hollow Brick Chimney 7'-6" x 175'-0"

Plate 1.

Shear.—Shear will seldom effect the design of a stack. It is well to investigate a section at the bottom and one through the breech opening. Taking a section 160 ft. from the top, we have

$$\text{Total wind load} = (160)(9.9)(20) = 31{,}700 \text{ lb.}$$

$$\text{Shear} = \frac{31{,}700}{(18.5)(144)} = 12 \text{ lb. per sq. in.}$$

And at the base

$$\text{Shear} = \frac{(175)(10.1)(20)}{3460} = 10 \text{ lb.}$$

Design of Foundation.—A chimney foundation should be built octagonal or circular in plan. A square footing produces such a high toe pressure at the corners when the wind is blowing on the diagonal of the footing that this shape is undesirable. The bearing pressure on the soil should be lower than one would use on the same soil for a stationary load of practically constant amount. In this case we will use a maximum pressure of 4000 lb. per sq. ft. The footing design for all chimneys is practically the same. In the case of the steel stack the weight of the

footing must be greater and in the case of the brick stack the footing may be lighter, than the footing for the crete stack. The weight of the earth fill and any other loads coming on the foundation should be included. Th bottom of the foundation should be well below frost line.

Weight of concrete shell = 495,00
Weight of brick lining = (120)(100)(0.785)[(8.17)² − (7.5)²] + ½[(8.83)² − (8.17)²] = 152,00

Total weight at top of footing = 648,00

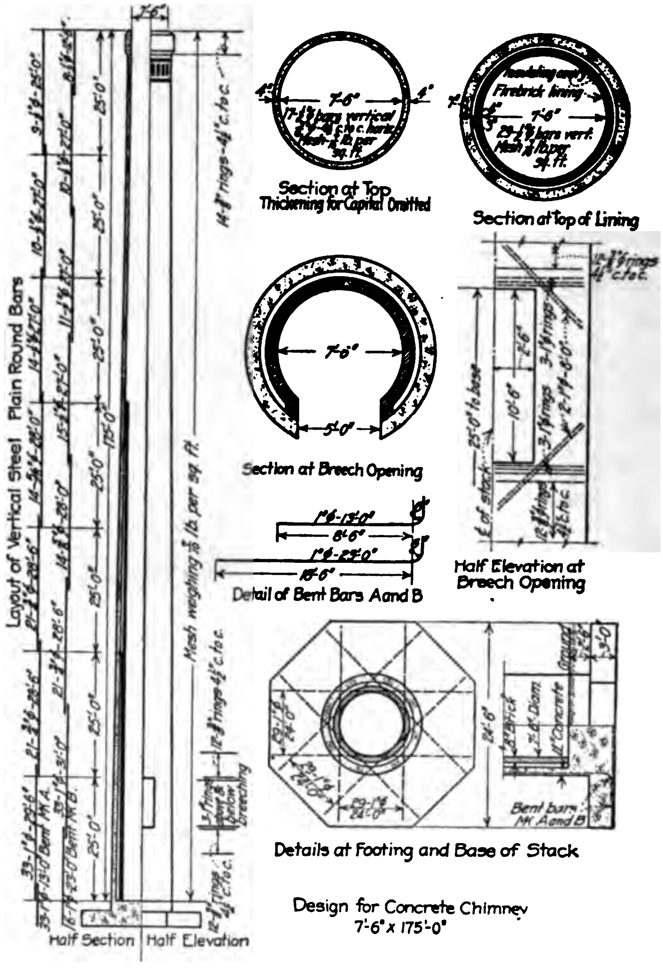

Layout of Vertical Steel Plain Round Bars
Half Section
Half Elevation
Mesh weighing ⅜ lb. per sq. ft.
Section at Top Thickening for Capital Omitted
Section at Top of Lining
Firebrick lining
Section at Breech Opening
Detail of Bent Bars A and B
Half Elevation at Breech Opening
Details at Footing and Base of Stack
Design for Concrete Chimney
7'-6" x 175'-0"

PLATE 2.

The kern of a circular footing has a radius equal to one-fourth of the radius of the footing. Also, the toe pressure is approximately twice the average pressure. Now, we can approximate the weight of the earth filling and footing. Assume 600 lb. per sq. ft. Then the area of the footing will be approximately $\frac{648{,}000}{4000 - 600} \times 2 = 381$ sq. ft. = on octagon having a width of 21 ft. 6 in., or a circle having a diameter of 22 ft. 0 in. We may take the radius of the kern, then, as 2 ft. 9 in. To avoid the negative pressure at any point in the base, the eccentricity must not exceed 2 ft. 9 in. Taking our assumed footing and cover on same at 600 lb. per sq. ft., we have a total load = 228,000 lb. So the total load at the bottom of footing = 876,000 lb. Now we found that M due to wind = 37,000,000 in.-lb. so the eccentricity $e = \frac{37{,}000{,}000}{876{,}000} = 42$ in., which is greater than the maximum eccentricity found permissible. We must, therefore, increase the area of the footing, or increase its weight, or both. Assume 700 lb. per sq. ft. for footing and cover, and take area = 500 sq. ft. Then weight of footing = 350,000, e = 37 in., and 500 sq. ft.

gives an octagonal footing of 24 ft. 6 in. × 24 ft. 6 in. With this size no negative pressure occurs. If the bottom course is made 3 ft. thick, we have a weight of 450 lb. per sq. ft. so we must have 2½ ft. of earth above the bottom course to obtain a total of 700 lb. as assumed.

Depth for punching shear at 120 lb. per sq. in. at edge of shaft =

$$\frac{495{,}000 \times 2}{(\pi)(12)(12)(120)} = 18.3 \text{ in.}$$

The depth assumed gives a maximum of 60 lb. per sq. in.

The footing will be reinforced with 4 bands of steel similar to the one indicated. The moment at the center of the section of stack wall bounded by *abc* is the moment of the soil pressure due to stack load on the figure *abde* about the line *de*. Now at *ab* we have a maximum pressure of $2 \times \frac{(648{,}000 + 350{,}000)}{500} = 4000$ lb. (approx.) and since the weight of the footing and fill amount to 700 lb., the unbalanced upward pressure is 3300 lb. per sq. ft. at line *ab*. We find by proportions that the upward pressure at *de* is 2350 lb. and the length of *de* is 4.35 ft. Also *cf* = 5.25 ft. and *cg* = 12.25 ft. and *ab* = 10.2 ft.

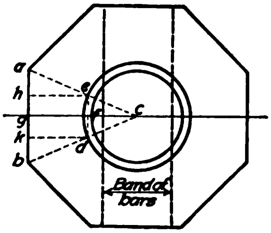
$$\begin{aligned} M \text{ at } ed &= (2350)(4.35)(7.0)(42) = 3{,}000{,}000 \text{ in.-lb.} \quad (M \text{ of area } edhk \text{ at } 2350 \text{ lb.}) \\ &+ (\tfrac{1}{2})(950)(4.35)(7.0)(56) = 810{,}000 \quad (M \text{ of area } edhk \text{ at } 950 \text{ lb. at } ab). \\ &+ (2)(2350)(\tfrac{1}{2})(7.0)(2.92)(56) = 1{,}750{,}000 \quad (M \text{ of area } aeh \text{ and } dbk \text{ at } 2350 \text{ lb.}) \\ &+ (2)(950)(\tfrac{1}{3})(2.92)(7.0)(63) = \frac{820{,}000}{6{,}380{,}000} \quad (M \text{ of area } aeh \text{ and } dbk \text{ at } 950 \text{ lb. at } ab). \end{aligned}$$

For $f_s = 16{,}000$; $f_c = 650$; and $n = 15$

$$d = 31 \text{ in.} \quad b \text{ required} = 62 \text{ in.}$$

The depth is satisfactory, $A_s = 14.7$ sq. in. Use 19 — 1-in. round bars in each band.

The stack is not large enough to cause any upward bending at *C* (Fig. 486) and so we will have no reinforcing in the top of the slab. We previously found that we required 49 — 1-in. round bars at the base of the stack. These must be carried a sufficient distance into the foundation to develop their strength. Since we have a depth of footing of only 3 ft., we must hook these bars as indicated. Total upward pressure on line *ed* = 130,000 lb. For 40 lb. shear, width required $= \frac{130{,}000}{(31)(40)} = 105$ in. This is much less than the stack diameter so we need not further provide for diagonal tension. All vertical steel in the stack should be lapped a sufficient distance to develop its strength in bond. At a bond stress of 80 lb. per sq. in., a lap or imbedment of 50 diameters is required. The lap in the bars must not all be made at any one section in the stack. Good practice is to lap half of the bars at any section as indicated. Some steel should be placed diagonally across the corners of the breech opening as illustrated. Two rods of the same size as the vertical steel is sufficient.



·Fig. 486.

296c. Steel Stacks.—It was pointed out previously that the sum of the compressive and tensile stresses in the steel must not exceed the allowable stress of 16,000 lb. per sq. in. In stack design it will be found satisfactory to use a stress of 10,000 lb. per sq. in. on the net section (rivet holes deducted) as this will result in a compression of only about 6000 lb. on the gross section.

Assuming a joint efficiency of 60% the design would resolve itself into designing the stack with 100% efficiency in the joints and using $f_s = 6000$ lb. on both the tension and compression sides. Similarly with an efficiency of 80% f_s becomes 7100 lb.

The design for the stack must be such that it will maintain its form against the tendency of the wind to flatten it. It must also be prepared so that the stresses resulting from combined bending and direct load are within the above limits.

Unless the stack is lined to the top and the lining carried on shelf angles, the dead weight of the stack itself may be omitted from the strength calculations.

Steel stacks are built cylindrical except for a section at the base which is made conical. It is desirable for the sake of economy to keep the breech opening above the conical portion. The sides of the breech opening must be reinforced with plates and angles to make up for the portion cut away, just as in the case of the concrete stack. The stack is set upon a cast-iron base in most cases and rigidly bolted down to the foundation by means of a series of bolts. A stress of 12,000 lb. per sq. in. may be used on the net section of these bolts. It is good practice to add from ⅛ to ¼ in. to the theoretical diameter to allow for corrosion. A large cast-iron washer is embedded in the foundation at the end of each bolt. The washer or bearing plate should be of such size that its area in contact with the concrete does not produce a bearing stress in excess of 400 lb. per sq. in. To prevent leakage through the stack joints, the rivet spacing should not exceed ten times the plate thickness. With this spacing and well driven rivets, it is not usually necessary to calk the joints. Plating less than ¼ in. in thickness should not be used. In fact, it is poor economy to use plate as thin as that on account of deterioration due to rust.

Design Formulas.—The section modulus $S = 0.098(D_1^3 - \frac{D_2^4}{D_1})$. Now $D_2 = D_1 - 2t$, where t = thickness of metal. Consequently

$$S = 0.098\ D_1^3 - \frac{(D_1^4 - 8D_1^3t + 24D_1^2t^2 - 32D_1t^3 + 16t^4)}{D_1}$$

The values of t^2, t^3, and t^4 are so small that we may write this equation

$$S = 0.098\ (D_1^3 - D_1^3 + 8D_1^2t) = 0.784D_1^2t$$

Omitting the dead load

$$M = fS = (6000)\ (0.784)\ (D_1)^2\ (t)$$

$$t = \frac{M}{(4704)(D_1)^2(12)^2}$$

and using a 20-lb. wind pressure

$$t = \frac{H^2}{5645\ D_1}$$

where D_1 is the diameter in feet, H the height in feet, and t the thickness in inches.

Table of Plate Thickness for Chimneys Based on 20-Pound Wind Pressure on Projected Area—Joint Efficiency 60%

Height	Diameter										
	5′ 0″	6′ 0″	7′ 0″	8′ 0″	9′ 0″	10′ 0″	11′ 0″	12′ 0″	13′ 0″	14′ 0″	15′ 0″
50	0.09	0.074									
60	0.127	0.106	0.09								
70	0.174	0.144	0.124	0.108							
80	0.227	0.19	0.162	0.142	0.126						
90	0.287	0.239	0.205	0.179	0.160	0.144					
100	0.355	0.295	0.252	0.220	0.196	0.176	0.16				
110	0.43	0.357	0.306	0.258	0.238	0.214	0.195				
120	0.508	0.424	0.363	0.318	0.283	0.255	0.232				
130	0.60	0.498	0.427	0.375	0.332	0.299	0.272	0.25			
140	0.693	0.58	0.496	0.433	0.386	0.347	0.315	0.29	0.267		
150	0.795	0.662	0.568	0.498	0.443	0.398	0.362	0.332	0.307	0.284	
160	0.905	0.752	0.646	0.565	0.503	0.453	0.412	0.378	0.348	0.324	0.302
170	.....	0.85	0.73	0.638	0.566	0.51	0.463	0.425	0.393	0.364	0.34
180	.....	.....	0.82	0.716	0.635	0.572	0.52	0.476	0.44	0.409	0.382
190	.....	.....	.....	0.796	0.706	0.638	0.58	0.53	0.49	0.455	0.425
200	.....	.....	.....	.....	0.788	0.71	0.645	0.592	0.545	0.506	0.472

For a joint efficiency of 80% use ¾ of values given above.

Bearing on shop driven rivets may be taken as 25,000 lb. per sq. in.—field, 20,000 lb. per sq. in. Shear on shop driven rivets may be taken as 12,000 lb. per sq. in.—field, 10,000 lb. per sq. in.

The total tension or compression per linear foot in the stack = (6000) (t) (12). From this we can determine the spacing and size of rivets necessary.

Let D_b denote the diameter of the center line of the holding down bolts, n the number of bolts, and d their diameter; then the size and number of bolts may be determined from the formula:

$$nd^2 = \frac{1}{0.615f_s}\left(W - \frac{M}{D_b}\right)$$

Assuming a convenient number of bolts n, then d, the bolt diameter, can be found. The designer must then add for the depth of thread and also add ⅛ in. to allow for corrosion.

Where f_s = 12,000 lb., the above formula becomes

$$nd^2 = \frac{1}{7380}\left(W - \frac{M}{D_b}\right)$$

296d. Guyed Steel Stacks.—Guyed steel stacks are designed to act as beams spanning between the base and the collar to which the guy wires are attached. The moment due to the cantilever action of the stack above the collar should be taken into account. Having found the maximum bending moments, apply the formula for the thickness of the plates

$$t = \frac{M}{(4704)(D_1)^2(144)}$$

The guys are usually attached at one-third of the height from the top. The collar to which the guys are attached should be stiff enough to withstand the tendency to buckle.

The guy wires will be designed to take the entire wind reaction at the collar. The maximum pull on a guy will occur when the wind blows directly along it. With the guys attached one-third H from the top, the reaction at the collar becomes

$$0.75DPH$$

So the pull on any guy wire becomes

$$0.75DPH \sec \alpha$$

where α is the angle the guy makes with the horizontal.

The foundation must be made large enough to take the vertical component of the tension on a guy in addition to the chimney weight.

296e. Ladders.—Permanent ladders must be built into all large chimneys. They are placed on the outside. In the case of some guyed steel stacks the ladder is omitted but a pulley is attached to the top and a steel cable left in place so that a painter can pull himself up.

296f. Lightning Conductors.—All self-supporting stacks should have a first class lightning conductor installed upon them.

DOMES

By Richard G. Doerfling

297. Definitions.—In a statical sense, and in contradistinction to plane structures like girders, trusses, and arches, where all external and internal forces are assumed to act within a plane, domes may be defined as space structures. Similar to plane structures, such structures may be divided into solid and framed domes.

Solid domes are curved shells of stone masonry, plain concrete, reinforced concrete, or riveted steel plate, while framed domes consist of compression and tension members either curved to the form of a shell and supporting a roof cover, or straight between panel points, but all panel points upon a curved shell surface. Framed domes may be built of timber, steel, or reinforced concrete.

Generally a surface of revolution is chosen as the dome surface, generated by a straight line or a curve revolving about a vertical axis. A straight line will thus generate a conical surface, an arc of a circle a spherical surface, and a quadrant of an ellipse a spheroidal surface. Other generating curves employed are the cubical parabola for economy in design and reversed curves, like the ogee and similar ones, for architectural reasons. All horizontal sections of domes of revolution are either circles or regular polygons; but domes have been built sometimes elliptical in plan and may indeed be built irregular in shape and simply defined as solid shells and framed polyhedrons.

298. Loads.

298a. Wind Pressure.—The wind pressure p upon a plane surface, at right angles to the direction of the wind, is taken generally as from 20 to 30 lb. per sq. ft. For any inclination between the surface and direction of the wind, p may be dissolved into two comp nents, normal and parallel to the surface and, with friction between surface and wind equa zero, it is only the normal component which acts as a load upon the surface. The rela between p, its normal component n, and the angle of inclination i, has been given differe by different experimenters, the simplest one, apparently the most rational, and the one mos employed is that by F. v. Loessl, namely:

$$n = p \sin i$$

It has also been observed and well established that the direction of wind may vary fro horizontal as much as 10 deg., and while such increase in i would affect the pressure upon verti cal and steeply inclined surfaces but slightly, it will gain in importance as the inclined surfa approaches the horizontal. Fig. 487 gives the normal components of $p = 20$ lb. for 9 divisio of a quadrant, and the following tabulation gives these values of n for surfaces of different slop:

For a slope =	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10
n =	16.4	13.8	11.9	10.5	9.5	8.8	8.1	7.6	7.3

298b. Snow Load.—If s is the snow load in pounds per square foot upon horizontal surface, then the snow load per square foot upon a surface inclined at an angle the horizontal is:

$$s' = s \cos v$$

For s = 20 lb. and v =	40 deg.	30 deg.	20 deg.	10 deg.	0 deg.
s' =	15.3	17.3	18.8	19.7	20.0

For v greater than 40 deg., snow will surely slide off and need not be considered.

298c. Wind and Snow Loads.—If separate calculations for wind and snow are not made, it is customary instead to consider a vertical live load of from 20 to 30 lb. per sq. ft. of roof surface.

298d. Dead Loads.—Framed domes of timber or steel with tar and gravel roofing will weigh from 10 to 15 lb. per sq. ft.; framed domes of steel or reinforced concrete, with 2½ in. concrete cover, from 40 to 50 lb. per sq. ft. A plastered ceiling will add about 10 lb. to these loads. After a preliminary design the actual dead load may be very closely determined and the size of all members corrected if necessary.

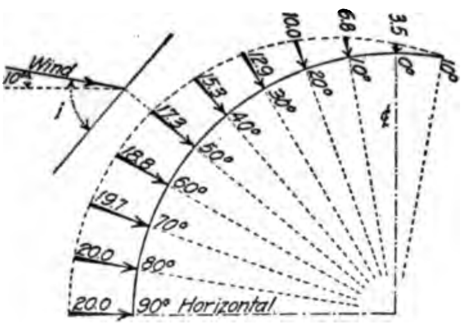


Fig. 487.—Distribution of wind pressure.

Solid domes of reinforced concrete have been built with a thickness of shell from 1/150 to 1/200 of the span, with a minimum thickness of 2½ in. The thickness is generally made uniform throughout, though the stresses call for a uniform increase in thickness from the crown towards the base.

299. Framed Domes.—Though admirable domes of masonry have been built in ancient times, the framed dome, with all its structural members upon a mantle surface, is an invention of modern times. The crude practice of constructing a dome of a number of radially placed trusses has not entirely vanished, neither the mistaken idea of designing dome ribs like arches. The forces acting upon a dome rib are non-coplanar, though for the sake of a simple analysis it is most convenient to proceed in steps from a coplanar system of forces to the forces outside the plane.

The structural members of a modern dome frame are the meridian ribs, the horizontal rings or belts, and the diagonal ties. Their typical arrangement is shown in Fig. 488. In order to avoid ambiguity of stress the ribs are not brought together at the crown but abut against a horizontal ring, termed the latern ring, though it need not necessarily carry a lantern as indicated in the figure. The lowest ring is termed the wall ring. It is not really a necessary member of the dome frame but introduced to counteract the horizontal components of the rib stresses, leaving all wall reactions vertical, each equal to the total load upon the rib above it.

The ribs and the lantern ring are under maximum compression, and the wall ring under maximum tension, when the dome carries its maximum loads. Any intermediate ring is under maximum tension (or minimum compression) when the part of the dome inside the ring carries its maximum load while the ring itself carries its minimum load. It is under maximum compression (or minimum tension) when this condition of loading is reversed. This is readily understood when considering that in the former case the ring receives its maximum outward push, increasing tension or reducing compression, while in the latter case it receives its maximum inward push, increasing compression or reducing tension (see Figs. 492 and 493).

Any diagonal cross tie finally must carry the diagonal component of the difference between the stresses of the ribs to the right and left of it Hence, the possible maximum difference between two adjacent rib stresses determines the maximum tension of the compensating diagonal. This maximum difference in rib stresses is found generally in the dome panels parallel to the direction of the wind, assuredly so under the somewhat severe assumption that the windward rib carries snow and wind while the leeward rib carries neither.

All loads are assumed to be concentrated at the panel points and the contributary load area for any panel point is determined by the dimensions to midway between adjacent panel points, as indicated by the hatched areas in Fig. 488. The weight of a lantern is carried by the panel points of the lantern ring while the loads upon the lower halves of the lowest ribs are carried directly to the points of support.

The stresses determined by the following methods are compressive and tensile stresses for members straight between panel points. For curved members a bending moment equal to the axial stress F times the rise of curve must be considered, and if the members act also as supporting beams for purlins or rafters, as they mostly do, the bending moment due to such beam action furnishes another component of stress. For rings in tension the sum of these two bending moments make up the resulting moment M, for rings in compression their difference, giving for the final design of a curved member a unit fiber stress of

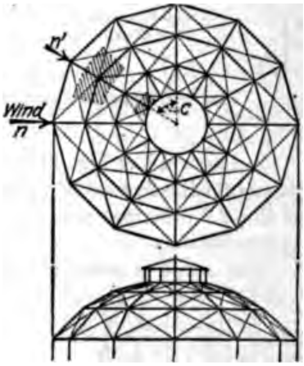


FIG. 488.—Plan and elevation of typical dome frame.

$$f = \frac{F}{A} \pm \frac{Mc}{I} \qquad (2)$$

This formula applies also to straight members with M due to beam action only. For a relatively long member, the bending moment due to its own weight may be important enough for consideration.

Though stress theory is based on freely turning joints, it is well to aim at rigidity of joints and provide a liberal amount of continuity across the panel points in both directions. Such departure from theoretic assumption is, in this case, on the side of safety.

299a. Stress Diagrams.—Let Fig. 489 represent a dome rib with panel loads P_1, P_2, P_3, P_4, and wall reaction P_{1-4}. Assume auxiliary horizontal forces H_1 to H_5 acting at the panel points 1 to 5 in the meridian plane of the dome rib, so that all forces immediately considered are coplanar. The lower part of the stress diagram can now be drawn in the usual way. Beginning with the 3 forces at panel point 1, draw the force triangle $P_1R_1H_1$. Proceeding to panel point 2, draw R_2H_2 and so on, until the rib stresses R_1 to R_4 and the auxiliary forces H_1 to H_5 have been determined, H_5 being the sum of H_1 to H_4, or the closing line of the force triangle for panel point 5. All that remains now is to resolve each one of the auxiliary forces H into its two component rings or belt stresses B which is done in the upper part of the diagram, the plan of the dome furnishing the direction of the B-lines.

Since the angle between the B-lines is often quite acute, the B-stresses may as well be de-

termined by simple computation. Thus let *b* be the length of any *B*-member and *h* its hori
distance from the center of the dome, then, by similar triangles,

$$\left.\begin{array}{r} \frac{B}{H} = \frac{h}{b} \text{ or } B_1 = H_1\frac{h}{b} \\ B_2 = H_2\frac{h}{b} \\ \text{etc.} \end{array}\right\}$$

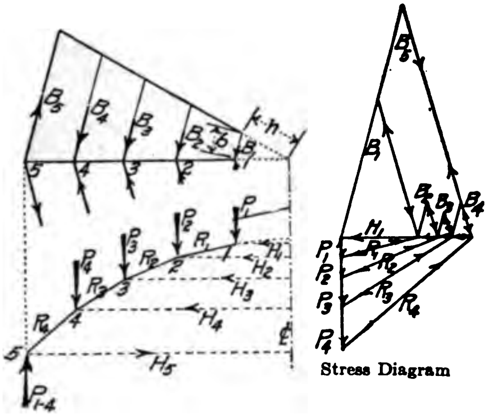


Fig. 489.—Plan and elevation of dome rib.

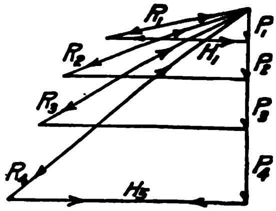
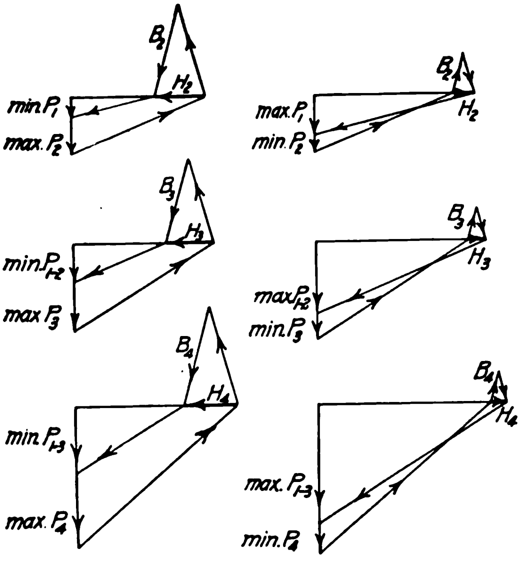


Fig. 490.—Maximum rib stresses and maxim for lantern and wall ring.

A diagram like Fig. 489 drawn for maximum dead and live loads will furnish the maxi stresses for the dome ribs, the lantern ring, and the wall ring. Fig. 490 is another stress dia for these 3 principal stresses, and though different in form from Fig. 489, it needs no fu explanation.

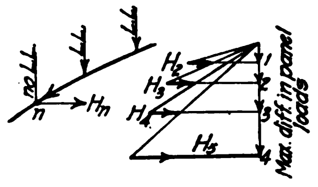
The sense or stress in dome ribs and lantern ring is always compressive, that of the ring is always tensile. The stress the intermediate rings, however, m either compression or tension acco to the distribution of load, shap dome, or position of ring. Fig. shows diagrams for determining i imum compression and maximum sion in these rings and are self-expl tory. A maximum difference bet any belt load and the loads inside belt is sometimes caused by snow, b is well to consider that during const tion, a roof covering (slate, for insta may be put on either from wall rin towards the crown or inversely, an the same way the mode of construc of a plastered ceiling may furnish critical case for maximum stresses intermediate rings. Fig. 491 might combined into one diagram, but multiplicity of lines would be somew confusing.



Max. compression in intermediate rings. Max. tension in intermediate rings.

Fig. 491.

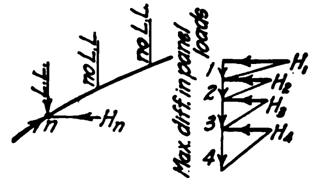
The maximum difference in pa loads between adjacent panel points is readily seen, will be given by a loading reaching to midway between such points. One pa point is carrying then $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of a full panel load and the other $\frac{1}{2} + \frac{1}{2} \times \frac{3}{4} =$ giving a difference of $\frac{3}{4}$ panel load. It is generally assumed, however, that one panel po may carry a full live load while the adjacent one carries none. On this assumption, st diagrams like Figs. 492 and 493 may be drawn giving maximum H for live load ring tension and ring compression. These must be combined with H for total loads, Fig. 489, in order to obtain the total maximum which was obtained directly by Fig. 491. The stress T in a diagonal tie is a maximum where the difference between two adjacent rib stresses is a maximum. This critical case of maximum difference may occur during construction while a roof cover or plastered ceiling is carried gradually around the frame; it may be furnished by a one-sided snow load, by wind, or by snow and wind. The maximum load difference for two adjacent ribs due to one-sided roof cover, plastered ceiling, or snow, is a ¾ panel load as before.



Max. outward push on n.

Max. H for live load ring tension.

FIG. 492.



Max. inward push on n.

Max. H for live-load ring compression

FIG. 493.

The maximum wind pressures (as given in Fig. 487) decrease horizontally around the dome to zero where the panels are parallel to the direction of the wind. Referring to Fig. 488

$$n' = n \sin c$$

or referring to Formula (1), the normal wind pressure for any point of a spherical surface is

$$n' = p \sin i \sin c \tag{4}$$

Designating a full panel wind load by nA, the maximum wind load difference between two adjacent panel points is

for regular polygons of	8	16	24	32 sides
nearly	½	⅓	¼	⅙ nA

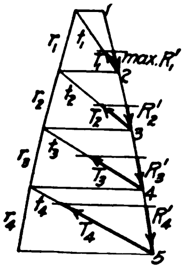


FIG. 494.—Max. tie stress construction upon dome panels developed.

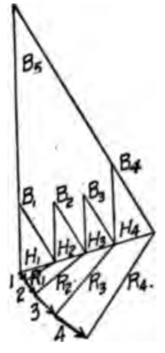


FIG. 495.—Wind stress diagram.

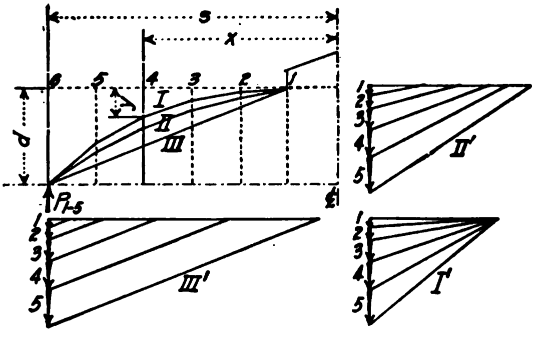


FIG. 496.—Relative stress economy due to difference in form only.

Considering that wind and snow will hardly be a maximum, at the same time, it seems reasonable to assume the maximum difference between adjacent rib stresses to be due to ½ live load, or ½ wind and snow load combined, and determine the maximum tie stresses accordingly. This may be done by projection upon the dome panels developed, as shown in Fig. 494, or by simple computation thus: If t be the length of a diagonal tie T, r the length of the adjacent ribs R, and R' the stress difference between them, then by similar triangles

$$\left.\begin{aligned} \frac{\text{max. } T}{\text{max. } R'} = \frac{t}{r} \text{ or max. } T_1 &= \text{max. } R_1' \frac{t_1}{r_1} \\ \text{max. } T_2 &= \text{max. } R_2' \frac{t_2}{r_2} \\ &\text{etc.} \end{aligned}\right\} \tag{5}$$

Fig. 495 gives a stress diagram for wind loads normal to the dome surface, while Fig. 496 may illustrate possible economy in design due to form only, span and rise being the same for the three dome sections shown. The panel points of I are upon a cubical parabola $\left(\frac{y}{d} = \frac{x^3}{s^3}\right)$, the panel points of II upon a circle, and those of III upon a straight line. The three stress diagrams, I′, II′, and III′, are drawn to the same scale and for the same dead loads. Comparing stress diagram I′ with II′, shows larger stresses for lantern ring and upper rib members, smaller stresses for lower rib members and wall ring, and zero stress for intermediate rings. The intermediate rings will be stressed, however, by variable loads, and the economical advantage of I over II is more theoretical than real. The lack in economy of III becomes evident by comparison with I or II. For a practical example, the location of the panel points for I, Fig. 496, may be computed as follows:

Let $s = 90$ ft. and $d = 30$ ft., then $y = \frac{d}{s^3}x^3 = \frac{30}{729{,}000}x^3 = 0.0000412x^3$, hence, with panel points 15 ft. apart horizontally

$$
\begin{aligned}
y_1 &= 0.0000412 \times 3{,}375 = 0.14 \text{ ft.}\\
y_2 &= 0.0000412 \times 27{,}000 = 1.11 \text{ ft.}\\
y_3 &= 0.0000412 \times 91{,}125 = 3.75 \text{ ft.}\\
y_4 &= 0.0000412 \times 216{,}000 = 8.90 \text{ ft.}\\
y_5 &= 0.0000412 \times 421{,}875 = 17.35 \text{ ft.}
\end{aligned}
$$

Figs. 489 to 496 will serve to show that graphical methods are quite general in application, giving quick results for any form of dome, convex or conical, bell shaped or onion shaped. By inverse operation, the shape of a dome may be altered to conform to a desired relation or result of stresses.

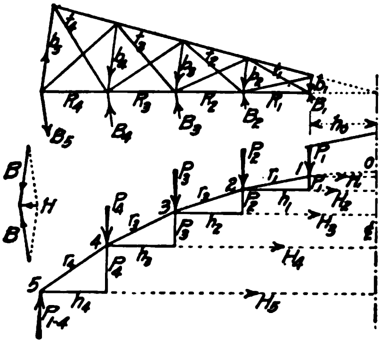


FIG. 497.—Plan and elevation of dome rib.

299b. Stress Formulas.—Stress formulas for domes are stated generally in terms of trigonometric functions, but since the slope angles, or their functions, must first be determined by operating with dimensions, or by scaling upon the dome drawing, it seems more direct and more convenient for the memory to give these formulas in terms of dimension or line ratios. Slope angle functions, however, may be readily substituted if desired.

Stress formulas for the intermediate rings will, for choice in application and a clearer comprehension, be given in two forms: (1) for direct maximum and minimum values, analogous to Fig. 491; and (2) for total loading and for maximum difference between adjacent panel loads, analogous to Figs. 489, 492, and 493.

Now let P = a maximum panel load.
D = a nominal dead load.
L = a nominal live load, such, that at any one time $P - D = L$ gives the maximum possible difference between adjacent panel points.

P_{1-2} be an abbreviation for $P_1 + P_2$.
P_{1-3} be an abbreviation for $P_1 + P_2 + P_3$, etc.
r_1 to r_4 be the length of rib members R_1 to R_4.
b_1 to b_4 be the length of belt members B_1 to B_4.
t_1 to t_4 be the length of tie members T_1 to T_4.
p_1 and h_1 be the vertical and horizontal projection of r_1, etc.
h_0 be the horizontal distance from center of dome to B_1.

Then by similar triangles throughout, and referring to Fig. 497, all formulas may be written as follows:

Rib Stresses:

$$\left.\begin{array}{ll} \dfrac{R_1}{P_1} = \dfrac{r_1}{p_1} \text{ or } R_1 = P_1\dfrac{r_1}{p_1} & R_2 = P_{1-2}\dfrac{r_2}{p_2} \\ R_3 = P_{1-3}\dfrac{r_3}{p_3} & R_4 = P_{1-4}\dfrac{r_4}{p_4} \end{array}\right\} \qquad (6)$$

Belt Stresses:

$$\frac{H_1}{P_1} = \frac{h_1}{p_1} \text{ or } H_1 = P_1\frac{h_1}{p_1}$$

$$\frac{B_1}{H_1} = \frac{h_0}{b_1} \text{ or } B_1 = H_1\frac{h_0}{b_1} = P_1\frac{h_1}{p_1}\cdot\frac{h_0}{b_1} = \text{compression in lantern ring} \qquad (7)$$

$$\left.\begin{array}{l} \text{max. } B_2 = \left[P_1\dfrac{h_1}{p_1} - (P_1 + D_2)\dfrac{h_2}{p_2}\right]\dfrac{h_0}{b_1} \\ \text{min. } B_2 = \left[D_1\dfrac{h_1}{p_1} - (D_1 + P_2)\dfrac{h_2}{p_2}\right]\dfrac{h_0}{b_1} \\ \text{max. } B_3 = \left[P_{1-2}\dfrac{h_2}{p_2} - (P_{1-2} + D_3)\dfrac{h_3}{p_3}\right]\dfrac{h_0}{b_1} \\ \text{min. } B_3 = \left[D_{1-2}\dfrac{h_2}{p_2} - (D_{1-2} + P_3)\dfrac{h_3}{p_3}\right]\dfrac{h_0}{b_1} \\ \text{max. } B_4 = \left[P_{1-3}\dfrac{h_4}{p_4} - (P_{1-3} + D_4)\dfrac{h_4}{p_4}\right]\dfrac{h_0}{b_1} \\ \text{min. } B_4 = \left[D_{1-3}\dfrac{h_4}{p_4} - (D_{1-3} + P_4)\dfrac{h_4}{p_4}\right]\dfrac{h_0}{b_1} \end{array}\right\} \qquad (8)$$

$$B_5 = P_{1-4}\frac{h_4}{p_4}\cdot\frac{h_0}{b_1} = \text{tension in wall ring} \qquad (9)$$

Tie Stresses:

$$\left.\begin{array}{ll} T_1 = L_1\dfrac{r_1}{p_1}\cdot\dfrac{t_1}{r_1} = L_1\dfrac{t_1}{p_1} & T_2 = L_{1-2}\dfrac{t_2}{p_2} \\ T_3 = L_{1-3}\dfrac{t_3}{p_3} & T_4 = L_{1-4}\dfrac{t_4}{p_4} \end{array}\right\} \qquad (10)$$

Positive values of Formulas (8) mean tension, while negative values mean compression, hence maximum and minimum applies in an algebraic sense. In other words: a maximum is either a high plus value (high tension) or a low minus value (low compression), while a minimum is either a high minus value (high compression) or a low plus value (low tension). Note that a load at any panel point does not influence the stress in any member above it, and that the formulas for maximum B are the same as for minimum B except that P and D have exchanged position. Compare this with Fig. 491, where maximum P and minimum P were used instead of P and D.

Note further that $L\dfrac{r}{p}$, Formula (10), means rib stress due to a nominal live load equal to the maximum possible difference between adjacent panel loads. Compare with Formula (5).

Formula (8) may be replaced by the following simpler forms:

$$\left.\begin{array}{l} B_2 = \left(P_1\dfrac{h_1}{p_1} - P_{1-2}\dfrac{h_2}{p_2}\right)\dfrac{h_0}{b_1} \\ B_3 = \left(P_{1-2}\dfrac{h_2}{p_2} - P_{1-3}\dfrac{h_3}{p_3}\right)\dfrac{h_0}{b_1} \\ B_4 = \left(P_{1-3}\dfrac{h_3}{p_3} - P_{1-4}\dfrac{h_4}{p_4}\right)\dfrac{h_0}{b_1} \end{array}\right\} \qquad (11a)$$

giving the stresses due to P. Plus values mean tension and minus values mean compression. These stresses must be combined with the stresses due to a maximum difference L between adjacent panel loads, namely, a tension for

$$B_2 = L_1\frac{h_2}{p_2}\cdot\frac{h_0}{b_1} \quad B_3 = L_{1-2}\frac{h_3}{p_3}\cdot\frac{h_0}{b_1} \quad B_4 = L_{1-3}\frac{h_4}{p_4}\cdot\frac{h_0}{b_1} \qquad (11b)$$

and a compression for

$$B_2 = L_2\frac{h_2}{p_2}\cdot\frac{h_0}{b_1} \quad B_3 = L_3\frac{h_3}{p_3}\cdot\frac{h_0}{b_1} \quad B_4 = L_4\frac{h_4}{p_4}\cdot\frac{h_0}{b_1} \qquad (11c)$$

Note that in (8) as well as in (11), $\frac{h_0}{b_1}$ is the constant multiplier which resolves all H-forces into B-stresses as in Formula (3).

It will readily be seen that all stress formulas may be looked upon simply as analytical expressions of stress-diagram lines; similar triangles are the simple bases of derivation for both, or the geometric links between form of structure, stress diagram, or formulas.

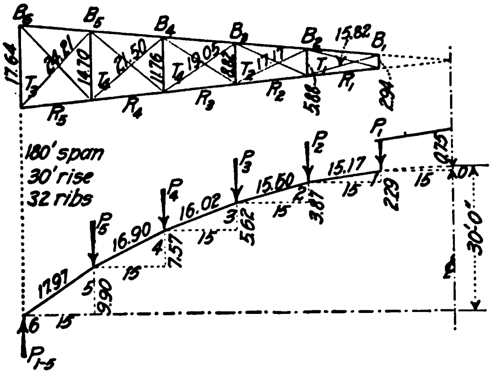


Fig. 498.—Part plan and elevation of dome.

299c. Numerical Example.—Let it be required to design a dome of 180-ft. span and 30-ft. rise with panel points upon a spherical surface. The radius of the generating circle $= \frac{90^2 + 30^2}{60} = 150$ ft. Choosing rings 15 ft. c. to c. horizontally and a corresponding arrey of 32 ribs, the length of all members c. to c. panel points, and other dimensions required, may be computed or scaled with sufficient accuracy from a skeleton drawing. These dimensions are given in Fig. 498. Assuming 15 lb. for framework with tar and gravel roofing, 10 lb. for plastered ceiling, and 25 lb. for snow and wind, or a total loading of 50 lb. per sq. ft. of dome surface, the stresses for total loading will be determined by Formulas (6), (7), (11a), and (9) as follows:

For panel point	1	2	3	4	5
The panel area =	50	90	139	194	256 sq. ft.
The panel load P =	2500	4500	6,950	9,700	12,800 lb.
Summing, P =	2500	7000	13,950	23,650	36,450 lb.

(All stresses are slide rule values)

Rib Stresses:

$R_1 = -(2500)(15.17/2.29) = -16{,}570$ (compression)
$R_2 = -(7000)(15.50/3.87) = -26{,}120$ (compression)
$R_3 = -(13{,}950)(16.02/5.62) = -39{,}700$ (compression)
$R_4 = -(23{,}650)(16.90/7.57) = -40{,}000$ (compression)
$R_5 = -(36{,}450)(17.97/9.90) = -66{,}200$ (compression)

Belt Stresses:

$B_1 = -(2500)(15/2.29)(15/2.94) = -83{,}600 =$ compression in lantern ring.
$B_2 = [(2500)(15/2.29) - (7000)(15/3.87)](15/2.94) = -54{,}800$ (compression)
$B_3 = [(7000)(15/3.87) - (13{,}950)(15/5.62)](15/2.94) = -51{,}600$ (compression)
$B_4 = [(13{,}950)(15/5.62) - (23{,}650)(15/7.57)](15/2.94) = -49{,}400$ (compression)
$B_5 = [(23{,}650)(15/7.57) - (36{,}450)(15/9.90)](15/2.94) = -42{,}100$ (compression)
$B_6 = (36{,}450)(15/9.90)(15/2.94) = 281{,}400 =$ tension in wall ring.

Increase or decrease in belt stresses due to a nominal live load L (or an assumed maximum difference between panel loads at the ring and those within the ring) of 10 lb. per sq. ft. due to ceiling construction or other causes.

For panel points	1	2	3	4	5
Nominal live load L =	500	900	1390	1940	2560 lb.
Summing L =	500	1400	2790	4730	lb.

By Formula (11b)
$B_2 = (500)(15/2.29)(15/2.94) = 16{,}750$ (tension)
$B_3 = (1400)(15/3.87)(15/2.94) = 27{,}700$ (tension)
$B_4 = (2790)(15/5.62)(15/2.94) = 28{,}200$ (tension)
$B_5 = (4730)(15/7.57)(15/2.94) = 47{,}900$ (tension)

By Formula (11c)
$B_2 = -(900)(15/3.87)(15/2.94) = -17{,}800$ (compression)
$B_3 = -(1390)(15/5.62)(15/2.94) = -18{,}900$ (compression)
$B_4 = -(1940)(15/7.57)(15/2.94) = -19{,}600$ (compression)
$B_5 = -(2560)(15/9.90)(15/2.94) = -19{,}800$ (compression)

Tie stresses due to a nominal live load L (or an assumed maximum difference between to adjacent panel loads) of ¾ of a 20-lb. snow load = 15 lb. per sq. ft.

For panel points	1	2	3	4	5
Nominal live load L =	750	1350	2085	2910	3840
Summing L =	750	2200	4285	7195	11,035

By Formula (10)

$$\begin{aligned} T_1 &= (750)(15.82/2.29) = 5{,}180 \text{ lb.} \\ T_2 &= (2200)(17.17/3.87) = 9{,}770 \text{ lb.} \\ T_3 &= (4285)(19.05/5.62) = 14{,}540 \text{ lb.} \\ T_4 &= (7195)(21.50/7.57) = 20{,}400 \text{ lb.} \\ T_5 &= (11{,}035)(24.21/9.90) = 27{,}000 \text{ lb.} \end{aligned}$$

Adding the two compressions for intermediate rings gives the maximum axial compression for

B_2	B_3	B_4	B_5
72,600	70,500	69,000	61,900 lb.

These ring members will also serve as supporting beams for wooden rafters, radiating with the rib members and carrying wooden sheathing and roof cover. Hence, in addition to the maximum axial compression, they will be subjected to a flexural stress due to beam loads P_2, P_3, P_4, and P_5, and should be designed, in agreement with Formula (2), giving a fiber stress

$$f = \frac{B}{A} \pm \frac{Pb}{8} \cdot \frac{c}{I}$$

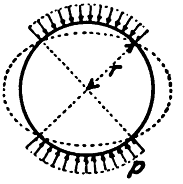


Fig. 499.—Bending action on lantern ring.

All dome members will be of steel and straight between panel points except the lantern ring which will be curved. The wooden rafters may be cut to the curvature of the dome without great expense.

The design of the lantern ring requires particular care. In addition to its maximum axial compression, it is subjected to bending by any inequality in thrust of the abutting rib members. It must hence be made stiff as a whole, both vertically and horizontally, and spliced to its maximum obtainable value so as to make it a continuous circular girder beam.

The bending action due to the horizontal components of thrust inequalities may be computed upon the severe assumption that the nominal live loads L act upon two opposite quadrants of the dome, while the other two quadrants carry no live load. Then, referring to Fig. 499, if r is the radius of the ring and p a uniform load per foot, the bending moment of the ring is

$$M = \frac{1}{5} pr^2 \tag{12}$$

For the present example, the horizontal thrust of R_1 for a nominal live load of 15 lb. per sq. ft. is (750)(15/2.29) = 4930 lb.

$$\frac{4930}{2.94} = 1680 \text{ lb. per ft. of lantern ring.}$$

$$M = (1/5)(1680)(15)^2 = 75{,}500 \text{ ft.-lb.} = 453 \text{ in.-tons}$$

The axial compression in the lantern ring is 42 tons. For a Bethlehem 12-in. 78-lb. H section, $A = 22.9$ and $\frac{I}{c} = 102.6$. Formula (2) gives a maximum fiber stress of

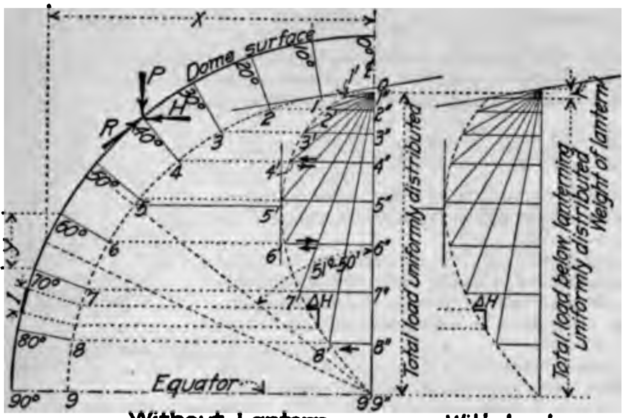
$$f = \frac{42}{22.9} + \frac{453}{102.6} = 6.25 \text{ tons per sq. in.}$$

300. Framing Material and Cover.—Although the framing material and cover are governed largely as for building construction in general, by economy, temporariness, permanency, architectural requirements, building laws, etc., it may here be emphasized that timber is a suitable material even for very large domes. With all purlins or rafters cut to the curvature of the dome and well connected to either a timber or a steel frame, good timber sheathing ¾ or 1 in. thick, and thoroughly nailed down in two diagonal layers, will supply a considerable amount of bracing, and for smaller domes perhaps the only bracing necessary. For steel dome frames, up to 50 ft. diameter and more, sufficient bracing may also be obtained by the use of gusset corner plates parallel to the dome surface at all panel point connections. A reinforced concrete shell upon a steel dome frame will naturally take the place of the diagonal panel bracing, but the spacing of either ribs or rings for such structures should be to accommodate a thin shell reinforced in two directions. For close rib spacing, alternate ribs may terminate halfway up. A steel frame entirely fireproofed with concrete seems an uneconomical structure if reinforced concrete ribs and rings of not much larger bulk will do the work. However, most reinforced concrete domes so far constructed are solid shells without ribs or rings except a lantern ring if not entirely continuous at the crown.

301. Solid Domes.—The analysis of solid domes is not essentially different from that of framed domes. If the ribs and rings of the latter are imagined to be spaced closer and closer, the stress conditions of a solid dome are practically realized.

301*a*. Graphical Method.—Fig. 500 (*a*) represents a stress diagram for a solid hemispherical dome analogous to Fig. 490 for a framed dome. The triangles 01′1″, 02′2″, 03′3″, etc. are force triangles of P, R, and H for points 1, 2, 3, etc., hence the curve 01'2'3' etc., a closing these force triangles, gives R and H for any point along the meridian section of the dome.

The area of a spherical segment is $2\pi ry$, hence all belt areas are proportional to their h. This, for a spherical dome of uniformly distributed loading p per sq. ft. of surface, permits the rapid plotting of diagrams like Fig. 500 (a), as indicated. The total weight of a hemisphere ($= 2\pi r^2p$) laid off to a convenient scale from the center of the dome along its vertical axis, and any equal or unequal division into belts projected upon it as shown, furnishes at once the complete load line without further computation. H is the horizontal shear across the shell as indicated by pairs of arrows. It reaches a maximum when the stress curve 01' 2'3' etc. returns, namely, at an angle between generating radius and vertical of 51 deg. 50 min. as shown. Above this point the belt stresses B are compressive, below it they are tensile.



Without Lantern (a) With Lantern (b)

Fig. 500.—Stress diagrams for solid domes.

The difference ΔH between two H-lines enclosing a belt is the radial horizontal thrust around this belt. $\frac{\Delta H}{2\pi x}$ equals thrust per unit circumference. To determine B, let Fig. 501 represent a unit length of a horizontal ring, largely exaggerated. Then by similar triangles

$$B: \frac{\Delta H}{2\pi x} = x: 1 \quad \text{or} B = \frac{\Delta H}{2\pi}$$

This gives the belt stress per foot of meridian if ΔH is taken accordingly, as shown in Fig. 500 (a). $\frac{R}{2\pi x}$ gives the meridian stress per foot of circumference of belt.

For practical application the load line is made equal to r^2p, thus eliminating 2π from the operation and obtaining:

$B = \Delta H$ = belt stress per foot of meridian.

and $\frac{R}{x}$ = meridian stress per foot of belt.

Compression concentrated in lantern ring = H at lantern.

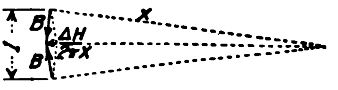


Fig. 501.—Determination of belt stress.

Tension concentrated in wall ring = H at wall.

The latter will be a maximum for a dome terminating at 51 deg. 50 min., where ΔH is zero. At 90 deg. H is zero and ΔH a maximum. Note, however, that the stress diagram may be continued for domes extending spherically below the equator where the wall ring stress would then be compressive.

Fig. 500 (a) is drawn for a dome continuous at the crown, while Fig. 500 (b) will show the slight difference for a dome with lantern.

For a dome shell increasing in thickness from crown to base, or for nonuniform loading, the load line is determined in the usual way, using belt areas ry.

In order to comprehend the stress conditions at the crown of a closed dome, imagine the lantern ring replaced by a solid plate which must necessarily be under compression in all directions.

For a conical dome the method is still simpler, but the stress diagram had better be drawn analogous to III' (Fig. 496).

301b. Analytical Method.—Dead plus live load per square foot of surface is designated by p. The area of a spherical cap above a plane cc (Fig. 502) equals $2\pi ry$.

The vertical reaction along circumference cc = total load above cc, that is, $2\pi xR \sin v = 2\pi ryp$, or since $x = r \sin v$ and $y = r\,(1 - \cos v)$

$$R = \frac{yp}{\sin^2 v} = \frac{rp(1 - \cos v)}{(1 - \cos^2 v)} = \frac{rp}{1 + \cos v} \tag{13}$$

R is the meridian stress per unit length of circumference of belt. At the crown $\cos v = 1$, hence $R = \frac{rp}{2}$. At the equator $\cos^2 v = 0$, hence $R = rp$. Now let B be the belt stress per unit length of meridian, then from the greatly exaggerated force plans of Fig. 502, in which Δv and Δh are very small angles.

$$H = 2B \sin \Delta h = 2B\frac{1}{2x} = \frac{B}{x}$$

$$\text{and} \quad 2R \sin \Delta v = 2R\frac{1}{2r} = \frac{R}{r}$$

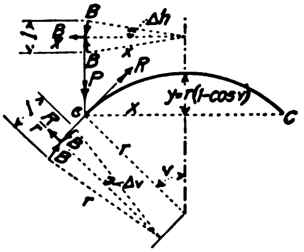


FIG. 502.—Dome shell.

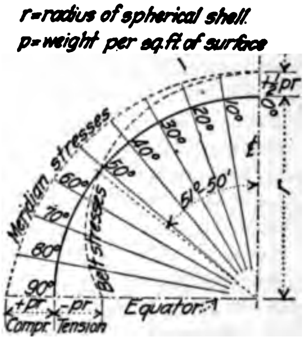


FIG. 503.—Stress values for solid domes.

The three forces B, R, and p upon unit area at c must be in equilibrium, hence their components in any direction = 0. This for direction r gives

$$\frac{B}{x}\sin v + \frac{R}{r} - p \cos v = 0$$

$$\text{and since } R = \frac{rp}{1 + \cos v} \text{ and } \frac{x}{\sin v} = r$$

$$B = rp\left(\cos v - \frac{1}{1 + \cos v}\right) \tag{14}$$

At the crown $\cos v = 1$, hence $B = \frac{rp}{2}$. At the equator $\cos v = 0$, hence $B = -rp$ (tension)

Following is a table of $(1 + \cos v)$ and $\left(\cos v - \frac{1}{1 + \cos v}\right)$ for convenient application of Formulas (13) and (14), and Fig. 503 is a graphical representation of these formulas.

Angle v	$(1 + \cos v)$	$\left(\cos v - \frac{1}{1 + \cos v}\right)$
0	2.000	−0.500
10	1.985	−0.482
20	1.940	−0.425
30	1.866	−0.331
40	1.766	−0.201
50	1.643	−0.034
51 deg. 50 min.	1.618	±0.000
60	1.500	+0.167
70	1.342	+0.403
80	1.174	+0.678
90	1.000	+1.000

The vertical and horizontal wall reactions per foot of wall ring are $R \sin v$ and $R \cos v$. The tension in the wall ring is $Rx \cos v$.

301c. Reinforcement.—The reinforcement is placed in direction of meridians and horizontal belts. Outside of wall ring or of the tension belt area below 51 deg. 50 min., the shell need only be lightly reinforced against shrinkage and temperature cracks, for the unit compression of the concrete will ordinarily be found very low. For a semispherical dome, for instance, of 100-ft. span, and 6-in. thickness of shell, and a loading of 72 lb. per sq. ft. in addition to its own weight, the compression and tension at the base $= \frac{144 \times 50}{72} =$ 100 lb. per sq. in., and the compression at the crown, one-half of this.

It is assumed generally that the pressure surface of a dome shell, analogous to the pressure line of a well designed arch, may oscillate within the middle third of the thickness of the shell, hence the maximum unit compression should not exceed one-half of the permissible compressive stress of the concrete. This is of less importance for architectural domes, for which as already stated, the compression of the concrete will hardly ever reach that amount, but for subterranean domes and domes for tanks under large earth or water loads, it will determine the thickness of the shell.

SECTION 4

GENERAL DESIGNING DATA

ARCHITECTURAL DESIGN

BY ARTHUR PEABODY

1. Theory of Design.

1*a*. Three Fundamentals.—In 1624 Sir Henry Wotton, an English architect, stated the requirements of good architecture in three words, "commoditie, firmeness and delight."

This covers the ground today as it did 300 yr. ago. A building that is commodious in the sense of being suitable and sufficient for the intended use, one that will withstand the effects of nature and the loads and strains to which it is subjected, and that is pleasing to the intelligent and unprejudiced observer, represents good architecture. None of the three primary elements are independent of the others. The plan must be sufficiently flexible to meet the demands of stability and appearance. The structural system must adapt itself somewhat to conditions and the artistic scheme must be perfected without seriously trenching upon the other elements.

1*b*. The Language of Design.—A design must declare its intention, so far as possible. It should indicate the character of the building as political, religious, domestic, etc. In the expression of this lies a good measure of its success. The several parts of a design must be in harmony if not in symmetry, and in scale—that is to say, commensurate one with the rest. Finally, good design requires grace of form, articulation of parts, dominant elements, proportion and emphasis. These qualities are dependent upon mass, outline, color and detail.

1*c*. Characteristics of Design.—A design may be simple, that is, consist of a few elements dominated by a single point of interest, as in a church spire. It may be complex, with similar parts symmetrical about a central axis like the Elizabethan Manor, or irregular, with sharply articulated masses arranged in a picturesque manner so as to bring about a pleasing result, as in the dormitory quadrangles of some of our Universities. The ordinary limits of the safe use of materials and structural methods should be kept in mind. Curious expedients for the solution of problems arouse criticism and usually reflect on the quality of the design. The element of apparent stability affects the impression of beauty. Apparent stability is ordinarily connected with mass. A stone column appears to be stronger than an iron post of equal structural value. The appearance of strength is therefore satisfied better in some instances by stone than iron. From the customary mental attitude toward them, columns attribute strength to a building although used in a purely decorative way. On the other hand, openings out of scale with the design, though constructed in a very stable manner, detract from apparent strength and reduce architectural value.

1*d*. Use of Elements.—In the matter of scale, small units may be made to increase the apparent size of a building, or large ones to diminish it. Architectural size is measured in terms of the human figure. It would be impracticable, however, to adhere closely to this unit, especially in sculptural decoration of buildings. It is necessary to increase such figures to avoid the appearance of diminution, due to juxtaposition with elements that must inevitably partake of unusual dimensions. The actual size of units must harmonize with the scale of the building. Very large stones appear out of place on a small cottage, or very small stones on a large building. Expecially on the interior, members of great strength must be considerably masked, to obviate their crushing effect. In short, "absolute frankness" would be as disastrous in architectural design as in everyday life. Vertical lines add also to slenderness. Horizontal lines increase strength. For this reason fluted Corinthian columns are used in upper stories, while the Tuscan order of the lower parts may be rusticated, along with the massive ashlar of the building.

1*e*. Color and Ornament.—Color is one of the important elements of design. The same building which in the purity of white marble would reflect and etherialize the intention of the architect, might be an abomination in cold red sandstone. The vagaries of certain Italian work are more or less glossed over by the magnificient color and quality of the materials. For this reason, in the use of mixed materials such as stone and brick, discretion is a saving virtue. In a general way delicate members are quite useless in materials of strong and especially of sombre colors. The play of light and shade is to a great extent lost, and members which would be adequate in light colored stone, appear weak and non-effective. The bright red of modern tile, or the variegated tints of rock faced slate, must be reckoned with in the completed color scheme of buildings.

Carved ornament, which may be thought of somewhat as a color decoration, must be placed where it will emphasize an idea. This it cannot do if placed where it will not be seen, or dissipated over a building in such a manner as to signify nothing in particular. Placed on a bracket it increases the effect of strength by its light and shadow and is therefore justified. The same use applies to the carving on a capital, which increases the apparent size and adds to architectural strength.

2. Architectural Style.—An architectural style is an assemblage of parts, ornaments and details forming a definite structural and ornamental system of design. It is formed partly on tradition, partly on structural methods. A new element introduced into an existing style may in time produce an entirely new style, as in the case of the Gothic, which owes its existence to the intelligent and persistent use of the pointed arch and vault, together with the supporting buttress, as new elements applied to the previous architectural system of the round arched style.

A style seldom becomes free from similarity to its predecessor. It tends to carry along, as purely ornamental features, elements which originally had a vital function. In this way the dentils and modillions of the Corinthian Order remain as obsolete members, the function of the bracket having been replaced by other structural elements.

2*a*. The Gothic System.—Gothic architecture as developed principally in France depended upon the arrangement of arch ribs, vaults, buttresses and flying buttresses so combined as to make a stable, constructive system. The problem of the vaulting was the whole matter. During the Romanesque period this was founded on the semi-circular arch, which from its nature fixed the height of the vault over a given width of nave. The adoption of the pointed arch freed the nave from this limitation. It might then be as high as the exigencies of constructive materials would permit. To resist the outward thrust of the main vault the expedient of the buttress was employed. As the height was gradually increased, by extending the wall of the clerestory, a second row of braces called flying buttresses was employed. The system was now complete. The buttresses took the place of the heavy walls of the previous Romanesqûe style and the spaces between were filled by thin enclosing walls pierced by great windows. Over the stone vaults a false roof of timber work kept off the rain. The progress of the style led to increased slenderness and more complicated decoration until the limit of resistance was reached in some cases.

Military Gothic grew out of the needs of the feudal system and was developed most completely in France. Based upon the art of warfare of the time, the castle, or chateau, consisted of a walled enclosure of considerable area, with great towers at points of advantage. The area was divided into the outer court, containing barracks and drill grounds and other buildings, and the inner court containing various buildings of good size, behind which was the great tower, donjon, or keep. The castle was ordinarily located on broken ground, for defensive purposes. The bank of a river, and particularly the land between a river and one of its branches, was thought to be desirable for this reason. The keep would be located at the point of intersection, and the plan of the works would describe an irregular triangle, the enclosing wall following the banks and the front wall closing the interval between them. The design of the chateau varied with the progress of military art up to the advent of gun powder in war. At a later date the buildings of the inner court, largely remodeled and beautified, became the chateau or country seat of the descendant of the feudal lord. Connection with civil architecture was thereby established and the effect on private architecture may be seen in modern French residences of large size.

2b. Ornaments of the Gothic Style.—The method of ornamentation and the detail of ornament in Gothic architecture is quite different from that of the Renaissance. It is less sophisticated, has less repose and is less commonly repeated in exactly the same form. It is bold, variable, constantly substituting equal values for identical forms, and is imbued with the virility and strain that is characteristic of the style. Among the continuous ornaments are moldings, derived largely from the grouping of slender shafts about a pier or at the jamb of a window. The intention of these is to produce a strong effect of verticality and of light and shade. During the early period of the Gothic this was the principal ornamental motive. In the decorated Gothic the moldings were interrupted by ornaments at intervals or formed to contain them within the concave members. These took the form of grotesque heads, or flowered bosses. In English Gothic a rounded ornament called the Tudor apple is spaced along the moldings, like a series of knobs. The forms of Gothic moldings are to some extent determined by the intention of serving as water drips. No large projections give room for decoration as with the Classic cornice. The label or lip moldings of the arches end in rosettes. The slender cylindrical shafts of the columns are decorated with molded bases and elaborately carved capitals. In the complicated interlace, derived from the Celtic, to the delicate leafage of the best period, the entire gamut of variety is run. The shafts are sometimes decorated with zig zag chevrons. The bases are frequently round, or octagonal, with deeply cut moldings.

From the Romanesque the diaper or lozenge pattern is carried into the style for decoration of flat surfaces. The intersections of vault ribs are ornamented with carved rosettes or pendants. Buttresses, at first plain, are later decorated with pinnacles bearing poppy heads. The flying buttresses, especially on their pinnacles, are ornamented with crockets.

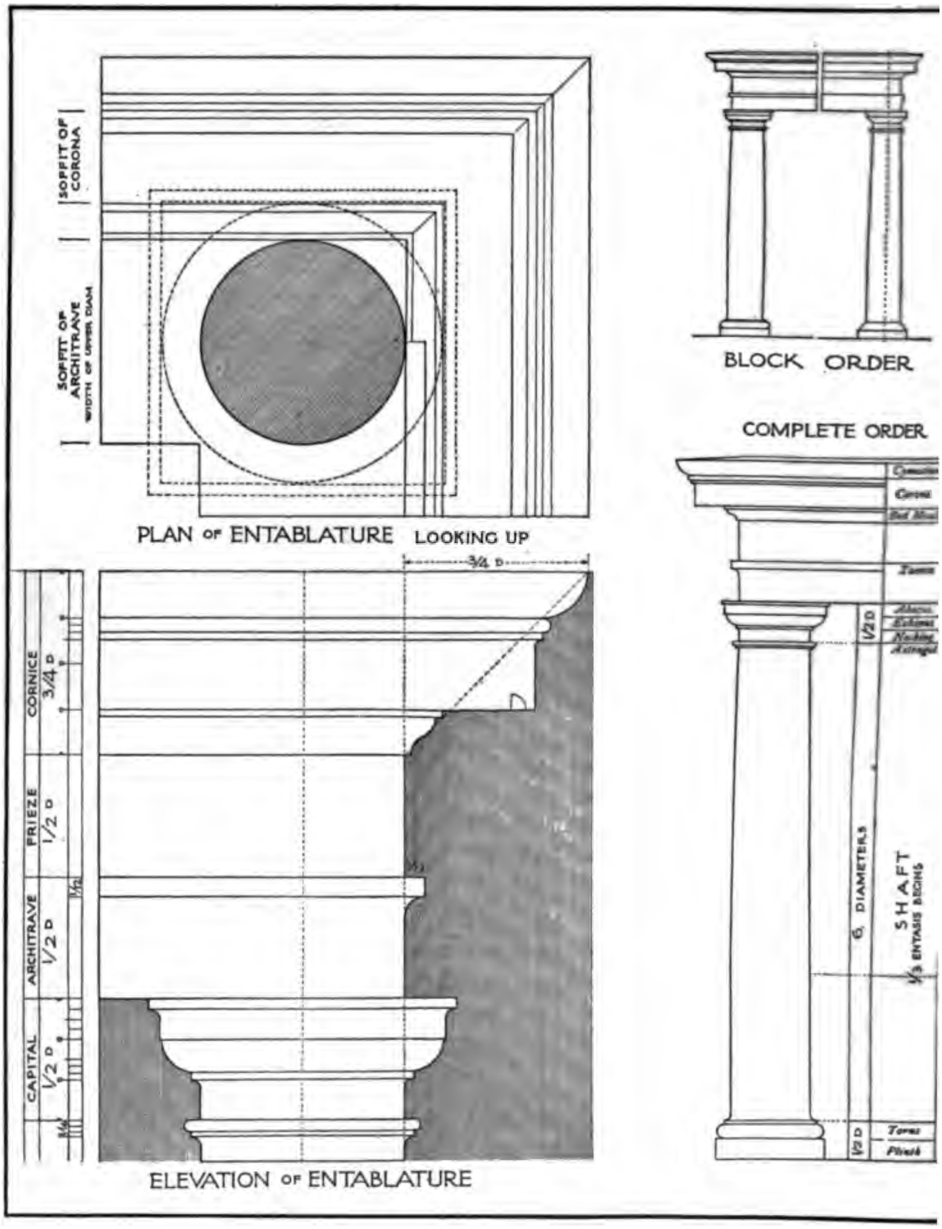
The Gothic window is ordinarily divided by stone mullions, which interlace at the arch level. From his arose the Gothic tracery of pierced stone work, which became one of the distinguishing features of Gothic decoration. At first geometrical, it presently developed into wonderful figures and wavering branchings. Traceries are called trefoil or "three leaved," quatre foil, cinq foil. In combination with stained glass of brilliant beauty, the Gothic window became a distinguishing feature of the style. Tracery, like every other excellent thing, was carried to its ultimate form in the lace-like stone draperies over the elaborate niches of the late period. It decorated not only openings, but spread over the surfaces of vaultings, ever increasing in complexity with the development of the Gothic style. In Spain it was crusted over with minute decorations and filagree. The effort toward slenderness and multiplicity ended with the extreme of possibility in chiseled stone. This applied not only to decoration, but to structure as well, until a halt was called by the final breaking down of parts.

2c. The Renaissance Style.—The Renaissance occurred in Italy in the 15th Century. The chief characteristic of the Renaissance style of architecture is the use of the Greek and Roman architectural orders and decoration. The models for these were derived from study of Roman remains in Italy by the architects Vignola, Palladio and others.

2d. Orders of Architecture.—An order is a principal element of style. Having represented, at first, the entire expression of a limited architectural scheme, it has at a later time shared with other similar orders in the development of the completed system. The term, order, is used only in connection with the Classic and Renaissance styles. In the Gothic style there are no such distinct demarcations, but examples are spoken of as being in the French or English Gothic, the flamboyant, or perpendicular, as the case may be.

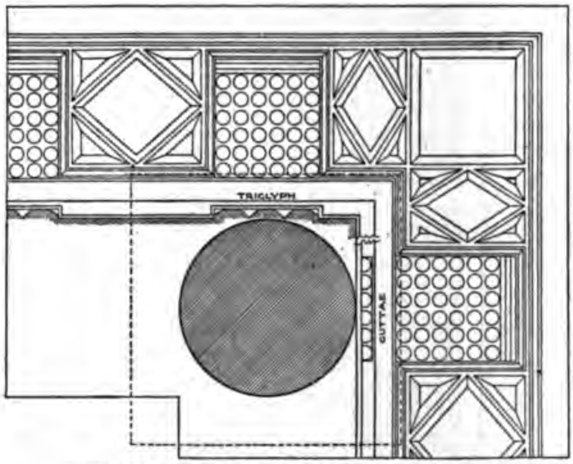
An order is made up of the column, with its base and cap, the architrave, frieze and cornice. Where the cornice is divided and extended along a gable to fit the pitch of the roof, it becomes a pediment. The space enclosed between the level cornice and the slanting portion is known as the tympanum. Any portion of an order may be ornamented according to customary use. The tympanum may be filled with sculpture. The best practice is to ornament alternate members only, leaving plain fillets or bands between. In the last period of Roman architecture, entire surfaces were covered, but the result is admitted to be inferior. The period of the Renaissance gave opportunity for experimentation with the detail of orders, which was carried out to its ultimate conclusion. Some of the more worthy variations are still employed. The rusticated Doric is one such. In this the raised surfaces of the adjacent stone work are repeated on the columns. In other ways, such as variations in the flutings or in the amount of entasis employed, the intention of the artist to modify or emphasize the value of parts is shown, as necessary to the harmony of the design. The illustrations of the orders here given are

TUSCAN ORDER

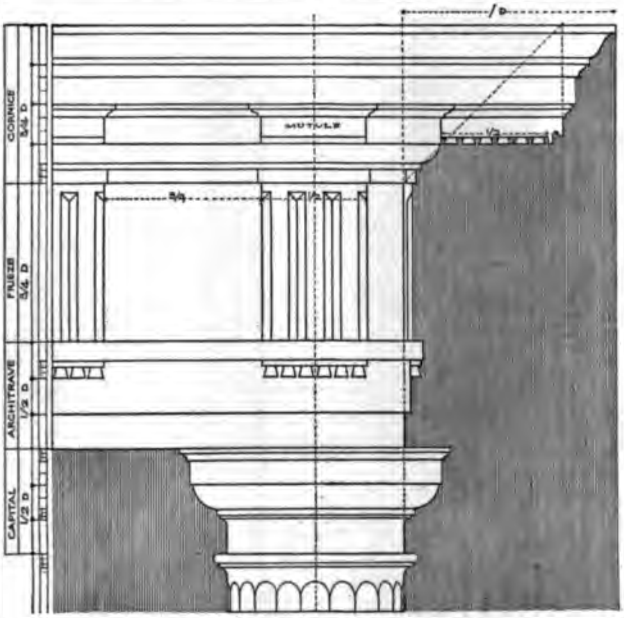


These diagrams of the classic orders are taken from The American Vignola, by permission of The Internati Text-book Company, Scranton, Pa., publishers, and correspond with the original drawings prepared by Giac Barozzi da Vignola.

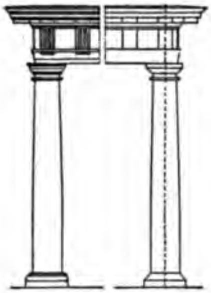
DORIC ORDER



PLAN OF ENTABLATURE LOOKING UP

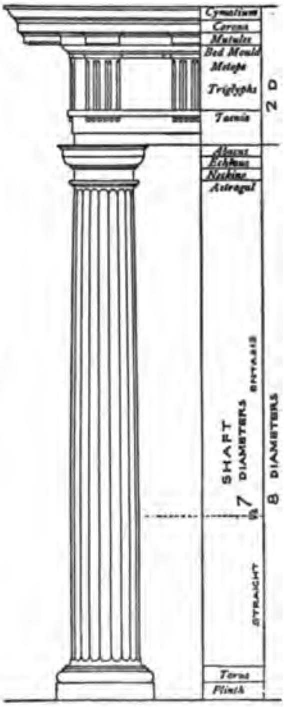


ELEVATION OF ENTABLATURE

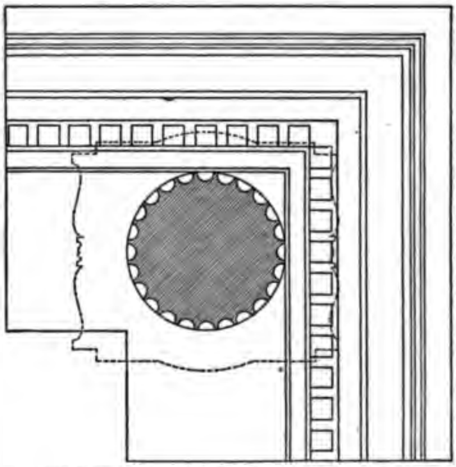


BLOCK ORDER

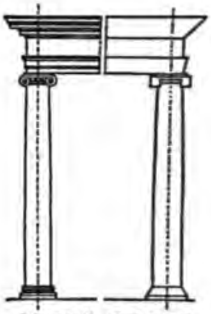
COMPLETE ORDER



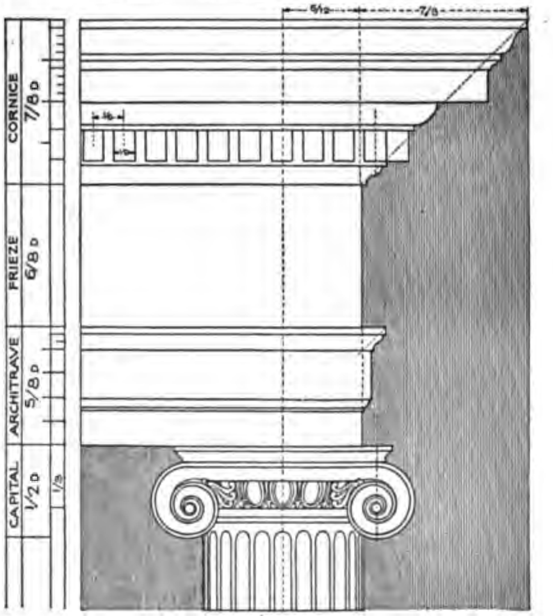
IONIC ORDER



PLAN OF ENTABLATURE LOOKING UP



BLOCK ORDER



ELEVATION OF ENTABLATURE

COMPLETE ORDER

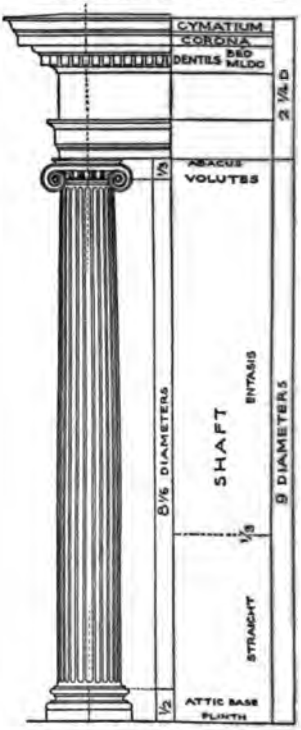
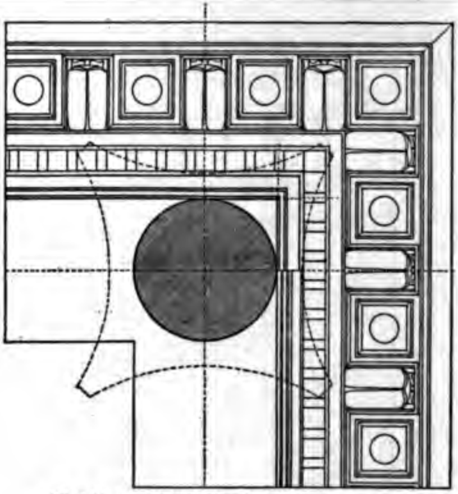
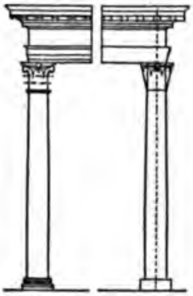


PLATE 1

CORINTHIAN ORDER

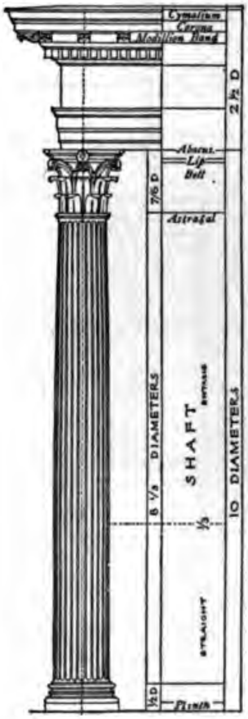


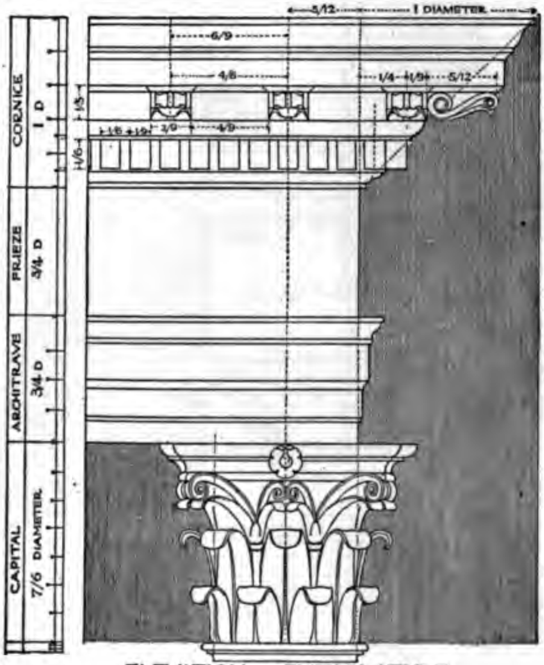
PLAN OF ENTABLATURE LOOKING UP



BLOCK ORDER

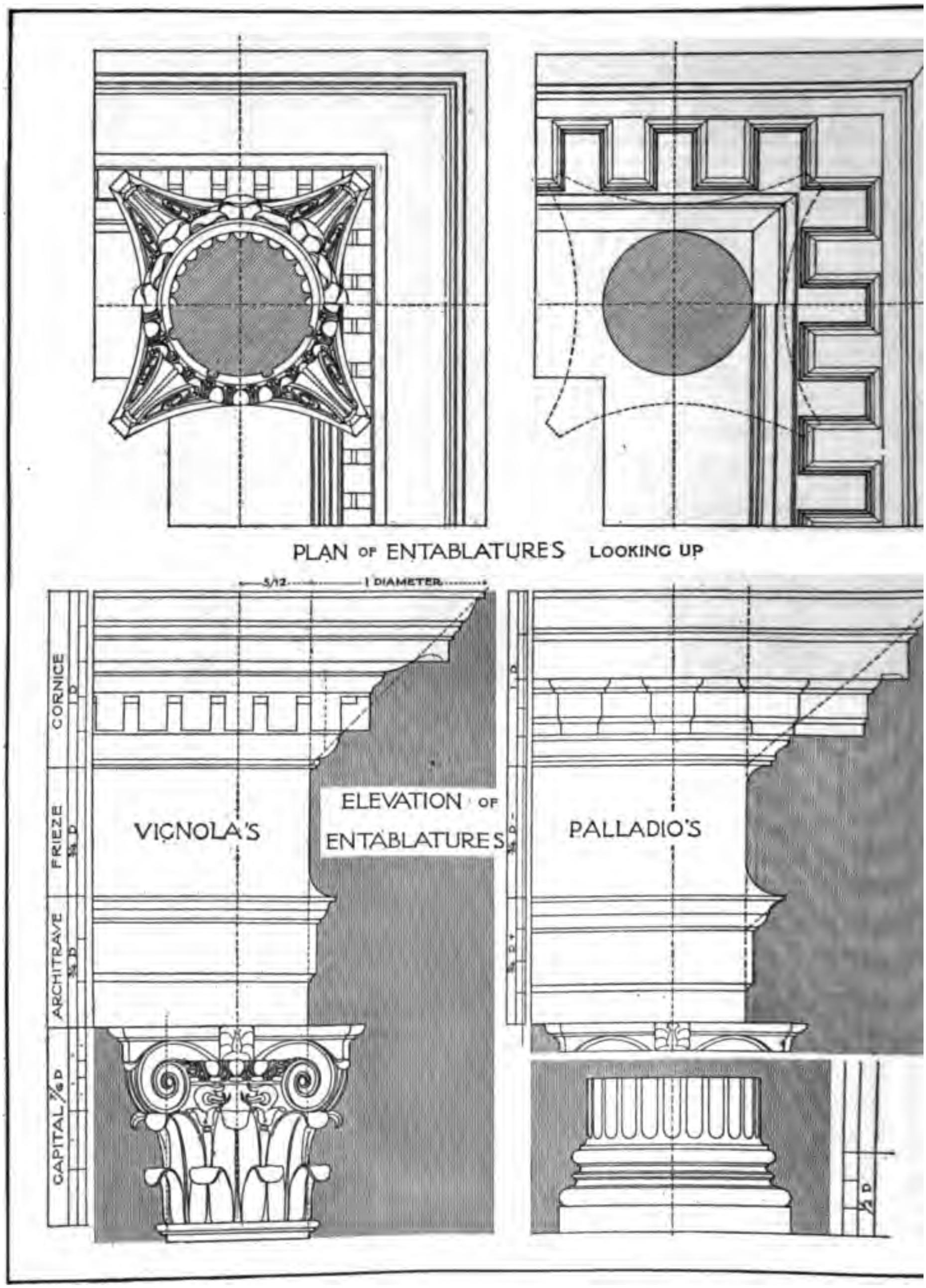
COMPLETE ORDER





ELEVATION OF ENTABLATURE

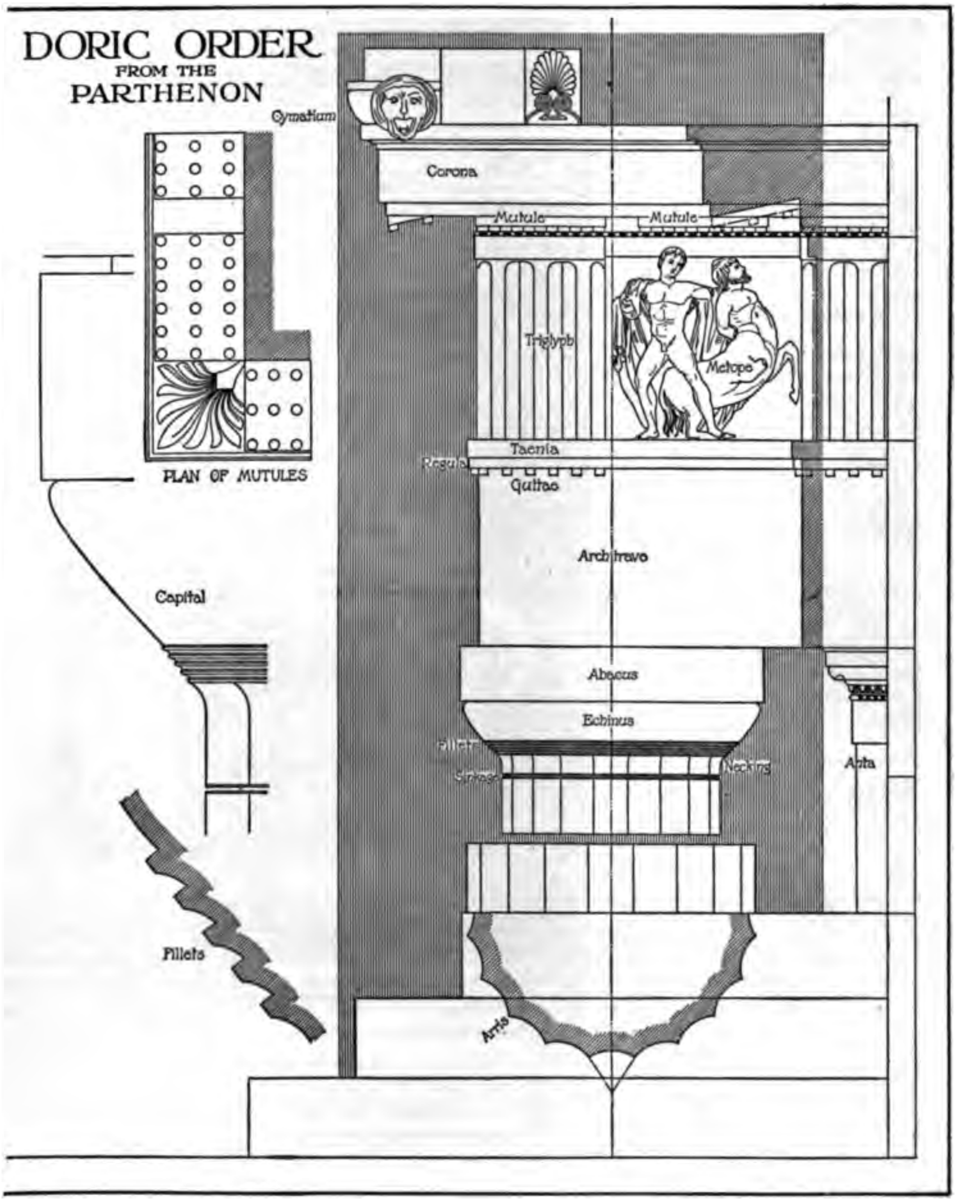
COMPOSITE ORDER



For general dimensions of this order, see Plate IV.

PLATE VI

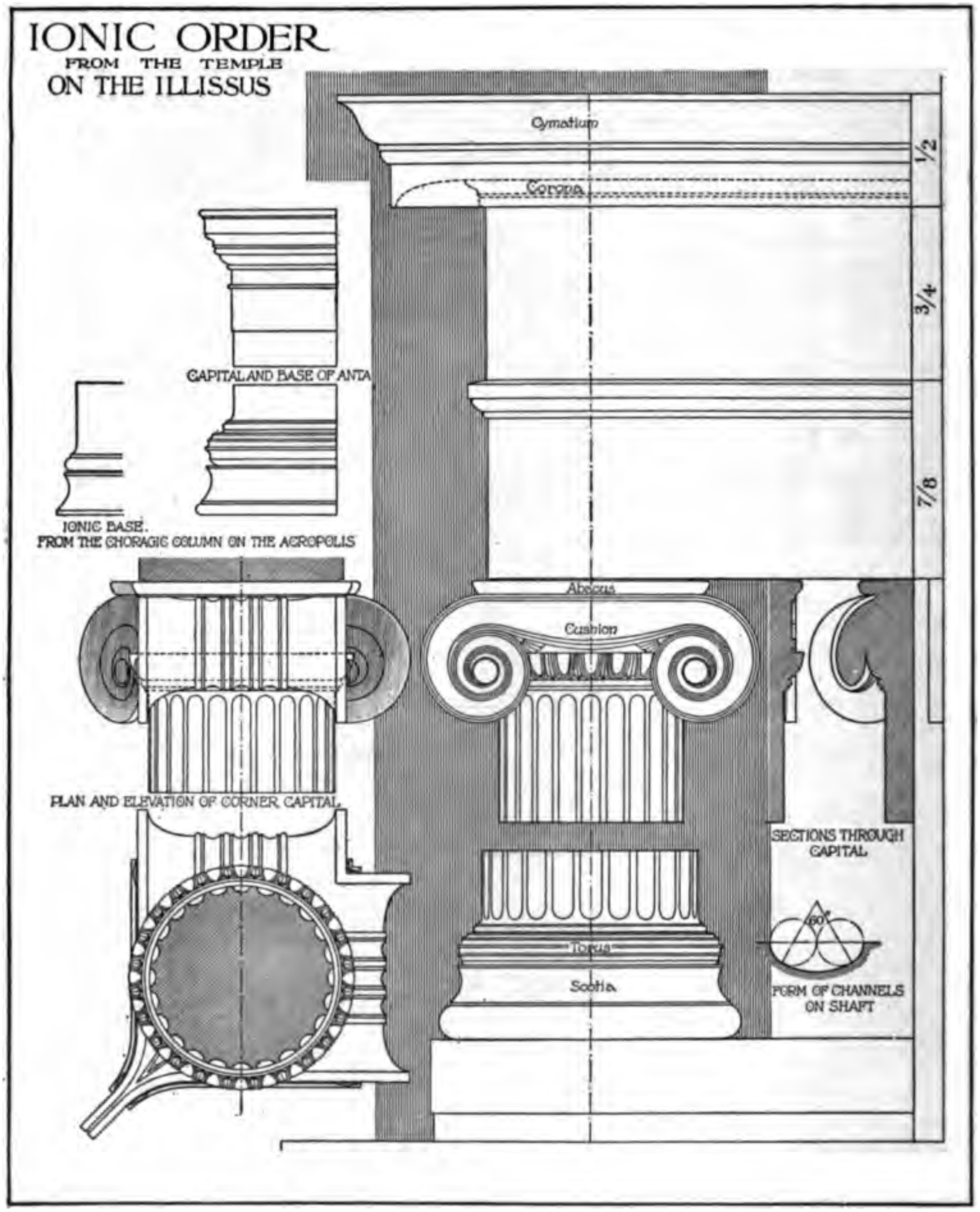
THE GREEK ORDERS



For height of column, see text, p. 721.

PLATE VII.

THE GREEK ORDERS



from the works of G. B. da Vignola, an Italian architect, 1507–1573, commonly regarded as an authority.

The orders of architecture as employed by the Romans are five in number, namely, Tuscan Doric, Ionic, Corinthian and Composite. Of these, the Tuscan is the most massive and simple. The other orders decrease gradually in mass and increase in height so that the Corinthian and Composite represent the most slender and ornate.

In the single storied temples of Greek and Roman days the order was of sufficient size to extend to the full height of the building. In larger structures they were sometimes placed one over another, corresponding with the stories of the building. This is called superposition. In this use the more massive Doric, or Tuscan, is employed in the lower portions and the slender Corinthian, or Composite, in the upper stories. In some buildings all five orders are used. In others two or three at will. Above an order there may be developed a story called the attic. This was employed by the Romans on their triumphal arches. It is now frequently used on a great building to increase the height of it, or of some prominent part, without increasing the scale of the order. Examples of the attic story may be seen on large buildings such as the new National Museum at Washington. The attic is a rather low structure, massive in detail, and may be crowned with a cornice molding. The surfaces are left plain or panelled, or may have openings. Pedestals, spaced at the same intervals as the columns below, may serve as bases for free statues or other ornamental forms. Instead of the attic story there is sometimes employed a parapet above the cornice, with pedestals and balustrades.

Beside the Roman orders the Greek Doric is sometimes used in modern work. This order differs from the Roman Doric, being more massive and severe. The column is without a molded base. The twenty flutes are broad and shallow, without fillets. The height of the column varies from 4½ diameters in buildings of the early period to 5½ in the best period, that of the Parthenon, and to 6 or more diameters in later examples. The cornice is simple and heavy, about two diameters in height. The other Greek orders are the Ionic and Corinthian. These differ from the Roman in certain details.

An order may be raised upon a pedestal, or the building may be set on a base or stylobate, upon which the order will then rest. The order may stand free, as on a portico, or may be engaged with the wall. It may extend through a single story or include several. It may be in connection with an arcade, under another code of customary use. Instead of columns, or in connection with them, rectangular shafts, known as pilasters, may be employed to bring about a complete design. In this use the question of entasis has given rise to some controversy among purists.

The various orders commonly include a certain ornamentation, such as molded bases and carved capitals and a cornice bearing a regular system of ornament as a minimum.

In Greek and Roman use the plainer orders were sometimes decorated with color and gold. Along with a fixed proportion of parts, an order contemplates a certain spacing of columns. These quantities vary with the different orders, the more massive Doric columns being set close together and the slender Corinthian farther apart. An appearance of slenderness is given to the columns by concave flutings on the shaft, while at the same time the optical illusion of central diminution, observed in a cylindrical shaft, is overcome by forming the columns with a convex curve diminishing to the top. This is referred to as entasis.

The orders have long since lost their character as primary supporting members, and have become almost wholly elements of design. The skilful use of them to indicate rather than to furnish actual strength is the province of the designer. This element of aesthetic values is one which prevents architecture from becoming an exact science. Such values cannot be determined by computation and set down in tables, like the safe working strength of steel beams. Within the rigid limits of customary use a wide field of variation is open to the designer.

2e. Architectural Ornaments of the Renaissance.—Renaissance moldings consist of curved surfaces, concave and convex, or of a multiple curvature, applied to the bases, capitals and cornices of this style of architecture. The surfaces of these moldings are frequently enriched by carved ornament, such as the acanthus leaf, the egg and dart, lambs tongue, bead and reel, flutings, the wave ornament, the guilloche or interlace, the honeysuckle, the garland and the Greek key or labyrinth. These are the most common of the continuous ornaments. Beside these a number of ornaments are employed such as the antefix or acroteria, sometimes employed as a cresting above a cornice, the lions head, the cadeuceus. Columns are sometimes replaced by standing female figures called caryatids, or male figures called Atlantes. The Doric frieze is ornamented with the trigylph. a vertical figure of three units placed regularly over the columns. Between these, in what are called metopes, are placed ornaments representing ox-skulls and garlands. Under the projecting portion of the cornice of this order a flat ornament is used, called the mutule. This is replaced in the Corinthian by a scroll bracket. Acroteria are placed at the peak of the gable or pediment and at the eaves. The Roman Doric is ornamented in a different manner from the Greek. Sculpture is used in various ways to decorate buildings in this style. Besides figures in relief on the frieze of the cornice, free statues may be placed at various points either on the stylobate, as on the Bureau of American Republics at Washington, or upon the parapet or attic story. In the case of the Triumphal Arch, horses and a chariot may crown the sructure. This is called a quadriga.

2f. Modern Styles.—The principal architectural styles in America are the Renaissance and the Gothic. Other styles have attained a temporary vogue at times through the exceptional merit of some designer. Among such is the Romanesque style as developed by H. H. Richardson, an architect of Boston.

The Renaissance was reëstablished in this country by the extraordinary display of talent at the Worlds Columbian Exposition in 1892.

The Gothic style for ecclesiastical buildings and for some of the universities, has been restored to favor by the excellent work of a few talented architects.

The successful application of these styles appears to depend largely on the proportions of the buildings in question. Where the main dimensions are horizontal, the Renaissance appears to be most commonly successful. For those exhibiting a preponderance of vertical masses the Gothic style seems to be well suited. Either of the styles is pleasing for buildings of certain types, where extremes of dimensions do not ordinarily occur. In this way the collegiate Gothic, so-called, is adaptable to school buildings faced with brick work. The absence of horizontal members, common to the Renaissance, affords considerable freedom, while the Gothic system of ornamentation gives room for emphasis of prominent parts. Many of these, however, can be treated equally well in simplified Renaissance. In private house work of the better class the designs follow the two principal styles in use. A number of actual reproductions of European dwellings, more or less accurate, exist, but the majority of designs follow a free Renaissance in so far as they are capable of being classified. Architecture in America is now passing through a transitional period and may easily develop into a new interpretation based on modern use and new structural materials such as concrete, steel, stucco and hollow tile.

PUBLIC BUILDINGS[1]—GENERAL DESIGN

By Arthur Peabody

3. Court Houses.—The typical court building, which may be enlarged to meet more complicated conditions, comprises a court room of good size, with chambers adjacent, sufficient to accommodate the several judges holding court at that place. A private office adjacent to each is required. Two or more jury rooms are necessary, of about 14 × 20-ft. dimensions; between these a sheriff's office with entrances to control both rooms. Waiting rooms for witnesses are required. One or more detention rooms are necessary, where convenient access to the jail is not provided. The offices of the county clerk, treasurer, surveyor and other officials will be located in the building, usually in the first story. The arrangement of the court room is that of a hall with the judge's desk on a platform, a space for attorneys, clerk and stenographers about a large table, and a space for witnesses. The twelve jury seats are at one side, frequently on the left, within a separate railing. The seats are raised above the floor on a stepped platform. The witness box is placed between this and the judge's platform, for convenient hearing. The room requires special lighting and ventilation, and should have good acoustic properties. The judges' suites should have separate toilets. Separate toilets should be provided for each jury room, detention or waiting room and for the public. A library room is desirable, but in small court houses is not imperative. The treasurer and the county clerk will require large storage vaults with a money vault for the treasurer.

Ordinary room sizes for small court houses:

Court room	30 × 50
Judges' chambers	14 × 20
Judges' private offices	14 × 20
Library	14 × 20
Jury rooms (2)	14 × 20
Sheriff's office	14 × 20
Witness waiting room	14 × 20
Detention rooms with private toilet	10 × 14
County clerk's office	14 × 20

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[1] Buildings for the United States Government are not included as these are usually designed by the Supervising Architect of the United States. State capitols are also omitted.

County clerk's private office	12 × 14
Treasurer's office	14 × 20
Treasurer's private office	12 × 14
Vaults for each	6 × 14 to 20
Surveyor's office	14 × 20
Health department	14 × 20
Assessor's office	14 × 20

4. Town Halls.—The town hall contains a large assembly room with a moderator's platform and desk. A space for the town clerk and other officials is railed off adjacent. The remainder of the hall is provided with seats for the voters at the rate of 8 sq. ft. per person. A visitors' gallery is desirable. At Bourne, Mass., the offices are in the front part and the hall at the rear. The offices should be of good size, with counters for the public. At North Hampton the hall is in the second story, the town business being conducted on the ground floor.

In some examples detention rooms are provided in the building for persons accused of misdemeanors. Such rooms should comply with the restrictions described under lockups. Most state laws forbid detention rooms in basements.

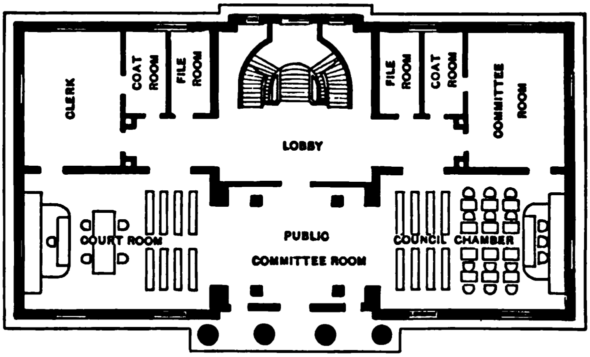


Fig. 1.—Plan of second floor of Municipal Building, Plainfield, N. J.

5. City Halls or Municipal Buildings.—The city hall is a development to meet the needs of the ordinary city government. The meeting room of the common council will require 50 sq. ft. per member. Ante-rooms and committee rooms are required, and offices for certain officials. The mayor's suite will comprise a waiting or reception room, general office, 16 × 24 ft., a private office and toilet. The other officials requiring one or more offices will be the city clerk, tax assessor, street commissioner, department of health, department of charities, department of building, city treasurer, city surveyor or engineer, and others.

Ordinary room sizes will be

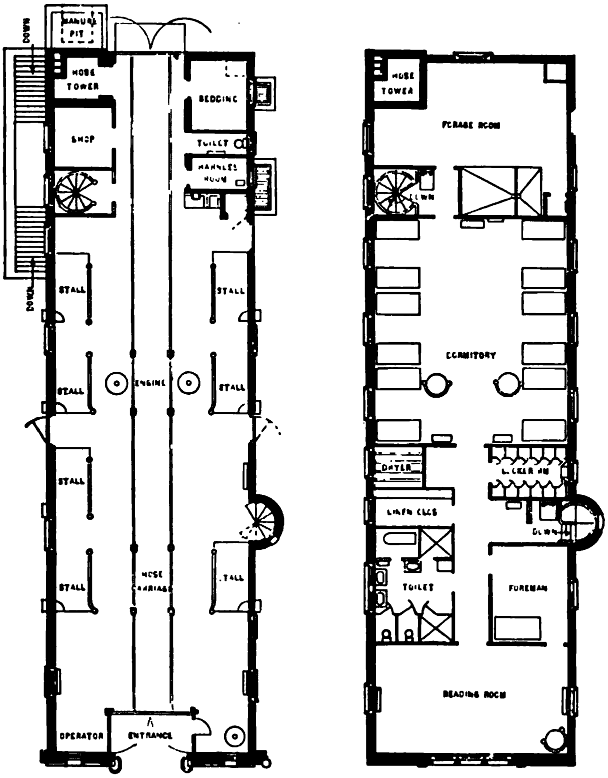
Council room	25 × 40
Committee rooms	12 × 25 to 20 × 25
Mayor's general office	20 × 45
Mayor's private office	20 × 28
City clerk's office	20 × 28
Assessor's office	20 × 28
Street commissioner's office	20 × 28
Department of health Department of charities Inspector of buildings City treasurer City engineer	Each 20 × 35
Private offices generally	12 × 14

6. Public Libraries.—The essential features of a library building are: the reading room, book room and delivery space. A typical arrangement has the delivery desk at the center of the public room, with the card catalogue conveniently placed, the children's reading room at one side, adults' at the other, and the book stacks at the rear. Open shelves are disposed along the walls of the reading rooms for reference books.

The book room will be equipped with metal stacks, self-contained and resting upon steel beams. The load imposed by the stacks will amount to 150 lb. per sq. ft. for each story of the book stack. The windows at the ends of the stacks light the intervals between them. Electric lights between each row are necessary. A book lift is provided in most libraries. Libraries are frequently provided with museum spaces and small lecture rooms equipped for stereopticon or moving picture talks. The working rooms comprise the librarian's offices, unpacking and repairing rooms, cataloguing room, manuscript rooms, rest rooms and travelling-library receiving and shipping rooms.

The construction of library stacks has become specialized to such an extent as to make it advisable to follow standard details. The open shelving in the reading rooms may be of wood construction to harmonize with the architectural treatment of the room.

7. Fire Engine Houses.—The first story will contain the steam fire engines, hose carts and chemical extinguishers. For these the following spaces are required:



FIRST FLOOR PLAN

SECOND FLOOR PLAN

Figs. 2 and 3.—Typical fire engine house.

Fire engines, each	8 × 24 ft.
Hose carts, each	8 × 24 ft.
Ladder wagons, each	8 × 55 ft.
Chief's wagon	8 × 20 ft.

Where horses are employed, for each piece of apparatus there should be three to four horse stalls along the sides or at the rear of the room.

Feed storage and litter space is required. Where auto-cars are used, the dimensions will be approximately the same. The office of the fire chief will be on the first story. A toilet should be provided for the firemen. The second story will contain sleeping rooms and dormitories for the men, together with a reading and lounging room. At some point in the building a tower or shaft is provided for handling hose.

8. Hotels.—The lobby is approached by a principal entrance and ladies' entrance. This contains the office, elevators, cigar and news stand, telephone and telegraph office and a small parlor for women. A private office for the manager is required. The other rooms are the dining room, cafe or tea room, with areas computed at 20 sq. ft. per sitting, the bar and lounge, the service room, with elevator, check room for coats and bags, trunk room and at a convenient point the barber shop and men's toilet. The street fronts may contain a drug store and furnishing store with entrances from the lobby. The dining room and cafe will be preferably on the first floor, or higher up according to the limits of the property. It is economical, as regards operation, to have the kitchen on the dining room level. The plan and equipment of the hotel kitchen and storage spaces is a highly specialized problem and should be studied in consultation with makers of kitchen equipment. Mechanical refrigeration is to be preferred.

Most hotels contain a ball room of about the area of the dining room. The second floor will contain the principal parlor and retiring room for women, which may be in connection with the ball room. There should be a small parlor and toilet for men in this case. The writing room may be on the first or second story. In the latter case a small writing room or alcove should be provided on the first floor adjacent to the lobby. Sample rooms for travelling salesmen should be 16 × 20 ft., well lighted.

The upper stories will be occupied by the hotel rooms. These will vary from 11 × 14 ft. to 16 × 20 ft. with a number of suites having private parlors, 20 × 24 ft. in some hotels. Besides there will be a linen room, utility room and maids' closets on each floor. The typical hotel room is designed on one of two plans. The most desirable arrangement is to place the bath on the outside wall, between rooms, with doors entering each. The closets are placed next to the corridor. In the other plan the bath is placed at the corridor end of the room and the closet next the entrance. This affords no light to the bath rooms and makes good ventilation more difficult. The bath room is intended to be available to either room at will. The adjustment of the closets may permit two rooms to be thrown together. The corridors will be 8 ft. wide. A space adjacent to the elevators is provided for the floor custodian. Helps' quarters are ordinarily at the top of the building. Segregation is necessary in this case, with ample bath and toilet rooms for both sexes.

9. Club Houses.—The general requirements of a club house are similar to those of a small hotel. The special features will depend upon the elements emphasized, such as athletics, golf, yachting. Dormitory rooms and suites are common to many clubs. The service provisions, kitchen and helps' quarters, the dining room, grill room, private dining rooms, game and card rooms, need ample spaces per capita. Cloak rooms and locker space for members should be convenient and of easy access.

10. Colosseums—Convention Halls.—The ordinary colosseum or convention hall will comprise an auditorium to contain a large number of seats. The rate of 7 to 8 sq. ft. per seat will be sufficient. The speaker's platform should be rather high and of sufficient size for seating perhaps 100 to 300 persons. The floor is usually flat, so that the building may be used for exhibitions and other activities, but may be designed with a moderate slope toward the platform. In other cases the building is provided with banked seats, a portion of which is constructed so that sections may be revolved toward the front, and the capacity of the hall reduced as desired. Galleries will be required where the general public must be admitted to certain parts of the hall, while delegates occupy the main floor space. The exits and toilets, provisions for safety, etc., will be controlled by city ordinances or state building codes. Judicious distribution of these utilities is necessary to avoid congestion. Ample committee rooms and administration offices must be provided, together with storage space for chairs not in use. The heating and lighting should be ample, but not excessive. Ventilation by gravity is sufficient.

11. Railway Stations.—The typical railway station, aside from large terminal stations, comprises a ticket office with a bay window overlooking the trackage, waiting rooms at either side for men and women, giving a space of 25 sq. ft. per person in the ordinary case; adjacent to these a baggage room and toilets for both sexes.

A restaurant or lunch counter is provided, convenient to the train platform or to the waiting rooms. The freight warehouse and office may be connected to the passenger station by a covered way. The information bureau and news stand is frequently combined in small stations. The stations are usually one story high except where, in the central portion, offices for the train master are placed overhead.

12. Universities.

12a. Ground Required.—The area necessary for a great university cannot be determined on the sole basis of utility. Other elements enter into consideration, such as the probable number and character of activities, the space required for an adequate and dignified approach, the necessity for light and air, and the desirability of a picturesque arrangement. The possible increase in attendance and of the number of courses to be offered in the near future affect the problem. It is advisable to secure as much land as possible at once and to see that no insurmountable obstacles will prevent enlargement. Advantage should be taken of a water front for a picturesque view and opportunity for water sports. Level ground for athletic fields, together with a rise of ground for the location of buildings, are among the elements of importance in selecting a site.

12b. Preliminary Design.—A preliminary design should be secured where a new university is contemplated or where considerable enlargements to an existing institution are at all probable. Such a plan will prevent unfortunate errors in the location of buildings, drives, walks, etc. It may not be necessary or desirable to fix absolutely the use of each building in a general design. Certain areas should be designated for the several colleges, within which a certain freedom of choice may be left to the future designer. The relation of the several colleges to each other should be carefully studied to secure convenience and efficiency.

12c. Buildings.—Modern universities comprise educational sections or colleges as follows: Letters and Science, Law, Medicine, Engineering, Art and Architecture, Agriculture, Military Science and Training, and University Extension. Besides these, other departments are as follows: Student Help and Recreation, Sports and Athletics, and Administration.

12d. College of Letters and Science.—For buildings in the colleges the following room sizes may be taken as an average:

Class rooms, 16 sq. ft. per student, at 30 per room.

Lecture rooms, 10 sq. ft. per student, at 100 per room.

Lecture halls, 8 sq. ft. per student, at 500 per room.

Offices should have 150 sq. ft. per man. Departmental libraries should have about 10,000 books capacity, with receiving desk for the attendant.

Laboratories for physics, chemistry, biology, etc., will be somewhat similar as regards requirements for space. Laboratory rooms will average 50 sq. ft. per student. Small laboratories for advanced work are necessary. The size 14 × 24 ft. may be taken as a unit. Lecture rooms and lecture halls require ample room for preparation, instruments and materials. The windows must be quite large, 1 sq. ft. to 5 of the floor space, arranged for darkening by shades or panels operated by hydraulic or electric motors. More than one exit from a large lecture room is required, and where possible, one should lead directly out of the building.

For chemistry the principal requirement is for chemical desks with acid proof tops with gas and water supply and waste, sinks at the ends and cupboards underneath; beside these, reagent shelves, fuming cabinets and balance rooms. A chemical store room and dispensary is necessary.

For physics laboratories absence of vibration is imperative. Concrete construction is advantageous. Physics desks are arranged along the walls under the windows and are equipped with electric outlets, gas and compressed air. Concrete piers are required, and special cabinets for apparatus. A mechanicians' shop is necessary with metal working machinery for the most part. Rooms for special apparatus are required for both chemistry and physics. Where photography is made part of the course in chemistry or physics, special equipment is necessary. There will be laboratories for the study of electricity, light, heat, sound, wireless telegraphy, liquid air and gas.

Biology requires microscope tables wider at one end and set at right angles to the windows which should be large, without cross bars of any sort. Chemical desks are needed; also ovens, fuming cabinets, refrigeration rooms, dark rooms, rooms for constant temperatures, green houses and glass covered laboratory rooms and animal houses partly under glass. Ponds open to the air are required and aquaria of various sorts; also a photographic room for recording results. An exhibit museum should be prominently located. A space on the first story, preferably a large entrance foyer, is ideal. The herbarium for botanical collections and the working museum of shells, skins, skeletons, and insects in the division of zoology, collections of alcoholics and specimens preserved in other liquids will require considerable space.

12e. College of Law.—The requirements of this college are lecture and class rooms, reading rooms, and the law library. A good number of offices are needed. The class rooms require more space, about 20 sq. ft. per student. Such class rooms are furnished to advantage with narrow desks to accommodate the text books which are large. In some cases two men are seated at one desk. Law students are older than students in the university courses and require larger furniture.

The law library should have a regular book stack for special texts and a large reading room with open shelves for standard works. The room should be very well lighted, ventilated and furnished with indirect lamps for evening work.

One or more lecture rooms of about 300 seats are required, according to the schedule of lectures.

12f. College of Medicine.—The theory of medicine includes anatomy, physiology and pharmacology. The laboratories will require tables or desks furnished with gas, compressed air, electric current. Microscope tables are extended under the windows which should have as few cross bars as possible. Special fuming cabinets strongly ventilated are necessary.

A gas crematory furnace is needed in the anatomical laboratory and vent flue to the roof. A refrigerated vault for subjects is required together with storage rooms for specimens in alcohol.

For all these laboratories there should be animal rooms. Open air runs for the dogs should be on the roof surrounded by brick walls not less than 8 ft. high. The drainage from these runs and all animal quarters should be well cared for, and provision made for hosing out at frequent intervals. Animals need out-of-door air and may be provided with winter and summer quarters. A small private lift from the laboratory floors to the roof is extended to the ground level. Cages for dogs have wire fronts and 30 sq. ft. area for each animal.

Clinic.—The clinic building should comprise offices for the head physician, a general waiting room, registration rooms and record rooms, 14 ft. square. The general waiting room to contain 50 persons at once will require 15 sq. ft. per person. A separate women's waiting room is desirable; also dressing and examination rooms, about 8 × 12 ft., sufficient for examining 20% of the capacity of the waiting room at one time. The temperature of the examination rooms will be kept to at least 74 deg. and the rooms must be light and well ventilated. Sound proof partitions between units should be provided.

The hospital or infirmary should have an adequate equipment, such as an elevator adjacent to the ambulance entrance, of sufficient size to receive a hospital cot. The corridor should be not less than 9 ft. wide and the room doors 4 to 4½ ft. wide so that a cot may pass them. The stairs, separated from the corridor by glass doors, should be 4½ ft. wide to permit a stretcher to be taken down. The nurses' stations on each floor will be perhaps 14 × 20 ft. with the call desk and signal service and the desks for each nurse keeping records. The hospital will be divided into two units per floor, shut off from the main stair corridor by glass doors. Each unit will require a fully appointed bath room and separate toilet, utensil room, linen closet, and locker room for street clothes. The rooms may be for single patients, or for two in a room, with wards of not over four beds as a maximum. Diet kitchens for each floor are required. The etherising and operating room should be near the elevator. The kitchen and store rooms in the basement will be sufficient with dumb waiters to the several stories, preferably to the diet kitchens direct. The other basement rooms will be X-ray room, baking room and one or two photographic rooms.

The research hospital will contain a number of laboratories. The division into isolated units will be more frequent than in the general hospital and more single rooms will be used.

12*g*. College of Engineering.—The class room building will be similar to the building for letters and science. The same areas per student will be required. Spaces in the basement may be used for instrument rooms, mechanicians shop and general utility rooms. Drafting rooms should be provided with indirect electric lighting for evening work. Laboratories should be quite separated from the academic building, and for that reason a limited provision for class rooms should be made in some of the laboratories.

Steam and Gas Engine Laboratory.—Preferably a long building about 40 ft. wide with spaces for engines on both sides of a central aisle. The engine foundations should be formed to permit ready installation and removal of engines of various types. There should be a basement underneath, for supply and exhaust piping, with ample head room under the piping; also an overhead electric crane for moving large units. Good ventilation, and overhead lighting by saw tooth roofs or otherwise, as well as efficient electric lighting are required. The building should be simple, like the machine room of a factory.

Engineering Shops.—Similar to the engine laboratory, but without a basement. Electric conduits are needed for individual drives of machines; also rooms for wood and metal working, forging, pattern making. casting and finishing.

Electric Engineering Laboratory.—Similar to the engine laboratory, without a basement, but with a central conduit for electric current main wires. Dark rooms for certain lines of study are needed; also laboratories for testing wires, conduits, lamps, etc., transmission of current and electric transmission of sound in telephones, telegraph, and the electric furnace.

Mining Engineering and Metallurgy.—A model ore dressing equipment and stamp mill require a height of approximately 25 ft. The furnaces are of masonry and quite heavy. Chemical laboratories in connection will take 50 sq. ft. per student.

Chemical Engineering is allied to the operative side of mining and metallurgy. The furnace work produces great volumes of acid gas. For the three branches above noted, it may be necessary to provide a masonry chimney for gas removal.

Materials Testing Laboratories for wood, metal, cement, stone, etc., will occupy as much space as the engine laboratories. The building should not be over two stories high and of heavy construction.

Testing Laboratories for pumps, fans, mills, and automatic machines will require as much space as the materials testing laboratory.

Hydraulic Engineering.—Laboratories should be provided with tanks of considerable size, arranged for the study of water power under constant or variable head. A lecture room with a demonstration table is needed.

Marine Engineering and Naval Architecture.—A special branch of steam and electric engineering. Separate laboratories for advanced work required, similar to other engineering laboratories. Naval architecture or ship design will require class and lecture rooms, drafting rooms and model laboratories similar to other engineering laboratories and a model testing pool of large size.

Aviation Engineering.—The class and lecture rooms will be similar to those for marine engineering. The laboratory work must be supplemented by field work involving a considerable area of ground and large shelter sheds for the machines.

12*h*. College of Architecture, Art, Music and Drama—*Studios for Architecture.*—For the study of architecture, class room provisions are required like those in letters and science—seminary and reading rooms for sections of the departmental library of books, photographs and plates, and rooms for models and casts and an exhibit room. The studio rooms, large and small, require correct lighting. These provisions may be taken as standard for all studios in the college as regards the academic or lecture side of the various branches of art.

Special conditions as to ceiling height, north lighting and work rooms in connection with studios will vary according to the special branch. In connection with studios, dressing rooms with locker spaces are imperative from the nature of the work.

Picture Studios.—Studios are for drawing and painting, including oil and water color work, charcoal drawings, etc. Lighting should usually be obained by the use of high ceilings and north illumination. Separate rooms for elementary and advanced work, life classes, etc., should be included. In large rooms division into alcoves is desirable.

Mural Painting, Scene Painting, Fresco Studios.—These should be broad and high to afford sufficient distance for ascertaining values.

Studios for Sculpture.—Rooms are needed for clay modeling, marble cutting by hand and power, gelatine molding, plaster casting, reducing and enlarging, bronze casting and finishing. Sculptures at large size require outside spaces for experimental mounting.

Studios for House Decoration.—These require spaces for experimentation at full size. For this purpose rooms which may be divided into alcoves 10 ft. square are desirable. The surfaces of the alcoves should be fitted to receive color decoration, wall papers, tapestries, etc., which may be removed at will. This branch of decorative art includes also furniture, hangings and floor coverings.

Decorative Art for Buildings.—This includes wood carving, mosaic work, scaggliolas, graffito, marble, metal and glass work.

Arts and Crafts.—These comprise the ceramic arts, designing and decoration of objects in clay, chinaand glass, small metal work, jewel grinding, cutting and mounting, and small wood carving. Power equipment is necessary for the last two arts.

Illustrative and Illuminating Arts.—Book illustration and illumination, the design and preparation of plates, printing blocks, engravings, half tones, photogravures and lithographs, plain and colored, leather tooling, book binding, gilding, etc., are included in this branch.

Posters and Advertisements.—Studios for this branch require good space and high ceilings.

Potrtrait Photographic Studios.—A general studio is needed with ample height and space with complete control of light, accessory electric lighting and flash light equipment; also dark rooms for developing, day light and electric printing space, filing space, fireproof, for materials and prints, storage rooms for scenic accessories. A portion of the space is arranged with seats for lecture purposes, arranged to secure absolute dark for certain work.

Music and Drama.—Studios would be small and numerous, 7 × 10 ft. area, suited for the study of music and oratory. Dramatic art, aesthetic dancing, moving picture photography require good space. For this part of a building a system of heating by warm air would obviate the transmission of sound through the piping incidental to steam heating apparatus. The floors, walls, ceilings of practice rooms should be insulated by sound deadening material. Care should be taken to preserve a certain resonance in the individual rooms. For solo, orchestra and dramatic practice, rooms of medium size, 20 × 28 ft., are required. Moving picture studios require sufficient length for proper focusing, ample room for the movement of actors. The photographic work in connection will require dark rooms 6 × 10 ft. and printing rooms for films, etc., and fireproof storage space.

12*i*. College of Agriculture.—The general course in agriculture will require laboratories for advanced work in various applied sciences. This college has connection with farmers, stock raisers, dairymen, and will hold institutes during the year in the main building. This building will contain the offices of the dean of agriculture and committee rooms for various purposes. The requirements for lighting and spaces will be similar to the academic buildings for letters and science. In all other buildings dressing and locker rooms are required, computed as in the case of gymnasiums.

Laboratories in the Agricultural College.—Soil study, mainly chemical in character but requiring large store rooms. About 25 sq. ft. per student.

Farm Engineering, for Demonstration and Study of Machines and Implements.—Floor areas large, for heavy loads. A freight elevator required.

Agronomy.—The study of seeds, grains, etc. Storage space in small bins, and laboratory rooms for study of seeds are needed. A space of 20 sq. ft. per student in laboratories is required.

Dairying.—Butter and cheese making. The work is partly applied chemistry. A machine laboratory is needed for demonstration of methods and processes. In connection a fully equipped dairy and cheese factory on a small scale with ample refrigeration and storage spaces should be included. The product is usually sold at retail so that a selling department is required. The computation of sizes will require study of the equipment intended to be installed.

Horticulture.—There should be ample storage spaces specially ventilated and darkened for fruits, vegetables, etc. The principal work will be on planting, grafting, budding and trimming of trees, vines and shrubs. There should also be a small laboratory for preparation of sprays, etc., about 20 sq. ft. per student.

Applied Entomology.—For the study of insects, noxious and beneficial to farms and orchards, cattle, etc., and in connection, the art of bee keeping, with outside space for apiaries.

Animal Husbandry.—The work in this course is conducted largely in the barns and fields. Dressing rooms, showers and lockers are necessary, with a number of reading or study rooms and a department library. Records, registers, pedigrees of animals, should be given fireproof space.

Stock Pavilion.—The minimum size of the elliptical arena for a stock pavilion is 175 ft. long by 67 ft. wide. Within this area horses of the various types can be exhibited. Riding, hurdling, etc., can be done. The entrance should be wide enough for wagons. About this arena a concrete amphitheatre of ten rows will seat 2500 people.

The other buildings in this department will be for horses, cattle, sheep and swine. The herds will not be large, but the buildings should follow the best practice as to construction and operation.

12*j*. Military Science and Training.—The buildings for military science and training may be combined where convenient. The drill hall should have an area of about 10,000 sq. ft., as nearly square as convenient. The dimensions of various drill rooms are as follows: 196 × 200 ft., 155 × 280 ft., 175 × 308 ft., 200 × 300 ft. Smaller armories have halls: 90 × 190 ft., 60 × 90 ft., 75 × 105 ft.

At the front or side or under the drill room should be showers, toilets, bowls. One or more rifle ranges are needed; also lecture rooms for instruction of officers and special corps, office rooms for the commandant and staff, and an armourer's work room.

The difficulty of maintaining a floor of large size will be minimised by having no basement under the drill room, and constructing a pavement of earth or asphaltum directly on the ground. The other portion of the building may be two or three stories in height. The great span over the drill room leads to excessive height, but the construction should be kept as low as practicable. Excessive sky lighting is not desirable. A ratio of 1 ft. of skylight to 8 or 10 ft. of floor space is sufficient.

Parade grounds should be as large as practicable up to 20 acres in extent.

12*k*. University Extension.—This department will offer courses to persons at a distance. The requirements comprise a number of working offices each about 14 × 20 ft., with filing spaces for documents and theses, library, and a book-room space, and an assembly room of 200 sittings. The department sends out package libraries, lantern slides, moving picture films, and other educational matter requiring storage space. The post office accommodation will occupy considerable room and mail chutes will be necessary from the upper stories of the building.

12*l*. Student Help and Recreation.—The buildings under this head are the dormitories, union, and commons.

The dormitory consists of a central portion containing the general parlor and visiting rooms, a post office room. The proctor or matron has a suite in the central portion. The remainder of the building contains the dormitory rooms 10 × 14 ft. for single, 12 × 14 ft. for double rooms. For one person in two rooms the bedroom is 7 or 8 × 14 ft. and the study 10 × 14 ft. For two persons in three rooms, another bedroom is added. Each bedroom contains a closet. Toilet and shower rooms are located on each floor. For women a certain number of bath tubs is added. The basement contains rooms for trunks and storage, dining and serving rooms and kitchen.

Dormitory quadrangles at some universities enclose a court accessible only to students. The dormitory units may be small, of about 24 rooms in three stories, or larger containing 50 rooms per story. The larger units are less expensive to build, but the smaller ones offer opportunity for individual donations of reasonable amount.

The commons, where meals are served, may take any convenient form. At the Harvard Memorial the dining room is quite large. In other cases the space is divided into several dining rooms. Cafeterias may be installed at several points, all served from a central kitchen. For dining spaces 15 ft. per person is ample. Serving rooms especially for cafeterias should be long and narrow, open on the front as in public cafeterias. Some room is gained by the use of balconies for dining space. The central kitchen will require space similar to what is common in hotels.

The union or clubhouse contains parlors, social rooms, smoking rooms, game rooms, billiard tables, bowling alleys, committee and society rooms and headquarters. It may have an assembly hall with or without a theater stage. It may contain a trophy room for prizes taken in athletic contests.

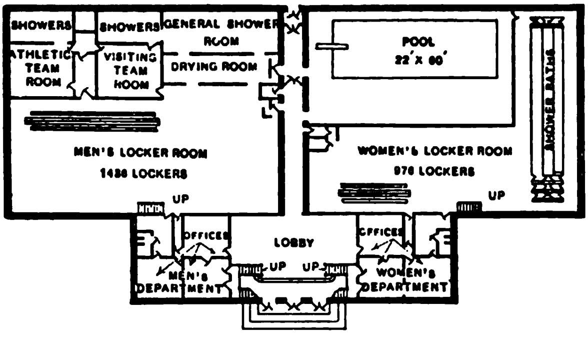
12*m*. Sports and Athletics.—University athletics comes under several heads. Indoor gymnasium class work, Individual work, Corrective work, Games, and Running.

The indoor work is done in the gymnasium and game rooms. Athletic education and development is constantly changing, but the regular equipments and spaces are still maintained in good measure. The minimum floor area for a standard gymnasium is determined by the standard dimensions of a basketball field. These are 90 ft. long between goals by 55 ft. wide. The space on the side lines should be at least 3 ft. and at ends 6 ft. Outside of this area spaces for bleachers are needed. The space per sitting on a bleacher is 20 × 27 in. A gymnasium room should be computed on the basis of 50 sq. ft. per person.

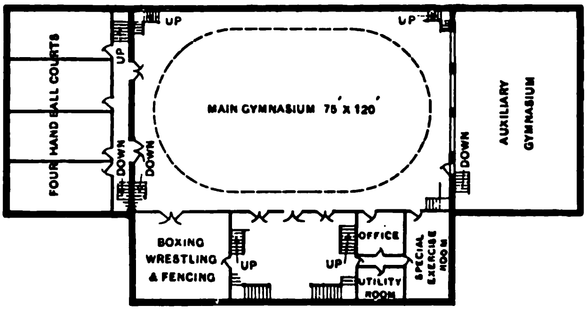
The running track should give 11-ft. head room underneath. The track is 6 ft. wide, circular at the en should be of such length, measured on the line of travel, that a certain number of laps will make a mile. Th is banked sharply around the ends, diminishing as the curve meets the side runs. The usual banking is 1½ ft. high point. Around the edge of the ning track is a railing, the spaces bet posts filled with smooth wire an Care is taken to have no projecting b or points about the railing. In some nasia, a single row of seats is plac the running track balcony inside the of the track, overlooking the bask field.

Gymnasium rooms are from 40 ft. for a small Y. M. C. A. to 60 X as a standard and 75 X 120 ft. for a gymnasium. The story height is fr to 22 ft. The entrances and stairs be at one or preferably both ends. cent to these are the director's offic paratus store rooms, locker and s rooms for visiting teams and stu and toilets for both sexes. Whe gymnasium is used for women as w men, locker and shower rooms mu duplicated. Toilets should be at th of one to twenty students, based c number in any class.

The swimming pool may be u turn by both sexes. The appr should be well separated to avoid c sion. Between the dressing room the pool, the shower rooms will inte Men's shower rooms are quite ope shower heads being along the sides room. Women's showers must be vided with individual stalls with dr stalls 4 X 4 ft. in size. Lockers s not be placed in these stalls but separate room. Where the numb students is quite large a system of wire baskets 12 in. wide, 12 in. high and 15 in. deep to contain gymnasium is economical. In this case lockers for the number of students in the classes at any hour will be sufficie at most double the number so as to permit one class to dress while another is on the floor. The locker wire ba are stored in racks in a basket room with an attendant. The rack system will accommodate three times as students as the individual locker system.



FIRST FLOOR PLAN



SECOND FLOOR PLAN

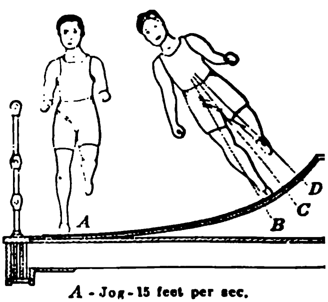
FIGS. 4 and 5.—Suggestion for large college or university gymnasium.

Shower stalls should be enclosed in separate rooms to prevent steam from entering the locker and dressing rooms and swimming pool room. The ventilation of the rooms is difficult so that a blast fan system is desirable. The exit vents should be placed near the ceiling, with other valved openings near the floor. Some form of vent hood having strong suction power should be placed on the exhaust to operate when the fans are not running.

Game rooms 20 X 40 ft. for hand ball, volley balls, squash, etc., must be plain, well ventilated and lighted. The number of these will depend on conditions, but it is well to be conservative about introducing too many.

Corrective gymnastics require moderate sized rooms similar to game rooms.

Stadia and Baseball Bleachers.—The standard dimensions of a football field are 300 ft. long between goals and 160 ft. wide. The running track is outside of this area. The length is 1320 ft. around the track for a quarter mile track measured at one foot from the inside. The width of the track is 20 ft. The straight away leads off from one side. The front rail of the stadium is about 65 ft. from the outside of the running track. In front of the rail is the band platform 64 X 20 ft. and a row of players' s The stadium is constructed of wood, steel or concrete, usually in the form of a horse shoe or open ellipse, to a sun and wind to enter. The dimensions of the seats, etc., are described under grandstands in State Fair P p. 733. At the top of the stadium a space for the reporters' stand is desirable. The entrances and exits of stadium will be placed as most convenient and must be adequate for large gatherings.



A - Jog - 15 feet per sec.
B - ¼ mile - 22 feet per sec.
C - ½ mile - 20 feet per sec.
D - 100 yard - 30 feet per sec.

FIG. 6.—Theoretical angles for a runn radius of 25 feet.

The baseball grandstand is shaped along two sides of a right angle parallel to the ball field and about 50 ft. from it. It may be two stories high. The front is screened with wire netting to prevent accident from stray baseballs. They are constructed of steel, for large stands, and have the usual dimensions per sitting. Chairs are employed to decrease the elevation of the stand which is formed with banks to afford a perfect view of the field from all points. The baseball diamond is 90 × 90 ft. and the playing field 300 × 300 ft.

Field Houses.—Where the grandstand does not give space for dressing rooms, etc., a field house is necessary for the teams. A foot ball eleven or a baseball nine may include an equal number of substitutes so that space for 18 to 22 men on each team should be provided. Dressing rooms, a shower for each four men, two closets, urinals and bowls for each team are adequate. The fixtures should be arranged to drain out in winter. A separate heating apparatus is necessary, where steam cannot be brought from a neighboring plant. An emergency room is required. A women's field house requires individual dressing stalls, shower stalls, etc.

The usual water sports at a university are swimming, canoe paddling, shell racing, skating, ice hockey. For these, a shore bath house and a boat house are necessary.

The bathhouse will cover a good number of dressing stalls 4 ft. wide by 6 ft. long as a maximum, furnished with locked doors opening upon an aisle 5 to 6 ft. wide. A water tap and foot tub in each stall is desirable, and a number of hooks for clothes and towels. Life lines and safe limit marks are necessary to this sport. The boat house, for rowboats and canoes will be arranged in units about 17 ft. wide, with canoe racks 3 ft. 6 in. wide by 2 ft. high on each side of a center aisle 8 ft. wide. Each unit should have a doorway on the center aisle leading to the platform, 10 ft. wide, and an apron extending to the water and furnished with rollers. Between each apron a landing pier 8 ft. wide extends perhaps 60 ft. into the water. A boat keeper's room with a pay counter is required. In some places a sleeping room is necessary. In connection with the boat house a life saving power patrol boat is necessary. It is an error to locate passenger boat landings in close proximity to a boat house or bath house. The congestion due to discharge of passengers and the danger of running down small boats or swimmers is a serious objection to the plan.

Winter sports, such as skating, skate-sailing, ice boating, and games on the ice may be accommodated by the bathhouse building, especially if it can be warmed. For evening skating, electric light poles at reasonable intervals are necessary. The skating areas should be marked with flags or other signs to prevent accidents.

12n. Administration.—The president's suite comprises a general office perhaps 16 × 24 ft., a private office and stenographer's room. The registrar requires a considerable office, 16 × 40 ft., with a counter for ordinary business; a private office for consultation, private stenographer's room, general stenographer's room for about six persons, a record and filing room 10 × 24 ft. or larger, for student records, bulletins, catalogues, etc.

The offices of the deans are usually located in the main building of their college, and consist of a general office perhaps 20 × 24 ft., a private office 14 × 20 ft. and a stenographer's room.

The offices of the business manager and staff will comprise a general office 16 × 24 ft., private office 12 × 16 ft., and stenographer's room 12 × 16 ft., and the regents' or trustees' meeting room 20 × 32 ft., and ante-rooms 14 × 20 ft.

The bursar will require a business office 16 × 40 ft. with counter and private office, accountants' business office of about the same size, with paymaster's counter. The purchasing agent will need about the same space.

Service Building.—The maintenance and repair of buildings and grounds requires a building of about 25,000 sq. ft. of floor space. The building should have a freight elevator.

Central Heating Station.—The central heating station, of four or five thousand horse power capacity, will require about 15,000 sq. ft. of area for boilers, engines, dynamos, etc. A plant of these dimensions must be designed by a heating engineer.

13. Normal Schools.—The typical normal school comprises courses in general education and pedagogy. In connection with this there is required a training school. Certain schools specialize on particular branches of education.

There will be required buildings for

(*a*) General education and pedagogy, including library and assembly hall.
(*b*) Training or practice school including kindergarten.
(*c*) Gymnasium with pool.
(*d*) Central heating station.
(*e*) Dormitories.
(*f*) Buildings for special branches, such as (1) agriculture, (2) manual training and (3) music and art.

The main building will be somewhat similar to a modern high school building of the first class. The training school will be similar to a grade school, with some high school rooms. Beside these there will be a series of rooms to be used as observation rooms by students in pedagogy. These open into class rooms. The gymnasium and heating station, dormitories and other buildings noted will be similar to the same type of buildings at universities, but adapted in capacity to the attendance usual at normal schools.

14. Public Schools.[1]—Public schools in America may be classed as rural schools, grade schools and high schools.

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[1] See also chapter on School Planning."

Rural Schools.—The one-teacher rural school building contains a single class room of standard dimensions 23 × 32 ft. with cloak rooms adjacent. Such a building will accommodate 40 pupils. The window lighting is on one side of the room only. Heating is done by a jacketted stove, connected to a duct which admits fresh warmed air to the building. A vent duct adjacent to the smoke flue carries away the foul air. Provide separate cloak rooms for boys and girls, a fuel closet and book closet. In the best buildings of this class the basement is excavated for a furnace, and inside toilets are provided for both sexes. The remainder of the basement space may be used as play room in severe weather.

The two-teacher room represents the usual limit of the rural school house development. This contains two class rooms and, in the best examples, a library, lunch room, toilets for both sexes, domestic science and manual training rooms. In some examples the two class rooms may be thrown together for special occasions, by means of multiple doors or sliding wood curtains. One and two-teacher school buildings sometimes serve the community for social purposes. Where the school is isolated, so that to go from a boarding place to the school house in winter would be a hardship, two-teacher schools are arranged with an upper story divided into a small apartment to be occupied by the teachers. In other examples a cottage is built near the school house.

These buildings are of frame construction or of brick, hollow tile, or stone masonry according to conditions. The requirements for ventilation, 1200 to 1800 cu. ft. per person per hour measured at the vent duct, and of window lighting (1 ft. of glass to 5 or 6 sq. ft. or floor area), and of exits, and the separation of sexes apply to these buildings. In the case of state aid schools these requirements are imperative.

Grade Schools and High Schools.—The standard primary and grade school building is from two to three stories high and contains six to nine class rooms on each floor for buildings in cities. A gymnasium and assembly hall are usual accessories. Domestic science and manual training rooms are commonly provided, as well as play rooms. Toilets are located in the basement or ground floor. The buildings are frequently symmetrical about an axis with the gymnasium and assembly hall in the rear court. The class rooms are of the standard dimensions, 23 × 32 ft. or affording 16 to 18-ft. area for each person, with a ceiling height of 12 ft. Main corridors are from 10 to 14 ft. wide. Glass areas equal to one-fifth to one-sixth of floor areas are required. Stairways and exits at or near to each end and central stairways in addition are usually provided. The buildings are heated by steam and provided with mechanical ventilation affording from 1200 to 1800 cu. ft. per person per hour. Later buildings of this type are fireproof. Fireproof corridors at least are required in two story buildings in most states. In others the first floor must be fireproof. The roofs are usually of timber construction. Risers in stairs may vary from 6 in. high by 11 in. in grade schools to 7 in. high by 11 in. wide in high schools. Stairs and corridor floors are frequently finished in terrazzo. The same style of floor finish is employed in toilet rooms.

Class rooms commonly have a wood floor finish, maple being preferred, laid upon the concrete floor, and fastened to nailing strips spaced about 16 in. on centers. Such floors may be given a durable finish by a flowing coat of linseed oil with a small amount of turpentine, applied to the wood while at a boiling heat, and the surplus removed after 12 hr. Basement floors are left to show a finish surface of concrete.

The toilet provisions for schools comprise individual closets, one for 15 to 20 female and one for 20 male scholars, with one urinal for 20 males, wash basins, one for 30 scholars, and bubble fountains, two on each floor, with one additional for each 100 scholars. Schools having a gymnasium provide separate toilets and shower bath stalls computed on the number in gymnasium classes.

Ventilation of school buildings may be done by gravity, with window inlets for fresh air; by blast, with fresh air warmed by steam; by recirculation and air washing. The first is the least expensive and, where practicable, fairly satisfactory. The second is the most common in large buildings. The third is the most costly for installation, but most economical of coal and most healthful and agreeable.

The most recent development is the one story school building about a court. Portions of these schools are two stories in height. The different units are connected by covered walkways or enclosed corridors. The plan necessitates considerable areas of ground, but not greatly in excess of the ordinary arrangement.

15. Fair Park Buildings and Grounds.—The design of a fair grounds concerns the management of large gatherings of people and their direction and transportation in considerable masses. The exhibition period is short so that the values must be obtained quickly. Everything that will simplify and facilitate the conduct of the enterprise is important. Among the things to avoid are congestion, discomfort, useless effort on the part of visitors, and needless expense to the exhibitors. Classification of kindred exhibits is desirable and the location of the most popular in a suitable place. A general design should cover all matters of transportation entrance, exit, circulation within the enclosure by walks and drives, architectural treatment, landscape work, exhibit fields, amusement spaces, buildings for administration, exhibits, catering, amusements, public comfort and service. It should be supplemented by an engineering design covering all underground work, surface drainage, lighting, power, fire protection, water supply and waste and sanitation.

Transportation and Entrance.—The entrance should be at the point most easily reached by transportation facilities, street cars, automobiles and the like. There should be a large unloading space capable of holding a number of street cars at once, planned to unload and take on passengers without obliging them to cross tracks or to stand in streets open to traffic. Automobile stands should be separated from street car stations. This class of transportation may properly approach the grounds at a subordinate entrance or at some point as near to the main entrance as convenient. Considerable space should be afforded for discharging passengers. A separate area for parking cars should be provided so that the space about the entrance will not become congested. The entrance for freight trucks and railway cars should be at another point on the grounds. The main entrance should be marked by a structure of more or less spectacular appearance, sufficient to indicate the place of entry and to carry decorations of flags, lights and placards. The actual gateways may extend considerably beyond the space covered by such a structure.

Drives and Walks.—It has been the policy to limit the use of automobiles within the fair enclosures. Drives, bridges and gateways must be designed, however, with reference to supporting the weight of cars and affording adequate room for turning and passing. Wherever possible, steps and sharp inclines in walks must be avoided where large crowds are customary.

The enclosure of the fair grounds should be made sufficiently difficult to prevent climbing.

Building Design.—As a general rule of planning, one story buildings should be considered. A few structures of good height should be included for spectacular effect, but the upper portions have but little value for exhibits.

Public Comfort Stations.—At various points on the grounds public comfort stations should be installed. The first units should be designed so that considerable additions may be made, perhaps to three or four times their original size. Stations intended for both sexes should be given particular attention as to approach. It is hardly practicable to provide the number of units customary in permanent buildings, but at least one toilet to 250 persons should be installed in the locations most commonly congested. This would give service to one person in twenty per hour.

Band Stands.—The ordinary band stand should be about 200 sq. ft. in area for a band of twenty pieces and should be elevated sufficiently above the ground.

Administration Building.—The business of carrying on the fair should be located near to the entrance. The building should be of permanent character and should have fireproof record rooms for documents. Beside a general business office, there should be a committee room of good size, and offices for each department of the exhibition. The building will be used considerably during the year and should be heated, lighted and provided with the regular equipments of an office building.

Service Buildings.—The care of the grounds during the exhibition period and at other times requires a building for the superintendent and his corps of men. It is generally necessary for the superintendent to live on the grounds at least during the summer and in some cases the entire year. The building should provide quarters for a family and a number of dormitories for workmen. The barns should be ample and capable of future expansion. Sheds for mowers, rollers, concrete mixers and garden implements should be conveniently near. A service yard, paved with concrete or macadamized, is desirable.

Greenhouses.—A fully appointed fair grounds would include a series of propagating pits for starting annuals and for protecting ornamental plants in a severe climate.

Crating Yard.—An enclosure for storing crates will save considerable expense to exhibitors and will keep the grounds in good order during the exhibit period. A portion of it at least should be roofed over.

Power Station.—Where electric current for light and power is accessible, as from the power lines of the electric railway, it is usually preferable to buy the current. The fair period is of such short duration that the investment and maintenance of a power station is unwarranted where reasonable rates of purchase can be had. The computation of current required would determine the capacity of a power station in other cases. The building would need to be of permanent materials designed with special reference to keeping the equipment in good condition during the idle period, as well as to providing a reasonable working space during operation.

Race Tracks and Grand Stands.—The vogue of horse racing is not what it has been in the past, the interest in machine racing having taken its place to some extent. In any event a grand stand of large dimensions is usually necessary to fair grounds.

The concrete grand stand, or one constructed of steel, is the only safe structure for the purpose. Temporary grand stands can be maintained for about eight years if constantly inspected and thoroughly repaired. The danger of fire and collapse are always present with a wooden structure, and only the most rigid inspection, renewal and policing will make one measurably safe.

A grand stand of reinforced concrete or of structural steel and concrete involves a large expenditure, but in some cases the ground space underneath can be utilized for exhibits. Upper spaces have no value of this kind. A concrete grand stand costs from $9.50 to $15.00 per seat, in the ordinary case, where the seats are left uncovered. The seats are arranged in steps about 17 in. in height, where the step forms the seat, or from 8 to 14 in. where plank seats are provided, supported on brackets. The latter plan is superior as requiring less total height and being easier of access. The usual width of the steps is 23 to 25 in. In any case a plank seat about 11 in. wide is necessary for comfort. Chair bodies are preferable to planks.

Entrance to the grand stand may be made at several points. A broad walkway is required between the grand stand and the track, from which steps lead to the rows of boxes. Where entrance to the stand is from the front, no other provision is required. Entrance from the back may be made by walkways under the stand extending to the front on the ground level, or by inclines leading to the higher levels and entering the stand through archways.

Restaurant Buildings.—The lunch counter is the normal fair grounds restaurant, compared with which all other types are at a disadvantage. Waiter service is in considerable use, however. The buildings are usually of frame construction and one story in height. The area, outside of the kitchen, will not exceed 15 sq. ft. per person. The kitchen is much reduced in area over the usual restaurant kitchen and will contain the range, vegetable cooker, soup kettles, work table, steam table, refrigerator and store pantry.

Concessionaires Buildings.—These are structures for the sale of small objects. They are generally open on the sides and front, with wooden shutters for closing at night.

Exhibit Buildings.—The principal exhibits at a state fair are: farm machinery, other machinery, processes, automobiles, trucks, tractors, vehicles, fruits, vegetables, grains, dairy products and animals. Galleries and second stories are worthless for exhibit spaces. The ordinary visitor will not go up to a second story at all, and seldom to a gallery. The floors of the buildings are marked off into convenient units called booths with aisles between for visitors. Ample daylight is necessary and electric lighting for evening use. A small business office is provided at some point. Sky lighting is necessary in the usual case. The area of glass surface in these buildings should be not less than 1 ft. to 3 ft. of floor area. Buildings for the exhibit of animals differ from others in that great attention must be paid to sanitation, and there must be provision for feeding, watering and protecting the animals from injury and disease.

16. Expositions.—The designing of world's expositions is affected by the same problems as with state fairs, but on a greatly magnified scale. There is opportunity for architectural effect not possible with the smaller enterprise. Otherwise no essential difference obtains. The same elements go to make up the ultimate result. There is the spectacular field, the exhibit field and the field of amusement. Accessory to these are the fields of states and foreign countries. The same problems of administration, transportation, circulation, public comfort, sustenance, safety and police protection obtain.

17. Park Buildings.—Parks are of two types. The grand park will contain plant houses of large size for palms and other exotics. Beside this there will be the animal, bird and reptile houses, aquarium buildings and outside spaces in connection, completing the zoological garden, a refectory of considerable size, public comfort buildings, boat houses and landings and waiting rooms at transportation terminals. The service buildings will be the central heating station, the administration building, gardeners' cottages, barns, sheds and greenhouses.

The small park will contain buildings for amusements such as a gymnasium with dressing rooms for men and women, dancing rooms, game rooms, a simple theater stage, lecture and reading rooms. Adjacent to it or in connection will be the bath building with showers, indoor swimming pool, open air swimming and wading pools. Playing fields will be provided, baseball and children's playgrounds fitted with swings and other amusement apparatus. Picnic grounds provided with concrete camp fire places are common in the best parks.

18. Theaters and Music Halls.—The theater for the drama and opera consists of an auditorium having a pitched or slanted floor, usually one or more galleries, and a series of private boxes at each side of the proscenium arch. The orchestra pit in front of the stage is depressed sufficiently to avoid blocking the view. The entrance or foyer contains the box office and cloak and toilet rooms for both sexes. The seating capacity varies from 800 in small theaters to 2000 in those of average size and 3300 for large theaters.

The Stage.—The proscenium opening should be of such width as to leave at each side a space on the stage about one-third as wide as the proscenium. The height of the stage to the gridiron should be at least 2 ft. more than twice the height of the proscenium opening. The gridiron or rigging loft consists of a series of beams spaced closely together by which the pieces of scenery may be supported. It should have a walkway and service stair on each side of the stage. The head room above the gridiron should be 7 ft. Under the stage a working space is required not less than 8 ft. high. The floor of the stage is constructed of members parallel to the proscenium so constructed as to permit easy removal or change of parts. In this a regular number of traps are framed out and covered. The trap mechanism resembles a short elevator, counterbalanced and formed with a platform to permit raising or lowering at will. At the back or one side a large doorway is needed to receive scenery and properties. A series of dressing rooms of small size and two large dressing rooms are necessary. The electric switch cabinet is placed at one side of the stage to control the stage and auditorium lights. A large ventilator to carry off smoke and gases in case of fire is now required on all stages.

The Auditorium.—The building codes usually require 36 in. of opening in exits per hundred seats. The exits are required to be distributed at fairly even distances about the auditorium and to be marked by signs, lights, etc. The height of the ground floor above the public streets adjacent is usually not over 3 ft.

Theater seats are regularly 19, 20, 21 and 22 in. wide. Minimum spacing 2½ ft. back to back, and average 2¾ ft. Seating space in theaters is computed at 6 to 8 sq. ft. per person including aisles, with 7 sq. ft. on curves. The ideal width of theaters is about 75 ft., the height 55 to 60 ft. above the stage or 3½ ft. more above the floor level, proscenium width, not over 40 ft., and stage depth not over 60 ft. The pitch of the main floor and balconies is graduated to secure a uniform view of the stage from all points.

Theater Scenery.—A minimum complement of scenes for a very small theater would be, one exterior, one interior, one street scene and one "cut wood scene," all with proper wings and sky borders, one set of "tormenters" or fronts, one drop curtain. These are attached by elevating strips counterbalanced to the gridiron, and operated by ropes. In low stages the scenes must be rolled up from the bottom, which is undesirable. Besides these, other forms called flats are used. In these the scenery is attached to hinged frames.

Moving Picture Theaters.—This type of building differs from the ordinary theater mainly as regards the stage, which may be brought to a minimum practicable depth of perhaps 10 ft. Provision for safety against fire is necessary on account of the inflammable nature of the picture films in use. The shape of the building is controlled primarily by the distance necessary for the best optical effects. The picture booth should be of fireproof materials and should have special ventilation. The exits, seating and other accessories will be the same as for regular theaters.

The Concert stage is usually enclosed with wood panelling for resonance. The organ may be arranged in parts at each side of the proscenium with the movable console on the stage. The chairs for singers are disposed on benches rising consecutively toward the back, sometimes in the arcs of circles. The benches should be about 3 ft. wide to serve for orchestra purposes as well. An orchestra of 60 pieces will require 800 sq. ft. Small orchestras somewhat more per man. A great organ will require from 450 to 900 sq. ft. of area and a height of 36 to 40 ft.

Temporary Stages.—The best form of movable stage is one composed of stout tables firmly bolted to each other. The table tops should be made without overhang and the frames bored for thumb screws with large grips. The units may be 3 × 6 ft. in size for easy handling. The units for the flat portions will have legs of uniform height. The rear sections will be taller to form the stepped areas. A stage of this kind may be made up of different sizes at will. Along the front and about the sides iron stanchions and rails may be clamped for safety and good appearance. The steps should be self-contained, clamped to the stage, and have stout hand rails.

Open Air Theaters.—The Greek theater has been the model in most cases. The theater at Berkeley, California, is typical. In this the seating is of concrete, partly seated with chairs. The capacity will depend partly on the character of the ground, a sloping hill side giving the greatest convenience. The stage and proscenium may be architectural. Other scenery is not commonly used. A simple theater may be designed by accommodating the slope to the line of vision, elevating the seats continuously to give a good view of the stage. The seats may be secured to timbers anchored to the ground. The stage should be of timber work with a wood floor, covered if desired with canvas. The background may be of canvas supported on frames, or of trees and shrubs set thickly together. A railing at the back and sides is necessary for safety. The stage area should be about the same as for a small theater and the proscenium opening will be formed by a frame at each side covered with canvas. This affords support for the stage lighting which will be suspended on wire cables. Simple dressing rooms are required, with canvas divisions. The auditorium will be enclosed with a canvas screen supported on posts.

19. Dance Halls and Academies.—The usual form of dance halls is that of the lecture hall, rather longer than wide. In addition to the dancing floor, retiring rooms, cloak rooms and toilets for both sexes are required and a good sized foyer or gathering room. Over these rooms the visitors' gallery is placed, and in some halls narrow refreshment galleries extend along the sides of the room. The dancing room should be high studded and well ventilated. The musicians' gallery may be at the front, but not too high above the floor. In dancing cafes the refreshment tables are on the dancing level. A dancing academy will require a suite of business offices and special rooms for individual instruction.

20. Military Buildings.—The description of drill halls in Art. **12*j***, will be sufficient for similar buildings in this section. Beside these are the riding school buildings, rather similar in the main, but requiring a dirt or bark floor for horses. In connection there will be the stables, for which see "Animal Husbandry," under Art. **12*i***. Other buildings will be the barracks, officers' quarters, toilet buildings, ammunition buildings, quartermasters' buildings and the post exchange.

The barracks at the cantonments in the United States during 1916–18 were of frame construction, two stories high, resting on a foundation of concrete posts. The space between posts was closed in to the ground with boarding. The typical barracks plan comprised a central hallway with stair, and dormitories at each side, computed on the basis of 85 sq. ft. per man. A sergeants' room for each dormitory room was placed near the entrance. The buildings were heated with jacketted stoves, and lighted by electricity. Some of the barracks at Camp Grant, Illinois, were heated by steam, the mains being carried overhead from a central heating station.

The toilet buildings were located adjacent to the barracks, one for each building, and contained the shower rooms with heaters, closets, urinals and washing troughs. The heating and lighting apparatus was similar to the barracks equipment. The floor was of concrete, carried up two to three feet on the side walls. Barracks and toilets were boarded on the outside, lined with building paper and ceiled inside with boarding three feet high and with "compo" board or heavy pasteboard above. The construction was extremely light. Roof ventilators were provided on the buildings. Windows and doors were of stock form.

Buildings for naval reserve cantonments were similar, but arranged in groups in some instances. These barracks were disposed about a square. One unit of nine buildings was adjacent to a double mess hall. The buildings contained 112 men each; the mess halls 500 men each. Two toilet and shower buildings served the group. Separate units were provided for probationers. There were ten officers' barracks with separate toilet and shower buildings. The barracks were 161 ft. long by 25 ft. wide. The hospital group contained four wards with four toilet buildings, a hospital corps dormitory, officers' quarters, nurses' quarters. The other buildings were the administration building, army library, camp theater for 2700 men, the commissary, Y. M. C. A. and K. C. near the entrance of the grounds.

21. Public Comfort Stations.[1]—The public comfort station for both sexes requires segregation. A common waiting room would be feasible under the best circumstances, otherwise not.

[1] See also chapter on "Public Comfort Stations."

The station will be composed of an ante-room, sometimes with two types of accommodation common and first class. There would be no difference in the fixtures. Compartments should be lined with marble or other enduring material. In the women's side a table for dressing children is needed. The building may preferably be above ground, but in cities basements or other underground spaces are most available. The computation of fixtures required will depend upon custom. A reasonable computation may be based on the number of persons one fixture will serve. Taking 4½ min. as the average time of occupancy for fixtures of all sorts one fixture will serve 13⅓ persons per hour. An equipment of four closets for women, two closets and two urinals for men would serve 107 persons per hour. The addition of two urinals would give an increased capacity of 40 persons per hour.

22. Tombs, Memorials, and Halls of Fame.—Memorials are of two principal types. The first is purely sculptural or mortuary. The mortuary crypts will be similar to those of public mausoleum. The second intended primarily as a memorial, partakes of secondary characteristics such as a museum, art gallery or chapel. All such buildings should have some feature to indicate the idea of a memorial. A bronze tablet may hardly meet the requirement. In some examples the foyer or some central room is made to give expression to the memorial idea. In this a statue or portrait may be placed. The design and detail of the memorial portion should be carried out in materials of permanent character and excellent appearance, and to a considerable extent constitute a chief attraction of the building. The remaining portions should be well done and of enduring materials, rather than to be so large as to necessitate cheap expedients. The hall of fame has a certain resemblance to a museum of sculpture. The central portion is designed partly for architectural effect. It will contain statues of celebrated men to whom particular honor is intended. The subordinate parts of the building will give space for portrait busts of men of various degrees of distinction. The Pan American Building at Washington partakes to some extent of the nature of a hall of fame.

23. Civic Centers.—The community building is an important element of a small town or of a neighborhood in a city. It partakes of the character of a club house, but the uses are somewhat different. No living quarters are required except for the caretakers. Rather large banquets and other social functions will be served but the kitchen provision may be simple if sufficiently spacious. Game rooms and especially bowling alleys are desirable. The principal room, frequently on the second story, will be used for lectures, dances, mass meetings and on occasion for religious services. There should be toilet and retiring rooms for both sexes. The first story will contain the offices and social rooms, billiard room, magazine room, etc. In smaller examples the street front is occupied by small stores for cigars, soda and mineral waters, or a women's exchange. The advantage of this arrangement is that the burden of carrying on the building is lessened and convenience is served at the same time. The entire first story should not be so occupied, but only a small area on each side of the front entrance.

24. Buildings for Sepulchres.—The public mausoleum in which compartments are sold, consists of a central mass of reinforced concrete, formed into cells or crypts 2½ x 2½ x 7 ft. with walls about 4 in. thick, arranged in 4 or 5 tiers. The smaller buildings of about 60 crypts comprise a central hall of good height, in which burial services may be held, with crypts in wings at each side, arranged along a corridor 8 to 10 ft. wide. Special crypts or rooms containing crypts are placed in the main portion. The crypts are closed upon occupation, with a 3 in. slab of concrete grouted into place. The crypt is provided with a lead drainage tube and ventilating tube leading to a central receptacle containing a powerful disinfectant. From there the ventilating pipe extends to the outside. The building is composed of masonry faced usually with cut stone. The interior is lined with marble on walls and floors. The ceilings are of plaster or other decorative material. Doors and window sash are of bronze. The intention of these buildings is to conserve the remains placed in them for a long time. To do this, the building itself must be of enduring materials. Everything of an ephemeral nature should be avoided and precaution taken against the effects of time and the elements, especially rain and frost. The buildings are lighted by windows in the ends of the corridors. Roof lights or transoms in the roof are sources of water leaks. The buildings are warmed by hot air furnaces if at all.

A receiving vault with metal supports for caskets may be connected to these buildings, in a compartment with a separate entrance. A crematory with furnaces of special design is provided in some cases.

Similar provisions as to the construction of individual mausoleums are necessary whether the structure be simple or elaborate. The tendency to collect moisture and to create water pockets which cause damage by freezing is the most frequent source of decay of these buildings.

25. Churches.—Church buildings in America fall into two classes, those for services which require an altar and a liturgy, and those that do not. In this respect the Roman, Greek, Lutheran and Episcopal church buildings are more or less similar. In the same way all other church buildings are somewhat alike, one to another. The service of the altar, the processionals and other functions hold the seating in straight lines and to a long and comparatively narrow building with a level floor.

The Roman Catholic Church.—Buildings of this type owe their form to the buildings of the early Christian Church, which were based on the scholæ or halls common in the cities of the Roman Empire. These were of rectangular form, narrower than long, with semicircular apse, or chancel, at the end opposite the entrance. In the Roman Church the altar stands free from the wall of the chancel affording a passage or ambulatory behind.

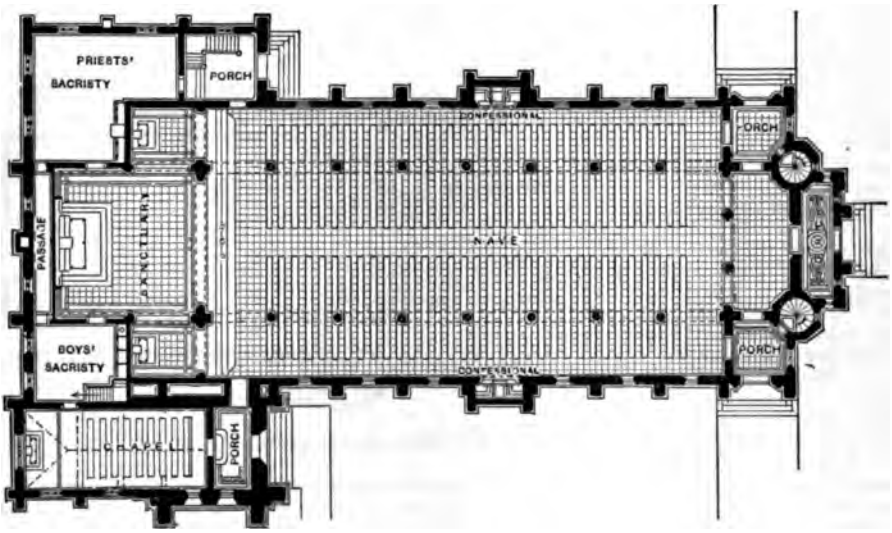


Fig. 7.—Typical plan of Roman Catholic church.

The chancel is raised above the floor of the church and is considerably elaborated according to the size and importance of the church. The main portion of the building is called the nave. The roof of this portion is supported on columns. The spaces between them and the side walls are called the aisles. The walls of the nave are higher than of the aisles, giving a clerestory, the windows of which light the central portion. At each side of the chancel arch are the low altars. The end containing the chancel is known as the east end, without regard to the actual points of the compass. The entrance, at the west end, admits to the vestibule, or narthex from which stairs lead to the gallery overhead. This gallery contains the organ and choir and, in some churches, a number of sittings. The font is placed either in the vestibule or the nave or in a bapistry on the north side. Along the sides of the church at regular intervals are the stations of the cross, more or less elaborated, and near to the front the confessionals. The chancel is provided with one or more sacristies, 8 × 10 ft. as a minimum, usually two, beside a choir sacristy and other necessary rooms. The building may have transepts or wings adjacent to the chancel wall. They are not so common in the Roman Church as in the English type. The basement may be used for a parish room, Sunday school, and other activities. In the usual examples the tower is centrally located, over the entrance, but duplicate towers, after the cathedral arrangement are common. The arrangement of pulpit, lectern and other accessories should be carefully studied to conform to the usage of the church. Adjacent to the nave and extending by the chancel may be one or more chapels. The church building, parish house and rectory complete the church plant to which may be added the parochial school.

The Lutheran Church follows the tradition of the Roman as to the main plan of the building. The altar is retained, but the arrangement of the chancel is somewhat modified.

The Protestant Episcopal Church follows the English tradition and use. The nave, aisles and vestibules are similar to the Roman type. Transepts are more common and larger. The chancel is set farther back, the choir intervening between it and the nave. The chancel may be octagonal, though of recent years, rectangular chancels have come into use. The altar is placed against the wall, with a dossal or reredos and a foot pace like the Roman altar. The chancel rail separates this portion from the choir, which is again railed off from the nave. The choir benches face to the center line of the church leaving a broad space in front of the altar. The choir is raised above the nave by one to six steps, as required. The organ is located on one or both sides of the choir with the console facing the center. The lectern on the south and the pulpit on the north are placed at the railing of the choir. In some examples the nave is separated by a rood screen at the choir front, or a single rood beam indicates the separation. The sacristy and other adjacent rooms are similar to the Roman type. The font is similarly placed. The morning chapel at one side contains a small altar and seats for forty or more people. The parish house, common to both Roman and Episcopal churches is used for the various guilds of the church, and contains an assembly room, kitchen, choir practice room, choir vesting rooms, etc.

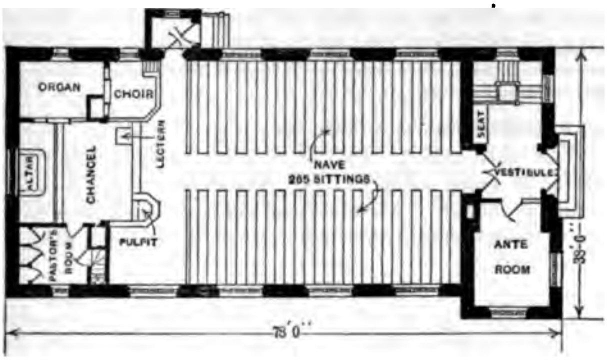


Fig. 8.—St. Mark's English Lutheran Church, Roxbury, Mass.

The Protestant Churches not using a liturgy have adopted a different form of building in many examples. The nave, aisles and chancel are replaced by a broad auditorium, with or without a gallery, facing a raised platform with the pulpit and the seats for the clergy. Back of this is the organ and choir gallery occupying the place of the chancel in the liturgical churches. The main floor is usually slanted toward the front. Immediately in front of the platform is the communion table. Perfect vision and hearing are required and, for this, all columns and other obstacles have been eliminated from the body of the church except for the gallery supports. This involves the use of wide spans of roof carried by a more or less complicated system of trusses. The other notable development of these churches is the Sunday School building at one side or the rear of the church. This is arranged to be opened into the church by sliding partitions on occasion. The Sunday school room is planned on circular lines, with class room alcoves around. The basement is divided into parlors, kitchen and rooms for various activities. In the completed plant a parish house and rectory are included.

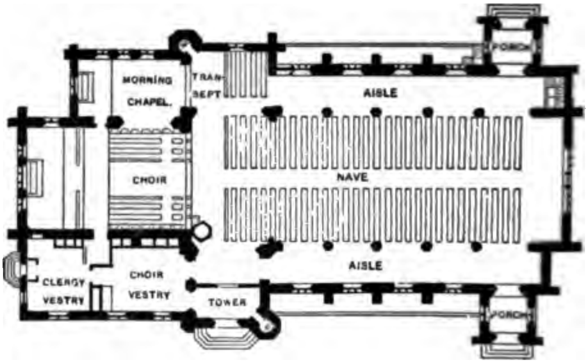


Fig. 9.—Protestant Episcopal Christ Church, New Haven, Conn.

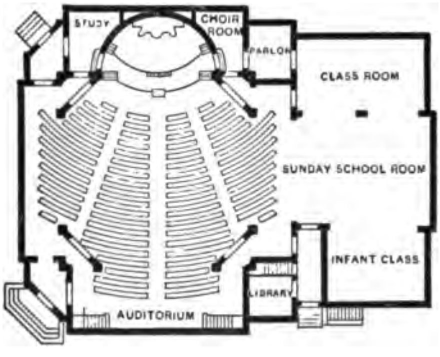


Fig. 10.—"Other Protestant Churches," West Presbyterian Church, Binghampton, N. Y.

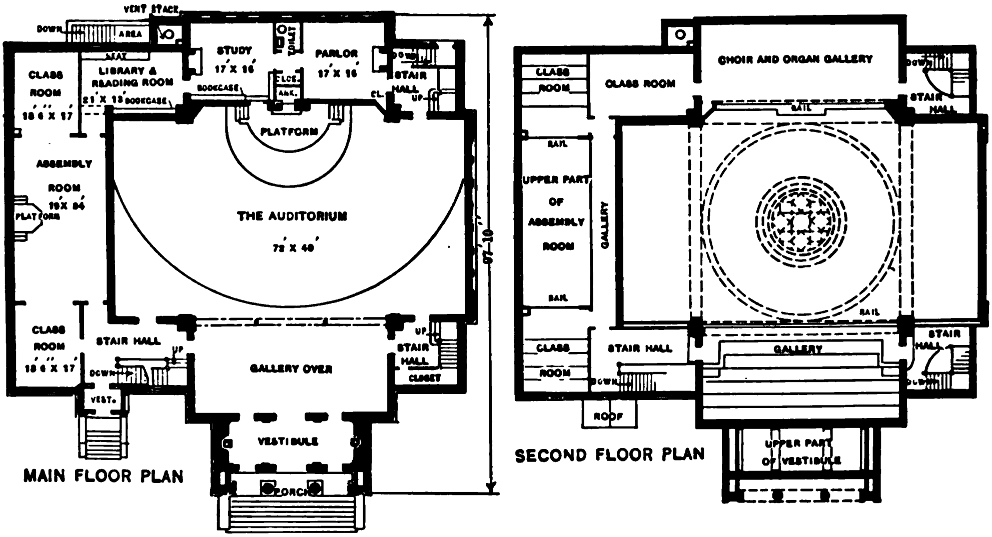
The Baptist Church building is similar to the above except that a baptismal pool is required. This is of good size, perhaps 100 sq. ft in area, and of convenient depth. Provision for warming the water is necessary. The pool is closed off or covered over except as needed.

The Unitarian Church plan is that of an auditorium with a platform in front and a choir gallery at the back or on one side. Committee rooms and social rooms are required.

The Christian Science temple is similar in plan. The equipment of reading rooms, study rooms, etc , is larger than for other buildings of this class.

The Synagogue plan is that of a square covered by a flat dome. At the center of the east side is the altar platform and in the orthodox synagogue the recess for the ark of the covenant. The main entrance and vestibules will be on the west. The reader's desk is on the main floor, quite advanced from the altar precinct. At one side of the platform is the private room of the rabbi, 14 × 14 ft. and a similar room for the reader on the other. A chapel 14 × 18 ft. to 16 × 25 ft. may be located at one side of the front. School rooms 16 × 25 ft. may be at one side or in the basement. Beside these are the library, 14 × 25 ft., assembly and parlor, 24 × 35 ft. In the orthodox synagogue no organ or separate choir are employed. The architectural design follows the Byzantine, affected by the Saracenic, and the decoration will employ Hebrew symbols, the seven branched candlestick and six pointed star and the geometric designs growing out of it.

Beside the orthodox, there are the conservative and the modern or reformed synagogues, in which the ancient practice and liturgy is somewhat modified. In these buildings the reader's desk is placed on the altar platform. The pipe organ and choir are employed, in a gallery on the east side. The altar platform is considerably enlarged to admit of the more elaborate service. Some of the modern synagogues contain large upper galleries so that the total capacity may exceed the ordinary audience. In these buildings, very complete cloak rooms, etc., are introduced. The style of architecture is considerably modified, tending to the Classic, but the central dome is contained for practical and aesthetic reasons.



Figs. 11 and 12.—Floor plans of The Temple (Synagogue), St. Paul, Minn.

The *Cathedral* as related to the church is the official place of service of the Bishop. Of large size and noble appearance, it has nothing of difference from other church buildings other than in size. The basement or crypt may contain special chapels. There is sometimes a church school or college in connection, which will not differ greatly from other schools. Notable examples of cathedrals are in New York, Baltimore and other large cities.

Student Chapels in theological seminaries are sometimes seated in lines parallel to the main axis of the building. The building is in this case an enlarged choir with the chancel at the end.

26. Detention Buildings.

26a. The Lockup.—The lockup is intended for temporary detention of persons accused of minor offenses or crime. It is used also for shelter of vagrants and other persons in severe weather. The laws of the different states vary in accordance with conditions, as whether there be a large colored population. In the usual case the building is required to contain two rooms so that the sexes may be segregated. Minimum dimensions are 22 × 40 × 10 ft. The women's room is furnished with a cot; the men's room with standard steel cells, 5 × 7 × 7 to 8 ft. in dimensions, provided with a cot or plank bed. A typical plan with four cells is here shown.

The building is of masonry or concrete, and is equipped with light, preferably electric, and with prison closets. A stove is used for heating. Detention rooms in a court house or other building may be constructed adjacent to a main exit, but not in a basement below ground.

26b. Police Stations.—The police station is a development to answer the requirements of a town or city. The detention portion is enlarged to contain a number of cells and an office portion for police and other officials. In no case should a police station be located in the basement of a building. The plan of a police station includes a cell room for men, one or more detention rooms for women and for juvenile offenders, and a room for vagrants and persons seeking shelter in severe weather. All these rooms should be on the first floor and as near the street level as possible. Two or more stories of cells and all expedients involving the movement of persons up or down stairs are impracticable.

Cell Room.—Cells must be 5 × 7 ft. size, with prison closets, and may have washbowls with bubble fountains combined.

Detention rooms for women are similar to cell rooms. Separate rooms of not less than 80 sq. ft. area are desirable, with prison closets, wash bowls and bubble fountains and cots. Each room should be ventilated by a separate duct.

Juvenile Rooms.—The detention of juveniles requires rooms like those for women.

Tramp Rooms.—The room for vagrants and persons seeking shelter require a prison closet, wash bowl and bubble fountain. Sleeping platforms made of smooth wood resting on heavy cleats about 6 inches high should be provided. The room should be above ground, well ventilated, heated and lighted. Shower baths may be added.

The office portion of the police station will contain the muster room, captain's office, clerk's office, a fireproof vault for storage of records, a large sitting room. In the second story, offices for the sergeants, roundsmen and detectives and the section or dormitory rooms for policemen, with toilets and showers.

At one side, on the ground level, will be the patrol barn with stalls for horses, harness rooms, grain and hay storage, or a garage equipment where motor vehicles are used.

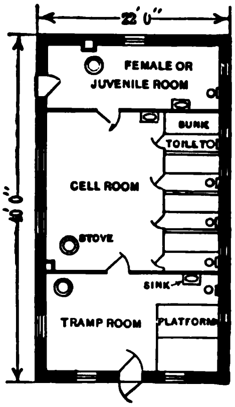


Fig. 13.—Typical lockup.

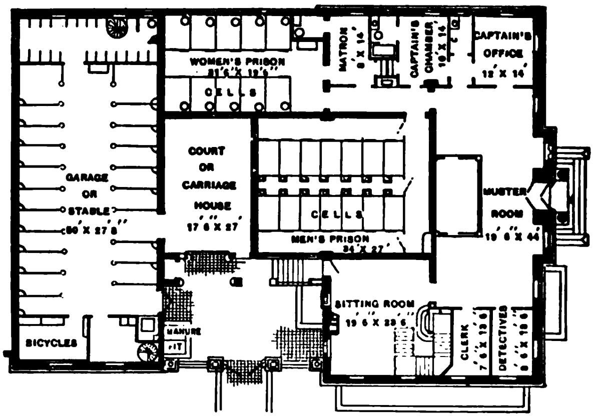

WOMEN'S PRISON 21' 6" X 19' 6"
CELLS
MATRON 8' X 14'
CAPTAIN'S CHAMBER 16' X 14'
CAPTAIN'S OFFICE 12' X 14'
GARAGE OR STABLE 50' X 27' 8"
COURT OR CARRIAGE HOUSE 17' 6" X 27'
CELLS
MEN'S PRISON 34' X 27'
MUSTER ROOM 19' 6" X 44'
SITTING ROOM 19' 6" X 23' 6"
CLERK 7' 6" X 13' 6"
DETECTIVES 8' 6" X 13' 6"
BICYCLES
MANURE PIT

Fig. 14.—Typical police station.

26c. Jails.—This class of buildings contemplates the continued detention of the inmates, and requires a complete equipment for cooking and serving meals. The cells must be arranged with bunks. Sick wards or hospital cells are necessary. Opportunity for bathing should be provided, preferably by shower baths. The requirements for protection, security, segregation, accessibility and sanitation as for police stations, are imperative. There should be ample sunlight in every part.

Witness Rooms.—It may be necessary to detain witnesses for a time, and the jail serves as the most convenient place. Special rooms for such detention, 8 × 10 ft. in size with good windows toilet and wash bowl and vent flues are required. While these rooms need not be cells, they should be secure. Meals will be served from the jail kitchen.

Jailer's Residence.—The jail plant includes a residence for the official in charge, separated from other portions by standard fire doors and standard fire walls.

26d. Workhouses.—These institutions are intermediate between the jail and the penitentiary. The workhouse in a city location must resemble the jail in point of security against escape. The interior arrangement will be like that of an industrial school, with work buildings located in an enclosed space protected by walls or fences as circumstances demand. Separation of sexes, protection against fire, proper sanitary equipment, heat, ventilaton, etc., are imperative. For dormitories and sleeping rooms, the required areas per person are, for one

80 sq. ft., for two 120 sq. ft., for three 160 sq. ft., and for four or more 45 sq. ft. for each person. For dining room 15 sq. ft. per person are required. Exercise rooms are required equal to the dining room in area. Assembly rooms should have 6 sq. ft. per person. School rooms for the primary education of illiterates are necessary; also private quarters for officials include dining rooms, reading rooms and dormitories.

Where located in the country the description of industrial schools will apply in general for the workhouse.

26e. Industrial Schools.—Institutions of this class are most advantageously located away from cities where a considerable area of ground can be obtained. In this case the items of accessibility from town and provision for adequate water, sewer, light, heat and power must be kept in mind (see "Institutions Isolated from Town and Cities," Art. 29). The tendency is to divide the inmates into groups, housed in cottages, grouped around central buildings containing the dining room, kitchen, assembly hall, etc. In some of these institutions a walled enclosure is necessary. Open dormitories are suitable for younger inmates. Quarters for attendants and hospital spaces are necessary. The directors of the institution and certain other officials should have separate cottages for residence. In so far as buildings of considerable size are built, they should be of fireproof materials with a minimum of woodwork. One-story cottages may be of less substantial character.

26f. Industrial Homes for Women.—Detention institutions for this class of offenders resemble workhouses for men. They will require somewhat different buildings. There will be the administration building, reception building, maternity building and hospital, cottages, refectory and assembly hall, industrial buildings, superintendent's residence, employees' cottages and central heating plant.

The administration building will contain offices for the superintendent, accountant, and other business employees, parlors and visiting rooms, a committee or board meeting room, ante rooms to the same.

Receiving Building.—This building should contain, record rooms, 16 × 24 ft.; medical examination rooms, 10 × 14 ft.; detention rooms for individuals, 10 × 14 ft.; bathing and toilet rooms; kitchen or serving room, 12 × 18 ft.; and matron's suite. The building will require barred windows and locked doors.

The maternity building though small will be like other maternity hospitals.

The cottages should be not over two stories high, for groups of not more than 30 persons in single or double rooms. Provisions against escape are generally necessary on windows and doors.

Industrial Building.—While a number of the inmates may be engaged in housework or the kitchen, a working building may be desirable in large institutions. The principal industries would be sewing, preserving, drying and other light work.

The refectory and assembly hall will contain the kitchen and storage rooms, etc. Its size will be controlled by the expected occupation on the basis of 20 ft. per person in the dining room. The kitchen and dining room should be wholly above ground. The assembly hall will require at least 8 sq. ft. per person.

The Superintendent's Residence.—The house should be isolated from the other buildings and have its own enclosure so that the family will not be intruded upon by the inmates. It should have about eight rooms.

The employees' cottages will be smaller, five or six rooms being sufficient, each with its own enclosure, or the buildings may be in a group enclosure outside the area accessible to inmates.

Central Heating Plant.—The necessities for the production of heat, light and power will determine the size and location of the plant. In severe climates the use of exhaust steam for heating has resulted in great economies. Ample coal storage space is imperative. The building should be capable of enlargement without difficulty both as to heating and power equipment.

Minor Buildings.—Small dairy barns, sheds, silos, swine pens and poultry houses are needed in the ordinary case.

Enclosures.—Some institutions have no enclosing fences. While this may be practicable in certain locations, a low wall or a fence that cannot be scaled is preferable for many reasons aside from prevention of escape.

26g. Reformatories and Penitentiaries.—No essential difference obtains as between these types of institutions. There will be an administration building, cell buildings, dining and kitchen building, central heating and power station, school, various shops, store houses, barns, a hospital and a women's building. The buildings will be surrounded by a wall from 15 to 35 ft. high, having a main entrance with guard houses; gates for wagons and railway cars. All buildings will be fireproof. For an institution of this kind a plot of ground 1000 ft. square will suffice, although larger areas are not unusual. A portion of the ground is used for gardens, etc.

The administration building will contain the offices of the warden, receiving and recording rooms and other business offices, committee and board rooms, officers' dining rooms, living rooms for minor officials, barber shop and bath rooms, school rooms and an auditorium or assembly hall sufficient for the entire number of inmates at 8 sq. ft. per inmate in large rooms.

Cell blocks are composed of individual cells of standard size, 5 × 7 × 7 ft. high, arranged in three or four stories, constructed of reinforced concrete or of brick with concrete floors. The block is double faced, with a utility corridor about 3½ ft. wide between. About the cell block on both sides and ends there will be a corridor about 14 ft. wide. A basement for pipes will extend over the whole area. The upper tiers of cells will be reached by iron stairs leading to balconies along the fronts. Stairs and balconies are of iron work or concrete, or may be paved with terrasso. The ceiling and roof over the building will be of concrete. The masonry walls, about 3 ft. thick, will contain large windows extending from about 5 ft. above the floor to the top of the upper cell openings, or sufficiently to give excellent light to all parts. The window sash are opened by multiple operators. The steel cell fronts are held in place by bolts extending through to the utility corridor. The locking device is such that all cells in a tier may be locked by throwing a lever at the end of the block. At the same time any cell may be separately locked or unlocked. Each cell contains a prison water closet, combined wash bowl and bubble fountain, electric light and folding iron cot with mattress. The lighting service will be switched so that the entire control, divided into several sections, for the cells, corridors, etc., will be on the main floor. The system of water supply and waste, ventilation and lighting will be exposed in the utility corridor. Blast and exhaust fans are required for ventilation. The heating by fresh air is supplemented by direct radiation. Each cell has a separate vent. In some cell buildings the masonry is plastered; in others, faced with pressed brick. The exit from the cell room will be at the grill leading to the corridor between cell buildings. An emergency door is placed on one side of the wing.

Disciplinary Cells.—Provision should be made for disciplinary confinement either in a small wing or separate building. The detail will be the same as in the regular cell house.

Hospital Cells.—The prison hospital differs from the ordinary only in the use of the "cell front" on the hospital rooms. Examination rooms, a dispensary and dentists' office are required. There should be a number of cells for prisoners suspected of insanity. A sun porch for tubercular patients should be of iron and glass. The work must be equal to the regular cell in security.

While the standard cell house is employed in most prisons, it is not universal. The cells of the prison at Guelph, Canada, are arranged along the outside walls with a central corridor. At Joliet, Illinois, the cell house is circular with cells along the outside. A central watch tower enables a guard to look directly into each cell, which may be closed on the front by steel and glass to secure privacy to the prisoner from all persons but the guard.

The Dining Hall.—A large hall connected with the kitchen. The tables are arranged in rows, the prisoners facing forward. About 15 sq. ft. per man is allowed including aisles. In some institutions tables are set in the ordinary way with men all around, allowing 20 sq. ft. per man. The halls accommodate 800 to 1000 persons and are without posts. A music platform is a feature of some dining halls.

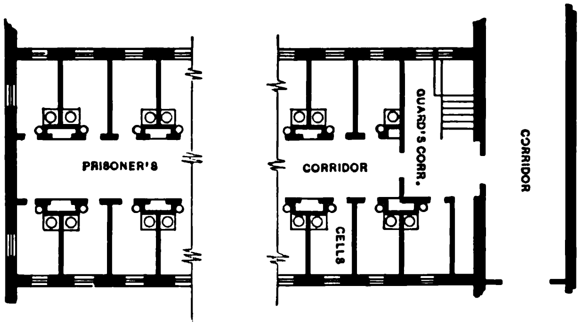


FIG. 15.—Typical cell block.

Kitchen and pantry arrangements are similar to what is usual in hotels. Storage spaces for meats, milk, etc., are provided with artificial refrigeration.

The heating and power station will be furnished with equipment adequate for spaces to be heated, and the lighting and power required for the institution. The heating will be done by exhaust steam in part. The power equipment will depend upon the size of the shops and the demands for power to open and close gates, move cars, etc., on the grounds. A chimney of a capacity considerably in excess of the boiler power first installed should be erected and the power house and coal storage arranged to permit future extensions without disturbance to previous equipment.

The number of immigrants from other countries, as well as native illiteracy, makes a school necessary, especially in reformatories for young men. The school will be for instruction in reading, writing, English language and arithmetic. Standard class rooms about 23 × 32 ft. with full lighting, ventilation and regular equipment are required. The furniture should be adapted to the use of grown men.

Barns, shops and storehouses should be designed on modern lines.

The women's prison or ward is composed of separate rooms, about 8 × 10 ft. with doors of metal, barred on upper portions, and windows in outside walls. The plumbing and ventilation will be similar to what is installed in the ordinary cell buildings. The rooms will be furnished with beds. A separate kitchen, dining room and storage pantry with refrigeration is necessary and hospital cells isolated and sound proof, a physician's office, a small dispensary, social rooms, a visitors' reception room and small visiting rooms. Also a suite for the matron and staff.

Prison Walls.—The enclosing walls of a prison are of masonry or concrete, from 15 to 35 ft. high. The most common height is 22 ft. No wall will prevent escape unless guarded, so that excessive height is quite useless. A number of wall heights are as follows:

Thomaston, Maine; Alcatras, Calif. (U. S.)	15 ft.
Elmira, N. Y.; Windsor, Vt.; Boise, Idaho; Ionia, Mich.; McAlester, Okla.	16 ft.
San Quentin, Calif.; Rawlins, Wyo.	17 ft.
Granite, Okla.; Sante Fe, N. M.; Weathersfield, Conn.; Salem, Oregon	18 ft.
Sioux Falls, S. D.; Deer Lodge, Mont.; Folsom, Calif.; Salt Lake City, Utah; Trenton, N. J.	20 ft.
Ossining, N. Y.	21 ft.
Concord, Mass.; Hutchinson, Kan.; Charleston, Mass.; Jackson, Mich.; St. Cloud, Minn.; Waupun, Green Bay, Wis.	22 ft.
Philadelphia, Pa.; Jeffersonville, Ind.	35 ft.

The desirable features of a first-rate wall are depth in the ground, not less than 6 ft., smoothness and the absence of projecting parts, or buttresses. Nothing should be attached to the walls, such as lighting fixtures, wires, etc., which would serve as holding places for a rope by which a prisoner might attempt escape. The top should be rounded. In some examples, the top is formed with a projecting roll on the inside. In the design of such walls, wind pressure must be taken into account. A wall 22 ft. high will need to be about 3 ft. thick at the bottom and 1½ ft. thick at the top, in an exposed location, to resist overturning under the force of a heavy wind. The prison at Rahway, N. J., has a reinforced concrete wall, quite thin, with buttresses on the outside.

Guardhouses.—These may be of steel and concrete, or of timber work and should be large enough to shelter the guard in severe weather. The windows should extend to the floor. From the guard house a walk, 2 ft. wide, to about 30 ft. in each direction is desirable. The walk may be on top the wall or along the outside, with a railing for safety. The guard house requires a stove or other heater and a toilet. The approach to the guardhouse should be from the outside of the prison yard or by a steel door on the inside. From this a ladder or spiral stair leads to the top.

Wagon Gates.—The gates from the prison yard will be double. The first gate opens into a walled enclosure to contain a wagon and team and the second to the outside. They may be formed to swing, slide or lift. The gate should be the full height of the wall or the wall should be carried over, as high as at other points. The gates should be smooth, formed with solid surfaces without gratings or catch points for climbing upon, and strong enough to resist forcing.

Railway enclosures will be of sufficient size to contain three or four railway cars. The rules of the railway companies as to clearance will determine the width. The clear height of these gates does not usually conform to the 26 or 28 ft. of head room demanded by the railway company, but so far as practicable should do so. The size makes the gates difficult to operate by hand. A system of gears and cranks will diminish the difficulty but power is desirable. The custom of delivering cars only into the gate enclosure makes a yard engine or a cable hauling system necessary for moving cars to the heating plant, storehouse and shops.

Yard Lighting.—The enclosing walls are usually illuminated at night. The best form of yard lighting is by flood lighting or by lamp posts set 10 to 12 ft. from the walls and furnished with reflectors to throw the light upon it. The wiring should be underground and the control switches located conveniently to the official in charge of lighting. Other parts of the prison yards, all walks, drives, entrances, etc., may be lighted in the same way. In some places lights may be attached to buildings. The approaches and the front portions of the prison grounds should be lighted adequately for good appearance.

Water Supply and Sanitation.—This type of institution is usually located away from large towns and public systems of water supply and waste, electric current supply so that these utilities must be provided independently.

Prison Camps.—It is the practice to send prisoners from penietntiaries to places within the state to be employed in grading, ditching and farming. The buildings required for this are: a headquarters building 20 × 24 ft. for the guards and superintendents, a bunk house with 85 sq. ft. per person, refectory and store house. The buildings will be of frame, very simple in construction. A camp on a prison farm would be more permanent and better constructed. Most of the work of construction would be done by the prisoners who may be quartered in tents for a time.

26h. Insane Asylums and Homes for Feeble-minded and Epileptics.—In the older institutions of the United States the various classes of patients are placed in one large building. This is objectionable from many standpoints. Separate cottages are superior to large buildings. Greater attention to fire prevention and provision against accident is necessary than with institutions sheltering persons of normal mentality. No buildings of inflammable nature should be occupied by insane persons even in small groups. Where both sexes are admitted, segregation must be carried to completion. Persons of defective mentality and all who are afflicted with insanity require hospital conditions in the buildings they occupy. The portions of these buildings devoted to violent wards require protection about windows and doors, stairways, etc.

The list of principal buildings for industrial homes will apply to these institutions. The ordinary cottages, so called, will be as follows:

Class 1: For persons slightly affected; for voluntary patients.
Class 2: For severe cases; for cripples and bed-ridden.

Class 1.—Furnished with day rooms, for the entire group on each floor, dormitory rooms, single or multiple, linen and supply closets, attendants' rooms, toilets and bath. Voluntary patients are housed separately from others.

Class 2.—Similar to Class 1, but having a dining room and kitchen, diet kitchen. Latrines are substituted for ordinary closets. Cripples and bed-ridden patients are housed separately from severe cases.

Farm Colonies.—Certain groups of feeble-minded and epileptics are capable of working and may be formed into farm colonies. The colonies should be close to the main institution so that medical supervision is not lost sight of by reason of the inconvenience to the attending physicians.

Separate houses for the director and certain officials are necessary. An insane asylum or feeble-minded home is an undesirable place to bring up a family of children.

27. Charitable Purpose Buildings.

27a. Homes for Dependent Children.—Inmates of this type will include infants, children and youths. The normal children are quite commonly adopted into families, and defectives as they approach maturity are placed in institutions for epileptics, feeble-minded, tubercular or insane. The inmates are formed into small groups according to their degree of mentality; segregation is necessary. Primary education is afforded for those able to learn. The work of the hospital is to secure nutrition and growth, and to cure such defects as club foot, spinal deformity, tuberculous joints and the like. Hospital conditions are necessary, and the same types of buildings, on a smaller scale, as for other custodial institutions.

27b. Poorhouses, Homes for the Aged and Infirm.—In the first of these institutions a certain number of inmates will be of defective mentality. For them a separate building should be provided where custodial care may be maintained. The other buildings will be similar to family hotels with single and double rooms, social and dining rooms, etc. An assembly room is provided for amusements and for religious services, where a separate chapel is not built. The cottage system is most advantageous for these institutions, with an administration building containing the offices and other public rooms, dining rooms, etc. The cottages may contain 40 rooms as a maximum. Aged couples capable of maintaining good conditions may be assigned rooms together. Otherwise sex separation is practiced.

27c. Veterans' Homes.—This type of institution follows the general scheme of homes for the aged and infirm. The desirable arrangement would comprise an administration building, central heating and power plant, large and small cottages. The small cottages will be occupied by married couples and persons desiring to be independent. The larger will accommodate such as require continuous care.

27d. Schools for the Deaf and Blind.—This form of education requires intimate personal instruction and care. The institutions provide housing, hospital care and recreation facilities, as well as teaching, and are commonly under boards of control or charities. The buildings will be similar to those for able-bodied defectives except for special arrangements to meet the peculiar limitations of the pupils. For schools for the deaf it will be necessary to install sight signals and for the blind, those based on sound. Class rooms will be about half the standard size. Classes of mutes number from four to twelve. For the blind the classes are about the same for most work. The younger pupils will be provided with open dormitories. The older ones should have individual or double rooms. Segregation is, of course, necessary outside the class rooms and dining halls. Vocational instruction is usually given. Shop buildings are necessary with manual training benches, etc. Among the persons attending these schools a certain percent will be of defective mentality, but as these are gradually removed to other institutions, no special provision is made for them. As in other institutions the system of small units about a main building is superior to large structures. In some examples the buildings are formed into quadrangles enclosing recreation spaces. Blind schools offer instruction in music and will require organ space in the assembly hall. Special provision against accident is necessary, such as railings about points of danger.

28. Hospital Purpose Buildings.

28a. General Hospitals.—These are usually large buildings in which the separation or isolation of parts is brought about by wings or closed bridges. Between different wings glazed doors or fireproof doors are used for isolation. The usual divisions are: medical wards, surgical wards, obstetrical wards, children's wards.

The administration portion will contain the general office, waiting rooms, examination rooms, physicians' offices, matron's suite, the general kitchen and dining rooms for patients, officers and help (see Art. 22*f*). The ward spaces will be divided into single rooms, small and large wards. In each ward, a utensil room, linen room, locker room with individual lockers for each patient, diet kitchens, general and private toilets. A laundry for patients and a separate laundry for attendants. The minimum single room should be 10 × 14 ft., double room 14 × 14 ft. and wards 85 sq. ft. per person including aisles. Lighting, heating and ventilation should be: one foot of glass to six of floor space; 70 deg. temperature, humidified if possible; 1800 cu. ft. of fresh air per person per hour. Hot water heat is decidedly preferable, on account of excellent control. Local humidifiers are capable of maintaining desired conditions. Special electric signal systems for nurses are provided. Live steam at 30-lb. pressure is used for sterilization and the kitchen requirements. For this service a small boiler is desirable. A large general steriliser in the basement is used for mattresses, clothes, etc., smaller ones in each utensil room and a special steriliser for bandages and instruments in operating rooms. The corridors, utensil rooms, operating rooms and toilets should be capable of extreme sterilization and cleaning. Patients' rooms, if brought to the same condition, are apt to be depressing. No materials should be employed, however, that would be damaged by ordinary cleaning.

The elevators and the doors to them should be of a capacity to pass a full size cot. Push button control is necessary where a regular elevator man is not employed. The elevator should be convenient to the ambulance entrance on the ground level. It should not be immediately adjacent to patients' rooms.

Laboratories, Operating Rooms, Etc.—It is customary to provide one or more laboratory rooms, X-ray rooms, baking rooms and for other special service. These may be in the basement. The operating room should be not less than 300 ft. area, to contain the necessary fixtures and should be very well lighted, with top lighting subject to control. The ætherizing room may be adjacent or where most convenient. This will be somewhat less in area than the operating room.

Soundproof Rooms.—The obstetric ward should be divided by soundproof walls and partitions and should have soundproof doors. Otherwise the rooms and wards are not different from ordinary.

Sunporches enclosed with glass for convalescents are desirable especially in severe climates. They should be provided with ample venting panels.

Screens and Weatherstrips.—All parts of hospitals and sanitariums of every sort should be screened on windows and doors. Metal weather strips are necessary to prevent drafts.

Nurses' Dormitories.—Separate buildings for nurses and attendants are necessary in order to maintain efficiency, and prevent infection. One or more social rooms are necessary and single and double sleeping rooms with general toilets and baths. The room sizes will be similar to those in wards. The basement spaces should not be used for sleeping rooms.

28*b*. Hospitals for the Treatment of Tuberculosis.—The same advice as to the location of other public institutions will apply to sanitarium for tuberculosis with the additional precaution that quiet and freedom from dust is necessary to successful treatment.

Grounds.—Ample grounds should be provided, shielded from the north and west but open to the sunshine from other points of the compass.

Buildings.—The plan arrangement in tuberculosis sanitariums will differ from other hospitals in that exposure to the outside air and sunshine is essential to cure. For this reason large window spaces and ample porches are required. Rooms facing to the north or otherwise deprived of sunshine are not suited for the work. Such spaces should be assigned to corridors, toilet and bath rooms and other utilities.

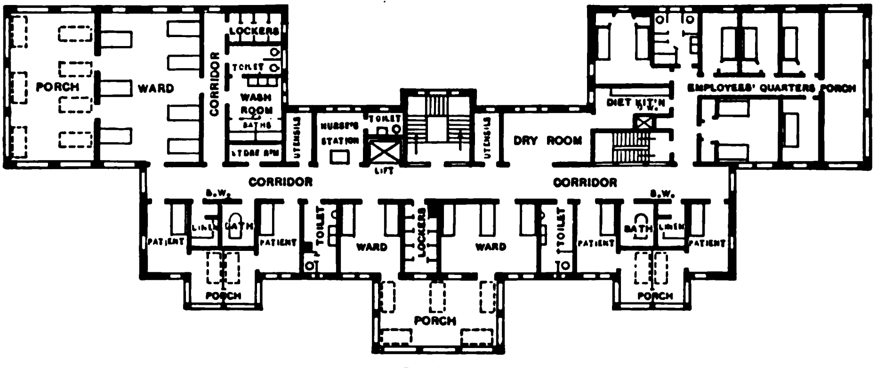


FIG. 16.—Typical sanitarium.

Rooms and Wards.—Patients' rooms should be exposed to sunshine and protected from the north wind. A room 7 ft. wide by 13 ft. long is a minimum. Ceiling heights above 10 ft. are not necessary. A French window extending to the floor, and not less than 4½ ft. wide should be provided, so that the cot may be moved out upon the porch. Such windows can be made weather tight by the use of metal strips. Double rooms should be 10 × 12 ft. and the adjacent porch space should be 10 × 12 ft. in size.

Porches.—All porches should be covered and screened and provided with sliding curtains of canvas to protect against rain. Large wards should be divided by screens into alcoves where practicable. In the same way, the spaces on porches may be broken up so that the long row of hospital beds will not be visible to all patients. The screens should be held up from the floor about a foot and extend to 6 ft. in height.

Administration.—The administration spaces will be similar to those at other hospitals. The laundry should be equipped with a steriliser, and none but patients' clothes should be treated in the general laundry.

Residences and Cottages.—Institutions for tubercular patients should provide houses for the superintendent and the employees, and a separate building for nurses and attendants.

Convalescent Camps.—Tubercular patients may be sent to a convalescent camp for final treatment. Such camps should be situated in places where food supply, fuel, sewage disposal and medical care can be readily obtained. Very simple cottages, a dining hall and work shop are required. The best location will be in the neighborhood of the regular sanitarium, where the same physicians can oversee the progress of the inmates.

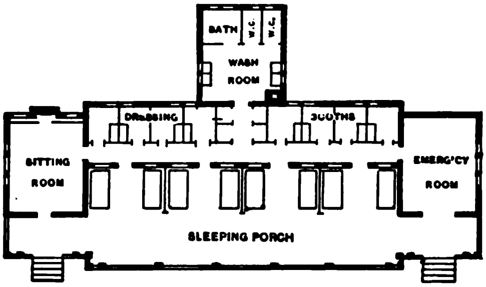


FIG. 17.—Convalescent open cottage.

29. Institutions Isolated from Towns and Cities.—Public institutions are not always located where advantage can be taken of the protection and the conveniences of a city. In this case everything included under the head of public utilities must be provided by the institutions themselves. The fundamental necessities are transportation, water, drainage, heat, light, enclosure, fire protection and police service. Besides these are such elements as soil qualities, climate, exposure, safety from the violence of nature. Subordinate provisions are for storage, refrigeration, industries and amusements. All such general provisions are accessory to the main object of the institution which may be disciplinary, military, social religious or political.

Transportation must be by railway, in the ordinary case. To attempt to maintain communication by wagon roads is expensive and hazardous in a severe climate. Where possible to obtain it, a railway side track will save from $3000 to $10,000 per year for a large institution.

Water supply for domestic use and for fire protection is of first importance. This involves drilling a deep well, or maintaining a storage reservoir from unfailing springs or making use of some large body of water, known to be safe. A knowledge of the geology and water supply of the neighborhood is therefore imperative.

Heat and Light.—The first building for an isolated institution will be the heat and power station, one or more units of which should be ready for service upon completion of the first buildings. The heating and power station will make the system of water supply available and may be necessary for pumping the effluent of the septic tanks.

Drainage is second only to water supply. The clearing of the ground of surface water and the disposal of waste water by natural means is fundamental. Septic tanks for the treatment of sewage are necessary to avoid pollution of lakes and streams. The system of drains should be determined upon as soon as the general disposition of buildings is made.

Enclosure in an isolated location will vary from the farm fence to the masonry wall with or without guards as conditions require.

Fire protection depends directly on the power plant and water supply for efficiency. The most effective fire protecting device is the sprinkler system which involves the construction of a tower and tank at least 25 ft. higher than the loftiest building. The tank may be of 50,000-gallons capacity supported by a steel frame or masonry tower. The water stored in the tank must be warmed by a special heater in winter. Large water mains are extended to various points with fire hydrants at intervals.

Police service, from the single watchman in the best locations to a considerable force, in exposed places, must be taken into account. Permanent police service will require guard houses, etc.

Soil qualities are important to institutions contemplating self-support. Soil analysis should be obtained where possible.

Climate and exposure will effect the design of grounds and buildings, especially where a period of years is expected to intervene before completion. In this case the first buildings should be grouped in such a way as to be convenient in operation at once, leaving future development to work into the scheme in an orderly manner.

Storage depends upon conditions, but will concern first the coal supply which may be delivered during the summer season and must be conveniently placed.

Refrigeration by ice or mechanical means is imperative and may be extensive. Ice storage may be employed in some cases. The supply storage and ice storage is sometimes combined.

Industries and amusements are essential to many isolated institutions. The character of the institution will determine the types of buildings to be erected for these purposes.

Future Development.—In any institution enlargement should be anticipated. While a natural barrier on one or more sides may be an advantage, there should be always a practicable outlet by which future growth may take place without disproportionate expense. This involves a general study of the lands adjacent.

ACOUSTICS OF BUILDINGS

By F. R. Watson

Increased attention has been paid in late years to the acoustical disturbances in buildings with the desire on the part of architects and builders to avoid these defects as far as possible. This desire has led to scientific investigations of the subject that have solved some fundamental problems and given formulas and data for guidance.

Acoustical disturbances are due first, to the sound generated within a room, which gives rise to echoes and reverberation; and second, to sounds outside that are transmitted into the room through walls, ventilating ducts, and other paths, and cause confusion. The sound in a room may be controlled by the proper design of the volume and shape of the room and by the use of a calculated amount of absorbing material, while the extraneous sounds may be minimized by properly constructed walls, doors, and windows. The problem may therefore be considered in a two-fold aspect: the acoustics of rooms and the insulation of rooms.

30. Acoustics of Rooms.

30a. Action of Sound in a Room.—When a sound is generated in a room it proceeds outward from the source at the rapid rate of about 1200 ft. per sec. and, by successive reflections from the boundaries, very quickly fills a room of ordinary dimensions. Fig. 18 shows the position of a pulse of sound in a room 60 × 40 ft., $\frac{1}{60}$ sec. after it started from the source. Fig. 19 gives the same pulse $\frac{1}{60}$ sec. later and shows the increasing reflections and interferences.

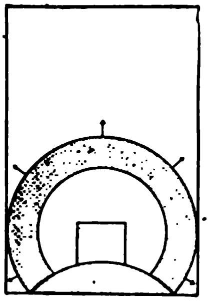


Fig. 18.—Pulse of sound in a room $\frac{1}{60}$ of a second after leaving the source.

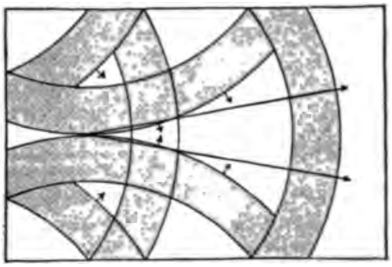


Fig. 19.—The same pulse $\frac{1}{60}$ of a second later, showing reflections and interferences.

The imagination readily pictures the conditions $\frac{1}{10}$ sec. later when the entire volume of the room is filled with sound proceeding in every direction. The width of the sound pulse should be much wider than shown if it is to represent actual conditions, because speech sounds take at least $\frac{1}{10}$ sec. for their generation[1] and musical sounds are frequently prolonged a second or more. In the meantime, the energy of the pulse is diminished at each reflection by the absorption of a fraction of the incident sound, so that it is used up after a number of reflections, depending on the absorbing efficiency of the surfaces it strikes.

30b. Conditions for Perfect Acoustics.—Perfect acoustical conditions for hearing require that the sound shall rise to a satisfactory intensity which shall be equal in every part of the room, with no echoes or distortion of the original sound, and that it shall then die out in a suitably short time so as not to interfere with the succeeding sounds. Unfortunately, these ideal conditions are not fulfilled in rooms. The reflections of sound give rise to distortions and unequal intensities in different parts of the room and, except for special cases, it is impossible to secure simultaneously a suitable intensity and a proper time of reverberation. It will be shown, however, that while the ideal is rarely found, satisfactory acoustics may be obtained for auditoriums of usual shape and size.

30c. Formula for Intensity and Reverberation.—Reasoning in the manner just described, Sabine[2] developed an equation for the reverberation in a room, a simplified form

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[1] Scripture, "The Study of Speech Curves," Carnegie Institution Publication, 1906.

[2] American Architect, 1919.

of which for practical use is given in a succeeding paragraph. Later, Jäger,[1] using a different constant, deduced the formula in a somewhat different form and discussed its applications to an auditorium. Thus, he developed the formula: $E = E_0 e^{-ant}$, where E is the intensity of the sound per unit volume t seconds after the initial intensity E_0 has been built up, n being the number of reflections that have taken place, and a the fraction of the energy absorbed at each reflection. More completely, the formula may be written:

$$E = \frac{4A}{avs} e^{-avst/4W}$$

where the initial intensity, $E_0 = 4A/avs$, is seen to depend on A, the energy given out by the source in one second; v, the velocity of sound; s, the area of all surfaces exposed to the action of the sound; and a, the average sound-absorbing coefficient of these surfaces. Inspection of the relation shows that the intensity may be increased by making the source of sound, A, more intense; also, for a given A, the intensity may be reduced by increasing the absorption, as.

The decadence of the sound is given by the factor: $e^{-avst/4W}$. The time of reverberation, t, is increased by increasing the volume, W, of the room, so that large rooms may be expected to have excessive reverberation. A decrease in t may be brought about by increasing the absorbing power, as, and thus improve the reverberation, but this procedure cannot be carried too far because an increase in the absorption decreases the initial intensity, as shown previously. The conclusion is drawn that only in special cases can both suitable intensity and time of reverberation be obtained for the same conditions in an auditorium.

30*d*. Correction of Faulty Acoustics.—The practical solution of the problem of correcting faulty acoustics, has been made by Sabine[2] whose scientific work has established the fundamental facts of the subject. Assuming a sound of average intensity, he developed the simple formula: $t = kW/as$, where t is the time of reverberation; W, the volume of the room; as, the absorbing power of all the interior surfaces; and k, a constant, depending on the units used, being equal to 0.164 when W is measured in cubic meters and s is taken in square meters. The term as is the sum of all the various absorbing agencies in the room and may be expressed as:

$$as = a_1s_1 + a_2s_2 + a_3s_3 + \ldots\ldots$$

where s_1 may be taken as the area of all the plaster surfaces, and a_1 as the absorbing coefficient of unit area of plaster surface; s_2 the area of all the wooden surfaces and a_2 the corresponding absorbing coefficient, etc., until all the absorbing surfaces are included.

In a series of investigations lasting several years, Sabine determined the absorbing coefficients of the various materials commonly used in building construction. His values are as follows, assuming that unit area of open window space has perfect absorbing power and that its coefficient is taken as unity:

TABLE 1.—SOUND ABSORBING COEFFICIENTS

Material	*Coefficient*
Wood sheathing, (hard pine)	0.061
Plaster on wood lath	0.034
Plaster on wire lath	0.033
Plaster on tile	0.025
Glass	0.027
Brick set in Portland cement	0.025
Audience	0.96
Oil paintings, (inclusive of frames)	0.28
House plants, per cubic meter	0.11
Carpet rugs	0.20
Oriental rugs, extra heavy	0.29
Cheese cloth	0.019
Cretonne cloth	0.15
Shelia curtains	0.23
Hair felt, 2.5 cm. thick, 8 cm. from wall	0.78
Cork 2.5 cm. thick loose on floor	0.16
Linoleum, loose on floor	0.12

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[1] "Zur Theorie des Nachhalls." Sitzungsberichte der Kais. Akad. der Wissensch. in Wien, Math-Naturw. Klasse; Bd. CXX. Abt. 2a, Mai, 1911.

[2] "Architectural Acoustics." A series of papers in the American Architect, 1900, and later papers.

Material	*Absorbing Power*
Audience, per person	0.44
Isolated man	0.48
Isolated woman	0.54
Plain ash settees, each	0.039
Plain ash settees per seat	0.0077
Plain ash chairs, "bent wood"	0.0082
Upholstered settees, hair and leather, each	1.10
Upholstered settees, per single seat	0.28
Upholstered chairs similar in style, each	0.30
Hair cushions, per seat	0.21
Elastic felt cushions, per seat	0.20

It should be noted that plaster, wood, and glass, the materials that usually form the interior surfaces of auditoriums, have small absorbing power, thus accounting for the faulty reverberation found in any large auditorium. Hairfelt, on the other hand, which is used extensively for acoustical correction, has a large coefficient. To be efficient as acoustical correctives, materials should have a coefficient of at least 0.10. When judged by this standard, any type of plaster wall in common use is seen to be practically useless as an absorber. The desirable qualities in an absorber are porosity and compressibility. The energy of sound incident on such a material is converted partly into heat by friction in the pores, and partly into mechanical energy by compressing the substance, the amount of energy so converted constituting the absorption. An audience is a good absorber of sound undoubtedly because of the clothing worn. When making an acoustical correction for an auditorium, the absorbing power of the audience is figured as an important factor. By the use of these coefficients and Sabine's formula, calculations may be made indicating how much absorbing material should be introduced into a room to give satisfactory acoustics for average conditions. These calculations may be made from the building plans so that the acoustics may be provided for in advance of construction.

In rooms used only for speaking purposes, the time of reverberation should be shorter than for music alone, because a longer time of reverberation is desired for music. When the room is to be used for both music and speaking, a time of reverberation is chosen that will be fairly satisfactory for both; the auditorium thus being made somewhat too reverberant for speaking, and not quite reverberant enough for music.

30e. Echoes in an Auditorium.—Other defects than the reverberation may exist in an auditorium. An echo is set up when an auditor hears a sound coming direct from a nearby speaker and then again at a later time when it is reflected from a distant wall. Figs. 20 and 21 show the reflections of sound in the Auditorium at the University of Illinois and how echoes were caused. This room is nearly hemispherical in shape with several large arches and recesses which break the regularity of its inner surface. Because of its large volume, 425,000 cu. ft., and curved walls of hard plaster, it was afflicted with both reverberation and echoes. An investigation[1] lasting several years, yielded an analysis of the acoustical defects, on the basis of which, action was taken to correct the faults. The echoes were located experimentally by sending a small bundle of sound successively in different directions and noting its path after reflection. A ticking watch was used as a source of sound. When backed by a reflector, this gave definite data, as did also a metronome enclosed in a box so that the sound could escape only through a directed horn; but the results were not conclusive. A satisfactory method was found that involved the use of an alternating-current arc light as the source of sound. This gave a hissing sound that traveled the same path as the light of the arc. The light and sound were reflected by a parabolic reflector to distant walls where an observer could see where the sound struck. The walls causing echoes were then readily located.

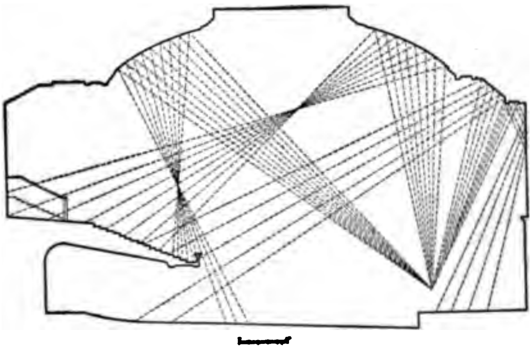

Fig. 20.—Reflection of sound in an auditorium.

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[1] Bull. 73 on "Acoustics of Auditoriums" by F. R. Watson and Bull. 87 on "Correction of Echoes and Reverberation in the Auditorium, University of Illinois" by F. R. Watson and James M. White. Published by the Univ. of Ill. Eng. Exp. Sta.

For a distinct echo, Tallant[1] estimates that the time difference between the direct and reflected sounds should be about 1/15 sec., depending on the acuteness of hearing of the auditor. For the practical avoidance of echoes, this would mean that the difference in paths of the direct and reflected sounds should not exceed 70 ft.

30*f*. Interference and Resonance.—Another acoustical defect is created when sound waves, reflected from the walls of the room, meet the oncoming waves in such a manner that pronounced interference takes place. Thus, a sustained musical sound may produce undue loudness in some places and a corresponding dearth of sound elsewhere. A further defect, called resonance, is caused when the original sound is amplified by the vibration of wooden paneling and by the reinforcement from alcoves or window recesses. In the practical correction of the acoustics of rooms, it is very desirable that the absorbing material introduced to reduce the reverberation, be placed so as to minimize the echoes and other faults.

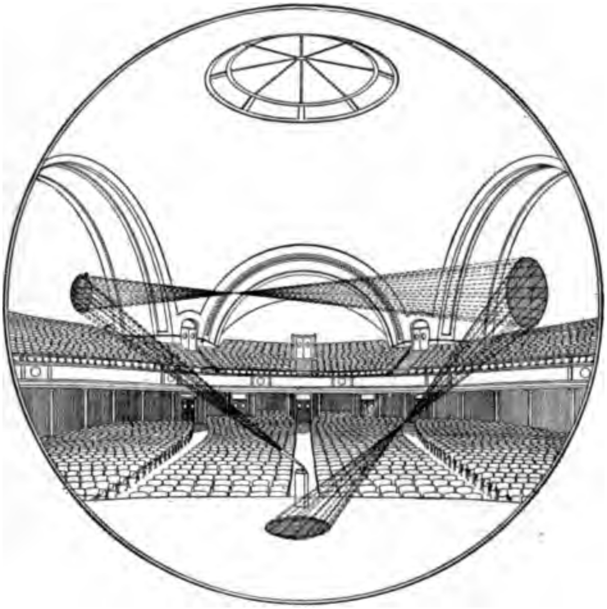

FIG. 21.—How echoes are set up by reflection of sound.

30*g*. Wires and Sounding Boards.—A statement should be made concerning the acoustical effect of wires and sounding boards, since these appeal to the popular mind as effective correcting agencies. Wires are of practically no effect.[2] They have much the same effect that a fish line in the water has on the water waves. To be effective, the obstacle should be large enough to be comparable with the wave length of the sound. An instance is recorded where five miles of wire were installed in an auditorium without acoustical effect, so it was removed and absorbing material put in for correction.

Sounding boards are useful in special cases where it is desired to direct sound.[3] Sounding boards, relief work on walls, galleries and other obstacles serve to break up the regular reflection of sound and prevent the formation of echoes, but their effect in acoustical correction is small compared with the absorption of energy by absorbing material.

30*h*. Modeling New Auditoriums After Old Ones With Good Acoustics.—A suggestion often made is for architects to model auditoriums after those already built that have good acoustical properties. It does not follow that halls so modeled will be successful, because the materials used in construction are not the same year after year. For instance, it was the usual custom years ago to build wooden structures; but modern practice requires the use of steel, concrete, and plaster thus forming walls that transmit and absorb less sound. Furthermore, a new auditorium is changed somewhat to suit the ideas of the architect or the particular circumstances of the new building, and it is quite probable that the changes will affect the acoustics.

30*i*. Effect of the Ventilation System.—It would seem at first thought that the ventilation system in a room would affect the acoustics. The air is the medium that transmits the sound. It has been shown that the wind has an action in changing the direction of propaga-

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[1] "Hints on Architectural Acoustics," The Brickbuilder, 1910.

[2] Sabine, Arch. Quarterly of Harvard University, March, 1912. Watson, Science, Vol. 35, May, 1912.

[3] Watson, "The Use of Sounding Boards in an Auditorium," The Brickbuilder, June, 1913.

tion of sound.[1] Sound is also reflected and refracted at the boundary of gases that differ in density and temperature.[2] It is found, however, that the effect of the usual ventilation currents on the acoustics in an auditorium is small. The temperature difference between the heated current and the air in the room is not great enough to affect the sound appreciably, and the motion of the current is too slow and over too short a distance to change the action of the sound to any marked extent.[3]

Under special circumstances, the heating and ventilating systems may prove disadvantageous. A hot stove or a current of hot air in the center of the room will seriously disturb the action of the sound. Any irregularity in the air currents so that sheets of cold and heated air are set up will modify the regular progress of the sound and produce confusion. The object to be striven for is to keep the air in the room as homogeneous and steady as possible. Hot stoves, radiators, and currents of heated air should be kept near the walls and out of the center of the room. It is of some small advantage to have the ventilation current go in the same direction as the sound since a wind tends to carry the sound with it.

31. Non-transmission of Sound.

31a. How Sound is Transmitted.—The second large problem in the acoustics of buildings is the transmission of sound. Sound may be transmitted from one part of a building to other parts in a variety of ways. The vibrations of pianos, cellos, etc., that rest on the floor, and the noise of motors, pumps, and other instruments that are placed in intimate contact with the building structure, are transmitted with surprising efficiency through the continuity of structure and are hindered in their progress only when encountering a discontinuity in elasticity or density, a large change of this kind being a transition from masonry to air. These disturbances may give rise to unexpected sounds by causing thin walls, partitions, desks, and other objects in contact with the building structure to vibrate and set up sound waves in the air. The action is quite similar to that of a speaking tube, the sound vibrations in this case being confined in the walls by the totally reflecting air boundary about them.

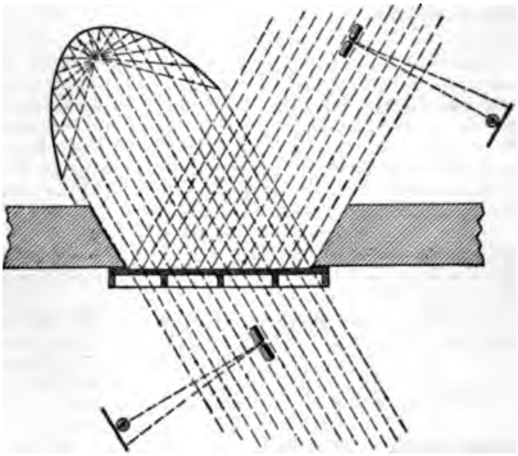

Fig. 22.—Apparatus for measuring sound transmitted and reflected by a partition.

Other types of sound that set up vibrations in the air, such as those produced by the voice, violin, etc., continue their progress in the air through ventilator ducts, open windows, spaces between doors and their casings, incompletely closed pipe openings, partition joints, or, in general, wherever there is a continuous air passage. On meeting thin walls and partitions they may cause these to vibrate and thus create sound vibrations on the further side.

The foregoing considerations show that vibrations may pass from one part of a building to other parts along paths not easy to trace and introduce extraneous sounds that are undesirable.

31b. Experimental Investigations.—Investigations that have led to some definite results, have been inaugurated to solve the difficulties, but there remains much to be done.[4] The comparative intensities of sound transmitted and reflected by partitions of different materials have been measured by the writer.[5] A sound of constant pitch blown by a steady air pressure, is directed by means of a parabolic reflector against the partition as shown in Fig. 22. Part of the sound is reflected and part transmitted, the intensity of each part being measured by a Rayleigh Resonator. The Rayleigh Resonator is a brass tube tuned to the sound and has

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[1] Osborne Reynolds, Proc. of Royal Soc., Vol. XXII, p. 531, 1874.

[2] Jos. Henry, Report of Lighthouse Board of U. S., 1874.
J. Tyndall, Phil. Trans., 1874.

[3] Sabine, Eng. Rec., Vol. 61, p. 779, 1910. Watson, Eng. Rec., Vol. 67, p. 265, 1913.

[4] Sabine, The Brickbuilder, Feb., 1915.

[5] Physical Review, Jan., 1916.

a glass disc hung inside by a quartz thread. The disc deflects under the action of the sound, the angle of deflection being proportional to the intensity of the sound. This arrangement allows quantitative, comparative measurements to be obtained independently of the ear.

A preliminary investigation gave the following results:

TABLE 2.—TRANSMISSION AND REFLECTION OF SOUND

Material	Deflections of resonator for					
	Transmission			Reflection		
Thickness in layers	1	2	3	1	2	3
½ in. hairfelt	22.6	15.4	10.4	4.9	6.6	10.5
¼ in. cork board	7.9	3.75	2.9	15.7	22.0	22.6
¾ in. cork board	1.15	2.05	0.85	25.9	21.2	22.1
¼ in. paper lined felt	5.0	21.7	3.8	20.7	5.9	10.0
¾ in. paper lined felt	6.5	1.95	0.4	10.4	6.6	9.3
¾ in. flax board	2.25	0.55	0.1	22.5	20.0	20.0
¼ in. pressed fiber	0.32			23.2		
¾ in. pressed fiber	0.2					

Inspection of the results shows that a porous material like hairfelt, transmits much sound. Lining it with paper stops the pores and introduces air spaces between successive layers and thereby diminishes the transmission. Dense materials transmit less sound, as shown by the results for the pressed fiber. The law of transmission for a homogeneous material, like hairfelt, states that the intensity of the transmitted sound decreases exponentially with the increasing thickness. Doubling the thickness does not double the amount of sound cut off; that is, if 1 in. of the material stops 10% of the sound entering the material, 2 in. stop 19%, 3 in. stop 27%, etc. For non-homogeneous walls, such as cork sheets with air spaces between, or compound walls, such as plaster partitions, there is no simple law of transmission. When a partition is elastic, it vibrates under the action of the incident sound and may be set in vigorous motion if in tune with the incident waves. This creates compressional waves on the further side of the partition and thus transmits the sound energy. Thick walls may act in this way as well as thin ones. Vibrations with amplitudes of one-thousandth of an inch and less are capable of producing audible sounds.[1]

The reflection of sound increases usually with the thickness of a homogeneous material, but the law is not a simple one. The reflection is large when the transmission is small unless the material is a good absorber. When a partition vibrates, the reflection may be smaller than expected, as in the case of the ¼ in. paper lined felt. Reflection is greater for rigid, heavy partitions than for elastic, thin ones.

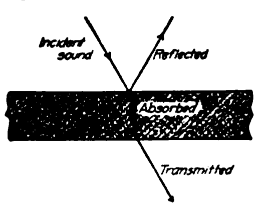

FIG. 23.—Action of a material in reflecting, absorbing, and transmitting sound.

The experiments just described point the way to further work and this has already been started with improved methods and apparatus. The complete solution of the problem involves the absorption of sound. Fig. 23 indicates how the incident sound is reflected, absorbed, and transmitted in varying amounts depending on the nature of the material, the construction of the partition and the possibility of vibration.

The transmission of sound has been measured by Sabine[2] who tested the sound-insulating efficiencies of hair-felt, sheet iron, and combinations of these materials by a method involving the use of the ear in listening for the faintest trace of sound. He found that hairfelt transmitted considerable sound but that the rigid, dense sheet iron was more efficient. Alternate layers of sheet iron and hairfelt gave quite satisfactory insulation. His experiments were preliminary to a more extended investigation of standard constructions and were intended to establish methods and principles.

Jäger[3] states from theoretical considerations that thin walls of small mass and easily capable of vibration transmit sounds quite readily; also that low pitched sounds pass through partitions more easily than high pitched ones. Tufts[4] concluded from his experiments that porous materials transmit sound in much the same proportion that they allow air to pass.

31c. Sound-proof Rooms.—The foregoing conclusions indicate the constructions best suited for making rooms sound-proof. What is desired are walls that are rigid and heavy

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[1] See "Vibrations of Buildings," Art. 31d.

[2] "The Insulation of Sound," The Brickbuilder, Feb., 1915.

[3] See previous reference, p 748.

[4] Amer. Jour. of Science, Vol. 2, p. 357, 1901.

with some sort of discontinuity, such as an air space. It appears of advantage to place sound-absorbing material in this air space. Unfortunately, it is not possible in practice to have a complete air discontinuity about a room, because the walls make a more or less intimate contact at the floor where they are supported. It is also apparent that any ventilation openings or cracks about doors, pipes, and partitions that will give a continuous air passage, will allow transmission of sound and should be avoided as far as possible. Further, steam and water pipes convey sounds of distant pumps, motors, and furnaces and are likely to pass these sounds to the air in the room.

31*d*. Vibrations in Buildings.—Another problem in the transmission of sound arises because of the vibrations of walls, floors, and other portions of the building which are apt to give forth sound. A systematic investigation of this subject was carried out by Hall[1] in San Francisco. He used a modified seismograph pendulum that recorded vibrations in three directions, two horizontal vibrations at right angles to each other and a third vertical vibration. The results showed that buildings vibrate in all three directions to a greater or less extent because of machinery, street traffic, and other causes. The magnitude of the vibrations is generally small, varying in Hall's observations from about 0.0014 to 0.00004 in.; but it is likely that vibrations of factory floors exceed these values. The frequencies of the vibrations varied from about 2 to 9 per sec.

Vibrations of walls are capable of producing sound waves in the surrounding air, that will be audible if the amplitude of vibration is large enough. There appears to be no data for this particular case, but some idea of the action may be gained from experiments by Shaw[2] who found that a telephone receiver membrane vibrating with small double amplitudes gave sounds when held to the ear as indicated in Table 3.

TABLE 3.—SOUNDS PRODUCED BY A VIBRATING TELEPHONE MEMBRANE

Double amplitude	Result
0.000006 in.	sound "just audible"
0.0004 in.	sound "just comfortably loud"
0.008 in.	sound "just uncomfortably loud"
0.04 in.	sound "just overpowering"

Hall's values lie within these limits but the sounds produced would be considerably fainter because they are not conveyed so directly or so efficiently to the ear as in Shaw's experiment.

More recently, this problem has been extended by others[3] from the economic standpoint, since it appears that these vibrations, particularly in factories, affect the physical welfare and efficiency of the employees. The results of the investigations described lead to the following recommendations for reducing vibrations: (1) to minimize the vibration at the source by using properly balanced machines, and by mounting them on separate foundations or on heavy, rigid floors; and (2) to reduce transmission of vibrations by introducing materials to produce changes in the elasticity and density of the building structure, thus following the principles already set forth in regard to non-transmission of sound.

32. Conclusion.—A number of related problems in the acoustics of buildings remain unsolved. There is need for further information on the construction of sound-proof rooms; how different constructions reflect, absorb, and transmit sounds of different frequencies; and how pipes and ventilating ducts may be modified to prevent transmission of vibrations. There is need also for efficient, fireproof, sound-absorbing materials that are comparatively inexpensive and capable of being cleaned without impairing their acoustical efficiency.

While it is seen that a number of problems await solution, it is also apparent that a considerable fund of information is at hand for guidance in securing satisfactory acoustics, particularly in auditoriums. It would therefore seem desirable for architects and builders to avoid the acoustical defects so prevalent in existing buildings by specifying in advance of construction of new buildings that proper action be taken to secure satisfactory acoustics as far as possible.[4]

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[1] "Graphical Analysis of Building Vibrations," Elec. World, Dec. 18, 1915. Also earlier papers.

[2] Proc. of Royal Society, Vol. 76A, p. 360, 1905.

[3] Maurice Deutsch, Consulting Engineer, New York City. (See pamphlet entitled "The Effects of Vibration in Structures" published by the Aberthaw Construction Company, Boston.)

[4] Those who desire to read further in the subject of Acoustics of Buildings will find ample material and bibliography in the references given in footnotes throughout the chapter.

SCHOOL PLANNING

By James O. Betelle

School Planning has made very rapid and marked development in the last decade, and, on account of the changes brought about by the world war, even greater and more marked developments are looked for within the next few years. Courses of studies are changing, new ones are being added, and some old ones being abandoned. This means changes in the usual school building plan to properly take care of these new conditions. It also means that new buildings shall be so constructed that changes may easily be made after the school is built, as no school building can be up-to-date for a very long period during these times of rapid adjustment in school administration.

33. Educational Surveys.—Farshigted communities who wish to locate and to build their schools scientifically, and with a look to the future, are beginning to see the importance of having an educational survey made of their town or city by experts who make a specialty of such work. As a result of what is learned regarding existing conditions and probable future trend and increase in population, a building program for the next 5 to 10 years is planned out, sites acquired, and building work started. For typical examples of these surveys see the reports of the surveys made of Portland, Oregon, Omaha, Neb., St. Paul, Minn., and Cleveland, Ohio.

34. School Sites.—The recently enacted physical training and military training laws in many of our states, as well as a more enlightened public opinion, has made larger school sites necessary.

For the average elementary school accommodating about 800 pupils, a site of not less than 4 acres is recommended; and for the intermediate school of 800 pupils, a site of not less than 5 acres is recommended. In an intermediate school the playground requirements become more important and an experimental school garden is often included.

For the high school accommodating about 1000 pupils a site of 10 acres, or more, is recommended. This will include not only space for games, such as tennis, hand ball, basket ball, etc., but also a complete athletic field with base ball and football fields, running track, and bleachers for spectators. It is very desirable to have the school athletic field adjoin the high school, as the games, drills, and exercises can be more easily supervised. The gymnasium in the building with its lockers, showers, and other dependencies are readily available and classes can be easily drilled or exercised in the open air when the weather is suitable, instead of in the enclosed gymnasium.

Sites should also be selected with due regard for healthful conditions, accessibility, absence of noise, dangerous approaches, good moral surroundings, etc. The Minnesota school building regulations recommend that even on the smallest sites, not more than 20% of the entire area be used for the building.

It seems to be agreed among educators that school buildings should be so located that no scholar attending the primary school shall have more than ¾ of a mile to walk, and if attending an intermediate school not farther that 1½ miles. High school scholars can travel as far as 2½ miles, but a limit of 2 miles is to be preferred. In special cases scholars do travel farther to school than these distances, but trolleys or other special means of transportation are used.

35. Program of Studies.—No school can be properly designed until the superintendent of schools furnishes the architect with a program giving the course of studies to be taught, length of class periods, number, size, and kind of rooms desired, and number of pupils to be accommodated in each subject. This will permit the architect to so design the school as to suit the particular subjects to be taught, rather than make the program of studies fit in with the building after it is built.

36. School Building Laws of Various States.—Many states have laws which apply to the construction of school buildings. Copies of these laws and any rules relating to building or grounds which have been adopted by the State Board of Education, should be obtained and carefully followed in designing the building. Where state laws exist it is usually required that all plans and specifications of school buildings be submitted to the State Board of Education for approval before starting to build. The requirements of these laws vary widely from nothing at all in some states, to very rigid requirements in Ohio. The appointment of a federal commission is being advocated to standardize these laws in the various states and bring about some semblance of uniformity. The control over existing school buildings and the plans for new buildings is usually enforced by State Boards of Education through their control of state money which they apportion and distribute to the various communities, and which they may withhold unless certain standards are lived up to.

37. School Organization.—The school life of children is divided into 12 years and generally designated as 1st to 12th grades. Sometimes grades are designated as 1B–1A, 2B–2A, etc., where there is a promotion at midyear. The further division has been general of housing the first 8 years or grades in a grade school and the last 4 years in a high school, the grades in the high school being called 1st, 2d, 3d, and 4th years. A change in this organization is now being made by placing a Junior High School between the lower grades and the senior high school. This organization and its advantages, will be treated under the description of the junior high school.

A tendency to make an intensive use of the school plant has been very marked in recent years. The use of the various buildings only 5 hours a day for 200 days each year is giving way to twice as much use, or more. If we are to have the necessary school plant and equipment, which modern education demands, and still keep taxes within reasonable limits, economy must be practiced. There is no easier way to economize than to make more use than formerly of the facilities we already have.

The so-called "Gary" plan is a scheme for the more intensive use of the school plant, the accommodation of more children, and at the same time giving a more diversified and attractive course of study, work, and play. The "alternating plan" and "platoon system" are only modifications of the "Gary" plan, to solve the problem of some special community. From lack of construction of new school buildings to take care of the normal growth in population during the past few years and the excessive cost of new construction, communities have been forced into this new scheme of organization. The other alternative is to place a portion of the scholars on part time, which every one hesitates to do. Briefly the schemes are about as follows: One-half the scholars report at school, say at 8:30 A. M. After the first period spent in the class rooms these pupils move on to the special rooms, such as shops, gymnasium, auditorium, or playgrounds, and leave the class rooms vacant for the second section or platoon. The program of the school is therefore rather complicated but very ingenious. The school day is longer than under the usual program because there are periods of supervised play, gymnasium, swimming pool, etc. The first plattoon's day comes to an end around 4:00 o'clock and the second platoon one period later, or around 4:40 o'clock. To run a school on this intensive basis makes it necessary to operate it on the departmental plan and the school building must be very complete in its various departments. The reason more scholars can be accommodated in this type of school than in the ordinary one is because several classes at a time are taken into the auditorium and given a singing lesson, an illustrated lecture, or something that can be taught in large groups; other large groups at the same time go to the playgrounds, to the gymnasium, etc., so that while the first platoon is absorbed in these special activities, the second platoon has the use of the recitation and class rooms; thus the platoons alternate throughout the day. In some instances, groups of children are sent once a week for religious instructions to nearby churches designated by the Protestant, Catholic, and Jewish organizations, thus making still more room in the school for more pupils.

38. Kinds of Schools.—(1) Primary School, (2) Intermediate or Junior High School, (3) Senior High School, (4) Manual Training or Commercial High School, (5) Vocational School.

39. Primary Schools.—Primary schools accommodate children from kindergarten age (4 to 6 years) up to and including the 6th grade, where there is a junior high school, and up to the 8th grade where no junior high school exists. The plan of the building is very simple and consists principally of class rooms accommodating 40 to 42 pupils each. It may or may not have an auditorium. If it has an auditorium, it need only be large enough to accommodate ½ to ⅔ of the pupils at one sitting. A few years difference in the ages of children at this period means considerable difference in their mental development. It is not possible to talk to the entire group of 1st to 8th grades, without talking over the heads of the smaller children or beneath the older ones. For this reason, they are assembled in groups of only a few years' difference in age, and not so large an auditorium is needed. There is no objection, however, other than the cost to having an auditorium seating the entire school, as it is often desirable to get all the pupils together for some special occasion, such as at Christmas time, or for other entertainments.

A play room or exercise room is provided to take care of the children at recess and before school during stormy weather, and equal in area to about 1500 sq. ft. It is not usually called a gymnasium because little or no apparatus is used.

The primary school is organized on the simplest basis and the children do not go from room to room as in the departmental scheme, but remain in the same class room and under the same teacher all the time.

In schools including the 7th and 8th grades, a few special rooms are included, such as manual arts room, household arts room, drawing room, etc.

40. Intermediate or Junior High School.—The junior high school is an innovation in the school organization which is being received with great favor. Educators claim many advantages both from a financial and an educational standpoint. Where junior high schools exist, the entire school system is organized on one of several ways, such as the 6–6, the 6–2–4, or the 6–3–3 plan, the latter meaning 6 years primary school, 3 years junior high school, and 3 years high school. The other schemes are adopted to meet certain special situations as, for instance, in the 6–6 plan, the community may have a large and well organized high school which is large enough to include the 7th and 8th grades. The pupils are therefore put in with the high school and no additional building is needed. As the high school enrollment increases, however, instead of enlarging the high school building, the junior high school can be built in which will be accommodated the 7th and 8th grades and the first year high school class, thus relieving not only the high school but the grade schools as well. The school system will then be organized on the 6–3–3 plan, which seems to be the most desirable.

The following claims are made in favor of junior high schools:

1. Children in the adolescent stage are best housed in separate buildings away from the extremely young pupils as well as the more mature.

2. As the junior high schools are usually run on the departmental plan or to a great extent on that basis, it provides an easy break between the very much supervised primary school, and the high school where the student is thrown on his own resources and responsibility.

3. The children are often kept a little longer at school and instead of leaving on completion of the 8th grade, as they probably would under the ordinary 8–4 organization, they are encouraged to complete the junior high school course, which includes the 9th grade or 1st year high school. There is also the chance that having gone through the 9th grade, the pupil will be interested to go further.

4. It is possible to give better instruction under the departmental plan where the pupils go to special teachers for certain subjects than it is where one teacher instructs in all subjects. Pupils have a more diversified course of study and wider experiences in a junior high school organisation, and are therefore better equipped to go out into the world's struggle than they are under the 8–4 system. Promotions are usually made by subjects and not by grades; this makes for efficiency and permits the pupil especially bright in any subject to progress more rapidly.

(5) The 9th grade or first year high school class is always the largest in a high school, and more pupils drop out during or at the end of this year than at any other time in the high school course. The three upper high school classes are of a more even number, and the chances are that a pupil entering the second year will complete the high school course. It is therefore more economical from a building standpoint to house this large number of pupils in the lowest high school grade in a building which is not so costly or elaborately equipped as a modern senior high school building.

(6) The number of pupils in a class is generally reduced from 42 to 35, thus placing it on the high school basis and furnishing more individual instruction to each pupil in the class.

More special rooms are provided than in a primary school but not so many or so elaborately equipped as in a senior high school.

41. Senior High School.—Almost everyone is familiar with the usual senior high school, its organization and general arrangement. The tendency is to have more elective courses and place special emphasis on the difference between the courses for those going to college and those whose education ends upon graduation from the high school. Many special rooms are included and these will be described in detail under separate headings.

42. Manual Training and Commercial High Schools.—These are specially planned and equipped schools for the teaching of special subjects. A manual training high school should not be confused with a vocational school. In the manual training school the pupil gives attention to many subjects in order to have a variety of experiences, and a trained eye and hand as well as a trained mind. In a vocational school the pupil gives special attention to one certain vocation and its allied studies with a view of taking up the subject for his life's work.

The commercial high schools specialize on subjects similar to the ordinary business college, such as bookkeeping, typewriting, stenography, letter writing, business arithmetic, business law, customs, etc.

It is the tendency at the present time to concentrate the various different departments in one large high school and not split them up into a number of separate units teaching special subjects. These are known as "comprehensive," or "cosmopolitan" type of high schools. It is claimed that the pupil has a better chance to make an intelligent choice of his life work by being in close association with pupils in various courses and if he decides, as time goes on, that it is best to make an adjustment or change in his course of studies, it can easily be arranged without the necessity of changing schools.

43. Vocational Schools, and Smith-Hughes Bill.—The object of the vocational school is to fit an individual to pursue effectively a recognized profitable employment. It is intended for persons over 14 yr. of age who are preparing for a trade or industrial pursuit. It is not intended to take the place of the regular schools, but in a large measure is intended to keep interested and at school, pupils who would otherwise leave and go to work. The number of pupils leaving school and seeking employment at the early age of 14 yr. is alarming, and the cause of their leaving and starting to work is not always an economic one. With courses of studies, such as the vocational school will provide, a great number of these pupils can be kept at school 2 or 3 yr. longer and receive sufficient training in useful occupations to take them out of the unskilled labor class.

The U. S. Government has recognized the need of more skilled artisans, and realizes that it is not a local matter. A pupil may be born in California, get his training in Massachusetts, and later spend his days as a machinist in Indiana. In recognition of the above condition Congress passed the Smith-Hughes Bill establishing the Federal Board for Vocational Education and renders financial aid to the various states where vocational schools are established. The bill, however, does not grant any money for the building or equipment; this must be taken care of entirely by the community. The financial aid from the Government is to be devoted toward payment of the teachers' salaries and the training of teachers.

Vocational schools are built separately for boys and girls and it is important to give an actual shop atmosphere to the building and its work rooms. Its shops should be amply large and flexible so as to take care of changing conditions. In most vocational schools special attention is given to local industries. Boys' schools include such courses as plumbing, electrical work, pattern making, sheet metal work, automobile and gas engines, printing, brick laying, carpentry, sign painting, blacksmithy, machinery, etc. Girls' schools, such courses as dressmaking, millinery, suit and cloak making, children's clothing, novelty work, electric power machine operating trades, feather and paper working, weaving, glove-making, straw hat making, embroidery, hemstitching, sample mounting, etc. About 70% of the girls forced to become wage earners in the skilled trades take up some form of dressmaking. All of the shops and work rooms should be laid out as near actual working conditions in the trade as possible. The advice of the instructor in the various shops, as well as advice of heads of large and successful local industries, should be sought and followed by the school board and architect in designing and equipping the school building.

Vocational schools and junior high schools are the two newest types in schools that have been developed in the past 10 years, and indications point to rapid development in these two types in the immediate future.

44. Continuation or Part-time Classes.—Continuation classes are for the purpose of continuing a pupil's education for a certain time longer, when he has been permitted to leave and go to work at an early age. Part-time classes are organized when an employer recognizes the advantage to him of improving the skill and training of his workmen. He permits certain of his younger employees, at his expense and during working hours, to go to school for special instruction in his particular line of work.

45. Wider Use of School Buildings.—In line with the more intensive use of school buildings for instruction purposes, has come the wider use of these buildings for community or neighborhood purposes. In designing the building, the architect should keep in mind this wider use and arrange certain rooms which are likely to be used by the community so that they are easily accessible without disturbing the school while in session, or so that these rooms can be used at nights or holidays without the necessity of opening up the entire building.

Among the rooms most likely to be used by the community are the following: (1) The auditorium for lectures' moving pictures, plays, concerts, political meetings, etc.; (2) the kindergarten for small dances, receptions by teachers, the home and school association, or similar bodies; (3) the gymnasium for large dances and receptions, for men or boys' gymnasium classes, for neighborhood basketball teams, for boy scouts, etc.; (4) the library as a circulating branch from the central public library; (5) the domestic science room by the Red Cross or other society, a community kitchen, or preparing refreshments for socials or receptions held in other parts of the building; (6) a room so arranged with an outside entrance, or that is near one, so it can be used on election days as a voting place; and (7) toilet and shower rooms made accessible for playing grounds so they can be used during summer vacations, and at hours when school building is closed. These are all uses separate and distinct from the day or evening schools, and should be provided for not only to stimulate interest and pride in the school, but to develop and maintain the best American citizenship.

46. Height of School Buildings, and One-story Schools.—It is an axiom in school construction to have as few stories as possible. Basements with floor lines below grade level are being eliminated. These basement stories contain very much waste space, are oftentimes damp and are always poorly lighted. When the school becomes crowded, classes are practically always placed in these unsuitable quarters either permanently or temporarily, until a new school can be built. In many large cities, notably Boston, New York, Chicago, and Cleveland, basements are being eliminated and the first floor placed one step above the general grade level. This makes the rooms in the lowest story as well lighted and as dry and usuable as any in the building. The heating plant is sometimes placed in a small basement under the ground floor, but as this makes a very deep excavation necessary, it is better to place it in the building on the ground floor, or in an extension outside the main building.

It seems agreed that a building 3 stories or 2 flights of stairs high is about the limit for any school. While in very large cities schools are sometimes built higher, it is an exception, and it is only the congested districts and immense value of land that makes it necessary. Grade schools which are built 2 stories of 1 flight of stairs high are preferred to a higher building.

A recent development in school buildings is the large 1-story schoolhouse. This idea is confined principally to primary or grade schools and has many advantages. It eliminates stairs and in many cases each room has a exit door to the outside on grade, besides being connected to the school corridors, thus making each class room more or less of a unit in itself. Numerous examples of this type of school have been built in California, Oregon, Kansas City, Minneapolis, Rochester, and around Chicago.

The advantages claimed are as follows: (1) Safety from fire and panic; (2) quicker and cheaper to build; and (3) elastic in plan, with additions easily made.

Its one great disadvantage is the size of the plot of ground required and the added cost of this land.

While 12 rooms with auditorium and kindergarten seems to be the average maximum size, the City of Cleveland is building 1-story schools considerably larger in size, made necessary principally by the drastic requirements of the Ohio school building code.

Many of the 1-story schools have a minimum amount of light admitted from the side walls, with the majority of the light coming from an overhead skylight. This has a special advantage in those rooms facing south, where during a greater part of the day the window shades have to be pulled down on account of the sun shining into class rooms. The skylight is built on the principle of the saw-tooth factory roof, and faces north. No sun can shine into the room through this type of skylight and yet the desk farthest from the outside windows is as well lighted as those next to the windows.

It is predicted that the 1-story schools, with floor on grade, without basements, will come into very general use in our smaller cities, for medium size grade school buildings.

47. School Building Measurements.—In order to bring about a standard of comparison as to cost, pupil capacity, cubature, etc., the following report has been adopted by the American Institute of Architects, and also by the Committee on Standardization of School Buildings of the National Educational Association. It is recommended and urged that these directions be closely followed in preparing data on school costs, etc.

For the purpose of obtaining comparable data upon the educational utility and cost of school buildings, they shall be classified, measured, and defined as follows:

Educational Classification: School buildings shall be classified, educationally, as: lower elementary, upper elementary, high, or secondary.

Lower Elementary: Shall be defined as a building containing class and kindergarten rooms, together with the usual accessory rooms, such as principal's office, teachers' rooms, play rooms, toilets, etc., and used for the lower elementary grades only.

Should a school building of this type be provided with assembly room, gymnasium, or other special rooms, it shall fall into the next classification.

Upper Elementary: Shall be defined as a building containing lower or upper elementary grades, and in addition to the regular class and accessory rooms, an assembly hall, gymnasium, and such special rooms as may be included for upper grade or special work, which may include elementary science, elementary industrial training and household arts.

This classification would thus include the Junior High School, the Elementary Industrial or other types of special elementary schools.

High or Secondary: Shall be defined as a building containing class rooms, recitation rooms, laboratories, and such special rooms as are necessary for classical, technical, commercial, industrial, household arts, normal, agricultural, or other purposes required for secondary or junior college education.

Construction Classification

Type A.—A building constructed entirely of fire resistive materials, including its roof, windows, doors, floors and finish.

Type B.—A building of fire resistive construction in its walls, floors, stairways and ceilings, but with wood finish, wood or composition floor surface, and wood roof construction over fire resistive ceiling.

Type C.—A building with masonry walls, fire resistive corridors and stairways, but with ordinary construction otherwise, *i.e.*, combustible floors, partitions, roofs and finish.

Type D.—A building with masonry walls, but otherwise ordinary or joist construction and wood finish.

Type E.—A frame building constructed with wood above foundation with or without slate or other semifireproof material on roof.

Note: Should buildings of any of the above classifications be erected without complete ventilating systems or other mechanical equipment, due note should be made of such fact in reporting its cost data.

Cost Units

To determine educational utility of the building, obtain the *cost per pupil.*

To determine construction cost of building, obtain the *cost per cubic foot.*

The divisor to be used to determine the *cost per pupil*, shall be determined by the number of pupils normally accommodated in rooms designed for classes only. In arriving at the number of pupils, special rooms are to be figured at the actual number of pupils accommodated for one class period only. Auditorium or assembly rooms are to be ignored, but gymnasiums may be figured for one or two classes, as the accommodation may provide. No gymnasium, however, shall be accredited with two classes, if below 40 × 70 ft. in size.

Cost per Cubic Foot.—To obtain the cube of a school building, multiply the area of the outside of the building at the first floor level by the height of the building from 6 inches below the general basement floor to the mean height of the roof. Parapet walls, stacks and other projections beyond the mean height of the roof, as well as balconies and porches not contributing to the actual usable floor of the building, are to be ignored.

Where portions of the building are built to different heights, each portion is to be taken as an individual unit and the rule as above applied.

Cost Items

The cost of school buildings shall be divided into four general items:

First.—Cost of land and grading.

Second.—Cost of building construction.

Third.—Cost of furniture and fixed equipment.

Fourth.—Cost of architects', engineers', brokers' and supervision services.

First.—*Cost of land and grading* should include the cost of the site and the necessary grading to place it in condition to receive the building. Should the site be abnormal and require piling, filling, quarrying, or other unusual expenditures to place it in normal condition to receive the building, such costs are also to be charged up against the site and not the building.

Second.—*Cost of building* should include (*a*) general contract and any sub-contracts pertaining to the general construction of the building, as, for example, excavating, masonry, fireproofing, steel construction, carpentry, cabinet work, sheet metal work, roofing, painting, etc.

(*b*) All contracts for electrical work, plumbing, vacuum cleaning, sewage disposal, heating and ventilating, clock systems, blackboards, elevators, or any other contract for any part of the building not included above, necessary to complete the same, ready for occupancy.

(*c*) The cost of all site improvements, such as walks, drives, yard paving, fencing, and landscape gardening.

Third.—*Cost of furniture and fixed equipment:* (*a*) Should include cost of all portable furniture and cabinets; all laboratory and shop equipment; and all other equipment which would not be classified as "Educational Supplies."

(*b*) All decorations, including special painting or decoration of any kind that may not be included in the general painting contract. Hangings, rugs, pictures, casts, and other forms of decorations furnished at the time of the occupancy of the building which are not classified as "Educational Supplies."

Fourth.—*Cost of architects', engineers', brokers' and supervision services* should include the cost of all plans and specifications, architects', engineers', landscape gardening and supervision and all other experts' services and expenses.

48. Orientation of Building.—In cities where ground space is limited and streets laid out, it is already settled which way the building will face. In rural sections and on large sites more choice is possible. It is generally agreed that where possible all rooms should have sunshine some time during the day. This can best be done if the building is faced midway between the cardinal points. Otherwise, the majority of the rooms should face either east or west. Southern exposured is objectionable because the curtains have to be lowered most of the day; this reduces the light to considerable extent and is otherwise annoying. Suushine is not objectionable in laboratories, in fact is quite desirable; bilateral light in these rooms is also satisfactory. The pupils move around in various positions and are not confined to one spot, as in a class room. This free movement of the pupils in laboratories and shops permits them to adjust the light to the work they are doing and under these circumstances there is no objection to bilateral lighting.

49. Class Rooms.—The unit of the school is the class room and the building is built primarily to accommodate these rooms. Laws of different states vary as to the number of square feet and cubic feet to be allowed per pupil in class rooms. In Pennsylvania and New York it is 15 sq. ft. of floor space and 200 cu. ft. of air space per pupil. In New Jersey it is 18 sq. ft. floor space and 200 cu. ft. air space. In Ohio, 16 sq. ft. and 200 cu. ft. for primary grades, 18 sq. ft. and 225 cu. ft. for intermediate grades, and 20 sq. ft. and 250 cu. ft. for high schools. In grade schools 40 to 42 pupils are usually accommodated in a standard class room while in a high school 30 to 35 pupils is the custom. The minimum height of class rooms is usually placed at 12 ft. In New York State, 13⅓ ft. is the minimum and is arrived at by dividing 15 sq. ft. per pupil into the required 200 cu. ft. per pupil which gives a result of 13⅓ ft. Sizes of class rooms vary slightly, a room 24 × 30 ft. accommodating 40 pupils in New Jersey. A standard class room in New York City is 24 × 28 ft., in Pittsburgh, 24 × 32 ft. 6 in; in Boston, 23 × 29 ft. for lower and upper elementary grades and 26 × 32 ft. for junior high schools. Recitation rooms in Boston are made 16 × 26 ft., or one-half a class room. It would seem that 24 ft. is the maximum width that can be recommended for a class room, while 22 or 23 ft. is a more desirable width.

Where wood joists are used in floor construction, economy dictates it should be planned so the stock lengths of 22, 24, or 26-ft. timbers can be used. A maximum length for a class room of not over 34 ft. is good practice. In rooms longer than this the teacher's voice reaches the last rows with difficulty, and scholars have trouble in reading work placed on front blackboards.

Unilateral or lighting from windows only on the long side of the room on the left side of the pupil is the best practice, and is insisted upon in most states where there are any requirements at all. Under certain conditions bilateral lighting is permitted, with a minimum of light in rear of room at back of pupils. Light should never be admitted through windows in front of rooms, with children facing it. In kindergartens, shops, playrooms, gymnasiums, laboratories, bilateral lighting is permitted. Window heads should be kept close to ceiling so as to project the light as far across the room as possible; it is a good rule that the width of the room shall not exceed 1½ to twice the distance from floor to head of window. The net glass area, after deducting all area occupied by frame, sash, muntins, etc., should not be less than 20% of the floor area of the space which it illuminates.

Blackboards of slate ¼ in. thick should be installed on all available wall surfaces. In primary grades the chalk trough is placed 26 in. above the floor; in intermediate grades 30 in.; and in high schools 33 in. Slate blackboards come in stock widths of 3 ft. 6 in., 4 ft., and 4 ft. 6 in. Boards 4 ft. wide are to be preferred. Near front end of room a bulletin board should be installed in same frame as the blackboard. This is usually made of cork so exhibits and notices can be pinned to them—size of panel about 4 ft. high by 3 to 5 ft. long.

Window openings on inside should have trim omitted and plaster returned into jambs and heads. Plain wood sill and aprons are generally used, but slate or bull-nosed glazed brick sills are very desirable so growing plants can be placed in windows, or windows left open and no varnished woodwork to be repainted when damaged by water.

Floors should be of maple, rift sawn yellow pine, or other good hard wood depending upon local conditions—plain wood base about 7 in. high, with quarter-round molding top and bottom. If glazed brick or slate base can be afforded, it is desirable on account of washing compounds used on floors which eat off the varnish of the wood base.

A minimum amount of plain wood trim should be used, either of oak, chestnut, or similar hardwood depending upon locality. Picture molding should be used in all rooms and corridors. Combined bookcase and stationery closet is required in each class room, also steel or wood lockers for teachers' wraps. Special color finishes on woodwork are to be discouraged. The raw wood should be stained slightly to make it approximate the color of "golden oak." This permits furniture of standard shade to be purchased and match wood trim of room and also avoids trouble later on when any additional furniture is needed in matching same with the special color of the finish in the room. Plaster walls and ceilings should have a smooth finish; sand finished surfaces are not desirable for sanitary reasons. Painting of walls should be included in the building contract.

One door to corridor at teacher's end of the room where it is under control is sufficient. Doors should be 3 ft. 2 in. to 3 ft. 6 in. wide and 7 ft. high with small clear glazed panel in upper part. Door should open out from class room into corridor. Transoms are seldom used.

50. Wardrobes.—Provision has to be made to take care of pupils' clothing and a distinction is usually made between wardrobes and cloak rooms. A wardrobe is a shallow closet and a part of the room, while a cloak room is a separate room about 5 to 6 ft. wide located at one end of the class room. Cloak rooms have been preferred up to recent years when economy has encouraged the use of the wardrobe scheme. A saving in length of a class room unit of about 4 ft. is accomplished in the use of wardrobes, as these occupy a width of 2 ft. against 5½ to 6 ft. for cloak rooms. This amounts to quite an appreciable saving in a building 4 or 6 class rooms in length. In either the wardrobe or cloak room scheme, thorough ventilation should be provided.

51. Corridors.—*Width.*—Minimum 8 ft. where serving 4 class rooms, 10 to 12 ft. wide where more class rooms are taken care of. Width of corridor increases in proportion to its length and distance between staircases. Where staircases occur at ends of corridor, 10 ft. is the minimum width. Some authorities recommend extremely wide corridors up to 14 to 16 ft. There is no objection to this, in fact, it is desirable if it can be afforded. A compromise plan is to make the side corridors the minimum width, with a front corridor 12 or 14 ft. wide that can be used for various purposes, such as exhibition space, reception hall, etc. High school corridors should be wider than those in grade schools, so as to afford proper room for circulation, as the high school classes change and pupils move in different directions every 40 minutes.

Light.—Direct outside light and air desirable, at least enough so that no artificial light will be required under ordinary conditions.

Floors.—May be wood, asphalt, cement, terrazzo, tile, heavy linoleum glued down, composition, or any material that will withstand heavy wear, and that is non-slipping, noiseless, and sanitary.

Wainscoting.—The lower part of plaster walls gets excessively heavy wear, a protective wainscoting of some kind is desirable, may be glased brick or tile, or the cement or composition floor continued up the side walls. Fabric glued to the walls is not satisfactory on account of tendency to peel up at joints. Wood should not be used on account of fire hazard.

52. Stairways.—*Location.*—Should be properly distributed in order to serve equally all parts of building. Where located at ends of corridors, have the advantage of being always in sight and saving space in building.

Number.—Laws of different states vary. Two staircases are sufficient when there is not more than 8 class rooms on second and third floors. Building with 9 or more rooms on upper floors should have 3 or more staircases depending upon size of building. Another rule is sufficient stairs to empty building within 3 min., counting that 120 pupils can pass a given point two abreast in 1 min.

Width.—Should be sufficient for two pupils walking side by side, but too narrow for three. Ordinarily 4 to 5 ft. wide for each run. Wider stairs should be at least 8 ft. wide for each run with hand rail down center made continuous around landings.

Construction.—Should themselves be fireproof if possible, even in frame buildings, and always enclosed in fireproof walls with smoke screens separating them from the corridor. May be iron with slate or other treads, or reinforced concrete with iron safety treads. High balustrades at center between runs, open if iron or solid if concrete. Hand rails both sides of all runs. Stairs should have two runs to each story, with landing in center and one flight returning on the other. Rise of steps should be 6 to 7 in. No winders permitted. Where boys' and girls' toilets are located in basement, two staircases shall extend to basement. No closets for storage purposes permitted under stairs. Where small differences in levels occur between different portions of building, an inclined plane or ramp should be used instead of a few steps. At bottom of stairs should be a vestibule between it and the outside air. Vestibule provided with heat to prevent cold outside air from coming directly into staircase enclosure and making the temperature in same appreciably different from the temperature in corridor. Other special types of staircases are used, such as the duplex stairs in New York City, and the smoke proof factory tower used in Philadelphia.

53. Toilet Rooms.—*Location.*—In grade schools, principally on lower floor accessible from indoor playroom and outdoor playgrounds. Also desirable to have minor emergency toilets on upper floors. In high schools where classes change every 40 min., toilets are best distributed throughout the building, where they are easily accessible when classes change.

Number of Fixtures.—Opinions differ as to correct number of fixtures for a given number of pupils. The tendency is to install too many fixtures, rather than too few, with a corresponding waste of money. Good practice seems to dictate one W. C. to each 25 boys and one urinal to every 25 boys. For girls, one W. C. to every 25. Two or three lavatories for each toilet room depending upon the size.

Type of Fixture.—Water closets should be seat action, and as near "fool proof" as possible. Open front seats recommended. Individual porcelain urinals preferred to slate or soapstone. Urinal flushed automatically from tank, and turned off at night. Continuous range water closet and trough urinals should not be used.

Floors.—Some nonabsorbent materials such as cement, asphalt, or tile. Also desirable to wainscot room, with brick, tile, or cement.

Lighting.—Plenty of light and air are essential and more important than in many other rooms.

54. Kindergartens.—*Location.*—On lowest class room story, corner room with southern exposure preferred, bilateral lighting permitted.

Size.—Larger than a regular class room and equal to an area of 1000 to 1500 sq. ft. Often arranged so it can be divided into several smaller rooms with folding doors so class can be separated into small units.

Design and Equipment.—Usually made more attractive than a class room, walls paneled with high wainscot, plaster walls above painted and stenciled and often decorated with nursery scenes. Fireplace sometimes installed at one end of room. Plaster casts and pictures of juvenile subjects hung on walls. Flower boxes placed in windows. To give greater area to room, a bay window is often installed, in which is located a low down window seat. A separate entrance is desirable, as the kindergarten should be a separate unit in itself so that the small children have no reason to go into the main part of building, either for entrance, dismissal, or otherwise. It should have its own wardrobe and toilet room fitted up with juvenile size fixtures, also wardrobe space for two or three teachers. A drinking fountain, set down low so it can easily be reached, should be located in room. Plenty of storage space in closets or lockers should be provided for toys and material. Little blackboard space is necessary, but cork display boards for tacking up exhibits should be plentiful.

55. Gymnasiums.—Many states have enacted physical training and military training laws and are requiring instructions in same as part of the course of study in the school. This makes necessary large gymnasiums and playgrounds for drill and exercise purposes.

Location.—The gymnasium can be located either on the ground floor or upper story; the ground floor having the preference, because it has direct access to the playground, and can also be used more conveniently at night for community purposes.

Size.—In high schools it should be large enough to be used by the community at night for playing basketball. The minimum size of a basketball court is 35 × 60 ft. while the maximum size is 50 × 90 ft. At least 3 ft. should be allowed on all sides of the court. If companies of pupils drill in the gymnasium, it should be at least 50 × 70 ft. in size or larger. In high schools of 800 or more pupils, one gymnasium is not sufficient to take care of all classes. In this case, economy can be effected by providing an additional exercise room. This room can be the area of about two class rooms and can be used efficiently for all ordinary purposes. The large gymnasium can be used by the boys and girls alternately, or at such times as they have basketball games or other special exercises.

Equipment.—In the larger gymnasiums, running tracks are sometimes installed, but the tendency is to do all the running possible in the open air. Galleries are provided for spectators to watch the interscholastic games.

Height.—The height of the room should not be less than 18 ft. nor more than 25 ft. If lower than 18 ft., there is not sufficient swing for the flying rings. If higher than 25 ft., the supports for these rings must be hung down to this level.

Floor.—A maple wood floor is practically always installed in a gymnasium.

Minor Rooms.—Off the gymnasium should be located the Physical Director's office and also the boys' and girls' locker rooms, toilets, and shower rooms. A drinking fountain should be installed to avoid the necessity of pupils going out of the room for water. A room should be provided to store apparatus when it is desired to clear the floor for basket ball, a dance, or other purposes.

56. Swimming Pools.—The importance of everyone knowing how to swim is becoming more and more realised as time goes on and made part of the high school curriculum. It is only the high cost of installation and maintenance that prevents the more universal use of this item of education.

Location.—On lower floor.

Construction.—Should be built in the most sanitary way, using impervious tile or glazed brick. It takes constant care and attention to keep a swimming pool sanitary under the best conditions, so that pools built of cement or any absorbent material should be avoided.

Size.—The length of the pool should be 45, 60, or 75 ft., or in any case a multiple of 3 ft., as swimming contests are always measured by yards. The pool need not be very wide, especially for beginners, who are more easily reached in case of need in a narrow pool, the width being usually from 20 to 25 ft. The desirable size pool for a high school is at least 20 × 60 ft. The depth of the pool at the shallow end averages 3 ft. 6 in., while at the deep end about 8 ft.

Minor Rooms.—In connection with the pool should be the locker and dressing rooms with their shower baths, toilets, towel supply room equipped with laundry tubs.

Temperature, Light, Etc.—The pool room should have plenty of natural light and ventilation and should be kept warmer than the ordinary class room. It must be remembered that many of the children using the pool are undernourished, and the temperature of the water should average around 74 to 76 deg. or more to avoid discomfort.

Equipment.—The pool must be equipped with heater to keep the water in the pool at the proper temperature, a pump to circulate the water, and a filter and sterilizer to purify the water. As the pool has a capacity of 50,000 to 60,000 gal., it necessarily cannot be emptied except occasionally; the average seems to be once per week where the pool is being used to any great extent. It usually takes about 24 hr. to fill the pool and to bring the water up to the proper temperature.

57. Library.—It should be decided whether the library is to be for the school only, or a circulating library run in coöperation with the central public library serving a community purpose.

Location.—If for the school only, it can best be located at some central point in the building near Study Hall. If for community purposes, it must be located on the ground floor near an entrance, as to be of the most use, it will have to be open at times when the school is closed.

Size.—The tendency is to give more space to the library and to require the pupil to get familiar with its proper use. Not less than 1000 to 2000 sq. ft., depending upon size of school and number of books in library. An attendant is always at hand to give assistance and very often a stock room and work room are also included.

Equipment.—Bookcases, reading tables, and chairs, magazine racks, card catalogs, librarian's desk. The room should be made attractive and given a library atmosphere.

58. Auditorium.—*Location.*—It should be centrally located and made accessible not only to the pupils, but to the general public.

Size.—In high schools it should accommodate the entire student body at one sitting, while in grade schools it may or may not accommodate the entire school, often ½ to ⅔ of the pupils will be sufficient, as the younger pupils are not usually brought into the auditorium at the same time as the older ones.

The seating capacity may be determined by dividing the area of the room in square feet, not including the stage, by 6½ sq. ft. for each person, which includes the necessary aisles. Seates are usually 19 or 20 in. wide and spaced at least 30 in. back to back. Width of center aisles are 3 ft. at their narrowest part and increased towards rear at the rate of 1½ in. for every 5 ft. in length. Side aisles 2 ft. 6 in. wide at narrowest part, increasing ¾ in. to every 5 ft.

Equipment.—Provision should be made for stage curtain and scenery for school and community plays. The stage should be liberal in size to take care of large graduating classes, community chorus, or orchestra, and should be accessible from the rear for the speakers and players without the necessity of their passing through the audience. An electric plug should be installed for stereopticon and moving picture lantern, a moving picture booth and a stereopticon curtain. Arrangements should be made for darkening the auditorium in the daytime.

Where the auditorium is used for study, lecture, or recitation purposes, several rows of seats in front should be provided with folding tablet arms so pupils can take notes or write. Every other seat should be thus equipped, leaving the intermediate seats for the pupils' books, etc.

Where the corridor extends along either side of the auditorium, openings can be cut through the wall and serve as an overflow space for the audience during commencement and other times. These openings should be closed with obscure glass windows so that the auditorium can be used and view from corridors cut off when desired.

59. Chemical Laboratory.—*Location.*—Usually on top floor, corner room, bilateral lighting.

Size.—Area of 1200 to 1500 sq. ft. for large schools.

Equipment.—Three long chemistry tables accommodating four pupils on each side, or total of 24 pupils. Fume hoods with special ventilation, and chemical storage closets against walls. Gas and water connection at tables for each pupil, also sinks at ends of table and against walls. Electric connection to each table. Blackboard and cork display board.

In connection with chemical laboratory should be a small instructor's office, a chemical stock room, and a preparation room.

60. Physical Laboratory.—*Location.*—Usually top floor and adjoining chemical laboratory.

Size.—1200 to 1500 sq. ft. for large schools.

Equipment.—Six physical laboratory tables accommodating two pupils on each side, total of 24 pupils. Electric and gas connections at table for each pupil. Provision for different kinds and voltage of electricity at each table usually obtained through motor generator set, and switchboard with proper instruments. Closets for instruments and equipment.

A store room for apparatus, a preparation room, and a photographic dark room equipped with sink, should adjoin and be part of the physical laboratory.

61. Combined Physical and Chemical Laboratories.—In schools where classes are small it is possible to combine the physical and chemical laboratories by equipping with combination furniture. At one end of room can also be placed an instructor's demonstrating table with tablet arm chairs in front of same, thus eliminating the science lecture room.

62. Science Lecture Room.—*Location.*—Adjoining or between chemical and physical laboratories.

Size.—Depending upon number of pupils in science department, usually large enough to seat two classes.

Equipment.—Tablet arm chairs on raised platforms, instructor's demonstrating table in front of room, with water, gas, and electric connections, fume hoods, stock cabinet and blackboard back of demonstrating table, steropticon electric outlet and stereopticon screen, also provision for darkening room in daytime.

63. Biological Laboratory.—*Location.*—Adjoining other laboratories on upper floors unilateral or bilateral lighting with one side southern exposure if possible.

Size.—Area of about 1200 to 1500 sq. ft. and accommodating 24 pupils.

Equipment.—Flat top tables and chairs, large soapstone sink, aquarium, exhibition and storage cases, instructor's demonstrating table in front of room. If school has a conservatory, it is located in connection with this laboratory.

64. Bookkeeping Room.—*Location.*—No special requirements.

Size.—Equal in area to 1200 sq. ft. or more, depending upon number of pupils to be accommodated.

Equipment.—Individual bookkeeping or commercial desks for each pupil, store closets for stationery, school bank enclosure located at one end of room.

65. Typewriting Room.—*Location.*—Connecting with bookkeeping room.

Size.—About same size as bookkeeping room.

Equipment.—Individual typewriting desk for each pupil, cases or closets for storing stationery, wash basin for washing up after changing typewriter ribbon or cleaning machine.

66. Stenography Room.—*Location.*—Between and connecting with bookkeeping and typewriting rooms.

Size.—Same as a recitation room, or one-half to two-thirds of a class room unit.

Equipment.—Tablet arm chairs for pupils. Clear glass partition between this room and typewriting room so teacher can teach class in stenography and at same time supervise pupils practicing on typewriters. Commercial arithmetic, business law and customs, etc., also taught in this room.

67. Cooking Room.—*Location.*—Upper floors preferred although often placed in basement. Southern exposure. May have bilateral lighting if a corner room.

Size.—May consist of one room where all grades are taught, or two rooms—one for elementary cooking and one for advanced work, usually accommodates 24 pupils at one time and should not be less in area than 1200 to 1500 sq. ft.

Equipment.—Flat tables with small individual gas stoves on top, or family size gas ranges, sinks, tables and cupboards when operated on the "unit" plan. Wardrobe for keeping pupils' caps and aprons, dressers, sinks, ice box, hot and cold water supply, pair of laundry tubs for washing out tea towels, etc., also storage closet. Special attention given to ventilation of room.

68. Model Apartment.—*Location.*—Connection with cooking room.

Size.—May consist of only a dining room or in more elaborate building, a complete apartment consisting of bed room, bath room, kitchen, and living room. Should be of similar sizes and arrangement to rooms found in pupils' homes.

Equipment.—Furnished complete same as rooms in private dwelling.

69. Sewing Room.—*Location.*—Preferably adjacent to cooking room.

Size.—Equal in area to 1200 or 1500 sq. ft. depending on number of pupils.

Equipment.—Flat top sewing and cutting tables, usually accommodating 24 pupils; sewing machines, wash basin, pressing tables and electric irons, cabinet with individual drawers for pupils' unfinished work. Curtained off alcove, or small room to be used as a Fitting Room.

70. Laundry.—*Location.*—In connection with other rooms of household arts department.

Size.—Equal in area to 750 to 1200 sq. ft.

Equipment.—Laundry tubs, steam clothes drier, ironing board, and electric irons.

71. Lunch Room and Kitchen.—*Location.*—May be in basement or upper floor adjoining household arts department.

Size.—Depends on number of pupils to be accommodated at one time. Allow 10 sq. ft. per sitting in lunch room.

Equipment.—Operated on "Cafeteria" or "Self-service" plan. Flat top lunch tables seating 4 to 8 each, serving counter at one end of room. Kitchen in connection with this room to be of size and equipment sufficient to take care of number of meals served.

72. Study Rooms.—*Purpose.*—Occurring in high schools which are run on departmental plan and are to accommodate pupils having no recitation during a certain period and whose home class room is occupied by another class at recitation.

Location.—Central and easily accessible from all parts of building.

Size.—Accommodating 35 to 100 or more pupils depending upon size of school.

Equipment.—Pupils' desks like those used in standard class rooms.

73. Music Department.—*Location.*—Should be isolated so noise of practising will not disturb pupils at recitation or study.

Size.—May be several rooms, for choral work, orchestra, band, with several practice rooms, depending on how comprehensive a music course has been developed.

Equipment.—Ordinary class room with chairs and music racks, blackboard for writing music, piano, and storage cases for music and instruments.

74. Bicycle Room.—*Location.*—On lower floor with incline leading to entrance door from outside, near locker rooms if such are included in building plan.

Size.—Depends upon probable number of bicycles used by pupils.

Equipment.—Racks against wall and elsewhere in order to accommodate as many bicycles as possible.

75. Store and Book Rooms.—*Location.*—Within easy access of Principal's office, stock closet in Principal's office for day to day supply, while store and book room accommodates bulk supplies.

76. Teachers' Rooms.—*Location.*—Easily accessible.

Size.—About one-half a class room in area.

Equipment.—Comfortable, furnished like a sitting room, with table, chairs, rug, couch, etc., also toilet room connected. Gas outlet for stove, dresser for dishes, and provision make so teachers can have warm lunch. Individual steel lockers for teachers' cloaks, unless provision is made to care for same in class room.

77. Medical Inspection Room.—*Location.*—Adjoining or near Principal's office.

Size. Area of about 300 sq. ft. divided into waiting room and office.

Equipment.—Flat top desk, chairs, scales, wash basin, toilet, first aid cabinet, and small stock closet. Walls and woodwork, enamel, painted white.

78. Dental Clinic Room.—*Location.*—Near medical inspection room and near minor entrance to building if used by pupils from other schools.

Size.—Area of about 300 to 400 sq. ft. divided into waiting room and office.

Equipment.—Dental chair, instrument and medical cabinet, wash stand, desk, chairs. Wall and woodwork, enamel, painted white.

79. Manual Training Rooms (Woodwork).—*Location.*—In basement or on lowest floor, corner room preferred with bilateral lighting.

Size. Area about 1200 to 1500 sq. ft.

Equipment.—Usually 24 work benches, large soapstone sink, gas outlet for glue pot, blackboard and cork display board, raised bank of seats for demonstration purposes, small room or rack for wood stock, small lock-up room or closet for tools, etc., teachers' closet, floors of wood, ceiling plastered, walls plastered or exposed brick painted.

80. Open Air Class Room.—*Location.*—On top floor of building, preferably a corner room, with windows on two sides. Sometimes adjoining roof which is used as a play, rest, or study space, and covered with awning in summer.

Size and Equipment.—About 750 to 1000 sq. ft. area with adjoining closets for storage of reclining chairs and blankets, small toilets for both sexes. Also small room used as diet kitchen, with refrigerator, sink, gas stove, and cupboards. Windows arranged to open 100% and room protected from driving rains, while windows still remain open. Desirable to arrange heat and ventilation so room may be used for regular class room if desired.

81. Board of Education Room.—*Location.*—Nearby and easily accessible from Secretary's office and Superintendent of School's office on main floor of building near entrance.

Size.—Depends upon number of members of Board, size of school system, and amount of room available.

Equipment.—Long board table and chairs, also chairs for public, and newspaper representatives. Toilet room accessible and provisions for taking care of members' cloaks.

82. Superintendent of School's Office.—*Location.*—Near main entrance and Board of Education room.

Size.—Depends upon size of school system. Should be an outer or clerk's office, and inner private office. Board of Education room sometimes serves as Superintendent's private office as well as Board room.

Equipment.—Fitted up with office furniture.

83. Secretary of Board of Education.—*Location.*—Near Superintendent's office and Board room, also near main entrance.

Size.—Depends upon size of school system and may or may not have both public and private offices.

Equipment.—Fitted up with office furniture including a large safe or built-in fireproof vault for records.

84. Principal's Office.—*Location.*—Near visitors' entrance to building on main floor.

Size.—Area of 300 to 400 sq. ft. and should have an outer or public space, and an inner private office.

Equipment.—Fitted up with office furniture, also ample supply closets and toilet facilities.

Provision should also be made for night school Principal and Truant Officer.

85. Rest or Hospital Room.—*Location.*—Some secluded and quiet place. Also advantage to have near teachers' room.

Size.—About 300 sq. ft. area.

Equipment.—Chairs, table, couch, medicine cabinet, toilet facilities.

86. Play Grounds.—Larger play space is being insisted upon. Space around building should not be less than 200 sq. ft. for each pupil accommodated in the building. Surface should be of rolled clay and sand mixed, which will drain quickly and easily after a rain, and not be muddy. Proper play ground equipment is desirable.

87. School Gardens.—Adjoining the play ground should be space for a school garden, laid off in plots for each class and pupil. If we are to make our future citizens appreciate the farm and its importance, we must stir up the pupil's interest in growing things by the actual experience of having part in raising something with his own hands.

88. Flag Pole.—State laws require generally that an American flag shall be displayed on a proper flag pole when school is in session and on legal holidays. The flag pole is therefore usually included in the building contract. It is better located on the school grounds rather than out of a window or on top of the buildings where it is bothersome to get at. On the ground it can be used as a rallying point, and at certain times the entire school lined up around it to salute the flag. The flag pole can be given a little dignity by a proper base of iron and concrete seat around same, rather than simply embedding it in the ground. Flag poles are usually of wood, 40, 50, 60 ft. or more in height. Steel flag poles are used in some cities with success, but care should be exercised to give them some diameter and not have them look like a pipe stem.

89. Fireproof, Semi-fireproof, Fire Protection.—Needless to say every effort should be made to have our new schools fireproof. Semi-fireproof usually means masonry outside walls and corridor walls, with fireproof floors in corridors, over boiler and manual training rooms, and fireproof stairs. The floor construction in class rooms and roof construction are in this case of heavy timber. The first essential is the safety of the life and limbs of the children. To this extent the semi-fireproof building is practically as safe as a fireproof one, inasmuch as a school building can be emptied within two minutes, if properly designed and frequent fire drills are held. There is an economic loss in a fire, that we should try to eliminate, and fireproof buildings at slightly higher cost will accomplish this, and at the same time cost less for maintenance and insurance. All schools should be equipped with fire alarms, fire stand pipes and hose, also chemical fire extinguishers; all of which should be frequently inspected and kept in good working condition.

90. Standardization.—Most cities where an architectural department is maintained to design all the schools, or where schools are constantly being built, have standardised their requirements and embodied them in book form for use in designing future building. The standards of Boston, New York, and Pittsburgh are available for the asking.

In order to determine upon standards which are acceptable to the country generally outside the large cities, the National Educational Association has a committee on the Standardization of School Buildings, whose reports are also available.

OFFICE BUILDINGS—ECONOMICAL PLANNING AND GENERAL DESIGN

By Frederick Johnck

The planning of an office building is entirely a problem of securing a sufficient amount of good light floor space on the site selected so that the net income will be large enough to make the investment on the land and building profitable to the owner. The plan must be such that the space can be divided into small or large offices to meet the tenants' requirements. To make this possible the elevators, smoke stack, pipe and wire shafts, and stairs are generally arranged along a dead or alley wall so as not to use good light space that can be more profitably used for offices. A very determining point in the location of the elevators, stairs, etc., is the entrance from the street. While it may be to the advantage of the offices to enter the building on the main street, it must be borne in mind that space thus taken for vestibule and corridors has a very high rental value as store space. In considering the plan it is quite safe to say that the rental space in the basement and in the first and second floors will be used for stores, a bank, or by an insurance company. The rental of these three floors should be enough to carry the investment.

In regard to the number and size of elevators to be installed, see chapter on "Elevators and Elevator Service" in Part III.

91. Toilets.—In the early office buildings erected, a large toilet for men and one for women were arranged on the top floor, but as this space was light it was too valuable. After that the toilets were arranged on the light court side on one of the lower floors. In some of the latter types, smaller toilets have been arranged on each floor. This is more desirable from a tenant's point of view and saves on elevator service for the building owner. In this scheme, a main toilet for men should be provided on one of the lower floors near which the barber shop can be located. A main toilet should also be provided for women and a small rest room should be maintained in connection with it. These main toilets will serve for the stores on the basement, first, and second floors. In the smaller type of office buildings, it is well to provide small toilets for men and women on alternate floors. When this is done, a small urinal toilet should be provided for men on all floors.

92. Pipe and Wire Shafts.—Pipe and wire shafts should run continuous from the basement to the top story. They should be conveniently located and accessible for repairs and installation of new work. In addition to the main pipe shaft, a number of smaller ones should be built so that lavatories can be placed in each office or suite of offices. A great deal of care should be taken in locating the wire shafts so that the conduits for each floor can enter the shafts without difficulty. If it is possible to have two wire shafts, one at each end of the building, it is well to do so as this will reduce the length of the home runs in the wiring and consequently reduce the cost of the building. All pipe and wire shafts should be enclosed in tile and have all openings protected with metal doors so as to reduce the fire risks.

93. Floor Finish.—In the office sections, it is customary to use a maple floor on sleepers. The top of the floor should be at least 4½ in. above the top of the floor construction, so as to give sufficient space for runs of pipe and conduits. Floors in corridors and in toilets should be of marble or tile.

94. Wire Molds.—Wire molds of ample size to conceal telephone and A.D.T. wires should be provided in the corridors, as these wires are constantly being changed. They can be run open in offices, although they are often concealed.

95. Type of Construction.—All office buildings should be of fireproof construction. The particular type of construction depends largely on the height of the building and the condition of the steel market. It is safe to say that all buildings 10 or more stories in height should be of the skeleton steel type with steel girders and beams, and tile arches. Buildings from 4 to 10 stories can be built with concrete columns, girders, and joists with tile fillers. The low live load required for buildings of this class make it rather uneconomical to construct them with concrete floor slabs, as by so doing the dead load is increased beyond the point of economy.

96. Arrangement of Offices.—For high office buildings in large cities, the arrangement of an outer and an inner office has been found to be the best from a rental point of view (see Fig. 24). If two or more tenants desire to have offices together, the dividing partitions between the inner offices can be omitted, as shown in Figs. 25 and 26. By this arrangement the tenants' expenses are decreased since the same telephone switchboard and stenographic force can be used jointly by the tenants. In the new four and five story office buildings that are now being erected in the smaller cities, the inner office is not considered a desirable rental feature due perhaps, to two reasons: (1) the office force for this class of tenants is smaller than for tenants in larger cities; and (2) on account of a smaller rental value, the maintenance on this waste space greatly reduces the net profits on the investment for the owner.

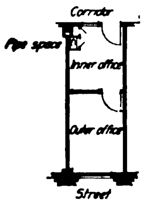


FIG. 24.—Single suite of inner and outer offices.

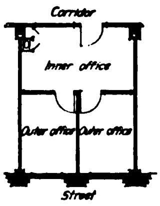


FIG. 25.—Double suite of inner and outer offices.

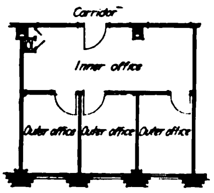


FIG. 26.—Triple suite of inner and outer offices.

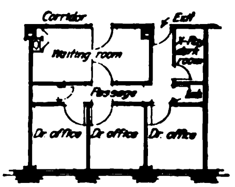


FIG. 27.—Doctor's suite of offices.

One other special feature in office planning is the arrangement of offices required by doctors. As it is very undesirable to discharge a patient through a general reception room, an inner passage connecting to the outer corridor should be provided, as illustrated in Fig. 27. In office buildings occupied by doctors and dentists, provisions should also be made for laboratories, and dark rooms for X-ray work. A space should also be arranged for a drug store.

97. Office Requirements.—In addition to the ceiling outlet, every office should have base plugs for desk lights and fans. A lavatory with hot and cold water should be provided in each suite of offices. These are sometimes concealed with a double wardrobe, one-half for the lavatory and the other half for clothes. The tops of these wardrobes should be left open to permit a free circulation of air. For doctors and dentists, it is also necessary to provide gas outlets, and compressed air. Lavatories in these offices should be of the pedal control type.

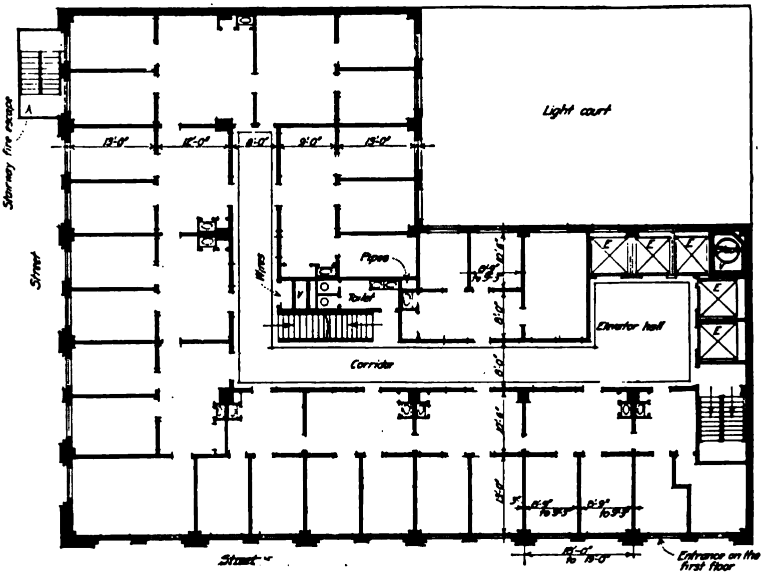


Fig. 28.—Typical plan of high office building on corner lot.

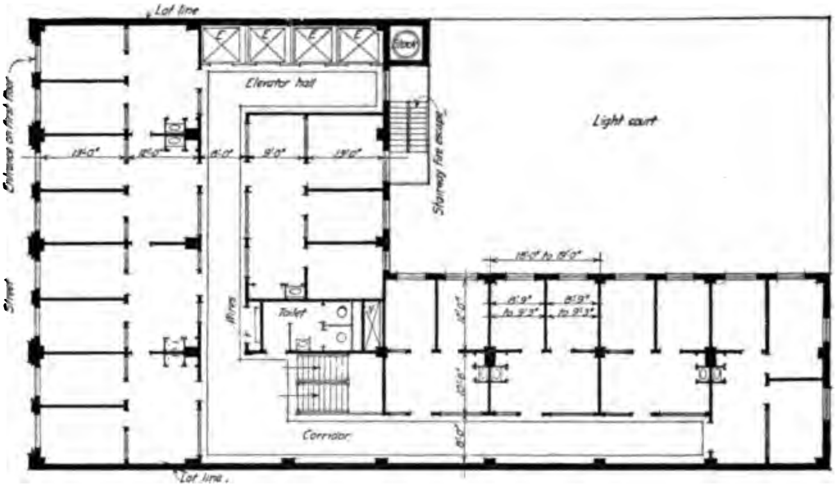


Fig. 29.—Typical plan of high office building on inside lot.

98. Story Heights.—First and second story heights in office buildings vary, depending upon the requirements of the tenants. If the first two floors are used for stores, the first story height can be from 15 ft. 6 in. to 17 ft. 6 in., the second story height from 12 ft. 6 in. to 14 ft., and the typical stories 11 ft. 6 in. to 12 ft. 5 in.

99. General Plan.—An office building on a corner lot naturally gives the maximum number of light offices. If the lot has a greater width than 50 ft. for a high building, a light court is necessary. For low buildings in smaller cities, a court is necessary in buildings wider than 25 ft. Fig. 28 shows a plan of a medium size high office building on a corner lot. In Fig. 29 is a plan of a high building on an inside lot. This scheme permits only a few offices on the street front while the greater portion of them are on the light court.

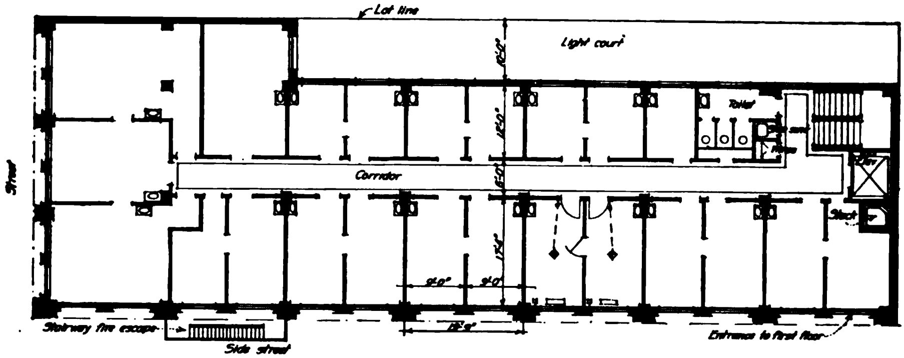


FIG. 30.—Typical floor plan of 4 or 5-story building on corner lot. Entrance on side street.

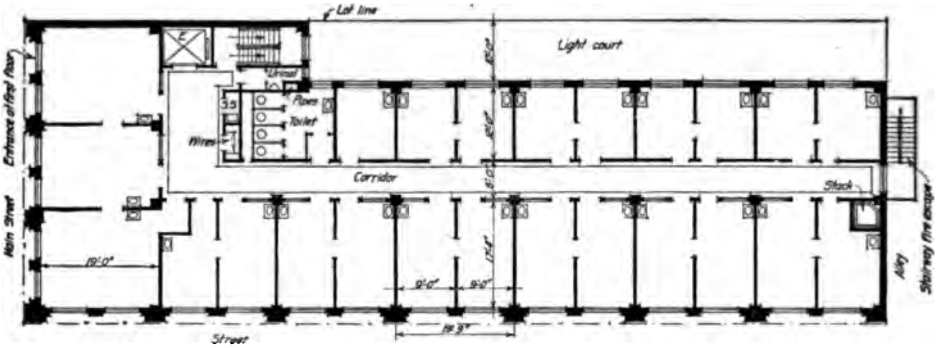


FIG. 31.—Typical plan of 4 or 5-story office building on corner lot. Entrance on main street.

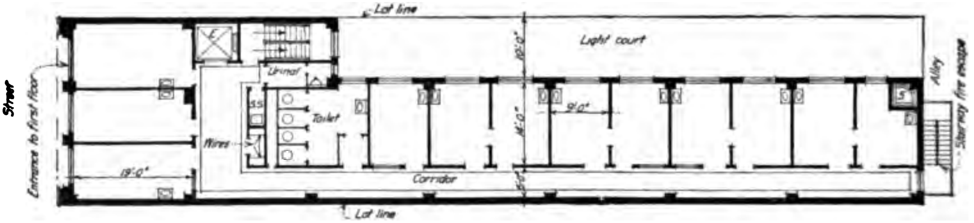


FIG. 32.—Typical plan of 4 or 5-story office building on inside lot.

In Fig. 30 is a plan of a low office building on a corner lot with the entrance on the side near the alley. Fig. 31 is a plan of a low office building on a corner lot with the entrance on the main or more important street. In Fig. 32 is illustrated a plan of a low office building on an inside lot.

100. Column Spacing.—The column spacing is determined by the width of the office required; the width and length of the lot for equal spacings; and the necessity of using economical sizes of steel beams and girders. A spacing of about 19 ft. has been found to be very good and permits two offices 9 ft. wide in each bay.

101. General Design.—The architectural treatment of the exterior is a problem in which cost and available material are important factors. In a general way the exterior design may be treated as a flat wall surface with terra cotta or stone cornices; or it may be designed with strong horizontal bands at the window sills and heads; or it may be treated with vertical piers with a Gothic effect. If the amount of money at hand is small, it is well to treat the main body of the building in a very simple dignified manner and only use ornamental and molded stone or terra cotta to mark the entrance to the building. The question of any particular style of ornament to be used is a matter of individual taste and opinion. In the designing and detailing of the ornament a human interest can always be worked in so as to give the building distinctive character.

PUBLIC COMFORT STATIONS

By Frank R. King

The term "public comfort station" denotes a structure planned for the convenience of the general public, in which the use of sanitary toilet facilities constitutes the principal service rendered. It is generally desirable to maintain rest rooms in connection with them. A public comfort station may take the form of a privy or an inside toilet room with washing facilities—the type depending upon the size of the community, the availability of water and sewerage connections, and the amount of funds at disposal for the purpose. Sanitary equipment of only the highest grade should be employed, inasmuch as constant public use makes the wear and tear more injurious than in the average toilet room.

As these stations are for the public's benefit, provision for their erection and maintenance should be regarded as a public function, supported by the funds of the state or municipality concerned. Such funds may be raised by direct taxation or bond issues.

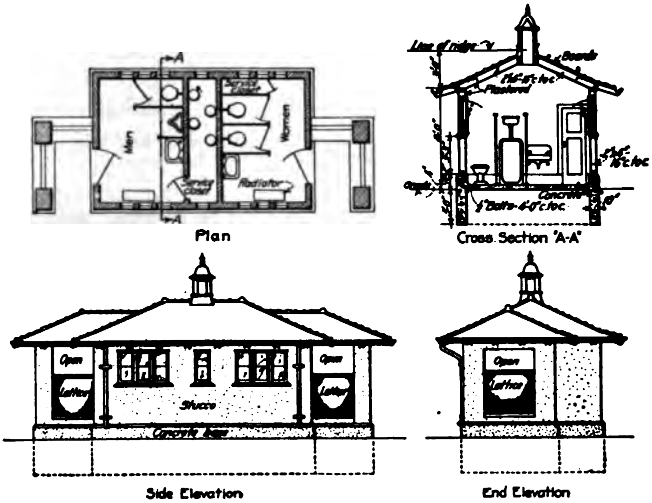


Fig. 33.—Comfort station of the independent building type, equipped with water-flushed conveniences, public water and sewer connections being available or, existing conditions permitting, private systems. Heating provided by basement plant or from adjoining building.

102. Location and Operation.—The maximum success of public comfort stations depends largely upon their central location, which means they should be established in the more congested districts and where they are easy of access. From the viewpoint of economy, ease of access, and central location, existing public buildings usually afford desirable sites for establishing comfort stations. Thus a municipality may utilize a court house, municipal building, school, fire, or police station, library, public market, or similar building. Other suitable sites are public squares, parks, playgrounds and bathhouses, cemeteries, bandstands, and bridge abutments. Semi-public places such as oiling stations and railroad stations are suitable for the purpose, and in some cases they may be housed satisfactorily in connection with other places of business, such as stores or similar mercantile centers (Figs. 37 to 44 incl.).

Another course open for communities, especially cities, is the erection of public comfort stations in the form of substantial, permanent, and artistic structures independent of existing buildings. There are possibilities for the development of this type of station as real municipal centers for public convenience. Following successful experience in many large cities, they may be made to pay, in part at least, the expense of operation through concessions, such as pay telephone booths, parcel check stands, vending machines, shoe shining stands, newspaper and magazine privileges, and counters for the sale of souvenirs, postcards, toilet articles, towels, soap, and auto conveniences. Primarily, however, the public comfort station should be regarded as a free, *public* institution, with toilet and washing facilities open to everybody, and the auxiliary features mentioned should in no way be allowed to supplant this free, public use nor to change in the slightest degree the *public* character of the stations.

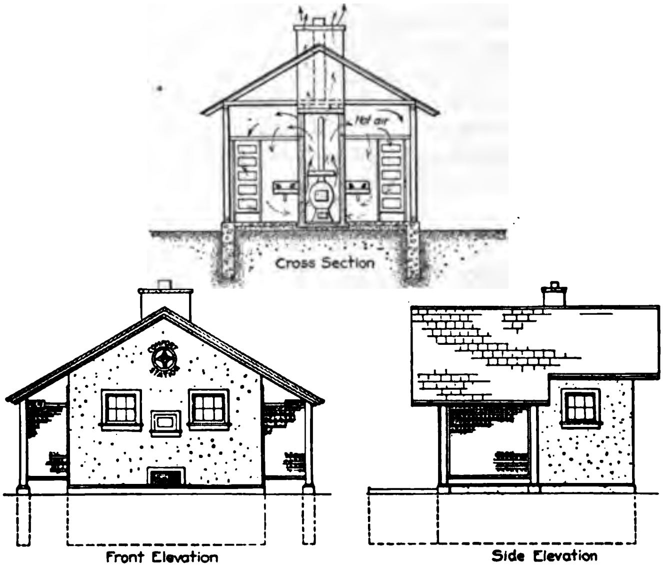


Fig. 34.—Small comfort station and rest room housed in a separate building, equipped with water-flushed toilets and heated by a hot-air heater, steam, or hot water system.

Obviously, public comfort stations should be cared for and supervised by regular attendants, clothed with adequate authority to enforce obedience to all rules and regulations governing use of the facilities.

The development of the public comfort station movement undoubtedly will witness the establishment of many stations along public highways for the convenience of the traveling public. This may well involve making the highway comfort station an integral part of the public highway system and using the highway patrol man as the caretaker or supervisor of the station.

Stations must not only be well located, but to serve their function best, should be marked with plainness. Signs should be clear and unmistakable and prominently placed, and yet be modest. The standard public comfort station sign (Fig. 45) is recommended for universal adoption. Like the red cross and the skull and cross bones, this symbol will convey its meaning wherever found. Once well fixed in the public mind, it should signify service, and imply a full degree of comfort, safety, and sanitation. The emblem was designed by A. C. Shaver, a domestic sanitary engineer of Pasadena, California, and was adopted by the American Society of Sanitary Engineers, June 4, 1912, as a universal public comfort station insignia. It is now used extensively throughout the country.

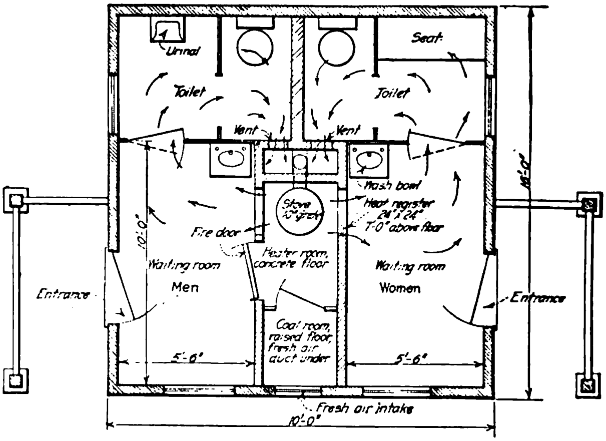


Fig. 35.—Floor plan of comfort station.

103. Submission of Plans.—Before proceeding with the location, design, and construction of a public comfort station or rest room, plans and specifications should be submitted for approval to the State Board of Health or other state or local authority vested with such power.

104. Supervision of Construction.—After approval of plans has been obtained, construction should proceed in accordance with the established regulations, and no changes in such plans should be made without permission from the proper authorities. All such work should be subject to inspection by the official authority.

105. Adequacy of Toilet and Washing Accommodations.—Toilet accommodations to serve the needs of the community depend for their adequacy upon local conditions, so that no definite rule can be laid down. Information available, however, indicates that under normal conditions at least, there should be one closet for every 1000 females and at least one closet and two urinals for every 1000 males in the community, assuming that the population, or the number deemed likely to frequent the station, be divided in the ratio of 40% females and 60% males.

Certain municipalities or resorts where there are frequently large gatherings naturally need more accommodations than places where the people do not fluctuate or assemble to much extent. In the lack of definite information, therefore, and because of possible changes in the development of communities, provision always should be made for increasing the size of the building or room and for installing additional fixtures should the original accommodations become inadequate.

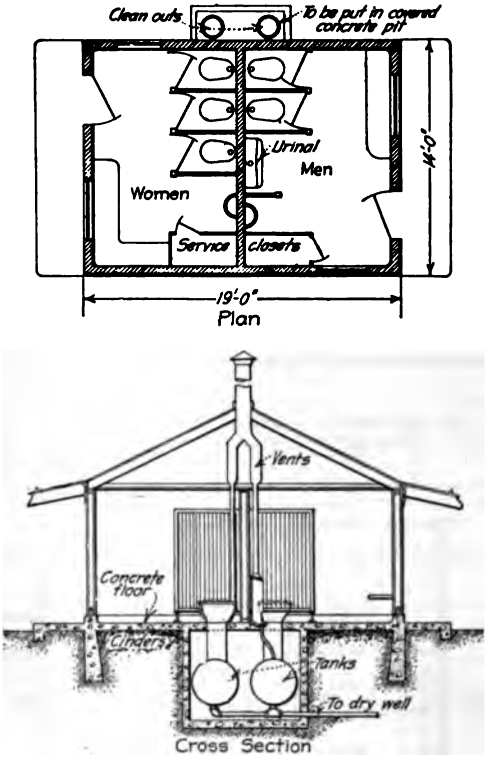


FIG. 36.—Comfort station equipped with chemical closets.

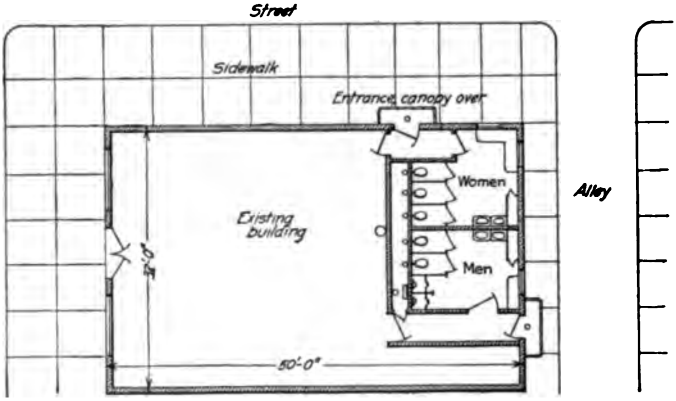


FIG. 37.—Station housed on the ground floor in connection with a heated store or other place of business. Note the approaches, entrances and general arrangement.

Each comfort station should be equipped with adequate washing facilities. There should be at least one lavatory for every 5 fixtures (closets and urinals), or fraction. One lavatory for every 2 or 3 fixtures is recommended.

106. Entrance Screen.—The entrances to the toilet rooms should be properly separated by screens or other means, and wherever possible should be at least 20 ft. apart or otherwise located with due regard to privacy for users.

107. Uniform Sign Required.—Every public comfort station should have displayed in a conspicuous position the standard public comfort station sign. In conjunction with this em-

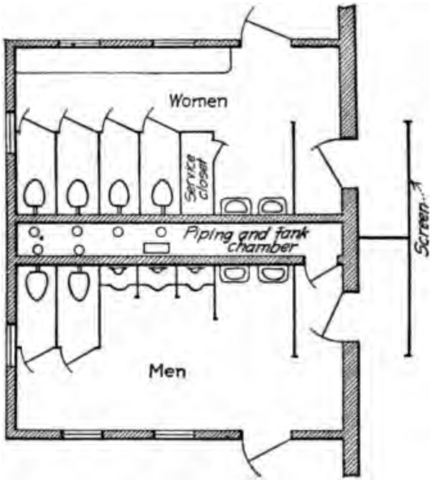


Fig. 38.—Station housed in an addition to an existing library or similar building.

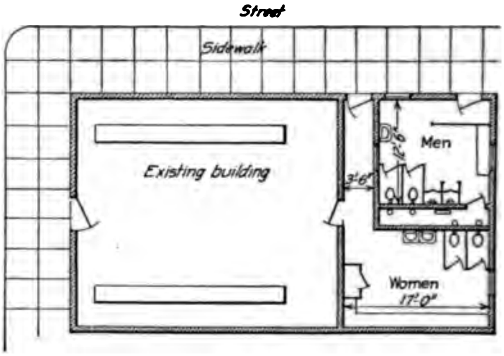


Fig. 39.—Station in connection with a small store building or similar structure.

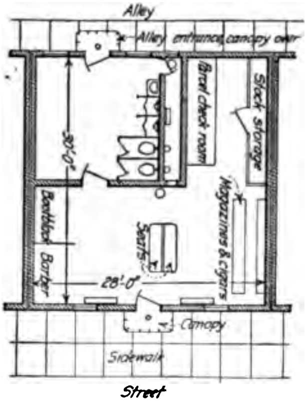


Fig 40.—Station for men only, having a concession annex. Toilets may be both of the free and pay type.

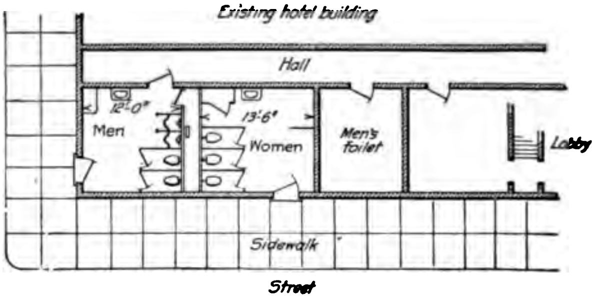
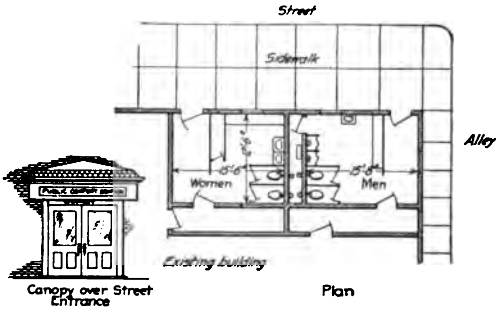


Fig. 41.—Station in connection with hotel building.



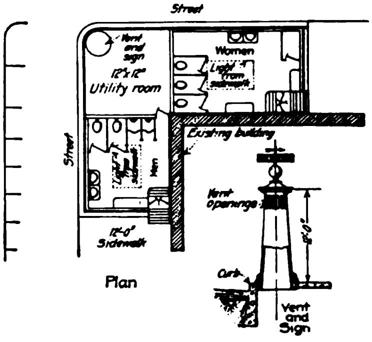


Fig. 42. Fig. 43.

Fig. 42.—Station in connection with a mercantile establishment. Entrances from building and street, with canopy over exterior entrance and approach.

Fig. 43.—Station houses below the street sidewalk. Water, sewer, lighting and heat from adjacent buildings; heating system may be an independent plant. Ventilation by means of an ornamental hollow column equipped at its base with a heating coil, air expulsion fan or its equivalent, and the top surmounted by a ventilator, comfort station mark, and weather-vane.

blem there should be placed a mark indicating women's entrance, and one indicating men's entrance. The uniform sign should be placed also at such other points as are best adapted for guiding the public to these stations.

The signs should be of uniform design throughout the state and not less than 8 × 12 in. in size, except where a larger sign obviously is preferable. Consistent uniformity should, however, be the rule. The universal sign consists of a green circle 5 in. in diameter on the outside and 1 in. wide, with a white center in which is set a four-pointed orange colored star. The body of the sign is white and the border and lettering are a deep blue (Fig. 45).

108. Ventilation and Light.—When housed within a building, a public comfort station should be so placed as to afford light and air by windows or skylights, or open directly upon a street, alley, court, or vent shaft. Every such vent shaft should have a horizontal area of at least 1 sq. ft. for each water closet or urinal adjacent thereto, but the least dimension of such shaft, if one story high, should not be less than 3 ft.; if two stories high, not less than 4 ft.; and 1 ft. additional for each extra story.

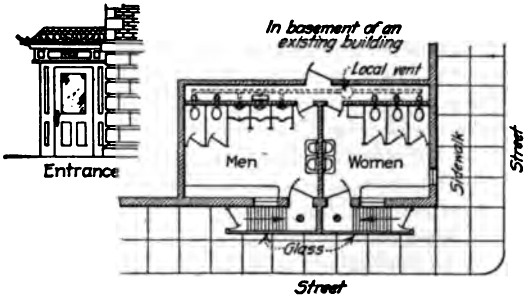


FIG. 44.

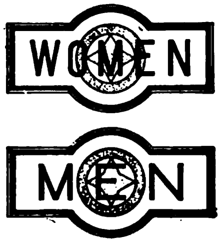


FIG. 45.

FIG. 44.—Station houses in the basement of a building with entrances from the sidewalk, hooded over with a canopy. Ventilation of room and fixtures made effective by one of various approved methods. Sewer, water, and lighting obtained from the building, or direct from the street services, or by independent systems. Heating from the building's heating plant or a separate system. Supervision and janitor service may be furnished by the occupant of the building.

FIG. 45.—Public comfort station mark.

The glass area for a toilet room containing one closet or urinal should be at least 4 sq. ft., with 2 sq. ft. additional for each additional closet or urinal.

In addition to the windows required, each toilet room containing more than three fixtures (closets and urinals) should have a vent flue of incombustible material, vertical or nearly vertical, running through the roof, surmounted by a cap or hood of the siphonic type, and the vent should be not less than the following size:

Four fixtures	8-in. pipe.
Five or six fixtures	10-in. pipe.
Seven to ten fixtures	12-in. pipe.

If the windows or skylights cannot be opened, vent pipes also should be placed.

No toilet room in a public comfort station should have a movable window or ventilator opening upon any elevator shaft or court which contains windows or sleeping or living rooms above; except that a toilet room containing not more than two closets may have a movable window on such court, provided the toilet room has a vent flue extending above the roof.

Except upon written approval by the proper officials, no public comfort station should be located in an interior room, nor in such position that it cannot be given outside light and ventilation.

Every public comfort station should be artificially lighted during the entire period the building is open for use, when adequate natural light is not available, and in such manner that all parts of the room may easily be visible.

109. Size.—Every public comfort station should have at least 10 sq. ft. of floor area and at least 100 cu. ft. of air space for each water closet and each urinal, together with adequate waiting room area.

110. Floor.—The floor and base of every public comfort station should be made of material (other than wood) which does not readily absorb moisture and which can easily be cleaned. Such floors should be of concrete faced with a cement, tile, or marble surface, or equivalent material.

To make a concrete floor non-absorbent, the concrete and cement top dressing must be a dense, rich mix, finished smooth, and kept well painted.

111. Floor Drains.—Toilet rooms of this type should be provided with a hose faucet and the floor graded toward a drain equipped with an adequate 4-in. trap. This trap should have a movable floor grate or strainer.

112. Walls and Ceiling.—The walls and ceilings should be completely covered with smooth cement or gypsum plaster, glazed brick or tile, galvanized or enameled metal, or other smooth, non-absorbent material. In the less frequented or inexpensive stations, wood may be used if well covered with two coats of body paint and one coat of enamel paint or spar varnish. But wood should not be used for separating walls or partitions between toilet rooms, nor for partitions which separate a toilet room from any room used by the opposite sex. All such partitions should be as nearly soundproof as possible.

113. Partitions Between Fixtures.—Adjoining water closets should be separated by partitions. Every individual urinal or urinal trough should be provided with a partition at each end and at the back to give privacy. Where individual urinals are arranged in batteries, a partition should be placed at each end and at the back of the battery. A space of 6 to 12 in. is required between the floor and the bottom of the partition. The top of the partition should be from 5½ to 7 ft. above the floor. Doors, of the same height as required for partitions, should be installed for water closet compartments used by women. Doors at least 24 in. high, with the center about 3 ft. above the floor, should be provided for water closet compartments used by men. All partitions and doors should be of material and finish as prescribed for walls and ceilings. Wood is not recommended; if used, it should be hardwood.

114. Service Closet.—Each toilet room in such stations should have a service closet, supplied with broom, mop, bucket, soap, toilet paper, toweling, lime or other disinfectant, and any other materials necessary for maintaining cleanliness and serving the public's needs (Fig. 33).

115. Depositories.—Men's and women's toilet rooms should be equipped with a depository so designed as to make the contents readily removable, and of such material and construction as to enable it to be kept in a clean condition.

116. Fixtures—*Water Closets.*—All water closets should be made of porcelain or vitreous chinaware. The bowl and trap should be of the combined pattern in one piece, and should hold a sufficient quantity of water and be of such shape and form that no fecal matter will collect on the surface of the bowl. All water closets should be equipped with adequate flushing rims, so as to flush and scour the bowl properly when discharged. The bowl should be of the heavy pattern, large throatway, siphonic action type.

Frost-proof closets should be installed only in compartments which have no direct connection with any building used for human habitation. The soil pipe between the hopper and the trap must be of cast iron, 4 in. in diameter and free from offsets. This type of closet should be used only in buildings subject to extreme frost conditions. When frost-proof closets are installed, the bowl must be of vitreous chinaware or iron enameled inside and outside, of the flush rim pattern, provided with an adequate tank, automatically drained to guard the fixtures and piping against frost. The installation and use of this type of fixture should be discouraged as much as possible. Under the most favorable conditions little can be said for this closet from a practical and sanitary standpoint.

Urinals.—Urinals should be made of material impervious to moisture, and of such design, materials, and construction that they may be properly flushed and kept in a sanitary condition. If cast iron is used in the construction of urinals, it must be enameled on the inside of the trough or bowl and coated with a durable paint or enameled on the outside. Trough and lip urinals should have a floor drain placed below the urinal, and the floor should be graded toward the drain. Individual urinals rising from the floor, with the floor pitched toward the urinal, made of porcelain or vitreous chinaware, and equipped with an effective automatic, or equivalent, flushing device and adequate local vent, are recommended.

Sinks and Wash Basins.—Sinks and wash basins in comfort stations should be made of earthenware, vitreous chinaware, enameled iron ware or other impervious material, and equipped with adequate traps and self-closing faucets.

Flush Tanks.—All flush tanks or flushometer valves should have a flushing capacity of not less than 3 gal. for water closets and not less than 1 gal. for urinals, and should be so installed that they are protected against frost, tampering, etc.

Open Plumbing.—All plumbing fixtures should be installed or set free and open from all enclosing work. Where practicable, all pipes from fixtures, except fixtures with integral traps rising from the floor, should be run to the wall. It is essential that all plumbing fixtures for this type of service be of high grade, and of such design and construction and so installed as to be practically fool-proof.

Piping.—Wherever practicable, the piping, tanks, flushing devices, traps, etc., should be installed exposed in a utility chamber, and so arranged that they are accessible for the removal of stoppages (Fig. 38).

Protection Against Frost.—All water closets and urinals and the pipes connecting therewith should be protected properly against frost, either by a suitable insulating covering or by an efficient heating apparatus, or in some other approved method, so that the facilities will be in proper condition for use at all times. Toilets should be adequately heated in cold weather. Heating equipment should be arranged to permit cleaning of floors and walls.

117. Where Water and Sewerage Systems Are Not Available.—In localities lacking public systems of water and sewerage, the disposal of human wastes may be accomplished as follows:

(1) By an efficient water system of the "compressed air storage" or "air pressure delivery" type and a proper sewage treatment tank and disposal units, as existing conditions may require.

(2) By outdoor privies or other toilet conveniences permitted by federal, state, or local authorities, when local conditions make it impractical to install a water supply and sewage disposal system (see Part III, Sect. 4, on "Waterless Toilet Conveniences"). Fig. 36 shows such a station equipped with chemical closets.

FARM BUILDINGS—GENERAL DESIGN

By Arthur Peabody

118. Cattle Barn.—Manufacturers of cattle stanchions and feed and litter carriers have developed the plan arrangement of the standard cattle barn. The stalls are in two lines, the cattle facing on the center aisle, by which the feed and water is distributed. In some barns the cattle are faced to the outside wall, with feed alleys between the stalls and the windows. The stalls are formed of concrete, pitched slightly to the back where a gutter extends the length of the building. The finished level of the stall floor should be even with the bottom of the manger. The stalls may be paved with cork bricks or creosoted blocks. The block paving is not imperative where ample bedding is provided. The stanchions and stalls are formed of iron pipe. The fabrication of this equipment has been specialized so as to be adjustable to different sized cattle. The concrete manger is formed in the floor structure. Separating partitions of metal prevent the cattle from robbing each other. The partitions are operated by a lever at the end of the row of stalls. Watering basins of cast iron are placed in each stall. These are automatic, self-filling, and are said to be non-freezing. Feed carriers hung to overhead railways, and litter or manure carriers, also on overhead rails, facilitate rapid attendance on the cattle. The manure carrier rails are extended to a distance outside the barn so that the carrier is automatically dumped and returned. Hay and grain are stored on the second floor of the barn, the structure of which is such as to permit a hay loader operating on a rail to fill the barn nearly to the top. A grain mixing room, on the first story, is connected to iron lined grain bins overhead by chutes. The hay is delivered by chutes to the first floor. The silo is at the end, or on one side of the barn. It is from 10 to 18 ft. in diameter according to the size of the barn, and from 20 to 45 ft. high. One side is closed with a series of doors connecting by a chute to the first story. The silage consisting of chopped corn stalks or other fodder finely cut, is delivered to the silo by a metal tube through which the silage is blown by a powerful fan to the top. Just enough silage is taken out for each day's feeding. The food capacity of silos is given in the following table.

Table of Standard Interior Dimensions of Silos for Feeding Cattle Six Months and Eight Months

Number of cattle	Tons required for		Diameter, (feet)	Height	
	6 mo.	8 mo.		6 mo. (feet)	8 mo. (feet)
10	36	..	10	28	..
20	72	96	12	31	39
30	108	144	14	34	41
40	142	192	16	34	42
50	180	240	18	34	47

The silo may be of wooden staves bound with iron rods, or formed of heavy wooden rings sheathed inside and out with vertical matched boarding, or of vertical studs covered with horizontal lap siding bent to the circle. It may be of hollow clay tile, laid in mortar, or of concrete

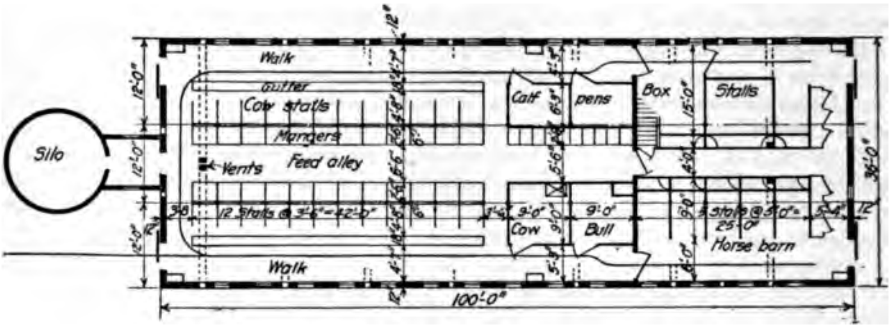


Fig. 46.—Typical general purpose farm barn.

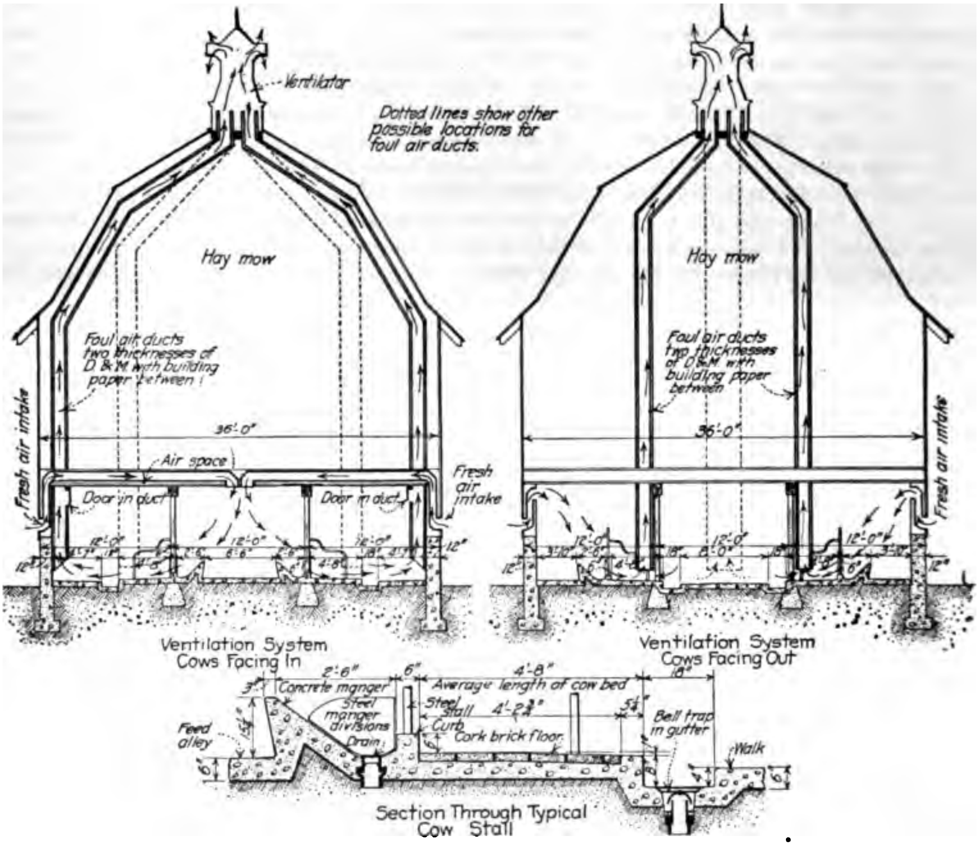

Ventilator
Dotted lines show other possible locations for foul air ducts.
Hay mow
Foul air ducts two thicknesses of D & M with building paper between
36'-0"
Air space
Door in duct
Door in duct
Fresh air intake
Fresh air intake
12'-0"
6'-6"
4'-8"
12"
Hay mow
Foul air ducts two thicknesses of D & M with building paper between
36'-0"
Fresh air intake
12'-0"
8'-0"
18"
12"
Ventilation System Cows Facing In
Ventilation System Cows Facing Out
2'-6"
6"
4'-8"
18"
3"
Concrete manger
Average length of cow bed
Steel manger divisions
Steel stall
4'-2¾"
5¼"
Feed alley
15½"
Drain
Curb
Cork brick floor
Bell trap in gutter
Walk
6"
6"
Section Through Typical Cow Stall

Fig. 47.—Typical sections showing ventilation systems and dimensions for general purpose farm barn.

reinforced with vertical and horizontal rods. The silo, whether of wood or masonry, should rest on a concrete or masonry foundation carried 2 ft. above ground and deep enough to prevent frost action. Claims are made for wood silos that they are more resistant to freezing than masonry. A continued period of cold weather will, however, freeze the silage around the outside wall in any construction. In a windy location the wooden silo is likely to be blown down or bent over on account of its light weight.

The ventilation of the cattle barn is done by a gravity system consisting of inlet ducts entering the outside of the walls midway between floor and ceiling, and discharging into the barn near to the ceiling in front of the stock. Control dampers are required. The ducts are distributed at intervals of 10 or 12 ft. on the walls. The out take ducts are large, and fewer in number, placed in such a manner that the air will be drawn under the stock from front to rear. The foul air enters the ducts near the floor and passes in as nearly a vertical line as possible to the ridge of the barn. A special form of vent cap prevents back draft and the entrance of wind and snow. Control dampers are desirable, but it should not be possible to close the ducts entirely, otherwise the cattle will not obtain sufficient fresh air.

The number and size of the outlet and inlet ducts depends on the number of animals housed.

The number of cubic feet of air required per head per hour, with the average relative humidity of fresh country air at 65 % or less, is as follows:

	Cu. ft. per hr. per head	Assumed weights per head (pounds)
For horses..............................	4924	1200
For cows................................	3953	1100
For swine...............................	1510	160
For sheep...............................	929	100
For hens................................	37	3

With different weights per head, the amounts of air would change in proportion.

The flow of air in a square outtake duct will have at least an average velocity of 250 ft. per min., without mechanical forcing or the aid of heat other than that derived from the animals in the space to be ventilated.

An outtake ventilating duct for 30 cows would require 30 × 3953 = 118,590 cu. ft. of air per hr. We will assume an air movement of 250 ft. per min., or 15,000 ft. per hr. To ascertain the cross section area of the outtake duct required for the cows, it is only necessary to divide the number of cubic feet of air required for 30 cows, by 15,000, thus,

118,590 cu. ft. ÷ 15,000 = 7.906 sq. ft., or 1138.5 sq. in. requiring either one duct 34 × 34 in., 2 ducts 24 × 24 in. each, or 4 ducts 12 × 24 in.

Stronger currents through the ventilators will be secured by making one or more larger ones than where many small ones are provided, and it is usually best to have as few as possible, yet not leave the impure air in distant parts of the barn.

For every outtake flue there should be a number of intake flues whose combined area exceeds that of the outtake flue by 10 %, even in view of the unavoidable leakage of air through the walls and around the windows and doors.

Thirty cows require an outtake duct of 1138.5 sq. in. area; then these cows should have an intake of 1138.5 sq. in. plus 10 % which would be 1252.4 sq. in. Assuming 20 intakes, each would have to be 1252.4 ÷ 20 = 62.7 sq. in. area, or about 8 × 8 in. square. It is better to have many small openings than a few large ones, because the cold air is better distributed, lessening drafts. All intake flues should be equipped with registers, so the air is at all times in control of the party in charge. Intake flues may be made of galvanzied sheets or wood.

The nominal area of a register or register face should be about 50 % greater than given by this computation; actual areas of commercial registers are given in the accompanying table.

Size of register face (inches)	Effective area (square inches)	Size of register face (inches)	Effective area (square inches)
6 × 8	32	12 × 12	96
6 × 10	40	12 × 14	112
6 × 12	48	14 × 14	130
6 × 14	56	6 round	19
8 × 8	42	7 round	25
8 × 10	53	8 round	33
8 × 12	64	9 round	41
8 × 14	75	10 round	51
10 × 10	66	11 round	62
10 × 12	80	12 round	74
10 × 14	93	14 round	100

A good form of ventilating flue is made of two layers of number 1 matched stuff, ⅞ in. thick, with building paper or deadening felt between, to make it as nearly as possible a perfect non-conductor, thus preventing rapid cooling of the air in the flue. This form of construction also makes the flue air-tight, which is essential, for every hole and crack lessens the ventilating power.

The most common and probably most suitable material for barn construction is wood. Concrete foundations and floors are advantageous and the concrete walls may be carried up a few feet above the floor or to the window sills. Above this the wood construction is started. There would seem to be no reason why the entire first story

and the floor of the second story should not be of reinforced concrete. In the event of fire the cattle might be saved by this construction.

A plan arrangement which would store the hay in a separate building might be the means of saving a valuable herd. This would require a special mechanism for bringing the hay into the cattle barn. In this case the roof of the barn should be built to resist the cold of winter.

119. Manure Pit.—The pit for storage of manure will be concrete formed into a shallow tank. It should be covered with a roof and screened from flies. The overhead railway from the barn will extend through the pit so that the manure may be dumped automatically. The pit should be large enough to contain the winter's production of fertilizer except what is spread directly on the fields.

120. Horse Barn.—For the powerful horses used on a farm, stalls of considerable strength are needed. The usual type is formed with cast-iron or steel posts and 2-in. oak or elm plank sides resting in channel forms bolted to the posts. Concrete posts will not endure the effect of constant kicking. The concrete pavement of the stall is covered with planking formed into movable platforms by metal straps secured to the under side. Elm is preferred for these platforms. Above the height of 5 ft., metal guards of the usual form are required. Where the hay is chuted down, it should not be confined by the iron gratings, but allowed to flow freely into the plank manger. Iron oat boxes and iron edgings to wood mangers are desirable. The stalls should be 9 ft. long and not over 4 ft. wide for standing horses or less then 5 ft. where horses are to lie down. The concreted aisles of horse barns should be left rough to prevent slipping. Deep grooving is objectionable for cleaning. Wood block paving, not creosoted sufficiently to be slippery, is useful. The slanted ways into a horse barn should not slope over 1 ft. in 5 ft., especially for brood mares. Harness and carriage rooms should be separated from the stall room to avoid the ammonia fumes.

121. Swine Barns.—The swine barn in a severe climate should have not over 10-ft. clear height. It should face to the south to secure ample sunlight. In mild climates windows in the roof may supplement those in the south wall, but the arrangement is not suitable for cold winters. The barn is divided into pens about 8 × 10 ft. by wood partitions or iron pipe railings of standard type. The fronts of these are provided with swinging feed gates hinged at the top. A wood platform 5 ft. square is laid on the concrete in each pen for the swine to lie on. The building is ordinarily of frame construction, warmly built, with swine doors that may be closed by the attendant. Standard barn ventilation is necessary. A feed cooking kettle is provided in the feed mixing room at one end. The space in the roof is used for hay storage. Along the sides containing the swine doors, concrete platforms 3 ft. wide are extended to prevent rooting next to the building.

INDUSTRIAL PLANT LAYOUT AND GENERAL DESIGN

By Harry L. Gilman

The design of a modern industrial plant is an important and complicated problem. From the selection of the site to the turning out of the first finished product, every step must be carefully thought out. The work should be entrusted only to an engineer of wide and general experience; to one who is constantly taking up and solving new problems in transportation, handling of materials, routing of work, power generation and transmission, fire prevention and protection, foundations, structures and materials. In addition to the above prerequisites, the engineer in charge should also have a good working knowledge of manufacturing processes and machinery in all lines, as this frequently enables him to approach a new problem to better advantage than the specialist. But it should not be inferred that the engineer himself should have the complete knowledge necessary to enable him to build alone any kind of a manufacturing plant. In a chemical works, for instance, he must turn to the manufacturing specialist for help in working out processes and equipment.

The work of the engineer in designing industrial plants is outlined in a general way in this chapter.

122. Locating An Industry.—The engineer will frequently be called upon to assist in the important matter of locating an industry. There are several factors which enter into the selection of the location of a factory, and upon which the engineer is called to report, such as sources of raw materials, labor, power, market for finished product, and shipping facilities. Paper mills, for instance, particularly those using wood, are best located near forests and on rivers which furnish water for use in the processes, power for operating the machinery, and the cheapest means of bringing logs to the mill. They must also have suitable railroad or other transportation facilities. In general, a plant using large tonnage of raw material should be located near the source of this material. Again, a plant requiring a large amount of power should be located where cheap power is available.

Industries in which labor produces a great part of the value, as in cotton mills, shoe factories, etc., require a good labor market near at hand of the class of employees desired. For this reason several cities have become large centers for special industries, as Lowell, Lawrence, and Fall River, Mass., in the textile industry; and Lynn, Mass., for shoes, etc. However, some of the advantages of such places as these have been lost on account of increasing labor troubles.

Other industries require an isolated location on account of obnoxious or dangerous fumes, or danger from explosions; others require large cheap areas on account of the amount of ground covered. Factories which consume semi-finished materials, such as clothing, printing, binding, etc., use a large portion of hand labor and are usually located in large cities where labor is plenty. Ordinarily in these plants the tonnage of product is not such as to require the best shipping facilities.

123. Selecting A Site.—Local considerations entering into the selection of a site for an industry are: transportation facilities; side tracks on to property if tonnage is large; and separate tracks for receiving and shipping where the business is extensive. The area selected should be ample for present and future needs, and the site should be convenient to suitable residential sections for employees. This is important and many manufacturers are investing much capital to provide suitable and attractive homes for their employees, with the object of reducing the labor turnover and improving both quantity and quality of output from the well-housed, and therefore better contented labor, with a probable reduction of labor troubles. The nature of the land effects the construction cost of the plant. Cheap land requiring expensive filling and pile foundations is often more expensive than more costly land offering good foundations. Borings and tests should be made and the cost of foundations investigated. The accessibility of public facilities should be considered in selecting a site; as fire and police protection, water, gas and electrical supplies, and street railways all have a direct bearing on the problem and effect efficient operation.

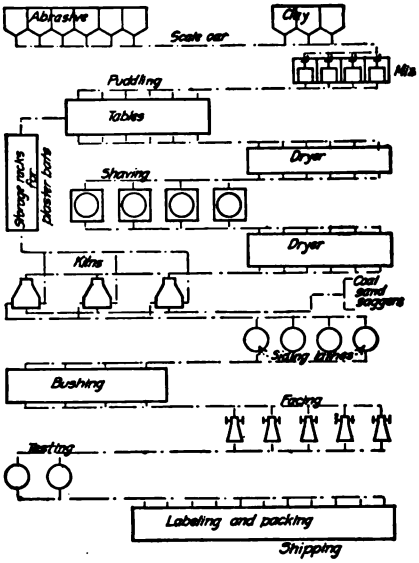


Fig. 48.—Routing diagram, vitrified grinding wheel works.

A plant located in or near a large city has both advantages and disadvantages. It has a large labor market, but the labor is not so reliable and labor troubles are more frequent. However, an industry in which the labor requirement fluctuates at different seasons is probably better located near a large labor market. It should be noted, however, that the most efficient employees are those trained in the plant, living in homes which they own and with surroundings which induce a feeling of contentment, remaining year after year.

124. Preparation of Plans.—The engineer should first obtain all necessary information relative to machinery and processes, quantity of raw materials to be handled, and finished product to be turned out. A flow sheet should be prepared particularly for plants where one or more materials pass through several continuous operations. This is best explained by the example

(Fig. 48) flow sheet for a plant for the manufacture of vitrified grinding wheels. With this should be determined the number, capacity, makes, etc., of the various units of equipment required. This is the simplest form of flow sheet, merely showing sequence of operations. It is followed either by a routing diagram, or by a complete flow sheet showing tentatively the location of machinery and means of handling the material from one process to the next, as elevators, conveyors, gravity chutes, etc. In Fig. 49 is shown such a sheet for a crushing, washing and roasting plant for abrasives for the Compagnie General des Meules, Paris, France. This flow sheet determines the necessary height of the buildings, and from it the floor plans may be worked out, as shown in Fig. 50.

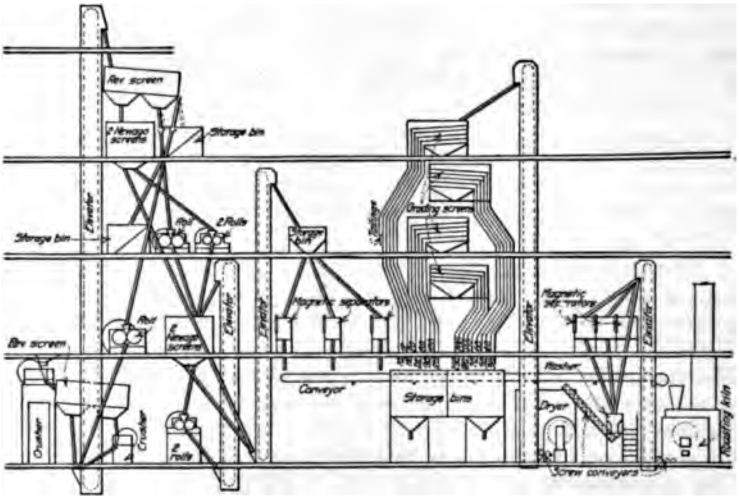


Fig. 49.—Flow sheet for crushing plant, Compagnie General Des Meules, Paris, France.

With this flow sheet and a survey of the site, the engineer will make up a block plan of the proposed plant, with sketches from which an estimate of cost can be made. The survey should include tests or borings of the soil, particularly if heavy foundations are to be built. It is of great importance that costs and a general idea of the arrangement and operation shall be thoroughly understood by all parties interested, so as to avoid expensive changes after work is started.

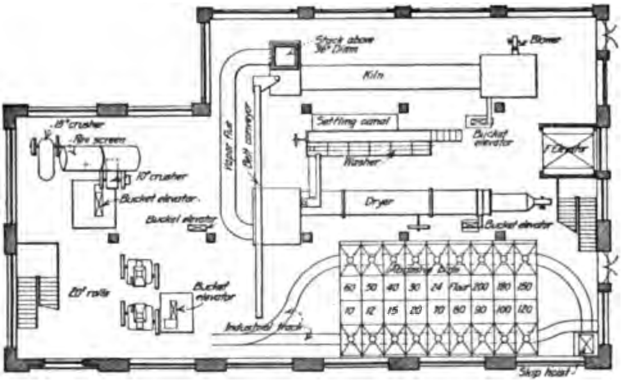


Fig. 50.—First floor of crusher building for Compagnie General Des Meules, Paris, France.

125. Shipping Facilities.—Ample side tracks should be provided both for receiving and shipping. Frequently a separate siding is installed for receiving fuel; in any event this should be so arranged that coal may be unloaded at the proper point without interfering with handling of other incoming or outgoing material. The track layout and block plan for a large machine works, shown in Fig. 50*A*, is a good illustration of trackage required for a plant handling in and out some 300 tons per day.

Shipping accommodations should be worked out in connection with the flow sheet and routing diagrams, with due consideration to the kind of materials to be handled; for instance, a foundry should have its track covered by

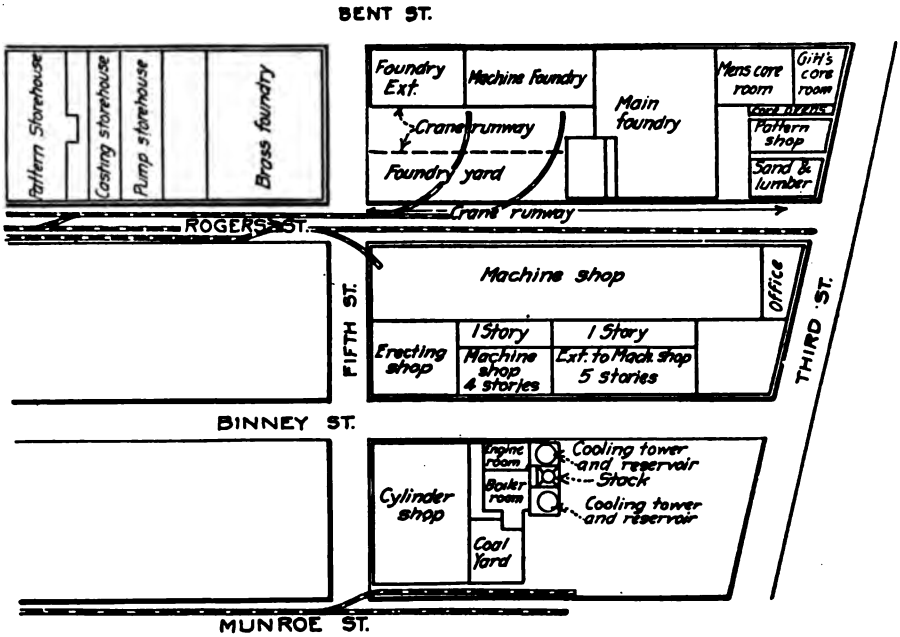


Fig. 50A.

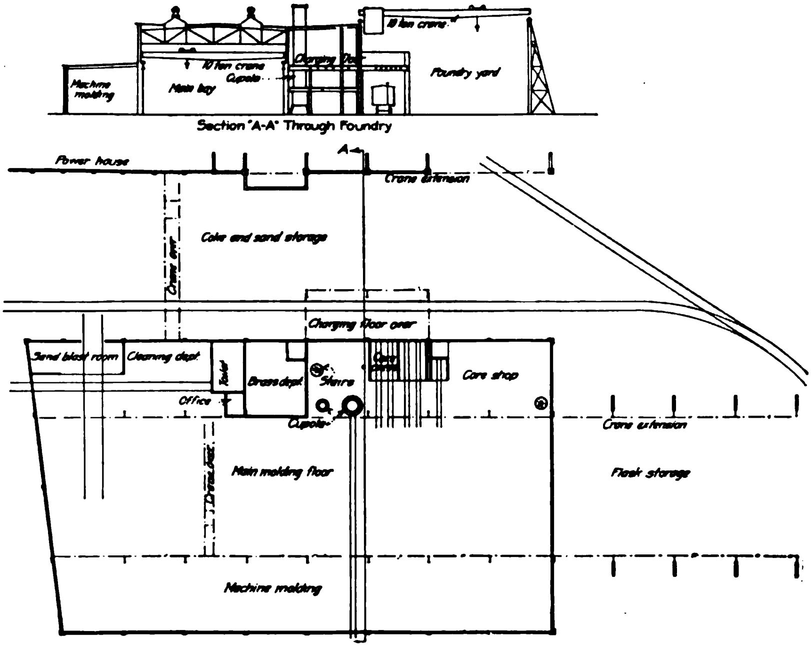


Fig. 51.—Foundry of Putnam Machine Co.

a travelling or other crane unloading the iron with an electro-magnet, which will also serve to load the same material to the charging platform, as shown in plan and section of the Putnam Machine Company foundry (Fig. 51). Other material must be loaded from a shipping platform alongside the freight house, which may, if quantities and other arrangements will permit, serve both for shipping and receiving.

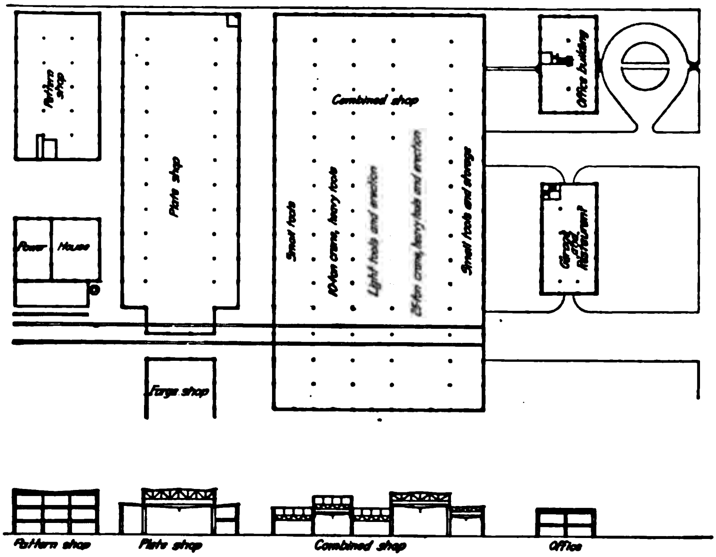


FIG. 52.—Mead-Morrison Mfg. Co.

126. Type of Buildings.—The type of buildings is determined to a great extent by the character of work to be done, or the machinery to be housed. Plants equipped with heavy machinery or making heavy product are usually one story buildings; as rolling mills, large machine works, foundries, paper mills, etc. (see Figs. 52, 53, 54, and 55). Heavy machines, erecting, etc., are located in the bays served by travelling cranes, while the light machines are in the side bays which frequently have a second or mezzanine floor, as in Figs. 53 and 54. These buildings are well lighted by windows in monitors and in the high bays above the roofs of the lower side wings. Paper mills usually have one story and basement. The machines which are up to 200 ft. in length require substantial foundations, and basements are used for pumps, machine drive shafts, stuff chests, etc.

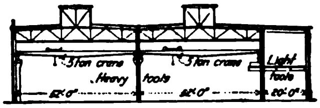


FIG. 53.—Blake-Knowles cylinder shop.

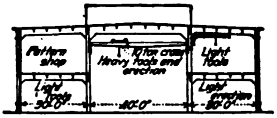


FIG. 54.—Reinforced concrete machine shop.

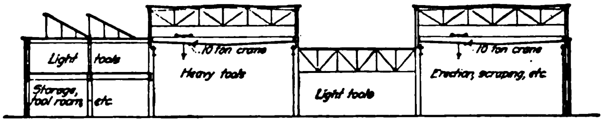


FIG. 55.—Putnam machine shop—cross section.

Another type of building much used for nearly all classes of light manufacturing is the one story saw-tooth bulding, which from its method of lighting, may be of any width and length. This type is well adapted to weave sheds of textile mills which require good lighting; in fact, it was originally developed for that purpose. They are well suited to any class of manufacture adapted to single floor operation, where heavy overhead cranes are not required and where the cost of land is not prohibitive.

The machine shops shown in Figs. 52 and 55 have a combination of saw-tooth and monitor construction, making excellently lighted shops of large floor area, bringing all related departments in close and convenient touch with each other instead of being in isolated buildings. The small automobile plant shown in Fig. 56 is a one-story construction, saw-tooth roof, long span trusses eliminating columns, grouping all operations in the several wings in such manner that all material flows through from the assembled parts to the finished car.

High or multi-story buildings are necessary where processes are continuous, so that material may be elevated to the top and flow by gravity from one process to another, as in crushing plants, flour and sugar mills, etc. Multi-story buildings are also necessary on expensive city land. The height of the building, unless governed by the requirements of the processes, will be fixed by the cost of construction or by the city building laws. They are also better adapted to many classes of industries, as textile mills (except weave sheds), paper box, candy, furniture factories, etc. The cost per square foot of floor space (exclusive of foundations) does not differ greatly from the cost of one-story saw-tooth buildings. The total cost of each depends much on the foundations.

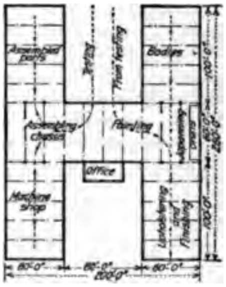


Fig. 56.—Small automobile factory.

127. Loft Buildings, Industrial Terminals.—This class of buildings erected in the larger cities for the housing of several small industries for light manufacturing purposes, is usually designed without regard to any particular industry, but to give good lighting and as large and unobstructed floor area as possible. They are usually of fireproof construction, with large windows and must have ample elevator service, stairways, fire escapes and exits to provide safe and easy access and egress in case of fire or panic. Ample electric lighting and power service should be provided.

The Industrial Terminal, a development of recent years and now in operation in several large cities, consists of a large group of buildings for manufacturing and storage, built with the idea of giving to smaller individual firms all the facilities of the largest industrial plants. It has a large central power plant to furnish heat, light, and power at the lowest cost to tenants. Freight and express houses are maintained, with a large force of employees to render every service required. The buildings should be of the most modern fireproof construction, usually of reinforced concrete, six to ten stories in height. Floor space of such area as desired is rented to various firms with all facilities furnished. The cost of insurance, watchmen's service, fire protection, teaming, and freight handling are much reduced over that in the smaller individual plant. Some of the larger loft buildings furnish this service to a great extent. These buildings should be designed with high ceilings, the greatest possible amount of window space, and a width of 60 to 80 ft. The storage buildings may be wider if desired.

Ample elevator service, both passenger and freight, wide stairways, and streets sufficiently wide to allow good lighting of the lower stories, should be provided. If buildings are intended for the lightest class of manufacturing 150 lb. live load per sq. ft. is sufficient, but for general purposes loads should not be restricted to less than 200 lb. per sq. ft. The larger plants, besides furnishing tenants with electricity and heat, also furnish gas for fuel, steam, water, and compressed air, all from the central plant. Naturally these terminals must be located near ample housing area for employees and in large shipping centers.

128. Materials of Construction.—In selecting materials for construction of an industrial plant, the engineer will be guided by the type of buildings required, limits of cost, and local material market. For the multi-story building, reinforced concrete is one of the best and most economical materials. It makes the least expensive entirely fireproof building, and withstands fire with the least damage, as proven by the Baltimore and San Francisco conflagrations, and the fire in the Edison Phonograph plant.

The various systems of concrete floor, beam, and column construction are treated in other chapters. Outside walls, while sometimes built of concrete, more often have a skeleton of concrete columns, spandrel beams, lintels, etc., and panels filled in with brick, terra cotta hollow tile, or cement stucco on metal lath. It is desirable for heat insulation, as well as to prevent moisture working through, to have an air space in the curtain walls. Sections shown in Fig. 57 indicate the most common methods of constructing certain walls. Hollow tiles give excellent insulation and may be either plastered outside with cement mortar (in which case the scored tiles for plastering should be used) or smooth face tile may be laid with good joints and left without further finish, if low cost is an object. Another method is to lay a 4-in. face of brick, bonded to hollow tile backing. These tiles are made from 2 to 12 in. thick.

For one-story machine shops of the type shown (Fig. 58), brick, concrete, or hollow tile curtain walls are used with either brick or concrete piers or steel columns encased in brick. Interior columns are of steel, as are trusses and purlins with concrete roof slabs, or heavy timber purlins with plank roof. In buildings where sprinklers must be installed on account of the contents, the wood roof will be the cheaper, but in cases where sprinklers must be installed only on account of the wood roof, the concrete roof will, as a rule, be found the more economical.

There are also several concrete tile and gypsum tile roofs on the market which are used to some extent. Reinforced concrete is not adapted to replace the long span steel trusses required in this type of building. A roof span of 40 ft. is probably about the practicable maximum for concrete with 30-ft. span for floors; in some cases, however, longer spans have been found practicable. Fig. 54 shows a machine shop 100 ft. wide, built entirely of reinforced concrete, of a practical and economical design.

Brick and heavy timber buildings of the so-called "slow-burning" construction, as developed in New England (adapted to either one-story or multi-story buildings), are treated in Sect. 3.

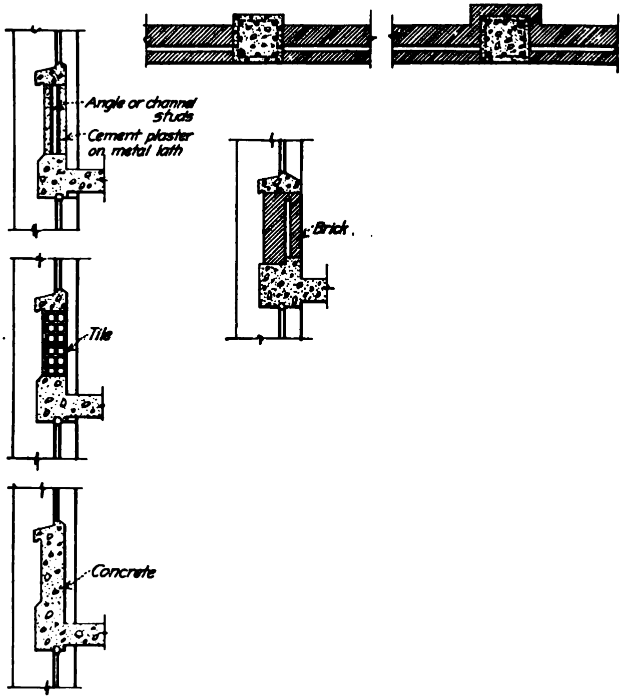


Fig. 57.—Spandrel sections.

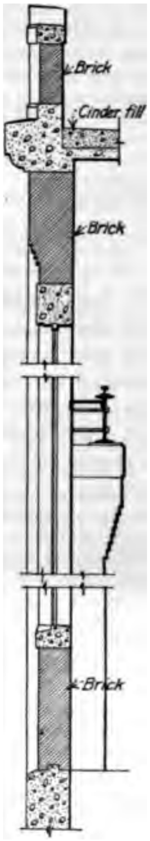


Fig. 58.—Spandrel section, Blake Knowles Brass Foundry.

The one-story saw-tooth building is generally built with brick, tile, or concrete walls, and either with long span trusses spaced about 20 ft., or columns carrying girders and purlins and spaced 20 to 25 ft. each way. Steel trusses of 60-ft. span, thus eliminating two-thirds of the columns, have been found by the writer to be, as a rule, as inexpensive as the column type without trusses. The roof may be of concrete on steel purlins or plank on wood purlins.

Concrete is used to some extent in saw-tooth construction but on account of complicated form work is rather expensive.

129. Foundations.—Care must be taken that foundations for heavy machinery are ample to absorb vibrations. If vibration is considerable, as in steam or power hammers or jarring machines for foundries, the foundations should be separated entirely from all building structures or other foundations.

130. Floors.—Floors should be designed to provide for any future changes that may be foreseen, particularly if the floors are of reinforced concrete, and sleeves should be setin floors where pipes, etc., are to run. Conduits should be properly placed and openings provided for belts, shafting, etc., properly protected. Where apparatus must be taken through floors, ample openings and trap doors or removable floor slabs should be provided.

131. Lighting.—Provision for lighting should be carefully worked out, always remembering that daylight is cheapest and most efficient. Windows should be wide, as a rule placed about 4 ft. above the floor and the tops as close to the ceiling as possible. One-story saw-tooth buildings should have the saw-tooth windows facing north, to avoid direct sunlight. Steel sashes of which there are now several standard makes on the market, should always be considered in designing a factory. The light area of steel sashes is 80 to 90% of the total window area, against 50 to 70% for wooden windows and frames. The cost of steel sash is no greater and is often less than for wooden windows. Ventilation with steel sashes may be as large as desired. With equal care (proper painting) steel windows will outlast wood. Two types of steel window lighting are shown in Figs. 59 and 60. One type has large windows between brick or concrete piers; the other type has steel wall columns and sashes set outside the line of columns to form continuous sashes. Artificial lighting is covered in the chapter on "Electric Lighting and Illumination" in Part III, Sect. 17.

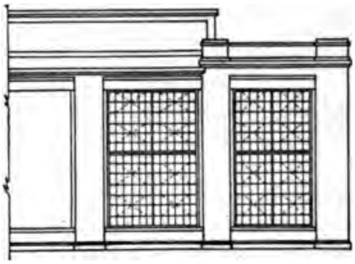


FIG. 59.—Shop with steel sash and brick pilasters.

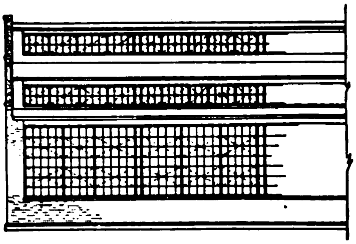


FIG. 60.—Shop with continuous sash.

132. Heating and Ventilation.—This is discussed in Part III. However, the engineer should use care in placing heating apparatus, to occupy as little as possible of important working space. The writer has seen large heaters so located in foundries and machine shops as to displace several important machines, reducing the production of the plant that amount. Care should be taken to see that pipes do not interfere with the operation of cranes and other apparatus. This applies also to plumbing, compressed air, oil piping, etc. All piping and wiring plans should be carefully checked with structural and layout plans to see that there is no interference. A composite plan, locating all apparatus on one sheet, will assist in checking clearances.

133. Cranes.—Attention should be paid to obtaining the proper clearance and ample support for all cranes, monorail hoists, jib cranes, etc., and contract drawings of apparatus should be checked over to see that proper clearances have been allowed. Shop drawings of structural steel work should be carefully checked for the same reason.

134. Conduits.—Conduits, panel boxes, and other electrical apparatus should be located to clear other apparatus, also to secure ease of operation and accessibility for repairs and alterations. Outlets should be provided wherever they may be needed. Conduits for wires may usually be placed in concrete floors before pouring of concrete, but care should be taken not to place them where openings may be made in floors.

135. Transportation.—The handling of materials (raw, finished, and in process) is a subject which requires careful study. Handling by manual labor is generally the most costly method. Conveyors should be installed wherever they will displace sufficient manual labor to warrant the investment, and this must be determined by the engineer in each case. Frequently plants requiring continuous operation may utilize gravity for a large part of the handling, as indicated by flow sheet of the Abrasive Crushing Plant (Fig. 49).

Granular materials are handled by bucket elevators, belt, scraper, screw conveyors, etc. Logs, wood, bags, and similar materials are handled by belt, endless chain, or gravity conveyors of various types. Gravity conveyors consist of a series of rollers close together and on a slight pitch, so that materials will be carried down by their own weight. These conveyors require no expense for power; hence are economical. A good example of a conveyor system, which saves sufficient labor to pay for the installation every two to three years, is shown on the plans of the Blake-Knowles Brass Foundry (Figs. 61 to 64 inclusive).

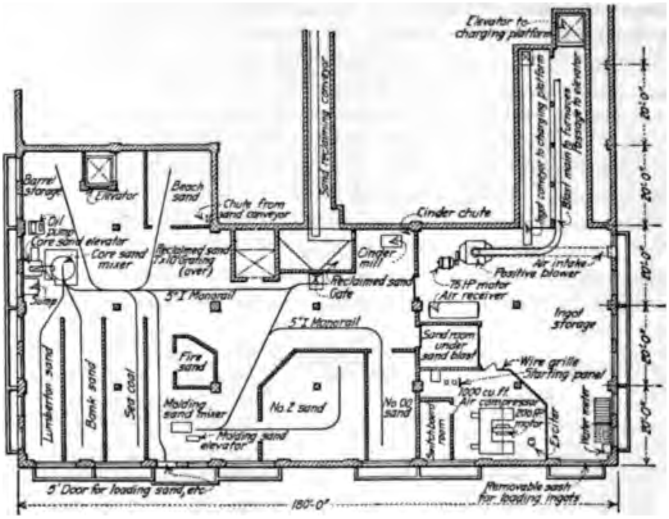


FIG. 61.—Plan of basement, Blake-Knowles brass foundry, Cambridge, Mass.

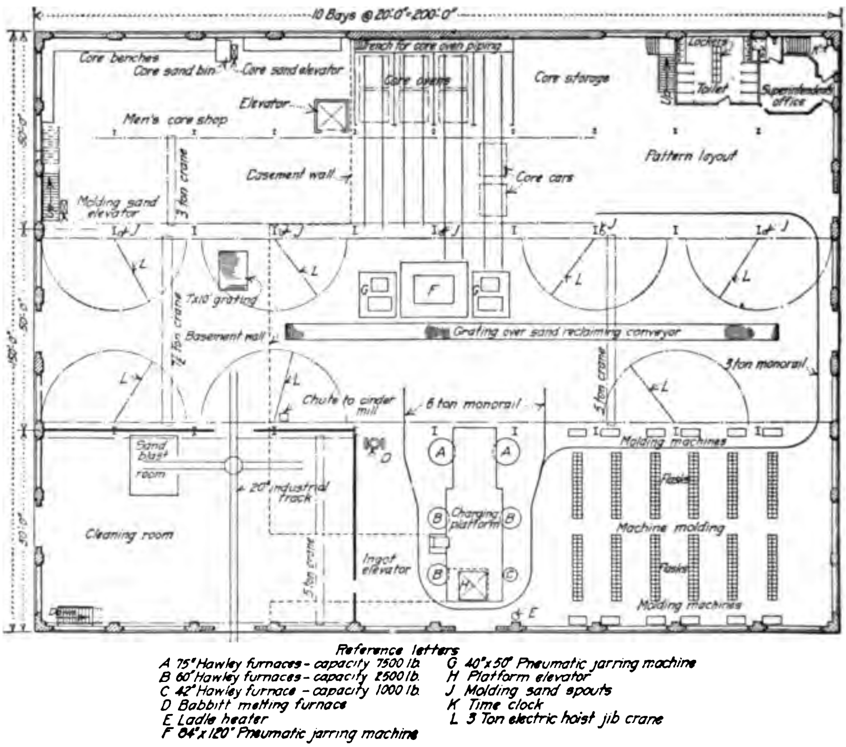


FIG. 62.—Plan of first floor, Blake-Knowles brass foundry, Cambridge, Mass.

Another important labor saving appliance is the elevating truck, of which there are now many on the market. These are used in factories of all kinds, materials in process being piled on movable platforms or racks, an elevating truck backed under, the load raised from the floor and moved on to the next operation, or wherever desired. Steel core oven racks built so as to be handled by these tracks have proved a very efficient system in at least one large foundry installation designed by the writer.

136. Fire Prevention and Fire Protection.—Important considerations in the design of industrial plants are the prevention of fires and the confining of fires which do start to the smallest possible areas. The following from a pamphlet of the Factory Mutual Insurance Companies are excellent rules to follow, whatever the class of building:

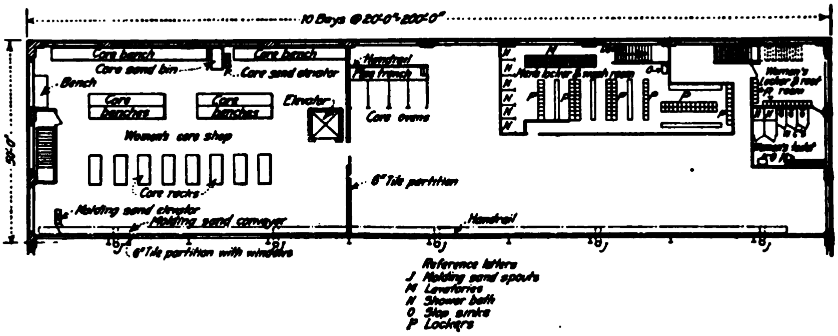


FIG. 63.—Plan of second floor, Blake-Knowles brass foundry, Cambridge, Mass.

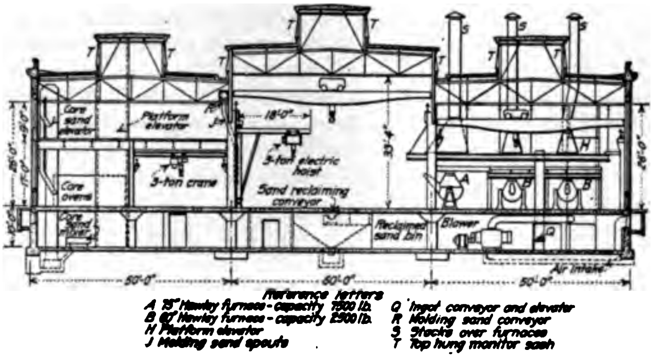


FIG. 64.—Cross section, Blake-Knowles brass foundry, Cambridge, Mass.

Hazardous processes should be located in detached buildings, or in rooms cut off from the remainder of the buildings by fire walls. Buildings of large area should be divided by fire walls, especially when containing combustible materials, in order to limit the extent of any fire that may start. Although reinforced concrete construction can withstand a severe fire without great damage, an automatic sprinkler system with adequate water supply is necessary to protect the contents, if combustible. Sprinklers will extinguish or control most fires at the start and protect the building as well as contents. Buildings subject to fire exposure (outside) should have exterior door openings protected by fire doors, and window openings protected by wired glass in metal frames, shutters, or open sprinklers, or by a combination of these, depending on character of buildings and severity of exposure.

Experience shows that in concrete construction, corners are a source of weakness when exposed to fire, and should be avoided wherever possible. The round column is the better design.

137. Planning For Future Growth.—One very important point for the engineer to consider in designing an industrial plant is provision for future growth. All departments should be so designed when possible that they may be enlarged at any time with the least expense and interference with operation of the plant. The plan of the paper board mill (Fig. 65) is an example of plant design with a view to future growth, even to four times its present capacity, without disturbing the present arrangement nor interrupting the operation.

The present plant uses only waste paper stock and makes a common grade of "New Board," some wood fiber being used for liners or outside surfaces to strengthen board for making heavy packing cases. Provision has been made for a future rag house for preparing rags, sorting, dusting, cutting, and boiling, ready for the beater room. A new paper machine of the Fourdrinier type will be installed in the present machine building, for making higher grade or rag papers. Provision is made for extending power house, beater room, a new machine room, and finishing room, and in these can be added two more paper machines with the other equipment required, of such type as will fill the demands of the market. While the present capacity is 75 tons per day the additions will bring the capacity up to 150 or 300 tons per day, depending on the class of machinery installed and the kind of paper produced.

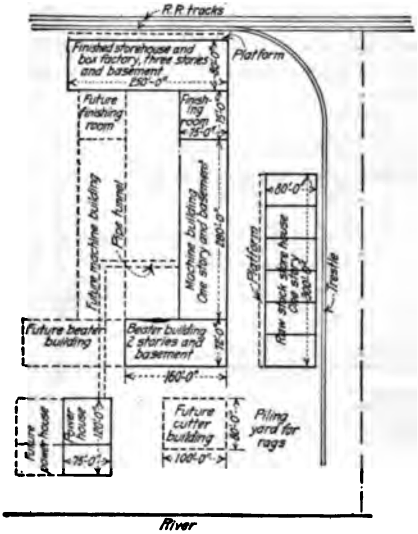


Fig. 65.—Paper board mill—block plan.

138. Power Plants.—The determination of power requirements in general is usually fixed by the location of the industry. As stated, some industries require large amounts of cheap power and so are located where water power is available, either by purchase from a power company or by the construction of a hydraulic power plant. Other plants, if quite extensive or if isolated, have their own steam plants, and many smaller or moderate sized ones buy their power from a local electric company. The design of power and lighting facilities requires, first, careful study of power requirements; that is, amount of power required and how it is to be distributed, whether by line shafting belts and gears, direct from the engines or water wheels, or by electric motors. In most industrial plants today electric current is distributed about the plant by wiring system, and machines are driven either singly or in groups by motors. Alternating current with induction motors is most used for constant speed drives on account of their simplicity and durability and freedom from sparking. For travelling cranes, hoists, and machines requiring variable speed drive, direct current motors are used more frequently at present.

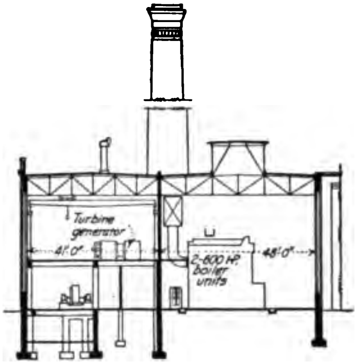


Fig. 66.—1500 kw. Blake-Knowles power plant, East Cambridge, Mass.

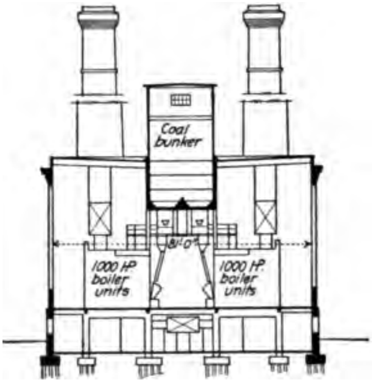


Fig. 67.—4000 hp. boiler house.

The steam power plant for larger industries will usually consist of water tube boilers of 300 to 600 hp., in batteries of two each, and with steam turbines and generators. The installation of condensing engines will, to a great extent, depend on the amount of steam used for heating and other purposes. In some cases an air compressor is installed in the power plant and compressed air piped to the buildings. Proper and ample coal storage and handling facilities should be installed. The usual type of power houses is shown in Figs. 66, 67, and 68.

Fig. 66 shows cross sections of a steam power plant of 1500 kw. capacity, with 1200 hp. of water tube boilers. There are two 750 kw. turbines with condensing equipment. The turbines are on the messanine floor which is served by a 5-ton travelling crane. Auxiliary machinery, with a 1000 c.f.m. air compressor, is on the ground floor.

Fig. 67 shows a typical boiler house with a double row of water tube boilers facing a center aisle, overhead coal bunker and automatic stokers.

Where space is limited, vertical water tube or the Manning type boilers are frequently installed, as in Fig. 68, where the width has been reduced to 30 to 35 ft.; and even less is possible. The overhead coal bunker in a boiler house calls for substantial construction and the installation of elevating and conveying machinery for handling coal.

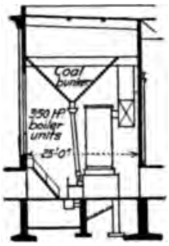


Fig. 68.—Section of 4200 hp. boiler house with vertical boilers.

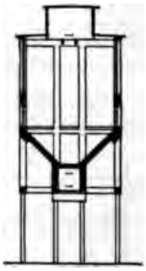


Fig. 69.—Section of a large concrete coal pocket.

There are several types of bunkers of reinforced concrete carried on steel columns, while that in Fig. 68 is a steel suspension bunker lined with concrete. In Fig. 69 is shown a large concrete coal pocket of 5000 tons capacity, 300 ft. long, designed to give additional storage capacity to the plant shown in Fig. 67.

139. Metal Working Industries.—The metal working industries are probably the most important as well as the most varied of the industries. The industrial engineer is interested particularly in machine works, foundries, and factories producing metal goods from the semi-finished material. Machine works are usually housed in a group of buildings, each one designed especially for its particular department. The iron or steel foundry is practically always in a one-story building with one or more bays or aisles of sufficient height to contain travelling cranes for handling heavy flasks, ladles and castings. There should be sufficient clearance under the crane hook to allow of turning the largest flasks to be used. The melting department is usually in the center of a side bay with a charging floor at the proper height for charging the cupola. The foundry building should be of fireproof construction, and provide for ample light and ventilation to remove troublesome fumes and smoke.

140. Foundries.—Much of the manual labor formerly required in foundries has been displaced by modern machinery and appliances. Molding machines are made suitable for practically all small or moderate sized work; in fact, the writer has installed turnover molding machines up to 44 × 56 in., and larger sizes are made and used successfully. Jarring machines may be installed up to 10 ft. square or larger, saving much labor, and allowing of a greater tonnage production per square foot of molding floor. Careful study should be given the problem of handling materials. In iron and steel foundries the pig iron and scrap should be stored where it is easily accessible to a travelling crane with electro-magnet, or other means to place the metal as required directly on the charging floor.

In the Putman Foundry (Fig. 51) a gantry crane serves to unload metal from the cars to pile it in the yard, and also to load small dump cars on the cupola charging floor. Coke is handled by the same crane with a grab bucket. Molding sand should be stored where it will require the least amount of shovelling and wheeling. A mixing, tempering, and screening machine should be installed, where it may be used for screening the used sand and mixing new and used sand in proper proportions. Conveying machinery will usually be found a good investment for handling the molding and core sand. The economical handling of sand is illustrated in the plans and description of the Blake-Knowles Brass Foundry (Figs. 61 to 64 inclusive).

An allotment of space for the various departments of a foundry will be determined by the character of the work. Metal and fuel storage is usually outside the building, if the metal is iron or steel, and as stated before, convenient to the cupola and furnace charging floor. Brass and other costly metals should be stored where only the furnace man or other authorized person has access to them. The melting department should be placed both with reference to the storage of raw materials and to the handling of molten metal to the molding floor. For heavy castings the cupola should be so placed as to run the metal into a ladle held by the travelling crane which will carry it directly to the mold.

Usually the heavy molding is done in a central bay which is served by travelling cranes for handling flasks and metal. The light work is usually done in side aisles or bays which will be equipped with such molding machines as the character of the work demands. The side bays should be served by light travelling cranes or monorail system.

The core shop, with the core ovens, is usually located in a side bay or wing. It is well to so locate the core shop that the ovens may include one or more large ones directly accessible to the main molding floor, for drying out large loam molds. The core shop in the Blake-Knowles Brass Foundry (with core sand mixer in the basement, and elevator bringing the sand either to the first floor or to the women's core shop on the mezzanine floor) is well arranged. In many cases a separate core shop for small cores to be made by women has been installed with good success, as in the one noted. Ample core storage and pattern layout space should be provided, convenient to the molding floor.

Toilet rooms, ample and convenient, with lavatory and shower bath equipment, are important and are required by law in some states, as are also individual lockers for the men.

The cleaning department is the one most frequently neglected or insufficiently provided for. Its size and equipment depend much on the class of work done. One or more sand blast rooms are required, and provision should be made for handling heavy pieces. This department should be located nearest to the machine shop, as castings are usually taken directly there for finishing.

141. Machine Shops.—The design of machine shops depends much on the character of work to be handled. Shops producing heavy machinery should be one-story buildings served by travelling cranes, as in Figs. 52, 53, 54, and 55. Fig. 52 shows a complete plant, producing coal and ore handling machinery of the heaviest type. The machine shop of this plant is 215 ft. wide, with five bays, three of which are served by travelling cranes. All machine tools as well as erecting, finishing and shipping departments are in this building, tracks into the building bringing in castings and shipping the finished machines. The building is lighted by large steel sash in walls, monitors, and saw-tooth windows. The plateshop is also arranged for efficient handling of materials from the cars in the end of the building, to and from the machines.

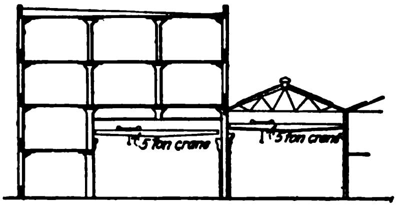


Fig. 70.—Cross section of reinforced concrete machine shop with high crane bay.

Fig. 53 shows section of a machine shop for handling only heavy work, and requiring very limited space for small tools, office, tool room, etc. Fig. 54 is a reinforced concrete machine shop for the average work. This is an economical type of structure; the center bay is lighted by saw-tooth windows and the side bays have two floors well lighted by side windows. Wider spans than those shown will not, as a rule, prove practicable in reinforced concrete. Fig. 55 shows a cross section of a machine shop of the Putnam Machine Company, where light and heavy machine tools are produced and where the lighting is excellent in a wide building housing all departments conveniently.

Before determining the type of building, a machinery layout should be prepared. Cardboard templates of machines, cut out to the scale of the plan to be made, will be of assistance in making the layout. With these, aisles, storage spaces, and machine locations can be determined. Heavy machines should be placed where they may be served by cranes, and light tools in side bays. Ample space should be allowed for passage and for storage of waiting and finished material near the machines. The tool room should be placed where the least amount of travel will be required of the employees.

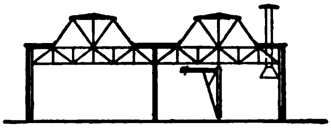
It should be remembered that castings must come in from the foundry, usually first to planers and then on through the operations of boring, milling, drilling, etc., to the erecting shop. Also forgings are brought from the forge shop, and shafting and bar stock from storage, and these all go through the necessary operations, all finally going to the erecting shop, or, in the case of smaller parts, perhaps to storage for finished parts. It is common practice to use one end of the machine shop, where the heavier work is done, for erection of the machines. This holds true only with the heavier machinery requiring travelling cranes for handling. Light machines or metal products, as phonographs, sewing machines, etc., usually have a separate room or building for assembling and erection.

Works for the manufacture of lighter machinery or apparatus from metal may be of the one-story saw-tooth construction type covering large areas, or multi-story buildings of many types. However, the tendency has been to build substantial plants of the best type of fireproof construction, as usually the value of material housed from raw to finished product is several times that of the buildings, so that reducing the fire hazard not only gives greater security but saves heavy insurance expense. Many plants use, or require, both one-story and multi-story buildings.

142. Forge Shops.—Forge shops are one-story buildings with ample means for ventilation and the removal of smoke. Heavy hammers should have foundations separate from the structure, and should be placed convenient to the heating forge. Trusses supporting the roof should be designed to carry the top bearing of jib cranes which serve hammers and forges. Fig. 71

shows a good design for forge shop, the sloping sides of the monitor having top hung continuous steel sash, for ventilation as well as good lighting.

143. Pattern Shops.—The pattern shop and pattern storage are sometimes in the same building, but usually the pattern storage building is an isolated fireproof building on account of the valuable and inflammable nature of its contents. The value of the patterns may not be great but the loss occasioned by the time required to replace them might be extremely heavy. The pattern shop is merely a small wood working shop equipped with machines and benches for the pattern makers, and may be a separate building or a room in a single-story or multi-story building, but it should be well lighted, and means should be provided for continuous removal of wood shavings and waste, which being from dry lumber, is of an inflammable character.



Cross Section of Forge Shop

Fig. 71.

Paint shops and storage and shipping buildings should be designed to suit the requirements of the materials or uses.

144. Wood Working Shops.—Some machine works require extensive wood working shops, and in general, the rules for design of machine shops apply to these, except that as a rule no travelling cranes are required. Planing mills and railroad car shops are generally housed in one-story buildings, except that the lighter work may be done in two or three-story buildings. The lumber passes through different operations, as does iron and steel in machine shops. There is, however, the important difference that the inflammable character of the material, as well as the value of the product in proportion to the space required for the work, does not as a rule justify the expenditure for costly fireproof buildings. The practice most justified seems to be to build wood-working shops at least partly of wood, and then use every means to prevent fires and to promptly extinguish them when they do start. Proper exhaust or blower systems should be installed for removing sawdust and shavings as fast as they are produced. Different departments should be divided by brick fire walls and be in isolated buildings, the finished product being in storehouses, which should be fireproof if possible. Automatic sprinklers in all buildings, hose houses, and yard hydrants with a fire squad trained for prompt action in case of fire, are the best means of preventing loss.

145. Pulp and Paper Mills.—Wood pulp and chemical fiber mills require a large amount of power and water, and also consume large quantities of wood; hence, they are as a rule located convenient to the lumber supply, on rivers which furnish not only water for use in the processes, but power and a means of bringing logs from forest to mill. Chemical fiber mills require specially designed structures; for instance, sulphite digester buildings are 140 to 170 ft. high and of heavy construction, usually brick, with a steel frame. The substructure of grinder houses and wood mills usually contains water wheels directly connected, or belted to the machines. Other buildings are usually of brick mill construction, with rather heavy floor loads (200 to 300 lb. per sq. ft.).

The beater building is of two or three stories. Those using rags or waste paper have sorting and cutting departments on the second floor; beaters, mixers, Jordan engines on the first floor; and stuff chests in the basement. Concrete is an excellent material for at least the basement and first floor of this building, on account of the amount of water used, and the fact that floors are likely to be continuously wet. The machines are heavy and require ample support; otherwise, floor loads are not heavy. The machine room, containing the paper machine or machines, is usually one story and basement. A machine room for two machines should be 60 to 75 ft. wide, depending on the width of machines. Length varies with the machines, which may be 150 to 225 ft. long. The roof is carried on trusses and should have monitors and ventilators for the removal of steam from the drying cylinders.

The finishing building, usually a continuation of the machine room, contains machinery for cutting the paper into sheets, or slitting and rewinding into smaller rolls.

Paper warehouses must be designed to carry heavy loads, ranging from 300 to 500 lb. per sq. ft. of floor, and in one case in the writer's experience a mill storehouse was loaded with 750 to 800 lb. per sq. ft., the paper being piled in rolls from 12 to 15 ft. high.

146. Chemical Industries.—Chemical industries are so varied that only a general treatment can be given here. As a rule, the buildings are one story except those in which gravity may be used for handling the materials in continuous operation, similar to the abrasive crushing plant

shown in Fig. 49. Some plants require small buildings isolated for certain processes, on account of the dangerous character of the contents or obnoxious fumes. Some buildings require all iron work to be heavily protected from the corrosive action of fumes or liquids. Most of these buildings must be designed with special reference to the apparatus which they are to house.

147. Textile Mills.—The design of cotton and woolen mills has been standardized to a great extent, on account of the slight variation in the process of making any grade of cotton cloth or woolen goods. Each department contains a group of a few to hundreds of identical machines, all of which are arranged in a certain definite manner. Furthermore, all makes of textile machines vary little in dimensions. The drive, usually by motors running groups of machines, presents little difficulty. Space will not allow a description of processes and layout.

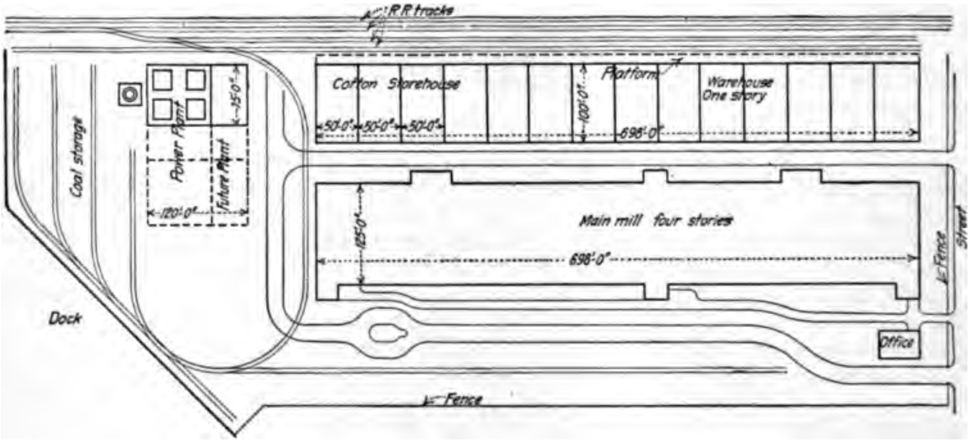


FIG. 72.—Block plan of cotton mill.

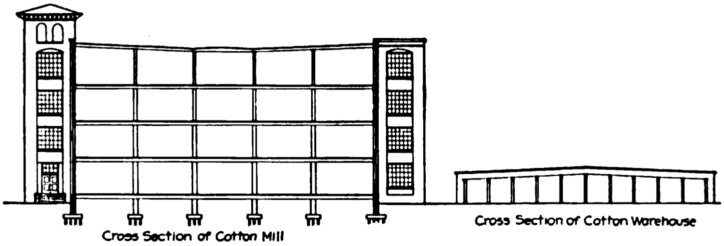

Cross Section of Cotton Mill
Cross Section of Cotton Warehouse

FIG. 73.

Textile mill buildings are generally three or more stories in height and of good width—60 to 125 ft. One exception is the weaving, which in many modern mills is housed in a one-story saw-tooth building, on account of the better lighting which is important in this operation. The floor loads in textile mills are light, the actual load on some floors being not over 30 to 40 lb. per sq. ft., and rarely over 75 ft. per sq. ft. on any floor.

Brick walls with heavy timber frame and plank floors and roof (known as "Mill Construction") are economical, durable, and command a low insurance rate. However, some recent mills have been constructed of reinforced concrete and have proven very satisfactory, although opinions differ, some claiming that the dust and rigidity of the structure shorten the life of machinery. The concrete floor does not present an ideal working surface for the operatives, but this may be overcome with wood, asphalt composition, or other surfaces.

Storehouses for cotton in bales, where ground is available, are usually one-story brick with mill construction roof, well protected by automatic sprinklers. These buildings are usually 100 ft. wide and divided by fire walls into 50-ft. sections. A standard cotton storehouse is shown in section in Fig. 73. When large capacity is required in small space the cotton storehouse may be either of mill construction or reinforced concrete, the former 4 to 8 stories high, and the latter as much as 10 stories. The height of each story is usually about 8 ft. from floor to floor.

Figs. 72 and 73 show a typical cotton mill with all operations in one building 125 × 698 ft., with one-story storehouse serving both for cotton and finished goods storage.

148. Shoe Factories.—In general, the same construction is used for shoe factories as for textile plants, except that the buildings are usually not so wide. On account of the lighting required for nearly all processes, 40 to 50 ft. is about the proper width. Floor loads are generally 150 lb. per sq. ft., and the buildings vary from 3 to 6 stories in height. Fig. 74 shows a shoe factory of reinforced concrete, consisting of a main building with wings, all of flat slab construction.

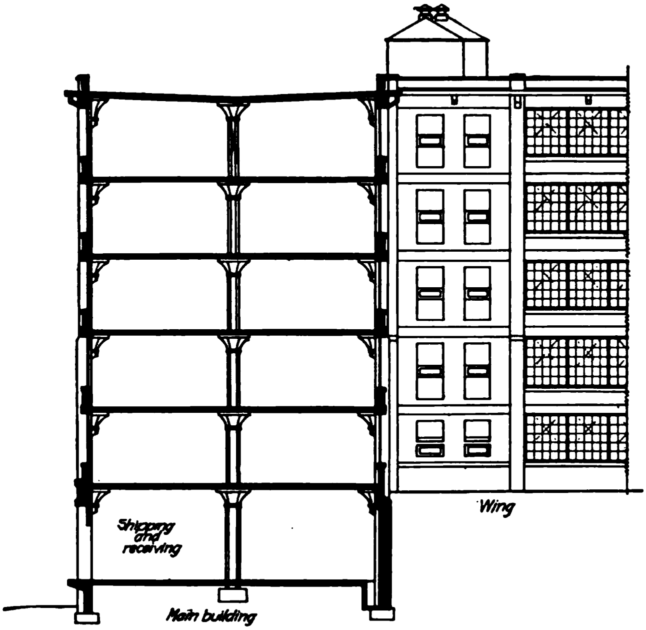


Fig. 74.—Concrete shoe factory.

STANDARDIZED INDUSTRIAL BUILDINGS

By Chas. D. Conklin, Jr.

149. Origin.—The trend of the great industrial organizations for the past few years, throughout the world, has been toward a standardization of output. Even before the recent war produced such enormous demands for vast quantities of products, the large industries realized that "standardization" was the solution of many difficult problems of production. A new significance was given the principle of standardization by the great and hurried demands for all classes of material growing out of the war. It is now a well established fact that in all lines of industrial enterprise, standardization of methods, parts or complete products results in both economical and increased quantity production.

Noting the success of the motor companies and other manufacturing organizations through their standardized products, pioneers in building construction conceived the idea of standardized industrial or factory buildings. Heretofore, it had been the practice to design a special building for every requirement, the result being an enormous amount of detail work and expense for each construction job. While some of this detail work and expense was necessary for very special problems, the greater part could have been eliminated by the use of standardized buildings designed to meet the average requirements of many industries. The result of the study of these pioneer builders was the production of a series of standard designs from which it was believed that by a careful selection, most requirements of industrial building could be met. There are cases of building construction which require special design and study to produce the best results, and in which the use of a standardized building is advisable, but by far the greater percentage of industrial construction may be economically and rapidly accomplished by the use of standardized products.

150. Types.—There are two types of standardized buildings in extensive use at the present time. The first type consists of the permanent, substantial, up-to-date building designed for heavy service over a period of years. They embody all the features of the best types of modern building construction. The second type consists of the lighter, cheaper form of construction which might be termed portable buildings and which are intended more for temporary occupancy rather than permanent use. With proper care, the second type will last for years and fulfill every requirement usually expected of the light steel mill building.

151. General Design.—In the design of both types of standardized buildings described above, the object sought was to produce a series of buildings which would meet the requirements of the average industrial enterprise. Widths, clear heights, units of length, kinds of material, loading, arrangement of lighting and ventilating sash, and many other problems were carefully studied and averaged, so as to obtain finished designs which would suit most conditions. Basic building units were designed which admit of the greatest flexibility, thus permitting their use in numerous combinations. Spans, spacing, and general arrangement were so selected as to use materials up to their safe limit, thus securing a minimum of waste and an economical design.

152. Standardized Method of Construction.—The following description is taken from the catalog of The Austin Company of Cleveland, Ohio, a pioneer company in the construction of standardized factory buildings. The method of this company, known as "The Austin Method," consists of the following:

A method of erecting permanent and substantial factory buildings in the fewest number of working days, eliminating by standardization and quantity production, delays otherwise unavoidable.

A method which provides for various industrial types of construction by standardized designs and specifications. The time ordinarily required for the preparation of special plans is saved.

A method of preconstruction work which prepares and holds stocks of fabricated steel, steel sash, roofing, lumber, and other materials at strategic points and delivers them to any job with dispatch.

A method of figuring costs which places the production of industrial buildings on a definite price basis by lump sum, cost plus percentage, or cost plus fee contracts.

A method which delivers a thoroughly satisfactory building, meeting every requirement of the business, with the least expenditure of the owner's time and money.

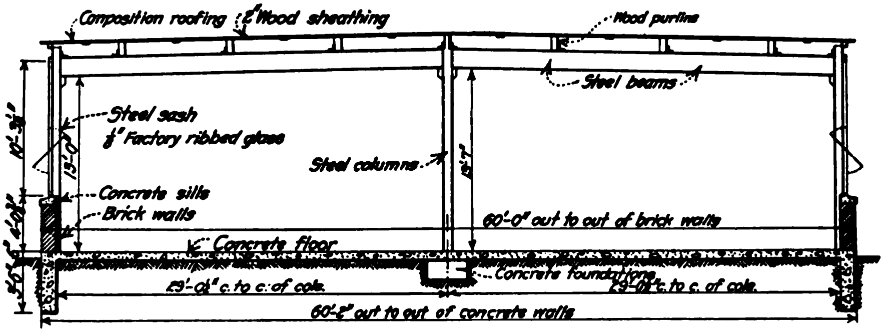
153. Advantages of Standardized Construction.—One of the principal advantages of standardized buildings lies in the time saved over usual methods of construction. Economy in time means economy in labor and capital because of the shorter period during which labor and capital will be tied to one job and because of the hastening of production. Ballinger and Perrot of Philadelphia, describe their standardized buildings as "Quick-Up" buildings, a term well chosen to point out their chief advantage over usual construction. Plans and specifications have been prepared well in advance of construction and the time ordinarily required for special architecture, engineering, preparation of designs, plans, estimates and other matters of detail is saved. Practically all preliminary work is eliminated and construction work can be started immediately upon awarding of contract. All essential materials required for the standardized building are carried in stock and are ready for immediate shipment and can be sent to the job with little or no delay. Material lists for all minor materials not in stock, are already prepared. Continuous contracts are usually carried with material contractors for such and all materials can thus be readily supplied to the workmen. By purchasing materials ahead of construction and carrying same in stock, the builder is able to buy to much better advantage during periods of low market price, thus permitting more economical construction.

Again, workmen are trained in every step and branch of standardized buildings. They know every move to make and make few useless ones. The scheme of construction has been worked out to perfection so that all operations are coordinated and several trades work together at the same time without undue interference. The workmen do not need to spend useless time studying plans and specifications as they are perfectly familiar with the work at hand due to their training in standardized building construction. The work proceeds smoothly and with unnecessary haste and the result is a first-class building, every detail of which is just right due to experience gained from numerous previous similar buildings. By the above described method of construction, buildings have been erected in 30 working days that have ordinarily taken from 3 to 6 months to build, the result being increased production and profit, time, and money saved. To quote again from the catalog of The Austin Company:

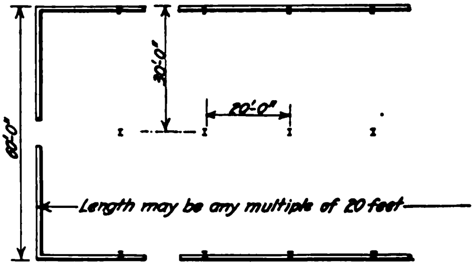
> Standardized construction has automatically placed costs on a more solid foundation. Frequent repeating of the same building operations establishes basic cost figures and eliminates guess work. By the Austin Method, factory buildings can be purchased with the same certainty as machinery or other equipment.

The work is so well organized and developed that delivery can be guaranteed under a penalty and bonus contract.

154. Illustrations.—No attempt will be made here to show sketches of all standard buildings on the market, as there are many of such. A few typical illustrations will be given, sufficient to show the general nature of standardized buildings. There are several organizations advertising and constructing standardized industrial buildings at the present time, and the following sketches are taken from their catalogs in an effort to present briefly some points in the work of each of these organizations. For a more extensive treatment of this subject, the reader is referred to the catalogs of the various companies mentioned in this chapter.



Cross-Section



Plan

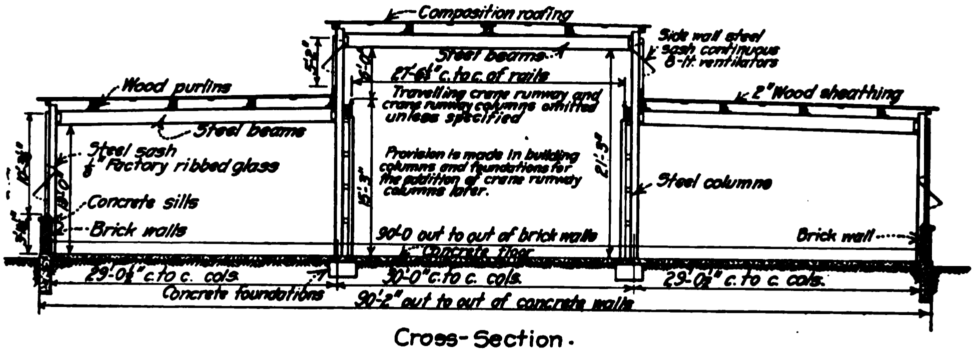
Fig. 75.—Austin No. 1 standard.

Austin Standard Factory Buildings. The Austin Company of Cleveland, Ohio, has worked extensively along the line of standardized construction and, through several years of experience, has adopted ten basic standard designs of permanent, sturdy factory buildings of concrete, brick, and structural steel. "These ten Austin standards, together with their innumerable adaptations and combinations, cover a large variety of industrial structures. Practically every typo of building from the light manufacturing and storage types to the heavy machine and assembling shops will be found in the standard designs. While each style has been standardized, they are sufficiently flexible to meet a great variety of construction requirements." In addition to the ten standard designs mentioned above, The Austin Company has several standard designs for railway buildings and storage buildings, including warehouses, freight stations, repair shops and round houses, which apply Austin standard units of constructions. In most of these standard designs, expansion is possible in width or length in standard multiple and the height may be varied to suit special requirements It will be noted that the longitudinal distance between columns or pilasters, for the large majority of standard buildings, is 20 ft. This distance (usually called the bay) is found to be the most economical one for heavy types of buildings and a very convenient one to use for engineering and construction purposes.

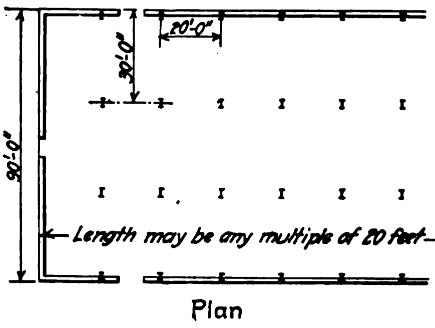
Fig. 75 shows Austin No. 1 Standard Building. The cross section of the building and plan are almost self-explanatory. This building is very similar to "The Miracle" type building as constructed by the Crowell-Lundoff-Little Company of Cleveland, Ohio, the difference being mostly in points of detail. This building is also similar to Type E as designed by Ballinger and Perrot of Philadelphia, the chief difference being in the addition of a monitor for lighting and ventilating purposes. This building is ideal for small machine and assembly shops, carpenter and pattern shops, paint shops, storage, light manufacturing or laboratories. An important point in the design of this and other types of standard buildings lies in the fact that the steel beams or trusses overhead should be made amply strong to support all ordinary shafting loads.

Fig. 76 shows section and plan of Austin Standard No. 2 building. The width of this building may be increased in multiples of 30 ft. or less and the length may be any multiple of 20 ft. This building is suited to many lines of manufacture as it is well lighted and amply ventilated. It is ideal for light foundry service. This building is very similar to "The Monitor," a standard building constructed by the Crowell-Lundoff-Little Co., the latter having a 40-ft. center aisle with light steel truss above instead of the 30-ft. aisle with I-beam rafter in the above No. 2 building.

Fig. 77 is a cross section and part plan of the Austin No. 3 Standard Building. It has proven to be one of the most popular of Austin standards and adaptable to a great variety of purposes. It has been called the Universal type because it has been used for so many operations in the manufacturing field. "It is ideal for lighting conditions, ease of installation of shafting and for its wide area of unobstructed floor space, 2000 sq. ft. per column." The space in the monitor at either end of the building has been used frequently for well-lighted and ventilated office and drafting rooms, also for toilet and washrooms. The open space between the trusses on the side aisles is available for heating, lighting, plumbing and power equipment, leaving the entire floor space free for actual manufacturing. This No. 3 Standard is very similar to Type F building as constructed by Ballinger and Perrot of Philadelphia and somewhat similar to "The Monarch" as constructed by Crowell-Lundoff-Little Co.


Composition roofing
Wood purlins
Steel beams
27'-6½" c. to c. of rails
Travelling crane runway and crane runway columns omitted unless specified
Provision is made in building columns and foundations for the addition of crane runway columns later.
Side wall steel sash continuous 6-ft. ventilators
2" Wood sheathing
Steel sash
⅛" Factory ribbed glass
Concrete sills
Brick walls
Steel columns
Brick wall
90'-0 out to out of brick walls
Concrete floor
29'-0½" c. to c. cols.
30'-0" c. to c. cols.
29'-0½" c. to c. cols.
Concrete foundations
90'-2" out to out of concrete walls
21'-5"
15'-5"
5'-6"

Cross-Section.



Plan

FIG. 76.—Austin No. 2 standard.

Fig. 78 shows the exterior of an Austin No. 3 Standard Building built for the International Motor Company at Allentown, Pa., in 34 working days.

In all the standard buildings above described, either continuous side wall sash with steel columns, or non-continuous side wall sash with brick pilasters may be used. The former gives slightly the better lighting conditions.

Brief specifications covering the above standard buildings are as follows:

Length—Any multiple of 20 ft.

Minimum clearance—13 ft.

Excavation and grading—On normal site, excavation for standard foundations and grading within 3 ft. of outside.

Foundations—Concrete (1 part cement, 3 parts sand, and 5 parts coarse aggregate).

Floor—5-in. concrete base with monolithic finish.

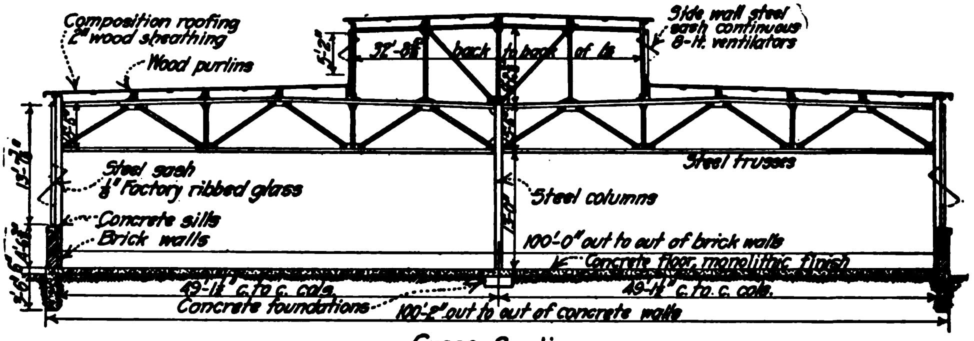
Side walls—Common brick, selected for facing, laid in lime-cement mortar.

Window sills—Concrete (usually precast).

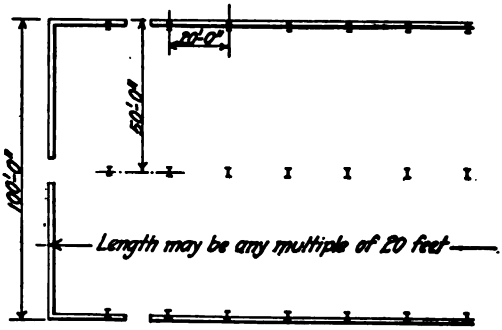
Columns—Structural steel.

Roof structure—Steel beams or trusses with 6 × 12-in. yellow pine purlins, carrying 2 × 6-in. dressed and matched yellow pine sheathing.

Waterproofing—Four-ply built-up felt, pitch, and slag roofing or equal.



Cross-Section



Plan

FIG. 77.—Austin No. 3 standard.

Sash and ventilation—Side wall steel sash with ¼-in. factory ribbed glass, push bar or chain operated. Ventilated sections in monitors mechanically operated

Painting—Structural steel and steel sash, one shop coat and one field coat. Exterior wood work, two coats lead and oil. Interior walls and ceiling, two coats of mill white paint.

Miscellaneous—Sheet metal gutters and downspouts, plumbing, heating, lighting and sprinklers are not usually standardised but are furnished on special order.

Other Standard Buildings.—Fig. 79 shows the section and plan of "Bessemer 70" building of the Crowell-Lundoff-Little Co. It is especially adapted to housing of forging and foundry operations, for rolling mills, machine shops, heavy assembling shops, power houses, and similar structures. Bessemer 50 and 60 are very similar to Bessemer 70, the numeral indicating in each case the distance in feet center to center of crane rails. The Austin Company's Nos. 5, 6, and 7 Standards are very similar to the "Bessemer" building shown, the general type being the same, the dimensions being somewhat different with slight differences in the details.

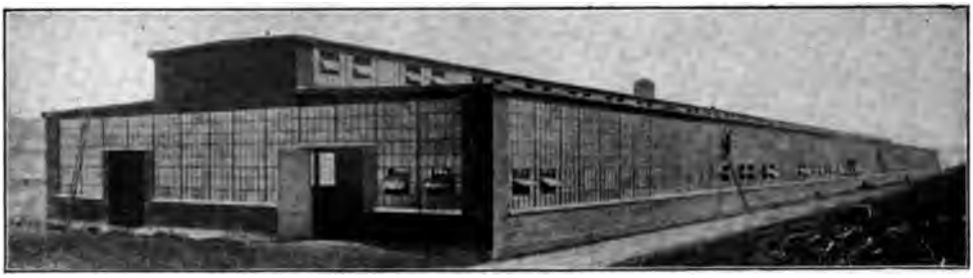


FIG. 78.—Austin No. 3 standard building, 100 × 660 ft.

Fig. 80 is a cross section and part plan of Type C building as constructed by Ballinger and Perrot of Philadelphia. It is a long span saw-tooth building, "the skylights facing north, affording exceptional lighting and ventilation with unobstructed floor space. For many industries this is ideal. Length may be any multiple of 20 ft., and

widths in multiples of 50, 60, 75, and 100 ft. By omitting certain interior columns, this type may be arranged to give unobstructed floor space in units of 75 × 60 ft."

Fig. 81 represents a standard multiple story, flat slab reinforced concrete building, "Gibraltar Type" as erected by the Crowell-Lundoff-Little Co. It is very similar to The Austin Company's No. 9 Standard and is ideal for factories, warehouses, storage buildings, stores, and office buildings. This type of building is economical, fireproof, permanent, sanitary, and free from vibration, and possesses all the advantages of the flat slab building.

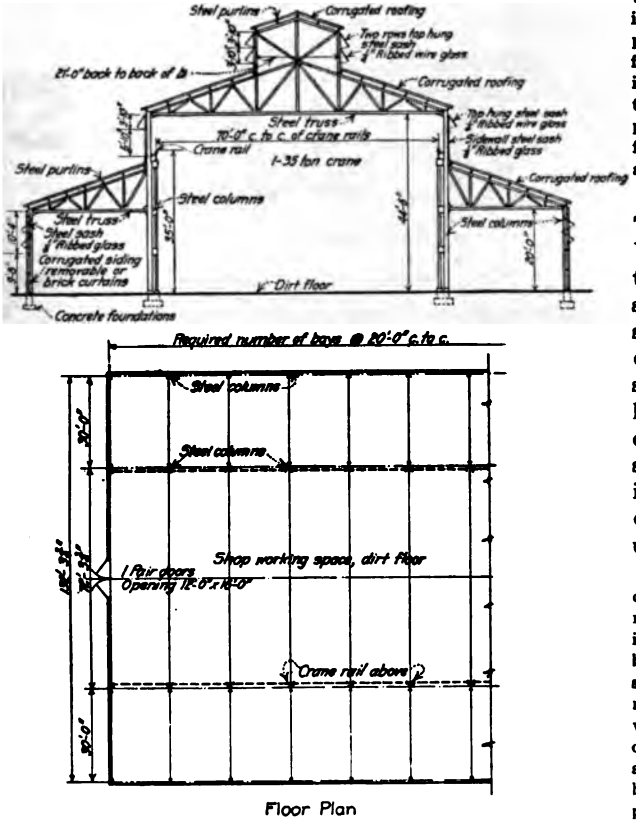


FIG. 79.—"Bessemer 70" building of the Crowell-Lundoff-Little Co.

Truscon Steel Buildings.—

The Truscon Steel Company of Youngstown, Ohio, manufactures and erects a series of semi-permanent buildings "constructed of standard units, every one of which is made of steel." The design of each part has been carefully studied in order to develop maximum strength. Every pound of steel is utilized; there is no waste in either material or labor of manufacturing.

The walls of Truscon buildings consist of standard steel wall units made in various heights, which are interchangeable with doors and may be furnished either with or without steel windows. Field connections are made with a slotted bolt and wedge, very easily assembled and just as easily dismantled, thereby making it simple and inexpensive to move a Truscon building. Hence they are very good portable buildings, especially adapted for temporary use and can be and are used extensively for permanent structures. These buildings are particularly adaptable for storage and light manufacturing, and inasmuch as they have a reputed salvage value of 100%, they are readily re-saleable. The Truscon building is very quickly erected as all units are carried in stock and can be delivered at the site by the time the foundations have been built.

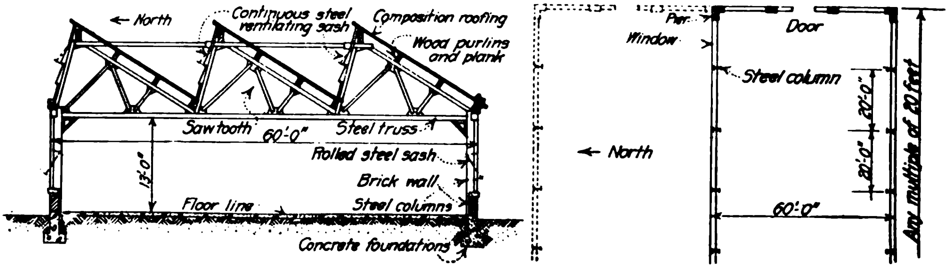
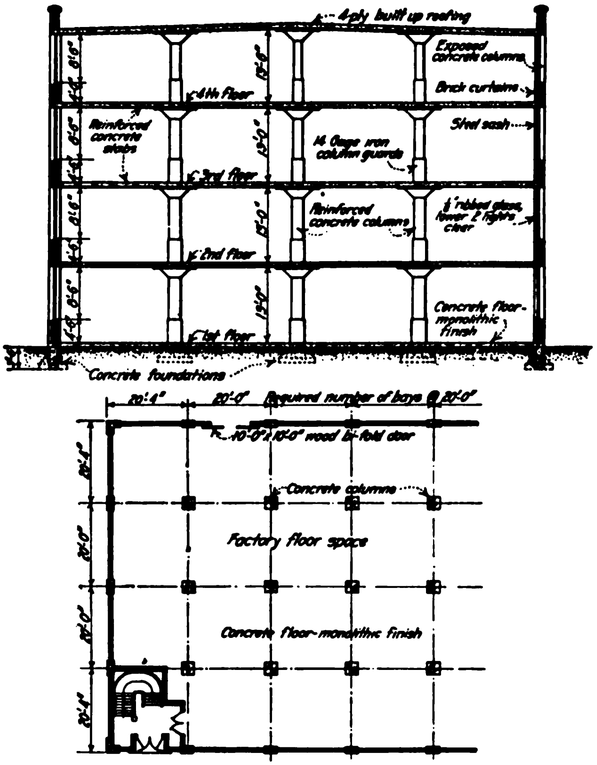


FIG. 80.—"Type C" building of Ballinger and Perrot.

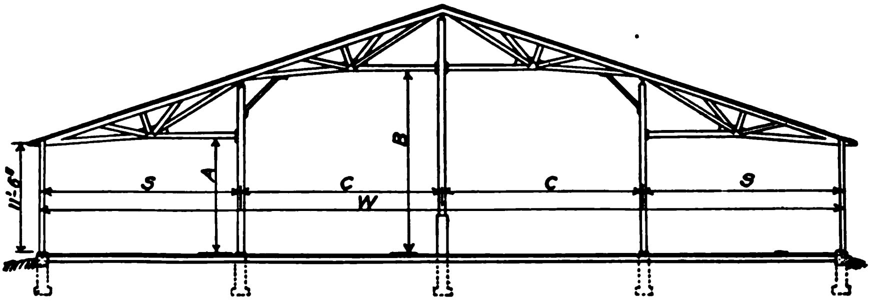
Fig. 82 shows the cross section of one of the several types of standard Truscon buildings. Many variations and adaptations of these types are possible.

155. Conclusion.—As stated above, there are a great variety of standardized buildings on the market at the present time. Only a few of the many have been given, sufficient to


4-ply built up roofing
Exposed concrete columns
Brick curtains
Steel sash
Reinforced concrete slabs
14 Gage iron column guards
Reinforced concrete columns
¼" ribbed glass, lower 2 lights clear
Concrete floor-monolithic finish
4th floor
3rd floor
2nd floor
1st floor
Concrete foundations
8'6"
6'6"
15'6"
15'0"
20'4"
20'-0"
Required number of bays @ 20'-0"
10'-0" x 10'-0" wood bi-fold door
Concrete columns
Factory floor space
Concrete floor-monolithic finish

Ground Floor Plan

FIG. 81.—"Gibraltar type" of the Crowell-Lundoff-Little Co.



Cross-Section

Type 4 Buildings have three lines of columns spaced 16' c. to c. along the length.

Width of building W	Width of side bays S	Width of center bays C	Clear height of side bays A	Clear height of center bays B
80'-0"	20'-0"	20'-0"	11'-10⅝"	18'-6⅝"
100'-0"	25'-0"	25'-0"	11'-11⅝"	20'-3⅝"

FIG. 82.—Truscon standard building (type 4).

convey a clear idea of the principles and methods of standardization. In selecting a building for a definite purpose, careful consideration should be given to the requirements of the case and a standard building only used when it fits the particular need. There are numerous cases where the standard building will answer every requirement. There are other cases where the standard building will not fit the conditions. Efficiency in operation of plant should not be sacrificed by the use of a standard building when the latter is clearly not adapted to the industry to be housed. In the numerous cases in which standardized buildings are adaptable, the results are very satisfactory.

CLEARANCES FOR FREIGHT TRACKS AND AUTOMOBILES

By Allan F. Owen

156. Clearances for Freight Loading Tracks.—When a railroad switch track enters a building, the clearances at the side and overhead and the radius of the curves of the track must be approved by the railroad to which the switch track is to be connected. The tendency is to use larger and larger engines for switching and the curves must have longer radii for the larger engines. Some railroads demand a minimum curvature of 18 deg., and prefer 14 deg. Very few will not allow a 24-deg. curve.

Degree of curvature..........	14	15	16	17	18	19	20	21	22	23	24
Radius in feet...	410.3	383.1	359.3	338.3	319.6	302.9	287.9	274.4	262.0	250.8	240.5

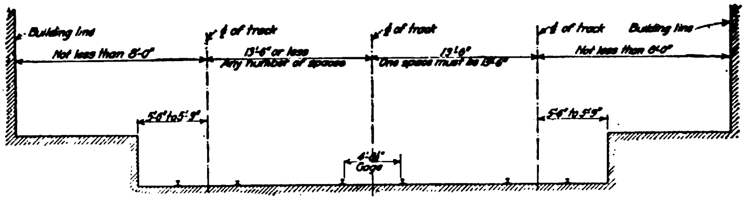


Fig. 83.—Clearances allowed by the State Public Utilities Commission of Illinois for freight loading tracks.

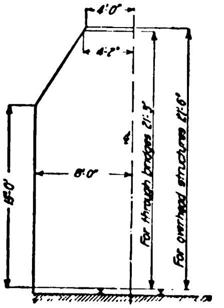


Fig. 84.—Clearances for main and subsidiary freight tracks.

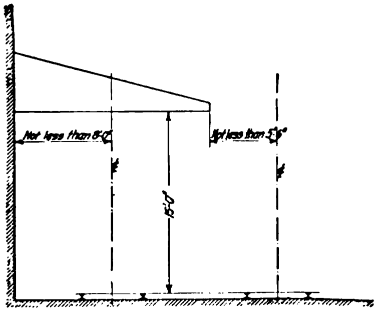


Fig. 85.—Clearances for awnings and canopies.

Note: All awnings and canopies not owned by R. R. companies are subject to the approval of The State Public Utilities Commission of Illinois.

A 42-ft. length of track should be allowed for each freight car that is to be loaded or unloaded.

Clearances allowed by The State Public Utilities Commission of Illinois are given in Figs. 83, 84, and 85.

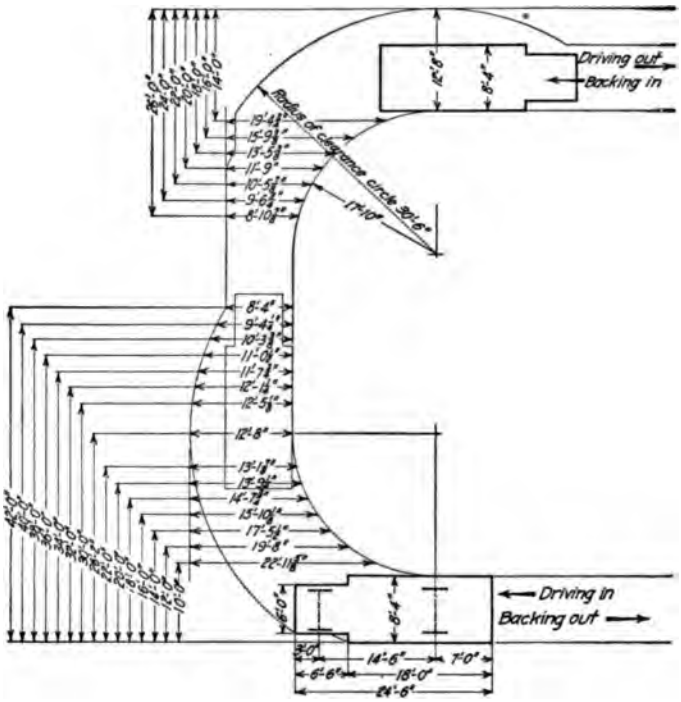


FIG. 86.—Auto truck clearance lines.

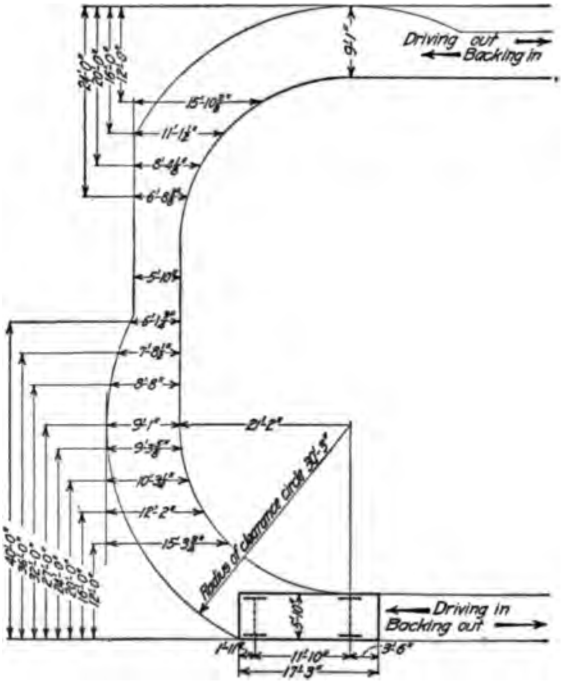


FIG. 87.—Touring car clearance lines.

Loading platforms should be 3 ft. 9 in. above the top of rail. This height will allow doors of refrigerator cars to open. Car platform heights vary from 3 ft. 9 in. to 4 ft. 2 in.

157. Automobile Sizes and Clearances.—Doors to public garages which have to accommodate every kind of automobile truck should be 14 ft. high. Entrances to truck backing-in spaces should be of the same height. Doors to many such garages are 11 ft. high and these will take all but the very largest trucks. Doors should be at least 9 ft. wide and must be wider if they are nearer than 40 ft. from the opposite side of the street or alley. Fig. 86 gives the clearance lines for a truck of the following dimensions: Length over all, 24 ft. 6 in.; width over all, 8 ft. 4 in.; front overhang, 3 ft. 0 in.; wheel base, 14 ft. 6 in.; rear overhang, 7 ft. 0 in., tread—front wheels, 5 ft. 0 in.; tread—rear wheels, 5 ft. 6 in.; radius of clearance circle, 30 ft. 6 in.; body size, 8 ft. 4 in. × 18 ft. 0 in.; width over front fenders, 6 ft. 0 in.

The manufacturers have standard sizes of chassis but there is no standard for bodies; so when it is necessary to provide for particular trucks, it is best to get the dimensions from the owner or builder and lay out the clearance lines.

Touring cars do not require so much room as trucks. Doors should not be less than 8 ft. wide nor lower than 8 ft. unless the garage is made to fit one small car. The diagram of clearance lines for a touring car is given in Fig. 87 for a car of the following dimensions: Length overall, 17 ft. 3 in.; width overall, 5 ft. 10 in.; front overhang, 1 ft. 11 in.; wheel base, 11 ft. 10 in.; rear overhang, 3 ft. 6 in.; tread—front and rear, 4 ft. 8 in.; radius of clearance circle, 30 ft. 3 in.

The following table gives the required dimensions of a few passenger cars

Name	Capacity (number of passengers)	Length	Width	Height	Wheel base	Rear overhang	Radius of clearance circle	Weight (pounds)
Packard 3–25	7	15′ 9″	5′ 9″	7′ 0″	10′ 8″	......	22′6″	4435
3–35	7	16′ 5″	5′ 9″	7′ 0″	11′ 4″	......	23′6″	4490
Locomobile 48–2	7	17′ 3″	5′10″	6′10″	11′10″	3′ 6″	30′3″	5000
38–2	7	17′ 0″	5′10″	6′10″	11′ 7″	3′ 6″	30′3″	5000
Pierce-Arrow 48 H.P.	7	17′10″	5′ 9″	7′ 9″	11′10″	......	25′0″	5500
38 H.P.	5	16′10″	5′ 7″	7′ 4″	11′ 2″	......	21′6″	4400
Stanley Steamer 735	7	17′ 0″	6′ 0″	7′ 6″	10′10″	4′ 0″	25′0″	3800
Ford "T"	......	12′ 4″	5′ 6″	7′ 0″	8′ 4″	2′10″	14′0″	1500
"TT"	....	......	.....	.....	10′ 4″	......	23′0″	

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